UPDATING MECHANISM FOR LIFELONG INSURANCE CONTRACTS SUBJECT TO MEDICAL INFLATION

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November 18, 2016

Abstract

This paper proposes a practical way for indexing level premiums in lifelong medical insurance contracts, in order to take into account observed medical inflation. The indexing can be achieved by considering only premiums, without explicit reference to reserves. This appears to be relevant in practice as reserving mechanisms may not be transparent to policyholders and as some mutual and shareholder insurers do not compute contract-specific reserves, managing the whole portfolio in a collective way. Note that the similarity with the existing literature on life insurance comes from the lifelong nature of the contracts considered in our setting. However, in the case of health insurance covers the value of future benefits is random.
The present study originates from a proposal for indexing lifelong medical insurance level premiums in Belgium. As an application, we study the impact of various indexing mechanisms on a typical medical insurance portfolio on the Belgian market.

Key words and phrases: health insurance, reserving, inflation, premium update, solvency evaluation.

1 Introduction

Consider a portfolio of lifelong health insurance contracts covering medical expenses (in excess of Social Security, say, as it is often the case in continental Europe). We deal here with SLT health insurance contracts, where SLT stands for "Similar to Life Techniques" in the context of Solvency 2. We assume that the contracts stipulate that no surrender value is paid out in case of policy cancellation. The (unpredictable) increase of medical costs in the future generates a systematic risk for the health insurance provider. Therefore, medical inflation is usually not guaranteed when setting the level premiums of the contracts at policy issue. Instead, premiums and eventually also reserve (also known as mathematical reserve or policy value) are regularly updated, accounting for observed medical inflation over the previous years.

In this paper, we propose a simple but actuarially sound method that takes into account the observed medical inflation ex-post via a yearly recalculation of the premium levels. The premium-updating mechanism is based on a medical inflation index. This index quantifies the global increase in prices of medical goods and services and may thus differ from the classical consumer price index. Notice that a one-step version of the formula used in the present paper has been derived by Schneider (2002, Section 8) in the particular case of no reserve update. Here, this formula is extended to a multi-period setting, allowing for premium and/or reserve revisions. The case where the reserve is also updated is studied in Pitacco (2014, Section 5.4). To the best of our knowledge, the present paper is the first to provide a comprehensive study of the updating mechanism in lifelong health insurance cover, including sensitivity analysis and a comparison between the individual and the aggregate methods.

Lifelong health insurance contracts and related premium updating mechanisms have been investigated in Vercruysse, Dhaene, Denuit, Pitacco and Antonio (2013) as well as in Dhaene, Godecharle, Antonio and Denuit (2015). In the current paper, we aim to derive a practical indexing method and to assess the approximation suggested by the indexing mechanism adopted for mutual and shareholder insurers in the Belgian market. This approximation consists in applying a factor '1.5' to the medical inflation in order to take into account the increase of the reserve. Specifically, we show that indexing can be achieved by considering only premiums, without explicit reference to reserves. This appears to be relevant in practice as reserving mechanisms may not be transparent to policyholders and as some insurers do not compute contract-specific reserves, managing the whole portfolio in a collective way. The present study originates from a proposal for indexing medical insurance premiums in Belgium. As an application, we study the impact of various indexing rules on a typical medical insurance portfolio on the Belgian market.

The remainder of this paper is organized as follows. In Section 2, we describe the actuarial model for the health insurance contracts considered in this paper. Section 3 presents the one-step revision of the premium amount and/or of the accumulated reserve, as a consequence of medical inflation. Section 4 extends this one-step formula to periodic revisions during the coverage period. In Section 5, we consider the indexing mechanism recently proposed in Belgium. We show that the simple rule implemented after a Royal Decree dated March 2016 allows insurers to update premium amounts accounting for necessary reserve revaluations. In Section 6, we replace the individual revision formula with a collective one, considering all policyholders who entered the portfolio during a given year (i.e. a cohort of new contracts).

In Section 7, the indexing is performed for the whole portfolio, accounting for new business and lapses. The final Section 8 concludes the paper, revisiting some assumptions.

2 Actuarial model

2.1 Two-decrement model

The origin of time is chosen at policy issue. Time t stands for the seniority of the policy (i.e., the time elapsed since policy issue). The policyholder's (integer) age at policy issue is denoted by x, so that upon survival at time k, he or she has reached age $x+k$. We denote the ultimate integer age by ω, assumed to be finite. This means that survival until integer age ω has a positive probability, whereas survival until integer age $\omega+1$ has probability zero.

We describe the lifelong health insurance policy considered in the previous section in a two-decrement Markov model, with states "active" (i.e. policy in force), "withdrawn" (i.e. policy has been cancelled) and "dead", abbreviated as "a", "w" and "d", respectively. Let X_k be the status of the contract at time k, starting from $X_0 = a$. The stochastic process $\{X_k,\ \ k = 0, 1, 2, \ldots\}$ describes the history of the contract.

For j and $k \in \{0, 1, 2, \ldots\}$, we define the sojourn (or non-exit) probability ${}_jp^{aa}_{x+k}$ as

$$ {}_jp^{aa}_{x+k} = \Pr[X_{k+j} = a | X_k = a]. \tag{2.1}$$

In words, the quantity defined in (2.1) is the probability that a policy in force at age $x+k$ is still in force j years later. In accordance with standard actuarial notation, we omit the index j when it is equal to unity. The ultimate integer age ω is such that $p^{aa}_{\omega-1} > 0$, while $p^{aa}_{\omega} = 0$.

The probability that a policy in force at age $x+k$ has ceased j years later (due to death or withdrawal), is denoted by ${}_jp^{a\bullet}_{x+k}$. This "exit" probability can be expressed as

$$ {}_jp^{a\bullet}_{x+k} = \Pr[X_{k+j} \neq a | X_k = a] = 1 - {}_jp^{aa}_{x+k}. \tag{2.2}$$

We also introduce the probabilities ${}_jp^{ad}_{x+k}$ and ${}_jp^{aw}_{x+k}$, which are defined by

$$ {}_jp^{ad}_{x+k} = \Pr[X_{k+j} = d | X_k = a] \text{ and } {}_jp^{aw}_{x+k} = \Pr[X_{k+j} = w | X_k = a]. \tag{2.3}$$

These are the probabilities of leaving the portfolio due to death and withdrawal, respectively, between ages $x+k$ and $x+k+j$.

2.2 Benefits and level premiums

The expected annual health cost at age $x+j$, which is denoted by b_{x+j}, is clearly random due to medical inflation. Let $b^{(0)}_{x+j}$ be an estimate at time 0 for the expected medical expenses in year $(0, 1)$ for a person aged $x+j$ at time 0. At time 0, the insurer needs an estimate of the future costs for premium calculation, starting from the current expected cost $b^{(0)}_{x+j}$ for individuals aged $x+j$, increased by the assumed inflation rate. The insurer assumes a constant yearly medical inflation $f \geq 0$ over the coming years. Hence, $b^{(0)}_{x+j}\ (1+f)^j$ is an estimate at time 0 of the expected medical expenses in year $(j, j+1)$ for a person aged $x+j$

in the beginning of that year. The results that we present hereafter can easily be generalised to the case of non-constant but deterministic estimates for future inflation in the coming years.

Throughout the paper, a superscript "(k)", $k = 0, 1, 2, \dots$, indicates that the quantity under consideration is based on information about medical costs available at time k. Hereafter, $\pi_{x,j}^{(k)}$ denotes the premium to be paid at time k for a contract that was underwritten at time $j \leq k$ at age x.

The tariff $\pi_{x,0}^{(0)}$ is determined from a technical basis, i.e. from assumptions about mortality, surrenders, interest and medical inflation. In this note, in order to emphasize on the medical inflation risk, we assume that the realizations of the technical basis follow the assumption. This means that the assumptions about mortality, surrenders and interest rates are not subject to revision. In addition, we assume that adverse selection has been ruled out by the insurer using an appropriate underwriting policy. We do not go into details on the aspects of adverse selection and its impact, however, we refer to Newhouse (1996) and Ellis et al. (2000) for more information about the topic in the health insurance market. On the other hand, the uncertainty about future medical inflation levels induces systematic risk. Therefore, the amount of future premiums is revised on a yearly basis, using the observed inflation in the past year. We assume that in later years there is no update of the constant inflation scenario f that was used for premium calculation at time 0 so that the whole process is based on the tariff $\pi_{x,0}^{(0)}$ known at policy issue.

The level yearly premium $\pi_{x,0}^{(0)}$ for a health insurance contract underwritten at current time 0 on an insured aged x is determined by means of the equivalence principle. Let $v(0,j)$ be the discounting factor over the period $(0,j)$. The expected present value (or actuarial value) $B_x^{(0)}$ of the benefits paid by the insurer is then given by

$$B_x^{(0)} = \sum_{j=0}^{\omega-x} b_{x+j}^{(0)} \ (1+f)^j \ {}_jE_x^{aa} \tag{2.4}$$

where ${}_jE_x^{aa}$ is the actuarial discounting factor accounting for mortality, lapses and interest, over the period $(0,j)$, i.e.

$${}_jE_x^{aa} = v(0,j)_j p_x^{aa}.$$

Furthermore, let $\ddot{a}_x^{aa}$ be the actuarial value of an annuity-due paying a unit amount per year, as long as the policy is in force, i.e.

$$\ddot{a}_x^{aa} = \sum_{j=0}^{\omega-x} {}_jE_x^{aa}. \tag{2.5}$$

We then have

$$\pi_{x,0}^{(0)} = \frac{B_x^{(0)}}{\ddot{a}_x^{aa}}. \tag{2.6}$$

In the following sections, we present an actuarially sound methodology for revising the level of the premium as inflation emerges over time. In order to ease the notations, we drop the superscript "^{aa}" for the actuarial discounting factor and the annuity value and simply denote ${}_jE_x^{aa}$ as ${}_jE_x$ and $\ddot{a}_x^{aa}$ as $\ddot{a}_x$.

3 Adapting the premium and/or the reserve level at time 1

3.1 Accumulated reserve

Suppose that we have arrived at time 1 and that the policy that was underwritten at age x at time 0 is still in force. This means that at time 1, a positive prospective reserve

$$V_{x+1}^{(0)} = (1+f)\, B_{x+1}^{(0)} - \pi_{x,0}^{(0)}\, \ddot{a}_{x+1}, \tag{3.1}$$

is required for the insurer now age $x+1$, where $B_{x+1}^{(0)}$ and $\ddot{a}_{x+1}$ are defined similarly to (2.4) and (2.5), respectively.

Taking into account that the premium $\pi_{x,0}^{(0)}$ was determined via the equivalence principle (2.6), the prospective expression (3.1) for $V_{x+1}^{(0)}$ at time 1 can be transformed into the retrospective expression

$$V_{x+1}^{(0)} = \left(\pi_{x,0}^{(0)} - b_x^{(0)}\right) \left({}_1E_x\right)^{-1}, \tag{3.2}$$

which stands for the available reserve of the policyholder.

3.2 Revision of benefits

Suppose that the inflation for medical expenses observed during the first year is given by $f^{(1)}$. This means that at time 1, due to the observed medical inflation in the past year, the expected annual medical expenses $b_{x+1+j}^{(0)}$ have to be updated to

$$b_{x+1+j}^{(1)} = \left(1+f^{(1)}\right) b_{x+1+j}^{(0)}, \qquad j = 0, 1, 2, \ldots \tag{3.3}$$

Notice that we assume in (3.3) that medical inflation over the past year is age-independent. This assumption and how it can be relaxed is discussed in the closing Section 8.

