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CONSUMPTION AND PORTFOLIO CHOICE OVER THE LIFE-CYCLE

by

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Consumption and Portfolio Choice over the Life-Cycle*

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Abstract

We augment a life-cycle model of consumption with given undiversifiable labor income risk (estimated from the PSID), to include portfolio choice between two assets: a safe asset as in the standard consumption literature, and a risky asset. The agent is not allowed to borrow at either the riskfree or risky rate. We find that the share of savings invested in the risky asset is hump-shaped over life, a pattern which seems to be supported by the data. Also, labor income risk crowds out asset holding risk. The consumption profile we obtain is similar to the one in the consumption literature.

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1 Introduction

The issue of portfolio choice over the life-cycle is encountered by every investor. Popular finance books (e.g. Malkiel, 1996) and financial counselors generally give the advice to shift the portfolio composition towards relatively safe assets, such as T-bills, and away from risky stocks as the investor grows older and reaches retirement. But what could be the economic justification for doing so?

A seminal reference addressing the problem of consumption and portfolio choice over the life-cycle is Samuelson (1969). The concept of 'businessman's risk' (i.e. holding risky stocks is only advisable for young businessmen, not for widows) is explored and rejected as invalid. However this conclusion is reached under the assumptions of independently and identically distributed returns and of no labor income. In later work Samuelson analysed the consequences of mean-reverting return processes to portfolio composition over the life-cycle (1991). It is intuitive that the length of an investor's horizon matters when facing asset returns that exhibit mean-reversion.

Another crucial element one needs to consider when discussing portfolio choice over the life-cycle is labor income and the risk associated with it. To the extent that the level and risk of the labor income stream change over the life-cycle, and to the extent that portfolio choice depends on these factors, the presence of labor income can provide a rationale for age-varying investment strategies. This is the route we explore in this paper.

A natural point of departure to model these issues is the recent consumption literature (Deaton, 1991, Carroll, 1997 and Gourinchas and Parker, 1996). Gourinchas and Parker find that in a life-cycle model young prudent agents typically behave as buffer-stock agents,[1] saving only enough to buffer adverse labor income shocks, while older agents, facing a less steep labor income profile, save mainly for retirement in the spirit of the Permanent Income Hypothesis. Our concern is which combination of assets is used for these different saving purposes. To that end, we use a life-cycle model of consumption behavior in the spirit of Gourinchas and Parker (1996) or Carroll (1997), with given undiversifiable labor income risk (estimated from the PSID), augmented to include the choice between two assets, a safe asset as in the standard consumption literature, and a risky asset. We extend Deaton's liquidity constraint by ruling out borrowing at either the riskfree or risky rate (no short-selling). We also allow for positive

[1] More precisely: buffer-stock behavior arises when the agent's expected income growth or discount rate are high relative to the rate of return on savings, so that in the absence of uncertainty and of capital market frictions he would be willing to borrow against future income.

correlation between shocks to 'labor income' and innovations to the return on the risky asset.[2]

In order to understand the effects of labor income risk on portfolio allocation, it is important to have in mind that a labor income stream constitutes the implicit holding of an asset. The crucial question is whether these implicit asset holdings are perceived as a closer substitute for riskfree asset holdings rather than for risky asset holdings. By examining the policy rules giving the optimal share of the portfolio allocated to the risky asset at a given age as a function of wealth, we address this question to find that labor income acts as a substitute for riskfree asset holdings, if labor income risk is uncorrelated with stock market risk. Subsequently, the evolution of these policy functions over life allows us to understand the simulation results.

We find for most parametrizations considered, that the share of savings invested in the risky asset is hump-shaped over life. Early in life the annuity value of future labor income rises because of the steep labor income profile. Moreover the analysis of the policy functions indicates that this labor income stream, though risky, substitutes for riskfree asset holdings. This induces the agent to gradually increase her proportional stock holdings early in life, as her implicit (i.e in the form of labor income) riskfree asset holdings rise. On the other hand, households approaching retirement decrease the share invested in risky assets since they are more dependent on accumulated wealth to finance future consumption and want to make up for the loss in implicit riskfree asset holdings. We also find that labor income risk crowds out asset holding risk, whether this risk is due to an increased variance of labor income shocks or to increased correlation between shocks to labor income and innovations to the return on the risky asset. When comparing portfolio allocations across different education groups, we obtain that the share of savings invested in stocks is highest for college graduates, a result which is due solely to differences in estimated labor income profiles.

The consumption profiles we obtain are similar to the ones in the modern consumption literature, as are the effects of increasing the variance of labor income shocks. We find however that the presence of positive correlation between shocks to labor income and innovations to the return on the risky asset has almost no effect on the consumption-saving decision, only on portfolio choice.

Recently a number of papers examined the issue of portfolio choice over the life-cycle empirically. Although one has to be careful when taking a stylized model as the one solved here to the data, the hump-shape of the portfolio share in the risky asset, as predicted by our

[2] In a recent paper Heaton and Lucas (1997b) emphasize the importance of proprietary or entrepreneurial income as a determinant of saving behavior and portfolio choice. One important characteristic of proprietary income is that it is positively correlated with the return on stocks.

model, seems to be a recurrent finding. Also the result that labor income risk crowds out asset holding risk is supported by the existing empirical evidence.

There are several decision-theoretic models which study the effects of labor income risk on portfolio composition. These models differ among other things in the length of the investors' horizon. Heaton and Lucas (1997a) solve for the optimal consumption and portfolio choice of an infinitely-lived household who faces uninsurable labor income risk, borrowing and short-selling constraints. The most striking result, which holds for most of the parametrizations they consider, is the difficulty in generating positive holdings of the riskless asset. In a similar spirit Koo (1995) evaluates the effects of uninsurable labor income risk on the consumption and portfolio decisions of an infinitely-lived household who is liquidity constrained. He finds that labor income risk and liquidity constraints tend to decrease both consumption and investment in risky assets. By their stationary nature, infinite-horizon models do not address life-cycle issues. More precisely, one of our findings is that an important determinant of portfolio composition is the ratio of accumulated wealth to expected future labor income. To the extent that this ratio changes during an investor's life so does the optimal portfolio allocation. These effects (and the resulting hump-shape in life-cycle portfolio choice) cannot be captured by the steady-state of an infinite horizon model.[3]

At the other end of the spectrum, in terms of investor horizon, there are the static portfolio choice models studied in the theoretical literature on background risk (Pratt and Zeckhauser (1987), Kimball (1993), Gollier and Pratt (1996), Gollier and Zeckhauser (1996)). These models involve a static portfolio choice problem in which an exogenously given level of wealth must be allocated between a risky and a riskless asset. The literature provides conditions on the class of utility functions for which the addition of some undesirable background risk causes the investor to decrease his holdings of the risky asset.

Bertaut and Haliassos (1997) and Gakidis (1997) are the papers closest to ours. Bertaut and Haliassos solve a three-period model of consumption and portfolio choice, interpreting each period as a real-life twenty-year period, with a focus on long-run precautionary motives for saving (e.g. career uncertainty). Our model and the one in Gakidis are full life-cycle models in which consumption and portfolio composition are chosen each year, and therefore capture both short- and long-run motives for saving. More importantly, they predict that the share of savings invested in stocks is hump-shaped over life. However, neither Bertaut and Haliassos nor Gakidis present or interpret policy functions, which is crucial for understanding

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[3] Viceira (1997) obtains an approximate analytical solution for an infinite-horizon model with non-tradable labor income. Retirement effects are captured through a constant probability of zero labor income forever.

the effects of labor income and the risk associated with it. Finally, contrary to both of these papers, we impose borrowing constraints, which have important pricing implications as shown by Constantinides, Donaldson and Mehra (1997).

The rest of the paper is organized as follows. Section 2 discusses the model's assumptions and set-up. The calibration and parametrization is presented in section 3. Section 4 looks at the solution of our model without permanent labor income shocks in terms of policy functions. This allows to isolate intuitively the forces at work without having to worry about the effects of persistence in shocks. Sections 5 and 6 give the simulation results with and without permanent labor income shocks. In section 7 these results are discussed in the light of the empirical literature on portfolio choice over the life-cycle. Finally, section 8 concludes.

2 The Model

2.1 Assumptions

In this section we present the assumptions we make and briefly discuss them.

Assumption 1. We let t denote adult age. The investor is adult for T periods, of which he works the first K. Both T and K are assumed to be exogenous and deterministic.

Assumption 1 is made primarily for simplicity. For most individuals the retirement age is neither exogenous nor known in advance. While this is certainly true, we believe that it is reasonable to assume that the typical individual has a good idea of the age at which he will retire.

It certainly is more questionable to assume that individuals know the age at which they will die. It is possible to extend our model to allow for uncertainty in T in the manner of Hubbard, Skinner and Zeldes (1994).

Assumption 2. The investor's age t labor income, $\widetilde{Y}_t$, is exogenously given by:

$$\log(\widetilde{Y}_t) = \begin{cases} f(t, Z_t) + \lambda\widetilde{v}_t + \widetilde{\varepsilon}_t & \text{for } \ t \leq K \\ trf & \text{for } t > K \end{cases} \tag{1}$$

where $f(t, Z_t)$ is a deterministic function of age, t, and of a vector of other individual characteristics, Z_t; $\widetilde{\varepsilon}_t$ is distributed as $N(0, \sigma_\varepsilon^2)$ and $\widetilde{v}_t$ is given by

$$\widetilde{v}_t = v_{t-1} + \widetilde{u}_t \tag{2}$$

where $\widetilde{u}_t$ is distributed as $N(0, \sigma_u^2)$ and is uncorrelated with $\widetilde{\varepsilon}_t$; trf is the amount of exogenous deterministic transfers the individual receives in each of the retirement years. Below we will consider two different specifications, one with $\lambda = 0$ and another with $\lambda = 1$.

The first case will allow us to identify the main forces at work at the different stages of life. In the second case, the process for $\widetilde{v}_t$ is taken be a random walk, following Carroll (1997) and Gourinchas and Parker (1996). Hubbard, Skinner and Zeldes (1994) and Chamberlain and Hirano (1997) estimate an AR(1) specification and find the autocorrelation coefficient to be very close to one.

The assumption that labor income is exogenous is made primarily for simplicity. In reality individuals must decide how many hours to work and how much effort to put on the job, decisions that will influence the amount of labor income received. In particular, by having exogenous labor income we rule out the possibility that an individual who has had a bad portfolio return (or labor income) realization works more hours to compensate for it.[4]

Assumption 3. We assume that there are two assets in which the agent can invest:

- A riskless asset with gross real return $\overline{R}_f$. We call this riskless asset *T-bills* and denote the dollar amount of T-bills the investor has at time t by B_t;
- A risky asset with gross real return $\widetilde{R}_t$. The excess return on the risky asset, $\widetilde{R}_{t+1} - \overline{R}_f$, is given by:

$$\widetilde{R}_{t+1} - \overline{R}_f = \mu + \widetilde{\eta}_{t+1} \tag{3}$$

where $\widetilde{\eta}_{t+1}$, the period $t+1$ innovation to excess returns, is assumed to be i.i.d. over time and distributed as $N(0, \sigma_\eta^2)$. We allow innovations to excess returns to be correlated with innovations to labor income, and denote the coefficient of correlation by $\rho_{\widetilde{\varepsilon},\widetilde{\eta}}$. We call the risky asset *stocks* and denote the dollar amount the investor has in stocks at time t by S_t.

Assumption 4. The investor's preferences are described by the time separable utility function:

$$\sum_{t=1}^{T} \delta^t u(C_t) \tag{4}$$

[4] The issue of labor supply and portfolio choice has been studied in the context of a life-cycle model by Bodie, Merton and Samuelson (1992).

where $u(.)$ is the investor's felicity function, C_t is the level of date t consumption and $\delta < 1$ is the discount factor. We assume that $u'(0) = \infty$ and that the individual derives no utility from leaving a bequest.[5]

Assumption 5. The investor faces the following borrowing constraint:

$$B_t \geq 0 \tag{5}$$

i.e., the amount the investor has in T-bills is non-negative at all dates.

Additionally we assume that no short sales of the risky asset are allowed,

Assumption 6. The investor faces the following short-sale constraint:

$$S_t \geq 0 \tag{6}$$

The borrowing constraints preclude the individual from capitalising future labor income. They can be motivated using the standard moral hazard and adverse selection arguments. It is straightforward to allow for a negative limit in (5) or (6). What is important for our results is that the individual is to some extent liquidity constrained in the early years of his adult life. We believe that this is the case for most households.

2.2 Setup of the problem

In each period t the timing of the events is as follows. The investor starts the period with wealth W_t. Then labor income Y_t is realized. Following Deaton (1991) we denote *cash-on-hand* in period t by $X_t = W_t + Y_t$. Then the investor must decide how much to consume, C_t, and how to allocate the remaining cash-on-hand (savings) between stocks and T-bills. We denote the proportion of savings invested in stocks by α_t. Next period wealth is then given by:

$$\widetilde{W}_{t+1} = \widetilde{R}^p_{t+1}(W_t + \widetilde{Y}_t - C_t) \tag{7}$$

where $\widetilde{R}^p_{t+1}$ is the return on the portfolio held from period t to period $t+1$:

$$\widetilde{R}^p_{t+1} \equiv \alpha_t \widetilde{R}_{t+1} + (1 - \alpha_t)\overline{R}_f. \tag{8}$$

We now state the problem the investor faces:

It is straightforward to extend our analysis to incorporate bequest motives if we take the utility of terminal wealth to be additively separable.

$$\underset{\{C_t\}_{t=1}^{T},\{\alpha_t\}_{t=1}^{T}}{Max} \quad E_1 \sum_{t=1}^{T} \delta^{t-1} u(C_t)$$

$$s.t. \quad \widetilde{W}_{t+1} = \widetilde{R}^{p}_{t+1}(W_t + \widetilde{Y}_t - C_t)$$

$$\widetilde{R}^{p}_{t+1} = \alpha_t \widetilde{R}_{t+1} + (1-\alpha_t)\overline{R}_f$$

$$\widetilde{R}_{t+1} - \overline{R}_f = \mu + \eta_{t+1}$$

$$\log(\widetilde{Y}_t) = \begin{cases} f(t, Z_t) + \lambda \widetilde{v}_t + \widetilde{\varepsilon}_t & \text{for } t \leq K \\ trf & \text{for } t > K \end{cases}$$

$$\widetilde{v}_t = v_{t-1} + \widetilde{u}_t$$

$$S_t \geq 0, \; B_t \geq 0, \; C_t \geq 0$$

The control variables of the problem are $\{C_t, \; \alpha_t\}_{t=1}^{T}$. The state variables are $\{t, \; X_t, \; v_t\}_{t=1}^{T}$. The problem is to solve for the policy functions as a function of the state variables, i.e., $C_t(X_t, v_t)$ and $\alpha_t(X_t, v_t)$.

