Updating mechanisms for lifelong health insurance contracts with reserve- or premium-based surrender values

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Abstract

For lifelong health insurance covers, medical inflation not sufficiently incorporated in the level premiums determined at policy issue requires an appropriate increase of these premiums, the corresponding reserves or both during the term of the contract. Such a premium or reserve update is necessary to maintain the actuarial equivalence between future health benefits and surrender values on the one hand, and available reserves and future premiums on the other hand. In Vercruysse et al. (2013) and Denuit et al. (2015), premium and reserve indexing mechanisms were proposed in a discrete-time framework where medical inflation is only taken into account ex-post as it emerges over time and where the reserves are not transferable in case of policy cancellation. In this paper, we extend this work by investigating the more general situation where a surrender value is paid out in case of policy cancellation. Reserve-based as well as premium-based surrender values are considered.

Key Words: medical expense insurance, lifelong contract, medical inflation index, withdrawal, surrender value.

1 Introduction

In this paper, we investigate private health insurance contracts with periodic premiums covering lifelong medical expenses. Examples of such contracts can be found in the Belgian market, see Devolder et al. (2008). At contract inception, the level premium is calculated so that the contract is actuarially fair, i.e. the actuarial value (or expected present value) of premiums over the contract duration is equal to the actuarial value of the health benefits and the possible surrender value paid out to the insured in case of policy cancellation. As medical expenses typically rise over the lifetime, a level premium contract generates premium surpluses in the early years which lead to asset accumulation in a reserve, while the shortfall of premiums in the later years is covered by these assets. As a result, the well-known hump-shape of the reserves becomes apparent. We refer to Dickson et al. (2013) (Chapter 7) for a discussion of reserves with a focus on life insurance products.

Unpredictable changes in prices for medical goods and services impact the health benefits that will be paid over the years for a lifelong health insurance policy. Given the long-term nature of health insurance contracts and the impossibility to predict or hedge against medical inflation, insurers are generally not able to properly account for this medical inflation in the calculation of the yearly premium level at policy issue. Therefore, these lifelong contracts are usually designed in such a way that the insurer is allowed to adapt the premium amounts at regular times (e.g. yearly) to account for medical inflation not taken into account at policy issue, based on some predefined medical inflation index. This practice is used in several EU member countries (for instance in Belgium and Germany, see Haberman and Pitacco (1999) and Milbrodt (2005)).

Vercruysse et al. (2013) and Denuit et al. (2015) consider the problem of premium indexing for lifelong health insurance contracts with non-transferable reserves. Non-transferability of the reserves means that the reserve is not paid out (neither fully nor partially) to the insured when he or she lapses the contract. Non-transferability of the reserves has a premium-reducing effect in case the insurer accounts for lapses in his premium calculations. This is often considered as controversial since lapse rates may depend on economic factors and may give rise to systematic risk. We refer the reader to Section 8.8 in Dickson et al. (2013) for a discussion. A drawback of the non-transferability is that it binds the insured to his insurer, especially at times when the reserve is relatively high. Hofmann and Browne (2013) provide empirical evidence of the lock-in consumers face when premiums are front–loaded.

Although non-transferability of reserves is actuarially fair (if it is appropriately taken into account in the premium calculation), consumers may feel this lack of liquidity of their contract as a serious drawback. Baumann et al. (2008) explore which part of the reserve can be transferred in case of surrender without imposing premium changes on the policyholders staying in the contract. These authors do not consider medical inflation in their study.

In case of a contract with transferable reserves, the surrender benefits should be clearly defined in the policy. In this paper, we introduce two possible definitions for the surrender value. The first definition is based on the reserve built up by the policyholder until the moment of surrender. The second definition takes the premiums paid up to surrender into account. We consider the problem of medical inflation in a context of private health insurance contracts with fully or partially transferable reserves by generalizing the results of Vercruysse et al. (2013). In this setting, several ways exist to restore the actuarial equivalence. The insurer can either increase premiums, or increase the reserve, or combine both approaches. In the first case it is the insured who carries the burden of increased costs due to medical inflation, in the second case it is the insurer, while in the third case they share the burden. This general approach is demonstrated in numerical examples, based on Belgian data.

Modeling and choosing appropriate lapse rates is a delicate issue. Kuo et al. (2003) explore the impact of unemployment and interest rate on lapse rates. Hofmann and Browne (2013) show that policyholders generally lapse less in case of higher premium front-loading and Christiansen et al. (2014a) find that premium development, premium adjustment frequency and the sales channel impact lapse rates. In the present work, we investigate the influence of the choice of the lapse rates on the numerical results by means of a sensitivity analysis.

The remainder of this paper is organized as follows. In Section 2, we describe the lifelong health insurance contract under study. In Section 3, we extend the framework of Vercruysse et al. (2013) to take into account (partially) transferable reserves. We describe how contracts may be adapted over time to take unanticipated medical inflation into account. In Section 4, we consider the special cases of reserve- and premium-dependent surrender values, respectively. Section 5 discusses detailed numerical examples. Section 6 concludes the paper.

2 The lifelong health insurance contract

2.1 Health benefits and surrender values

The origin of time is chosen at policy issue. Time t stands for the seniority of the policy (i.e. the time elapsed since policy issue). The policyholder's (integer) age at policy issue is denoted by x, so that upon survival at time k, he or she has reached age $x+k$. We denote the ultimate integer age by ω, assumed to be finite. This means that survival until integer age ω has a positive probability, whereas survival until integer age $\omega+1$ has probability zero.

The superscript "(0)" will be used to denote quantities estimated or known at policy issue (i.e. time 0). The average health-related benefit to be paid out in the year $(k, k+1)$, $k \in \{0, 1, \ldots, \omega - x\}$, is denoted by $b^{(0)}_{x+k}$. We assume that health-related benefits are paid at the beginning of the year, which is a convenient and conservative assumption in our context. Furthermore, in case the policyholder cancels the contract in year $(k, k+1)$, the surrender value $w^{(0)}_{x+k+1}$ is paid out at the end of the year. We set $w^{(0)}_{\omega+1} = 0$, which means that the surrender value in the last possible year of survival is equal to zero. In Sections 4.1 and 4.2, we suggest two possible ways of defining the surrender benefits in the policy. The first one expresses surrender values in terms of a linear function of the available reserve (also referred to as *retrospective reserve*) of the contract at the moment of surrender, whereas the second one defines the surrender value as a fraction of the accumulated value of the premiums paid so far.

The health benefits that will be paid over the years are subject to medical inflation. Medical inflation, as we define it, is assumed to account for the full increase of medical costs, not only the increase of these medical costs above the inflation taken into account by the usual consumer price index. We suppose that medical inflation is unpredictable and hence, at policy issue, an assumption has to be made about this inflation. Here, we assume that the actuary includes a future medical inflation of f per year in the premium calculation. This means that

$$b^{(0)}_{x+j} = \overline{b}^{(0)}_{x+j} \times (1+f)^j \,, \tag{1}$$

where $\bar{b}^{(0)}_{x+j}$ is the average health benefit paid out to an insured aged $(x+j)$ in year $(0,1)$. We assume that appropriate estimates for the values $\bar{b}^{(0)}_{x+j}$ are available at time 0. Furthermore, $w^{(0)}_{x+k+1}$ is the payment in case of surrender at time $k+1$, under the assumption of a future medical inflation of f per year. Analogously, the level premium $\pi^{(0)}$ determined at policy issue incorporates a medical inflation of f per year. In the context of temporary health covers, f is often set equal to 0. Due to the long term nature of the contracts considered in this paper, we consider it appropriate to work with a more realistic estimate $f > 0$ for medical inflation.

Obviously, observed medical inflation may and will depart from the assumed f. Therefore, the premium level and the available (i.e. retrospective) reserve should be rebalanced every year according to the observed medical inflation, in order to restore the actuarial equivalence between available reserve and future premiums on the one hand, and health benefits and future surrender values paid by the insurer on the other hand. This yearly rebalancing process gives rise to a sequence of yearly premiums $\pi^{(k)}$, $k = 0, 1, \ldots$, where the superscript "(k)" now denotes the updated values based on actual inflation observed up to and including time k, whereas future inflation is assumed to be f per year. Our contract assumptions stipulate that premiums, reserves and possibly also surrender values may be updated (on a yearly basis) according to a well-defined procedure, in order to restore the broken actuarial equivalence. The updated value for the health benefits at time $k \in \{0, 1, \ldots, \omega - x\}$ based on information available up to time k, will be denoted by $b^{(k)}_{x+k+j}$, $j = 0, 1, \ldots, \omega - x - k$. Hence,

$$b^{(k)}_{x+k+j} = \bar{b}^{(k)}_{x+k+j} \times (1+f)^j \,,$$

where $\bar{b}^{(k)}_{x+k+j}$ is the average health benefit to be paid out to an insured aged $(x+k+j)$ in year $(k, k+1)$. Appropriate estimates for the values $\bar{b}^{(k)}_{x+k+j}$ are available at time k. Furthermore, $w^{(k)}_{x+k+j+1}$ stands for the time k (hence the "(k)" superscript) updated value of the surrender value that could be paid at time $k+j+1$, taking into account the observed inflation until time k and an assumed inflation of f per year beyond that time.

The new series of values $b^{(k)}_{x+k+j}$ and $w^{(k)}_{x+k+j+1}$, available at time k, lead to updated premiums, reserves and surrender values and give rise to $\pi^{(k)}$, the new yearly premium to be paid from time k on. Throughout this paper, we set $w^{(k)}_{\omega+1} = 0$ in line with the assumption of ultimate age ω. The procedure of the yearly updating is considered in detail in Section 3.

Apart from the assumed medical inflation, other elements of the technical basis (interest, mortality and lapse rates) are in line with the reality that unfolds over time. As such, these elements do not require a yearly update in order to maintain actuarial equivalence. It allows us to isolate and investigate the effect of medical inflation on its own. However, the methodology proposed hereafter can easily be adjusted to take into account deviations of interest, mortality and lapse rates from the ones assumed in the technical basis. This issue will be discussed in Section 3. As a final comment, let us stress that we do not revise the assumed inflation f during the coverage period. This revision is achieved ex-post by the proposed indexing mechanism.

2.2 Discrete–time double decrement model

We describe the lifelong health insurance policy introduced in Section 2.1 as a two-decrement Markov model, with states "active" (i.e. policy in force), "withdrawn" (i.e. policy has been

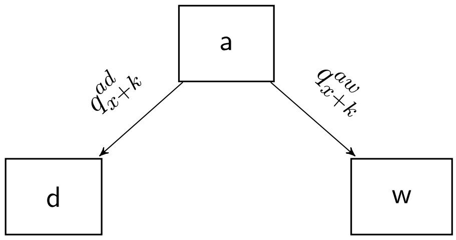

Figure 1: The 2-decrement model.

cancelled) and "dead", abbreviated as "a", "w" and "d", respectively. Figure 1 gives a graphical representation. X_k denotes the status of the contract at time k, starting from $X_0 = a$. The stochastic process $\{X_k,\ \ k = 0, 1, 2, \ldots\}$ describes the states occupied over time.

For j and $k \in \{0, 1, 2, \ldots\}$, we define the sojourn (or non–exit) probability ${}_jp^{aa}_{x+k}$ as

$$ {}_jp^{aa}_{x+k} = \Pr[X_{k+j} = a | X_k = a]. \tag{2} $$

In words, (2) is the probability that a policy in force at age $x + k$ is still in force j years later. The probability that a policy in force at age $x + k$ has ceased j years later (due to death or surrender), is denoted by ${}_jq^{aa}_{x+k}$. This "exit" probability can be expressed as

$$ {}_jq^{aa}_{x+k} = \Pr[X_{k+j} \neq a | X_k = a] = 1 - {}_jp^{aa}_{x+k}. \tag{3} $$

We also introduce the probabilities ${}_jq^{ad}_{x+k}$ and ${}_jq^{aw}_{x+k}$, defined as

$$ {}_jq^{ad}_{x+k} = \Pr[X_{k+j} = d | X_k = a] \text{ and } {}_jq^{aw}_{x+k} = \Pr[X_{k+j} = w | X_k = a]. \tag{4} $$

These are the probabilities of leaving the portfolio due to respectively death and surrender between ages $x + k$ and $x + k + j$.

