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ON THE INTERACTION BETWEEN TASTE AND DISTANCE AND ITS IMPLICATIONS ON THE VERTICAL DISTRIBUTION ARRANGEMENT

by

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On the Interaction between Taste and Distance and Its Implications on the Vertical Distribution Arrangement*

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Abstract

This paper studies the vertical distribution arrangement between manufacturers and retailers in a two dimensional horizontal product differentiation framework. Products are differentiated along consumers' taste and retailers' location. The objective of the paper is to analyse the incentive of manufacturers to impose an exclusive territory restriction. We construct a four-stage game. In the first stage manufacturers decide the optimal number of retailers. In the second stage manufacturers decide the wholesale prices. In the third stage retailers decide the retail prices. Finally, in the fourth stage consumers purchase goods. We show that the relative magnitude of the transportation and substitution costs is crucial in determining the decision of manufacturers to impose an exclusive territory restriction.

Keywords: vertical distribution arrangement, horizontal product differentiation, optimal number of retailers.

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1 Introduction

Manufacturers often have to make a choice between distributing their products directly or delegating the product distribution to independent retailers. The choice obviously depends on the cost of self distributing the brands relative to the cost of appointing independent retailers. These costs will be determined by the scale of the production process, the size of the market faced by manufacturers, and the handling cost. Indeed, the cost of distributing products might be prohibitively high such that it does not pay-off for manufacturers to handle the product distribution themselves. This is what we often see in reality as well. For instance, car manufacturers more often distribute their brands using outside independent retailers rather than distribute the brands themselves (see Mycielski, et.al., 1997). Given the decision of manufacturers to use outside independent retailers, the next crucial issue to deal with is the design of the optimal distribution arrangement with retailers. This paper will cover this issue. In particular it analyses exclusive territory arrangement. Exclusive territory is the right given to a single retailer by a manufacturer which allows the retailer to act as a sole distributor in a specific territory.

Manufacturers' incentive to choose strategically the number of retailers to distribute their products within a specific territory will depend on the toughness of competition between manufacturers. The toughness of competition between manufacturers is influenced by the degree of product differentiation. We present an analysis of the vertical distribution arrangement in a two dimensional horizontal product differentiation model to capture this setting. Products are differentiated along consumers' taste for different brands and retailers' location. The interaction between these two dimensions will affect the decision of a consumer to buy a product, and hence it will have an important implication on the strategic decision of manufacturers.

Suppose a consumer wants to buy a product, and she has to choose between two brands (A and B). Each of these brands is distributed by retailers. A consumer will take into account her product taste and retailers' location in deciding which brand to buy. If the product she wants is available from the nearby retailer, she will incur less transportation cost. If instead the product is not available from the nearby retailer, she will have to travel to the retailer which sells the product and pays a higher transportation cost. Which product will eventually be bought depends on the dominance of either the consumer's taste or the transportation cost. If the taste difference dominates the transportation cost, then a consumer who resides sufficiently far might still be willing to buy from the retailer carrying her preferred product. However, if the taste difference is not important, then the transportation cost will determine to which retailer the consumer is going to shop.

A manufacturer can influence the transportation cost by changing the number of retailers in the market.[1] Adding more retailers shrinks the distance between two retailers, and thus reduces the transportation cost for consumers. Consequently, the existence of either the taste dominance or the transportation cost dominance can partly be influenced by manufacturers.[2] This can be explained as follows. Suppose we initially have a situation in which the transportation cost dominates the taste difference. In this situation adding new retailers shrinks the distance between retailers and thus decreases the transportation cost for consumers. If the transportation cost decreases quite substantially we might switch from the case of transportation cost dominance to the case of taste dominance. This would mean that a consumer who prefers product A to product B will go to the retailer which sells product A. In the previous case of transportation cost dominance, this consumer will buy product B despite the fact that she prefers product A to product B. The high travelling cost deters her from buying product A.

However, adding new retailers has a competitive effect. It makes the retail competition tougher. This could put pressure on manufacturers' profits. Hence, the feasibility of adding (reducing) new retailers will depend on the relative benefits of the transportation cost reduction to the cost of a tougher competition.

This implies that manufacturers face two choices, on the one hand they might want to reduce the number of retailers (possibly to a single retailer) to take advantage of the maximum monopolistic position due to a high transportation cost (a high search cost for consumers). In this case, manufacturers also avoid intense competition among them. On the other hand, they might also want to increase the number of retailers to reduce the transportation cost, and thus to supress the search cost for consumers with the hope of attracting more consumers. The crucial issue here would be the identification

[1] In this paper we assume that manufacturers have all the bargaining power. There are many ways in which the manufacturers can influence the number of retailers (Katz,1989). Manufacturers can appoint directly the retailers they would like to carry the brands. Alternatively, manufacturers can indirectly control the number of retailers using vertical restraints. Note that retailers here can also be interpreted as outlet stores. In this case, we can safely assume that the number of outlet stores can be fully influenced by manufacturers.