The Bellman equation for this problem is given by:

$$\begin{aligned} V_t\left(X_t, v_t\right) &= \underset{C_t,\alpha_t}{Max} \left[U(C_t) + \delta E_t V_{t+1}(\widetilde{X}_{t+1}, \widetilde{v}_{t+1})\right] \text{ for } t < T \\ \text{where } \widetilde{X}_{t+1} &= \widetilde{Y}_{t+1} + \left(X_t - C_t\right)\left(\alpha_t \widetilde{R}_{t+1} + (1-\alpha_t)\overline{R}_f\right) \end{aligned}$$

The problem cannot be solved analytically. We derive the policy functions numerically by discretizing the state-space and by using backward induction. More details on the numerical solution technique are given in the appendix.

3 Calibration

3.1 Labor Income Process

We used the PSID to estimate equations (1) and (2) which give labor income as a function of age and other characteristics. We used the family questionnaire since it provides a disaggregation of labor income and asset income. The families that were part of the Survey of Economic Opportunities subsample were dropped to obtain a random sample. Because the age-profile is potentially different for households with a female head of household and therefore requires a separate estimation, the sample was split according to the gender of the head of household. However there were too few observations for the subsample with female head of household, so that the estimation was only done for households with male head of household. From this subsample we eliminated retirees, non-respondents, students and housewives.

We took a broad definition of labor income so as to implicitly allow for (potentially endogenous) ways of self-insuring against pure labor income risk. If one were to include only labor income, the risk an agent faces would be overstated for several reasons: multiple welfare programs effectively set a lower bound on the support of non-asset income available for consumption and savings purposes, both the agent and his spouse can vary their labor supply endogenously, help from relatives and friends might be used to compensate for bad labor income shocks and so on. For these reasons we defined labor income as total reported labor income plus unemployment compensation, workers compensation, social security, supplemental social security, other welfare, child support and total transfers (mainly help from relatives), all this for both head of household and if present his spouse. Observations which still reported zero for this broad income category were dropped. Labor income defined this way was then deflated using the Consumer Price Index, with 1992 as baseyear.

The estimation controls for family-specific fixed effects. We chose this technique over the synthetic-cohort approach, because the latter one might overstate the variance of the income shocks as many sources of heterogeneity are not properly accounted for.[6] We were not able to properly identify families in the 1969 wave of the PSID, and start the sample in 1970. A household appears therefore at most 23 times in our sample. We do not remove households with less observations and estimate an unbalanced panel.

To control for education the sample was split in three groups: the observations without highschool education, a second group with highschool education but without a college degree, and finally college graduates. The reason for doing so is the well-established finding that age-profiles differ in shape across education groups (see e.g. Attanasio, 1995 and Hubbard, Skinner and Zeldes, 1994). Doing so in a fixed-effects context is potentially problematic if education changes endogenously over the life-cycle. However we have only three different education groups and found few households switching from one education group to another. Consequently we considered the household as a new entity once its education changes.

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[6]We estimated the income profiles with the synthetic-cohort technique as well. Although the number of degrees of freedom is substantially larger with this technique, the shape of the profiles [illegible] ones obtained with family fixed-effects. The estimated variance is of course larger.

For each education group we assume that the function $f(t, Z_t)$ is additively separable in t and Z_t. The vector Z_t of personal characteristics other than age and the fixed household effect, includes marital status and household composition. Household composition equals the additional number of family members in the household besides the head and (if present) spouse. Ideally one should also control for occupation. Using PSID data this is problematic because from the 1975 wave onwards the majority of the unemployed report no occupation, and are categorized together with people who are not in the labor force. Obviously, modelling unemployment as a switch in occupation is not appropriate for our purposes as we believe that the possibility of getting laid off is one of the main sources of labor income risk.

The logarithm of labor income was then regressed on dummies for age, family and marital status, and on household composition. We used households whose head was between 20 and 65 years old (except for the third education group where the lowest age included in the sample was 22).

In Table 1 we report the results for the three education groups. The coefficients of the age dummies are clearly significant and the results match intuition and stylized facts. In this table we also report the estimated variance of shocks to labor income for the three education groups. In the subsequent numerical analysis, we also consider a lower variance of labor income taking into account potential biases due to measurement error.

The age profiles passed on to the simulation exercise are smoothed versions of the estimates above. We fitted a third-order polynomial to the age dummies to obtain the profiles reported in Table 2 and Figure 1. The results are similar for a fifth-order polynomial. The income profile generated mimics the results of Gourinchas and Parker (1996), Attanasio (1995) and Carroll and Summers (1991).

Next we estimated the second specification for the labor income process (with $\lambda = 1$). Our procedure follows closely Carroll and Samwick (1997). Defining r_d as

$$r_d \equiv \log(Y^*_{t+d}) - \log(Y^*_t) \tag{9}$$

where Y^*_t is given by

$$\log(Y^*_t) \equiv \log(Y_t) - \widehat{f}(t, Z_t), \tag{10}$$

then

$$Var(r_d) = d * \sigma^2_u + 2 * \sigma^2_\varepsilon. \tag{11}$$

We can then combine any two different series of r_d's to get estimates of σ_u^2 and σ_ε^2, by running an OLS regression of $Var(r_d)$ on d and a constant term (for all d). In our estimation we included all possible series of r_d's to maximize efficiency gains. The results are reported in Table 3.

The level of transfers received upon retirement, trf, is set to a fixed amount per year. This amount was estimated as the average of our labor income variable defined above for retirees in a given education group.

3.2 Other Parameters

The mean equity premium μ is 4.25% for the benchmark case but we will also consider 5.75% (the value observed in the data). The risk-free rate is 2.00% and the standard deviation of innovations to the risky asset is set to 0.157.

The discount factor was set to 0.94. Adult age starts at age 20 for the first and second education groups and at age 22 for the third education group. The age of retirement is set to 65 for all education groups. The investor dies at the age of 75. We take the felicity function to be isoelastic, with coefficient of relative risk-aversion (γ) equal to 10.

4 Policy Functions

Before looking at simulated life-cycle paths for consumption and portfolio choice, we discuss the policy functions underlying these results. This allows to highlight the main forces at work and to gain intuition on the determinants of consumption and portfolio choice. All the results presented in this section are for the second education group without permanent labor income shocks. The policy functions behave in a similar manner for the other education groups.

4.1 Benchmark Case

The consumption function, giving optimal consumption as a function of current cash-on-hand, is concave as derived analytically by Carroll and Kimball (1996). In the first phase of the life-cycle (roughly until age 35 to 40, see Figure 2a), the consumption function shifts upward as the agent ages. The reason is that her permanent income increases during this part of the life-cycle, due to the steep slope of the labor income profile. As households approach retirement and as their labor income profile becomes negatively sloped this pattern is reversed in Figure 2b. Finally at retirement, the consumption functions in Figure 2c become more kinked and less *strictly* concave relative to the functions for young agents in Figure 2a. The part to the left of the kink is characterized by a marginal propensity to consume of near unity, reflecting the liquidity constraint. To the right of the kink, the marginal propensity to consume is strictly less than one, but increases with age, to approach one in the last period of life. The fact that the consumption function becomes less concave and kinked during retirement relative to early in life reflects the decreased precautionary savings motive (the only source of uncertainty remaining is the one due to the (endogenously chosen) portfolio return). The same logic explains why the policy functions for age 65 and age 35 or 50 cross in Figure 2b: as cash on hand increases, the buffer motive disappears and the marginal propensity to consume will mainly reflect the length of the horizon ahead. Therefore the marginal propensity to consume is higher for older agents except when the value of cash on hand is low (i.e. where the agent is likely to be liquidity constrained). This explains the crossing of the consumption functions.

Let us now study the households' portfolio choice. We are interested in the shape of the policy function giving the optimal fraction of the portfolio invested into the risky asset (α), as a function of cash-on-hand and for a given age. For simplicity, it is useful to consider a two-period set-up, where the agent starts off with some initial wealth or cash-on-hand, and decides how much and how to save for the second (and final) period, in which he also receives labor income. If labor income received in the second period is certain, then it is similar to the payoff of a riskless asset. The presence of this certain labor income stream will then decrease the agent's demand for the riskless asset and increase his demand for stocks. Put differently, a certain labor income stream acts as a substitute for T-bills, not for stocks. This is exactly the intuition provided by Jagannathan and Kocherlakota (1996). By the same logic, as cash-on-hand increases for a given certain amount of future labor income, or as future labor income decreases for a given level of initial wealth, the agent wants to invest a higher proportion of his portfolio in the riskless asset. This suggests that the policy function for the share in stocks is decreasing in initial wealth for a given level of second-period labor income. Once the labor income stream is uncertain, it is not obvious that payoff still mimics the payoff of a riskless asset more closely than the one of the risky asset, i.e. that the policy function is still decreasing in cash-on-hand. In Cocco, Gomes and Maenhout (1997, henceforth CGM) however it is shown that for reasonable amounts of labor income risk, labor income is still perceived as a closer substitute for the riskless asset than for stocks in the two-period set-up described above.

The issue now is how these results carry over to the multi-period case. The relevant analogue of second-period labor income seems to be some measure of permanent income. In a complete-markets setting, where future labor income can be capitalized, this would be the annuity value of labor income (defined as the annuity factor times the expected present discounted value of labor income). In an incomplete-markets setting, He and Pearson (1991) show that there does not exist a unique way to compute the annuity value of labor income. The intuition provided by the complete-markets setting will prove useful for the understanding of our results.

The corresponding wealth concept for the multi-period model is the annuity value of wealth (defined as the product of an annuity-factor and of cash-on-hand). As one ages, the annuity factor rises, and hence the annuity value of wealth rises for a given level of cash-on-hand.

With this in mind, consider first the retirement stage. In this phase of the life-cycle, we model 'labor income' as being constant and certain. In retirement the annuity value of permanent labor income is constant, while for a given level of cash-on-hand the annuity value of wealth increases as the agent ages. Moreover, the income stream is certain and hence clearly substitutes for T-bill holdings. Therefore, as one approaches death, the relevant next-period labor income concept falls relative to the relevant wealth concept, and for a given level of cash-on-hand the agent responds to this by holding a larger proportion of her portfolio in the riskless asset. This implies that the policy function for portfolio choice shifts downwards as one ages in retirement (Figure 3a). Early in life, matters are more subtle. It is of course still true that for a given level of cash-on-hand the annuity value of wealth rises with age. However, with a steep labor income path, the annuity value of permanent labor income actually increases with age. The first effect tends to make the proportional demand for the risky asset decrease (provided that labor income is a closer substitute for T-bills than for stocks), while the second tends to render the opposite effect. Which effect would we expect to dominate? Figure 3b shows that the first effect is stronger early in life, i.e. that the policy functions for α shift out. It is interesting to note that this happens exactly as long as the annuity value of labor income increases (roughly until age 34).

There is however a second effect at work. Comparing Figure 3a with Figure 3b, we also see that the policy functions for portfolio choice become steeper (i.e. more negatively sloped) as the agent ages. The intuition for this slope-effect is as follows. For a given annuity value of permanent labor income, the fraction of this annuity value that stems from a riskless labor income stream is higher for an older agent than for a young agent. That is, given a fixed annuity value of permanent income, this annuity is 'riskier' for young agents. Remember also that a negatively sloped policy function for α indicates that labor income is perceived as a closer substitute for T-bills than for stocks. Therefore the clockwise rotation of the functions in Figure 3a relative to Figure 3b, or the increase in the (absolute value of the) slope is intuitive: as the agent grows older, and as she derives a higher fraction of her permanent labor income from a riskless labor income stream, she perceives this permanent income increasingly as a closer substitute for T-bills than for stocks, and hence the policy functions become more negatively sloped. In mid-life finally (Figure 3c), we do not see a lot of action in terms of policy function changes. The reason is that the agent is mainly at corner solutions. Nearing retirement however, the policy function for α shifts in.

4.2 Some Comparative Statics

4.2.1 Less labor income risk ($\sigma_\varepsilon = 0.20$)

First of all, note that the problem faced by a retiree under this scenario is the same as the one under the benchmark parameters. Therefore the policy functions coincide during retirement. Earlier in life, we expect to see a much weaker precautionary savings motive. This is confirmed in Figure 4a, where the consumption function is higher than under the benchmark variance of labor income, especially at low levels of cash-on-hand, and early in life, i.e. where the precautionary motive is most relevant. Of course, as the agent ages, or as the level of cash-on-hand increases, the difference between the optimal consumption of an agent with high labor income risk and of an agent facing less labor income risk decreases.