The following relations are well known:

$$ {}_jp^{aa}_{x+k} + {}_jq^{ad}_{x+k} + {}_jq^{aw}_{x+k} = 1 \tag{5} $$

and

$$ {}_{j+1}p^{aa}_{x+k} = \prod_{l=0}^{j} p^{aa}_{x+k+l}. \tag{6} $$

In accordance with standard actuarial notation, we omit the left subindex when it is equal to unity, e.g. ${}_1p^{aa}_{x+k} = p^{aa}_{x+k}$. The ultimate integer age ω is such that $p^{aa}_{\omega-1} > 0$, while $p^{aa}_{\omega} = 0$.

2.3 Actuarial values, premiums, reserves and the equivalence principle

In this paper, we assume a constant yearly technical interest rate i, and denote the corresponding annual discount factor $(1 + i)^{-1}$ by v. Premiums are paid at the beginning of the year, as long as the policy is in force. Let

$$ \ddot{a}^{aa}_x = \sum_{l=0}^{\omega-x} {}_lp^{aa}_x \, v^l \tag{7} $$

be the actuarial value at policy issue of an annuity-due paying an amount of 1 per year to an insured aged (x) as long as the health insurance contract is in force. Furthermore, let

$$B_x^{(0)} = \sum_{j=0}^{\omega-x} {}_jp_x^{aa} \; v^j \; b_{x+j}^{(0)} \tag{8}$$

be the actuarial value at time 0 of all health-related benefits, and let

$$W_x^{(0)} = \sum_{j=0}^{\omega-x} {}_jp_x^{aa} \; q_{x+j}^{aw} \; v^{j+1} \; w_{x+j+1}^{(0)} \tag{9}$$

be the actuarial value at time 0 of the surrender value.

At policy issue, the level premium $\pi^{(0)}$ for a policyholder aged x is then determined from the equivalence principle:

$$\pi^{(0)} \; \ddot{a}_x^{aa} = B_x^{(0)} + W_x^{(0)}. \tag{10}$$

As outlined in Section 2.1, the superscript "(0)" indicates that the calculation is based on the information available at policy issue (i.e. at time 0), taking into account a deterministic medical inflation of f per year.

Solving equation (10) for $\pi^{(0)}$ leads to the level premium to be paid yearly in advance. It is important to notice that the equivalence relation (10) does not always provide an explicit expression for this premium. This is the case for instance when the surrender value, and hence also $W_x^{(0)}$, is defined in terms of the available reserve or in terms of the premiums paid so far. In Sections 4.1 and 4.2, we consider reserve-dependent and premium-dependent surrender values, respectively. We present a methodology that leads in both cases to an explicit expression for $\pi^{(0)}$.

3 Updating the health insurance contract

This section explains the general updating mechanism for the lifelong health insurance contract as described in Section 2.

3.1 Actuarial values

Before explaining the mechanism of yearly updating the health insurance contract, we generalize the notation introduced in Section 2.3. For $k \in \{0, 1, 2, \dots, \omega - x\}$ and $j \in \{0, 1, 2, \dots, \omega - x - k\}$, let

$$B_{x+k+j}^{(k)} = \sum_{l=0}^{\omega-x-k-j} {}_lp_{x+k+j}^{aa} \; v^l \; b_{x+k+j+l}^{(k)} \tag{11}$$

be the actuarial value at time $k + j$ of the health benefits to be paid at time $k + j$ and beyond for a policy still in force at time $k + j$. Similarly, let

$$W_{x+k+j}^{(k)} = \sum_{l=0}^{\omega-x-k-j} {}_lp_{x+k+j}^{aa} \; q_{x+k+j+l}^{aw} \; v^{l+1} \; w_{x+k+j+l+1}^{(k)} \tag{12}$$

be the actuarial value at time $k+j$ of the surrender benefits for a policy still in force at time $k+j$. As outlined in Section 2, (11) and (12) use the superscript "(k)" and hence take into account the observed medical inflation until time k, while a constant yearly medical inflation of f is assumed from time k on.

3.2 Updating mechanism: two fundamental relations

In our lifelong health insurance contract the deviation between the observed and the assumed medical inflation is taken into account ex-post as it emerges over time, by adapting the premium, the surrender values and the available reserve from year to year.

The proposed updating mechanism is based on recursive formulas for the reserve that are well known in the actuarial literature, in the context of life insurance, see Dickson et al. (2013) (Chapter 7). We extend these formulas to the setting of lifelong health insurance contracts with surrender values, and develop an actuarial equivalence methodology based on these relations. More specifically, we use two fundamental recursions (see further) and restore the equivalence by bringing the available reserve (i.e. the available assets or the *retrospective reserve*) to the level of the required reserve (i.e. the actuarial liabilities taking into account future premiums or *prospective reserve*). We denote the available reserve by the capital letter V and the required reserve by $\vec{V}$. On the one hand, there is the relation

$$V_{x+k}^{(k-1)} = \left(V_{x+k-1}^{(k-1)} + \pi^{(k-1)} - b_{x+k-1}^{(k-1)} - q_{x+k-1}^{aw}\ v\ w_{x+k}^{(k-1)}\right)\left(v p_{x+k-1}^{aa}\right)^{-1}, \tag{13}$$

which allows us to determine the available reserve at time k (before updating) from the available reserve $V_{x+k-1}^{(k-1)}$ at time $k-1$ and cash in- and outflows in the past year. On the other hand, we can express the required reserve at time k as the expected present value of the benefits minus the expected present value of the premiums:

$$\vec{V}_{x+k}^{(k)} = B_{x+k}^{(k)} + W_{x+k}^{(k)} - \pi^{(k)}\ \ddot{a}_{x+k}^{aa}. \tag{14}$$

If the actuarial assumptions in the technical basis are fulfilled over time, the available (or retrospective) reserve $V_{x+k}^{(k-1)}$ and the required (or prospective) reserve $\vec{V}_{x+k}^{(k)}$ are known to coincide. When reality departs from the assumptions made in the technical basis, which may be the case for the benefits and surrender values in our scenario, the available reserve $V_{x+k}^{(k-1)}$ may differ from the required reserve $\vec{V}_{x+k}^{(k)}$ at time k. This breaks the actuarial equivalence. We restore this equivalence by updating the available reserve to the level of the required reserve: $V_{x+k}^{(k)} = \vec{V}_{x+k}^{(k)}$ or

$$V_{x+k}^{(k)} = B_{x+k}^{(k)} + W_{x+k}^{(k)} - \pi^{(k)}\ \ddot{a}_{x+k}^{aa}. \tag{15}$$

This updated version of the available reserve, $V_{x+k}^{(k)}$, depends on the updated premium $\pi^{(k)}$. In turn, we use it to determine the available reserve at time $k+1$ through the recursive scheme (13).

3.3 Updating mechanism: a detailed explanation

Suppose that the policy is still in force at time $k \in \{1, 2, 3, \ldots, \omega - x\}$. Reevaluations up to time $k-1$ have led to the updated values $b_{x+k+j}^{(k-1)}$ and $w_{x+k+j+1}^{(k-1)}$, $j \in \{0, 1, 2, \ldots, \omega - x - k\}$, for the health benefits and the surrender values, respectively. Here, $b_{x+k+j}^{(k-1)}$ and $w_{x+k+j+1}^{(k-1)}$ are based on the observed medical inflation until time $k-1$, while assuming a medical inflation of f per year from time $k-1$ on. Furthermore, premiums have been adapted from year to year and have reached level $\pi^{(k-1)}$ at time $k-1$.

We assume that at each time $1, 2, \ldots, k-1$, the available reserve, the surrender benefits and the premium have been reset in such a way that the available reserve and the required reserve are equal. In particular this means that at time $k-1$ we restore the actuarial equivalence $V_{x+k-1}^{(k-1)} = \overrightarrow{V}_{x+k-1}^{(k-1)}$ where

$$\overrightarrow{V}_{x+k-1}^{(k-1)} = B_{x+k-1}^{(k-1)} + W_{x+k-1}^{(k-1)} - \pi^{(k-1)} \, \ddot{a}_{x+k-1}^{aa}, \tag{16}$$

with $B_{x+k-1}^{(k-1)}$ and $W_{x+k-1}^{(k-1)}$ defined according to (11) and (12), respectively. The right-hand side in (16) is the actuarial value at time $k-1$ of the future liabilities of the contract under consideration, based on the information available at that time. Splitting the payments related to year $(k-1, k)$ from the other payments and taking into account (6), we obtain the recursive equation for the required reserve:

$$\begin{aligned} \overrightarrow{V}_{x+k-1}^{(k-1)} &= b_{x+k-1}^{(k-1)} + q_{x+k-1}^{aw} \, v \, w_{x+k}^{(k-1)} - \pi^{(k-1)} \\ &\quad + p_{x+k-1}^{aa} \, v \left(B_{x+k}^{(k-1)} + W_{x+k}^{(k-1)} - \pi^{(k-1)} \ddot{a}_{x+k}^{aa} \right). \end{aligned} \tag{17}$$

Similar recursive expressions are well known in the context of life insurance, see Dickson et al. (2013) for example. Having arrived at time k, the available reserve for a policy still in force at age $x+k$, taking into account all information up to time $k-1$, $V_{x+k}^{(k-1)}$, is given by (13). Combining this equation with the restored actuarial equivalence and (17) we find the following expressions for the available reserve $V_{x+k}^{(k-1)}$, prospectively:

$$V_{x+k}^{(k-1)} = B_{x+k}^{(k-1)} + W_{x+k}^{(k-1)} - \pi^{(k-1)} \, \ddot{a}_{x+k}^{aa}. \tag{18}$$

Thus, the available reserve (left hand side) and the required reserve (right hand side) at time k are equal, provided the technical basis that was used at time $k-1$ is still adopted at time k. This result is known to hold in general, see e.g. Dickson et al. (2013).

Suppose now that medical inflation during year $(k-1, k)$ was such that each future health benefit $b_{x+k+j}^{(k-1)}$, $j \in \{0, 1, \ldots, \omega - x - k\}$ determined at time $k-1$ has to be replaced by the corresponding adapted health benefit $b_{x+k+j}^{(k)}$, determined at time k. The latter assumes a future yearly medical inflation f, while taking into account observed medical inflation up to time k. In particular, due to medical inflation, the actuarial value of future health benefits $B_{x+k}^{(k-1)}$, which is based on observed medical inflation until time $k-1$, has to be replaced by $B_{x+k}^{(k)}$, which is based on observed medical inflation until time k, see (11). Due to this change in the health benefits, the actuarial equivalence is broken at time k, in the sense that the available reserve $V_{x+k}^{(k-1)}$ is different from the actuarial value of future liabilities (i.e. the required reserve) at that time.

3.4 Restoring the actuarial equivalence: premium updates, reserve updates or shared burden

At time k, we update the premium level $\pi^{(k-1)}$ and the available reserve $V_{x+k}^{(k-1)}$ to $\pi^{(k)}$ and $V_{x+k}^{(k)}$, respectively. As such, the actuarial equivalence $V_{x+k-1}^{(k-1)} = \overrightarrow{V}_{x+k-1}^{(k-1)}$ is restored and we go from (18) to (15). Note, however, that equality (15) may be obtained in many ways, in the sense that an infinite number of pairs $\left(V_{x+k}^{(k)},\ \pi^{(k)}\right)$ satisfy relation (15).