[2] It is partly, because the relative magnitude of the transportation costs per unit distance to the utility costs from consuming the most preferred product will also determine the existence of either transportation costs dominance or taste dominance. This relative magnitude is exogeneous in our model.

of the conditions for manufacturers to prefer one choice to another. We are going to discuss this issue in the present paper.

We use the Salop's model on two dimensional horizontal characteristics. On the contrary to the traditional one-dimensional product differentiation framework, the literature on the multi dimensional product differentiation has only been recently developed. There are several such studies, for instance Economides (1989) and Neven and Thisse (1990). They develop a two characteristics product differentiation model which combines horizontal and vertical product differentiation. These two characteristics are independent of each other. Tabuchi (1994) analyses Hotelling's model of spatial duopoly on two-dimensional spaces in a two- stage game. The first stage is the locational game and the second stage is the price game. Degryse (1995) analyses a model that combines horizontal and vertical product differentiation and applies it to banking. Ansari, Economides, and Steckel (1998) extend the model to the case of three-dimensional model. In our model, there is an indirect interaction between the two horizontal dimensions. As is mentioned before a manufacturer can add new retailers and influence the transportation cost, which will result in a change in the relative importance of the transportation cost to the taste difference.

We construct a four-stage game. There are three parties involved, i.e. consumers, retailers, and manufacturers. In the first stage manufacturers decide the optimal number of retailers. In the second stage manufacturers choose the wholesale prices, and in the third stage retailers choose the retail prices. Finally, in the fourth stage consumers purchase goods.

To the best of our knowledge there is only a paper by Besanko and Perry (1994) which tries to analyse the vertical distribution arrangement in a spatial competition framework. They use a logit model of product differentiation. In this paper we use a model in the spirit of Neven and Thisse (1990). Furthermore, Besanko and Perry (1994) stress more on the comparison between exclusive dealing and non exclusive dealing arrangement. Our paper will focus more on the optimal number of retailers. We assume that exclusive dealing prevails. In their analysis, Besanko and Perry (1994) show that exclusive dealing will generate higher profits for manufacturers, retail prices and transportation costs for consumers than non exclusive dealing. Hence, manufacturers will have incentives to adopt exclusive dealing. In this paper we do not make an explicit comparison. However it can be easily deduced that in our framework non exclusive dealing will often be less preferred by manufacturers. In the non exclusive dealing case, both adjacent retailers will carry the same brands. This situation, on the one hand, is beneficial for consumers because consumers can easily switch to a different retailer without incurring a high transportation cost. On the other hand it is less beneficial for manufacturers because it creates a tougher competition. Manufacturers' profits will likely be smaller in this case. We want to concentrate the analysis on the optimal number of retailers, thus there is no loss of generality from assuming that exclusive dealing prevails.

A paper by Martinez-Giralt and Neven (1988) is also closely related. As a matter of fact our paper can be considered as an extension of their paper. They use the conventional Salop's one-attribute spatial competition model, while we use two-attributes spatial competition model. They analyse the incentive of manufacturers to establish retail outlets, and show that the need to dampen price competition will outweigh the need to segment the market. In equilibrium, they show that firms will not install multiple outlets, and thus use only a single retail outlet. On the contrary, we show that in a two-attributes framework this is not always the case. It depends on the relative importance of consumers' taste difference and the transportation cost.

Another related paper is Mycielski, et.al. (1997). This paper discusses the link between product differentiation and the vertical distribution arrangement. The choice of the optimal number of retailers is also discussed in the paper. However, instead of using an explicit product differentiation model, the paper uses the cross-price elasticity of demand as a measure of the degree of product differentiation. We take up the same issue and somewhat restrict the focus of the discussion, but we introduce an explicit product differentiation model.

Our results show that when the transportation cost dominates the substitution cost, it may be better for manufacturers to make the distance between retailers sufficiently far. In other words, manufacturers have to make the transportation cost even more dominant. This can be done by appointing a single retailer to serve the whole city. This might explain why quite often the case that exclusive dealing is coupled with exclusive territory. However, when the substitution cost dominates the transportation cost, then the number of retailers in the market is not important for manufacturers. The number will be determined by the free entry equilibrium at the retailer level. Hence, there could potentially be many retailers in the market.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 gives the solutions for retailers. Section 4 discusses the solutions for manufacturers. Section 5 analyses the optimal number of retailers. Section 6 discusses the results. Finally, section 7 concludes.

