SOME COMMENTS ON QIS3

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1. Introduction

In this paper we discuss some of the issues proposed in QIS3. In particular, we comment on and discuss the "AISAM-ACME study on non-life long-tail liabilities; reserve risk and risk margin assessment under Solvency II". In the latter paper, the reserve risk calculation of non-life long-tail insurers is investigated based on a sample of 45 supervised insurance companies. In Section 2, we define the different risk measures used in a solvency environment. In Section 3, we show that the proposed Value-at-Risk measure is the solution of a general optimisation problem.

In Section 4, we confirm the findings in the AISAM-ACME study that a loading for solvency by 15% of the reserves might be too high. Because the basic idea of QIS3 is to find a VaR for determining a loading on the calculated reserve (best estimate), a probabilistic approach is needed and mechanical methods or parameter-free methods cannot give information about the tail of the distribution. Hence the remark in the AISAM-ACME study concerning the non-applicability of the methods is correct.

The one-year volatility concept is discussed in Section 5. In Section 6, the relationship between a long-term VaR and the corresponding short-term VaR is explored. In Section 7, we give some simple illustrations of the fact that a long-tail business should in many cases lead to a lower solvency capital requirement than a short-tail business with a comparable amount of liabilities. Section 8 concludes the paper.

2. Risk Measures

We define S as the sum of claims to be paid out over the reference period and the provisions to be set up at the end of the reference period, minus the sum of provisions available at the beginning of the reference period. The valuation principles on whose basis the value of the assets (represented by the available provisions, the premiums received and investment income generated) and, in particular, the liabilities (represented by the provisions to be set up and the claims to be paid out) is determined are left unspecified in this paper; our setup is compatible with any particular valuation basis.

A portfolio might run into problems in the case its loss S is positive. In this case, the obligations to the policyholders cannot be completely covered. Solvency reflects the financial capacity of a particular risky business to meet its contractual obligations. To protect policyholders from insolvency, the regulatory authority imposes a solvency capital requirement $\rho[S]$, which means that the available capital in the company has to be at least equal to $\rho[S]$. This capital can be employed when premiums and provisions, together with the investment income, turn out to be insufficient to cover the policyholders' claims. In principle, $\rho[S]$ will be chosen such that one can be "fairly sure" that the event "$S>\rho[S]$" will not occur.

The base probability measure could be the "physical probability measure", but could also be another (for example, subjective or risk-neutral) probability measure. Two well-known risk measures used for setting solvency capital requirements are the Value-at-Risk and the Tail-Value-at-Risk[1]. For a given probability level p, they are denoted by VaR_p (or Q_p) and by TVaR_p, respectively. They are defined by

$$\text{VaR}_p[S] = Q_p[S] = \inf\left\{x \mid P[S = x] = p\right\}, \quad 0 < p < 1, \tag{1}$$

and

$$\text{TVaR}_p\left[S\right] = \frac{1}{1-p}\int_p^1 Q_q\left[S\right] dq, \quad 0 < p < 1. \tag{2}$$

The shortfall of the portfolio with loss S and solvency capital requirement $\rho[S]$ is defined by:

$$\max\left(0, S-\rho\left[S\right]\right) \equiv \left(S-\rho\left[S\right]\right)_+ . \tag{3}$$

The shortfall can be interpreted as that part of the loss that cannot be covered by the insurer. It is also referred to as the *residual risk*, the *insolvency risk* or the *policyholders' deficit*.

As is well-known (see e.g. Dhaene et al. (2004)), $\text{TVaR}_p\,[S]$ can be expressed as a linear combination of the corresponding quantile and its expected shortfall:

$$\text{TVaR}_p\left[S\right]=\text{VaR}_p\left[S\right]+\frac{1}{1-p}\text{E}\left[\left(S-Q_p\left[S\right]\right)_+\right], \tag{4}$$

where the expectation is taken with respect to the base probability measure P.