From time k on, the level premium $\pi^{(k-1)}$ that was determined at time $k-1$, is replaced by the updated level premium $\pi^{(k)}$. These premium updates $\pi^{(k)} - \pi^{(k-1)}$ are financed by the policyholder. Also, at time k, the available reserve $V_{x+k}^{(k-1)}$ is updated to $V_{x+k}^{(k)}$. Obviously, this reserve update is financed by the insurer. The reserve update depends on the updated premium level and vice versa through (13). When $\pi^{(k)} = \pi^{(k-1)}$ the insurer caries all the effects of medical inflation, whereas $V_{x+k}^{(k)} = V_{x+k}^{(k-1)}$ lays all the uncertainty with the policyholder. Both parties can also share the burden. An increase in the available reserve can be considered an additional benefit and as such could justify a premium increase for the policyholder, i.e. $\pi^{(k)} > \pi^{(k-1)}$. In practice, a reserve increase may be financed by technical gains on interest, mortality and surrenders. For example, technical gains can realize in case interest rates obtained are higher than the ones assumed in the technical basis. Mortality gains may originate in case actual mortality is higher than mortality assumed in the technical basis or increasing longevity may postpone late-life costs to higher ages, so caution is needed when setting mortality assumptions. Although not considered in this paper, technical gains can arise also when actual expenses are lower than the expenses assumed for the premium calculation. In our general setting, the former surrender values $w_{x+k+j}^{(k-1)}$ are replaced by revised values $w_{x+k+j}^{(k)}$, based on the information about medical inflation until time k. A corresponding increase $W_{x+k}^{(k)} - W_{x+k}^{(k-1)}$ is financed by the insurer (via a reserve increase) and/or the policyholder (via increased premiums).

Let us briefly discuss two extreme cases where the effect of inflation is entirely borne by one of the agents, either the insurer or the policyholder.

Example 1. *When the premium is kept unchanged, i.e. when $\pi^{(k)} = \pi^{(k-1)}$, we find from (18) and (15) that*

$$V_{x+k}^{(k)} - V_{x+k}^{(k-1)} = \left(B_{x+k}^{(k)} - B_{x+k}^{(k-1)}\right) + \left(W_{x+k}^{(k)} - W_{x+k}^{(k-1)}\right), \tag{19}$$

which means that the health benefit and surrender payment increases are completely financed by the insurer via an increase of the available assets.

Example 2. *When the insurer does not increase the available reserve, i.e. when $V_{x+k}^{(k)} = V_{x+k}^{(k-1)}$, the health benefit and surrender payment increases are completely financed by the policyholder via increased premium payments. In this special case we find from (18) and (15) that*

$$\pi^{(k)} - \pi^{(k-1)} = \frac{\left(B_{x+k}^{(k)} - B_{x+k}^{(k-1)}\right) + \left(W_{x+k}^{(k)} - W_{x+k}^{(k-1)}\right)}{\ddot{a}_{x+k}^{aa}}. \tag{20}$$

This means that the premium increase $\pi^{(k)} - \pi^{(k-1)}$ introduced at time k can be interpreted as the level premium for an insurance contract with yearly benefits equal to the health benefit increases and with surrender values equal to the surrender value increases of the original contract.

3.5 Generalizing the recursions: from 1 to j-step

We give additional insights on the reserve calculations in this paragraph by generalizing some of the one-step recursions developed in Sections 3.2 and 3.3. This will be useful in the next section. Starting from the available reserve $V_{x+k}^{(k)}$ at time k, we introduce the notation $V_{x+k+j+1}^{(k)}$ for the available reserve of the contract at time $k+j+1$ where $j = 1, 2, \ldots, \omega - x - k - 1$, in case of a future yearly medical inflation of f in the interval $(k, k+j+1)$ as predicted at time k. Assuming the technical basis at time k, the available reserve follows from the forward recursion

$$V_{x+k+j+1}^{(k)} = \left(V_{x+k+j}^{(k)} + \pi^{(k)} - b_{x+k+j}^{(k)} - q_{x+k+j}^{aw} \; v \; w_{x+k+j+1}^{(k)}\right) \left(p_{x+k+j}^{aa} \; v\right)^{-1}, \tag{21}$$

or equivalently,

$$\begin{aligned} v \; V_{x+k+j+1}^{(k)} &= V_{x+k+j}^{(k)} + \pi^{(k)} - b_{x+k+j}^{(k)} + q_{x+k+j}^{ad} \; v \; V_{x+k+j+1}^{(k)} \\ &\quad - q_{x+k+j}^{aw} v \left(w_{x+k+j+1}^{(k)} - V_{x+k+j+1}^{(k)}\right), \end{aligned} \tag{22}$$

which holds for $j \in \{0, 1, \ldots, \omega - x - k - 1\}$. The initial value $V_{x+k}^{(k)}$ is given by (15), where $V_x^{(0)} = 0$. Hereby, $V_{x+k+j}^{(k)}$ is an expresses the available reserve per policy in force at time $k+j$, based on the information available and the technical assumptions used at time k.

It is easy to verify that the solution of recursion (21), with initial value $V_{x+k}^{(k)}$ can be expressed in the following retrospective form:

$$\begin{aligned} V_{x+k+j}^{(k)} &= V_{x+k}^{(k)} \left({}_j p_{x+k}^{aa} \; v^j\right)^{-1} \\ &\quad + \sum_{l=0}^{j-1} \left(\pi^{(k)} - b_{x+k+l}^{(k)} - q_{x+k+l}^{aw} \; v \; w_{x+k+l+1}^{(k)}\right) \left({}_{j-l} p_{x+k+l}^{aa} \; v^{j-l}\right)^{-1}, \end{aligned} \tag{23}$$

for $j \in \{0, 1, \ldots, \omega - x - k\}$. Taking into account the restored actuarial equivalence (15), the reserves $V_{x+k+j}^{(k)}$ can also be expressed prospectively:

$$V_{x+k+j}^{(k)} = B_{x+k+j}^{(k)} + W_{x+k+j}^{(k)} - \pi^{(k)} \ddot{a}_{x+k+j}^{aa}, \tag{24}$$

with $\ddot{a}_{x+k+j}^{aa}$, $B_{x+k+j}^{(k)}$ and $W_{x+k+j}^{(k)}$ defined in (7), (11) and (12), respectively. In particular, we find that

$$V_\omega^{(k)} = b_\omega^{(k)} - \pi^{(k)}, \tag{25}$$

which means that the available reserve at the last attainable integer age is equal to the health benefit minus the premium to be be paid at that time.

3.6 Discussion

Our technical basis includes a constant yearly technical interest rate i and a constant yearly medical inflation f. However, all results can be generalized to include varying deterministic technical interest rates, i.e. replacing i with a sequence $i_1, i_2, \ldots$, and varying deterministic yearly medical inflation rates $f_1, f_2, \ldots$.

The methodology explained in this paper can be used in very different situations, depending on the elements of the technical basis that are guaranteed and those elements that are subject to revision according to policy conditions. Any adverse departure from guaranteed components represents losses for the insurer that cannot be recovered by increasing premiums for the existing portfolio. On the contrary, premiums may be adapted in case no guarantee has been granted to the policyholder. Let us illustrate these two different situations with two extreme examples.

Consider first a health insurance contract where no element of the technical basis is contractually guaranteed. In this case, the contract is similar to a pooling agreement. The contract is now updated at time k, starting from the recursion (13), but $b^{(k-1)}_{x+k-1}$, q^{aw}_{x+k-1}, v and p^{aa}_{x+k-1} correspond to observed values over the past year. Premium and reserves are then updated according to (15), for some appropriately chosen technical basis for the required reserve at time k.

Second, consider the situation where the policy guarantees the technical basis, except for the health benefits. In this case, the contractual reserve at time k is determined from (13), where the q^{aw}_{x+k-1}, v and p^{aa}_{x+k-1} are those specified in the contract. In this case, it may happen that the accumulated assets for the contract differ from the contractual reserve $V^{(k-1)}_{x+k}$. If the guaranteed technical basis turns out to be conservative, accumulated assets exceed the contractual reserve, resulting in technical profits. The contract can then be updated according to (15), where $V^{(k)}_{x+k}$ is the sum of $V^{(k-1)}_{x+k}$ and the participating gain awarded at time k. This profit sharing mechanism has a reducing effect on the new premium level $\pi^{(k)}$.

Above we considered two extreme cases (no guarantees, and all elements of technical basis, except medical inflation, guaranteed). Of course, intermediate cases, where e.g. mortality and interest are guaranteed, but inflation and lapse rates are not, may be considered in the same framework.

4 Defining the surrender values

This section introduces and investigates two possible ways of defining the surrender values in the lifelong health insurance contract. For both scenarios we calculate the initial premium and show explicitly how to solve the actuarial equivalence.

4.1 Reserve-dependent surrender values

We consider the case where upon surrender in year $(k, k+1)$, the surrender value that is paid out at time $k+1$ is a linear function of the available reserve:

$$w^{(k)}_{x+k+1} = (1-\beta_{k+1})\, V^{(k)}_{x+k+1} - \alpha_{k+1}, \qquad k = 0, 1, 2, \ldots, \omega - x - 1, \tag{26}$$

with $w^{(0)}_{\omega+1}$ equal to zero and where $\alpha_{k+1} \geq 0$ is a reserve-independent penalty and $0 < \beta_{k+1} \leq 1$ is the non-transferred percentage of the available reserve in case of policy cancellation. The quantities α_{k+1} and β_{k+1} are fixed at policy issue. Benefits of the form (26) have been studied in a continuous-time setting by Christiansen et al. (2014b), without allowance for medical inflation. The reserve $V^{(k)}_{x+k+1}$ is a function of $\pi^{(0)}, \pi^{(1)}, \ldots, \pi^{(k)}$. Therefore, at contract initiation, this reserve and hence, the surrender value $w^{(k)}_{x+k+1}$ is in general unknown. However, when the surrender option is exercised in year $(k, k+1)$, the reserve $V^{(k)}_{x+k+1}$ and the surrender value are known at time $k+1$, see (13).

Initial premium Let us first determine the level premium $\pi^{(0)}$ at policy issue. In order to be able to determine this premium from the equivalence principle (10), we choose 'time 0' observable values for the future surrender values. We propose to estimate the surrender payment in case of surrender in year $(j, j+1)$, $j \in \{0, 1, 2, \dots, \omega - x - 1\}$, by

$$w_{x+j+1}^{(0)} = (1 - \beta_{j+1}) V_{x+j+1}^{(0)} - \alpha_{j+1}, \tag{27}$$

where $V_{x+j+1}^{(0)}$ is the estimate for the available reserve at time $j+1$ defined by the recursion (21), with initial value $V_x^{(0)} = 0$.

Taking into account the reserves (and hence, also $W_x^{(0)}$) depend on the premium $\pi^{(0)}$, the equivalence relation (10) does not lead to an explicit expression for the initial premium $\pi^{(0)}$. In order to find such an explicit expression, we insert the values (27) of the surrender benefits $w_{x+j+1}^{(0)}$ in the recursion (21). Re-arranging the terms in this recursion leads to the transformed recursion

$$V_{x+j+1}^{(0)} = \left(V_{x+j}^{(0)} + \pi^{(0)} - b_{x+j}^{(0)} - \overline{q}_{x+j}^{aw}\ v\ \overline{w}_{x+j+1}\right) \left(\overline{p}_{x+j}^{aa}\ v\right)^{-1}, \tag{28}$$

which holds for any $j \in \{0, 1, \dots, \omega - x - 1\}$, and with initial value $V_x^{(0)} = 0$, where

$$\overline{q}_{x+j}^{aw} = \beta_{j+1}\ q_{x+j}^{aw}, \quad \overline{w}_{x+j+1} = -\frac{\alpha_{j+1}}{\beta_{j+1}} \quad \text{and} \quad \overline{p}_{x+j}^{aa} = 1 - q_{x+j}^{ad} - \overline{q}_{x+j}^{aw}. \tag{29}$$

Furthermore, we set

$$\overline{q}_{\omega}^{aw} = 0 \text{ and } \overline{w}_{\omega+1} = 0. \tag{30}$$

As $\beta_{j+1} \in (0, 1]$, we have $\overline{q}_{x+j}^{aw} \in [0, 1]$ so that this quantity and $\overline{p}_{x+j}^{aa}$ can be interpreted as probabilities.

We can conclude that at any time $j+1$, the reserve $V_{x+j+1}^{(0)}$ of the health insurance contract, defined by recursion (21) with initial value $V_x^{(0)} = 0$, is identical to the reserve of an artificial health insurance contract with transformed surrender values $\overline{w}_{x+j}$ and transformed probabilities $\overline{q}_{x+j}^{aw}$ and $\overline{p}_{x+j}^{aa}$. The reserves of this artificial contract follow from recursion (28) with initial value $V_x^{(0)} = 0$. In the following proposition, we derive an explicit expression for the initial premium level of the original contract.