2 The Model Setting

We consider a vertical structure between manufacturers and retailers. There is a duopoly setting at the manufacturer level. Manufacturers (firm 1 and firm 2) produce differentiated products $i \in \{A, B\}$. The distance between the two products is normalized to 1. Product A is located at 0 and product B is located at 1. A consumer's taste over the two products is represented by a location somewhere between these two products.

We assume that there is a circular city with a perimeter equal to 1. There are an equal number of identical retailers (n) which are located equidistantly along the circle carrying each product. Exclusive dealership prevails, thus two neighbouring retailers will carry different brands. Thus, there is an interlaced structure between retailers who carry product A and product B. For instance, a retailer which distributes product A will be located at $\frac{j}{n}$ and a retailer which distributes product B will be located at $\frac{(j-\frac{1}{2})}{n}$ for all $j = 1, 2, 3, ..., n$ (see figure 1). This can be justified as follows. Suppose a retailer is located between two other retailers selling brand A, then it is better for the retailer to sell brand B rather than brand A. Selling brand B will result in higher sales than selling brand A (see also Besanko and Perry (1994) which use similar justification).[3]

Consumers are distributed uniformly with a density equal to 1 along the product space. For simplicity, we assume that consumers purchase only 1 unit of a product. A consumer will have to pay a unit transportation cost of t for a visit to a retailer. In addition, a consumer has to spend a unit substitution cost of s in utility terms for substituting her most preferred brand with the actual available brand.

Obviously if inter-brand competition is absent, i.e. there is only a monopolist manufacturer which produces a brand (say brand A) and distributes the brand through n retailers, we will have the familiar one dimensional Salop model. This Salop model can then be interpreted as a model of pure intra-brand competition. The distance between two retailers will be $\frac{1}{n}$ instead of $\frac{1}{2n}$.

To proceed we construct a four-stage game. In the first stage manufacturers decide the optimal number of retailers. In the second stage manufacturers compete and choose the wholesale prices. In the third stage retailers compete and decide the retail prices. Finally, in the last stage consumers purchase goods. We apply backward induction approach to solve the game.

[3] Using this justification, we assume that the above interlaced and equidistant structure is still preserved when the number of retailers changes.

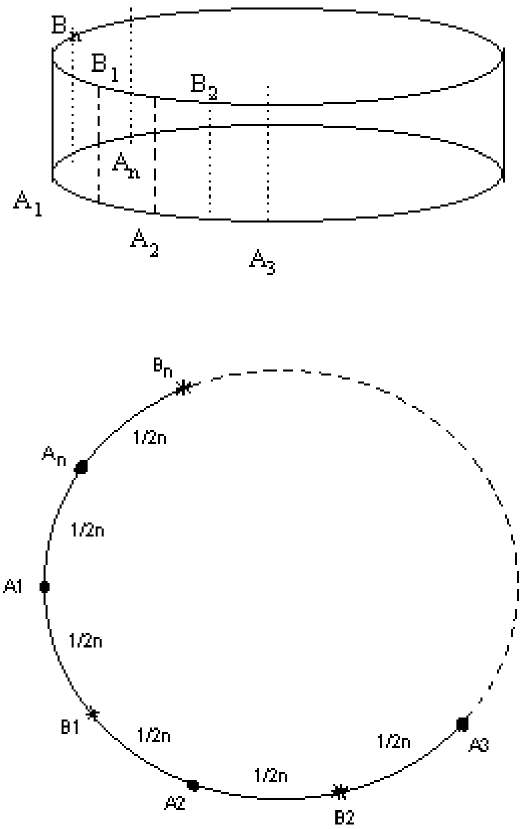


Figure 1: The Retail Structure

2.1 Derivation of the Demand Function

Let us assume that retailers on the left side and on the right side sell product A and B respectively. We denote x as the location of a consumer who is indifferent between buying from the left retailer and the right retailer, and y as the location of a consumer who is different between buying product A and B. A consumer who is located at (x, y) derives the following indirect utility from buying 1 unit of product A from the left-side retailer.

$$U_A(x, y) = v - p_A - tx - sy \tag{1}$$

Where v denotes the reservation value of the consumer and it is a positive constant. We assume that v is sufficiently large so that all consumers will draw a positive utility from buying a good, otherwise consumers will simply refrain from buying the good.[4] We denote the retail prices with p_i for $i \in$

[4] This assumption will ensure that the market can be covered by both manufacturers.