Taking into account this assumed uniformity of medical inflation over all ages, we find that at time 1, the actuarial value of future benefits $(1+f)\, B_{x+1}^{(0)}$, which was based on estimates available at time 0, has to be updated to

$$B_{x+1}^{(1)} = (1+f^{(1)}) B_{x+1}^{(0)}. \tag{3.4}$$

3.3 Premium and/or reserve update

The required (prospective) reserve thus becomes

$$B_{x+1}^{(1)} - \pi_{x,0}^{(0)}\, \ddot{a}_{x+1},$$

which coincides with the available (retrospective) provision $V_{x+1}^{(0)}$ in (3.1) only if the observed inflation $f^{(1)}$ in the first year is equal to the assumed inflation f at time 0. This means that, due to the update of the actuarial value at age $x+1$ of future medical expenses, the available provision $V_{x+1}^{(0)}$ that was determined without knowing the observed medical inflation in the first year, turns out to be insufficient to cover future liabilities in case $f^{(1)} > f$. In order to restore the actuarial equivalence, the premium $\pi_{x,0}^{(0)}$ and/or the available provision $V_{x+1}^{(0)}$ will have to be updated to levels $\pi_{x,0}^{(1)}$ and $V_{x+1}^{(1)}$, respectively. Any pair $\left(V_{x+1}^{(1)},\ \pi_{x,0}^{(1)}\right)$ satisfying the equality

$$V_{x+1}^{(1)} = B_{x+1}^{(1)} - \pi_{x,0}^{(1)}\ \ddot{a}_{x+1} \tag{3.5}$$

will perform the task of resetting the actuarial equivalence. Hence, updating the premium and the available provision at time 1 can be performed in an infinite number of ways. Notice that (3.5) is the prospective reserve at time 1, based on updated benefits and premiums.

Subtracting (3.5) from (3.1), we find that for any pair $\left(V_{x+1}^{(1)},\ \pi_{x,0}^{(1)}\right)$ which restores the actuarial equivalence, the new premium level $\pi_{x,0}^{(1)}$ at time 1 is given by

$$\pi_{x,0}^{(1)} = \pi_{x,0}^{(0)} + \left(f^{(1)} - f\right)\ \pi_{x+1,0}^{(0)} - \frac{V_{x+1}^{(1)} - V_{x+1}^{(0)}}{\ddot{a}_{x+1}} \tag{3.6}$$

where

$$\pi_{x+1,0}^{(0)} = \frac{B_{x+1}^{(0)}}{\ddot{a}_{x+1}} \tag{3.7}$$

is the level premium at time 0 for a health insurance contract underwritten at that time on a person aged $x+1$.

Remark 3.1. *In the special case where $f = 0$ and the insurer decides to update the premium according to the observed medical inflation $f^{(1)}$, i.e.*

$$\pi_{x,0}^{(1)} = \left(1 + f^{(1)}\right) \pi_{x,0}^{(0)},$$

we find from (3.1), (3.4) and (3.5) that

$$V_{x+1}^{(1)} = \left(1 + f^{(1)}\right) V_{x+1}^{(0)}.$$

This means that in case no inflation is taken into account to determine the initial premium level $\pi_{x,0}^{(0)}$, indexing the premium according to the observed medical inflation $f^{(1)}$ requires the same proportional update of the available reserve.

3.4 Adapting the premium, only

Let us now assume that the level of the available provision is left unchanged, i.e.

$$V_{x+1}^{(0)} = V_{x+1}^{(1)}. \tag{3.8}$$

This means that the deviation of observed inflation $f^{(1)}$ from assumed inflation f in the first year is completely financed by the policyholder. From (3.6) it follows then that the new premium level at time 1 is given by

$$\pi_{x,0}^{(1)} = \pi_{x,0}^{(0)} + \left(f^{(1)} - f\right)\ \pi_{x+1,0}^{(0)}. \tag{3.9}$$

A similar formula has been obtained by Schneider (2002) in the particular case $f = 0$. Formula (3.9) shows that the premium increase $\pi_{x,0}^{(1)} - \pi_{x,0}^{(0)}$ at time 1 can be interpreted as the level premium corresponding to a "new" insurance contract underwritten at time 1 offering benefits with actuarial value equal to the benefit increases $\left(f^{(1)} - f\right) B_{x+1}^{(0)}$. This can be intuitively explained as follows: due to the increase in future medical costs from $(1+f)\, B_{x+1}^{(0)}$ to $(1+f^{(1)})B_{x+1}^{(0)}$, the policyholder now aged $x+1$ must virtually buy at time 1 a supplementary insurance policy, covering the benefit increase $\left(f^{(1)} - f\right) B_{x+1}^{(0)}$, whose price is $\left(f^{(1)} - f\right) \pi_{x+1,0}^{(0)}$ adding to $\pi_{x,0}^{(0)}$ in (3.9).

Formula (3.9) is a simple rule for updating the premium level at time 1: the new premium level $\pi_{x,0}^{(1)}$ follows from the original premium, the observed inflation over the past year and the insurer's tariff at time 0. The premium formula (3.9) can be rewritten in the following form:

$$\pi_{x,0}^{(1)} = \left(1 + \frac{\pi_{x+1,0}^{(0)}}{\pi_{x,0}^{(0)}} \left(f^{(1)} - f\right)\right) \pi_{x,0}^{(0)}.$$

This expression shows that the actual indexing for the original premium $\pi_{x,0}^{(0)}$ is $\left(f^{(1)} - f\right) \frac{\pi_{x+1,0}^{(0)}}{\pi_{x,0}^{(0)}}$. In case no inflation assumption is made at policy issue, i.e. $f = 0$, the proportional increase of the premium will be different (and usually higher) than the observed medical inflation $f^{(1)}$ over the first year. Also notice that in case the inflation assumption in the first year was too conservative, i.e. $f^{(1)} < f$, the premium level may be reduced at time 1.

4 Adapting the premium level at time k

4.1 Accumulated reserve

Suppose that we have arrived at time $k = 2, 3, \ldots$ and that the policy that was underwritten on the person aged x at time 0 is still in force. The observed medical inflation up to time $k-1$ has been taken into account by restoring the actuarial equivalence and updating the premium levels at times $1, 2, \ldots, k-1$. Suppose that the deviations of observed inflation from assumed inflation f are completely financed by the policyholder, which means that the available provisions are not updated. Let $V_{x+k-1}^{(k-1)}$ and $\pi_{x,0}^{(k-1)}$ be the available provision and the premium level determined at time $k-1$. They were set such that the actuarial equivalence at time $k-1$ was restored:

$$V_{x+k-1}^{(k-1)} = B_{x+k-1}^{(k-1)} - \pi_{x,0}^{(k-1)} \ddot{a}_{x+k-1}. \tag{4.1}$$

In this formula, $B_{x+k-1}^{(k-1)}$ is the actuarial value at time $k-1$ of the future health benefits related to an insured of age $x+k-1$ at that time, i.e.

$$B_{x+k-1}^{(k-1)} = \sum_{j=0}^{\omega-x-k+1} b_{x+k-1+j}^{(k-1)} \ (1+f)^j \ {}_jE_{x+k-1},$$

where $b^{(k-1)}_{x+k-1+j}$ is the expected health benefit in year $(k-1,k)$ for a person aged $x+k-1+j$ in the beginning of that year, based on the information available at time $k-1$:

$$b^{(k-1)}_{x+k-1+j} = b^{(0)}_{x+k-1+j} \prod_{l=1}^{k-1} (1+f^{(l)}).$$

Notice that we assume that inflation is age-independent. Furthermore, the ${}_jE_{x+k-1}$ are the appropriate actuarial discount factors, accounting for mortality, lapses and interest, while $\ddot{a}_{x+k-1}$ is an annuity-due paying an amount of 1 per year to the insured with current age $x+k-1$, as long as the policy remains in force.

The available provision at time k for the policy still in force at that time is then given by

$$V^{(k-1)}_{x+k} = \left(V^{(k-1)}_{x+k-1} + \pi^{(k-1)}_{x,0} - b^{(k-1)}_{x+k-1}\right) \left({}_1E_{x+k-1}\right)^{-1}. \tag{4.2}$$

The available provision acts as a savings account, which first builds up by the premium surpluses in the early years (when $\pi^{(k-1)}_{x,0} > b^{(k-1)}_{x+k-1}$), whereas it melts away in later years due to the premium shortfalls in these years (when $\pi^{(k-1)}_{x,0} < b^{(k-1)}_{x+k-1}$).

Taking into account the restored actuarial equivalence (4.1) at time $k-1$, the available reserve $V^{(k-1)}_{x+k}$ at time k can be expressed in the following prospective form:

$$V^{(k-1)}_{x+k} = (1+f)\, B^{(k-1)}_{x+k} - \pi^{(k-1)}_{x,0}\ \ddot{a}_{x+k}, \tag{4.3}$$

with

$$B^{(k-1)}_{x+k} = \sum_{j=0}^{\omega-x-k} b^{(k-1)}_{x+k+j}\ (1+f)^j\ {}_jE_{x+k}.$$

4.2 Premium update

Due to the observed medical inflation during the k-th year, the actuarial value of future health benefits $(1+f)\, B^{(k-1)}_{x+k}$ based on an evaluation at time $k-1$ has to be updated to $B^{(k)}_{x+k}$ which, under the age-uniform medical inflation $f^{(k)} \geq 0$, is given by

$$B^{(k)}_{x+k} = (1+f^{(k)}) B^{(k-1)}_{x+k}.$$

At time k, the premium level $\pi^{(k-1)}_{x,0}$ and/or the available provision $V^{(k-1)}_{x+k}$ have to be replaced by $\pi^{(k)}_{x,0}$ and $V^{(k)}_{x+k}$, respectively, in order the restore the actuarial equivalence:

$$V^{(k)}_{x+k} = B^{(k)}_{x+k} - \pi^{(k)}_{x,0}\ \ddot{a}_{x+k}. \tag{4.4}$$

From (4.3) and (4.4), we find that for any actuarial equivalence restoring pair $\left(V^{(k)}_{x+k}, \pi^{(k)}_{x,0}\right)$, the updated premium $\pi^{(k)}_{x,0}$ can be expressed as

$$\pi^{(k)}_{x,0} = \pi^{(k-1)}_{x,0} + \left(f^{(k)} - f\right)\ \pi^{(k-1)}_{x+k,k-1} - \frac{V^{(k)}_{x+k} - V^{(k-1)}_{x+k}}{\ddot{a}_{x+k}}, \tag{4.5}$$

where $\pi^{(k-1)}_{x+k,k-1}$ is given by

$$\pi^{(k-1)}_{x+k,k-1} = \frac{B^{(k-1)}_{x+k}}{\ddot{a}_{x+k}}, \tag{4.6}$$

which is the initial level premium for a lifelong health insurance contract underwritten at time $k-1$ on a person of age $x+k$ at that time.

Assuming again that the observed inflation $f^{(k)}$ is solely financed by the policyholder, i.e.

$$V^{(k)}_{x+k} = V^{(k-1)}_{x+k},$$

the premium updating formula (4.5) reduces to

$$\pi^{(k)}_{x,0} = \pi^{(k-1)}_{x,0} + \left(f^{(k)} - f\right) \pi^{(k-1)}_{x+k,k-1}. \tag{4.7}$$

Hence, the updated premium $\pi^{(k)}_{x,0}$ at time k is equal to the premium $\pi^{(k-1)}_{x,0}$ paid the year before, augmented by the product of the medical inflation deviation $\left(f^{(k)} - f\right)$ observed over the past year and last year's premium for a new contract that was issued on a person of age $x+k$.

The updated premium $\pi^{(k)}_{x,0}$ can also be written as

$$\pi^{(k)}_{x,0} = \left(1 + \frac{\pi^{(k-1)}_{x+k,k-1}}{\pi^{(k-1)}_{x,0}} \left(f^{(k)} - f\right)\right) \pi^{(k-1)}_{x,0}. \tag{4.8}$$

This expression clearly shows that the proportional premium increase at time k is different from the medical inflation deviation $\left(f^{(k)} - f\right)$ that was revealed over the past year. Obviously, the proportional premium increase depends on the age x at policy issue as well as on the number k of years that the contract has been in force so far. The proportional increase of the premium will usually be larger for policies that are longer in force.