Proposition 1. *An explicit expression for the initial premium level $\pi^{(0)}$ of the health insurance contract with reserve-dependent surrender values (27) is given by*

$$\pi^{(0)} = \frac{\underline{B}_x^{(0)} + \underline{W}_x^{(0)}}{\underline{\ddot{a}}_x^{aa}}, \tag{31}$$

with

$$\begin{aligned}
\underline{B}_x^{(0)} &= \sum_{l=0}^{\omega-x} {}_l\overline{p}_x^{aa}\ v^l\ b_{x+l}^{(0)} \\
\underline{W}_x^{(0)} &= \sum_{l=0}^{\omega-x} {}_l\overline{p}_x^{aa}\ \overline{q}_{x+l}^{aw}\ v^{l+1}\ \overline{w}_{x+l+1} \\
\underline{\ddot{a}}_x^{aa} &= \sum_{l=0}^{\omega-x} {}_l\overline{p}_x^{aa}\ v^l.
\end{aligned}$$

In these expressions, the $\overline{q}^{aw}_{x+l}$ *and* $\overline{w}_{x+l+1}$ *are defined by (29) and (30). Furthermore,* ${}_0\overline{p}^{aa}_{x+j} = 1$ *and for* $l > 0$*, we have*

$$ {}_l\overline{p}^{aa}_x = \prod_{k=0}^{l-1} \overline{p}^{aa}_{x+k} $$

with the $\overline{p}^{aa}_{x+k}$ *defined in (29).*

Proof. Following Section 3.5, the recursion (21) for $k = 0$ with initial value $V_x^{(0)} = 0$ leads to the retrospective expression (23) with $j = \omega - x$ for $V_\omega^{(0)}$. In a similar way, the transformed version (28) of this recursion with the same intial value leads to the following expression for $V_\omega^{(0)}$:

$$ V_\omega^{(0)} = \sum_{l=0}^{\omega-x-1} \left(\pi^{(0)} - b^{(0)}_{x+l} - \overline{q}^{aw}_{x+l}\ v\ \overline{w}_{x+l+1}\right) \left({}_{\omega-x-l}\overline{p}^{aa}_{x+l}\ v^{\omega-x-l}\right)^{-1}. $$

On the other hand, from (25) we know that $V_\omega^{(0)} = b_\omega^{(0)} - \pi^{(0)}$. Hence,

$$ \sum_{l=0}^{\omega-x} \left(\pi^{(0)} - b^{(0)}_{x+l} - \overline{q}^{aw}_{x+l}\ v\ \overline{w}_{x+l+1}\right) \left({}_{\omega-x-l}\overline{p}^{aa}_{x+l}\ v^{\omega-x-l}\right)^{-1} = 0. $$

Multiplying each term in this expression by ${}_{\omega-x}\overline{p}^{aa}_x\ v^{\omega-x}$ leads to

$$ \pi^{(0)} \underline{\ddot{a}}^{aa}_x = \underline{B}_x^{(0)} + \underline{W}_x^{(0)}, $$

which proves the stated result. □

Obviously, $\underline{B}_x^{(0)}$ and $\underline{W}_x^{(0)}$ can directly be determined at policy issue. This means that (31) is indeed an explicit expression for the initial premium level.

Updating mechanism Suppose now that we have arrived at time $k \in \{1, 2, \ldots, \omega - x\}$, and that the contract is still in force. At that time, the actuarial value of future health benefit payments, taking into account medical inflation up to that time is given by $B^{(k)}_{x+k}$, which is defined in (11). As before, the updated value of the available reserve at time k is denoted by $V^{(k)}_{x+k}$, whereas the new level premium to be determined at that time is denoted by $\pi^{(k)}$. We propose to update the surrender values at time k using the information about observed inflation until time k, and assuming a yearly medical inflation of f for future years:

$$ w^{(k)}_{x+k+j+1} = (1 - \beta_{k+j+1})\, V^{(k)}_{x+k+j+1} - \alpha_{k+j+1}, \qquad j = 0, 1, , \ldots, \omega - x - k - 1, \tag{32} $$

where $V^{(k)}_{x+k+j+1}$ is the available reserve at time $k + j + 1$ as defined in (21).

Taking into account that the reserves $V^{(k)}_{x+k+j+1}$ and hence also $W^{(k)}_{x+k}$, depend on the premium $\pi^{(k)}$, we find that restoring equivalence equation (15) does not give an explicit relation between the updated premium level and the available reserve at time k. In order to solve this problem, we insert the values (32) for the updated surrender values in the recursion (21). This leads to the transformed recursion for the available reserves:

$$ V^{(k)}_{x+k+j+1} = \left(V^{(k)}_{x+k+j} + \pi^{(k)} - b^{(k)}_{x+k+j} - \overline{q}^{aw}_{x+k+j} v \overline{w}_{x+k+j+1}\right) \left(\overline{p}^{aa}_{x+k+j}\ v\right)^{-1}, \tag{33} $$

which holds for any $j \in \{0, 1, \dots, \omega - x - k - 1\}$, with initial value $V^{(k)}_{x+k}$. The quantities $\overline{q}^{aw}_{x+k+j}$, $\overline{p}^{aa}_{x+k+j}$ and $\overline{w}_{x+k+j+1}$ are defined as before. Rewriting the recursion this way allows us to find an explicit relation between $V^{(k)}_{x+k}$ and $\pi^{(k)}$, as shown in the following proposition.

Proposition 2. *Consider the lifelong health insurance contract with reserve-dependent surrender values (26). Relation (15) at time k can be expressed in the following way:*

$$V^{(k)}_{x+k} = \underline{B}^{(k)}_{x+k} + \underline{W}^{(k)}_{x+k} - \pi^{(k)} \underline{\ddot{a}}^{aa}_{x+k}, \tag{34}$$

with

$$\begin{aligned} \underline{B}^{(k)}_{x+k} &= \sum_{l=0}^{\omega-x-k} {}_l\overline{p}^{aa}_{x+k}\, v^l\, b^{(k)}_{x+k+l} \\ \underline{W}^{(k)}_{x+k} &= \sum_{l=0}^{\omega-x-k} {}_l\overline{p}^{aa}_{x+k}\, \overline{q}^{aw}_{x+k+l}\, v^{l+1}\, \overline{w}_{x+k+l+1} \\ \underline{\ddot{a}}^{aa}_{x+k} &= \sum_{l=0}^{\omega-x-k} {}_l\overline{p}^{aa}_{x+k}\, v^l. \end{aligned}$$

In these expresssions, the $\overline{q}^{aw}_{x+k+l}$, ${}_l\overline{p}^{aa}_{x+k}$ and $\overline{w}_{x+k+l+1}$ are defined as in Proposition 1.

Proof. Following Section 3.5, the recursion (21) with initial value $V^{(k)}_{x+k}$ for the available reserves leads to the retrospective expression (23) with $j = \omega - x - k$ for $V^{(0)}_{\omega}$. The proof here follows a similar reasoning as in Proposition 1. □

This result makes restoring the actuarial equivalence as discussed in Section 3 easy.

4.2 Premium–dependent surrender values

Actuaries generally base surrender values on accumulated available reserves and this case has been thoroughly investigated in Section 4.1. However, this concept may seem obscure to many policyholders. Moreover, some insurers do not compute individual reserves but rather manage the entire portfolio as a collective. In order to overcome these concerns and problems, one might prefer to consider surrender values based on the premiums paid so far. In particular, we assume that in case of surrender in the year $(k, k+1)$, the surrender value paid out at time $k+1$ is given by

$$w^{(k)}_{x+k+1} = \beta_{k+1} \sum_{l=0}^{k} \pi^{(l)}\, (1+i')^{k+1-l} - \alpha_{k+1}, \qquad k = 0, 1, 2, \dots \tag{35}$$

Hence, the surrender value is equal to a time-dependent fraction β_{k+1}, $0 \leq \beta_{k+1} \leq 1$, of the accumulated value of the premiums paid until time k, minus a time-dependent penalty $\alpha_{k+1} \geq 0$. We assume that the quantities $\beta_1, \beta_2, \dots$ and $\alpha_1, \alpha_2, \dots$ are fixed at policy issue. The coefficients $\beta_1, \beta_2, \dots$ can be chosen such that they approximately mimic the accumulation of the savings premiums in the reserve, representing the part of the premiums paid but not consumed to finance past health benefits. The accumulation of the premiums is performed at a constant interest rate i', which may be different from the technical interest rate i. We could for instance set $i' = 0$ so that premiums enter the calculation at nominal values.

Initial premium At policy issue, the payment for surrender in year $(k, k+1)$ is in general unknown as it depends on the a priori unknown stream of future premium payments $\pi^{(1)}, \pi^{(2)}, \ldots, \pi^{(k)}$. However, when the surrender option is exercised in year $(k, k+1)$, the surrender value $w_{x+k+1}^{(k)}$ that is actually paid out is fully specified at time $k+1$, based on the information that is available at that time about previous medical inflation.

In order to be able to determine the initial level premium $\pi^{(0)}$ from the equivalence principle (10), we have to choose values for the future surrender values observable at time 0. We propose to estimate the payment in case of surrender in year $(j, j+1)$, $j \in \{0, 1, 2, \ldots, \omega - x - 1\}$ by

$$w_{x+j+1}^{(0)} = \beta_{j+1} \sum_{l=0}^{j} \pi^{(0)} \left(1 + i'\right)^{j+1-l} - \alpha_{j+1}. \tag{36}$$

This means that $w_{x+j+1}^{(0)}$ corresponds to the surrender value $w_{x+j+1}^{(j)}$ in case of a medical inflation of f per year and no adaptation of the premiums until surrender.

Proposition 3. *Assuming (36), an explicit expression for the initial premium level $\pi^{(0)}$ of the health insurance contract with premium-dependent surrender benefits (35) is given by*

$$\pi^{(0)} = \frac{B_x^{(0)} - \sum_{j=0}^{\infty} {}_j p_x^{aa} \ q_{x+j}^{aw} \ v^{j+1} \ \alpha_{j+1}}{\ddot{a}_x^{aa} - \sum_{j=0}^{\infty} {}_j p_x^{aa} \ q_{x+j}^{aw} \ v^{j+1} \ c_{j+1}^{(0)}} \tag{37}$$

with

$$c_{j+1}^{(0)} = \beta_{j+1} \sum_{l=0}^{j} (1 + i')^{j+1-l}. \tag{38}$$

Proof. We rewrite (36) as follows:

$$w_{x+j+1}^{(0)} = c_{j+1}^{(0)} \ \pi^{(0)} - \alpha_{j+1} \tag{39}$$

with the $c_{j+1}^{(0)}$ defined in (38). Inserting the surrender values $w_{x+j+1}^{(0)}$ in the actuarial equivalence relation (10), while taking into account (12), leads to the explicit expression (37) for the initial level premium $\pi^{(0)}$. □

Updating mechanism Suppose now that we have arrived at time $k \in \{1, 2, \ldots\}$ and that the policy is still in force. The available reserve $V_{x+k}^{(k-1)}$ at this moment is given by (13). Taking into account the information about medical inflation up to time k, the future health benefits are re-estimated and their updated actuarial value $B_{x+k}^{(k)}$ follows from (11). The previously chosen values $w_{x+k+j+1}^{(k-1)}$ for future surrender values are replaced by the values $w_{x+k+j+1}^{(k)}$, $j \in \{1, 2, \ldots\}$, which are defined by

$$w_{x+k+j+1}^{(k)} = \beta_{k+j+1} \sum_{l=0}^{k+j} \pi^{(\min\{k,l\})} \left(1 + i'\right)^{k+j+1-l} - \alpha_{k+j+1}. \tag{40}$$

At time k, the future surrender values are determined using the information about medical inflation until time k, while assuming a future inflation of f per year. To restore the actuarial equivalence at time k, the insurer updates the available reserve to level $V_{x+k}^{(k)}$ and the premium is replaced by $\pi^{(k)}$.