The properties of risk measures have been investigated extensively; see e.g., Goovaerts et al. (1984) and Denuit et al. (2006).

The desirability of the subadditivity property of risk measures has been a major topic for research and discussion; also see Section 3 of this paper. As is well-known, Value-at-Risk in general does not satisfy the subadditivity property (although it does in various particular cases), whereas for any p the Tail-Value-at-Risk measure is subadditive.

In general, the properties that a risk measure should satisfy depend on the risk preferences in the economic environment under consideration.

3. Optimality of VaR

This section is based on the ideas set out in Dhaene et al. (2008). Consider a portfolio with future loss X. As explained above, the regulator wants the solvency capital requirement related to X to be sufficiently large so as to ensure that the shortfall risk is sufficiently small. We suppose that, to achieve this goal, the regulator introduces a risk measure for the shortfall risk, which we will denote by φ:

$$\varphi\left[\left(X-\rho\left[X\right]\right)_+\right]. \tag{5}$$

From equation (5), we see that two different risk measures are involved in the process of setting solvency capital requirements: the risk measure ϱ that determines the solvency capital requirement and the risk measure φ that measures the shortfall risk.

We will assume that φ satisfies the following condition:

$$\rho_1[\text{X}]\leq\rho_2[\text{X}]\Rightarrow\varphi\left[\left(X-\rho_1\left[X\right]\right)_+\right]\geq\varphi\left[\left(X-\rho_2\left[X\right]\right)_+\right], \tag{6}$$

which means that an increase of the solvency capital requirement implies a reduction of the shortfall risk as measured by φ. A sufficient condition for (6) to hold is that φ is monotonic.

[1] Of these two, Value-at-Risk is currently by far the most popular risk measure in practice among both

Assumption (6) implies that the larger the capital, the better from the viewpoint of minimising $\varphi\left[\left(X - \rho[X]\right)_+\right]$. The regulator wants $\varphi\left[\left(X - \rho[X]\right)_+\right]$ to be sufficiently small. However, holding capital $\varrho[X]$ involves a capital cost $\varrho[X]\ i$, where i denotes the required excess return on capital. To avoid imposing an excessive burden on the insurer, the regulator should take this capital cost into account. For a given risk X, a given risk measure φ and a given number ε, $0 < \varepsilon < 1$, we consider the cost function $C(X, \varrho[X])$ given by

$$C\left(X,\ \rho[X]\right) = \varphi\left[\left(X - \rho[X]\right)_+\right] + \rho[X]\ \varepsilon, \tag{7}$$

which takes into account the shortfall risk and the capital cost. For convenience, we suppress in the notation the dependence of C on φ and ε. Here, ε can be interpreted as a measure for the extent to which the capital cost is taken into account. The regulatory authority can decide to let ε be company-specific or risk-specific. The optimal capital requirement $\varrho[X]$ can now be determined as the smallest amount d that minimises the cost function $C(X, d)$. In the limiting case that $\varepsilon = 0$, the capital cost is not taken into account at all and an optimal solvency capital $\rho[X] = \inf\left\{d \mid \varphi\left[\left(X - d\right)_+\right] = 0\right\}$ arises. Here, we use the convention that $\inf\{\phi\} = \infty$.

Increasing the value of ε means that the regulator raises the relative importance of the cost of capital. This will result in a decrease of the optimal capital requirement.

In the remainder of this section we will use the expectation to measure the shortfall risk, hence $\varphi[X] = \mathrm{E}[X]$.

Clearly, the choice $\varphi[X] = \mathrm{E}[X]$ satisfies condition (6). In this case, the shortfall risk measure can be interpreted as the net stop-loss premium that has to be paid to reinsure the insolvency risk. We state the following result:

Theorem 1 *The smallest element in the set of minimisers to the cost function C(X, d) defined by*

$$C\left(X, d\right) = E\left[\left(X - d\right)_+\right] + d\varepsilon, \quad 0 < \varepsilon < 1, \tag{8}$$

is given by

$$\rho[X] = Q_{1-\varepsilon}[X]. \tag{9}$$

Proof: See Dhaene, Laeven, Vanduffel, Darkiewicz & Goovaerts (2008). ■

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regulators and risk managers; see, for example, Jorion (2001).