From (4.7) which holds for $k = 1, 2, 3, \ldots$, we find that

$$\pi^{(k)}_{x,0} = \pi^{(0)}_{x,0} + \sum_{j=1}^{k} \left(f^{(j)} - f\right) \pi^{(j-1)}_{x+j,j-1}, \tag{4.9}$$

with

$$\pi^{(j-1)}_{x+j,j-1} = \frac{B^{(j-1)}_{x+j}}{\ddot{a}_{x+j}}. \tag{4.10}$$

Formula (4.9) has an intuitive interpretation. Indeed, the premium level $\pi^{(k)}_{x,0}$ to be paid at time k is equal to the initial premium level $\pi^{(0)}_{x,0}$, augmented with the extra premia for all the virtually added contracts covering the increases in medical costs in any of the first k years.

Using the assumption of an age-uniform medical inflation in each of the past years, we find that

$$B^{(j-1)}_{x+j} = B^{(0)}_{x+j} \prod_{l=1}^{j-1} \left(1 + f^{(l)}\right), \qquad j = 1, 2, 3, \ldots, \tag{4.11}$$

provided we set $\prod_{l=1}^{0}(1+f^{(l)})=1$, by convention. Taking into account (4.10), the expression above immediately leads to

$$\pi_{x+j,j-1}^{(j-1)} = \pi_{x+j,0}^{(0)} \prod_{l=1}^{j-1} \left(1+f^{(l)}\right), \qquad j=1,2,3,\ldots \tag{4.12}$$

It follows then from (4.9) that the updated premium level $\pi_{x,0}^{(k)}$ at time k can be written as

$$\pi_{x,0}^{(k)} = \pi_{x,0}^{(0)} + \sum_{j=1}^{k} \left(f^{(j)} - f\right) \pi_{x+j,0}^{(0)} \prod_{l=1}^{j-1} (1+f^{(l)}). \tag{4.13}$$

This is an expression for the updated premium level $\pi_{x,0}^{(k)}$ at time k for a contract underwritten to a person aged x at time 0, in terms of the observed inflation levels $f^{(1)}, f^{(2)}, ..., f^{(k)}$ in the past years and the insurer's tariff $\pi_{y,0}^{(0)}$ at policy issue.

5 Case study: The new indexing mechanism for the Belgian medical insurance market

5.1 Indexing rule imposed by the Belgian law

In Belgium, public health insurance is organized by the state through the Federal Agency RIZIV-INAMI and operated by several so-called "sickness funds" (non-profit organizations). In addition to this compulsory medical cover, individual or group private health insurance contracts are sold which pay (part of) the non-covered medical costs and supplements. These private insurance products are regulated via the so-called "Law Verwilghen" of 20 July 2007 and "Law Reynders" (also called the "Law Verwilghen II") of 17 June 2009, both named after the ministers in charge.

Individual private coverages are lifelong by law. In case of level premiums, the initial premium amount is fixed at policy issue and then linked to the CPI or to a specific medical index. The Federal Agency KCE studied different indexing mechanisms, see Devolder et al. (2008). The Royal Decree defining the premium indexing mechanism to be applied by insurance companies operating in Belgium has been cancelled on December 29, 2011, one of the reasons being that the updating mechanism for the premiums to adjust for observed but unanticipated inflation did not take into account the shortfall of the accumulated reserves.

Recently, a Belgian Royal Decree dated March 18, 2016 introduced a new updating mechanism for individual private coverages. The newly proposed mechanism, which is intended to be both appropriate for the insurers and transparent towards the clients, is given by

$$\pi_{x,0}^{(k)} = \left(1 + 1.5\ f^{(k)}\right) \pi_{x,0}^{(k-1)}, \tag{5.1}$$

subject to some restrictions that are not be considered in the present paper (as they only apply to very special cases, not encountered in our numerical examples). Here it is assumed that the level premiums are determined without assuming future inflation: in all our numerical illustrations, we always take $f=0$. Henceforth, the premiums calculated according to (5.1) are called the “1.5 rule premiums” and denoted by $\pi_{x,0}^{(k)}$ (150%). We compare these premiums with the ”exact premiums”, which follow from (4.8):

$$\pi_{x,0}^{(k)} = \left(1 + \alpha_{x,0}^{(k)}\ f^{(k)}\right) \pi_{x,0}^{(k-1)}, \tag{5.2}$$

with

$$\alpha_{x,0}^{(k)} = \frac{\pi_{x+k,k-1}^{(k-1)}}{\pi_{x,0}^{(k-1)}}, \tag{5.3}$$

which holds under the assumption that $f = 0$ and that reserves are not updated, i.e. $V_{x+k}^{(k-1)} = V_{x+k}^{(k)}$. In order to distinguish the exact premiums (5.2) from the premiums derived from the 1.5 rule, we often denote them by $\pi_{x,0}^{(k)}$ (exact).

Hereafter, we investigate whether using 1.5 instead of the correct indexing factor appears to be a sufficiently prudent approach for the insurance company and at the same time, a not too conservative approach for the insured. Let us mention that (5.1) determines the maximum premium update allowed by the law, so that we mainly adopt the insurer’s point of view and examine whether a rule like (5.1) may threaten its solvency or not.

5.2 Technical basis

In this subsection, we describe the technical basis and the assumptions made in our numerical calculations. The assumed discount factors correspond to a constant yearly interest rate $i = 1\%$. Mortality is assumed to obey the first Heligman-Pollard law, in the sense that the ”independent mortality rates” for ages $x = 25, 26, \ldots, 109$ are given by

$$\frac{q_x}{1-q_x} = A^{(y+B)^C} + De^{-E(\ln x - \ln F)^2} + GH^x \tag{5.4}$$

with $A = 0.00054$, $B = 0.017$, $C = 0.101$, $D = 0.00013$, $E = 10.72$, $F = 18.67$, $G = 1.464 \times 10^{-5}$ and $H = 1.11$. Furthermore, we fix the ultimate age to $\omega = 110$. For a justification of this mortality law, we refer to Pitacco (1999) and Vercruysse et al. (2013).

The corresponding “dependent mortality rates” p_x^{ad} in the two-decrement model satisfy the relation

$$p_x^{ad} = q_x \left(1 - \frac{p_x^{aw}}{2-q_x}\right),$$

which holds under the assumption of a uniform distribution of decrements in any year for each of the two single decrement models involved, see Section 8.10.2 in Dickson et al. (2013). The “dependent lapse rates” p_x^{aw} are assumed to be given by

$$p_x^{aw} = 0.1 - 0.002(x - 20)$$

at age $x = 25, 26, \ldots, 70$ and 0 otherwise.

The severity of medical claims is based on age-specific annual claim amounts including an accident-childbearing hump and a concave behavior near the end of the lifetime. The data have been normalized to fit the annual expected hospitalization cost provided by the

Belgian *Mutualité Chrétienne*; they are displayed in Figure 1. The minimum age to purchase a private medical insurance is assumed to be 25.

As mentioned, no medical inflation is taken into account when setting future premiums, i.e. $f = 0$. In Figure 2, the resulting insurer's tariff $\pi^{(0)}_{x,0}$, $x = 25, 26, ..., 109$, is depicted. We observe that the accident-childbearing hump and the concave behavior for higher ages visible on Figure 1 are impacting the tariff structure, causing the break right before age 40.

5.3 The 1.5 rule

In the remainder of Section 5, we will investigate the evolution over time of the successive premiums to be paid by an insured of age 25 at policy issue. Concerning the evolution over time, we assume that experienced interest rates, mortality rates and withdrawal rates are equal to their corresponding values in the technical basis. Furthermore, we assume an observed medical inflation of 2 percent, i.e. $f^{(k)} = 2\%$ for all $k = 1, 2, \ldots$

In Figure 3, we compare the correctly updated premiums $\pi^{(k)}_{25,0}(\text{exact})$, $k = 1, 2, \ldots$, determined from (5.2) with the premiums $\pi^{(k)}_{25,0}(150\%)$ updated via the 1.5 rule (5.1). We observe that the 1.5 rule appears to be conservative for the insurer as the related future premiums are always larger than the correctly updated premiums. In Figure 3, also the premiums arising from the 1.0 rule, where premiums are updated with a proportion equal to the observed medical inflation, i.e.

$$\pi^{(k)}_{25,0} = (1 + f^{(k)})\pi^{(k-1)}_{25,0}$$

are shown. Obviously, this 1.0 rule ignores the fact that built up reserves have to be taken into account in the updating process. As expected, Figure 3 shows that in this case the updating rule leads to insufficient premiums.

The previous figure does not necessary imply that the correct indexing factors $\alpha^{(k)}_{25,0}$ are uniformly smaller than 1.5. In order to verify this, we draw the correct indexing factor $\alpha^{(k)}_{25,0}$ as a function of k in Figure 4. We observe that $\alpha^{(k)}_{25,0}$ is initially smaller than 1.5, then it becomes slightly larger than 1.5 and finally decreases to become again lower than 1.5. Hence, the 1.5 rule is very conservative in the periods where the available reserve is relatively small. Overall, we can conclude that replacing $\alpha^{(k)}_{25,0}$ by 1.5 turns out to be conservative on the premium level for a person who underwrites the policy at age 25. Indeed, the intermediate period where $\alpha^{(k)}_{25,0}$ is slightly larger than 1.5 is more than compensated by the conservative premium increases in the initial period, where the exact factors $\alpha^{(k)}_{25,0}$ are (substantially) smaller than 1.5.

5.4 Sensitivity analysis

In this subsection, we perform a sensitivity analysis by varying the constant interest rate, the observed medical index and the age of the policyholder.

Apart from the base case $i = 1\%$ for the interest rate, we consider two additional scenarios, namely $i = 0.5\%$ and $i = 5\%$. Figure 5 shows the effect of these changes on the tariff at time 0. Obviously, the corresponding premiums are decreasing with the interest rate.

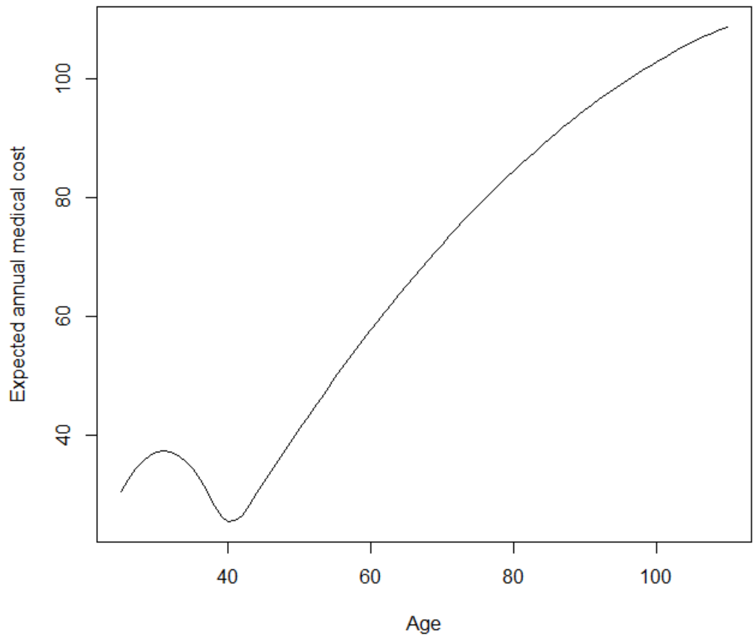

Figure 1: Expected annual medical costs $b_y^{(0)}$ as a function of age y.

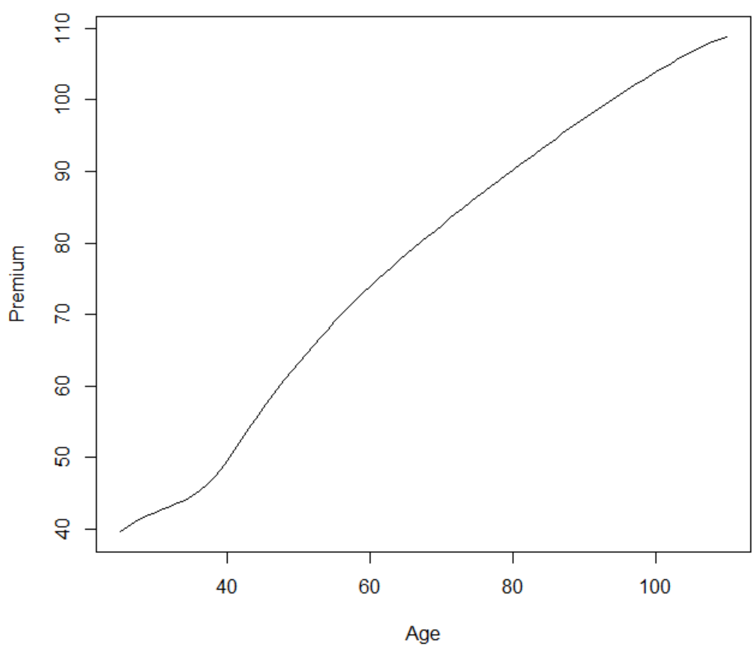

Figure 2: Tariff at policy issue: $\pi_{x,0}^{(0)}$ for $x \in \{25, 26, ..., 109\}$.

