

# On the transferability of reserves in lifelong health insurance contracts

Jan Dhaene, Els Godecharle, Katrien Antonio, Michel M. Denuit



# ON THE TRANSFERABILITY OF RESERVES IN LIFELONG HEALTH INSURANCE CONTRACTS

JAN DHAENE, ELS GODECHARLE, KATRIEN ANTONIO

Actuarial Research Group, AFI  
Faculty of Business and Economics  
KU Leuven  
Leuven, Belgium

MICHEL M. DENUIT

Institut de Statistique, Biostatistique et Sciences Actuarielles  
Université Catholique de Louvain  
Louvain-la-Neuve, Belgium

April 14, 2015

## **Abstract**

For lifelong health insurance covers, medical inflation not sufficiently incorporated in the level premiums determined at policy issue requires an appropriate increase of these premiums and/or of the corresponding reserves during the term of the contract. This premium and/or reserve update is necessary to maintain the actuarial equivalence between future health benefits and withdrawal payments on the one hand, and available reserves and future premiums on the other hand. In Vercruysse et al. (2013), premium and reserve indexing mechanisms were proposed in a discrete-time framework where medical inflation is only taken into account ex-post as it emerges over time and where the reserves are not transferable in case of policy cancellation. In this paper, we extend this work by investigating the more general situation where a surrender value is paid out in case of policy cancellation. Reserve-based as well as premium-based surrender values are considered.

**KEY WORDS:** medical expense insurance, lifelong contract, medical inflation index, withdrawal, surrender value.

# 1 Introduction

In this paper, we investigate private health insurance contracts covering medical expenses with lifelong cover and periodic premiums. At contract inception, the level premium is calculated so that the contract is actuarially fair, i.e. the actuarial value (or expected present value) of premiums over the contract duration is equal to the actuarial value of the health benefits and of the eventual withdrawal value paid out to the insured. As medical expenses typically rise over the lifetime, a level premium contract generates premium surpluses in the early years which lead to asset accumulation in a reserve, while the shortfall of premiums in the later years is covered by these assets. As a result, the well known hump-shape of the reserves becomes apparent.

The health benefits that will be paid over the years for a lifelong health insurance policy will be impacted by unpredictable changes in prices for medical goods and services. Given the long-term nature of health insurance contracts and the impossibility to predict or hedge against medical inflation, insurers are not able to appropriately account for this medical inflation in the calculation of the yearly premium level at policy issue. Therefore, these lifelong contracts are usually designed in such a way that the insurer is allowed to adapt the premium amounts at regular times (e.g. yearly) to account for medical inflation not taken into account at policy issue, based on some predefined medical inflation index. This practice is used in several EU member countries (for instance in Belgium and Germany, see Haberman and Pitacco (1998) and Milbrodt (2005)).

Vercruysse et al. (2013) considers the problem of premium indexing for lifelong health insurance contracts with non-transferable reserves. Non-transferability of the reserves means that the reserve is not paid out (neither fully nor partially) to the insured when he lapses the contract. Non-transferability of the reserves has a premium-reducing effect in case the insurer accounts for lapses in his premium calculations. Notice however that lapse-supported business, which means that the introduction of lapse rates in the calculations reduces the premiums, is often considered as a controversial technique because lapse rates may depend on economic factors and may give rise to systematic risk. We refer the reader to Section 8.8 in Dickson et al. (2013) for a discussion. A drawback of the non-transferability is that it binds the insured to his insurer, especially at times when the reserve is relatively high. Hofmann and Browne (2013) provide empirical evidence of the lock-in consumers face when premiums are front-loaded.

Although non-transferability of reserves is actuarially fair (if it is appropriately taken into account in the premium calculation), consumers may feel this lack of liquidity of their contract as a serious drawback. Baumann et al. (2008) explore which part of the reserve can be transferred in case of surrender without imposing premium changes on the policyholders staying in the contract. These authors do not consider medical inflation in their study.

This paper tries to solve the problem of medical inflation in a context of private health insurance contracts with fully or partially transferable reserves by generalizing the results of Vercruysse et al. (2013). In this setting, several ways exist to restore the actuarial equivalence. The insurer can either increase premiums, or increase the reserve, or a

combination of both. In the first case it is the insured who carries the burden of increased costs due to medical inflation, in the second case it is the insurer and in the third case they share the burden. This general approach is demonstrated in numerical examples, based on Belgian data.

Modeling and choosing appropriate lapse rates is a delicate issue. Kuo et al. (2003) explore the impact of unemployment and interest rate on lapse rates. Hofmann and Browne (2013) show that policyholders generally lapse less in case of higher premium front-loading and Christiansen et al. (2015) find that premium development, premium adjustment frequency and the sales channel impact lapse rates. In the present work, we investigate the influence of the choice of the lapse rate on the numerical results by means of a sensitivity analysis.

The remainder of this paper is organized as follows. In Section 2, we describe the lifelong health insurance contract under study. In Section 3, we extend the framework of Vercruyssen et al. (2013) to take into account (partially) transferable reserves. We describe how contracts may be adapted over time to take into account unanticipated medical inflation. In Sections 4 and 5, we consider the special cases of reserve- and premium-dependent withdrawal payments, respectively. Section 6 discusses detailed numerical examples. Section 7 concludes the paper.

## 2 The lifelong health insurance contract

### 2.1 Health benefits and withdrawal payments

The origin of time is chosen at policy issue. Time  $t$  stands for the seniority of the policy (i.e. the time elapsed since policy issue). The policyholder's (integer) age at policy issue is denoted by  $x$ , so that upon survival at time  $k$ , he or she has reached age  $x + k$ . We denote the ultimate integer age by  $\omega$ , assumed to be finite. This means that survival until integer age  $\omega$  has a positive probability, whereas survival until integer age  $\omega + 1$  has probability zero.

The superscript “(0)” will be used to denote quantities estimated or known at policy issue (time 0). The average health-related benefit to be paid out in the year  $(k, k + 1)$ ,  $k \in \{0, 1, \dots, \omega - x\}$ , is denoted by  $b_{x+k}^{(0)}$ . We assume that health-related benefits are paid at the beginning of the year, which is a convenient and conservative assumption in our context. Furthermore, in case the policyholder cancels the contract in year  $(k, k + 1)$ , the amount  $w_{x+k+1}^{(0)}$  is paid out at the end of the year. We set  $w_{\omega+1}^{(0)} = 0$ , which means that the surrender value in the last possible year of survival is equal to zero. In Sections 4 and 5, we consider two particular types of surrender values. The first expresses withdrawal payments in terms of a linear function of the available reserve of the contract at the moment of withdrawal, whereas the second defines the surrender value as a fraction of the accumulated value of the premiums paid so far.

The health benefits that will be paid over the years are subject to medical inflation. Given our adjusted definition, medical inflation is assumed to account for the full increase

of medical costs, not only the increase of these medical costs above the inflation taken into account by the usual consumer price index. We assume that medical inflation is unpredictable and hence, at policy issue, an assumption has to be made about this inflation. Here, we assume that the actuary includes a future medical inflation of  $f$  per year in premium calculation. This means that

$$b_{x+k}^{(0)} = \bar{b}_{x+k}^{(0)} \times (1 + f)^k,$$

where  $\bar{b}_{x+k}^{(0)}$  is the average health benefit to be paid out to a policyholder aged  $x + k$  in year  $(0, 1)$ . We assume that appropriate estimates for the values  $\bar{b}_{x+k}^{(0)}$  are available at time 0. Furthermore,  $w_{x+k+1}^{(0)}$  is the payment in case of withdrawal at time  $k + 1$ , under the assumption of a future medical inflation of  $f$  per year. The level premium  $\pi^{(0)}$  determined at policy issue is thus based on a medical inflation of  $f$  per year. It is worth to mention that all the results can easily be adapted to the case of a deterministic but varying assumed future medical inflation path, i.e. replacing the constant  $f$  with a given sequence of yearly medical inflation rates  $f_1, f_2, \dots$

Obviously, observed medical inflation may depart from the assumed  $f$ . Therefore, the premium and the available reserve should be rebalanced every year according to the observed medical inflation, in order to restore the actuarial equivalence between available reserve and future premiums on the one hand, and future surrender values and health benefits paid by the insurer on the other hand. This yearly process gives rise to a sequence of yearly premiums  $\pi^{(k)}$ ,  $k = 0, 1, \dots$ , where the superscript “ $(k)$ ” is used to denote that the updated values are based on the experienced inflation that was observed up to and including time  $k$ , whereas future inflation is assumed to be  $f$  per year. We suppose that the contract stipulates that premiums, reserves and eventually also withdrawal payments may be updated (on a yearly basis) according to a well-defined procedure, in order to restore the broken actuarial equivalence. The updated value for the health benefits at time  $k \in \{0, 1, \dots, \omega - x\}$  based on information available up to time  $k$ , will be denoted by  $b_{x+k+j}^{(k)}$ ,  $j = 0, 1, \dots, \omega - x - k$ . Hence,

$$b_{x+k+j}^{(k)} = \bar{b}_{x+k+j}^{(k)} \times (1 + f)^j,$$

where  $\bar{b}_{x+k+j}^{(k)}$  is the average health benefit to be paid out to a policyholder aged  $x + k + j$  in year  $(k, k + 1)$ . Again, we assume that appropriate estimates for the values  $\bar{b}_{x+k+k}^{(k)}$  are available at time  $k$ . Furthermore,  $w_{x+k+j+1}^{(k)}$  stands for the time- $k$  updated value of the withdrawal payment at time  $k + j + 1$ , taking into account the observed inflation until time  $k$  and an assumed inflation of  $f$  per year beyond time  $k$ .

Taking into account these new series of values  $b_{x+k+j}^{(k)}$  and  $w_{x+k+j+1}^{(k)}$  that are available at time  $k$ , premiums, reserves and withdrawal benefits are updated at that time and give rise to  $\pi^{(k)}$  for the new yearly premium to be paid from time  $k$  on. Throughout this paper, we set  $w_{\omega+1}^{(k)} = 0$ . The procedure of the yearly updating is considered in detail in Section 3.

We assume that, apart from the assumed medical inflation, the other elements of the technical basis (interest, mortality and lapse rates) are in line with the reality that unfolds

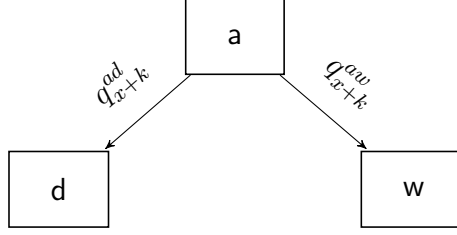


Figure 1: The 2-decrement model.

over time. This simplifying assumption implies that these elements do not require a yearly update in order to maintain actuarial equilibrium. It allows us to isolate and investigate the effect of medical inflation on its own. Notice however that the methodology proposed hereafter can easily be adapted to take into account deviations of interest, mortality and lapse rates from the ones assumed in the technical basis. This issue will be discussed in Section 7.

## 2.2 Discrete-time double decrement model

We describe the lifelong health insurance policy considered in the previous subsection in a two-decrement Markov model, with states “active” (i.e. policy in force), “withdrawn” (i.e. policy has been cancelled) and “dead”, abbreviated as “a”, “w” and “d”, respectively. A graphical illustration is in Figure 1. We denote as  $X_k$  the status of the contract at time  $k$ , starting from  $X_0 = a$ . The stochastic process  $\{X_k, k = 0, 1, 2, \dots\}$  describes the history of the contract.

For  $j$  and  $k \in \{0, 1, 2, \dots\}$ , we define the sojourn (or non-exit) probability  ${}_j p_{x+k}^{aa}$  as

$${}_j p_{x+k}^{aa} = \Pr[X_{k+j} = a | X_k = a]. \quad (1)$$

In words, the quantity defined in (1) is the probability that a policy in force at age  $x+k$  is still in force  $j$  years later. The probability that a policy in force at age  $x+k$  has ceased  $j$  years later (due to death or withdrawal), is denoted by  ${}_j q_{x+k}^{aa}$ . This “exit” probability can be expressed as

$${}_j q_{x+k}^{aa} = \Pr[X_{k+j} \neq a | X_k = a] = 1 - {}_j p_{x+k}^{aa}. \quad (2)$$

We also introduce the probabilities  ${}_j q_{x+k}^{ad}$  and  ${}_j q_{x+k}^{aw}$ , which are defined by

$${}_j q_{x+k}^{ad} = \Pr[X_{k+j} = d | X_k = a] \text{ and } {}_j q_{x+k}^{aw} = \Pr[X_{k+j} = w | X_k = a]. \quad (3)$$

These are the probabilities of leaving the portfolio due to respectively death and withdrawal between ages  $x+k$  and  $x+k+j$ .

The following relations are well-known:

$${}_j p_{x+k}^{aa} + {}_j q_{x+k}^{ad} + {}_j q_{x+k}^{aw} = 1 \quad (4)$$

and

$${}_{j+1}p_{x+k}^{aa} = p_{x+k}^{aa} \times {}_j p_{x+k+1}^{aa} = \prod_{l=0}^j p_{x+k+l}^{aa}. \quad (5)$$

In accordance with standard actuarial notation, we omit the index  $j$  when it is equal to unity. The ultimate integer age  $\omega$  is such that  $p_{\omega-1}^{aa} > 0$ , while  $p_{\omega}^{aa} = 0$ .