For values of $d \geq Q_{1-\varepsilon}[X]$, the marginal increase of the capital cost exceeds the marginal decrease of the expected shortfall. For values of $d \leq Q_{1-\varepsilon}[X]$, the opposite holds.

Remark 1 *From (4) it follows that the minimal value of the cost function in (8) can be expressed as:*

$$C\left(X, Q_{1-\varepsilon}[X]\right) = \mathrm{E}\left[\left(X - Q_{1-\varepsilon}[X]\right)_+\right] + Q_{1-\varepsilon}[X]\varepsilon = \varepsilon \mathrm{TVaR}_{1-\varepsilon}[X]. \qquad (10)$$

Theorem 1 provides a theoretical justification for the use of Value-at-Risk to set solvency capital requirements. Hence, to some extent the theorem supports the current regulatory regime for banking supervision established by the Basel Capital Accord and the Solvency II regulatory regime under construction. Indeed, both have put forward a Value-at-Risk-based capital requirement approach.

4. Estimation of volatility using IBNR reserves

4.1. Introduction

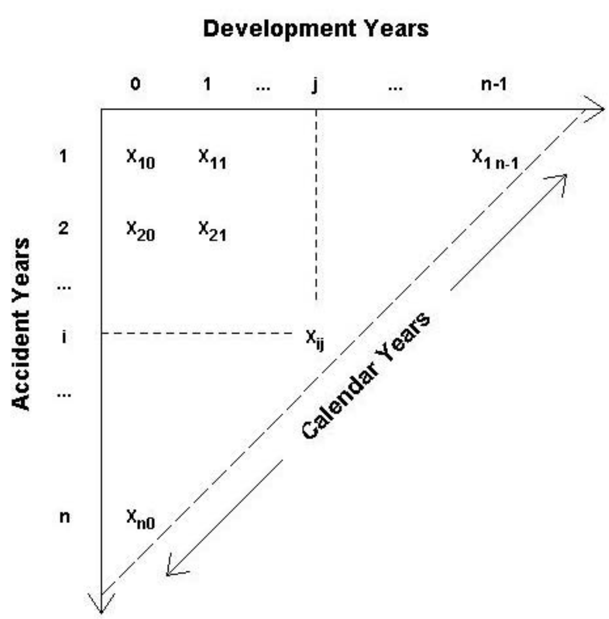

Figure 1: Run-off Triangle

An important problem in insurance, especially in the non-life, long-tail business, is to determine, at the end of an insurance period, how much provisions and how much capital should be set aside for claims already incurred but not reported yet (hence IBNR), or not fully paid. The past data used to construct estimates consist of numbers X_{ij}, where *i* is the risk year and j the development year, $j = 0,1,2,\ldots$ By the end of calendar year n, the known data are X_{ij} for $i \leq n$ and $j = 0,1,2,\ldots,n-i$. The purpose is to complete this run-off triangle to a square, and even to a rectangle if estimates are required pertaining to development years for which no data are recorded in the run-off triangle at hand. One method to do this is the traditional chain ladder method which can be described most easily as follows. Estimate the numbers α_i and β_j, denoting the total amount paid for risk year i and the fraction of it paid in development year j, respectively, in such a way that the recorded data X_{ij}, $i+j \leq n$ and their estimated values $\hat{\alpha}_i\hat{\beta}_j$ have the same row and column sums. Then the numbers $\hat{\alpha}_i\hat{\beta}_j$, $i+j > n$ are used to complete the square, and next, extrapolated values $\hat{\beta}_j$ serve as the basis for completing the required rectangle. A general treatment can be performed using GLIM-models (see Antonio (2007)).