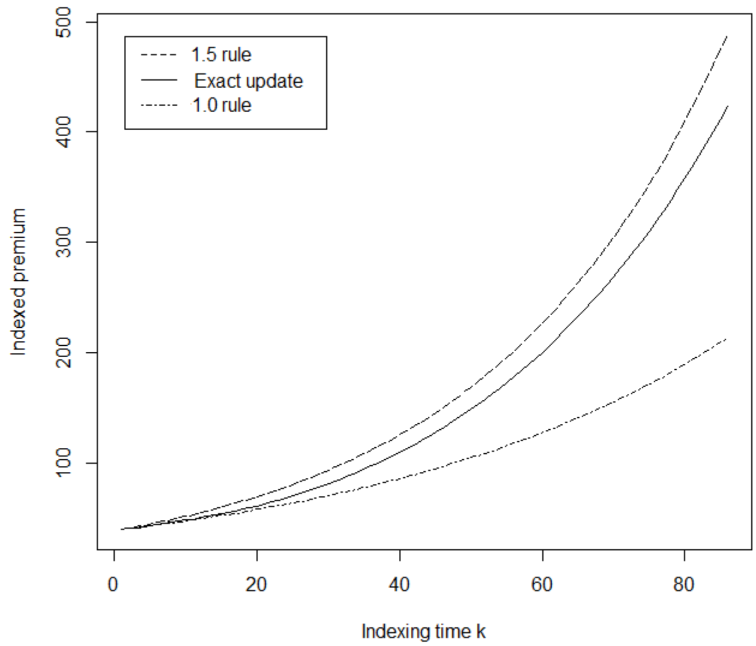

Figure 3: $\pi_{25,0}^{(k)}$ (exact), $\pi_{25,0}^{(k)}$ (150%) and $\pi_{25,0}^{(k)}$(100%) for $k = 1, 2, \ldots$.

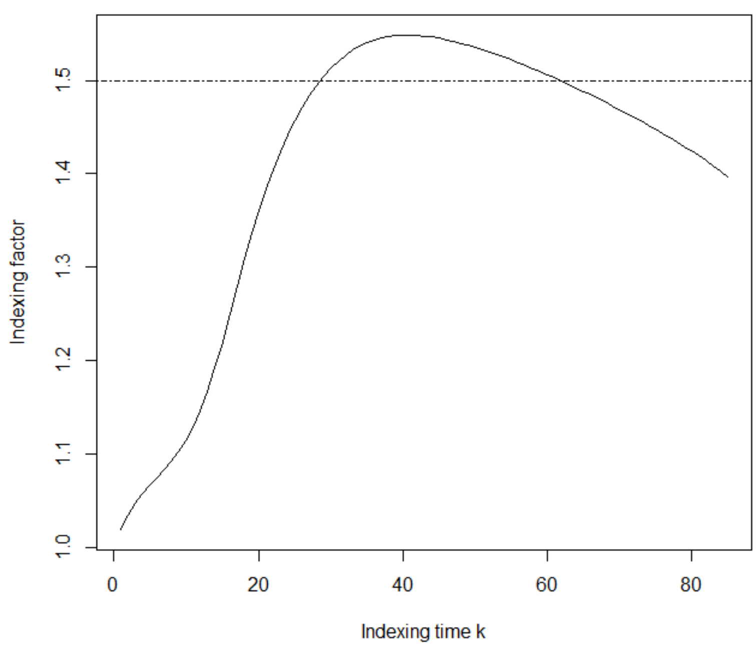

Figure 4: Indexing factor $\alpha_{25,0}^{(k)}$ as a function of time-since-issue k.

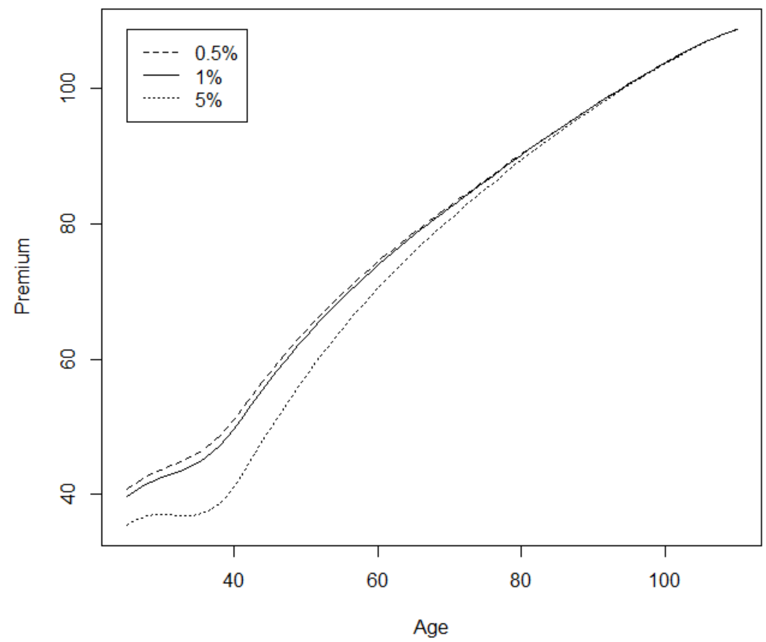

Figure 5: Tariff $\pi_{y,0}^{(0)}$ at time 0 as a function of age y for different technical interest rates.

In Figure 6 we depict the behavior of the exact indexing factor $\alpha_{25,0}^{(k)}$ for the three interest rate scenarios considered. Overall, the indexing factor increases with i. The 1.5 rule (5.1) seems to be more conservative in a context of low interest rates.

In Figure 7, we show ratio $\frac{\pi_{25,0}^{(k)}(150\%)}{\pi_{25,0}^{(k)}(\text{exact})}$ between the premium updated with (5.1) and (4.8), respectively, for different interest rate assumptions. The ratio is always greater than 1, meaning that in any of the considered cases the 1.5 rule is conservative.

Returning to the base case for the interest rate, i.e. $i = 1\%$, let us now vary the observed medical inflation according to the following scenarios: either $f^{(k)} = 0.5\%$ for all $k = 1, 2, \ldots$ or $f^{(k)} = 3\%$ for all $k = 1, 2, \ldots$. Figure 8 shows the evolutions of the indexing factors $\alpha_{25,0}^{(k)}$ for the two different medical inflation scenarios considered.

In Figure 9, we compare the premiums $\pi_{25,0}^{(k)}$ (exact) with the corresponding premiums $\pi_{25,0}^{(k)}$ (150%) for different inflation scenarios. In this case, the 1.5 rule is more conservative for higher medical inflation levels. Indeed, we can see in Figure 8 that for an observed inflation of 0.5%, the 1.5 approximation is under-estimating the exact update after the 25 first years of the coverage.

To end this section, we return again to the base case technical assumptions and consider a constant inflation scenario with $f^{(k)} = 2\%$. We compare the premium evolution of three insureds with initial ages 25, 35 and 50, respectively.

Figure 10 shows that $\alpha_{25,0}^{(k)}$ is initially smaller than both $\alpha_{35,0}^{(k)}$ and $\alpha_{50,0}^{(k)}$, but later becomes greater. Furthermore, for the 35 and 50 year old persons at policy issue, the correct index factors $\alpha_{35,0}^{(k)} < 1.5$ and $\alpha_{50,0}^{(k)} < 1.5$ for any k, showing that in these two cases, the 1.5 rule is always conservative. In Figure 11, $\pi_{x,0}^{(k)}$ (exact) and $\pi_{x,0}^{(k)}$ (150%) for $x = 25, 35$ and 50 are shown as a function of k. For the three ages, the premium based on the 1.5 rule is uniformly

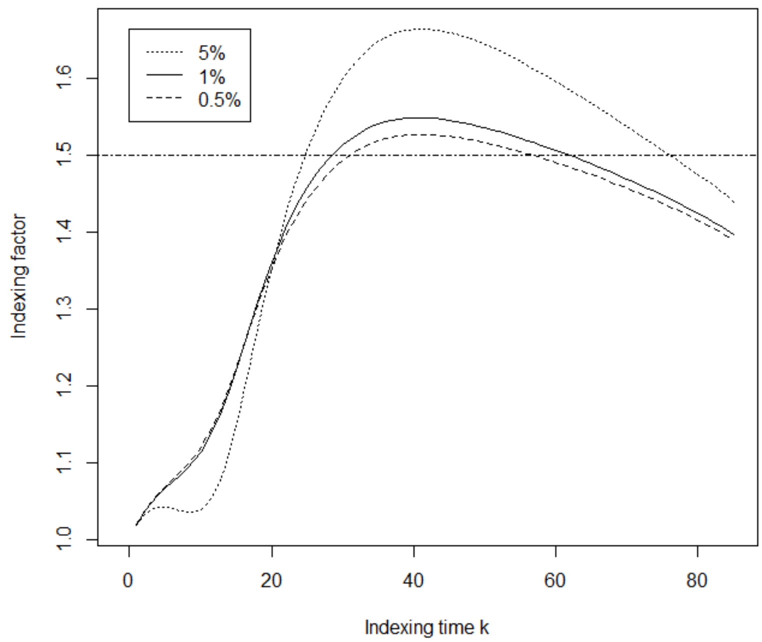

Figure 6: Indexing factor $\alpha_{25,0}^{(k)}$ for different technical interest rates.

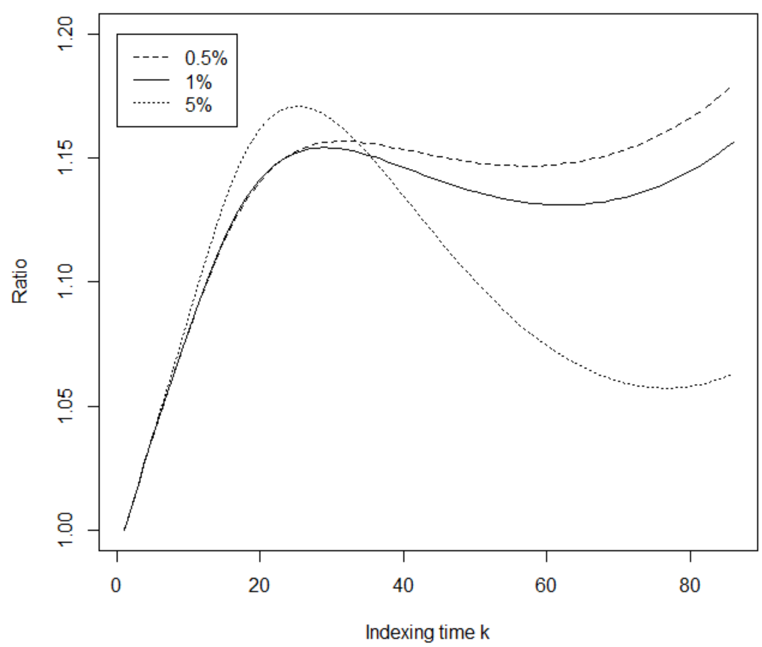

Figure 7: Ratio $\frac{\pi_{25,0}^{(k)}(150\%)}{\pi_{25,0}^{(k)}(\text{exact})}$ for three different technical interest rate assumptions.