$\{A, B\}$. A consumer who is located at $y - \epsilon$ will ideally prefer product A to product B. However if he is located at $x + \epsilon$ from the left retailer who sells product A, he will instead buy product B from the right side retailer. This is because the transportation cost will deter the consumer from buying product A.

Alternatively, if the consumer shops to the retailer on the right adjacent side and buys product B she receives the following indirect utility.

$$U_B(x, y) = v - p_B - t(\frac{1}{2n} - x) - s(1 - y) \tag{2}$$

The location of an indifferent consumer who is indifferent between buying product A from the left retailer and buying product B from the right retailer can be calculated as follows.

$$v - p_A - tx - sy = v - p_B - t(\frac{1}{2n} - x) - s(1 - y) \tag{3}$$

Solving for y and expressing it as a function of x we obtain.

$$y(x) = \frac{(p_B - p_A)}{2s} + \frac{1}{2} + \frac{t}{4ns} - \frac{t}{s}x \tag{4}$$

If $y(x)$ is bigger (smaller) than the right hand side expression, consumers will buy product A (product B) from the left-side (right-side) retailer. Thus, this function partitions consumers into two groups, one group will buy product A and another group will buy product B. We illustrate an example of this partition line in figure 2.

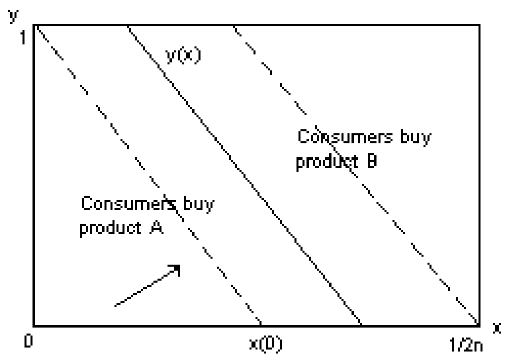


Figure 2: The Transportation Cost Dominance

The shape of the $y(x)$ depends on its slope. We have two mutually exclusive cases depending on the relative magnitude of the ratio of per unit transportation cost (t) to per unit substitution cost (s), and the number of retailers in the market. These two cases are the case of transportation cost dominance and the case of taste dominance. The case of transportation cost dominance (taste dominance) refers to a situation in which consumers' decision is influenced more (less) by the transportation cost than by the difference in taste. In the case of transportation dominance not all consumers who strictly prefer a product (say product A) are served. Some of them have to incur a sufficiently high transportation cost to go to the left retailer who sells product A, as a result they decide to buy product B instead. On the contrary in the case of taste dominance all retailers who strictly prefer product A are served and buy product A from the left retailer.

The aggregate demand consists of three different segments depending on the retail price. The first segment prevails at the high price range, the second segment prevails at the intermediate price range, and the last segment prevails at the low price range. First, we will look at the demand function for the case of transportation cost dominance.[5]

2.1.1 Transportation Cost Dominance

The First Segment

This case is depicted in figure 2. The first segment starts from a situation in which there is no demand at all for product A because the price is far too high. Reducing the price (see the direction of the arrow in figure 3) will attract additional consumers. This segment ends when the $y(x)$ curve touches the top left hand corner of the rectangular. The demand at this segment can be expressed as,

$$D_A^{'}(p_A, p_B) = \frac{1}{32n^2st}(2p_Bn - 2p_An + t + 2sn)^2 \tag{5}$$

The Second Segment

This segment begins right after the border with the first segment until the point where the $y(x)$ curve touches the bottom right hand corner of the rectangular. The demand at this segment is linear and can be expressed as,

$$D_A^{''}(p_A, p_B) = \frac{p_B - p_A}{2t} + \frac{1}{4n} \tag{6}$$

The Third Segment

The rest of the area consitutes the last segment of the demand. This segment is just the mirror image of the first segment. It can be expressed as,

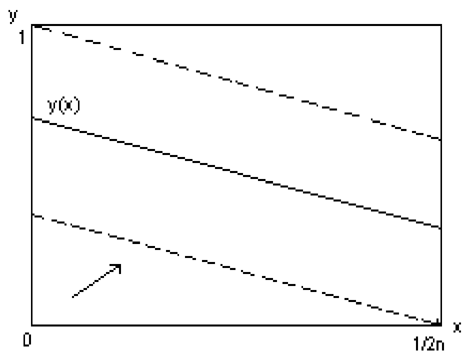
[5] The complete derivation can be found in appendix 1.

$$D_A^{'''}(p_A, p_B) = \frac{1}{2n} - \frac{1}{32n^2 st}(2np_A - 2np_B + 2ns + t)^2 \qquad (7)$$

The demand for the second product can easily be derived by substracting the demand for product A to the total demand. The total demand is always equal to $\frac{1}{2n}$, hence the demand for product B is $D_B(p_A, p_B) = \frac{1}{2n} - D_A(p_A, p_B)$.