### 2.3 Actuarial values, premiums, reserves and the equivalence principle

In this paper, we assume a constant yearly technical interest rate  $i$ , and denote as  $v = (1+i)^{-1}$  the corresponding annual discount factor. Notice however that all results hereafter can easily be generalised to include varying deterministic technical interest rates, i.e. replacing  $i$  with a sequence  $i_1, i_2, \dots$ . Premiums are paid at the beginning of the year, as long as the policy is in force. Let

$$\ddot{a}_x^{aa} = \sum_{j=0}^{\omega-x} {}_j p_x^{aa} v^j$$

be the actuarial value at policy issue of an annuity-due paying an amount of 1 per year, as long as the health insurance contract is in force. Furthermore, let

$$B_x^{(0)} = \sum_{j=0}^{\omega-x} {}_j p_x^{aa} v^j b_{x+j}^{(0)} \quad (6)$$

be the actuarial value at time 0 of all health-related benefits, and let

$$W_x^{(0)} = \sum_{j=0}^{\omega-x} {}_j p_x^{aa} q_{x+j}^{aw} v^{j+1} w_{x+j+1}^{(0)} \quad (7)$$

be the actuarial value at time 0 of the withdrawal option.

At policy issue, the level premium  $\pi^{(0)}$  for a policyholder aged  $x$  is then determined from the equivalence principle:

$$\pi^{(0)} \ddot{a}_x^{aa} = B_x^{(0)} + W_x^{(0)}. \quad (8)$$

Remember that the actuarial values  $B_x^{(0)}$  and  $W_x^{(0)}$ , as well as the yearly premium  $\pi^{(0)}$ , are determined under the assumption of a yearly medical inflation  $f$ .

Solving equation (8) for  $\pi^{(0)}$  leads to the level premium to be paid yearly in advance. It is important to notice that the equivalence relation (8) does not always provide an explicit expression for this premium. This is the case for instance when the withdrawal payments, and hence also  $W_x^{(0)}$ , are defined in terms of the available reserve or in terms of the premiums paid so far. In Sections 4 and 5, we consider reserve-dependent and

premium-dependent withdrawal payments and present a methodology that leads to an explicit expression for  $\pi^{(0)}$  in these cases.

Let us introduce the notation  $V_{x+j+1}^{(0)}$  for the available reserve of the contract at time  $j+1$ , in case of a yearly medical inflation  $f$  in the interval  $(0, j+1)$  as predicted at time 0. Assuming the technical basis used for determining the premium level at policy issue, these reserves can be determined from the forward recursion

$$V_{x+j+1}^{(0)} = \left( V_{x+j}^{(0)} + \pi^{(0)} - b_{x+j}^{(0)} - q_{x+j}^{aw} v w_{x+j+1}^{(0)} \right) (p_{x+j}^{aa} v)^{-1}, \quad (9)$$

or equivalently,

$$v V_{x+j+1}^{(0)} = V_{x+j}^{(0)} + \pi^{(0)} - b_{x+j}^{(0)} + q_{x+j}^{ad} v V_{x+j+1}^{(0)} - q_{x+j}^{aw} v \left( w_{x+j+1}^{(0)} - V_{x+j+1}^{(0)} \right), \quad (10)$$

which holds for  $j \in \{0, 1, \dots, \omega - x - 1\}$ . The initial available reserve  $V_x^{(0)}$  is given by

$$V_x^{(0)} = 0. \quad (11)$$

The quantity  $V_{x+j}^{(0)}$  can be interpreted as an estimate for the available reserve per policy in force at time  $j$ , based on the technical assumptions made at policy issue. Notice that recursions are common in actuarial calculations related to long-term contracts; see e.g. Giles (1993) for some examples. Often, explicit solutions are available.

One can easily verify that the solution of recursion (9), with initial value (11), can be expressed in the following retrospective form:

$$V_{x+j}^{(0)} = \sum_{l=0}^{j-1} \left( \pi^{(0)} - b_{x+l}^{(0)} - q_{x+l}^{aw} v w_{x+l+1}^{(0)} \right) (j-l p_{x+l}^{aa} v^{j-l})^{-1}, \quad (12)$$

for  $j \in \{0, 1, \dots, \omega - x\}$ . Taking into account the equivalence principle (8), the available reserve  $V_{x+j}^{(0)}$  can also be expressed prospectively:

$$V_{x+j}^{(0)} = B_{x+j}^{(0)} + W_{x+j}^{(0)} - \pi^{(0)} \ddot{a}_{x+j}^{aa}, \quad (13)$$

with

$$B_{x+j}^{(0)} = \sum_{l=0}^{\omega-x-j} l p_{x+j}^{aa} v^l b_{x+j+l}^{(0)} \quad (14)$$

$$W_{x+j}^{(0)} = \sum_{l=0}^{\omega-x-j} l p_{x+j}^{aa} q_{x+j+l}^{aw} v^{l+1} w_{x+j+l+1}^{(0)} \quad (15)$$

$$\ddot{a}_{x+j}^{aa} = \sum_{l=0}^{\omega-x-j} l p_{x+j}^{aa} v^l. \quad (16)$$

Notice that  $B_{x+j}^{(0)}$  in (14) includes the benefit payment at time  $j$ , whereas the first withdrawal benefit included in  $W_{x+j}^{(0)}$  is the one that will be paid out eventually at time  $j+1$ .



As we have assumed that  $w_{\omega+1}^{(0)} = 0$ , the last term in (15) is equal to zero. The superscript “(0)” in (14) and (15) indicates that both actuarial values are based on the information available at policy issue (i.e. at time 0), taking into account an assumed medical inflation of  $f$  per year. In particular, we find from (13) that

$$V_{\omega}^{(0)} = b_{\omega}^{(0)} - \pi^{(0)}, \quad (17)$$

which states that the available reserve at the last possible integer age  $\omega$  is equal to the difference of the health benefit and the premium to be paid immediately. Equations (11) and (17) provide us with initial and final values for the reserve trajectory  $j \mapsto V_{x+j}^{(0)}$ .

### 3 Updating the health insurance contract

Before explaining the mechanism of yearly updating the health insurance contract, we introduce some notations for actuarial values, which will be used throughout the remainder of this paper. For  $k \in \{0, 1, 2, \dots, \omega - x\}$  and  $j \in \{0, 1, 2, \dots, \omega - x - k\}$ , let

$$B_{x+k+j}^{(k)} = \sum_{l=0}^{\omega-x-k-j} {}_l p_{x+k+j}^{aa} v^l b_{x+k+j+l}^{(k)} \quad (18)$$

be the actuarial value at time  $k + j$  of the health benefits to be paid at time  $k + j$  and beyond for a policy still in force at time  $k + j$ . Similarly, let

$$W_{x+k+j}^{(k)} = \sum_{l=0}^{\omega-x-k-j} {}_l p_{x+k+j}^{aa} q_{x+k+j+l}^{aw} v^{l+1} w_{x+k+j+l+1}^{(k)} \quad (19)$$

be the actuarial value at time  $k + j$  of the future withdrawal option for a policy still in force at time  $k + j$ . The actuarial values (18)-(19) are calculated taking into account the observed medical inflation until time  $k$ , while a constant yearly medical inflation of  $f$  from time  $k$  on.

In the lifelong health insurance contract that we consider in this paper, the deviation between the observed and the assumed medical inflation is taken into account ex-post as it emerges over time, by adapting the premium, the withdrawal benefits and the available reserve from year to year. The procedure how to adapt these quantities is described below.

Suppose that the policy is still in force at time  $k \in \{1, 2, 3, \dots, \omega - x\}$ . Reevaluations up to time  $k - 1$  have led to the updated values  $b_{x+k+j}^{(k-1)}$  and  $w_{x+k+j+1}^{(k-1)}$ ,  $j \in \{0, 1, 2, \dots, \omega - x - k\}$ , for the health benefits and the withdrawal payments, respectively. Notice that  $b_{x+k+j}^{(k-1)}$  and  $w_{x+k+j+1}^{(k-1)}$  are based on the observed medical inflation until time  $k - 1$ , while assuming a medical inflation of  $f$  per year from time  $k - 1$  on. Furthermore, premiums have been adapted from year to year and have reached level  $\pi^{(k-1)}$  at time  $k - 1$ .

We assume that at each time  $1, 2, \dots, k - 1$ , the available reserve, the withdrawal benefits and the premium have been reset in such a way that the available reserve (i.e.

the available assets) and the required reserve (i.e. the actuarial liabilities) are equal. In particular this means that at time  $k - 1$ , the available reserve  $V_{x+k-1}^{(k-1)}$ , the withdrawal values  $w_{x+k-1+j}^{(k-1)}$ ,  $j = 1, 2, \dots$ , and the premium  $\pi^{(k-1)}$  have been chosen such that they satisfy the following relation:

$$V_{x+k-1}^{(k-1)} = B_{x+k-1}^{(k-1)} + W_{x+k-1}^{(k-1)} - \pi^{(k-1)} \ddot{a}_{x+k-1}^{aa}, \quad (20)$$

with  $B_{x+k-1}^{(k-1)}$  and  $W_{x+k-1}^{(k-1)}$  defined according to (18) and (19), respectively. The right-hand side in (20) is the actuarial value at time  $k - 1$  of the future liabilities (also called the required reserve at time  $k - 1$ ) of the contract under consideration, based on the information available at that time. Splitting the payments related to year  $(k - 1, k)$  from the other payments and taking into account (5), we can rewrite (20) as follows:

$$\begin{aligned} V_{x+k-1}^{(k-1)} &= b_{x+k-1}^{(k-1)} + q_{x+k-1}^{aw} v w_{x+k}^{(k-1)} - \pi^{(k-1)} \\ &\quad + p_{x+k-1}^{aa} v \left( B_{x+k}^{(k-1)} + W_{x+k}^{(k-1)} - \pi^{(k-1)} \ddot{a}_{x+k}^{aa} \right). \end{aligned} \quad (21)$$

Having arrived at time  $k$ , the reserve available for a policy still in force at age  $x + k$ , taking into account all information up to time  $k - 1$ , is denoted by  $V_{x+k}^{(k-1)}$ . It is given by

$$vV_{x+k}^{(k-1)} = \left( V_{x+k-1}^{(k-1)} + \pi^{(k-1)} - b_{x+k-1}^{(k-1)} + q_{x+k-1}^{ad} v V_{x+k}^{(k-1)} - q_{x+k-1}^{aw} v \left( w_{x+k}^{(k-1)} - V_{x+k}^{(k-1)} \right) \right),$$

or equivalently,

$$V_{x+k}^{(k-1)} = \left( V_{x+k-1}^{(k-1)} + \pi^{(k-1)} - b_{x+k-1}^{(k-1)} - q_{x+k-1}^{aw} v w_{x+k}^{(k-1)} \right) (vP_{x+k-1}^{aa})^{-1}. \quad (22)$$

It follows that we can express the available reserve  $V_{x+k}^{(k-1)}$  in the following prospective form:

$$V_{x+k}^{(k-1)} = B_{x+k}^{(k-1)} + W_{x+k}^{(k-1)} - \pi^{(k-1)} \ddot{a}_{x+k}^{aa}. \quad (23)$$

This expression states that the available reserve and the required reserve at time  $k$  are equal, provided the technical basis that was used at time  $k - 1$  is still appropriate at time  $k$ .

Suppose now that medical inflation during year  $(k - 1, k)$  was such that each future health benefit  $b_{x+k+j}^{(k-1)}$ ,  $j \in \{0, 1, \dots, \omega - x - k\}$  that was determined at time  $k - 1$  has to be replaced by the corresponding adapted health benefit  $b_{x+k+j}^{(k)}$ , determined at time  $k$ , taking into account observed medical inflation up to time  $k$ , while assuming a future yearly medical inflation  $f$ . In particular, we assume that, due to medical inflation, the actuarial value of future health benefits  $B_{x+k}^{(k-1)}$ , which is based on observed medical inflation until time  $k - 1$ , has to be replaced by  $B_{x+k}^{(k)}$ , which is based on observed medical inflation until time  $k$ , see (18). Due to this change in the health benefits, the actuarial equivalence is broken at time  $k$ , in the sense that the available reserve  $V_{x+k}^{(k-1)}$  is different from the actuarial value of future liabilities (i.e. the required reserve) at that time.

In order to restore the actuarial equivalence at time  $k$ , the premium level  $\pi^{(k-1)}$  and the available reserve  $V_{x+k}^{(k-1)}$  are updated to  $\pi^{(k)}$  and  $V_{x+k}^{(k)}$ , respectively. These changes

are chosen such that the available reserve  $V_{x+k}^{(k)}$  is again equal to the actuarial value of the future liabilities:

$$V_{x+k}^{(k)} = B_{x+k}^{(k)} + W_{x+k}^{(k)} - \pi^{(k)} \ddot{a}_{x+k}^{aa}, \quad (24)$$

where  $W_{x+k}^{(k)}$  is determined from the updated withdrawal payments  $w_{x+k+j}^{(k)}$ . The actuarial equivalence (24) may be obtained in many ways, in the sense that an infinite number of pairs  $(V_{x+k}^{(k)}, \pi^{(k)})$  satisfy relation (24).

From time  $k$  on, the level premium  $\pi^{(k-1)}$  that was determined at time  $k-1$ , is replaced by the updated level premium  $\pi^{(k)}$ . Notice that the premium increases  $\pi^{(k)} - \pi^{(k-1)}$  are financed by the policyholder. Also, at time  $k$ , the available reserve  $V_{x+k}^{(k-1)}$  is increased to  $V_{x+k}^{(k)}$ . Obviously, this reserve increase is financed by the insurer. In practice, the reserve increase may be financed by technical gains on interest, mortality and withdrawals. In our general setting, the former withdrawal payments  $w_{x+k+j}^{(k-1)}$  are replaced by revised values  $w_{x+k+j}^{(k)}$ , based on the information about medical inflation until time  $k$ . The corresponding increase  $W_{x+k}^{(k)} - W_{x+k}^{(k-1)}$  is financed by the insurer (via a reserve increase) and/or the policyholder (via increased premiums).