As a particular case one finds the loglinear cross-classified claims reserving methods as described, for instance, in Redant & Goovaerts (2000). These models recognise that there are influences at work which tend to make claim sizes vary by year of origin as well as by year of payment. The (i, j)-element in the run-off triangle is modelled by

$$X_{ij} \approx \alpha_i \times \beta_j \times \gamma_{i+j}$$

The parameters α_i and β_j are as above; the additional parameter γ_{i+j} denotes the calendar year effect (combining the effects of monetary inflation and changing jurisprudence). Techniques for solving the statistical problem of estimating these parameters are widespread since it is a standard generalised linear model in the sense of Nelder and Wedderburn (1972). Many statistical programmes can compute maximum likelihood estimates under various assumptions about the stochastic nature of the observations, using a logarithmic link between the mean of the observations and the linear predictor $\log\left(\alpha_i \times \beta_j \times \gamma_{i+j}\right)$. For a description of the IBNR-model, giving the statistical development of a lognormal model along the three time dimensions (i, j and $i + j$) of the model, we refer to Doray (1996) and Goovaerts et al. (1990). In fact, we assume one has found estimators for the following multiplicative model for (non-cumulative) loss figures:

$$X_{ij} \approx \alpha_i \times \beta_j \times \gamma_{i+j} \times \varepsilon_{ij}, \quad i+j \leq n$$

Using lognormal error terms ε_{ij}, a linear model can be solved to obtain values for $\hat{\alpha}_i$, $\hat{\beta}_j$ and $\hat{\gamma}_{i+j}$.

4.2. Distribution function of reserve

In order to find the distribution of the provision for all accident years in the triangle or for one accident year or calendar year we have to find the distribution for a sum of risks whose (lognormal) marginal distribution have been determined from the previous section (the risks in this sum represent payments made on different times in the future). Suppose we want to determine the distribution of the total IBNR-reserve at the end of the last accident year, let us say at time $t_0 = 0$. Suppose also that the payment X_{ij} ($i + j > n$) is performed at time t_{i+j-n}. Define $t_i - t_j$ as the difference in years between time t_i and time t_j. In order to find the distribution for the discounted sum we have to specify a return for our assets. Let us define $r(0,t)$ and $s(0,t)$, respectively, as the mean yearly expected return and the mean yearly volatility on the return between time 0 and time *t*. The actual value X_0 of a stochastic payment X_t (depending on actuarial risk factors and not on the return) made on time *t* is determined as follows:

$$X_0 = X_t e^{-X(t)},$$

where

$$X(t) \sim N\left(\left(r(0,t) - \frac{s(0,t)}{2}\right)t, t\, s(0,t)\right)$$

The IBNR reserve R_n can now be written as follows:

$$R_n = \sum_{i+j>n} X_{ij} e^{-X(i+j-n)},$$

with $X_{ij} \sim LogN\left(\mu_{ij}, \sigma_{ij}^2\right)$ and the $X(t)$ are normally distributed as defined above.

Stated in a somewhat more straightforward way, we have to find a distribution for a sum of risks *V*, with *V* equal to

$$V = X_1 e^{-X(1)} + X_2 e^{-X(2)} + \ldots + X_n e^{-X(n)}$$

where X_i is the risk which belongs to a cell in the loss-development array. In the model the marginal distribution function F_i of X_i is lognormal. The return process $X(t)$ is normally distributed, as defined before.

Because the dependencies between the risks X_i cannot easily be measured and because there is a strong positive dependency between the risks $X(t)$ we will replace *V* by the comonotonic upper bound *W* (see Goovaerts, Dhaene and De Schepper (2000)):

$$W = \sum_{i=1}^{n} F_{X_i}^{-1}(U) F_{e^{-X(i)}}^{-1}(V)$$

where *U* en *V* are mutually independent uniformly distributed random variables.