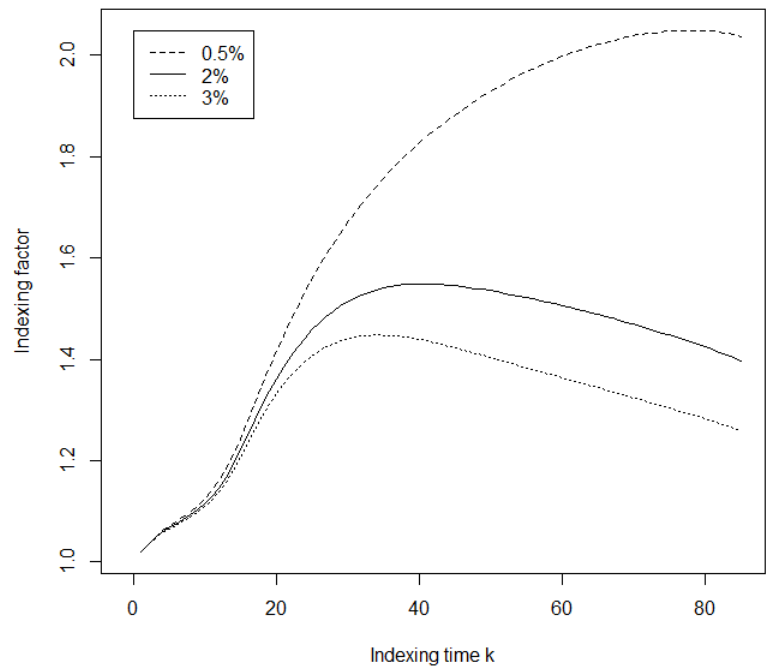

Figure 8: $\alpha_{25,0}^{(k)}$ as a function of k, for different inflation scenarios.

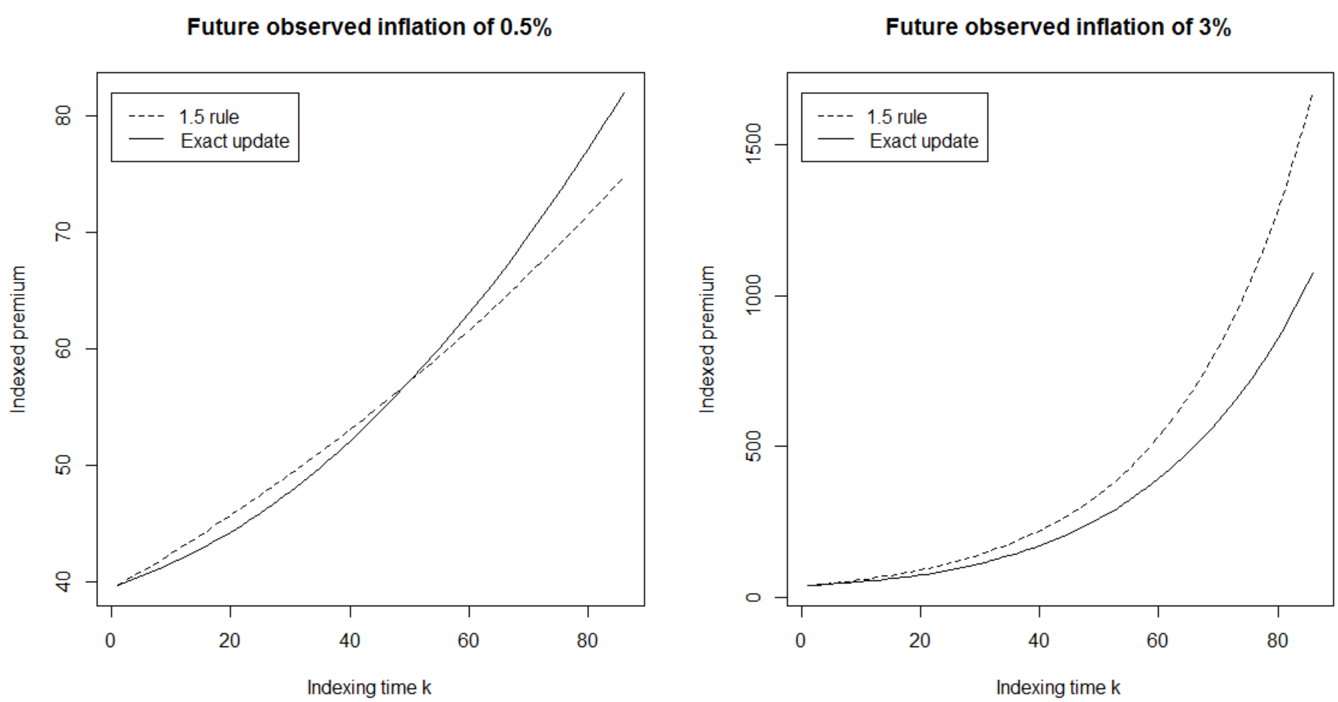

Figure 9: Premiums $\pi_{25,0}^{(k)}$ (exact) and $\pi_{25,0}^{(k)}$ (150%) as a function of k, for the two different medical inflation scenarios.

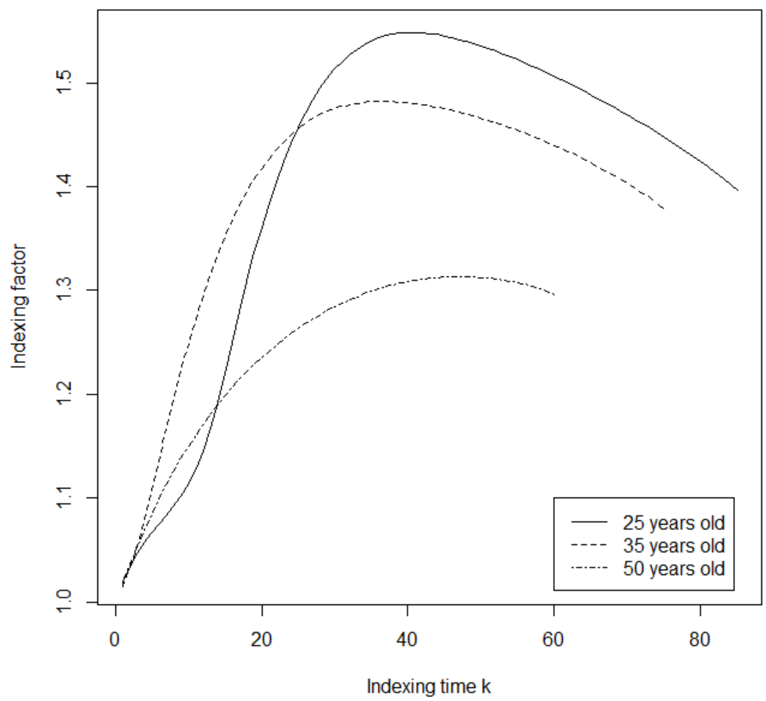

Figure 10: Indexing factors $\alpha^{(k)}_{25,0}$, $\alpha^{(k)}_{35,0}$ and $\alpha^{(k)}_{50,0}$ as a function of k.

larger than the corresponding exact premium, indicating that the 1.5 rule is a conservative rule in these three cases.

6 An aggregate premium indexing mechanism for a group of new entrants

6.1 Individual vs aggregate indexing approach

The exact premium indexing mechanism (4.8) that we considered so far is based on yearly restoring the actuarial equivalence on policy level. In this sense, we can call it an *individual* premium indexing mechanism. In this section, we will present an *aggregate* premium indexing mechanism, where the yearly restoring of the actuarial equivalence is performed at aggregate level for all insureds that have entered the portfolio at the same time.

Let us consider a portfolio of new entrants at time 0. We follow this portfolio over time, assuming that the technical basis for mortality, surrender and interest is in line with what is experienced over time. For each age x, let us denote by $l^{(k)}_{x,0}$ the number of persons who entered the portfolio at age x at time 0 and who are still in the portfolio at time $k = 0, 1, 2, \ldots$. Of course, we have $l^{(k)}_{x,0} = 0$ for $x + k > \omega$.

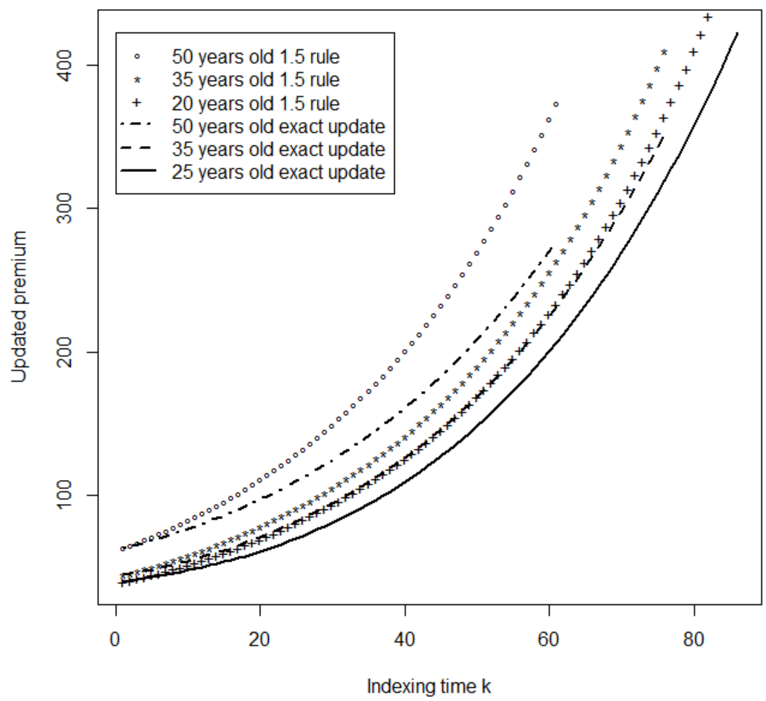

Figure 11: $\pi_{x,0}^{(k)}$ (exact) and $\pi_{x,0}^{(k)}$ (150%) for $x = 25, 35$ and 50 as a function of k.

6.2 Adapting the premium level at time 1

Suppose we are at time 1. In the previous sections, we adapt the premiums on an individual (contract per contract) basis, using the relation (4.8). This means that the premium is reset at time 1 in such a way that for each contract, the available provision $V_{x+1}^{(0)}$ and the required provision $V_{x+1}^{(1)}$ are equal at that time:

$$V_{x+1}^{(0)} = V_{x+1}^{(1)}, \tag{6.1}$$

with $V_{x+1}^{(0)}$ and $V_{x+1}^{(1)}$ given by (3.1) and (3.5), respectively.

In this section, we propose to restore the equivalence between available provision and required provision only on an aggregate level, i.e. we replace the individual equivalence relation (6.1) by the following aggregate equivalence relation:

$$\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)} \, V_{x+1}^{(0)} = \sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)} \, V_{x+1}^{(1)}, \tag{6.2}$$

where x_0 is the youngest age in the portfolio. Taking into account (3.1) and (3.5), we find that (6.2) can be rewritten as

$$\sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)} \, \pi_{x,0}^{(1)} \, \ddot{a}_{x+1} = \sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)} \, \pi_{x,0}^{(0)} \, \ddot{a}_{x+1} + \left(f^{(1)} - f\right) \sum_{x=x_0}^{\omega-1} l_{x,0}^{(1)} \, B_{x+1}^{(0)}. \tag{6.3}$$

This is an equality on "aggregate level" for the population at time 1 who entered at time 0. Without any further requirement, there exists an infinite number of premiums $\left(\pi^{(1)}_{x_0,0},\ \pi^{(1)}_{x_0+1,0},\ \ \dots,\ \pi^{(1)}_{\omega-1,0}\right)$ that satisfy this aggregate equivalence condition. In order to specify the new tariff, we assume now that at time 1, each premium is adapted by the same factor, namely

$$\pi^{(1)}_{x,0} = \left(1 + \alpha^{(1)}_0 (f^{(1)} - f)\right)\pi^{(0)}_{x,0}, \qquad x = x_0,\ x_0 + 1, \dots, \omega - 1. \tag{6.4}$$

Inserting these expressions for the $\pi^{(1)}_{x,0}$ in the equivalence relation (6.3) leads to

$$\alpha^{(1)}_0 = \frac{\sum_{x=x_0}^{\omega-1} l^{(1)}_{x,0}\ B^{(0)}_{x+1}}{\sum_{x=x_0}^{\omega-1} l^{(1)}_{x,0}\ \pi^{(0)}_{x,0}\ \ddot{a}_{x+1}}.$$

Taking into account (2.6), this expression for $\alpha^{(1)}_0$ can be transformed into

$$\alpha^{(1)}_0 = \frac{\sum_{x=x_0}^{\omega-1} l^{(1)}_{x,0}\ \pi^{(0)}_{x+1,0}\ \ddot{a}_{x+1}}{\sum_{x=x_0}^{\omega-1} l^{(1)}_{x,0}\ \pi^{(0)}_{x,0}\ \ddot{a}_{x+1}}.$$

The indexing factor $\alpha^{(1)}_0$ is applied at time 1 for updating the premiums of all policies that were underwritten the year before. Typically, the numerator exceeds the denominator, so that $\alpha^{(1)}_0 > 1$, and all premiums $\pi^{(1)}_{x,0}$ are increased by the factor $\alpha^{(1)}_0(f^{(1)} - f)$, which is larger than the difference between experienced and assumed inflation.