Let us briefly discuss two extreme cases where the effect of inflation is entirely borne by one of the agents, either the insurer or the policyholder.

**Example 1.** *When the premium is kept unchanged, i.e. when  $\pi^{(k)} = \pi^{(k-1)}$ , we find from (23) and (24) that*

$$V_{x+k}^{(k)} - V_{x+k}^{(k-1)} = \left( B_{x+k}^{(k)} - B_{x+k}^{(k-1)} \right) + \left( W_{x+k}^{(k)} - W_{x+k}^{(k-1)} \right), \quad (25)$$

*which means that the health benefit and withdrawal payment increases are completely financed by the insurer via an increase of the available assets.*

**Example 2.** *When the insurer does not increase the available reserve, i.e. when  $V_{x+k}^{(k)} = V_{x+k}^{(k-1)}$ , the health benefit and withdrawal payment increases are completely financed by the policyholder via increased premium payments. In this special case where at time  $k$ , the available reserve is not updated, we find from (23) and (24) that*

$$\pi^{(k)} - \pi^{(k-1)} = \frac{\left( B_{x+k}^{(k)} - B_{x+k}^{(k-1)} \right) + \left( W_{x+k}^{(k)} - W_{x+k}^{(k-1)} \right)}{\ddot{a}_{x+k}^{aa}}. \quad (26)$$

*This means that the premium increase  $\pi^{(k)} - \pi^{(k-1)}$  introduced at time  $k$  can be interpreted as the level premium for an insurance contract with yearly benefits equal to the health benefit increases and with withdrawal payments equal to the withdrawal payment increases of the original contract.*

Starting from the available reserve  $V_{x+k}^{(k)}$  at time  $k$ , we introduce the notation  $V_{x+k+j+1}^{(k)}$  for the available reserve of the contract at time  $k+j+1$ , in case of a future yearly medical inflation of  $f$  in the interval  $(k, k+j+1)$  as predicted at time  $k$ . Assuming the technical

basis used for resetting the actuarial equivalence (24) at time  $k$ , these reserves can be determined from the forward recursion

$$V_{x+k+j+1}^{(k)} = \left( V_{x+k+j}^{(k)} + \pi^{(k)} - b_{x+k+j}^{(k)} - q_{x+k+j}^{aw} v w_{x+k+j+1}^{(k)} \right) (p_{x+k+j}^{aa} v)^{-1}, \quad (27)$$

or equivalently,

$$\begin{aligned} v V_{x+k+j+1}^{(k)} &= V_{x+k+j}^{(k)} + \pi^{(k)} - b_{x+k+j}^{(k)} + q_{x+k+j}^{ad} v V_{x+k+j+1}^{(k)} \\ &\quad - q_{x+k+j}^{aw} v \left( w_{x+k+j+1}^{(k)} - V_{x+k+j+1}^{(k)} \right), \end{aligned} \quad (28)$$

which holds for  $j \in \{0, 1, \dots, \omega - x - k - 1\}$ . The initial value  $V_{x+k}^{(k)}$  is given by (24). Notice that  $V_{x+k+j}^{(k)}$  is an estimate for the available reserve per policy in force at time  $k + j$ , based on the information available and the technical assumptions used at time  $k$ .

It is easy to verify that the solution of recursion (27), with initial value  $V_{x+k}^{(k)}$  can be expressed in the following retrospective form:

$$\begin{aligned} V_{x+k+j}^{(k)} &= V_{x+k}^{(k)} (j p_{x+k}^{aa} v^j)^{-1} \\ &\quad + \sum_{l=0}^{j-1} \left( \pi^{(k)} - b_{x+k+l}^{(k)} - q_{x+k+l}^{aw} v w_{x+k+l+1}^{(k)} \right) (j-l p_{x+k+l}^{aa} v^{j-l})^{-1}, \end{aligned} \quad (29)$$

for  $j \in \{0, 1, \dots, \omega - x - k\}$ . Taking into account the restored actuarial equivalence (24), the reserves  $V_{x+k+j}^{(k)}$  can also be expressed prospectively:

$$V_{x+k+j}^{(k)} = B_{x+k+j}^{(k)} + W_{x+k+j}^{(k)} - \pi^{(k)} \ddot{a}_{x+k+j}^{aa}, \quad (30)$$

with  $\ddot{a}_{x+k+j}^{aa}$ ,  $B_{x+k+j}^{(k)}$  and  $W_{x+k+j}^{(k)}$  defined in (16), (18) and (19), respectively. In particular, we find that

$$V_{\omega}^{(k)} = b_{\omega}^{(k)} - \pi_x^{(k)}, \quad (31)$$

which means that the available reserve at the last possible integer age is equal to the health benefit minus the premium to be paid at that time.

## 4 Reserve-dependent withdrawal payments

In this section, we investigate reserve-dependent withdrawal benefits for the lifelong health insurance contract described in Sections 2 and 3. Specifically, we consider the case where upon surrender in year  $(k, k + 1)$ , the withdrawal benefit that is paid out at time  $k + 1$  is a linear function of the available reserve. In particular, we assume that the withdrawal payment is given by

$$w_{x+k+1}^{(k)} = (1 - \beta_{k+1}) V_{x+k+1}^{(k)} - \alpha_{k+1}, \quad k = 0, 1, 2, \dots, \omega - x - 1, \quad (32)$$

where  $\alpha_{k+1} \geq 0$  is a reserve-independent penalty and  $0 < \beta_{k+1} \leq 1$  is the non-transferred (or lost) percentage of the available reserve in case of policy cancellation. The quantities

$\alpha_{k+1}$  and  $\beta_{k+1}$  are fixed at policy issue. Furthermore, we set  $w_{\omega+1}^{(\omega)} = 0$ . Benefits of the form (32) have been studied in a continuous-time setting by Christiansen et al. (2014), without allowance for medical inflation. Notice that  $V_{x+k+1}^{(k)}$  is a function of  $\pi^{(0)}, \pi^{(1)}, \dots, \pi^{(k)}$ . This implies that at contract initiation, this reserve and hence, the surrender value  $w_{x+k+1}^{(k)}$  is in general unknown. However, when the surrender option is exercised in year  $(k, k+1)$ , the reserve  $V_{x+k+1}^{(k)}$  and the withdrawal payment are known at time  $k+1$ , see (22).

Let us first determine the level premium  $\pi^{(0)}$  at policy issue. In order to be able to determine this premium from the equivalence principle (8), we choose ‘time 0’ observable values for the future withdrawal payments. We propose to estimate the withdrawal payment in case of surrender in year  $(j, j+1)$ ,  $j \in \{0, 1, 2, \dots, \omega - x - 1\}$ , by

$$w_{x+j+1}^{(0)} = (1 - \beta_{j+1}) V_{x+j+1}^{(0)} - \alpha_{j+1}, \quad (33)$$

where  $V_{x+j+1}^{(0)}$  is the estimate for the available reserve at time  $j+1$  defined by the recursion (9), with initial value  $V_x^{(0)} = 0$ . Furthermore,  $w_{\omega+1}^{(0)}$  is set equal to zero.

Taking into account that the reserves (and hence, also  $W_x^{(0)}$ ) depend on the premium  $\pi^{(0)}$ , we find that the equivalence relation (8) does not lead to an explicit expression for the initial premium  $\pi^{(0)}$ . In order to find such an explicit expression, we insert the values (33) of the withdrawal benefits  $w_{x+j+1}^{(0)}$  in the recursion (9). Re-arranging the terms in this recursion leads to the transformed recursion

$$V_{x+j+1}^{(0)} = \left( V_{x+j}^{(0)} + \pi^{(0)} - b_{x+j}^{(0)} - \bar{q}_{x+j}^{aw} v \bar{w}_{x+j+1} \right) \left( \bar{p}_{x+j}^{aa} v \right)^{-1}, \quad (34)$$

which holds for any  $j \in \{0, 1, \dots, \omega - x - 1\}$ , and with initial value  $V_x^{(0)} = 0$ , where

$$\bar{q}_{x+j}^{aw} = \beta_{j+1} q_{x+j}^{aw}, \quad \bar{w}_{x+j+1} = -\frac{\alpha_{j+1}}{\beta_{j+1}} \quad \text{and} \quad \bar{p}_{x+j}^{aa} = 1 - q_{x+j}^{ad} - \bar{q}_{x+j}^{aw}. \quad (35)$$

Furthermore, we set

$$\bar{q}_{\omega}^{aw} = 0 \quad \text{and} \quad \bar{w}_{\omega+1} = 0. \quad (36)$$

As  $\beta_{j+1} \in (0, 1]$ , we have  $\bar{q}_{x+j}^{aw} \in [0, 1]$  so that this quantity and  $\bar{p}_{x+j}^{aa}$  can be interpreted as probabilities.

We can conclude that at any time  $j+1$ , the reserve  $V_{x+j+1}^{(0)}$  of the health insurance contract, which is defined by recursion (9) with initial value  $V_x^{(0)} = 0$ , is identical to the reserve of an artificial health insurance contract with transformed withdrawal payments  $\bar{w}_{x+j}$  and transformed probabilities  $\bar{q}_{x+j}^{aw}$  and  $\bar{p}_{x+j}^{aa}$ . The reserves of this artificial contract follow from recursion (34) with initial value  $V_x^{(0)} = 0$ . In the following proposition, we derive an explicit expression for the initial premium level of the original contract.

**Proposition 1.** *An explicit expression for the initial premium level  $\pi^{(0)}$  of the health insurance contract with reserve-dependent withdrawal benefits (33) is given by*

$$\pi^{(0)} = \frac{B_x^{(0)} + W_x^{(0)}}{\ddot{a}_x^{aa}}, \quad (37)$$

with

$$\begin{aligned}\underline{B}_x^{(0)} &= \sum_{l=0}^{\omega-x} {}_l\bar{p}_x^{aa} v^l b_{x+j}^{(0)} \\ \underline{W}_x^{(0)} &= \sum_{l=0}^{\omega-x} {}_l\bar{p}_x^{aa} \bar{q}_{x+l}^{aw} v^{l+1} \bar{w}_{x+l+1} \\ \underline{\ddot{a}}_x^{aa} &= \sum_{l=0}^{\omega-x} {}_l\bar{p}_x^{aa} v^l.\end{aligned}$$

In these expressions, the  $\bar{q}_{x+l}^{aw}$  and  $\bar{w}_{x+l+1}$  are defined by (35) and (36). Furthermore,  ${}_0\bar{p}_{x+j}^{aa} = 1$  and for  $l > 0$ , we have that

$${}_l\bar{p}_x^{aa} = \prod_{k=0}^{l-1} \bar{p}_{x+k}^{aa}$$

with the  $\bar{p}_{x+k}^{aa}$  defined in (35).

*Proof.* In Section 2.3, we have proven that the recursion (9) with initial value  $V_x^{(0)} = 0$  leads to the retrospective expression (12) with  $j = \omega - x$  for  $V_\omega^{(0)}$ . In a similar way, one can prove that the transformed version (34) of this recursion with the same initial value leads to the following retrospective expression for  $V_\omega^{(0)}$ :

$$V_\omega^{(0)} = \sum_{l=0}^{\omega-x-1} \left( \pi^{(0)} - b_{x+l}^{(0)} - \bar{q}_{x+l}^{aw} v \bar{w}_{x+l+1} \right) \left( {}_{\omega-x-l}\bar{p}_{x+l}^{aa} v^{\omega-x-l} \right)^{-1}.$$

On the other hand, from (17) we know that  $V_\omega^{(0)} = b_\omega^{(0)} - \pi^{(0)}$ . Hence,

$$\sum_{l=0}^{\omega-x} \left( \pi^{(0)} - b_{x+l}^{(0)} - \bar{q}_{x+l}^{aw} v \bar{w}_{x+l+1} \right) \left( {}_{\omega-x-l}\bar{p}_{x+l}^{aa} v^{\omega-x-l} \right)^{-1} = 0.$$

Multiplying each term in this expression by  ${}_{\omega-x}\bar{p}_x^{aa} v^{\omega-x}$  leads to

$$\pi^{(0)} \underline{\ddot{a}}_x^{aa} = \underline{B}_x^{(0)} + \underline{W}_x^{(0)},$$

which proves the stated result.  $\square$

Obviously,  $\underline{B}_x^{(0)}$  and  $\underline{W}_x^{(0)}$  can directly be determined at policy issue. This means that (37) is indeed an explicit expression for the initial premium level.

Suppose now that we have arrived at time  $k \in \{1, 2, \dots, \omega - x\}$ , and that the contract is still in force. At that time, the actuarial value of future health benefit payments, taking into account medical inflation up to that time is given by  $B_{x+k}^{(k)}$ , which is defined in (18). As before, the updated value of the available reserve at time  $k$  is denoted by  $V_{x+k}^{(k)}$ , whereas the new level premium to be determined at that time is denoted by  $\pi^{(k)}$ . We propose

to update the values for withdrawal payments at time  $k$  using the information about observed inflation until time  $k$ , and assuming a yearly medical inflation of  $f$  for future years:

$$w_{x+k+j+1}^{(k)} = (1 - \beta_{k+j+1}) V_{x+k+j+1}^{(k)} - \alpha_{k+j+1}, \quad j = 0, 1, \dots, \omega - x - k - 1, \quad (38)$$

where  $V_{x+k+j+1}^{(k)}$  is the available reserve at time  $k + j + 1$  as defined in (27). Furthermore, as mentioned before, we set  $w_{\omega+1}^{(k)} = 0$ .