2.1.2 Taste Dominance

This case is depicted in figure 3. *In this case, the first and the third segments of the demand function are similar to the previous case. However, the second segment is different.* This segment starts from the point where the $y(x)$ curve touches the bottom right hand corner of the rectangular until the point where the $y(x)$ curve reaches the top left hand corner.



The Demand at this segment is linear and can be expressed as,

$$D_A^{''}(p_A, p_B) = \frac{1}{4n}\left(1 + \frac{p_B - p_A}{2s}\right) \qquad (8)$$

The demand for product B at this segment is just $D_B(p_A, p_B) = \frac{1}{2n} - D_A(p_A, p_B)$.

3 Equilibrium Retail Prices

We will now solve for the pure-strategy equilibrium in retail prices. We consider only the interior solutions. The profit functions for retailers which sell product A and product B respectively are,

$$\pi_A(p_A, p_B) = 2D_A(p_A, p_B)(p_A - w_A)$$

$$\pi_B(p_A, p_B) = 2D_B(p_A, p_B)(p_B - w_B)$$

We assume for simplicity that there is no retailing cost. We have to check for all possible retail price equilibria for different segments of the demand function faced by retailers. We start from the first segment of the demand function for the case of transportation cost dominance.

3.1 Transportation Cost Dominance

Profit functions of both retailers can be expressed respectively as,

$$\pi_A(p_A, p_B) = 2D_A(p_A, p_B)(p_A - w_A) \tag{9}$$

$$\pi_B(p_A, p_B) = 2\left(\frac{1}{2n} - D_A(p_A, p_B)\right)(p_B - w_B) \tag{10}$$

Substituting equation (5) into expressions (9) and (10) we get the retail profit functions. Taking first order conditions of the retail profit maximizations with respect to retail prices and solving for the equilibrium retail prices we obtain (see the derivation in appendix 2),

$$p_A^1 = \frac{w_B + 7w_A}{8} + \frac{1}{16n}\left(2sn + t + \sqrt{Z}\right) \tag{11}$$

$$p_B^1 = \frac{3w_B + 5w_A}{8} - \frac{10sn + 5t - 3\sqrt{Z}}{16n} \tag{12}$$

$$p_A^2 = \frac{w_B + 7w_A}{8} + \frac{1}{16n}\left(2sn + t - \sqrt{Z}\right) \tag{13}$$

$$p_B^2 = \frac{3w_B + 5w_A}{8} - \frac{10sn + 5t + 3\sqrt{Z}}{16n} \tag{14}$$

In which $Z = \sqrt{(2n(w_A - w_B) - 2ns - t)^2 + 64nts}$. These retail price equilibria are difficult to interpret. However, at this point there is no need to worry about the interpretation of the results because we still have to check whether or not the pure-strategy wholesale price Nash equilibrium exists for these equilibrium retail prices.

Next, we calculate the equilibrium retail prices which will occur on the *second segment* of the demand function. Substituting expression (6) into expressions (9) and (10) we have the following retail profit functions.

$$\pi_A(p_A, p_B) = 2\left(\frac{p_B - p_A}{2t} + \frac{1}{4n}\right)(p_A - w_A) \tag{15}$$

$$\pi_B(p_A, p_B) = 2\left(\frac{1}{4n} - \frac{p_A - p_B}{2t}\right)(p_B - w_B) \tag{16}$$

$$p_A = \frac{2}{3}w_A + \frac{1}{3}w_B + \frac{1}{2n}t \tag{17}$$

$$p_B = \frac{2}{3}w_B + \frac{1}{3}w_A + \frac{1}{2n}t \tag{18}$$

Because the buying decision of a consumer is only influenced by the transportation cost, the substitution cost does not play a role. It is obvious that when the transportation cost (t) increases, retail prices will increase. This is plausible because the costlier to travel is, the more captive consumers they have. As a result retailers have more monopoly power. However, retail prices are decreasing in the number of retailers. It is because the more retailers in the market are, the closer the distance between retailers is, and thus the stronger the competition is. As a consequence retail prices will tend to decrease.

The prevailing retail price equilibria on the *third segment* of the demand function are just the *mirror images* of the prevailing retail price equilibria on the *first segment* of the demand function.