With respect to portfolio choice, the evolution of the policy functions over the life-cycle is similar to the one described above for the higher variance of labor income shocks. However, the literature on background risk (Pratt and Zeckhauser, 1987, Kimball, 1993, and Gollier and Pratt, 1996) suggests a level effect. Figure 4b shows that this is also the case in a life-cycle model. The intuition that labor income risk crowds out asset risk is vindicated.

4.2.2 Positive correlation between labor income shocks and return innovations

Next, we introduce positive correlation between labor income shocks and return innovations. This makes stocks less attractive through hedging motives. For sufficiently high correlation the agent might actually be willing to short the risky asset, as that would provide insurance (at an expected cost equal to the equity premium) against otherwise uninsurable labor risk. Interestingly, one can also expect a slope effect in terms of our policy function for α. Allowing stocks returns and labor income draws to be positively correlated is likely to induce the agent to perceive her labor income stream as a closer substitute for the risky asset than for the riskless asset. If this is the case, α will be an increasing function of cash-on-hand. Figure 5a shows both effects at work: for low values of cash-on-hand, i.e. where labor income matters, α

is increasing. Moreover, the demand for stocks is significantly lower: α is below 1 for all values of initial wealth until age 30. From then onwards, the composition effect comes into play (the fraction of the income stream that is risky and correlated with returns decreases) and less of an increasing pattern shows up. As retirement nears and as the future labor income stream shortens, the policy functions start shifting back in.

Finally in terms of consumption functions, roughly the same results emerge as in the benchmark case, with the consumption function shifting out until age 40, when the retirement motive for savings kicks in and makes the agent save more for all values of cash-on-hand (Figure 5b).

5 Simulation Results

Using the policy functions derived above, we simulated the consumption and asset allocation profiles of 10000 agents over the life-cycle. Below we present and discuss the means of these simulated profiles.

5.1 Benchmark Case

In Figure 6a the simulated income, wealth and consumption profiles are plotted. We see that households are liquidity constrained during, roughly, the first 15 years of their working lives. Consumption tracks income very closely and a small level of savings is accumulated to use as insurance-cushion against negative labor income shock. As labor income increases and when this profile becomes less steep the agent starts accumulating wealth for retirement. The consumption profile ceases to be increasing as the agent gets older, reflecting the fact that the liquidity constraint becomes less binding. We do not obtain the decreasing pattern as documented for instance in Gourinchas and Parker (1996), because under the benchmark parameters, the agent is not sufficiently impatient to choose a decreasing consumption profile given a labor income profile that is not increasing and given a high (endogenously chosen) return on her portfolio.

In Figure 6b we present the agent's simulated portfolio allocation. During the beginning of life, the agent holds most of her wealth in stocks but nevertheless finds it optimal to invest partially in the riskless asset. As she reaches around age 30 however, she already invest all of her wealth in stocks. What warrants this cautious investment behavior early in life? The agent faces a very steep labour income profile, and is hence discouraged to save, all else equal. The little savings that do accumulate, are in part due to a precautionary savings motive. A prudent individual finds it suboptimal to expose this small buffer-stock fully to stock market risk. As the labor income profile becomes less steep, the agent saves more and finds it now advantageous to fully invest his portfolio in the risky asset. In terms of the policy functions described above, what happens is that early in life both the policy functions shift out and the amount of cash-on-hand increases, i.e. there is a shift of the curve and a shift along the curve. The first effect tends to increase α, the second has the opposite effect. Until the age of 30, the first dominates. Therefore the crux to understanding the increasing proportional stock holdings early in life is the increase in the annuity value of labor income during that phase of the life-cycle. From then onwards, both the downward shifts of the curve and the increase in cash-on-hand act to decrease α. At this stage of the life-cycle, saving for retirement becomes a crucial determinant of the agent's behavior. Meanwhile the income profile becomes decreasing, and this continuous decrease in implicit riskfree-asset holdings induces the agent to hold more T-bills in his financial portfolio, and to decrease his proportional stock holdings. Roughly, this pattern seems to be consistent with the recommendations made by popular financial advisers.

5.2 Some Comparative Statics

5.2.1 Less labor income risk ($\sigma_\varepsilon = 0.20$)

We first consider the effect of a decrease in the standard deviation of the labor income shocks. We expect two effects. First of all, this decrease in labor income risk should reduce the precautionary savings motive. Secondly, the intuition that labor income risk crowds out portfolio risk suggests that the fraction of the portfolio allocated to the risky asset should increase. The first effect is at work in Figures 7a: the agent saves less early in life. This allows her to consume slightly more early in life. However, because the agent subject to more risk saves more, he is able to sustain a higher consumption path later on. Of course, this implies that the household facing less labor income risk has a smoother consumption path. And with respect to portfolio choice, we see in Figure 7b that the agent now invests significantly more into the risky asset. The agent is basically fully invested in stocks until age 50. Comparing with the results under the higher benchmark standard deviation, this implies that labor income risk crowds out asset risk. Again it is instructive to interpret this result using the policy functions. What happens as labor income risk increases, is that first of all the policy functions shift down, and secondly that the agent accumulates more wealth. Given the negative slope of the policy function, both effects reduce the demand for the risky asset.

A noteworthy result is that even during retirement labor income risk influences portfolio choice, even though both agents get exactly the same certain retirement income stream. The reason of course lies in the fact that the agent facing more labor income risk before retirement has accumulated more wealth. As the policy function for α is decreasing, this leads the agent that was subject to more labor income risk to invest less into stocks.

5.2.2 Positive correlation between labor income shocks and return innovations

When considering a coefficient of correlation of 0.10, we see that portfolio choice early in life is influenced substantially by the presence of positive correlation (Figure 8a). The intuition is straightforward. In the benchmark case, we argued that the young agent is cautious in allocating her portfolio, because she mainly saves for precautionary motives, i.e. to cushion adverse labor income draws. But then if it is the case that bad labor income shocks tend to coincide with poor returns on the risky asset, the risk properties of this asset are particularly unattractive. When the agent accumulates sufficient wealth not to be dependent on the labor and stock return realizations for his consumption, the effects of the positive correlation on portfolio choice are reduced.

As noted in CGM, consumption choice is not significantly affected by the introduction of positive correlation (Figure 8b).[7] The agent saves slighty more early in life. This can be attributed to a stronger precautionary motive given the presence of positive correlation.

5.3 Other Education Groups

Figure 9 plots simulated labor income profiles, invested wealth and portfolio allocation over the life-cycle for the different education groups. It is important to have in mind that in our stylized model an education group is characterized solely by the age at which working life begins, a given labor income profile and the variance of shocks to it. In particular, we ignore any informational costs of investing in stocks and how these might differ across education groups.

As Figure 9c shows the share of savings invested in stocks is hump-shaped over life for all education groups. The peak is attained, in terms of age, first for education group 1, then for education group 2 , and only later for education group 3. The order in which the maximum is attained can be easily understood if one has in mind the discussion of the previous section which identifies the increasing labor income profile as the reason for the share of savings

[7] CGM find that introducing negative correlation between labor income shocks and return innovations does influence savings decisions. The intuition is that saving and investing in the risky asset provides a powerful insurance device against income shocks.

invested in stocks to be increasing early in life. Looking at Figure 9a (which replicates Figure 1) we see that the maximum of labor income is attained earlier for lower education groups. Still concerning portfolio allocations early in life one should also have in mind that households in the third education group only begin their working lives at age 22, as opposed to 20 for the other education groups. This helps explain why the share of savings invested in stocks starts off higher for the third than the first education group, then lower, only to be higher again after age 35.

After age 38 and roughly until retirement, the share of savings invested in stocks is, for a given age, increasing in the level of education. This can be understood from our previous discussion which identifies the annuity values of invested wealth and future labor income as determinants of portfolio allocation. For a given age, the annuity value of future labor income is increasing in the level of education (Figure 9a). This means that the implicit riskless asset holdings (in the form of future labor income) are higher for more educated households, leading them to invest a higher proportion of their savings into stocks.

This is not the whole story however, since at each age the annuity value of invested wealth differs across education groups. Also, recall that a higher value for this annuity tends to reduce the share of savings invested in stocks. This second effect helps to explain why roughly after age 51 the difference in the share invested in stocks between education group 1, on one hand, and education groups 2 and 3, on the other hand, is reduced (Figure 9c). After this age households in the latter education groups accumulate significantly more wealth than in the first education group (Figure 9b).

After age 64, and for education group 2 compared to education group 1, the effect of a higher annuity value of invested wealth dominates the effect of higher annuity value of future labor income so that the latter group invests a higher share of savings in the risky asset. Even though the portfolio choice rule for the second education group is to the right of the one for the first education group, we obtain that the share invested in stocks is higher for the latter group, since the point where the former is evaluated is more to the right (and the policy function is decreasing).[8]

Figure 9c shows how, in the context of a stlylized model, differences in the (estimated)

[8] There may be some concern as to why in comparing potfolio choice for the different education groups we do not explicitly mention differences in the variance of shocks to labor income. The reason is that these differences are implicitly taken into account in our informal discussion of the anuity value of future labor income. The variances are similar for education groups 2 and 3, and somewhat larger for education group 1. This depresses the anuity value of future labor income for the latter education group especially early in life. As retirement nears the effect of the variance of labor income disappears.

income profile of different education groups can give the result that one would a priori expect, that better educated households invest on average a higher share of savings in risky assets.

6 Permanent Labor Income Shocks

In this section we introduce the possibility of permanent shocks to labor income, with the variances of transitory and permanent shocks equal to those estimated from the PSID and presented in section 3 above.

Figure 10a presents consumption and wealth profiles for the second education group with and without (benchmark case) permanent shocks to labor income. The agent subject to permanent shocks has a stronger precautionary savings motive and accumulates more wealth. These savings eventually have to be consumed, so that consumption later in life is slighty higher. Of course, this means the agent facing less labor income succeeds better in smoothing consumption over time.

In terms of portfolio choice, the main difference in mid-life is that the agent facing permanent shocks has accumulated more wealth. Because the policy functions are roughly the same (and downward-sloping), this delivers a lower fraction of savings allocated to the risky asset. Early in the life-cycle, there are two effects resulting from the introduction of the permanent shocks. First of all, we might expect the presence of permanent labor income shocks to crowd out asset holdings, because of the higher long-run variance. On the other hand, the lognormality of the permanent shocks increases the mean of future labor income relative to the case without permanent shocks due to Jensen's Inequality.[9] With permanent shocks this latter effect mainly matters early in life and increases the demand for the risky asset. Actually from Figure 10b, it is the dominating effect, and results in a higher share of savings invested in stocks, relative to the benchmark case. At retirement the problems of the agent with and without permanent shocks are exactly the same, but since the former has accumulated more wealth he invests slightly less in stocks.[10]

In Figure 10b we have that the share invested in stocks is constrained by one early in life.

[9] It is straightforward to correct for this Jensen's inequality effect, but we are interested in obtaining results for the labor income profiles as estimated.

[10] There is a small nonmonotonicity around age 65 for the case of permanent shocks. This is the result of two opposing effects: the inward shift of the curve and the shift along the curve (as wealth is run down). The exact pattern obtained here is not robust as can be seen from other simulations, and is likely to disappear if the agent is made to decumulate less fast, as will be the case if one realistically allows for more uncertainty during retirement (e.g. medical expenses, uncertain lifespan etc.).

In order to see if the hump-shape still obtains under the permanent shocks to labor income scenario we experimented with different equity premia (Figure 11). We see that when the investor is not constrained the hump-shaped pattern still obtains.

As expected, a higher equity premium shifts the portfolio allocation curves up. For a given age and level of cash-on-hand the investor finds it more attractive to invest in stocks the higher the equity premium. There is, however, a second effect at work which goes in the opposite direction. When the equity premium is higher the investor accumulates more wealth and therefore the policy functions are evaluated more to the right. This tends to attenuate the effect of the increased equity premium on the share of savings invested in stocks.

Figure 12 presents results for the different education groups. The reason why the shares invested in stocks increase at retirement for education groups 2 and 3 and not 1 is because the former groups accumulate more wealth during their life and therefore decumulate faster at retirement (this decumulation occurs faster than the inward shift of the policy functions). The higher level of accumulated wealth also explains why, from mid-life on, the share invested in stocks is slightly higher for education group 1 than for 2 (the policy functions are evaluated more to the right for the latter). Education groups 2 and 3 accumulate more wealth due to the combined effects of larger variance of permanent shocks and a higher drop in labor income at retirement.

Comparing Figures 9c and 12, the predictions for the first and second education groups are found not to be robust. However the result that the third education group chooses to hold a riskier portfolio carries over.

7 Empirical Literature on life-cycle portfolio choice

In this section we confront the predictions of our model with the empirical literature, with a focus on portfolio composition over the life-cycle.

The stylized model solved in this paper assumes that there are only two assets: a riskless and a risky asset. In comparing its predictions with the empirical literature on portfolio choice at least two problems arise. First our riskless asset is riskless in real terms and not in nominal terms as are cash and treasury bills. The asset which is most similar to the riskless asset in our model is an indexed bond, which was only recently introduced in the United States. Second, households own a variety of assets other than cash and stocks such as farms or family businesses, housing and other real estate, life insurance policies. In order to bring the predictions of the model to the data, we must first take a position on whether the riskless and risky assets should be interpreted literally as cash and stocks, respectively, or more broadly to include other assets as well. In the latter interpretation we must also decide which real assets resemble more closely each of our stylized assets.

Several predictions of the model deserve confrontation with the data. The shape of the underlying portfolio allocation rules can in principle be empirically tested. For households whose innovations to labor income are uncorrelated with innovations to stock returns, the share of savings invested in risky assets should be decreasing in the level of cash-on-hand. A particularly robust result of the model is that the share of wealth invested in the risky asset is hump-shaped over life. Furthermore, college educated households should invest a higher share of savings in risky assets. Another prediction of the model is that labor income risk crowds out asset holding risk. Finally, we want to compare the level of risky asset holdings in the model with the data.