Taking into account that the reserves  $V_{x+k+j+1}^{(k)}$  and hence also  $W_{x+k}^{(k)}$ , depend on the premium  $\pi^{(k)}$ , we find that the restoring equivalence equation (24) does not give an explicit relation between the updated premium level and the available reserve at time  $k$ . In order to solve this problem, we insert the values (38) for the updated withdrawal payments in the recursion (27). This leads to the transformed recursion for the available reserves:

$$V_{x+k+j+1}^{(k)} = \left( V_{x+k+j}^{(k)} + \pi^{(k)} - b_{x+k+j}^{(k)} - \bar{q}_{x+k+j}^{aw} v \bar{w}_{x+k+j+1} \right) (\bar{p}_{x+k+j}^{aa} v)^{-1}, \quad (39)$$

which holds for any  $j \in \{0, 1, \dots, \omega - x - k - 1\}$ , with initial value  $V_{x+k}^{(k)}$ . The quantities  $\bar{q}_{x+k+j}^{aw}$ ,  $\bar{p}_{x+k+j}^{aa}$  and  $\bar{w}_{x+k+j+1}$  are defined as before. The advantage of rewriting the recursive relation (29) in the form (39) is that it allows us to find an explicit relation between  $V_{x+k}^{(k)}$  and  $\pi^{(k)}$ , as shown in the following proposition.

**Proposition 2.** *Consider the lifelong health insurance contract with reserve-dependent withdrawal benefits (33). The actuarial equilibrium restoring equivalence relation (24) at time  $k$  can be expressed in the following way, which provides an explicit relation between  $V_{x+k}^{(k)}$  and  $\pi^{(k)}$ :*

$$V_{x+k}^{(k)} = \underline{B}_{x+k}^{(k)} + \underline{W}_{x+k}^{(k)} - \pi^{(k)} \underline{\ddot{a}}_{x+k}^{aa}, \quad (40)$$

with

$$\begin{aligned} \underline{B}_{x+k}^{(k)} &= \sum_{l=0}^{\omega-x-k} {}_l\bar{p}_{x+k}^{aa} v^l b_{x+k+l}^{(k)} \\ \underline{W}_{x+k}^{(k)} &= \sum_{l=0}^{\omega-x-k} {}_l\bar{p}_{x+k}^{aa} \bar{q}_{x+k+l}^{aw} v^{l+1} \bar{w}_{x+k+l+1} \\ \underline{\ddot{a}}_{x+k}^{aa} &= \sum_{l=0}^{\omega-x-k} {}_l\bar{p}_{x+k}^{aa} v^l. \end{aligned}$$

In these expressions, the  $\bar{q}_{x+k+l}^{aw}$ ,  ${}_l\bar{p}_{x+k}^{aa}$  and  $\bar{w}_{x+k+l+1}$  are defined as in Proposition 1.

*Proof.* We have proven that the recursion (27) with initial value  $V_{x+k}^{(k)}$  for the available reserves leads to the retrospective expression (29) with  $j = \omega - x - k$  for  $V_{\omega}^{(0)}$ . In a similar way, one can easily prove that the transformed version (39) of this recursion with the

same initial value leads to the following retrospective expression for  $V_\omega^{(k)}$ :

$$V_\omega^{(k)} = V_{x+k}^{(k)} (\omega_{-x-k} p_{x+k}^{aa} v^{\omega-x-k})^{-1} + \sum_{l=0}^{\omega-x-k-1} \left( \pi^{(k)} - b_{x+k+l}^{(k)} - \bar{q}_{x+k+l}^{aw} v \bar{w}_{x+k+l+1} \right) (\omega_{-x-k-l} \bar{p}_{x+k+l}^{aa} v^{\omega-x-k-l})^{-1}.$$

On the other hand, from (31) we know that  $V_\omega^{(k)} = b_\omega^{(k)} - \pi^{(k)}$ . This observation leads to

$$0 = V_{x+k}^{(k)} (\omega_{-x-k} p_{x+k}^{aa} v^{\omega-x-k})^{-1} + \sum_{l=0}^{\omega-x-k} \left( \pi^{(k)} - b_{x+k+l}^{(k)} - \bar{q}_{x+k+l}^{aw} v \bar{w}_{x+k+l+1} \right) \times (\omega_{-x-k-l} \bar{p}_{x+l}^{aa} v^{\omega-x-k-l})^{-1}.$$

Multiplying each term in this expression by  $\omega_{-x-k} \bar{p}_{x+k}^{aa} v^{\omega-x-k}$  gives rise to (40), which proves the stated result.  $\square$

As a consequence of the particular form of  $w_{x+k+1}$ , both  $\underline{B}_{x+k}^{(k)}$  and  $\underline{W}_{x+k}^{(k)}$  do not depend on  $V_{x+k+j}^{(k)}$  and  $\pi^{(k)}$ . This leads to an easy-to-handle relation between the updated available reserve  $V_{x+k}^{(k)}$  and the updated premium  $\pi^{(k)}$  which can be used to restore the actuarial equivalence at time  $k$ . Restoring this actuarial equivalence can be obtained in several ways (see also Vercruyssen et al. (2013)). The insurer could for instance first increase the available reserve by a certain percentage (financed by technical gains) from level  $V_{x+k}^{(k-1)}$  to level  $V_{x+k}^{(k)}$ , and then use (40) to determine the appropriate new level premium  $\pi^{(k)}$ . Another possibility is that the premium increases are contractually fixed as a function of observed medical inflation (e.g. the premium increase at time  $k$  is 100% of the medical inflation in the previous year). The updated available reserve  $V_{x+k}^{(k)}$  follows then from (40). In Section 6, both cases will be illustrated numerically.

## 5 Premium-dependent withdrawal payments

Actuaries generally base surrender values on accumulated reserves and this case has been thoroughly investigated in Section 4. However, this concept may seem obscure to many policyholders. Moreover, some insurers do not compute individual reserves but rather manage the entire portfolio as a collective. In order to overcome these concerns and problems, one might prefer to consider withdrawal payments based on the premiums paid so far. We will discuss this issue in this section. In particular, we assume that in case of surrender in the year  $(k, k+1)$ , the withdrawal benefit paid out at time  $k+1$  is given by

$$w_{x+k+1}^{(k)} = \beta_{k+1} \sum_{l=0}^k \pi^{(l)} (1+i')^{k+1-l} - \alpha_{k+1}, \quad k = 0, 1, 2, \dots \quad (41)$$

Hence, the withdrawal benefit is equal to a time-dependent fraction  $\beta_{k+1}$ ,  $0 \leq \beta_{k+1} \leq 1$ , of the accumulated value of the premiums paid until time  $k$ , minus a time-dependent



penalty  $\alpha_{k+1} \geq 0$ . We assume that the quantities  $\beta_1, \beta_2, \dots$  and  $\alpha_1, \alpha_2, \dots$  are fixed at policy issue. The coefficients  $\beta_1, \beta_2, \dots$  can be chosen such that they approximately mimic the accumulation of the savings premiums in the reserve, representing the part of the premiums paid but not consumed to finance past health benefits. The accumulation of the premiums is performed at a constant interest rate  $i'$ , which may be different from the technical interest rate  $i$ . We could for instance set  $i' = 0$  so that premiums enter the calculation at nominal values.

At policy issue, the payment for withdrawal in year  $(k, k + 1)$  is in general unknown as it depends on the a priori unknown stream of future premium payments  $\pi^{(1)}, \pi^{(2)}, \dots, \pi^{(k)}$ . However, when the surrender option is exercised in year  $(k, k + 1)$ , the surrender benefit  $w_{x+k+1}^{(k)}$  that is actually paid out is fully specified at time  $k + 1$ , based on the information that is available at that time about previous medical inflation.

In order to be able to determine the initial level premium  $\pi^{(0)}$  from the equivalence principle (8), we have to choose values for the future withdrawal payments observable at time 0. We propose to estimate the payment in case of withdrawal in year  $(j, j + 1)$ ,  $j \in \{0, 1, 2, \dots, \omega - x - 1\}$  by

$$w_{x+j+1}^{(0)} = \beta_{j+1} \sum_{l=0}^j \pi^{(0)} (1 + i')^{j+1-l} - \alpha_{j+1}. \quad (42)$$

This means that  $w_{x+j+1}^{(0)}$  corresponds to the withdrawal payment  $w_{x+j+1}^{(j)}$  in case of a medical inflation of  $f$  per year and no adaptation of the premiums until withdrawal.

**Proposition 3.** *Assuming (42), an explicit expression for the initial premium level  $\pi^{(0)}$  of the health insurance contract with premium-dependent withdrawal benefits (41) is given by*

$$\pi^{(0)} = \frac{B_x^{(0)} - \sum_{j=0}^{\infty} {}_j p_x^{aa} q_{x+j}^{aw} v^{j+1} \alpha_{j+1}}{\ddot{a}_x^{aa} - \sum_{j=0}^{\infty} {}_j p_x^{aa} q_{x+j}^{aw} v^{j+1} c_{j+1}^{(0)}} \quad (43)$$

with

$$c_{j+1}^{(0)} = \beta_{j+1} \sum_{l=0}^j (1 + i')^{j+1-l}. \quad (44)$$

*Proof.* We can rewrite (42) as follows:

$$w_{x+j+1}^{(0)} = c_{j+1}^{(0)} \pi^{(0)} - \alpha_{j+1} \quad (45)$$

with the  $c_{j+1}^{(0)}$  defined in (44). Inserting the surrender values  $w_{x+j+1}^{(0)}$  in the actuarial equivalence relation (8), while taking into account (15), leads to the explicit expression (43) for the initial level premium  $\pi^{(0)}$ .  $\square$

Suppose now that we have arrived at time  $k \in \{1, 2, \dots\}$  and that the policy is still in force. The available reserve  $V_{x+k}^{(k-1)}$  at this moment is given by (22). Taking into account the information about medical inflation up to time  $k$ , the future health benefits

are re-estimated and their updated actuarial value  $B_{x+k}^{(k)}$  follows from (18). In general, the actuarial equivalence will be broken at time  $k$ . Therefore, the insurer updates the available reserve  $V_{x+k}^{(k-1)}$  to level  $V_{x+k}^{(k)}$ , while the premium  $\pi^{(k-1)}$  is replaced by  $\pi^{(k)}$ . Furthermore, the previously chosen values  $w_{x+k+j+1}^{(k-1)}$  for future withdrawal payments are replaced by the values  $w_{x+k+j+1}^{(k)}$ ,  $j \in \{1, 2, \dots\}$ , which are defined by

$$w_{x+k+j+1}^{(k)} = \beta_{k+j+1} \sum_{l=0}^{k+j} \pi^{(\min\{k,l\})} (1+i')^{k+j+1-l} - \alpha_{k+j+1}. \quad (46)$$

This means that at time  $k$ , the future withdrawal payments are determined using the information about medical inflation until time  $k$ , while assuming a future inflation of  $f$  per year. The new values for the reserve and the premium are chosen such that the actuarial equivalence is restored, i.e. such that (24) holds.

**Proposition 4.** *Consider the lifelong health insurance contract with premium-dependent withdrawal benefits (41). The actuarial equilibrium restoring equivalence relation (24) at time  $k$  can be expressed in the following explicit relation between  $V_{x+k}^{(k)}$  and  $\pi^{(k)}$ :*

$$\begin{aligned} V_{x+k}^{(k)} &= B_{x+k}^{(k)} + \sum_{j=0}^{\omega-x-k} {}_jP_{x+k}^{aa} q_{x+k+j}^{aw} v^{j+1} d_{k+j+1}^{(k)} \\ &\quad - \pi^{(k)} \left( \ddot{a}_{x+k}^{aa} - \sum_{j=0}^{\omega-x-k} {}_jP_{x+k}^{aa} q_{x+k+j}^{aw} v^{j+1} c_{k+j+1}^{(k)} \right) \end{aligned} \quad (47)$$

with

$$c_{k+j+1}^{(k)} = \beta_{k+j+1} \sum_{l=k}^{k+j} (1+i')^{k+j+1-l} \quad (48)$$

$$d_{k+j+1}^{(k)} = \beta_{k+j+1} \sum_{l=0}^{k-1} \pi^{(l)} (1+i')^{k+j+1-l} - \alpha_{k+j+1}. \quad (49)$$

*Proof.* The updated withdrawal payment  $w_{x+k+j+1}^{(k)}$  can be rewritten as

$$w_{x+k+j+1}^{(k)} = c_{k+j+1}^{(k)} \pi^{(k)} + d_{k+j+1}^{(k)}, \quad (50)$$

with  $c_{k+j+1}^{(k)}$  and  $d_{k+j+1}^{(k)}$  defined by (48) and (49), respectively. From (19) and (50), it follows that the actuarial value of future withdrawal benefits  $W_{x+k}^{(k)}$  can then be expressed as follows:

$$W_{x+k}^{(k)} = \sum_{j=0}^{\omega-x-k} {}_jP_{x+k}^{aa} q_{x+k+j}^{aw} v^{j+1} \left( c_{k+j+1}^{(k)} \pi^{(k)} + d_{k+j+1}^{(k)} \right). \quad (51)$$

Combining (24) and (51) yields the announced result.  $\square$

Restoring the actuarial equivalence at time  $k$  can be obtained in several ways, as pointed out before. The insurer could for instance first increase the available reserve using technical gains and then deduce the new premium level from (47). Another possibility is that the premium increases are contractually fixed as a function of past medical inflation, the updated available reserve then following from (24). In Section 6, both cases will be illustrated numerically.