6.3 Adapting the premium level at time k

Let us suppose that we have arrived at time k and that at any times $1, 2, \dots, k-1$, we have reset the premiums on an aggregate level according to a similar procedure as the one performed at time 1. The time-k aggregate equivalence relation between available and required provision can now be expressed as follows:

$$\sum_{x=x_0}^{\omega-k} l^{(k)}_{x,0}\ V^{(k-1)}_{x+k} = \sum_{x=x_0}^{\omega-k} l^{(k)}_{x,0}\ V^{(k)}_{x+k}. \tag{6.5}$$

This means that the available provision at time k, aggregated over all policies that were underwritten at time 0 and are still in force at time k, is set equal to the required aggregate provision for this same set of policies. Taking into account (4.3) and (4.4), the equivalence relation (6.5) can be restated as follows:

$$\sum_{x=x_0}^{\omega-k} l^{(k)}_{x,0}\ \pi^{(k)}_{x,0}\ \ddot{a}_{x+k} = \sum_{x=x_0}^{\omega-k} l^{(k)}_{x,0}\ \pi^{(k-1)}_{x,0}\ \ddot{a}_{x+k} + \left(f^{(k)} - f\right) \sum_{x=x_0}^{\omega-k} l^{(k)}_{x,0}\ B^{(k-1)}_{x+k}. \tag{6.6}$$

Let us now again assume a uniform updating mechanism for all policies under consideration, i.e. the premiums $\pi^{(k)}_{x,0}$ are determined via

$$\pi^{(k)}_{x,0} = \left(1 + \alpha^{(k)}_0 \left(f^{(k)} - f\right)\right)\pi^{(k-1)}_{x,0}.$$

Inserting these expressions in (6.6) leads to

$$\alpha_0^{(k)} = \frac{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \, B_{x+k}^{(k-1)}}{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \, \pi_{x,0}^{(k-1)} \, \ddot{a}_{x+k}},$$

or, taking into account (4.6),

$$\alpha_0^{(k)} = \frac{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \, \pi_{x+k,k-1}^{(k-1)} \, \ddot{a}_{x+k}}{\sum_{x=x_0}^{\omega-k} l_{x,0}^{(k)} \, \pi_{x,0}^{(k-1)} \, \ddot{a}_{x+k}} \tag{6.7}$$

for the updating factor $\alpha_0^{(k)}$ that is applied to all policies that were underwritten at time 0 and are still in force at time k.

6.4 Numerical illustration

Let us illustrate the aggregate method explained in the previous subsections by a numerical example. Calculations are performed with the technical basis of Section 5.2. Again, we assume that experienced interest, mortality and lapse over time follows the technical basis. Furthermore, we assume an experienced medical inflation of 2% per year.

We consider two different portfolio compositions, see Figure 12. Portfolio 1 has a concentration of younger new entrants at time 0, which is often the case in practice. In Portfolio 2, the new entrants are uniformly distributed over the ages 20 to 50, followed by a decreasing number of new entrants up to age 55.

The number of new entrants in each portfolio at time 0 is equal to 10 000. As we observe from (6.7), the aggregate indexing factors $\alpha_0^{(k)}$ depend on the age distribution of new entrants at time 0.

In Figure 13 the aggregate index factor functions $\alpha_0^{(k)}$ are displayed for portfolio 1 and portfolio 2, as well as the individual index factor functions $\alpha_{x,0}^{(k)}$ for ages $x = 25, 35$ and 50. We observe that a younger portfolio of new entrants leads to larger aggregate index factors $\alpha_0^{(k)}$ in the major part of the curve. Also the individual index factor functions $\alpha_{x,0}^{(k)}$ are larger in the main part of their domain for younger new entrants. The aggregate index factors $\alpha_0^{(k)}$ turn out to be highest (in the major part of the curve) for a portfolio of entrants of age 25 only (as in this case the aggregate factor curve $\alpha_0^{(k)}$ coincides with the individual factor curve $\alpha_{25,0}^{(k)}$). We conclude that in this particular numerical illustration, younger entrants are better off with the aggregate method than older ones, unless the percentage of elderly new entrants is sufficiently high, which is usually not the case in practice.

In Figure 14, for ages $x = 25, 35$ and 50, the individual updated premium curve $\pi_{x,0}^{(k)}$ (individual) is compared with the aggregate updated premiums curves $\pi_{x,0}^{(k)}$ (portfolio 1) and $\pi_{x,0}^{(k)}$ (portfolio 2). We observe that

$$\pi_{25,0}^{(k)} \,(\text{individual}) \approx \pi_{25,0}^{(k)} \,(\text{portfolio 1}) > \pi_{25,0}^{(k)} \,(\text{portfolio 2})\,.$$

For policyholders aged 50 at policy issue, we find that

$$\pi_{50,0}^{(k)} \,(\text{individual}) < \pi_{50,0}^{(k)} \,(\text{portfolio 2}) < \pi_{50,0}^{(k)} \,(\text{portfolio 1})\,.$$

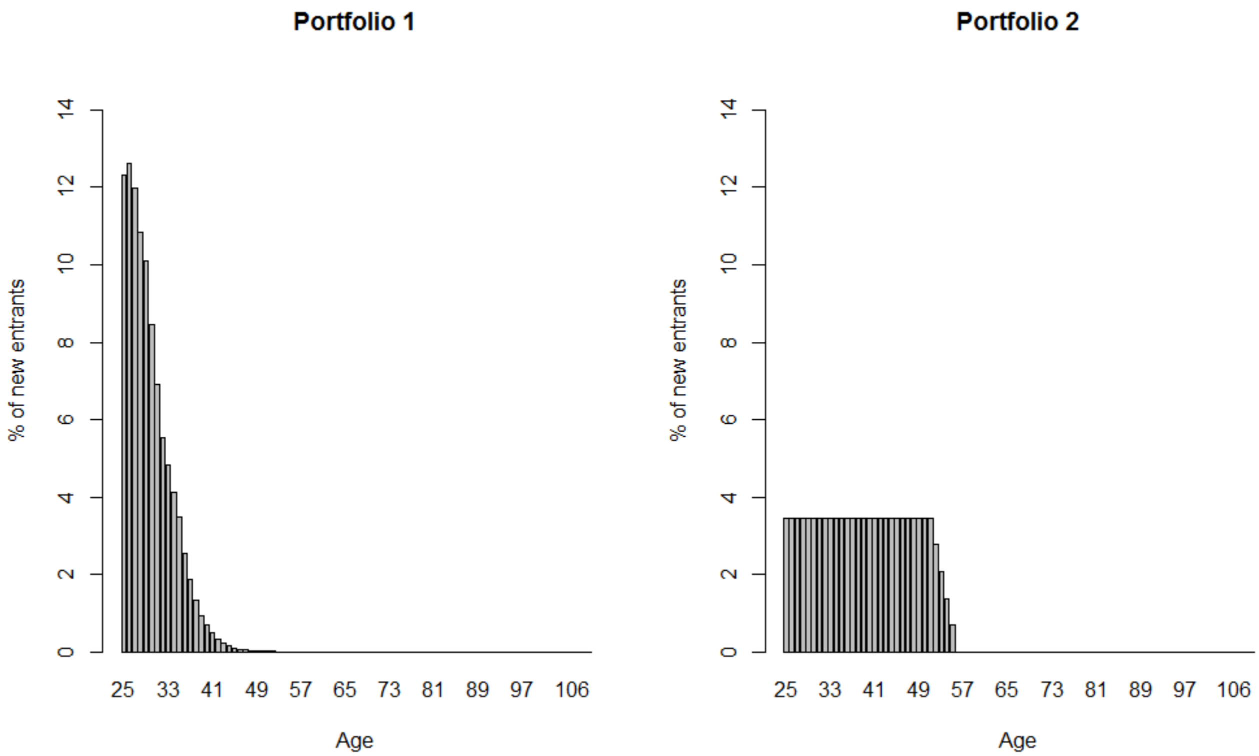

Figure 12: Relative numbers of new entrants $l_{x,0}^{(0)}$ as a function of age x.

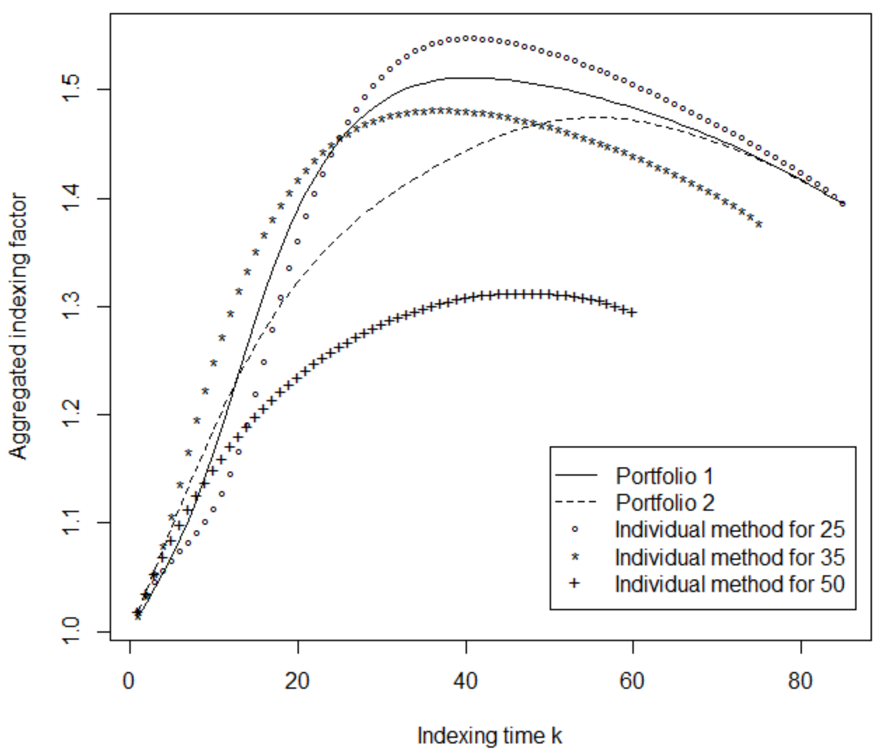

Figure 13: Aggregate index factors $\alpha_0^{(k)}$ and individual index factors $\alpha_{x,0}^{(k)}$ as functions of k.