With respect to the shape of the policy functions, the decreasing pattern seems to contradict casual evidence. However, one should remember that this pattern is obtained holding the characteristics of future labor income constant, whereas in practice wealthier households tend to have a higher level of future labor income. The latter will act so as to increase the share of cash-on-hand invested in risky assets. We are not aware of the existence of any work which estimates the share of savings invested in risky assets as a function of cash-on-hand, carefully controlling for future labor income.

Bertaut and Haliassos (1997) use several definitions of riskless and risky assets in their empirical work based on the 1992 Survey of Consumer Finances (SCF). For most of their discussion they include in stocks shares of publicly traded stocks, shares in stock mutual funds, and other stocks held in IRAs and Keogh plans. They also consider a broader definition of stocks that includes stocks held in trusts, managed investment accounts, and defined-contribution pension plans. Their riskless asset includes checking, saving, money market, and call accounts, certificates of deposit, savings and other bonds, and the cash value of life insurance less credit card balances, consumer loans and other non-real estate loans.

Whether the narrow or the broad definition of stocks is considered, the average share of financial net worth held in stocks is hump-shaped with age for households whose head has no college degree (excluding those households whose head is older than 80 years). For both education groups (less than highschool and highschool), the maximum share is obtained for the age group 40-49, later than predicted by our model. For households whose head has a college degree they do not find a clear pattern. With respect to differences across education groups, they find that highschool graduates invest a higher share of savings in risky assets than households in the first education group. For college versus highschool graduates there is no clear prediction. It is important to note that these results hold for average portfolios and that there is little variation in median portfolios since most households do not own any stocks.

Even considering only those households whose head has no college degree our model was only partially successful in mimicking the levels of portfolio choice of the average household in the Bertaut and Haliassos (1997) sample. The main difficulty is in the high level of riskless asset holdings, compared to what is predicted by the model.

Heaton and Lucas (1997b) use two sources of data: the 1989 SCF and the Tax Model. They find that the results in terms of portfolio composition are very sensitive to the definition of wealth used. Looking at the average share invested in stocks, bonds and cash relative to liquid assets (the amount invested in stocks, bonds, and cash) they find that the share invested in cash has an inverted hump-shape. This means that if we take stocks and bonds as "risky assets" the hump-shape predicted by the model obtains. The same pattern obtains for the share of liquid assets invested in stocks alone. The picture changes when they consider the share of stocks in financial assets (liquid assets plus housing and other real estate, and the value of all businesses, pensions, and trusts). This share is higher for the households who have reached retirement age because of the increasing importance of liquid assets.

A result that Heaton and Lucas (1997b) emphasize is that older households decrease the share of their wealth invested in risky assets, since they substitute stocks, bonds and cash for riskier proprietary business ownership. In a sense this is the retirement effect in our model. As the household ages and her ratio of wealth to expected future income increases, a larger fraction of future consumption must be financed through current wealth, and therefore the household shifts portfolio composition towards less risky assets.

Poterba and Samwick (1997) use the SCFs from 1983, 1989 and 1992 as repeated cross-sections, identifying six main categories of financial assets (among which taxable equity, tax-exempt and taxable bonds, and bank accounts). They run Probit and Tobit regressions for the probability of owning a particular asset category and for the share of total financial assets in a particular asset category respectively, on a constant, age and cohort dummies. They find the age coefficients to be often significant. Although the pattern is not always pronounced and depends on whether one controls for cohort effects or not, they do obtain the hump-shape predicted by our model for taxable equity, for all equity (including equity mutual funds) and for business net worth. They also document the U-shape implied by our model for the portfolio share in bank accounts, i.e. in the riskless asset. A second important finding of Poterba and Samwick is the significance of cohort effects. Our model is not able to explain these, although they might be partly due to changes in the characteristics of the labor income profile.

Finally, Guiso, Jappelli and Terlizzese (1996) use a sample of Italian households to study the effects of labor income risk on portfolio choice. They find that labor income risk reduces investment in risky financial assets such as stocks. Moreover education has a positive effect on the proportional demand for risky assets. From a life-cycle perspective they find that the share of risky assets is lowest for young households and increases throughout life to reach a maximum at age 61, later than predicted in our model. Of course our model was calibrated with U.S. data, not with Italian data.

8 Discussion

In this paper we solved numerically for the optimal consumption and portfolio decisions of a finitely-lived individual who faces labor income uncertainty and can invest in either a risky or a riskless asset. Given our assumption that the investor cannot borrow against future labor income, and that the estimated income profile used to calibrate the model is steeply sloped in the early stages of her life, the individual starts off acting as a buffer-stock agent. Because stocks are risky, the individual prefers to hold part of her buffer-stock in the safe asset. As the individual gets older, her labor income increases and she invests more of her wealth in the risky asset. As retirement nears, the agent has to rely more on her financial wealth to finance consumption and increases her T-bill holdings to compensate for the loss of labor income. This results in a portfolio allocation which is hump-shaped over life, a pattern for which we found supportive evidence in the recent empirical literature on portfolio composition.

Our results roughly support and rationalize the investment advice given by popular finance books and financial counselors, namely to shift the portfolio composition towards relatively safe assets as one ages. However early in life, the opposite advice is suggested by our model.

The model was calibrated and solved for three different education groups, split according to whether the head of the household had a college degree, highschool degree or no degree. Perhaps surprisingly, the solution to the model yielded the result which one would a priori expect, namely that the average level of stockholdings is increasing in the education level, for a given age. Surprisingly, because of the stylized nature of the model in which a given education group is characterized solely by the age at which working life begins, a given labor income profile and the variance of shocks to it. In particular, we ignore any informational or other cost of investing in stocks and how these might differ across education groups. A fixed cost of entering the market, not considered in our model, can help explain why less educated households may invest less in stocks: they save less and therefore the benefits of paying the cost are smaller. Also this cost may differ across education groups. We believe that this is a crucial element for understanding the difference in portfolio choice across education groups, and expect it to magnify the results obtained here.

The model solved in this paper is decision-theoretic and therefore it does not address the equity premium puzzle. However it suggests that life-cycle considerations combined with uninsurable labor income risk may help explain the puzzle. In order to understand how, consider the general equilibrium model of infinitely-lived agents presented in Heaton and Lucas (1995). They find that uninsurable idiosyncratic labor income risk has little effect on the equity premium as agents self-insure by accumulating wealth when labor income shocks are positive and by running down their assets in face of adverse shocks.

Our paper shows that life-cycle considerations limit the amount of self-insurance investors can achieve. A young household may not have accumulated sufficient wealth to allow for insurance against idiosyncratic labor income shocks. On the other hand, a household approaching retirement has accumulated significant wealth, but this wealth must be used to finance consumption in retirement years. These considerations limit the amount of risk that young and old households are willing to accept in their portfolio of assets and incorporating them into a general equilibrium setup may help explain the equity premium puzzle. This is the route taken by Constantinides, Donaldson and Mehra (1997). They construct an overlapping generations model of an economy in which agents face uninsurable labor income risk and borrowing constraints. In their model each generation lives for three periods. Within this framework they are able to explain a high equity premium.

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Appendix: numerical solution

The model was solved using backward induction. In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to compute the policy rules for the previous period and given these, obtain the corresponding value function. This procedure is then iterated backwards.

To avoid numerical convergence problems and in particular the danger of choosing local optima we optimized over the space of the decision variables using standard grid search. The sets of admissible values for the decision variables (consumption and portfolio allocation), were discretized using equally spaced grids. The state-space was also discretized. We used an equally spaced grid for cash-on-hand and, following Tauchen and Hussey (1991), approximate the density function for returns in the risky asset using Gaussian quadrature methods. The density function for both innovations to the labor income process were also approximated using Gaussian quadrature to perform the necessary numerical integration.[11] The upper and lower bounds for cash-on-hand and consumption were chosen to be non-binding in all periods. The grid for the level of the permanent component of labor income was limited to 9 points for computational reasons. Given this grid, no more than four consecutive realizations of either the highest or the lowest admissible values of u_t can occur (starting at the unconditional mean).

In order to evaluate the value function corresponding to values of cash-on-hand that do not lie in the chosen grid we used a cubic spline interpolation in the log of the state variable. This interpolation has the advantage of being continuously differentiable and having a non-zero third derivative, thus preserving the prudence feature of the utility function. The support for labor income realizations is bounded away from zero due to the quadrature approximation.[12] Given this and the non-negativity constraint on savings, the lower bound on the grid for cash-on-hand is also strictly positive and hence the value function at each grid point is also bounded below. This fact makes the spline interpolation work well given a sufficiently fine discretization of the state-space.

[11] The number of quadrature points used to compute the approximations for each of the density functions was set to 3, and the results were not sensitive to the choice of a higher value.

[12] In the case where we also include the permanent labor income shocks this lower bound is still strictly positive given the small dimension of the grid for v_t, as discussed above.

Table 1: Labor Income Process: Fixed-Effects Regression.

Independent Variable Log Real Income	No Highschool Coefficient	t-stat	Highschool Coefficient	t-stat	College Coefficient	t-stat
Family Size	-0.0176	-3.12	-0.0236	-7.42	-0.0228	-4.63
Marital Status	0.4008	18.48	0.4437	43.21	0.4831	30.50
Age Dummies						
21	0.0466	0.96	0.1311	5.20	-	-
22	0.1321	2.77	0.1828	7.44	-	-
23	0.1711	3.58	0.2401	10.01	0.3121	5.22
24	0.2124	4.42	0.3023	12.69	0.5504	9.67
25	0.1975	4.09	0.3465	14.59	0.6944	12.46
26	0.2803	5.78	0.4001	16.74	0.7963	14.38
27	0.2706	5.58	0.4043	16.90	0.8660	15.72
28	0.2995	6.11	0.4522	18.80	0.9301	16.88
29	0.3281	6.72	0.4597	19.04	0.9770	17.75
30	0.2990	6.07	0.4735	19.45	1.0222	18.60
31	0.3826	7.67	0.5151	20.92	1.0520	19.06
32	0.3825	7.60	0.5361	21.62	1.0767	19.44
33	0.4264	8.48	0.5621	22.38	1.1068	19.96
34	0.4329	8.42	0.5504	21.84	1.1492	20.70
35	0.4327	8.33	0.5794	22.79	1.1773	21.11
36	0.4139	7.85	0.5913	23.05	1.2077	21.60
37	0.4452	8.49	0.6047	23.31	1.2634	22.50
38	0.4539	8.51	0.6159	23.41	1.2649	22.51
39	0.4726	8.92	0.6268	23.61	1.2867	22.80
40	0.4562	8.55	0.6443	23.97	1.3323	23.55
41	0.4497	8.37	0.6688	24.36	1.3228	23.24
42	0.4099	7.82	0.6561	24.00	1.3439	23.63
43	0.3971	7.56	0.6890	24.92	1.3687	24.00
44	0.3762	7.08	0.6651	23.76	1.3801	23.93
45	0.4320	8.17	0.6559	23.21	1.3769	23.62
46	0.3377	6.34	0.6456	22.51	1.3936	23.79
47	0.3631	6.85	0.6505	22.43	1.4136	23.72
48	0.2736	5.14	0.6431	22.01	1.3999	23.52
49	0.3622	6.79	0.6363	21.73	1.3762	23.08
50	0.3028	5.62	0.6182	20.81	1.4015	23.25
51	0.3353	6.19	0.6089	20.10	1.3915	22.94
52	0.3207	5.91	0.6025	19.70	1.4373	23.48
53	0.3565	6.49	0.6071	19.62	1.3500	22.02
54	0.3037	5.55	0.5641	18.09	1.3786	22.31
55	0.2513	4.51	0.5149	16.91	1.3552	21.76

Table 1 (Continued): Labor Income Process: Fixed-Effects Regression.

Independent Variable Log Real Income	No Highschool Coefficient	t-stat	Highschool Coefficient	t-stat	College Coefficient	t-stat
56	0.2546	4.56	0.5834	18.21	1.2828	20.40
57	0.2753	4.91	0.5620	16.96	1.2865	20.53
58	0.2852	5.01	0.5492	16.38	1.3246	20.75
59	0.2556	4.48	0.5394	15.85	1.2780	19.94
60	0.1942	3.35	0.5341	14.95	1.2630	19.43
61	0.2638	4.54	0.4982	13.68	1.2481	18.72
62	0.1870	3.04	0.5130	12.81	1.2748	18.21
63	0.2126	3.29	0.4945	11.72	1.2683	17.60
64	0.2112	3.21	0.5160	11.24	1.1474	15.28
65	0.2257	2.95	0.5568	9.75	1.0804	13.19
Constant	2.6275	56.63	2.7004	118.27	2.3831	44.38
Number of obs	9475		26947		12273	
n	1104		2816		1110	
T-bar	8.58		9.57		11.06	
σ_ϵ^2	0.1583		0.1161		0.1189	
R^2 within	0.0648		0.1395		0.2609	
F-stat	12.27		83.10		87.23	

Table 2: Labor Income Process: Coefficients in the age polynomial.