## 6 Numerical illustration

### 6.1 Technical basis

We consider a contract issued to a policyholder aged  $x = 25$ . This contract covers medical expenses in excess of Social Security, as those commonly sold in Belgium. Additional background information can be found in the KCE reports 96 by Devolder et al. (2008). The technical basis assumes a yearly interest rate  $i$  of 2%.

Since health insurance contracts with a transferable reserve are not currently available on the Belgian market, we do not have relevant observed lapse probabilities at our disposal. Therefore we carry out a sensitivity analysis by varying the lapse probabilities according to the three following scenarios: considering a policyholder buying the contract at age 25, we consider one-year lapse probabilities  $q_y^{aw}$  at age  $y \geq 25$  given by

$$q_y^{aw_1} = 0 \tag{52}$$

$$q_y^{aw_2} = \begin{cases} 0.1 - 0.002 \cdot (y - 20) & \text{if } 25 \leq y \leq 70 \\ 0 & \text{otherwise} \end{cases} \tag{53}$$

$$q_y^{aw_3} = \begin{cases} 0.05 \cdot (\cos((y - 25) \cdot \frac{\pi}{95}) + 1) & \text{if } 25 \leq y \leq 120 \\ 0 & \text{otherwise.} \end{cases} \tag{54}$$

These lapse probabilities are displayed in Figure 2. Under the first set of lapse probabilities  $q_y^{aw_1}$ , policyholders never cancel the contract. The second set of lapse probabilities  $q_y^{aw_2}$  has been used by Vercruyssen et al. (2013). These probabilities imply a higher propension to lapse at younger ages for the 25-year-old policyholder under consideration, but no lapse after age 70. This is often taken as the central scenario on the Belgian market. Finally, under the third set of lapse probabilities  $q_y^{aw_3}$ , we have higher lapse probabilities at younger ages which then decline smoothly to 0 at the ultimate age  $\omega = 120$ .

Death probabilities are displayed in Figure 3, left panel. Notice that these are not the  $q_y^{ad}$  but yearly death probabilities  $q_y'^d$  based on observations at the general population level in a single decrement, two state setting, alive or dead. We recover the values for  $q_y^{ad}$  from the following relation:

$$q_y^{ad} = q_y'^d \left( 1 - \frac{q_y^{aw}}{2 - q_y'^d} \right), \tag{55}$$

which holds under the assumption of a uniform distribution of decrements in any year for each of the two single decrement models; see Section 8.10.2 in Dickson et al. (2013).

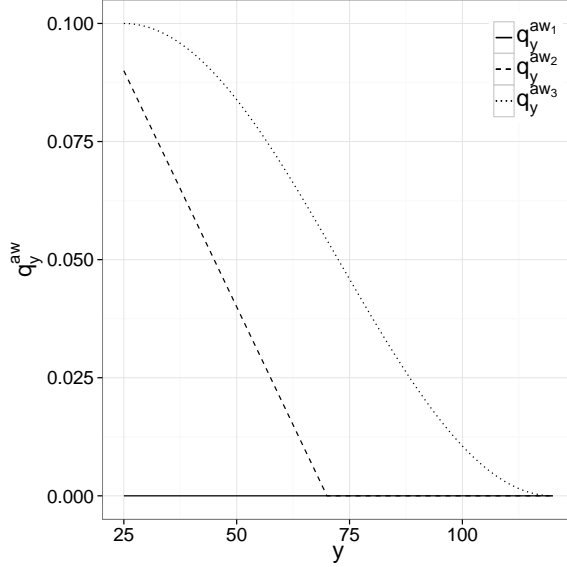


Figure 2: One-year lapse probabilities  $q_y^{aw1}$  (full line),  $q_y^{aw2}$  (dashed line), and  $q_y^{aw3}$  (dotted line).

Notice that lapse probabilities enter the calculation of death probabilities so that different scenarios of policy cancellation impact on all actuarial quantities. Under (52), we have  $q_y^{ad} = q_y^{ad}$  but these two probabilities differ under the other scenarios (53) and (54). Figure 3, right panel, displays the three sets of one-year death probabilities used in the numerical illustrations.

The dashed line in Figure 4 shows the average health benefit  $\bar{b}_y^{(0)}$  in function of policyholder's attained age  $y$ , in euros. The shape of  $y \mapsto \bar{b}_y^{(0)}$  is inspired from Belgian private health insurance market experience but the values have been rescaled for confidentiality reasons. The full line in this figure represents the average health benefits  $b_y^{(0)}$  when a medical inflation  $f$  of 2% per year is taken into account.

## 6.2 Surrender values

### 6.2.1 Reserve-dependent benefits

When withdrawal benefits depend on the available reserve, as discussed in Section 4, we set the non-transferred percentage and the reserve-independent penalty in the definition of withdrawal payments (32) respectively equal to

$$\beta_{k+1} = \begin{cases} 1 & \text{if } 0 \leq k \leq 4 \\ 0.2 & \text{if } 5 \leq k \end{cases} \quad \text{and} \quad \alpha_{k+1} = \begin{cases} 0 & \text{if } 0 \leq k \leq 4 \\ 150 & \text{if } 5 \leq k. \end{cases}$$

Early cancellations often cause significant losses for the insurer due to unrecovered administrative costs and commissions. Therefore no withdrawal benefits are paid in the first five

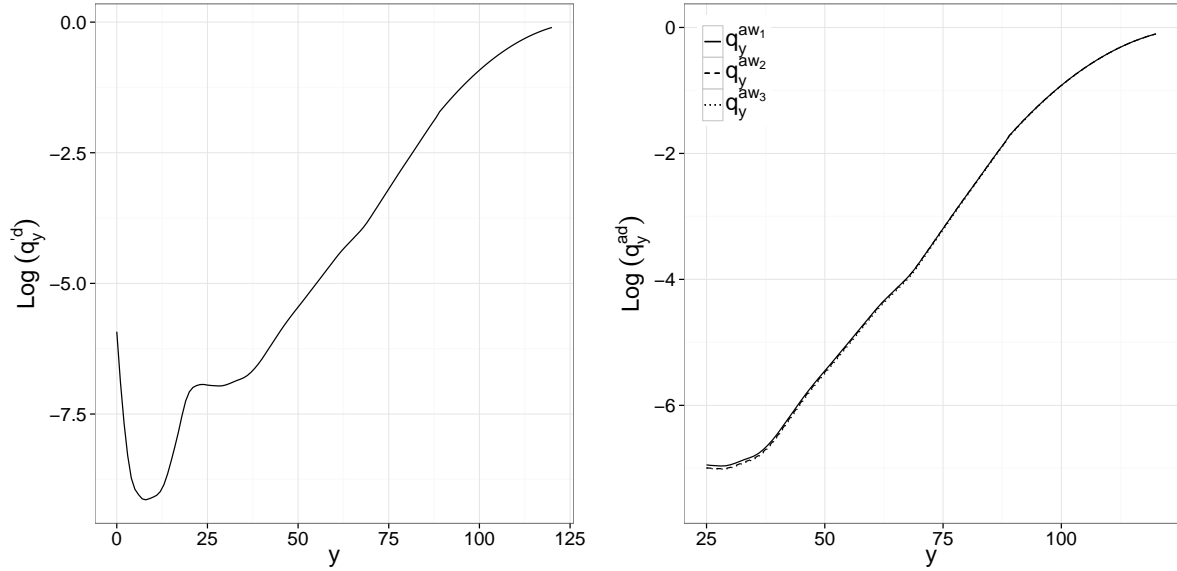


Figure 3: One-year death probabilities  $q_y^d$  on a log-scale (*left*) and one-year death probabilities  $q_y^{ad}$  on a log-scale corresponding to the three sets of lapse probabilities (*right*)  $q_y^{aw_1}$  (*full line*),  $q_y^{aw_2}$  (*dashed line*), and  $q_y^{aw_3}$  (*dotted line*).

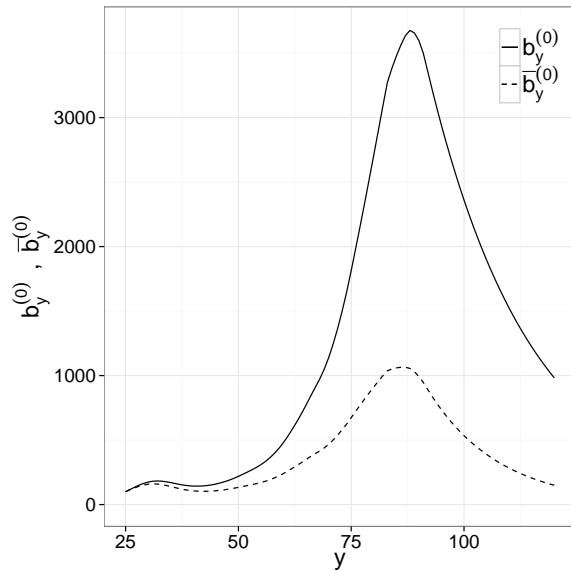


Figure 4: Average health benefit  $\bar{b}_y^{(0)}$  at age  $y$  (*dashed line*) and inflated values  $b_y^{(0)}$  (*full line*) at constant rate  $f$ .

years of the contract in this example. Afterwards we fix the non-transferred percentage at 20% and the reserve-independent penalty at 150.

## 6.2.2 Premium-dependent benefits

We also consider withdrawal benefits based on the premiums paid so far defined by (41). We set the interest rate on the accumulated premiums to  $i' = 1\%$ .

The time-dependent fractions  $\beta_k$  of the accumulated value of the premiums and the time-dependent penalties  $\alpha_k$  are chosen such that the surrender value at time  $k$  corresponds to the sum of the savings premiums of the contract with interest accumulation at rate  $i'$ . As these values need to be specified in the policy conditions, we propose to determine the  $\beta_k$  and  $\alpha_k$  as follows:

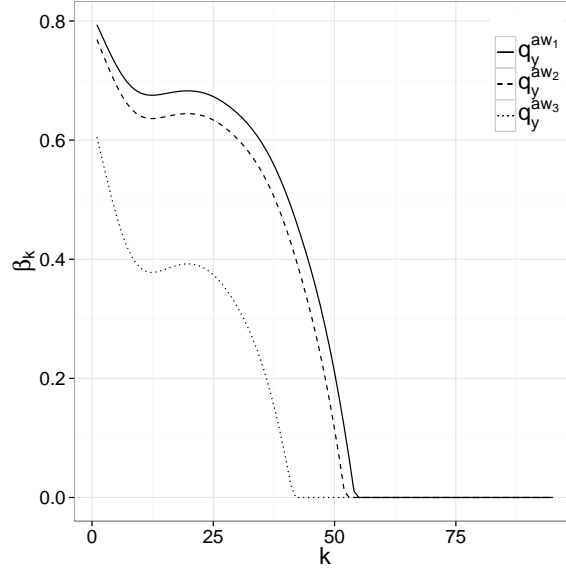


Figure 5: Values of the  $\beta_k$  defined from (56).

1. Define  $\beta'_k = 1$  and  $\alpha'_k = \sum_{l=0}^{k-1} b_{x+l}^{(0)} \cdot (1+i')^{k-l}$ . A contract with withdrawal benefits defined by (41) with parameters  $\beta'_k$  and  $\alpha'_k$  defines withdrawal benefits at time  $k+1$  as the sum of the savings premiums at policy issue with interest accumulation at rate  $i'$ :

$$w_{x+k+1}^{(0)} = \sum_{l=0}^k (\pi^{(0)} - b_{x+l}^{(0)}) \cdot (1+i')^{k+1-l}.$$

2. Calculate the initial premium  $\pi'^{(0)}$  for a contract with withdrawal benefits as defined in step 1. Use formula (43) from the approach described in Section 5.
3. Define

$$\beta_k = \max \left\{ 0, \frac{\sum_{l=0}^{k-1} (\pi'^{(0)} - b_{x+l}^{(0)}) \cdot (1+i')^{k-l}}{\sum_{l=0}^{k-1} \pi'^{(0)} \cdot (1+i')^{k-l}} \right\} \quad (56)$$

and  $\alpha_k = 0$  which leads to withdrawal benefits

$$w_{x+k+1}^{(0)} = \max \left\{ 0, \frac{\sum_{l=0}^k (\pi'^{(0)} - b_{x+l}^{(0)}) \cdot (1+i')^{k+1-l}}{\sum_{l=0}^k \pi'^{(0)} \cdot (1+i')^{k+1-l}} \right\} \cdot \sum_{l=0}^k \pi^{(0)} (1+i')^{k+1-l}. \quad (57)$$

These three simple steps base the definition of the withdrawal benefits on the sum of the savings premiums and ensure that the withdrawal benefits do not get negative. Moreover, this definition also ensures that the withdrawal benefits never exceed the sum of the premiums paid so far with interest accumulated at rate  $i'$  since  $\beta_k \leq 1$ . Figure 5 displays the  $\beta_k$  defined from (56).

### 6.3 Initial premium

Table 1 contains the initial premium  $\pi^{(0)}$  calculated for different types of withdrawal benefits: surrender value based on the reserve, surrender value based on the premiums, and no surrender value (i.e. the policyholder does not receive any benefit in case of policy cancellation). Obviously, the initial premium is identical under the first set of lapse probabilities  $q_y^{aw_1}$  as the definition of the withdrawal benefits is irrelevant in that case. The last column of Table 1 shows the initial premium in case the withdrawal benefits are always 0. Table 1 illustrates that given any lapse probability, a higher withdrawal benefit increases the initial premium.