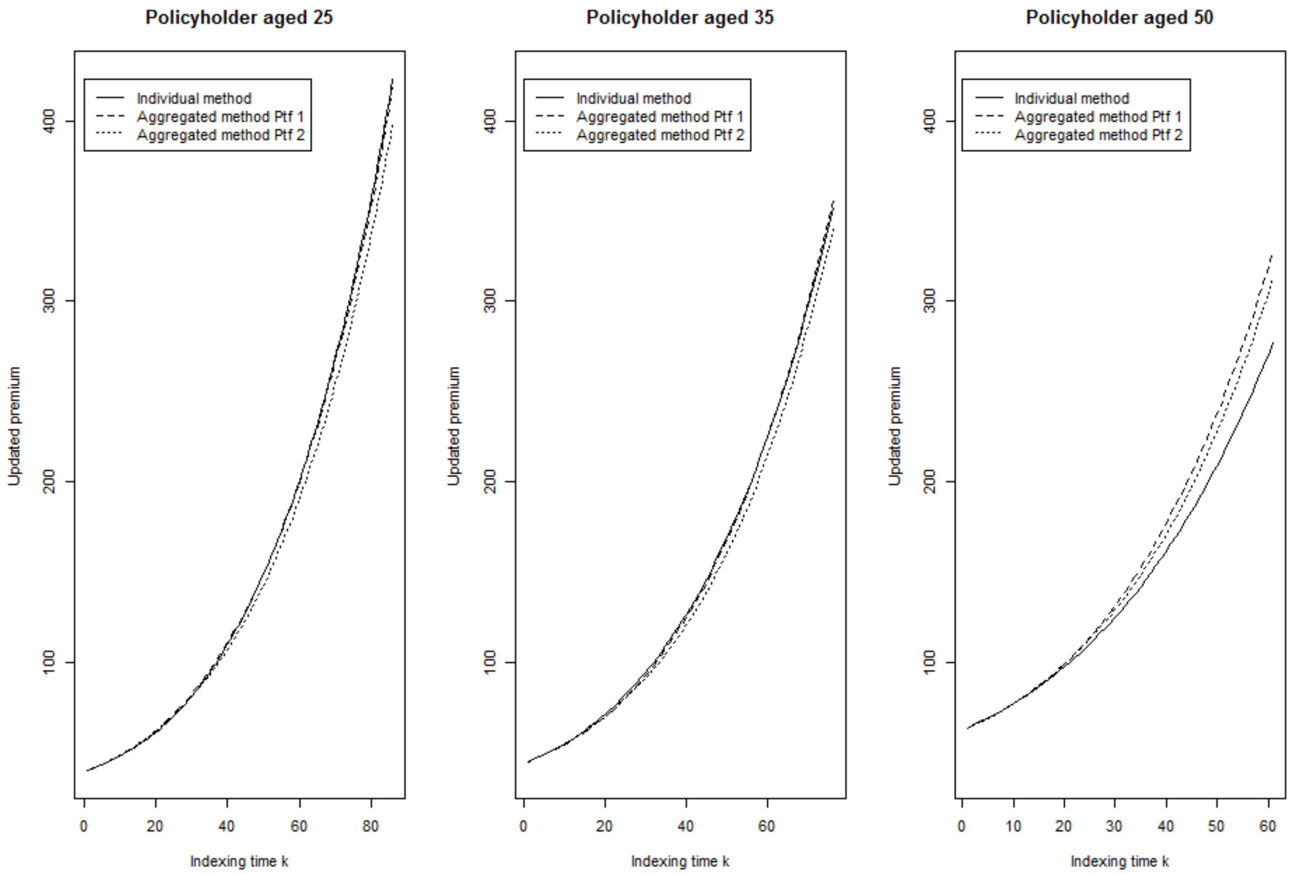

Figure 14: Updated premiums $\pi_{x,0}^{(k)}$ (individual) and $\pi_{x,0}^{(k)}$ (aggregate) for different ages and portfolio compositions.

We can again conclude that for young entrants ($x = 25$), the aggregate indexing method is to be preferred, in particular when there is a substantial number of older new entrants. On the other hand, for older policyholders ($x = 50$), the individual indexing mechanism always leads to lower premiums. Notice however that the impact on premiums remains moderate.

7 An aggregate premium indexing mechanism for a whole portfolio

7.1 Cohort vs portfolio indexing approach

In the previous section, we investigated an aggregate premium indexing mechanism, where aggregation was performed for a group of new entrants at a particular time. In this section, we consider an aggregate premium indexing mechanism, where the aggregation is performed at portfolio level. Each year, the actuarial equivalence will be restored by imposing an equality between the available and the required provisions for the whole existing portfolio at that moment. The related proportional increase of the premiums is chosen to be equal for all members of the portfolio at that moment.

Hereafter, we will denote by $l_{x,j}^{(k)}$ the number of persons observed in the portfolio at time k, who entered that portfolio at age x at time $j \leq k$. At time k, these persons have attained age $x+k-j$. Obviously, we have $l_{x,j}^{(k)} = 0$ for $x > \omega - k + j$.

7.2 Adapting the premium level at time 1

Suppose that we have arrived at time 1. According to the aggregate premium indexing mechanism at portfolio level, the premiums $\pi_{x,j}^{(1)}$ are chosen such that the available and the required aggregate provisions are equal:

$$\begin{aligned} &\sum_{j\leq 0} \sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)} \left((1+f)\, B_{x+1-j}^{(0)} - \pi_{x,j}^{(0)}\, \ddot{a}_{x+1-j} \right) \\ = &\sum_{j\leq 0} \sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)} \left((1+f^{(1)}) B_{x+1-j}^{(0)} - \pi_{x,j}^{(1)}\, \ddot{a}_{x+1-j} \right). \end{aligned} \tag{7.1}$$

We impose a uniform updating mechanism for all insureds in the portfolio at time 1. This means that the premiums $\pi_{x,j}^{(1)}$ satisfy

$$\pi_{x,j}^{(1)} = \left(1 + \alpha^{(1)}(f^{(1)} - f)\right)\pi_{x,j}^{(0)} \tag{7.2}$$

for an aggregate factor $\alpha^{(1)}$. Inserting (7.2) in the equilibrium equation (7.1) leads to the following expression for the updating coefficient $\alpha^{(1)}$:

$$\alpha^{(1)} = \frac{\sum_{j\leq 0} \sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)}\, B_{x+1-j}^{(0)}}{\sum_{j\leq 0} \sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)}\, \pi_{x,j}^{(0)}\, \ddot{a}_{x+1-j}}.$$

Taking into account that $B_{x+1-j}^{(0)} = \pi_{x+1-j,0}^{(0)}\, \ddot{a}_{x+1-j}$, we can rewrite the previous expression in terms of the premium structure at time 0:

$$\alpha^{(1)} = \frac{\sum_{j\leq 0} \sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)}\, \pi_{x+1-j,0}^{(0)}\, \ddot{a}_{x+1-j}}{\sum_{j\leq 0} \sum_{x=x_0}^{\omega-1+j} l_{x,j}^{(1)}\, \pi_{x,j}^{(0)}\, \ddot{a}_{x+1-j}}. \tag{7.3}$$

The updated premiums at time 1 follow from (7.2) and (7.3).

7.3 Adapting the premium level at time k

Let us assume that we have arrived at time k and that we have restored the actuarial equilibrium at times $1, 2, \ldots, k-1$ on an aggregate portfolio level, applying a procedure similar to the one applied at time 1. This has lead to the aggregate updating factors $\alpha^{(1)}, \alpha^{(2)}, \ldots, \alpha^{(k-1)}$. Now, having arrived at time k, the updated premiums $\pi_{x,j}^{(k)}$ are chosen such that the available and the required aggregate provisions for the whole portfolio are again equal at time k:

$$\sum_{j\leq k-1}\sum_{x=x_0}^{\omega-k+j} l_{x,j}^{(k)}\Big((1+f)\ B_{x+k-j}^{(k-1)} - \pi_{x,j}^{(k-1)}\ \ddot{a}_{x+k-j}\Big)$$
$$= \sum_{j\leq k-1}\sum_{x=x_0}^{\omega-k+j} l_{x,j}^{(k)}\ \Big((1+f^{(k)})\ B_{x+k-j}^{(k-1)} - \pi_{x,j}^{(k)}\ \ddot{a}_{x+k-j}\Big).$$

Assuming a uniform updating factor $\alpha^{(k)}$ for the premiums, i.e.

$$\pi_{x,j}^{(k)} = \Big(1+\alpha^{(k)}(f^{(k)}-f)\Big)\ \pi_{x,j}^{(k-1)} \tag{7.4}$$

for all j and k, the equivalence relation above leads to

$$\alpha^{(k)} = \frac{\sum_{j\leq k-1}\ \sum_{x=x_0}^{\omega-k+j}\ l_{x,j}^{(k)}\ B_{x+k-j}^{(k-1)}}{\sum_{j\leq k-1}\ \sum_{x=x_0}^{\omega-k+j}\ l_{x,j}^{(k)}\ \pi_{x,j}^{(k-1)}\ \ddot{a}_{x+k-j}}$$

or equivalently, taking into account that $B_{x+k-j}^{(k-1)} = \pi_{x+k-j,k-1}^{(k-1)}\ \ddot{a}_{x+k-j}$,

$$\alpha^{(k)} = \frac{\sum_{j\leq k-1}\ \sum_{x=x_0}^{\omega-k+j}\ l_{x,j}^{(k)}\ \pi_{x+k-j,k-1}^{(k-1)}\ \ddot{a}_{x+k-j}}{\sum_{j\leq k-1}\ \sum_{x=x_0}^{\omega-k+j}\ l_{x,j}^{(k)}\ \pi_{x,j}^{(k-1)}\ \ddot{a}_{x+k-j}} \tag{7.5}$$

Inserting this expression for $\alpha^{(k)}$ in (7.4) leads to updated premiums $\pi_{x,j}^{(k)}$ at time k.

7.4 Numerical illustration

Let us now illustrate the aggregate method described above by a numerical example. Again, the calculations are performed with the technical basis and assumptions of Section 5.2. In addition, we assume that the past observed inflation, i.e. the inflation in the years before time 0, was equal to 2%.

We assume that both portfolios exist since 20 years, i.e. since time −20. We consider three scenarios for the absolute number of new entrants in any year. In a first scenario, we observe a stable number of 10 000 new entrants per year. In a second scenario, we consider a linearly increasing number of new entrants over time, from 10 000 entrants at time −20 to 100 000 at time 65. In a third scenario, there is a linearly decreasing number of new entrants over time, from 100 000 entrants at time −20 to 10 000 at time 65.

Concerning the age-distribution of the new entrants, we consider two portfolios with the proportions of new entrants in any year as described in Figure 12 for time 0.

In Figure 15, the indexing factors $\alpha^{(k)}$ are shown for each scenario within each portfolio. We observe that in the first scenario, with 10 000 new entrants per year, the indexing factor converges to a fixed number. This is due to the use of time-independent probabilities which implies that in case of stable number of new entrants, the composition of the portfolio becomes constant from some k. Furthermore, an increasing absolute number of entrants leads to lower indexing factors than a decreasing absolute number.

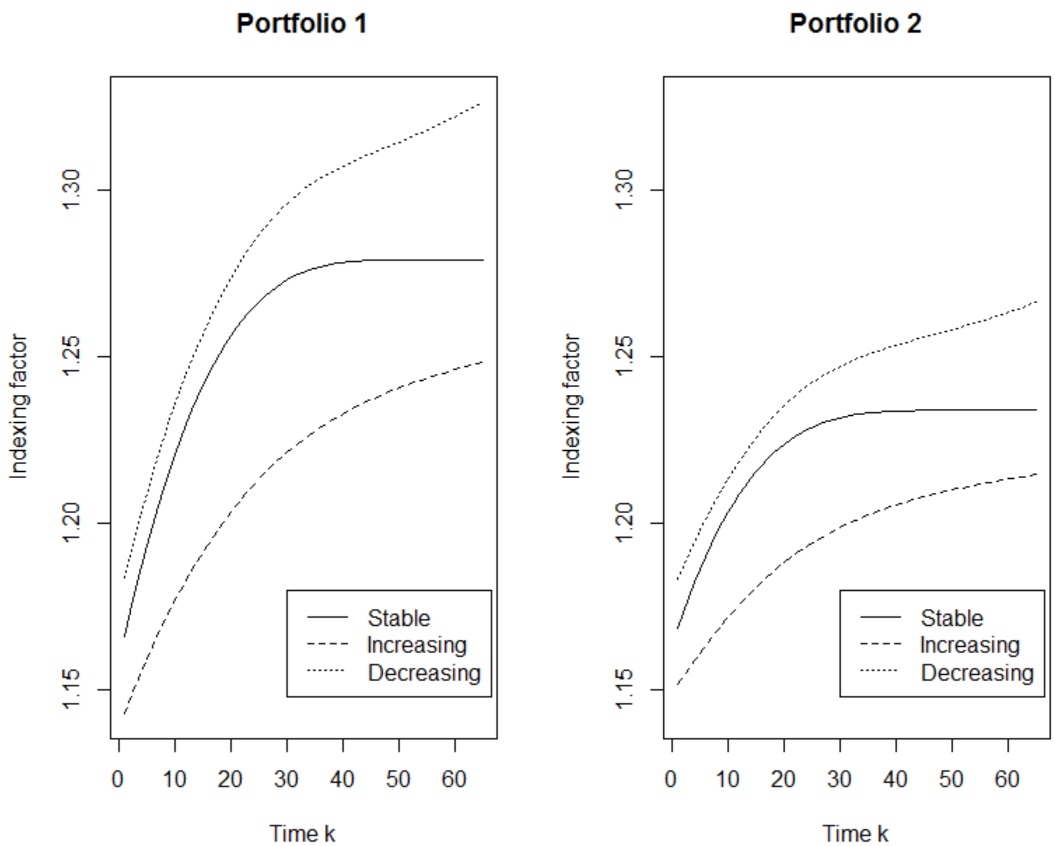

Figure 15: Aggregate indexing factors $\alpha^{(k)}$ at portfolio level for two portfolios.