	No Highschool 3rd order	5th order	Highschool 3rd order	5th order	College 3rd order	5th order
Constant	-2.1361	0.0549	-2.1700	-7.5185	-4.3148	-29.6153
Age	0.1684	-0.1277	0.1682	0.9046	0.3194	3.4476
Age2/10	-0.0353	0.1181	-0.0323	-0.4213	-0.0577	-1.5443
Age3/100	0.0023	-0.0359	0.0020	0.1007	0.0033	0.3439
Age4/1000	-	0.0046	-	-0.0121	-	-0.0377
Age5/10000	-	-0.0002	-	0.0006	-	0.0016
Retirement	2.7228	2.7228	2.9266	2.9266	3.4506	3.4506

Table 3: Variance Decomposition Estimates

	No High School	(t-ratios)	High School	(t-ratios)	College	(t-ratios)
σ_ε^2	0.1056	13.260	0.0738	21.962	0.0581	13.089
σ_u^2	0.0105	9.909	0.0106	24.258	0.0169	29.196

Figure 1 - Labor Income Profiles
(age dummies and fitted polynomials)

Figure 2a - Consumption Functions early in life
(Benchmark Case)

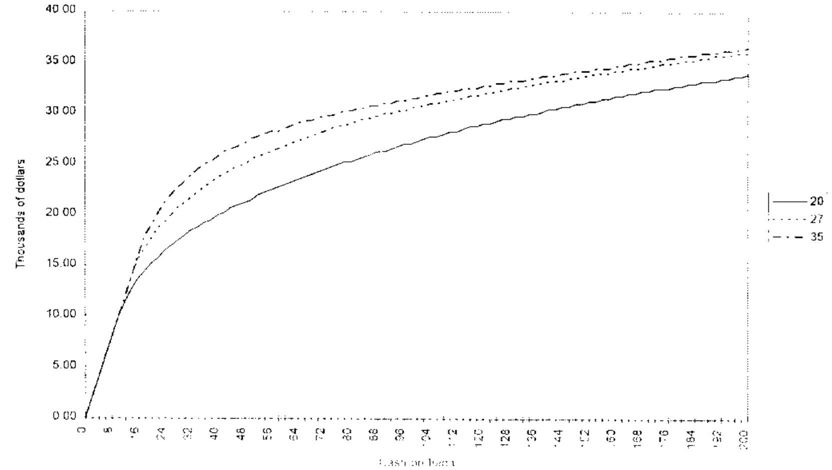


Figure 2b - Consumption Functions at mid-life
(Benchmark Case)

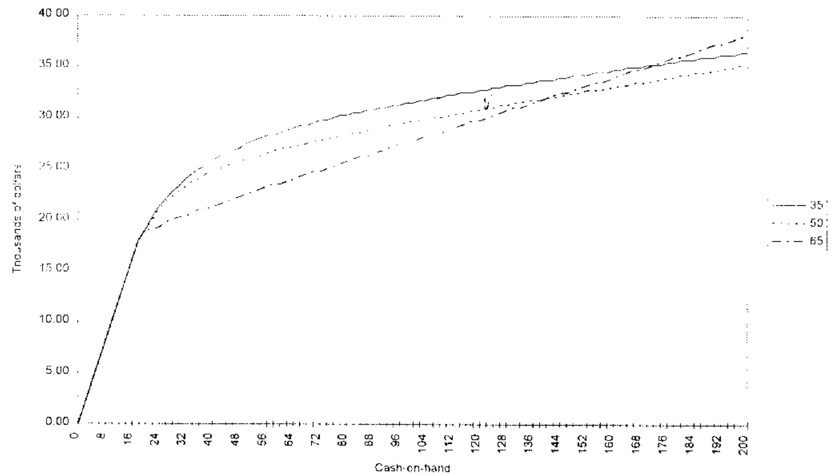


Figure 2c - Consumption Functions at retirement
(Benchmark Case)

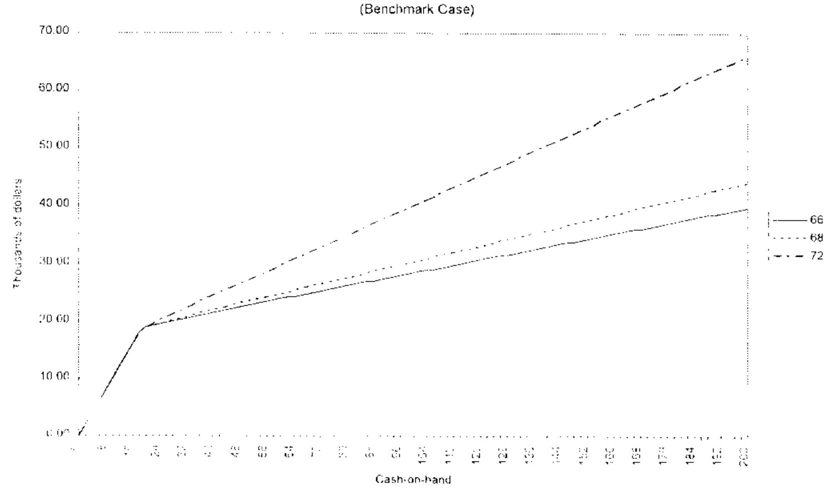


Figure 3a - Portfolio Allocation Functions at retirement
(Benchmark Case)

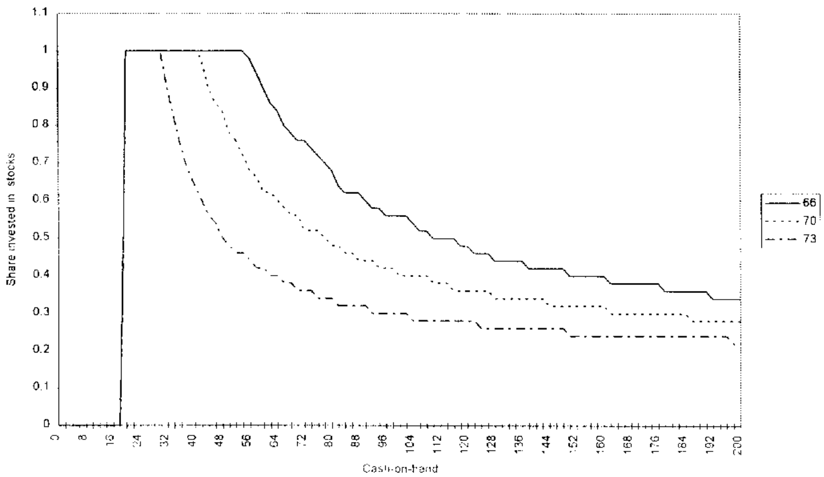


Figure 3b - Portfolio Allocation Functions early in life
(Benchmark Case)

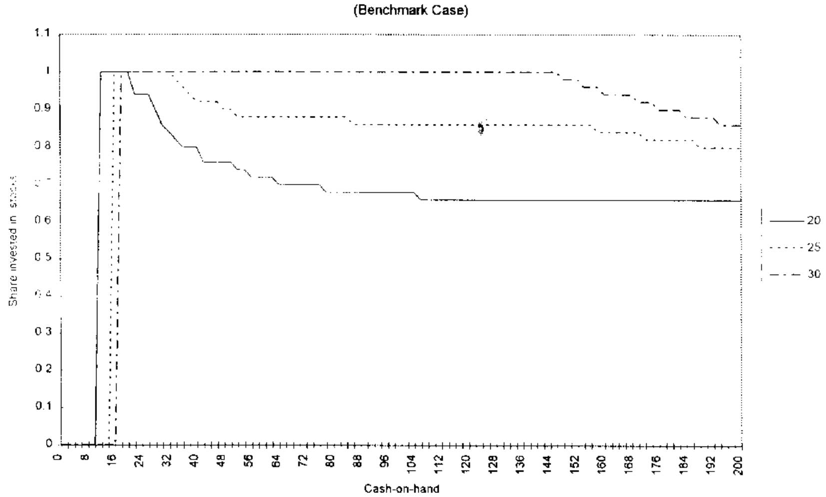


Figure 3c - Portfolio Allocation Functions at mid-life
(Benchmark Case)

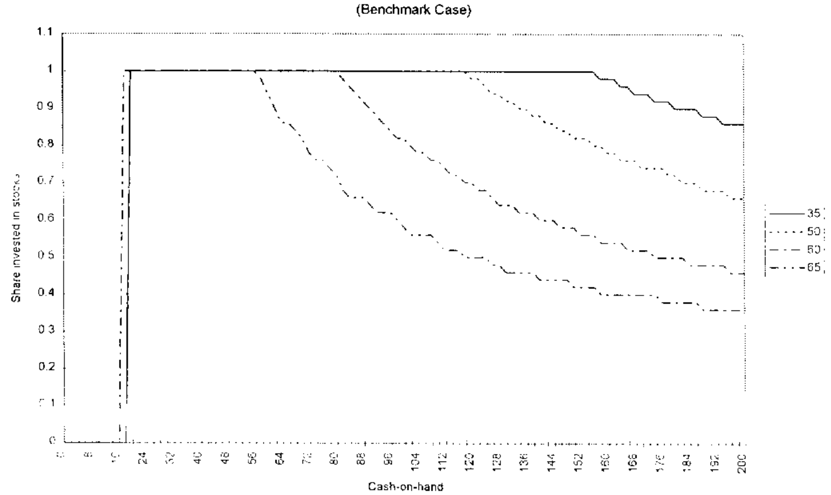


Figure 4a - Consumption Functions
(Benchmark and 20% Labor Income Standard Deviation)

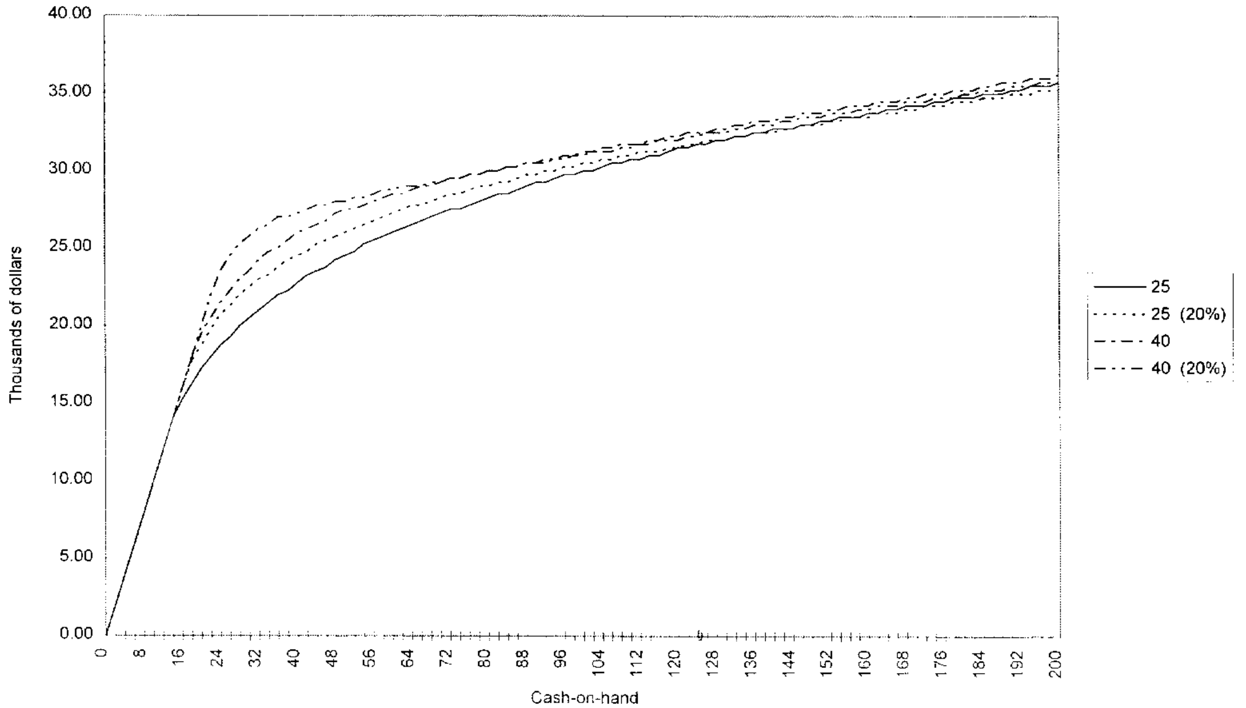


Figure 4b - Portfolio Allocation Functions
(Benchmark and 20% Labor Income Standard Deviation)

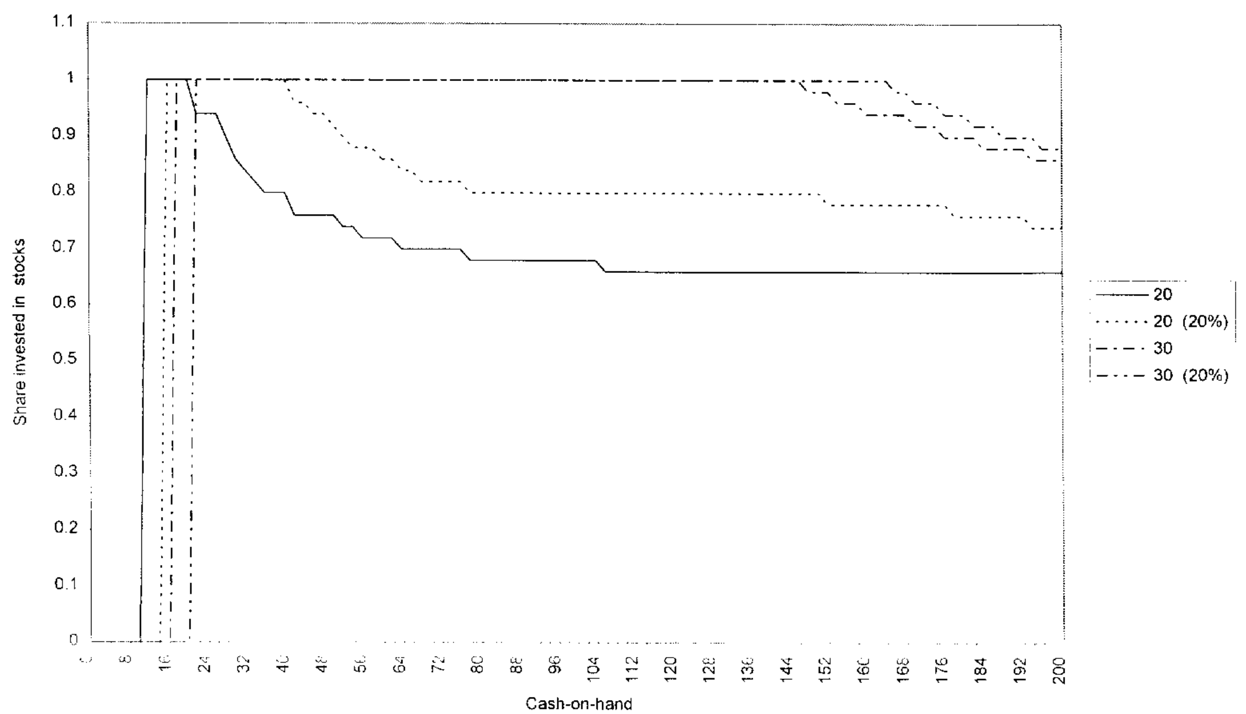


Figure 5a - Portfolio Allocation Functions
(10% Correlation between innovations to stock returns and to labor income)