Type of withdrawal benefits	Reserve-dependent	Premium-dependent	No benefits
Lapse probability $q_y^{aw_1}$	484.76	484.76	484.76
Lapse probability $q_y^{aw_2}$	415.50	431.15	267.18
Lapse probability $q_y^{aw_3}$	238.90	256.15	140.50

Table 1: Premium  $\pi^{(0)}$  at policy issue.

### 6.4 Medical inflation scenario

We illustrate the proposed methods by assuming an additional yearly medical inflation  $j_k^{[B]} = 1\%$  (for  $k \geq 1$ ) for health benefits, on top of the expected inflation  $f = 2\%$  incorporated in the premiums. In the notation of Section 2.1, the assumption of the expected inflation of  $f = 2\%$  translates to

$$b_{x+k}^{(0)} = (1 + f)^k \cdot \bar{b}_{x+k}^{(0)} = (1 + 2\%)^j \cdot \bar{b}_{x+k}^{(0)}. \quad (58)$$

The assumption of the additional medical inflation can be written as

$$\bar{b}_{x+k+j}^{(k)} = (1 + f) \cdot (1 + j_k^{[B]}) \cdot \bar{b}_{x+k+j}^{(k-1)} \quad (59)$$

such that

$$b_{x+k+j}^{(k)} = (1 + f)^j \cdot \bar{b}_{x+k+j}^{(k)} = (1 + 2\%)^j \cdot \bar{b}_{x+k+j}^{(k)} \quad (60)$$

or

$$b_y^{(k)} = (1 + j_k^{[B]}) \cdot b_y^{(k-1)} = (1 + 1\%) \cdot b_y^{(k-1)}. \quad (61)$$

## 6.5 Contract updating mechanisms

As explained earlier, there are many ways to update the contract to account for medical inflation. As in Verduyck et al. (2013) we denote the premium increase and the reserve increase at time  $k$  by respectively  $j_k^{[P]}$  and  $j_k^{[V]}$ . In this section, we consider the following two approaches:

**Mechanism 1:** The premium increase  $j_k^{[P]}$  is contractually fixed in function of past medical inflation, i.e.  $j_k^{[P]} = j_k^{[P]}(j_k^{[B]})$ . Premium updates are then obtained from

$$\pi_x^{(k)} = (1 + j_k^{[P]}(j_k^{[B]})) \cdot \pi_x^{(k-1)}. \quad (62)$$

The reserves are adjusted afterwards. For instance, policy conditions could specify that premiums are updated according to

$$j_k^{[P]}(j_k^{[B]}) = (1 + \gamma) \cdot j_k^{[B]} \quad (63)$$

where the additional  $\gamma$  accounts for the indexing of the accumulated reserve. In the numerical illustration in Sections 6.6, 6.7 and 6.8 we consider (63) with  $\gamma = 0$ .

**Mechanism 2:** The insurer now first increases the available reserve according to

$$V_{x+k}^{(k)} = (1 + j_k^{[V]}) \cdot V_{x+k}^{(k-1)} \quad (64)$$

and determines the corresponding new level premium afterwards. We illustrate this updating mechanism for  $j_k^{[V]} = 1\%$ , i.e. the insurer increases the available reserve by 1% each year.

## 6.6 No withdrawal benefits

We start with the case studied in Verduyck et al. (2013) where the policyholder receives no withdrawal benefit in case of surrender. Figure 6 shows the available reserve calculated with information available at time 0. Higher lapse probabilities tend to decrease the reserve. The reason is that when a policyholder lapses his contract, his reserve is transferred to the remaining policyholders.

Figure 7 illustrates the evolution of the reserves  $V_{x+k}^{(k)}$  and the premiums  $\pi^{(k)}$  over time  $k$  when nothing is paid in case of policy cancellation. In absence of withdrawal benefits, the result of updating mechanisms 1 and 2 described above is exactly the same so that we do not have to distinguish between the two mechanisms described above. This is because the expected present value of the withdrawal benefits (51) is zero which reduces actuarial equivalence (22) at time  $(k - 1)$  to

$$V_{x+k}^{(k-1)} = B_{x+k}^{(k-1)} - \pi^{(k-1)} \cdot \ddot{a}_{x+k}^{aa}. \quad (65)$$

When the additional medical inflation is  $j_k^{[B]} = 1\%$  and the reserve is updated by  $j_k^{[V]} = 1\%$  as in contract updating mechanism 2, the actuarial equivalence is restored by increasing



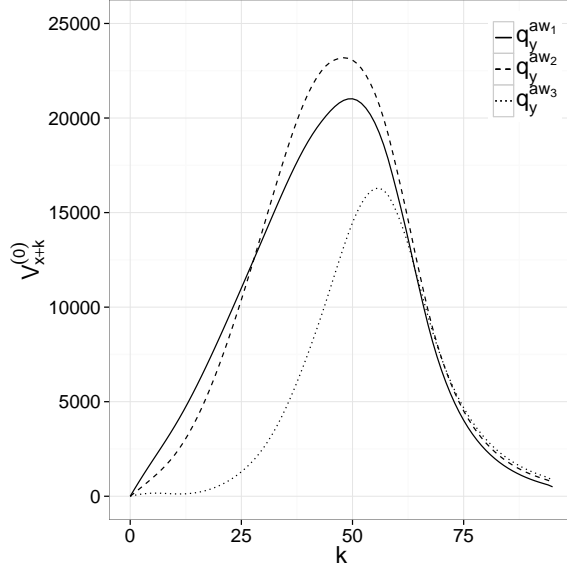


Figure 6: Available reserves  $V_{x+k}^{(0)}$  when no benefit is paid in case of withdrawal for the different types of lapse probabilities  $q_y^{aw1}$  (full line),  $q_y^{aw2}$  (dashed line),  $q_y^{aw3}$  (dotted line).

the premium by  $j_k^{[P]} = 1\%$ . This is the same premium update as for mechanism 1 as this mechanism sets the premium increase equal to the additional medical inflation, which we assume to be 1%. By a similar reasoning the reserve increase resulting from updating mechanism 1 equals  $j_k^{[V]} = 1\%$ . We conclude that both updating mechanisms have the same impact on the reserve and premium in this setting.

## 6.7 Reserve-dependent withdrawal payments

This section illustrates the strategy proposed in Section 4 where the withdrawal benefits depend on the available reserve. The first column of Table 1 shows the initial premium for a contract specifying reserve-dependent withdrawal benefits for the different lapse probability assumptions. The initial premium of the contract decreases as the lapse probability increases. This is a consequence of the definition and choice of parameters  $\beta_k$  and  $\alpha_k$  of the withdrawal benefits. As illustrated in Figure 8, the withdrawal benefits never exceed the available reserve. The part of the reserve not transferred in case of surrender can be added to the reserve of the remaining policyholders. Therefore, higher lapse probabilities have a premium reducing effect.

Figures 9 and 10 illustrate the evolution of the available reserve and withdrawal benefits over time under mechanisms 1 and 2, respectively. For both updating mechanisms the reserve increases over time due to the observed medical inflation. Therefore, the withdrawal benefits also increase because of their dependence on the reserve. The difference between both updating mechanisms is visualized in Figure 11. The graph on the right illustrates the yearly premium increase  $j_k^{[P]}$  when we use mechanism 2 to account for observed medical inflation. Under updating mechanism 1 this increase is fixed at the

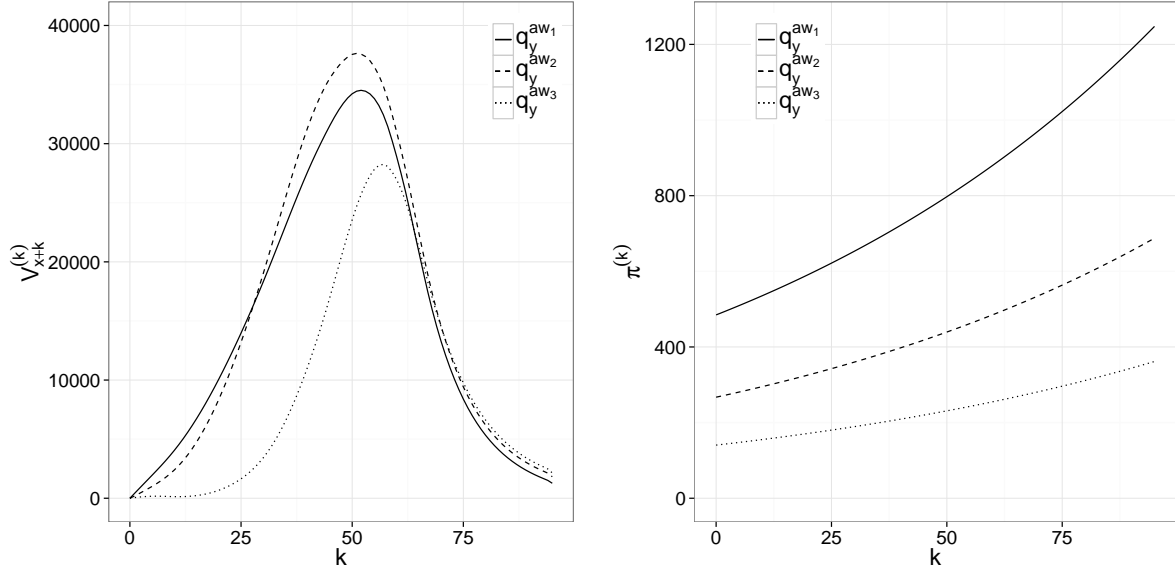


Figure 7: Available reserves  $V_{x+k}^{(k)}$  and premiums  $\pi^{(k)}$  when no benefit is paid in case of withdrawal for the different types of lapse probabilities  $q_y^{aw_1}$  (full line),  $q_y^{aw_2}$  (dashed line),  $q_y^{aw_3}$  (dotted line).

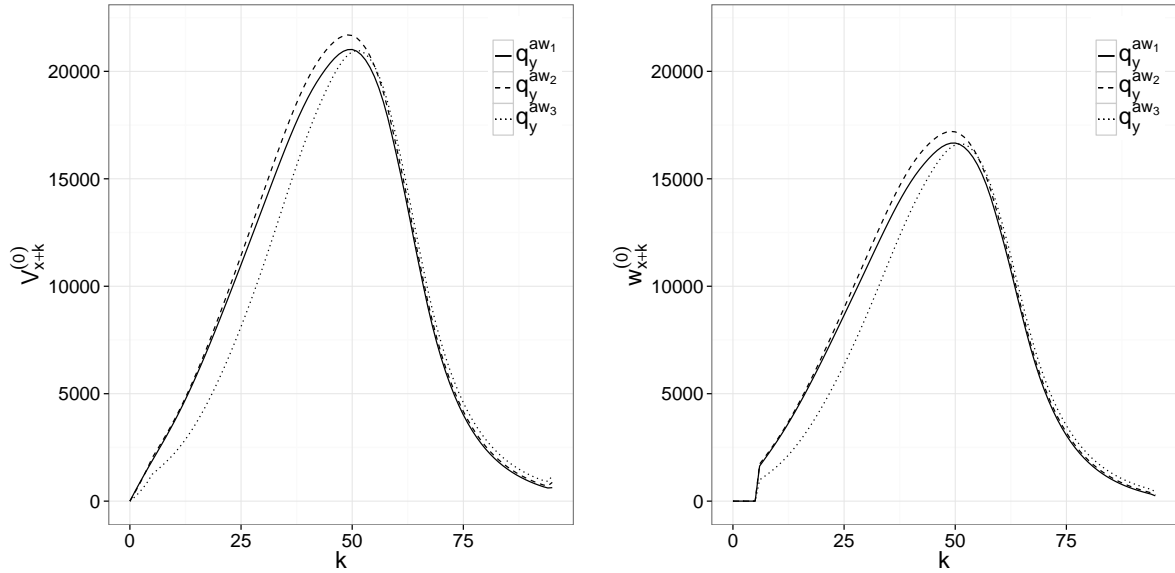


Figure 8: Available reserves  $V_{x+k}^{(0)}$  (left) and surrender values  $w_{x+k}^{(0)}$  (right) for the different types of lapse probabilities  $q_y^{aw_1}$  (full line),  $q_y^{aw_2}$  (dashed line),  $q_y^{aw_3}$  (dotted line).

additional medical inflation ( $j_k^{[B]} = 0.01$ ), whereas for updating mechanism 2 the increase may vary over time. The horizontal line at a premium increase of 1% corresponds to zero lapse probability  $q_y^{aw_1}$  as the expected present value of the withdrawal benefits is zero at

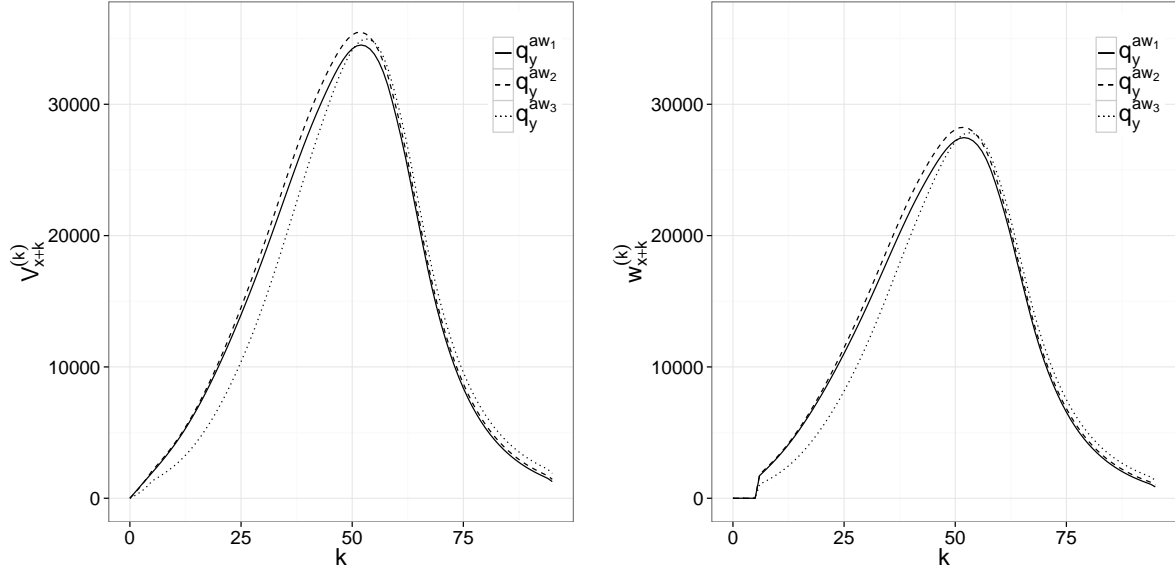


Figure 9: Contract updating mechanism 1: available reserves  $V_{x+k}^{(k)}$  (left) and surrender values  $w_{x+k}^{(k)}$  (right) for the different types of lapse probabilities  $q_y^{aw1}$  (full line),  $q_y^{aw2}$  (dashed line),  $q_y^{aw3}$  (dotted line).