Proposition 4. *Consider the lifelong health insurance contract with premium-dependent surrender values (35). Relation (15) can be expressed as:*

$$\begin{aligned} V_{x+k}^{(k)} &= B_{x+k}^{(k)} + \sum_{j=0}^{\omega-x-k} {}_jp_{x+k}^{aa}\, q_{x+k+j}^{aw}\, v^{j+1} d_{k+j+1}^{(k)} \\ &\quad -\pi^{(k)} \left(\ddot{a}_{x+k}^{aa} - \sum_{j=0}^{\omega-x-k} {}_jp_{x+k}^{aa}\, q_{x+k+j}^{aw}\, v^{j+1} c_{k+j+1}^{(k)} \right) \end{aligned} \tag{41}$$

with

$$c_{k+j+1}^{(k)} = \beta_{k+j+1} \sum_{l=k}^{k+j} (1+i')^{k+j+1-l} \tag{42}$$

$$d_{k+j+1}^{(k)} = \beta_{k+j+1} \sum_{l=0}^{k-1} \pi^{(l)} (1+i')^{k+j+1-l} - \alpha_{k+j+1}. \tag{43}$$

Proof. The updated surrender value $w_{x+k+j+1}^{(k)}$ can be rewritten as

$$w_{x+k+j+1}^{(k)} = c_{k+j+1}^{(k)}\, \pi^{(k)} + d_{k+j+1}^{(k)}, \tag{44}$$

with $c_{k+j+1}^{(k)}$ and $d_{k+j+1}^{(k)}$ defined by (42) and (43), respectively. Substituting (44) into (12) and combining with (15) yields the announced result. □

The explicit relation between $V_{x+k}^{(k)}$ and $\pi^{(k)}$ in Proposition 4 allows for a straightforward restoration of the actuarial equivalence (15).

5 Numerical illustrations

5.1 Contract and technical basis

We consider a contract issued to a policyholder aged $x = 25$. This contract covers medical expenses in excess of social security, as those commonly sold in Belgium. Additional background information is in Devolder et al. (2008). The technical basis assumes a yearly interest rate i of 2%.

The dashed line in Figure 2 shows the average health benefit $\bar{b}_y^{(0)}$ as a function of policyholder's attained age y, in euros. The shape of $y \mapsto \bar{b}_y^{(0)}$ is inspired by Belgian private health insurance market experience but the values have been rescaled for confidentiality reasons. The full line in this figure represents the average health benefits $b_y^{(0)}$, given by (1), when a medical inflation f of 2% per year is taken into account.

Since health insurance contracts with a transferable reserve are not currently available on the Belgian market, we do not have relevant observed lapse probabilities at our disposal. Therefore we carry out a sensitivity analysis by varying the lapse probabilities according to the following

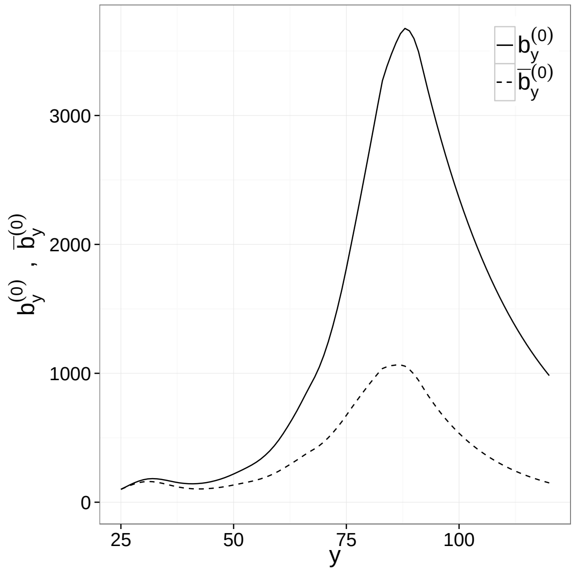

Figure 2: Average health benefit $\bar{b}_y^{(0)}$ at age y (*dashed line*) and inflated values $b_y^{(0)}$ (*full line*) at constant rate f for medical inflation.

three scenarios: for a policyholder buying the contract at age 25, we consider one-year lapse probabilities q_y^{aw} at age $y \geq 25$ given by

$$q_y^{aw_1} = 0 \tag{45}$$

$$q_y^{aw_2} = \begin{cases} 0.1 - 0.002 \cdot (y - 20) & \text{if } 25 \leq y \leq 70 \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

$$q_y^{aw_3} = \begin{cases} 0.05 \cdot \left(\cos\left((y - 25) \cdot \frac{\pi}{95}\right) + 1\right) & \text{if } 25 \leq y \leq 120 \\ 0 & \text{otherwise.} \end{cases} \tag{47}$$

These lapse probabilities are displayed in Figure 3. Under the first set of lapse probabilities $q_y^{aw_1}$, policyholders never cancel the contract. The second set of lapse probabilities $q_y^{aw_2}$ has been used by Vercruysse et al. (2013). These probabilities imply a higher propension to lapse at younger ages for the 25-year-old policyholder under consideration, but no lapse after age 70. This is often taken as the central scenario on the Belgian market. Finally, under the third set of lapse probabilities $q_y^{aw_3}$, we have higher lapse probabilities at younger ages which then decline smoothly to 0 at the ultimate age $\omega = 120$.

Death probabilities are displayed in Figure 4, left panel. Notice that these are not the q_y^{ad} but yearly death probabilities $q_y'^d$ based on observations at the general population level in a single decrement, two state alive or dead model. We recover the values for q_y^{ad} from the following relation:

$$q_y^{ad} = q_y'^d \left(1 - \frac{q_y^{aw}}{2 - q_y'^d}\right). \tag{48}$$

This relation holds under the assumption of a uniform distribution of decrements in any year for each of the two single decrement models ('active vs withdrawn' and 'active vs dead'; see Section 8.10.2 in Dickson et al. (2013). Notice that lapse probabilities enter the calculation of death probabilities so that different scenarios of policy cancellation impact on all actuarial quantities.

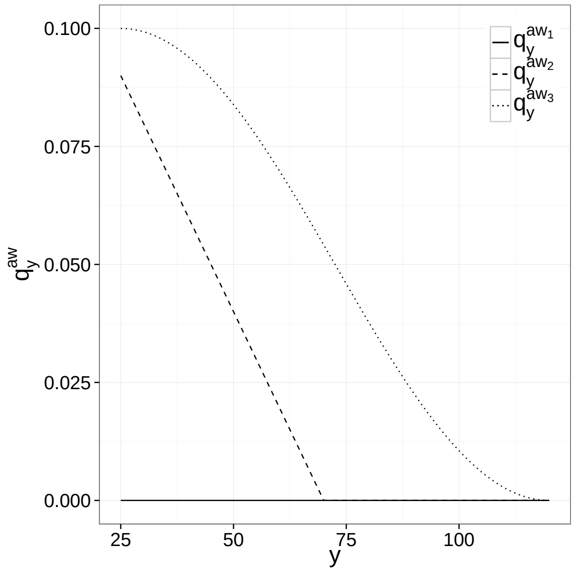

Figure 3: One-year lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), and $q_y^{aw_3}$ (*dotted line*).

Under (45), we have $q_y^{ad} = q_y^{\prime d}$. Figure 4, right panel, displays the three sets of one-year death probabilities used in the numerical illustrations.

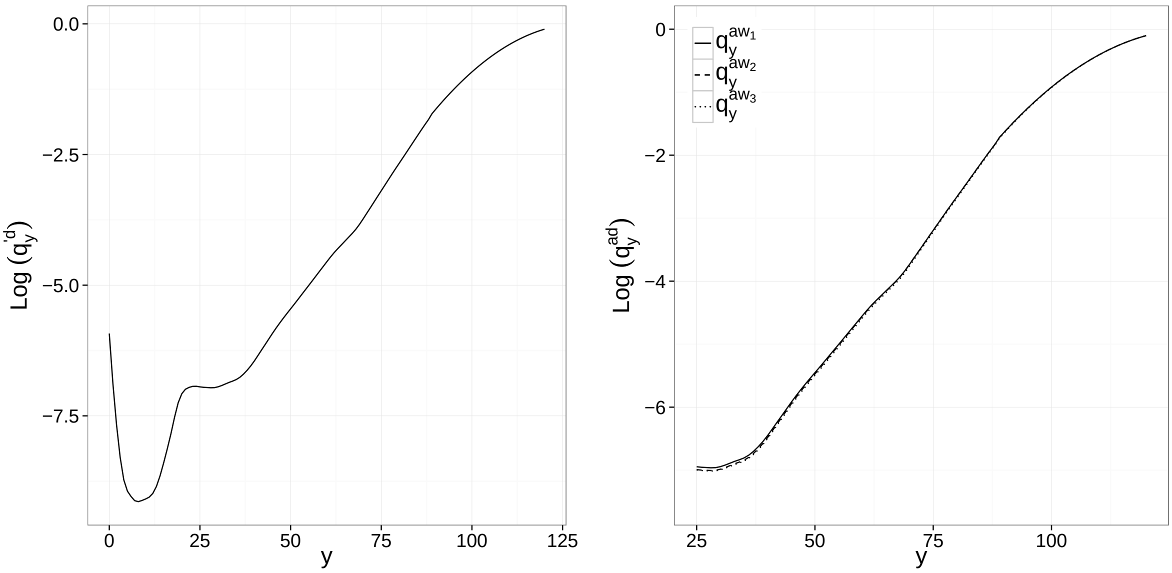

Figure 4: One-year death probabilities $q_y^{\prime d}$ on a log-scale (*left*) and one-year death probabilities q_y^{ad} on a log-scale corresponding to the three sets of lapse probabilities (*right*) $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), and $q_y^{aw_3}$ (*dotted line*).

5.2 Surrender values

5.2.1 Reserve-dependent surrender values

When surrender values depend on the available reserve, as discussed in Section 4.1, we set the non-transferred percentage and the reserve-independent penalty in the definition of surrender values (26) respectively equal to

$$\beta_{k+1} = \begin{cases} 1 & \text{if } 0 \leq k \leq 4 \\ 0.2 & \text{if } 5 \leq k \end{cases} \qquad \text{and} \qquad \alpha_{k+1} = \begin{cases} 0 & \text{if } 0 \leq k \leq 4 \\ 150 & \text{if } 5 \leq k. \end{cases}$$

Early cancellations often cause significant losses for the insurer due to unrecovered administrative costs and commissions. Therefore no surrender values are paid in the first five years of the contract in this example. Afterwards we fix the non-transferred percentage at 20% and take the reserve–independent penalty equal to 150.

5.2.2 Premium–dependent surrender values

We also consider surrender values based on the premiums paid so far, as discussed in Section 4.2. We set the interest rate on the accumulated premiums to $i' = 1\%$.

We propose to specify the time-dependent fractions β_k and time-dependent penalties α_k in (35) as follows in the policy conditions:

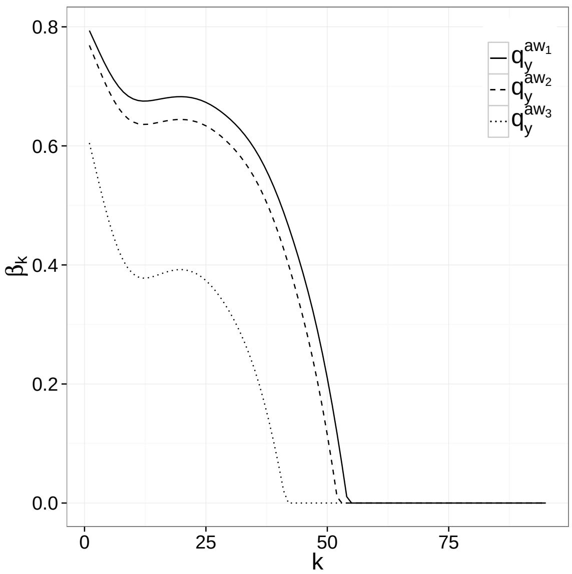

Figure 5: Values of the β_k defined from (49).