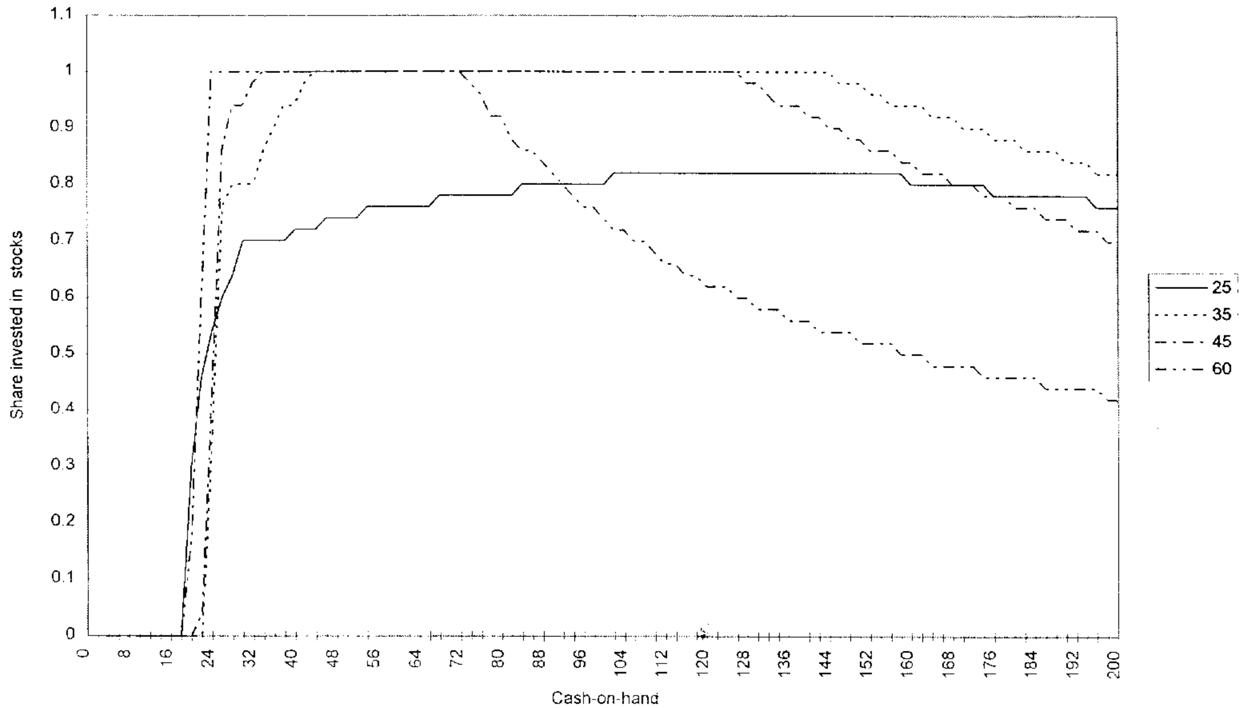


Figure 5b - Consumption Functions
(10% Correlation between innovations to stock returns and to labor income)

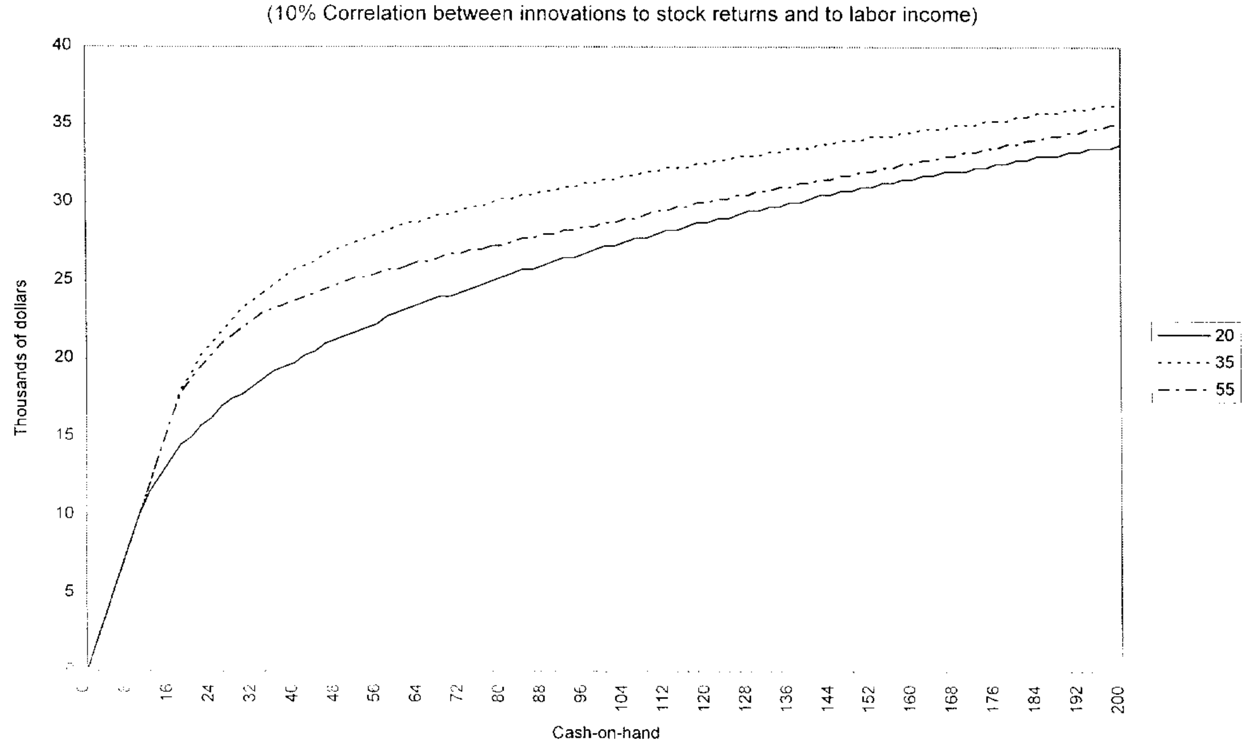


Figure 6a - Consumption, Income and Wealth over the Life-Cycle
(Benchmark Case)

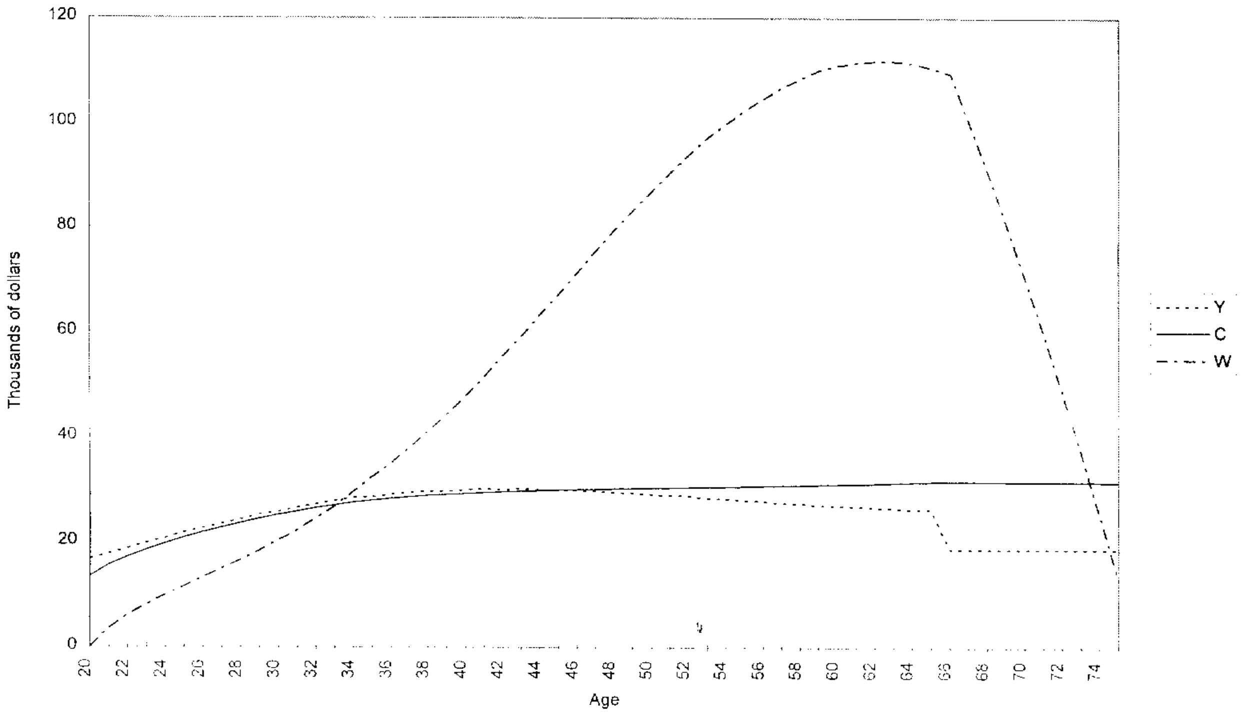


Figure 6b - Portfolio Allocation over the Life-Cycle
(Benchmark Case)

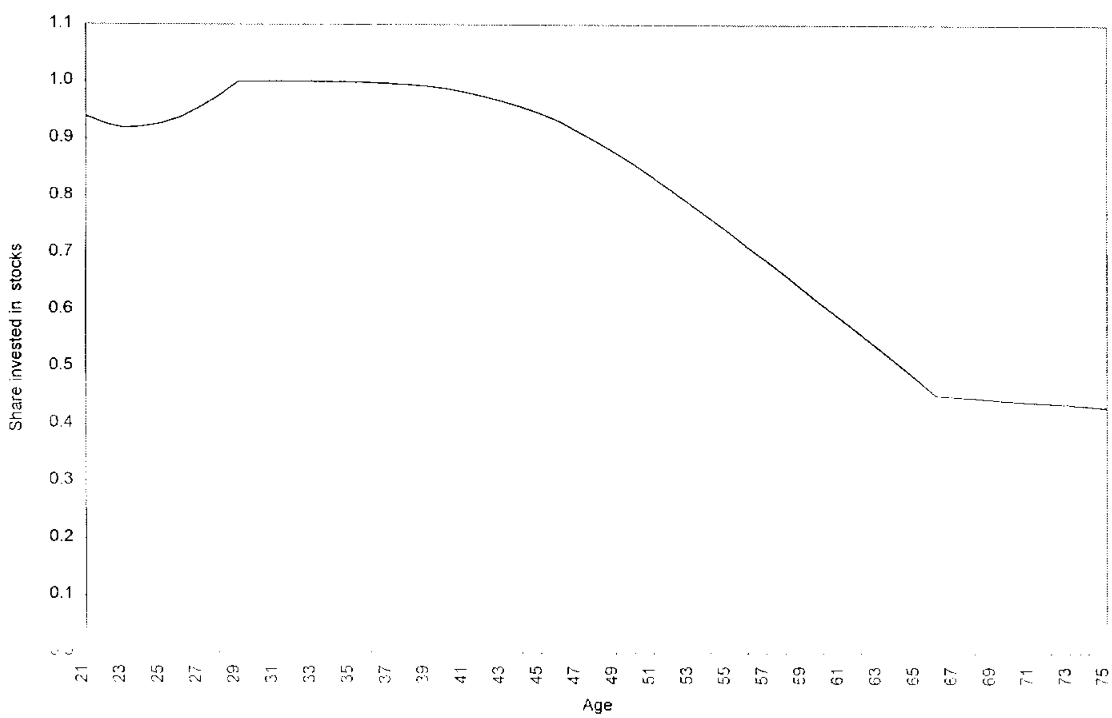


Figure 7a - Consumption and Wealth over the Life-Cycle
(Benchmark and 20% Labor Income Standard Deviation)