This approximation has a distribution function which is, in the sense of convex order, an upper bound for the original distribution. Once we have found the comonotonic upper bound for the IBNR reserve we can determine all the characteristics of the distribution, including the Values-at-Risk and stop-loss premiums. For more information concerning the theoretical background, the reader is referred to Goovaerts, Dhaene, De Schepper (2000).

In the loglinear case the residuals of the regression model are estimated in each cell. It follows that on the relevant diagonal of the triangle the IBNR reserve can be expressed as a sum over loglinear, weighted random variables, where the induced variances $\sigma_{ij}{}^2$ are different for each (*i,j*).

For the one-year distribution, along the diagonal (as explained in the AISAM-ACME study on non-life, long-tail liabilities (2007)), one obtains a distribution of a sum of lognormal variates. The Value-at-Risk at level 1 - ε (denoted as $\mathrm{VaR}_{1-\varepsilon}$) of such a sum cannot be written as the sum of the separate Values-at-Risk:

$$\mathrm{VaR}_{1-e}\left(\sum_{i+j>n} X_{ij} e^{-X(i+j-1)} e^{\frac{-\sigma_{ij}{}^2}{2}}\right) \neq \sum_{i+j>n} \mathrm{VaR}_{1-\varepsilon}\left(X_{ij} e^{-X(i+j-1)} e^{\frac{-\sigma_{ij}{}^2}{2}}\right).$$

Denote

$$\mu = E\left[\sum_{i+j>n} X_{ij} e^{-X(i+j-1)} e^{\frac{-\sigma_{ij}{}^2}{2}}\right]$$

and

$$\sigma^2 = \mathrm{Var}\left[\sum_{i+j>n} X_{ij} e^{-X(i+j-1)} e^{\frac{-\sigma_{ij}{}^2}{2}}\right].$$

Using these notations we can rewrite the inequality as follows:

$$\mathrm{VaR}_{1-e}\left(\sum_{i+j>n} X_{ij} e^{-X(i+j-1)} e^{\frac{-\sigma_{ij}{}^2}{2}}\right) \neq \mathrm{VaR}_{1-e}\left(\mu e^{\frac{-\sigma_{ij}{}^2}{2}+\sigma\Phi^{-1}(U)}\right)$$

with U a uniformly distributed random variable on the unit interval. However, the right-hand side of the last inequality is exactly what the QIS3 report is setting up by introducing the unnecessarily complicated formula (4.258). In this case, no diversification of the risk is taken into account.

The $\mathrm{VaR}_{1-\varepsilon}(R_n)$ has to be determined by

$$\Pr\left(R_n \geq \mathrm{VaR}_{1-\varepsilon}(R_n)\right) = \varepsilon$$

Hence the distribution function of R_n has to be approximated or simulated. This is the argumentation used in order to confirm the findings in the AISAM-ACME study that the loading for solvency by 15% of the reserves might be too high. Because the basic idea of QIS3 is finding a VaR for determining a loading on the calculated reserve (best estimate), a probabilistic approach is needed and mechanical methods or parameter-free methods cannot give information about the tail of the distribution. Accordingly, the remark in the AISAM-ACME study concerning the non-applicability of the methods is correct. The basic solvency loading is applied to the reserves at the beginning of the year. One is using the best estimate but no definition is given for this best estimate.

In all practical situations one uses a safety loading in the calculation of a best estimate. A problem might arise when using the following formula:

$$R_n \times \frac{R_n - P_{n+1} - R_{n+1}}{R_n} = R_n \times (\mathit{solvency\ loading}),$$

using estimates for R_n, R_{n+1} and P_{n+1}. In case a company applies a different estimation procedure, and for example reduces its reserve by 10% this would still lead to the same solvency loading:

$$\frac{0.9R_n - 0.9P_{n+1} - 0.9R_{n+1}}{0.9R_n} = \frac{R_n - P_{n+1} - R_{n+1}}{R_n}$$

However, the additional solvency margin for reaching the one-year 99.5% level is reduced by 10%. Hence an adequate, more appropriate estimation of the reserve is needed containing a more realistic safety loading. In all actuarial practice this is realised. We will apply a $\mathrm{VaR}_{0.75}$ to define the best estimate.