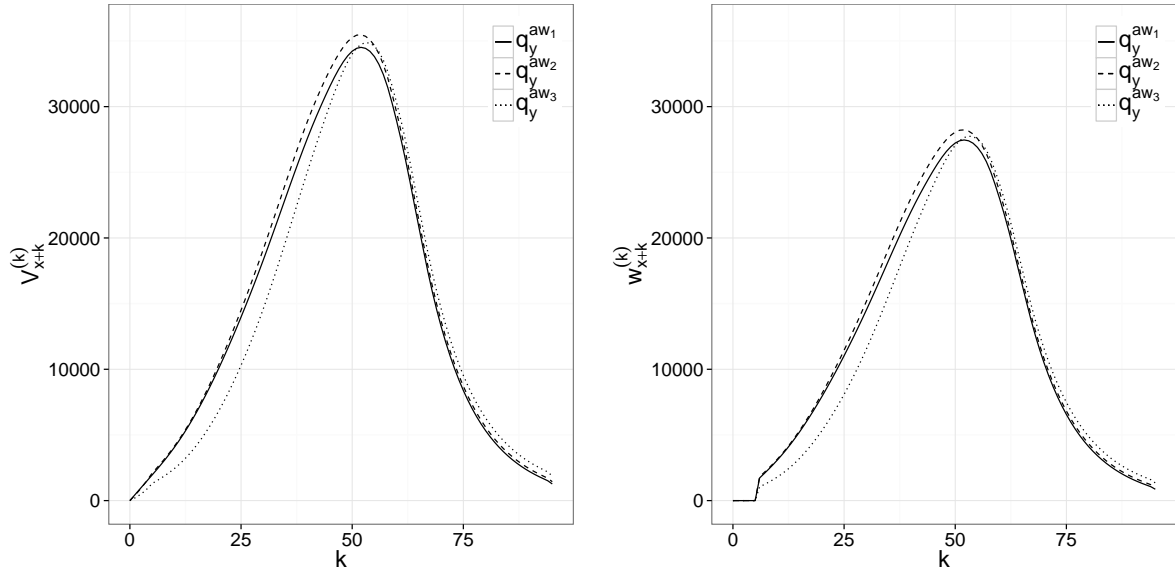


Figure 10: Contract updating mechanism 2: available reserves  $V_{x+k}^{(k)}$  (left) and surrender values  $w_{x+k}^{(k)}$  (right) for the different types of lapse probabilities  $q_y^{aw1}$  (full line),  $q_y^{aw2}$  (dashed line),  $q_y^{aw3}$  (dotted line).

any time which reduces the actuarial equivalence at time  $k - 1$  to (65). For a reserve increase of 1% and the same increase in  $B_{x+k}^{(k-1)}$  to account for observed medical inflation,

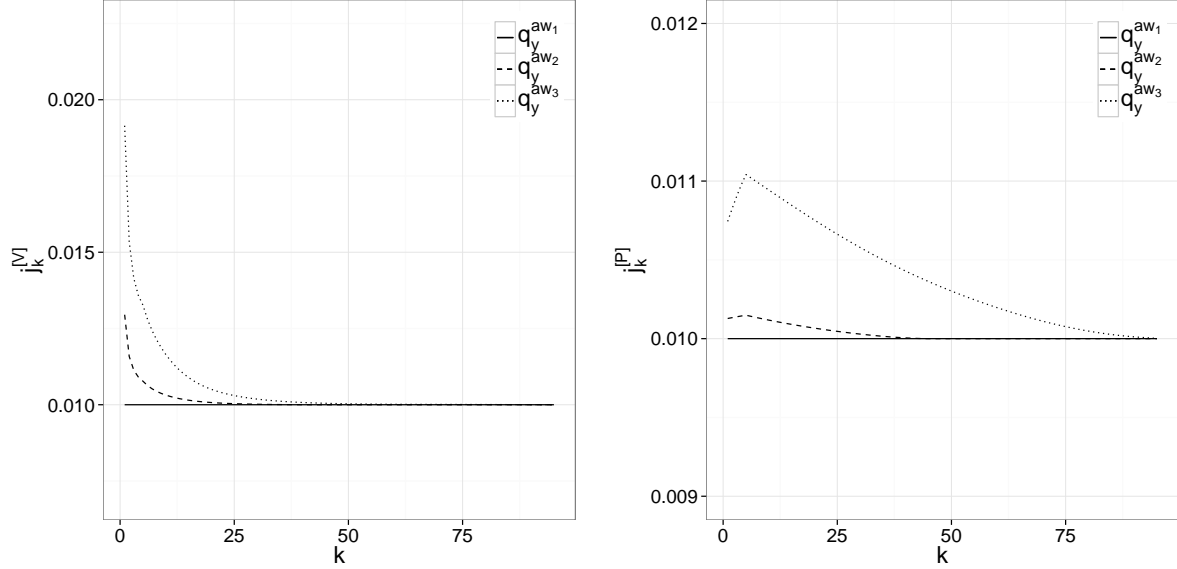


Figure 11: Yearly reserve increase  $j_k^{[V]}$  for mechanism 1 (*left*) and yearly premium increase  $j_k^{[P]}$  for updating mechanism 2 (*right*) for the different types of lapse probabilities  $q_y^{aw_1}$  (*full line*),  $q_y^{aw_2}$  (*dashed line*),  $q_y^{aw_3}$  (*dotted line*).

the equivalence is restored by a premium increase of 1%. The other lapse probabilities decrease and converge to 0 over time. Consequently, over time the expected present value of the withdrawal benefits has a smaller impact on the premium when restoring the actuarial equivalence. Therefore the curves corresponding to lapse probabilities  $q_y^{aw_2}$  and  $q_y^{aw_3}$  also converge to 1% over time. The point at which the percentage starts to decrease corresponds to the year after which the withdrawal benefits are strictly positive.

The graph in the left panel of Figure 11 shows the increase of the reserve  $j_k^{[V]}$  under mechanism 1. Updating mechanism 2 fixes this increase at 1%. As for the right figure, the reduced equivalence relation (65) explains why the reserve increase for  $q_y^{aw_1}$  is constantly equal to 1% and why the reserve increases for the other lapse probabilities flattens out at 1% in the left figure.

## 6.8 Premium-dependent withdrawal payments

The second column in Table 1 displays the initial premium  $\pi^{(0)}$  calculated at policy issue corresponding to the different lapse probability assumptions in (52), (53) and (54). The evolution of the available reserve  $V_{x+k}^{(0)}$  when medical inflation and contract updates over time are not taken into account is illustrated in the left graph of Figure 12. The right graph of this figure shows the evolution of withdrawal benefits (57) calculated at policy issue. The withdrawal benefits never exceed the available reserve. As a consequence, higher lapse probabilities result in lower premiums as demonstrated in Table 1. A lower premium implies that the sum of the savings premiums is lower and gets negative sooner,

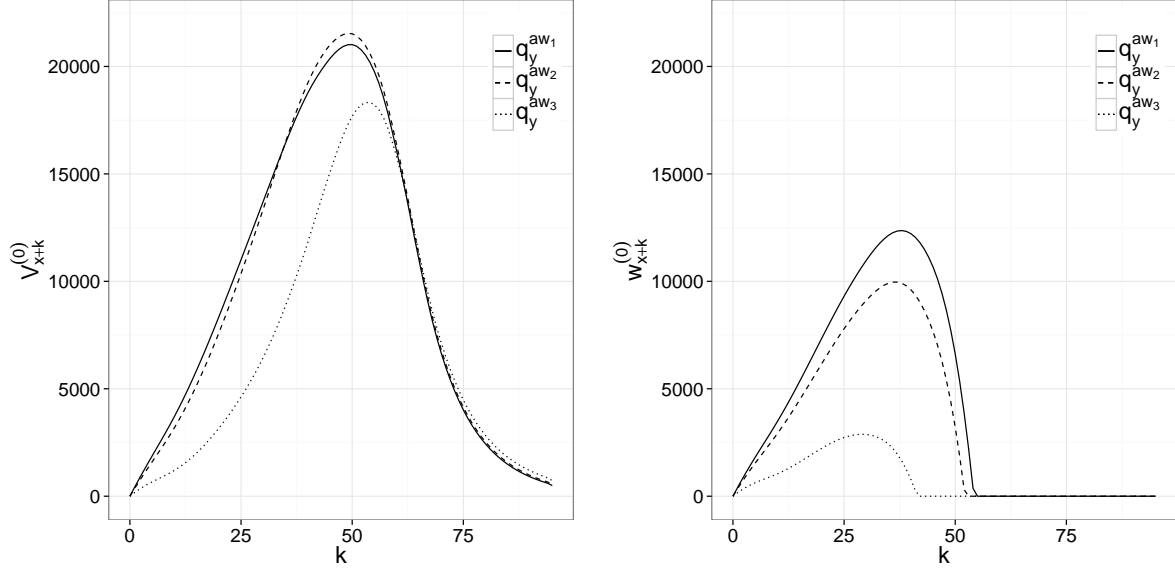


Figure 12: Available reserves  $V_{x+k}^{(0)}$  (left) and surrender values  $w_{x+k}^{(0)}$  (right) for the different types of lapse probabilities  $q_y^{aw1}$  (full line),  $q_y^{aw2}$  (dashed line),  $q_y^{aw3}$  (dotted line).

so the cap of 0 in definition (56) of  $\beta_k$  is reached more rapidly.

The impact of updating mechanism 1 and mechanism 2 on the reserves and withdrawal benefits is illustrated in Figures 13 and 14, respectively. As expected, for both mechanisms the reserves and withdrawal benefits have increased. However, withdrawal benefits never exceed the available reserve.

The difference between both updating mechanisms is highlighted in Figure 15. The right graph shows the yearly increase in the premium when we use updating mechanism 2 to account for observed medical inflation. Under updating mechanism 1, this increase is fixed at the additional medical inflation ( $j_k^{[B]} = 0.01$ ). The premium increase varies over time when using updating mechanism 2. When the lapse probability is zero for all ages, the expected present value of the withdrawal benefits (51) is zero which reduces the actuarial equivalence at time  $(k - 1)$  to (65). Additional medical inflation of 1% and a reserve update of 1% requires a premium increase of 1% to restore the actuarial equivalence. For the same reason the curves corresponding to the other lapse probabilities flatten out at zero: due to the cap of zero on the  $\beta_k$  the expected value of the withdrawal benefits drops to zero over time. Earlier in the contract, the premium increase is lower than the reserve increase.

Under updating mechanism 2 the reserve increase is fixed at 1%. The left panel of Figure 15 demonstrates that the reserve increase under mechanism 1 varies over time. Using the same reasoning as for the graph appearing in the right panel, the reserve increase for  $q_y^{aw1}$  is constantly equal to 1% and the reserve increases for the other lapse probabilities flatten out at 1%. Earlier in the contract the required reserve update for the non-zero lapse probabilities is lower than 1%.

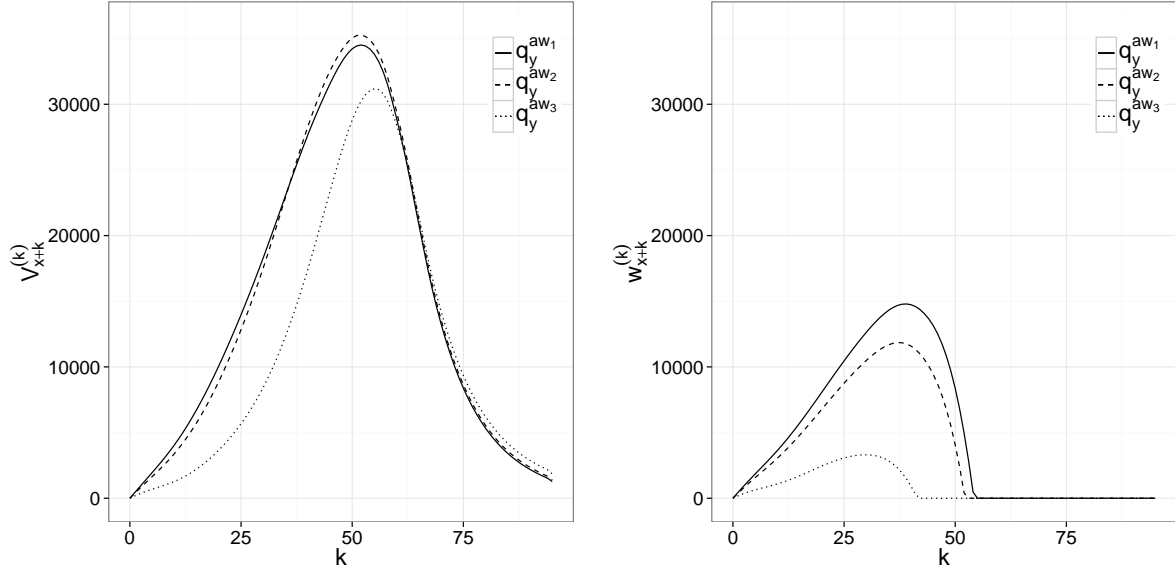


Figure 13: Updating mechanism 1: available reserves  $V_{x+k}^{(k)}$  (left) and surrender values  $w_{x+k}^{(k)}$  (right) for the different types of lapse probabilities  $q_y^{aw_1}$  (full line),  $q_y^{aw_2}$  (dashed line),  $q_y^{aw_3}$  (dotted line).