3.2 Taste Dominance

The equilibrium retail prices prevailing on the first and third segments of the demand function are the same as in the case of transportation cost dominance. However, the equilibrium retail prices occuring on the linear segment of the demand (the second segment) are different. These can be calculated from the retail profit maximization problems with respect to retail prices. Expressions for the retail profit functions can be obtained by substituting expression (8) into expressions (9) and (10).

$$\pi_A(p_A, p_B) = 2\left[\frac{1}{4n}\left(1 + \frac{p_B - p_A}{s}\right)\right](p_A - w_A) \tag{19}$$

$$\pi_B(p_A, p_B) = 2\left[\frac{1}{4n}\left(1 - \frac{p_B - p_A}{s}\right)\right](p_B - w_B) \tag{20}$$

Deriving first order conditions of the profit maximization and solving for the equilibrium retail prices we obtain the following,

$$p_A = \frac{2}{3}w_A + \frac{1}{3}w_B + s \tag{21}$$

$$p_B = \frac{1}{3}w_A + \frac{2}{3}w_B + s \tag{22}$$

The transportation cost does not play a role. A consumers decision is driven only by the substitution cost. It can be clearly seen that the higher the substitution cost is, the more reluctant the consumers are to switch to a different product, and thus retailers obtain more captived consumers. As a result retail prices will tend to increase.

4 Equilibrium Wholesale Prices

Having derived the equilibrium retail prices we will solve for the equilibrium wholesale prices. Again we start from the case of transportation cost dominance.

4.1 Transportation Cost Dominance

The profit functions of the two manufacturers can be expressed as follows,

$$\Pi_A(p_A, p_B) = 2nD_A(p_A, p_B)\, w_A \tag{23}$$

$$\Pi_B(p_A, p_B) = 2n\left(\frac{1}{2n} - D_A(p_A, p_B)\right) w_B \tag{24}$$

For simplicity let us assume that the production cost is zero and manufacturers do not have to incur the cost of setting-up a retailernet. We first solve for the equilibrium wholesale prices which occur when the equilibrium retail prices prevail on the *first* and *third segments* of the aggregate demand function. The following lemma summarizes the result.

Lemma 1 *For the retail price equilibria prevailing on the first and third segments of the demand function, there exist no pure-strategy wholesale price nash-equilibrium for manufacturers. Hence, we can restrict our attention to the case in which the equilibrium retail prices occur on the second segment of the demand function only.*

Proof. The proof is relegated to appendix 3.

Now, we will calculate the equilibrium wholesale prices when the equilibrium retail prices occur on the second segment of the demand function. Substituting expressions (17) and (18) into expression (6) and then substituting the result in expressions (23) and (24) we obtain the following profit functions for manufacturers,

$$\Pi_A (w_A, w_B) = \left(\frac{2nw_B - 2nw_A + 3t}{6t} \right) w_A \tag{25}$$

$$\Pi_B (w_A, w_B) = \left(\frac{2nw_A - 2nw_B + 3t}{6t} \right) w_B \tag{26}$$

Solving for the profit maximisation problems for both manufacturers we get the following equilibrium wholesale prices.

$$w_A = \frac{3}{2} \frac{t}{n} \tag{27}$$

$$w_B = \frac{3}{2} \frac{t}{n} \tag{28}$$

Substituting the equilibrium wholesale prices into the retail price equations (17) and (18) we obtain the following simple expressions of the equilibrium retail prices as a function of the transportation cost and the number of retailers.

$$p_A = 2 \frac{t}{n} \tag{29}$$

$$p_B = 2 \frac{t}{n} \tag{30}$$

4.2 Taste Dominance

Since the first and the third segments of the demand function for the transportation cost dominance case are the same as for the taste dominance case, the solutions for the equilibrium wholesale prices will also be the same for both cases. However, we have different equilibrium solutions for the second segment of the demand function.

Substituting expressions (21) , (22), and (8) into profit functions (23) and (24) we have the following,

$$\Pi_A(w_A, w_B) = \left(\frac{3s + w_B - w_A}{6s}\right) w_A \tag{31}$$

$$\Pi_B(w_A, w_B) = \left(\frac{3s + w_A - w_B}{6s}\right) w_B \tag{32}$$

Deriving first order conditions for the profit maximization problems and solving for the equilibrium wholesale prices we obtain,

$$w_A = 3s \tag{33}$$

$$w_B = 3s \tag{34}$$

Substituting expressions (33) and (34) into (21) and (22) we can express the equilibrium retail prices as a function of the substitution cost.

$$p_A = 4s \tag{35}$$

$$p_B = 4s \tag{36}$$

5 Optimal Number of Retailers

We now come to the first stage of the game. We have two cases in which equilibrium retail and wholesale prices exist. These are the cases where the equilibrium retail and wholesale prices occur on the second segment of the demand function for both the transportation cost dominance and taste dominance cases.