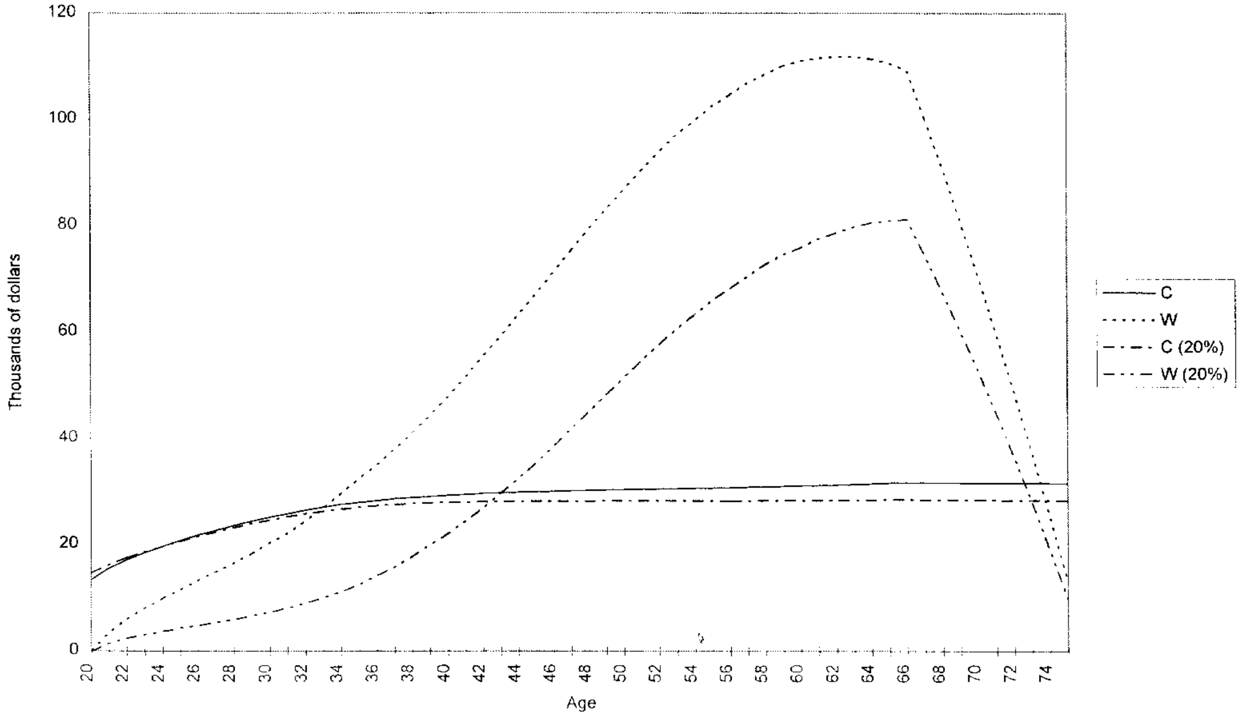


Figure 7b - Portfolio Allocation over the Life-Cycle
(Benchmark and 20% Labor Income Standard Deviation)

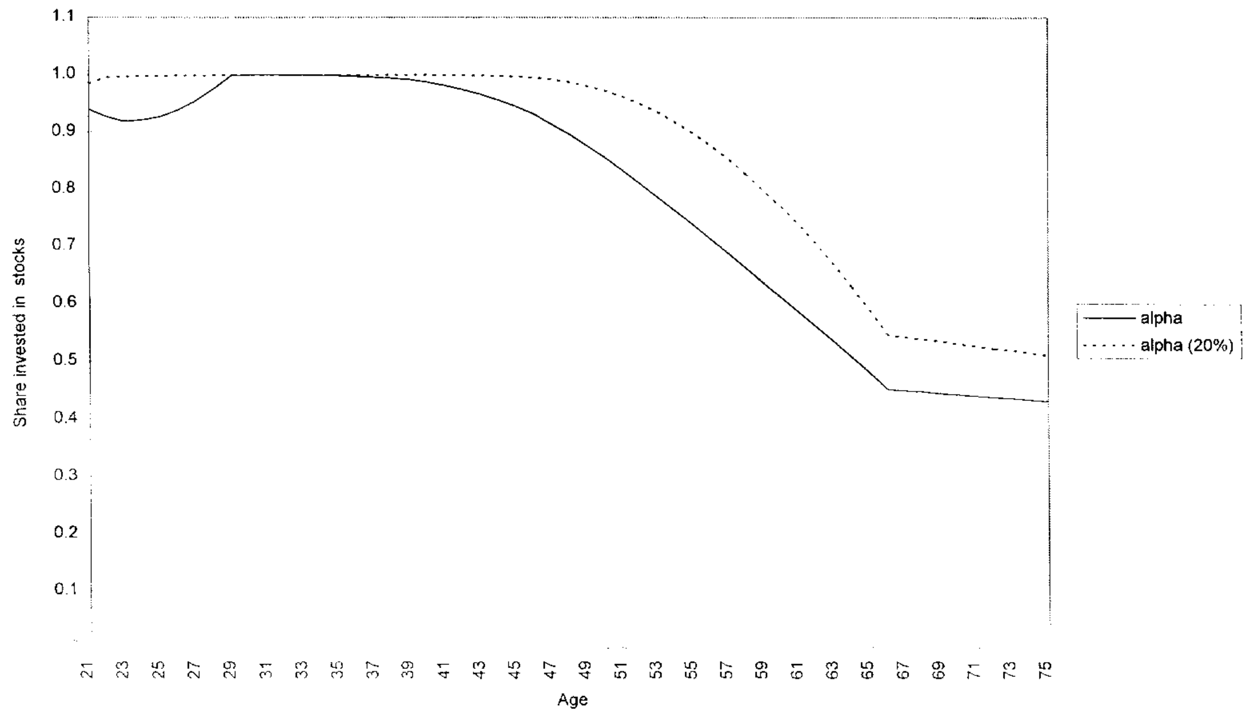


Figure 8a - Consumption, Income and Wealth over the Life-Cycle
(10% Correlation between innovations to stock returns and to labor income and Benchmark)

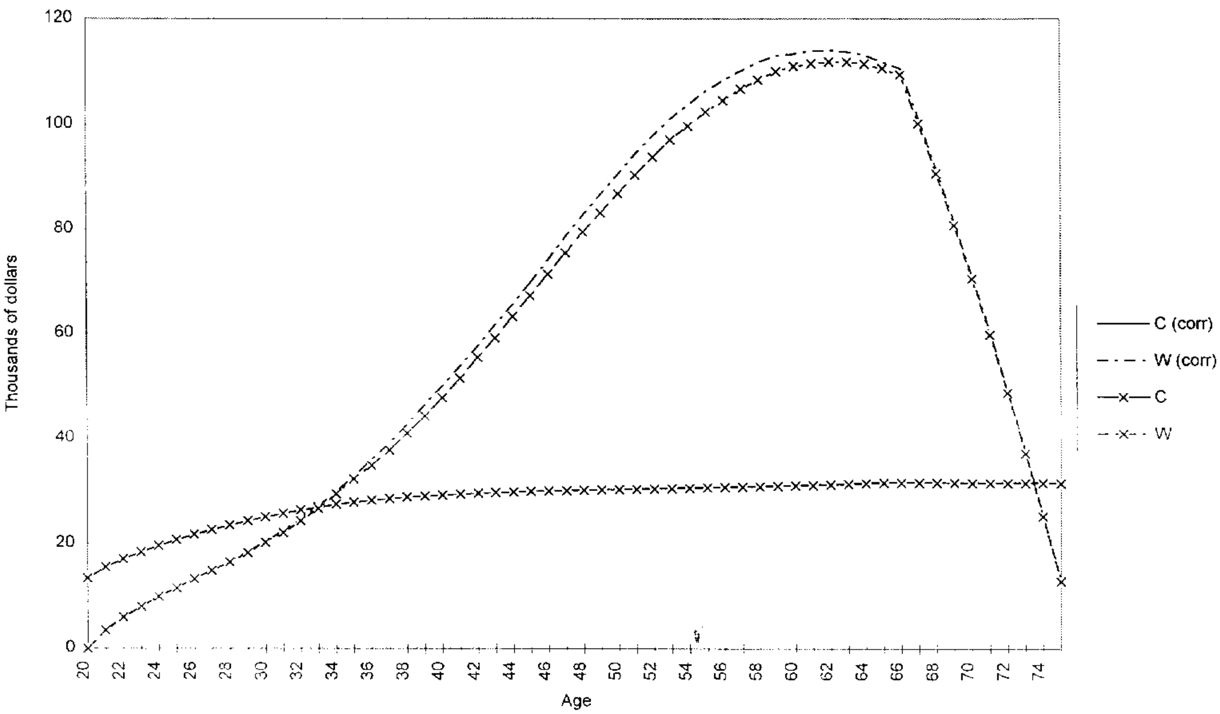
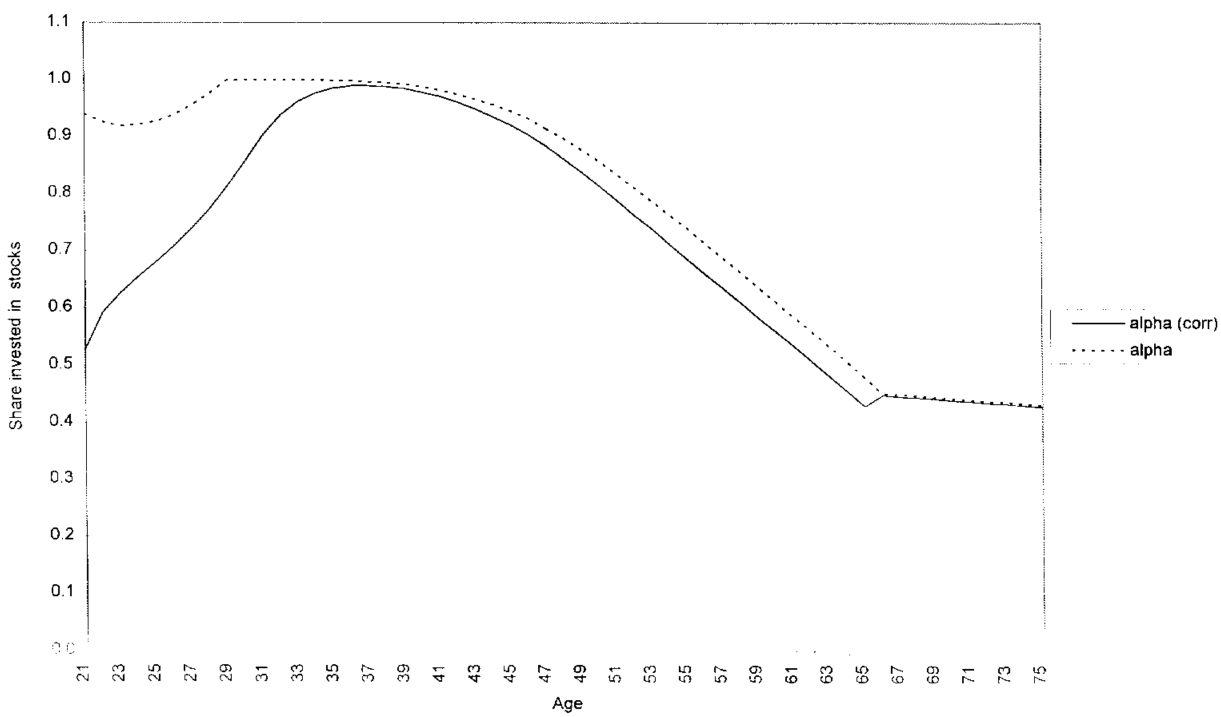


Figure 8b - Portfolio Allocation over the Life-Cycle
(10% Correlation between innovations to stock returns and to labor income and Benchnark)



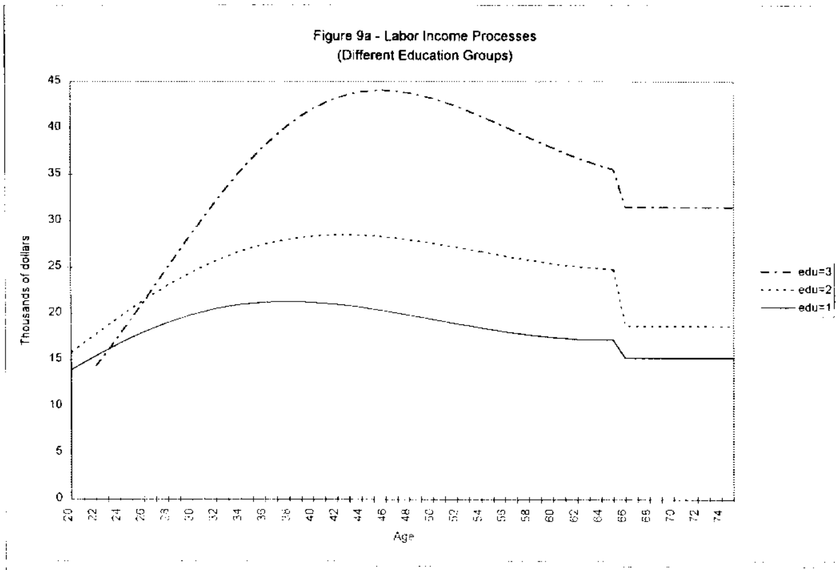


Figure 9a - Labor Income Processes
(Different Education Groups)
Thousands of dollars
Age
edu=3
edu=2
edu=1

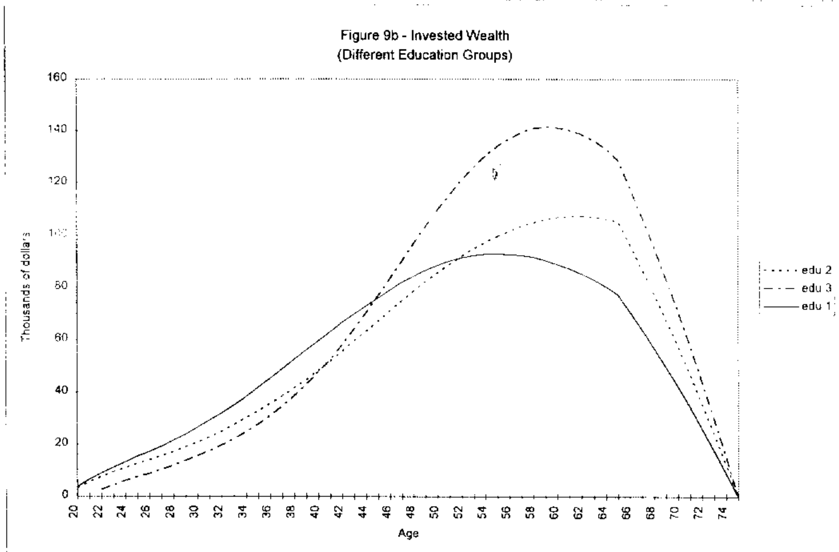


Figure 9b - Invested Wealth
(Different Education Groups)
Thousands of dollars
Age
edu 2
edu 3
edu 1