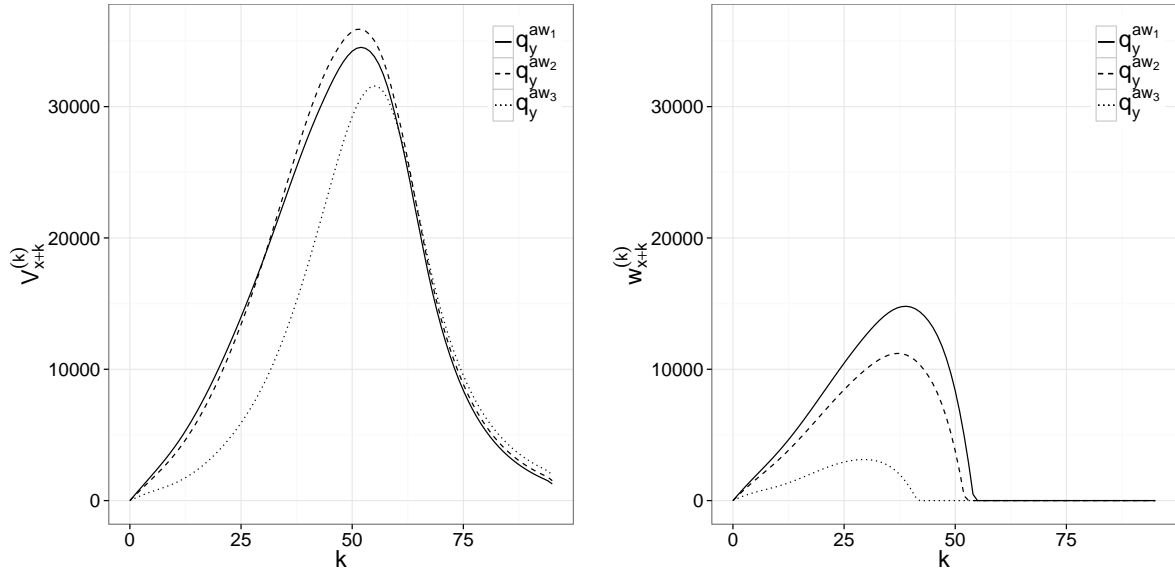


Figure 14: Updating mechanism 2: available reserves  $V_{x+k}^{(k)}$  (left) and surrender values  $w_{x+k}^{(k)}$  (right) for the different types of lapse probabilities  $q_y^{aw_1}$  (full line),  $q_y^{aw_2}$  (dashed line),  $q_y^{aw_3}$  (dotted line).

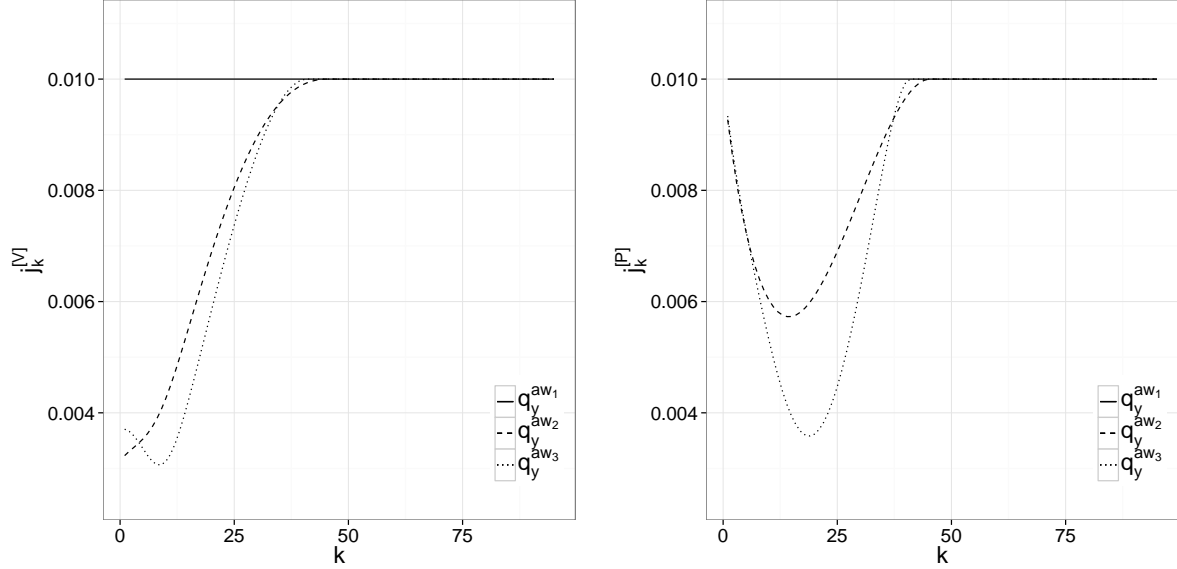


Figure 15: Yearly reserve increase  $j_k^{[V]}$  for mechanism 1 (*left*) and yearly premium increase  $j_k^{[P]}$  for updating mechanism 2 (*right*) for the different types of lapse probabilities  $q_y^{aw1}$  (*full line*),  $q_y^{aw2}$  (*dashed line*),  $q_y^{aw3}$  (*dotted line*).

## 7 Final discussion and conclusions

In this paper, we considered a lifelong health insurance contract with level premiums. In order to be able to determine premiums and reserves, a future medical inflation of  $f$  per year was assumed. The contract that we considered was such that a yearly update, based on the observed inflation in the past year, was possible. In order to maintain the actuarial equivalence from year to year, premiums and reserves were allowed to be adapted, according to a procedure specified in the policy. The other elements of the technical basis (interest, mortality and lapse rates) were assumed to be in line with the reality that unfolds over time, which implies that these elements do not give rise to a required update of the contract in order to maintain actuarial equilibrium. This simplifying assumption allowed us to isolate and investigate the effect of medical inflation on its own.

It is worth mentioning that our proposed approach easily extends to the case where other elements of the technical basis (mortality, interest, surrender) are modified during the term of the contract. Indeed, the available reserve (calculated retrospectively) and the required reserve (calculated prospectively) may well be computed with a totally different technical basis, differing from the one used for premium calculation. Consider for instance reserve-dependent surrender values as in Section 4, the accumulated reserve  $V_{x+k}^{(k-1)}$  at time  $k$  and the required reserve  $B_{x+k}^{(k)} + W_{x+k}^{(k)} - \pi^{(k)} \ddot{a}_{x+k}^{aa}$  may be obtained from two different sets of actuarial assumptions. Assuming that the basis used for calculating the required reserve continues to apply in the future, (39) still holds which shows that (40) remains valid even if other elements of the technical basis are revised, not only expected health benefits. The approach developed in the present paper is thus very general and allows

the actuary to deal with periodic revisions of the elements included in the technical basis during the coverage period.

The elements of the technical basis can even differ from year to year. In essence, the actuarial equilibrium methodology explained in this paper is only based on two fundamental relations. First, there is the relation

$$V_{x+k}^{(k-1)} = \left( V_{x+k-1}^{(k-1)} + \pi^{(k-1)} - b_{x+k-1}^{(k-1)} - q_{x+k-1}^{aw} v w_{x+k}^{(k-1)} \right) (vp_{x+k-1}^{aa})^{-1}, \quad (66)$$

which allows us to determine the available reserve at time  $k$  (before updating) from the cash flow movement in the past year. Second, there is the relation

$$V_{x+k}^{(k)} = B_{x+k}^{(k)} + W_{x+k}^{(k)} - \pi^{(k)} \ddot{a}_{x+k}^{aa} \quad (67)$$

which restores the actuarial equilibrium at time  $k$ , by resetting the premium level from  $\pi^{(k-1)}$  to  $\pi^{(k)}$ , and eventually also the available reserve from  $V_{x+k}^{(k-1)}$  to  $V_{x+k}^{(k)}$ .

The methodology explained in this paper can be used in very different situations, depending on the elements of the technical basis that are guaranteed and those elements that are subject to revision according to policy conditions. Any adverse departure from guaranteed components represents losses for the insurer that cannot be recovered by increasing premiums for the existing portfolio. On the contrary, premiums may be adapted in case no guarantee has been granted to the policyholder. Let us illustrate these two different situations on two extreme examples.

Consider first a health insurance contract where not any element of the technical basis is contractually guaranteed. In this case, the contract is similar to a pooling agreement. The contract is now updated at time  $k$ , starting from the recursion (66), but  $b_{x+k-1}^{(k-1)}$ ,  $q_{x+k-1}^{aw}$ ,  $v$  and  $p_{x+k-1}^{aa}$  can be considered as corresponding to observed values over the past year. Premium and reserves are then updated according to (67), for some appropriately chosen technical basis for the required reserve at time  $k$ , which may be different from the basis used to determine the available reserve.

The policy conditions may also guarantee the technical basis, except for the health benefits. In this case, the contractual reserve at time  $k$  is determined from (66), where the  $q_{x+k-1}^{aw}$ ,  $v$  and  $p_{x+k-1}^{aa}$  are those specified in the contract. In this case, it may happen that the accumulated assets for the contract differ from the contractual reserve  $V_{x+k}^{(k)}$ . If the guaranteed technical basis turns out to be conservative, accumulated assets exceed the contractual reserve, resulting in technical profits. The contract can then be updated according to (67), where  $V_{x+k}^{(k)}$  is the sum of  $V_{x+k}^{(k-1)}$  and the participating gain awarded at time  $k$ . This profit sharing mechanism has a reducing effect on the new premium level  $\pi^{(k)}$ .

Above we considered two extreme cases (no guarantees, and all elements of technical basis, except medical inflation, guaranteed). Of course, intermediate cases, where e.g. mortality and interest are guaranteed, but inflation and lapse rates are not, may be considered in the same framework.

This paper confines to deterministic inflation scenarios. The proposed approach nevertheless extends to stochastic medical inflation rates. Specifically, random departures from



the central medical inflation scenario adopted in initial premium calculation can be generated from a random walk with drift model, or another time series process. The resulting variations in premiums and reserves can then be computed according to the formulas derived in the present paper, so that the actuary has access to the whole distribution of these quantities and can compute credible intervals, for instance.

## References

- Baumann, F., Meier, V., Werding, M. (2008). Transferable ageing provisions in individual health insurance contracts. *German Economic Review* 9, 287-311.
- Christiansen, M., Eling, M., Schmidt, J.-P., Zirkelbach, L. (2015). Who is changing health insurance coverage? Empirical evidence on policyholder dynamics. *Journal of Risk and Insurance*, in press.
- Christiansen, M. C., Denuit, M. M., Dhaene, J. (2014). Reserve-dependent benefits and costs in life and health insurance contracts. *Insurance: Mathematics and Economics* 57, 132-137.
- Devolder, P., Denuit, M., Maréchal, X., Yerna, B.-L., Closon, J.-P., Léonard, C., Senn, A., Vinck, I. (2008). Setting a medical index for private health insurance contracts. *KCE Reports* 96.
- Dickson, D. C., Hardy, M. R., Waters, H. R. (2013). *Actuarial Mathematics for Life Contingent Risks*. Cambridge University Press.
- Giles, L. (1993). Life insurance applications of recursive formulas. *Journal of Actuarial Practice* 1, 141-151.
- Haberman, S., Pitacco, E (1998). *Actuarial models for disability insurance*. CRC Press.
- Hofmann, A., Browne, M. (2013). One-sided commitment in dynamic insurance contracts: Evidence from private health insurance in Germany. *Journal of Risk and Uncertainty* 46, 81-112.
- Kuo, W., Tsai, C., Chen, W.-K. (2003). An empirical study on the lapse rate: The cointegration approach. *Journal of Risk and Insurance* 70, 489-508.
- Milbrodt, H. (2005). *Aktuarielle Methoden der Deutschen privaten Krankenversicherung*. Verlag Versicherungswirtschaft.
- Pitacco, E. (2014). *Health insurance: Basic actuarial models*. Springer International Publishing.
- Vercruyse, W., Dhaene, J., Denuit, M., Pitacco, E., Antonio, K. (2013). Premium indexing in lifelong health insurance. *Far East Journal of Mathematical Sciences* 2013, 365-384.

**FACULTY OF ECONOMICS AND BUSINESS**  
Naamsestraat 69 bus 3500  
3000 LEUVEN, BELGIË  
tel. + 32 16 32 66 12  
fax + 32 16 32 67 91  
info@econ.kuleuven.be  
www.econ.kuleuven.be