Transportation Cost Dominance

Substituting wholesale prices (27) and (28) into expressions (23) and (24) we can express manufacturers' profits as,

$$\Pi_A = \frac{3}{8n} t \tag{37}$$

$$\Pi_B = \frac{3}{8n} t \tag{38}$$

Taste Dominance

Similarly, subtituting wholesale prices (33) and (34) into expressions (23) and (24) we obtain,

$$\Pi_A = \frac{3}{2}s \quad (39)$$

$$\Pi_B = \frac{3}{2}s \quad (40)$$

The following proposition summarizes the results.

Proposition 1 *(i) In the case of transportation cost dominance, the manufacturers' profits are decreasing in the number of retailers. The smallest number is* 1. *Hence, if the case of transportation cost dominance is optimal for manufacturers, each of them would gain the highest profits by appointing an exclusive retailer to serve the whole city.*[6] *(ii) If instead, the case of taste dominance is optimal for manufacturers, then manufacturers will not concern about the number of retailers. This number will then be determined by the zero profit equilibrium condition at the retailer level. As a result there could potentially be many retailers in the market.*

6 Discussions on the Results

We start by deriving the conditions for the existence of the case of transportation cost dominance and the case of taste dominance. From an inspection on figure 4, the condition for the transportation cost dominance can be established as follows.

$$x(y) = x(1) > 0 \quad (41)$$

In which the expression for $x(y)$ is obtained from re-arranging expression (4). Solving this condition and using the fact that the equilibrium retail prices are the same for both retailers we obtain,

$$n < \frac{t}{2s} \quad (42)$$

The condition for the case of taste dominance is,

[6] Obviously manufacturers could decide to distribute their product themselves rather than using an independent retailer network. In this case, an additional issue arises namely the issue of determining the boundary of the firm. Basically this decision will depend on manufacturers' net profits from using an independent distribution system relative to manufacturers' net profit from distributing their product directly. We assume here that direct distribution is very costly.

$$x\,(y) = x(1) < 0 \tag{43}$$

Again, upon solving this condition we obtain,

$$n > \frac{t}{2s} \tag{44}$$

We can easily see that the *two cases are mutually exclusive.*

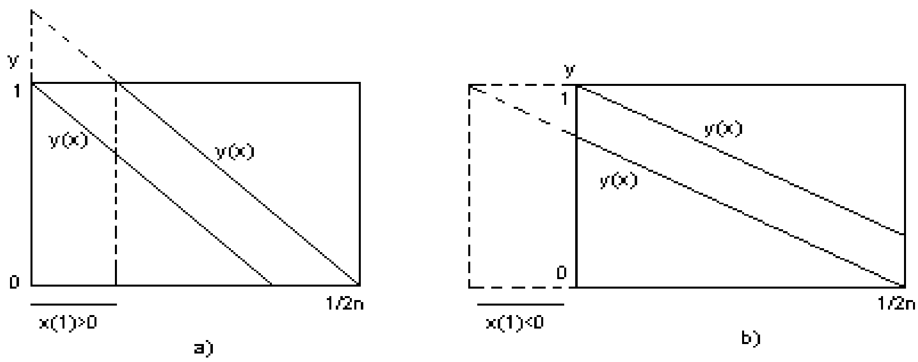


Figure 4: Conditions for a) the transportation cost dominance and b) the taste dominance.

Manufacturers will be indifferent between being in the case of transportation cost dominance or in the case of taste dominance if both cases give equal profits. Formally, this can be expressed as,

$$\Delta\Pi_i = \frac{3}{4n}t - \frac{3}{2}s = 0 \tag{45}$$

Which can be simplified into,

$$n = \frac{t}{2s}$$

From the previous results we know that the case of transportation cost dominance exists if $n < \frac{t}{2s}$,and the case of taste dominance exists if $n > \frac{t}{2s}$. Hence, it is clear that both cases will never give equal payoffs. The case of $n = \frac{t}{2s}$ is the *border case in which neither the transportation cost dominates the taste nor the taste dominates the transportation cost).*

6.1 Manufacturers' Incentive to Switch Between Cases

Suppose initially we have $n = \frac{t}{2s}$. If for an *exogeneous* reason per unit transportation cost (t) increases or substitution cost (s) decreases, ceteris paribus, we will have $n < \frac{t}{2s}$. This means that we are in the case of transportation cost dominance. If, instead, t decreases or s increases, ceteris paribus, we then have $n > \frac{t}{2s}$. We will be in the case of taste dominance.