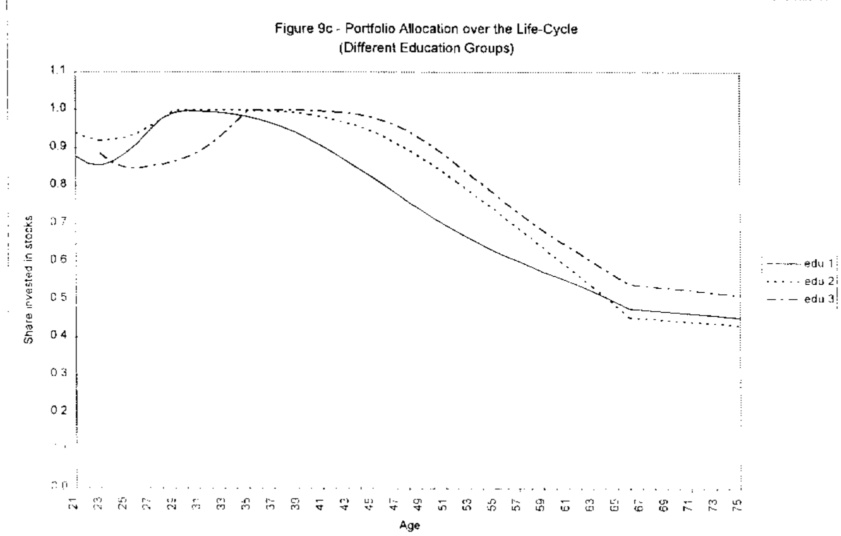


Figure 9c - Portfolio Allocation over the Life-Cycle
(Different Education Groups)
Share invested in stocks
Age
edu 1
edu 2
edu 3

Figure 10a - Consumption and Wealth
(Income Process with and without Permanent Shocks)

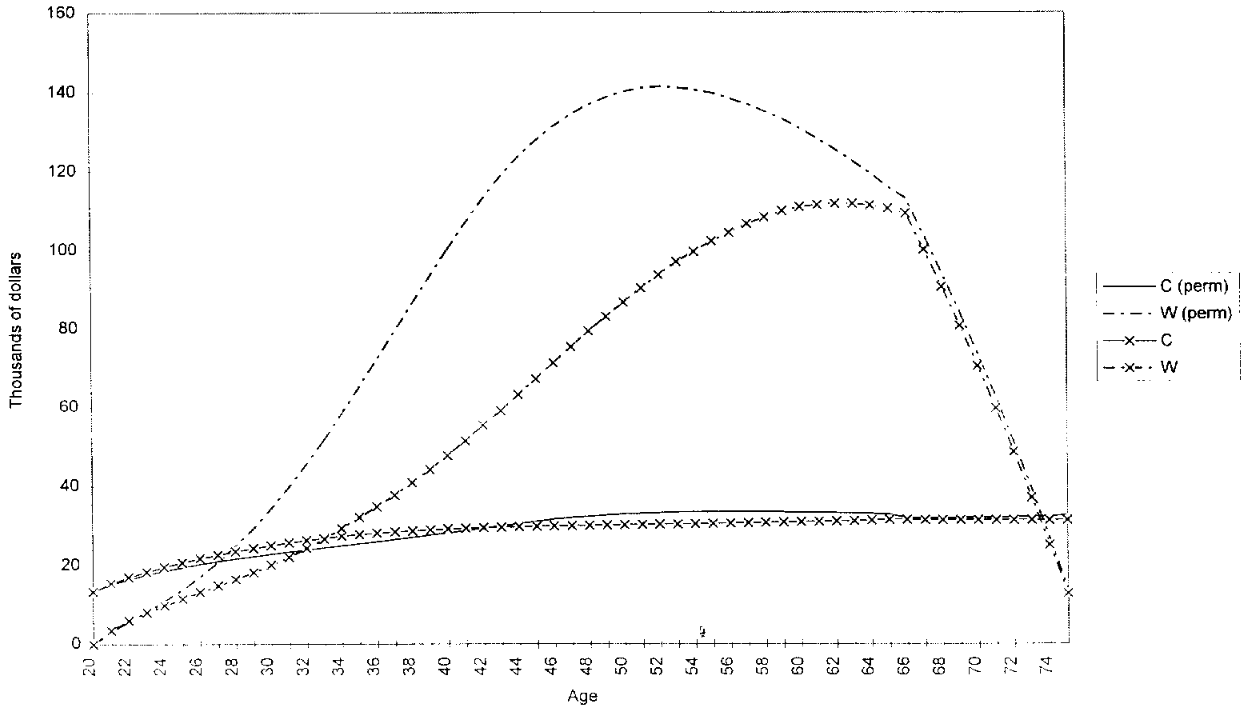


Figure 10b - Portfolio Allocation over the Life-Cycle
(Income Process with and without Permanent Shocks)

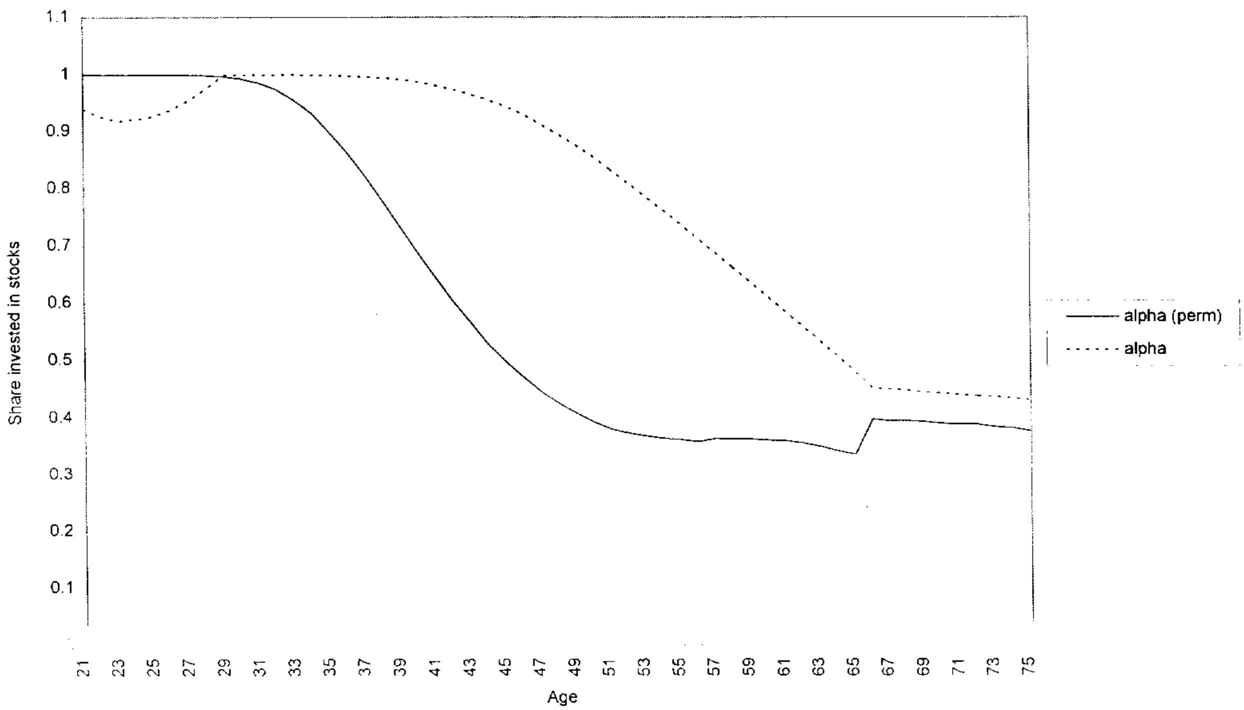


Figure 11 - Portfolio Choice over the Life-Cycle
(Different Excess Return Values, Permanent Shocks to Labor Income)

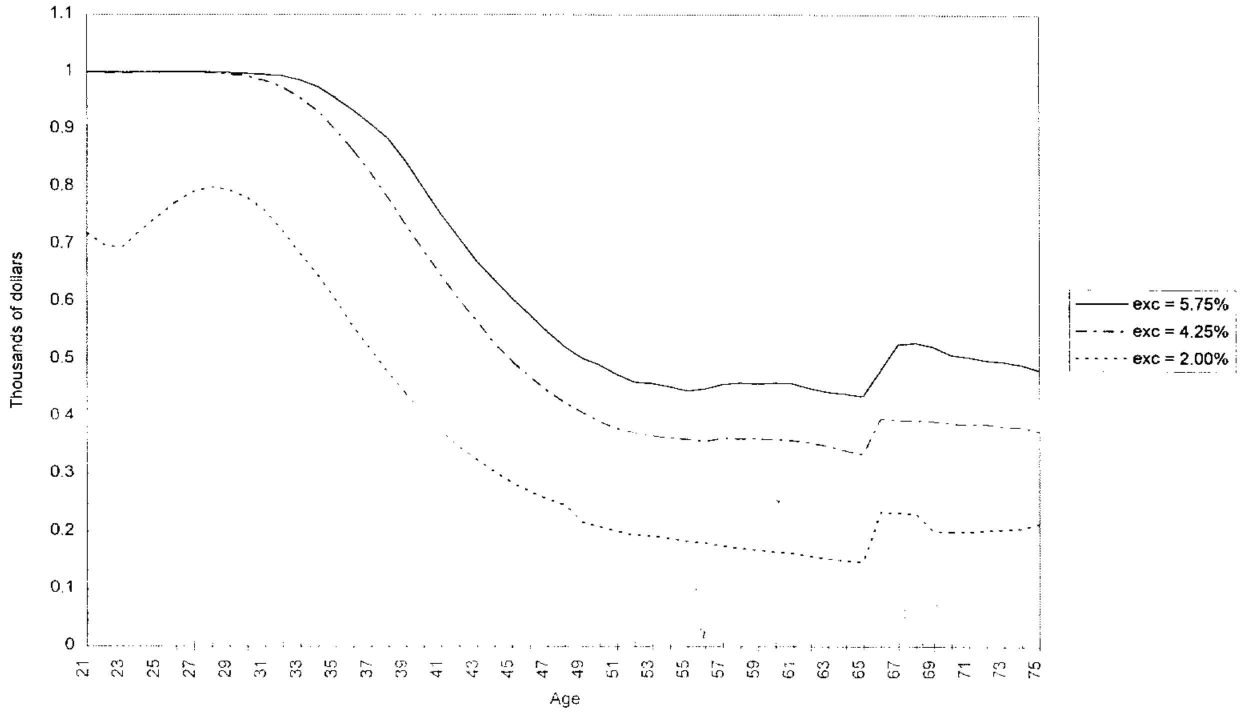
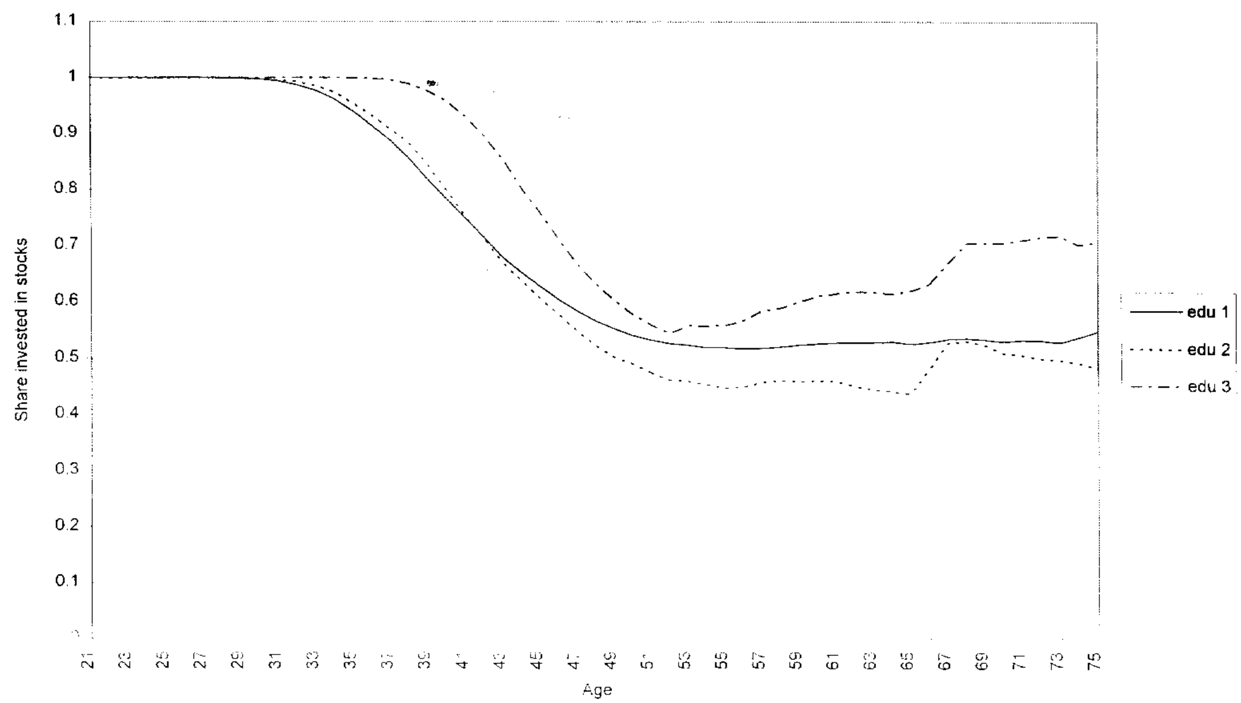


Figure 12 - Portfolio Allocation over the Life-Cycle
(Different Education Groups, Permanent Shocks to Labor Income)



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