1. Define $\beta'_k = 1$ and $\alpha'_k = \sum_{l=0}^{k-1} b^{(0)}_{x+l} \cdot (1 + i')^{k-l}$. A contract with surrender benefits defined by (35) with parameters β'_k and α'_k defines surrender benefits at time $k + 1$ as the sum of the savings premiums at policy issue with interest accumulation at rate i':

$$w'^{(0)}_{x+k+1} = \sum_{l=0}^{k} (\pi^{(0)} - b^{(0)}_{x+l}) \cdot (1 + i')^{k+1-l}.$$

2. Calculate the initial premium $\pi'^{(0)}$ for a contract with surrender values as defined in step 1. Use formula (37) from the approach outlined in Section 4.2.

3. Define
$$\beta_k = \max\left\{0, \frac{\sum_{l=0}^{k-1}(\pi'^{(0)} - b^{(0)}_{x+l}) \cdot (1+i')^{k-l}}{\sum_{l=0}^{k-1}\pi'^{(0)} \cdot (1+i')^{k-l}}\right\} \tag{49}$$
and $\alpha_k = 0$ which leads to surrender values
$$w^{(0)}_{x+k+1} = \max\left\{0, \frac{\sum_{l=0}^{k}(\pi'^{(0)} - b^{(0)}_{x+l}) \cdot (1+i')^{k+1-l}}{\sum_{l=0}^{k}\pi'^{(0)} \cdot (1+i')^{k+1-l}}\right\} \cdot \sum_{l=0}^{k}\pi^{(0)}(1+i')^{k+1-l}. \tag{50}$$

These three simple steps base the definition of the surrender values on the sum of the savings premiums and ensure that the surrender values do not become negative. Moreover, this definition also ensures that the surrender values never exceed the sum of the premiums paid so far with interest accumulated at rate i' since $\beta_k \leq 1$. Figure 5 displays the β_k defined from (49).

5.3 Initial premium

Table 1 contains the initial premium $\pi^{(0)}$ calculated for different types of surrender values: surrender value based on the available reserve, surrender value based on the premiums, and no surrender value (i.e. the policyholder does not receive any benefit in case of policy cancellation). Obviously, the initial premium is identical under the first set of lapse probabilities $q_y^{aw_1}$ as the definition of the surrender values is irrelevant in that case. The last column of Table 1 shows the initial premium in case the surrender values are always 0. Table 1 illustrates that given any lapse probability, a higher surrender value increases the initial premium.

Type of surrender values	Reserve-dependent	Premium-dependent	No payment
Lapse probability $q_y^{aw_1}$	484.76	484.76	484.76
Lapse probability $q_y^{aw_2}$	415.50	431.15	267.18
Lapse probability $q_y^{aw_3}$	238.90	256.15	140.50

Table 1: Premium $\pi^{(0)}$ at policy issue.

5.4 Medical inflation scenario

We illustrate the proposed methods by assuming an additional yearly medical inflation $j_k^{[B]} = 1\%$ (for year $k \geq 1$) for health benefits, on top of the expected inflation $f = 2\%$ incorporated in the premiums. In the notation of Section 2.1, the assumption of the expected inflation of $f = 2\%$ translates to
$$b^{(0)}_{x+k} = (1+f)^k \cdot \bar{b}^{(0)}_{x+k} = (1+2\%)^k \cdot \bar{b}^{(0)}_{x+k}. \tag{51}$$
The assumption of the additional medical inflation can be written as
$$\bar{b}^{(k)}_{x+k+j} = (1+f) \cdot (1+j_k^{[B]}) \cdot \bar{b}^{(k-1)}_{x+k+j} \tag{52}$$

such that

$$b^{(k)}_{x+k+j} = (1+f)^j \cdot \bar{b}^{(k)}_{x+k+j} = (1+2\%)^j \cdot \bar{b}^{(k)}_{x+k+j} \tag{53}$$

or

$$b^{(k)}_{x+k+j} = (1+j^{[B]}_k) \cdot b^{(k-1)}_{x+k+j} = (1+1\%) \cdot b^{(k-1)}_{x+k+j}. \tag{54}$$

5.5 Updating mechanisms

As explained earlier, there are many ways to update the contract to account for medical inflation. As in Vercruysse et al. (2013) we denote the premium increase and the reserve increase at time k by respectively $j^{[P]}_k$ and $j^{[V]}_k$. In this section, we consider the following two approaches:

Mechanism 1: The premium increase $j^{[P]}_k$ is contractually fixed in function of observed medical inflation, i.e. $j^{[P]}_k = j^{[P]}_k(j^{[B]}_k)$. Premium updates are then obtained from

$$\pi^{(k)}_x = (1 + j^{[P]}_k(j^{[B]}_k)) \cdot \pi^{(k-1)}_x. \tag{55}$$

The reserves are adjusted afterwards. For instance, policy conditions could specify that premiums are updated according to

$$j^{[P]}_k(j^{[B]}_k) = (1+\gamma) \cdot j^{[B]}_k \tag{56}$$

where the additional γ accounts for the indexing of the accumulated reserve. In the numerical illustration in Sections 5.6, 5.7 and 5.8 we consider (56) with $\gamma = 0$.

Mechanism 2: The insurer now first increases the available reserve according to

$$V^{(k)}_{x+k} = (1 + j^{[V]}_k) \cdot V^{(k-1)}_{x+k} \tag{57}$$

and determines the corresponding new level premium afterwards. We illustrate this updating mechanism for $j^{[V]}_k = 1\%$, i.e. the insurer increases the available reserve by 1% each year.

5.6 The case of no surrender values

We start with the case studied in Vercruysse et al. (2013) where the policyholder receives no surrender value in case of surrender. Figure 6 shows the available reserves calculated with information available at time 0. Higher lapse probabilities lead to a lower premium and, as a consequence, tend to decrease the reserve early in the policy. The reserve not paid out as a surrender value to a lapsing policyholder is used to cover this.

Figure 7 illustrates the evolution of the reserves $V^{(k)}_{x+k}$ and the premiums $\pi^{(k)}$ over time k when nothing is paid in case of policy cancellation. In absence of surrender values, the result of updating mechanisms 1 and 2 described above is exactly the same so that we do not have to distinguish between the two mechanisms described above. This is because the expected present value of the surrender values (12) is zero which reduces actuarial equivalence (18) at time $(k-1)$ to

$$V^{(k-1)}_{x+k} = B^{(k-1)}_{x+k} - \pi^{(k-1)} \cdot \ddot{a}^{aa}_{x+k}. \tag{58}$$

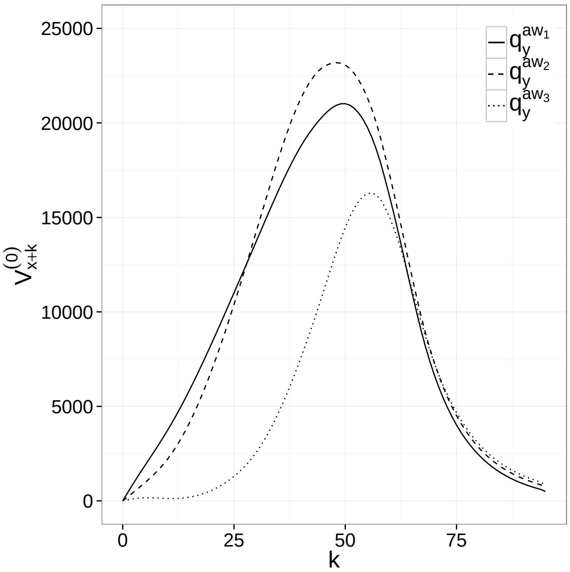

Figure 6: No surrender value: available reserves $V_{x+k}^{(0)}$ for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

When the additional medical inflation is $j_k^{[B]} = 1\%$ and the reserve is updated by $j_k^{[V]} = 1\%$ as in contract updating mechanism 2, the actuarial equivalence is restored by increasing the premium by $j_k^{[P]} = 1\%$. This is the same premium update as for mechanism 1 as this mechanism sets the premium increase equal to the additional medical inflation, which we assume to be 1%. By a similar reasoning the reserve increase resulting from updating mechanism 1 equals $j_k^{[V]} = 1\%$. We conclude that both updating mechanisms have the same impact on the reserve and premium in this setting.

5.7 Reserve-dependent surrender values

This section illustrates the strategy proposed in Section 4.1 where the surrender values depend on the available reserve. The first column of Table 1 shows the initial premium for a contract specifying reserve-dependent surrender values for the different lapse probability assumptions. The initial premium of the contract decreases as the lapse probability increases. This is a consequence of the definition and choice of parameters β_k and α_k of the surrender values. As illustrated in Figure 8, the surrender values never exceed the available reserve. The part of the reserve not transferred in case of surrender can be added to the reserve of the remaining policyholders.

Figures 9 and 10 illustrate the evolution of the available reserve and surrender values over time under mechanisms 1 and 2, respectively. For both updating mechanisms the reserve increases over time due to the observed medical inflation. Therefore, the surrender values also increase because of their dependence on the reserve. The difference between both updating mechanisms is visualized in Figure 11. The graph on the right illustrates the yearly premium increase $j_k^{[P]}$ when we use mechanism 2 to account for observed medical inflation. The horizontal line at a premium increase of 1% corresponds to zero lapse probability $q_y^{aw_1}$ as the expected present value of the surrender values is zero at any time which reduces the actuarial equivalence at

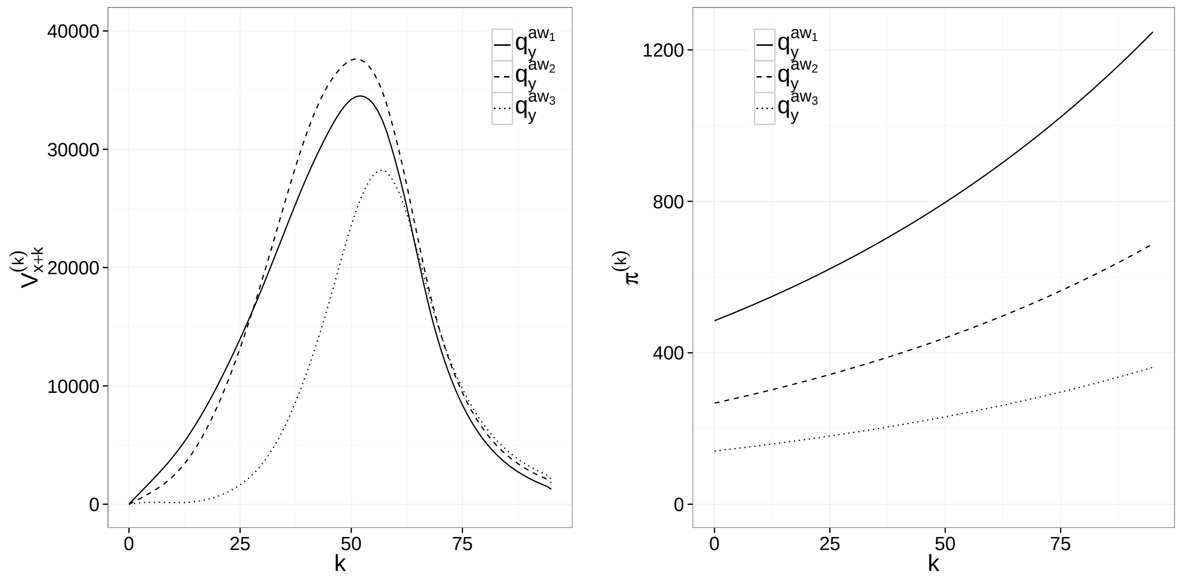

Figure 7: No surrender value: available reserves $V^{(k)}_{x+k}$ and premiums $\pi^{(k)}$ when for the different types of lapse probabilities $q^{aw_1}_y$ (*full line*), $q^{aw_2}_y$ (*dashed line*), $q^{aw_3}_y$ (*dotted line*).