4.3. Application to reserve risk

We can use the methodology described above to obtain an assessment of the reserve risk. Let R_n denote the total IBNR-reserve at current time 31.12.N. Let P_n denote the random amount representing the losses to be paid over the coming year or, in other words, the reserve for the (*n*+1)-th calendar year. As explained in the previous paragraph, we can determine the distribution function, and hence quantiles, of both R_n and P_n. For simplicity reasons, we will ignore the effect of interest rates and not use a return process in our examples by assuming that *r*(0,t) and *s*(0,t) are equal to zero.

Typically the regulator imposes a long-term provision requirement amounting to the 75% Value-at-Risk of the reserve R_n. We can assess the relative cost price of the one-year solvency requirement, with a probability of ruin of 0.5%, using the following formula:

$$\frac{\mathrm{VaR}_{99.5\%}\left[P_n\right]-\mathrm{VaR}_{75\%}\left[P_n\right]}{\mathrm{VaR}_{75\%}\left[R_n\right]}$$

The nominator is the difference between the amount of money needed to be able to cover all losses over the coming year with a probability of 99.5%, and the amount which would be set aside to cover these losses in case of the typical long-term solvency requirement.

Applying this to two example data sets leads to the following results:

	$\mathrm{VaR}_{99.5\%}\left[P_n\right]$	$\mathrm{VaR}_{75\%}\left[P_n\right]$	$\mathrm{VaR}_{75\%}\left[R_n\right]$	Relative Cost
Company 1	78,203	44,120	679,132	5.02%
Company 2	36,699	14,439	458,900	4.85%

5. The one-year volatility concept

Let R_n denote the reserve at current time 31.12.N. This amount is known, and hence deterministic. Let P_n denote the random amount representing the losses to be paid over the coming year [01.01.N+1, 31.12.N+1] and R_{n+1} the reserve to set up at 31.12.N+1. At current time the amount $P_n + R_{n+1}$ is unknown, and hence random.

Ignoring the effect of interest rates, the amount $P_n + R_{n+1}$ is the amount we will need at time 31.12.N+1. This amount can be expressed as follows:

$$R_n + R_n \frac{\left(P_n + R_{n+1}\right) - R_n}{R_n}$$

$$= R_n + R_n X = R_n \left(1 + X\right),$$

with X defined as the relative increase of the reserve over the coming year:

$$X = \frac{\left(P_n + R_{n+1}\right) - R_n}{R_n}.$$

To describe the reserve risk, we have to find an estimate of the volatility $\sigma\left[X\right]$ of this relative increase X.

In case the one-year solvency requirement is set as a 99.5% VaR of $R_n X$, we find that it is given by

$$\mathrm{VaR}_{99.5\%}\left[R_n X\right] = R_n \mathrm{VaR}_{99.5\%}\left[X\right].$$

For simplicity, let us assume that X is normally distributed. In this case we find:

$$\mathrm{VaR}_{99.5\%}\left[R_n X\right] = R_n \left(\mathrm{E}\left[X\right] + \sigma\left[X\right]\ \Phi^{-1}\left(0.995\right)\right).$$

Notice that E[X] and $\sigma\left[X\right]$ can be estimated from historical data.

It is important to note that on page 15 of the report, what is called "historical volatility" could be better called the "historical relative increase E[X] of the reserve". Notice that E[X] might be close to 0, or even negative in the case of a conservative setting of the reserves, whereas $\sigma\left[X\right]$ might be large.

This could explain the significant difference between the "volatility" estimated in the study and the "volatility" proposed in the QIS3 exercise.