Let us now consider three policyholders who enter the portfolio at time 0, at the ages of 25, 35 and 50, respectively. In Figure 16, we show the evolution of their premiums over time. For each of them, we compare their individually updated premiums (5.2) with the premiums coming from (7.4), updated via the portfolio-based aggregate method. Concerning the composition of the insurance portfolio, we consider each of the portfolios and scenarios described above. For policyholders aged 25 and 35 at policy issue, any of the aggregate indexing methods leads to lower premiums than the individual indexing method. For policyholders aged 50 at policy issue, the lowest premiums are obtained when the number of new entrants is increasing with an age distribution similar to Portfolio 2, while the highest premiums occur when the number of new entrants is decreasing following the age distribution of Portfolio 1.

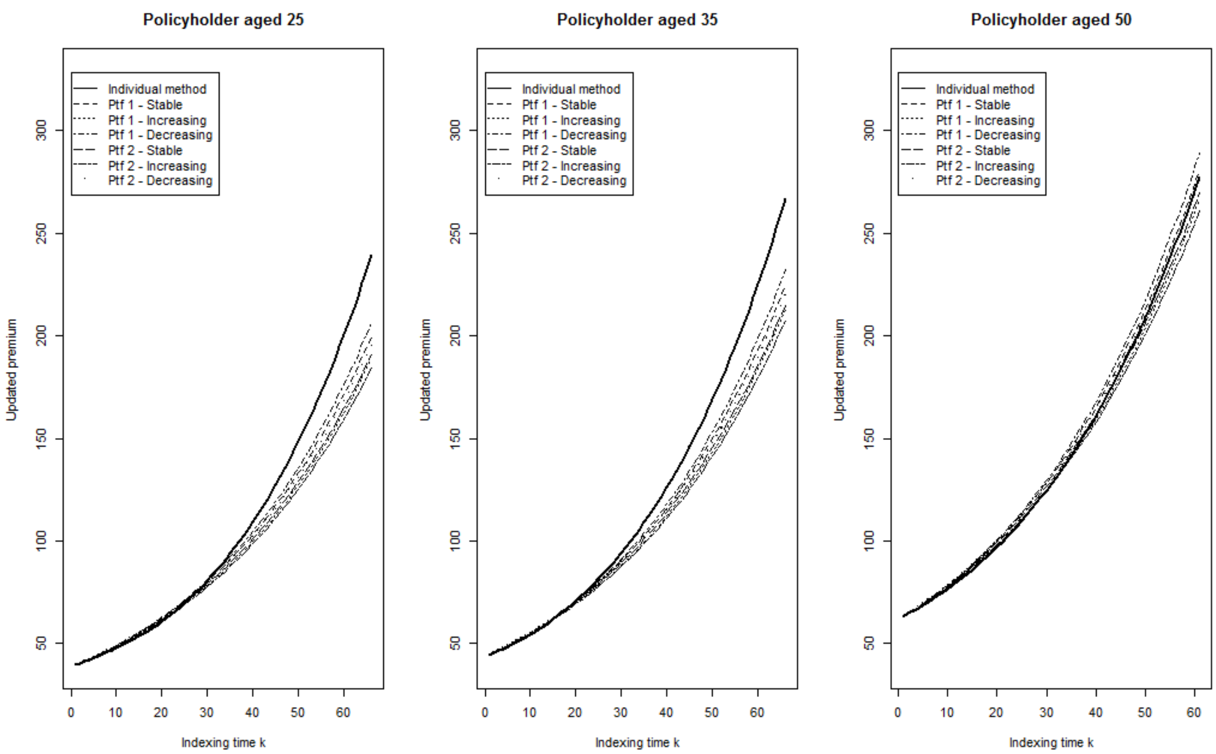

Figure 16: Updated premiums: individual vs. portolio aggregation based.

8 Final remarks

As an accurate prediction of future medical inflation is practically impossible, an insurer selling lifelong health insurance coverage usually does not make a guaranteed assumption concerning future inflation at policy issue, in order to avoid the risk of underestimating this inflation. Moreover, the systematic nature of medical inflation, affecting each policy in the same direction, implies that the Law of Large Numbers, which is the crucial concept on which insurance business is built, is not applicable. As a consequence, in lifelong health insurance, the uncertainty concerning medical inflation usually remains with the insureds, who will pay variable future premiums which are directly related to the level of inflation that will emerge over time.

In this note, we described a relatively simple but actuarially adequate individual updating mechanism (4.7), which can also be expressed as (4.8), for such lifelong health insurance contracts. The premium level is yearly updated, taking into account the observed inflation over the past year. From formula (4.8) it follows that the required proportional increase of the premium does not only depend on the difference between observed and assumed medical inflation in the previous year, but also on the age at policy issue and on the time since policy issue.

Although the proposed updating mechanism is technically correct, these dependency phenomena might not be easy to explain to consumers. Nevertheless, we are convinced that applying an indexing mechanism that causes less opposition from consumers but is at the same time less sound, is highly questionable, in particular in case of contracts of a lifelong nature. In the updating formula (4.8), one could e.g. replace the factor $\frac{\pi^{(k-1)}_{x+k,k-1}}{\pi^{(k-1)}_{x,0}}$ by a constant factor $(1+\alpha)$. Such a factor, which should depend on the composition of the portfolio, will likely induce subsidizing solidarity from the "newer" policies to the policies that are already longer in force.

In the previous sections, we have implicitly assumed that the technical basis assumptions concerning mortality, lapse and interest rates are identical to their respective experienced values that emerge over time. We have also assumed that the insurer's portfolio is not subject to adverse selection. This implies that the insurer makes no technical gain on these assumptions, and as a consequence, it was reasonable to assume that observed medical inflation is completely financed by the policyholders. In reality, the above-mentioned assumptions will be chosen in a conservative way, implying that the insurer will very likely make technical gains. These technical gains might be (partly) redistributed to the insureds via an increase of the available provisions, implying a partial financing of the observed medical inflation by the insurer.

Throughout this note, we made the simplifying assumption that in any year $k = 1, 2, \ldots,$ observed inflation is uniform over all ages, i.e.

$$b^{(k)}_{x+j} = (1 + f^{(k)})\, b^{(k-1)}_{x+j}, \qquad j = 1, 2, \ldots,$$

for some age-independent inflation factor $f^{(k)}$. Notice however that the results presented here can in a straightforward way be adapted to the case of age-specific medical inflation by replacing the inflation factor $f^{(k)}$ in the formula above by an age-dependent factor. In this case, we have that

$$b^{(k)}_{x+j} = (1 + f^{(k)}_{x+j})\, b^{(k-1)}_{x+j} \qquad j = 1, 2, \ldots$$

Once the age-specific inflation factors $f^{(k)}_{x+j}$ have been set, we can determine the global inflation factors $\overline{f}^{(k)}_{x+k}$ from

$$B^{(k)}_{x+k} = (1 + \overline{f}^{(k)}_{x+k})\; B^{(k-1)}_{x+k}.$$

Remark that the interpretation of the factors $\overline{f}^{(k)}_{x+k}$ is not straightforward, as it is a weighted average of the observed inflation factors for all ages from $x + k$, with weights that depend on age-specific expected health benefits and actuarial discount factors.

In the generalized setting with age-dependent inflation, the simple premium updating rule (4.7) has to be replaced by

$$\pi_{x,0}^{(k)} = \pi_{x,0}^{(k-1)} + \left(\overline{f}_{x+k}^{(k)} - f\right) \ \pi_{x+k,k-1}^{(k-1)},$$

with a similar interpretation as before: the updated premium $\pi_{x,0}^{(k)}$ at time k is equal to the premium $\pi_{x,0}^{(k-1)}$ paid the year before, augmented by the product of the deviation of global medical inflation factor $\overline{f}_{x+k}^{(k)}$ from the assumed inflation f and the initial premium for a new contract that was issued the year before on a person of age $x + k$.

Apart from the individual updating mechanism (4.8), where the actuarial equivalence is restored on a policy per policy basis, we also considered two aggregate updating mechanisms. The first aggregate method is based on a yearly renewed actuarial equilibrium that is applied to a group of insureds who all entered at the same time. The second aggregate method restores actuarial equilibrium based on portfolio level.

Summarizing, for any of the three methods the updated premium follows from

$$\pi_{x,j}^{(k)} = \left(1 + \alpha^{(k)}(f^{(k)} - f)\right) \ \pi_{x,j}^{(k-1)}.$$

For the individual method, we only consider the entry of the individual insured under consideration, and hence, $j = 0$, while the updating factor $\alpha^{(k)}$ follows from (4.8). For the first aggregate method, we consider the group of entrants at time 0, implying that $j = 0$ and $\alpha^{(k)}$ is given by (6.7). Finally, for the portfolio-based aggregate method, we consider all existing contracts in the current portfolio and hence $j = k, k-1, k-2, \ldots$, while the updating factor $a^{(k)}$ is given by (7.5).

Acknowledgment

The authors acknowledge the financial support of AG Insurance in the framework of the *AG Insurance Chair In Health Insurance* at KU Leuven. In addition, the authors wish to thank the anonymous referees for their valuable comments that helped to enhance the quality of the paper.

References

- Devolder, P., Denuit, M., Maréchal, X., Yerna, B.-L., Closon, J.-P., Léonard, C., Senn, A., Vinck, I. (2008). Construction d'un index médical pour les contrats privés d'assurance maladie. KCE Reports - Centre Fédéral d'Expertise des Soins de Santé, Volume 96B. Available from http://www.centredexpertise.fgov.be

- Dhaene, J., Godecharle, E., Antonio, K., Denuit, M. (2015). On the transferability of reserves in lifelong health insurance contracts. Submitted for publication.

- Dickson, D. C., Hardy, M. R., Waters, H. R. (2013). Actuarial Mathematics for Life Contingent Risks. Cambridge University Press.

- Haberman, S., Pitacco, E. (1999). Actuarial Models for Disability Insurance. CRC Press.
- Newhouse, J. (1996). Reimbursing health plans and health providers: Efficiency in production versus selection. Journal of Economic Literature 34, 1236-1263.
- Pitacco, E. (1999). Multistate models for long-term care insurance and related indexing problems. Applied Stochastic Models in Business and Industry 15, 429-441.
- Pitacco E. (2014). Health Insurance. Basic Actuarial Models. Springer.
- Schneider, E. (2002). The main features of German private health insurance. In the Proceedings of the 27th ICA Health Seminar, Cancun.
- Vercruysse, W., Dhaene, J., Denuit, M., Pitacco, E., Antonio, K. (2013). Premium indexing in lifelong health insurance. Far East Journal of Mathematical Sciences 2013, 365-384.
- Wynand P.M.M., Van De Ven, Randall P. Ellis (2000). Risk adjustment in competitive health plan markets. In Handbook of Health Economis, Volume 1, 755-845, Edited by A.J. Culyer and J.P. Newhouse.