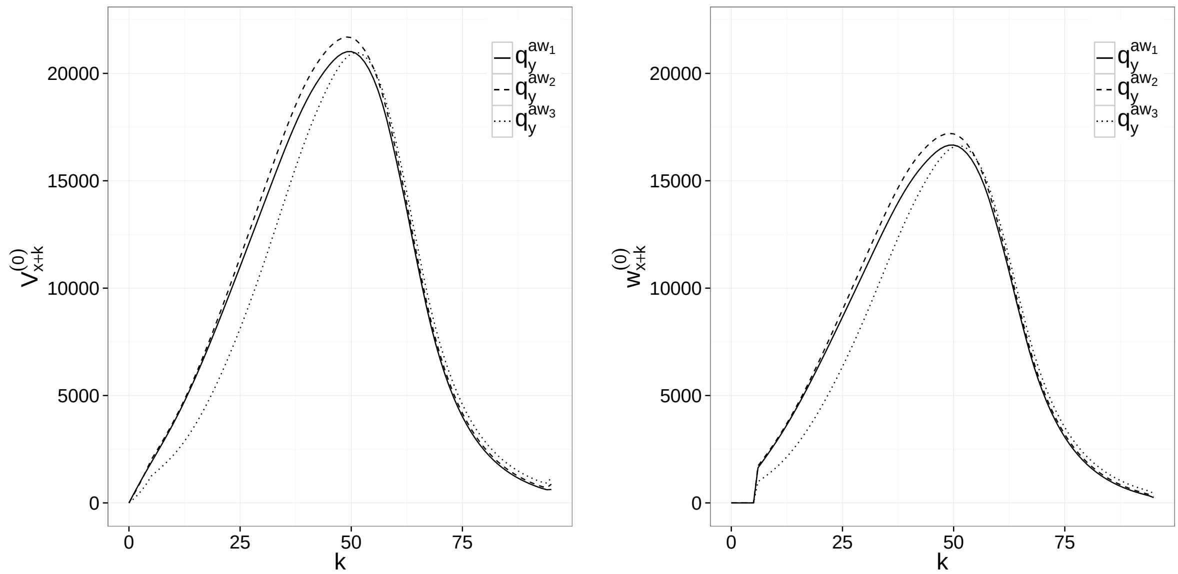

Figure 8: Reserve–dependent surrender value: available reserves $V^{(0)}_{x+k}$ (*left*) and surrender values $w^{(0)}_{x+k}$ (*right*) for the different types of lapse probabilities $q^{aw_1}_y$ (*full line*), $q^{aw_2}_y$ (*dashed line*), $q^{aw_3}_y$ (*dotted line*).

time $k-1$ to (58). For a reserve increase of 1% and the same increase in $B^{(k-1)}_{x+k}$ to account for observed medical inflation, the equivalence is restored by a premium increase of 1%. The other lapse probabilities decrease and converge to 0 over time. Consequently, over time the expected present value of the surrender values has a smaller impact on the premium when restoring the actuarial equivalence. Therefore the curves corresponding to lapse probabilities $q^{aw_2}_y$ and $q^{aw_3}_y$ also converge to 1% over time. The point at which the percentage starts to decrease corresponds

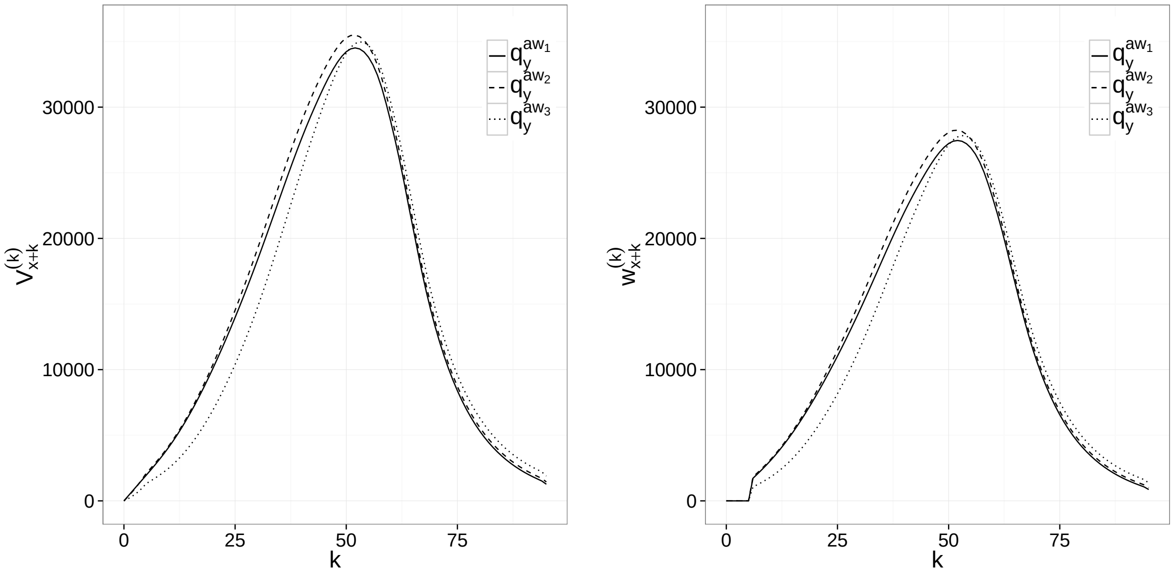

Figure 9: Reserve–dependent surrender value. Contract updating mechanism 1: available reserves $V_{x+k}^{(k)}$ (*left*) and surrender values $w_{x+k}^{(k)}$ (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

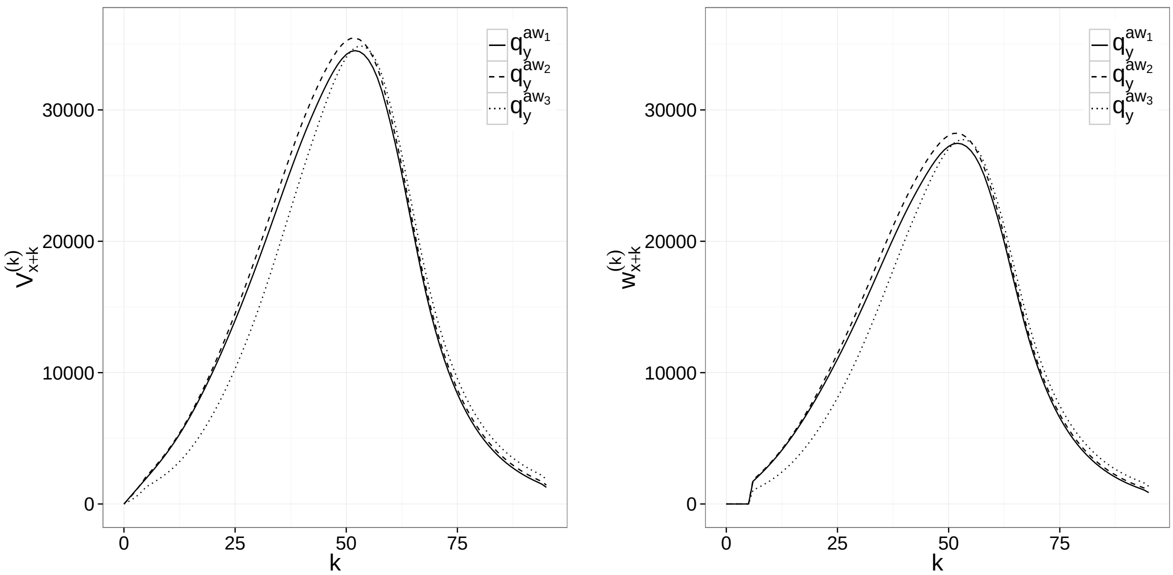

Figure 10: Reserve–dependent surrender value. Contract updating mechanism 2: available reserves $V_{x+k}^{(k)}$ (*left*) and surrender values $w_{x+k}^{(k)}$ (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

to the year after which the surrender values are strictly positive.

The graph in the left panel of Figure 11 shows the increase of the reserve $j_k^{[V]}$ under mechanism 1. As for the right figure, the reduced equivalence relation (58) explains why the reserve increase for zero lapse probability $q_y^{aw_1}$ is constantly equal to 1% and why the reserve increases for the

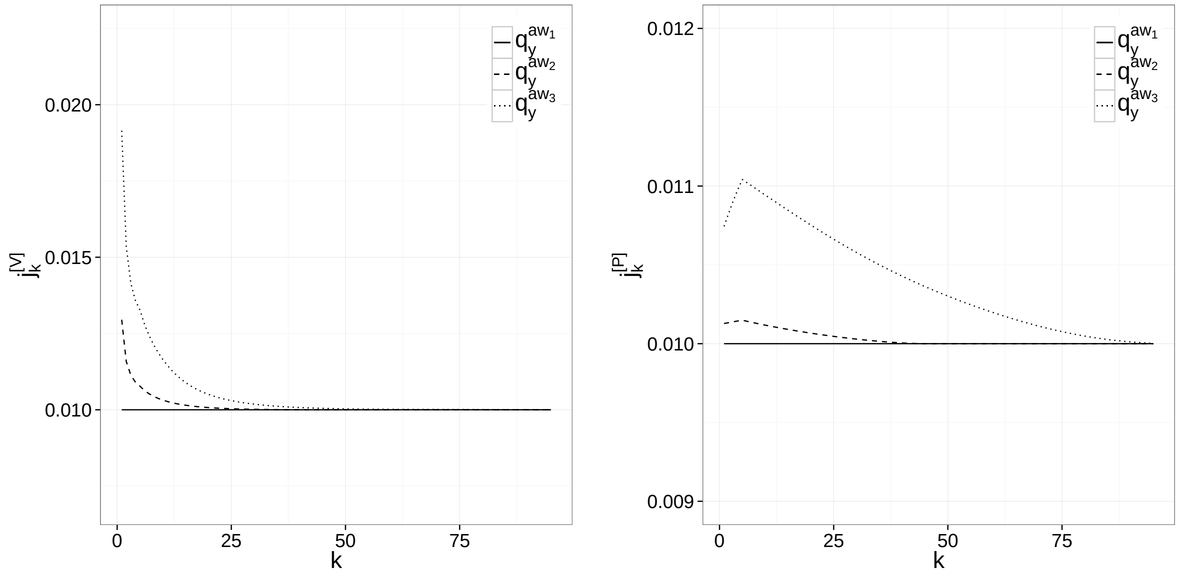

Figure 11: Reserve–dependent surrender value: yearly reserve increase $j_k^{[V]}$ for mechanism 1 (*left*) and yearly premium increase $j_k^{[P]}$ for updating mechanism 2 (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

other lapse probabilities flatten out at 1% in the left figure.

5.8 Premium-dependent surrender values

The second column in Table 1 displays the initial premium $\pi^{(0)}$ calculated at policy issue corresponding to the different lapse probability assumptions in (45), (46) and (47). The evolution of the available reserve $V_{x+k}^{(0)}$ when medical inflation and contract updates over time are not taken into account is illustrated in the left graph of Figure 12. The right graph of this figure shows the evolution of surrender values (50) calculated at policy issue. If the assumptions in the technical basis at policy issue realize over time, this figure demonstrates that the surrender values never exceed the available reserve. As a consequence, higher lapse probabilities result in lower premiums as demonstrated in Table 1. A lower premium implies that the sum of the savings premiums is lower and gets negative sooner, so the cap of 0 in definition (49) of β_k is reached more rapidly.

The impact of updating mechanism 1 and mechanism 2 on the reserves and surrender values is illustrated in Figures 13 and 14, respectively. As expected, for both mechanisms the reserves and surrender values have increased compared to their value computed at policy issue. However, surrender values never exceed the available reserve.