6. The one-year VaR versus the run-off VaR

Since the study deals with long-term liabilities, it is important to draw a distinction between short-term and long-term certainty levels. Suppose a given insurer has liabilities over a period of 40 years. In order to calculate the Value-at-Risk over the entire run-off period of 40 years, one has to decide on an appropriate choice of the long-term certainty level p. To do this, these long-term certainty levels, which correspond to survival over the entire run-off period where liabilities are due (in our example 40 years), have to be "translated" into short-term, yearly probability levels.

The following approximate rule can be applied to calculate the yearly probability p_{yearly} associated with a long-term survival probability over *n* years p_n:

$$\left(p_{yearly}\right)^n = p_n$$

Using this formula, a safety level of 70% over a period of 40 years corresponds e.g. to a yearly certainty level of 99.11%. The yearly survival probabilities related to a range of different long-term certainty levels p are given below:

certainty level p_{40} over a 40-year period	yearly certainty level p_{yearly}
65%	98.929%
70%	99.112%
75%	99.283%
80%	99.444%
81.83%	99.500%
85%	99.595%
90%	99.737%
95%	99.872%

From these figures, we see for example that calculating the Value-at-Risk at 81.83%, taking into account the liabilities over the entire run-off period, corresponds to a yearly certainty level of 99.50% or, in other words, the typical short-term ruin probability of 0.5%.

7. Long-term versus short-term liabilities

We will compare two situations. Suppose in Situation 1 we have a single liability of 100 in one year. On the other hand, in Situation 2 we have a liability of 10 each year over the next 10 years. In other words, the total amount of liabilities is the same in both cases, but the horizon over which they are due differs. Suppose we can invest in assets with an expected yearly return of 10%, and an expected yearly volatility of 15%.

EXAMPLE 1

In this example we will compute for the two cases mentioned above the minimal required amount of assets to be able to fulfil the future liabilities, with a yearly ruin probability of 0.5%. Note that, as explained in the previous section, this yearly certainty level of 99.5% corresponds in Situation 2 to a certainty level over the run-off period of 10 years equal to $p_{10} = (99.5\%)^{10} = 95.11\%$. We get the following results:

	Required assets
Situation 1	134.7
Situation 2	101.5

From the Table we see that the required assets in Situation 1 are significantly higher than in Situation 2.

EXAMPLE 2

Now suppose we have an amount of 100 as available assets. In this example, we compute the survival probability in the two aforementioned situations: given the available assets of 100, we determine the probability that all future liabilities can be fulfilled. This leads to the following results:

	Survival probability	
	entire run-off	yearly
Situation 1	72%	72%
Situation 2	94%	99.38%

The results in the table show that the survival probability in the second situation is much higher than in the first.

These two examples provide simple but clear illustrations of the fact that a long-tail business should in many cases lead to a lower solvency capital requirement than a short-tail business with a comparable amount of liabilities. Example 1 shows that the long-tail business requires significantly fewer assets to fulfil future liabilities, while taking the same yearly risk. Example 2 shows that the certainty level that can be achieved for a given amount of available assets is much higher when we consider the long-tail business.

8. Conclusion

This paper considers the problem of determining appropriate solvency capital requirements to be set by a regulatory authority. We have shown that Value-at-Risk arises as the "most efficient" solvency capital requirement in an intuitive minimisation problem with a cost function that balances the expected shortfall and the capital cost.

From a theoretical point of view, we have argued that a probabilistic method has to be used for calculating the provisions within the framework of liability risks in a long-tail business. We stressed the importance of defining the best estimate for the provision by means of a probabilistic model to obtain a solvency loading for the next year, based on Value-at-Risk. In our empirical results we used a 75% VaR for calculating the provision on two confidential run-off triangles for professional liabilities. In two real life cases, we find as a result that approximately 5% calculated on the best estimate is realistic for these types of portfolios.

8. Acknowledgments

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