The difference between both updating mechanisms is highlighted in Figure 15. The right graph shows the yearly increase in the premium when we use updating mechanism 2 to account for observed medical inflation. When the lapse probability is zero the actuarial equivalence at time $(k-1)$ reduces to (58). Additional medical inflation of 1% and a reserve update of 1% requires a premium increase of 1% to restore the actuarial equivalence. For the same reason the curves corresponding to the other lapse probabilities flatten out at zero. Indeed, due to the cap of zero

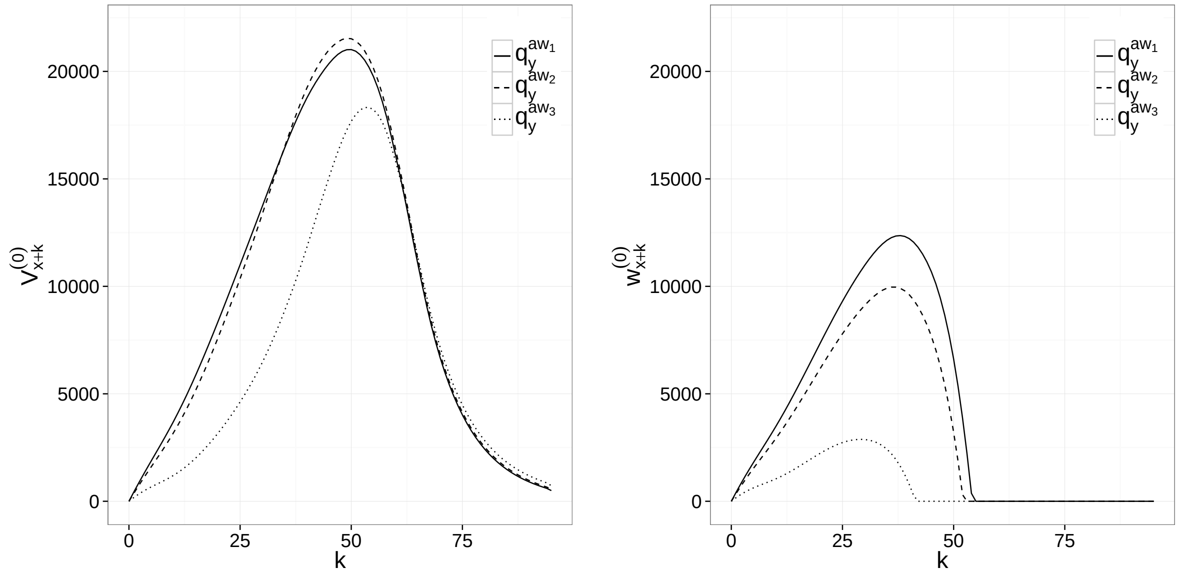

Figure 12: Premium–dependent surrender value: available reserves $V_{x+k}^{(0)}$ (*left*) and surrender values $w_{x+k}^{(0)}$ (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

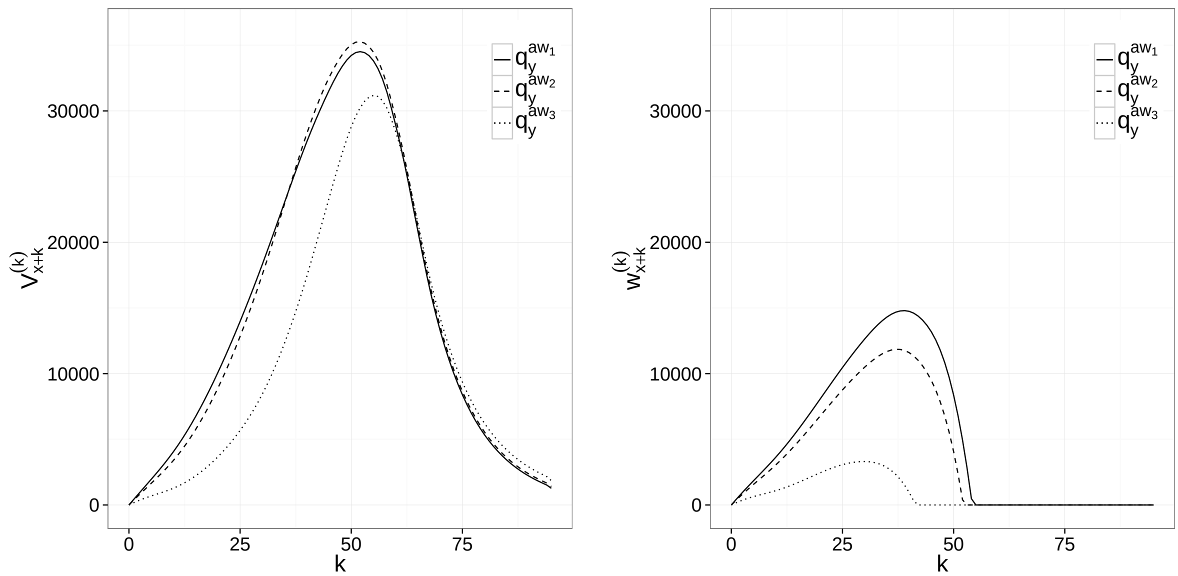

Figure 13: Premium–dependent surrender value. Updating mechanism 1: available reserves $V_{x+k}^{(k)}$ (*left*) and surrender values $w_{x+k}^{(k)}$ (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

on the β_k the expected value of the surrender values drops to zero over time. Earlier in the contract, the premium increase is lower than the reserve increase. Using a similar reasoning, the reserve increase for $q_y^{aw_1}$ is constantly equal to 1% and the reserve increases for the other lapse probabilities flatten out at 1% in the left panel. Earlier in the contract the required reserve

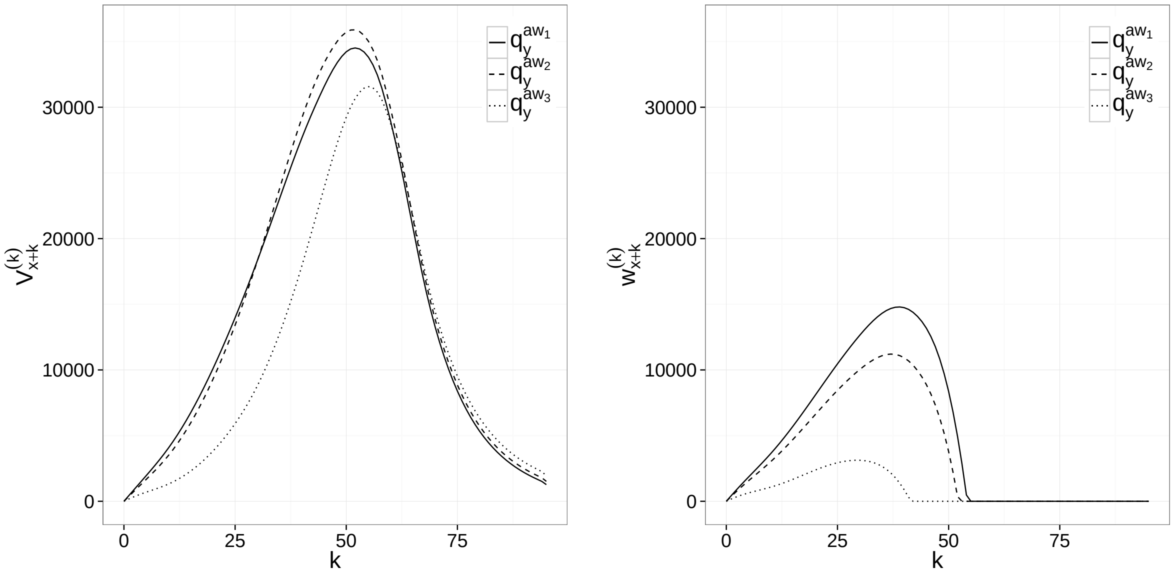

Figure 14: Premium–dependent surrender value. Updating mechanism 2: available reserves $V_{x+k}^{(k)}$ (*left*) and surrender values $w_{x+k}^{(k)}$ (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

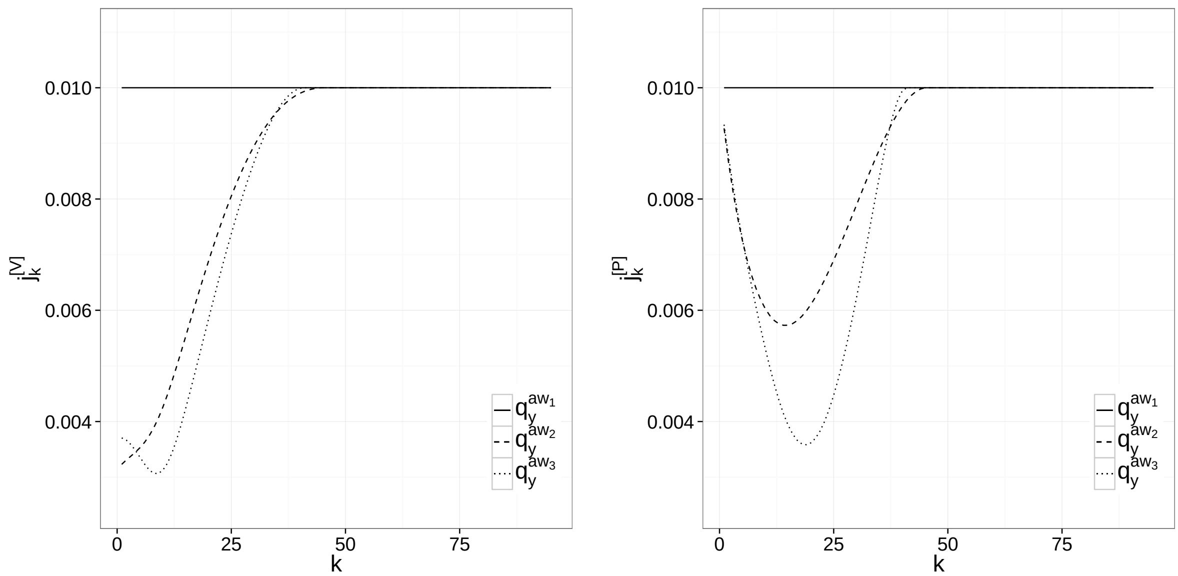

Figure 15: Premium–dependent surrender value: yearly reserve increase $j_k^{[V]}$ for mechanism 1 (*left*) and yearly premium increase $j_k^{[P]}$ for updating mechanism 2 (*right*) for the different types of lapse probabilities $q_y^{aw_1}$ (*full line*), $q_y^{aw_2}$ (*dashed line*), $q_y^{aw_3}$ (*dotted line*).

update for the non-zero lapse probabilities is lower than 1%.

5.9 Discussion

We put emphasis on the results of the sensitivity analysis regarding the lapse probabilities. Table 1 demonstrates that a higher lapse probability has a premium decreasing effect when, using information available at policy issue, the surrender benefit never exceeds the available reserve.

Figures 6 and 7 illustrate that when lapse rates increase the insurer expects a higher income from the part of the available reserve not paid out to the surrendering policyholder, the build up of the available reserve starts off slower. The impact of surrender probabilities is more complicated in Figures 8, 9, 10 and 12, 13, 14 as the reserve is indirectly influenced by the lapse probabilities not only through the initial premium, but also through the definition of the surrender values. Nevertheless, it is clear from these figures that the choice of lapse probabilities in the technical basis has a significant influence on the amount of the available reserve the insurer expects to hold over time.

Lastly, the consequences of the different updating mechanisms also clearly depend on the surrender probabilities as can be seen in Figures 11 and 15. The same updating mechanism can lead to very different relative increases to the available reserves and premiums under various lapse probability assumption.

6 Conclusion

In this paper, we considered a lifelong health insurance contract with level premiums. In contrast to Vercruysse et al. (2013) and Denuit et al. (2015) the policy under consideration allows for the transferability of reserves. To this end we suggest two possible ways to define the surrender value in case the policyholder decides to lapse the contract. First, the surrender value is defined as a proportion of the available reserve minus a penalty. This definition has as advantage that the insurer will never pay out more as a surrender benefit than what he has set aside for the policyholder as reserve. Disadvantages might be that not every insurer computes an individual reserve for each policyholder and that the concept of a reserve seems obscure to a policyholder. Second, the surrender value is defined as a proportion of the premiums paid up to the moment of surrender (possibly with interest) minus a penalty. This definition is more tractable and understandable by the policyholder. For the insurer it is, however, not straightforward to guarantee that surrender value will never exceed the available reserve for the policyholder.

In order to be able to determine premiums and reserves, a future yearly medical inflation was assumed. The contract that we considered was such that a yearly update, based on the observed inflation in the past year, was possible. In order to maintain the actuarial equivalence from year to year, premiums and reserves were allowed to be adapted, according to a procedure specified in the policy. The other elements of the technical basis (interest, mortality and lapse rates) were assumed to be in line with the reality that unfolds over time, which implies that these elements do not give rise to a required update of the contract in order to maintain actuarial equivalence. This simplifying assumption allowed us to isolate and investigate the effect of medical inflation on its own. The framework we propose, however, is easily extendable to include more flexibility regarding the adjustment over time of elements in the technical basis.

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