A manufacturer might also be able to *endogeneously switch* from one case to another by increasing or decreasing the number of retailers in the market (see figure 5 for the case of increasing n). We know that the distance between two retailers is $\frac{1}{2n}$. Consequently, if n increases (decreases) the distance between the two retailers decreases (increases), and thus we might switch from one case to another.

Before analysing manufacturers' incentive to switch, we will first provide the condition under which manufacturers are able to switch between cases.The following lemma states the condition.

Lemma 2 Manufacturers can switch from the case of transportation cost dominance into the case of taste dominance, and vice versa, if and only if $t > 2s$. Otherwise manufacturers can only choose between the case of taste dominance or the border case.

Proof. Let us suppose that $t \leqslant 2s$, thus $\frac{t}{2s} \leqslant 1$. The case of transportation cost dominance prevails iff $n < \frac{t}{2s}$. Since the smallest possible number of retailers is 1, hence if $\frac{t}{2s} \leqslant 1$ holds, manufacturers can never be in the case of transportation dominance because n can never be smaller than 1. *There are only two possible cases, namely the taste dominance case* $\left(n > \frac{t}{2s}\right)$ *or the border case* $\left(n = \frac{t}{2s}\right)$. Now, let us suppose that $t > 2s$,we will have that $\frac{t}{2s} > 1$. Under this condition, manufacturers can set the number of retailers (this has to be an integer number) such that $n > \frac{t}{2s}$, or $n < \frac{t}{2s} \leqslant 1$, or $n = \frac{t}{2s}$. If $n > \frac{t}{2s}$ prevails, we are in the taste dominance case. But if $n < \frac{t}{2s} \leqslant 1$ prevails, we are in the transportation dominance case. Finally if $n = \frac{t}{2s}$ prevails, we are in the border case. Hence, we establish that manufacturers can switch between cases iff $t > 2s$. ■

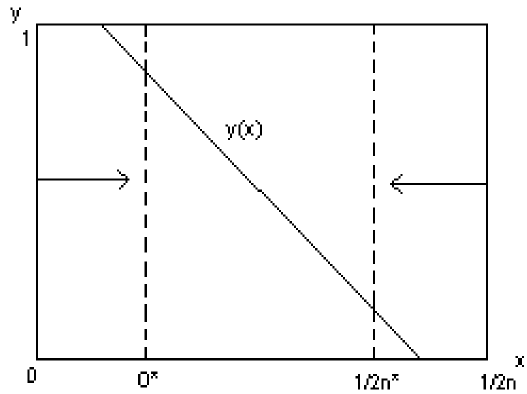


Figure 5: Adding More Retailers

Having established the above lemma, we will look at the case of $t > 2s$ and analyse the incentive of manufacturers to switch between the two cases.

Proposition 2 If per unit transportation cost exceeds per unit substitution cost, such that $t > 2s$, manufacturers prefer to be in the transportation dominance case, and manufacturers' profits will be the highest when each of them appoints an exclusive retailer to serve the whole city.

Proof. Suppose manufacturers prefer to be in the case of taste dominance and thus set n such that $n > \frac{t}{2s}$. The profits for manufacturers are $\frac{3}{2}s$. If instead manufacturers choose to be in the transportation dominance case, n will be set such that $n < \frac{t}{2s}$. Profits will be equal to $\frac{3}{4n}t$. Substituting $n = \frac{t}{2s}$ to $\frac{3}{4n}t$, we obtain $\frac{3}{2}s$. For any values of $n < \frac{t}{2s}$ we obtain profits which are *higher than* $\frac{3}{2}s$. Therefore, it is obvious that *if* $t > 2s$ *prevails, manufacturers would prefer to be in the transportation cost dominance.* ■

From our previous analysis we know that if the case of transportation cost dominance is optimal, manufacturers may want to impose an exclusive territory restriction. This gives us an interesting implication. If per unit transportation cost (t) exceeds per unit substitution cost (s) such that $t > 2s$, it implies that consumers will be more willing to substitute a product than to travel to a retailer which carries their preferred product. In this kind of circumstance, it is actually better for manufacturers to impose exclusive territory restriction, thus differentiating maximally with respect to the distance dimension. This seems to be intuitively plausible, if consumers perceive that it is better for them to switch to another product rather than to travel to a retailer just to pursue their preferred brand, then it implies that products face a tough inter-brand competition. To relax inter-brand competition, a manufacturer could assign an exclusive territory right to a retailer.[7] By doing this, the manufacturer can make it even costlier for consumers to shop to another retailer which carries their preferred brand. Thus, makes use of his maximal monopolistic advantage. As a result the manufacturer will get more locked-in consumers. It might be better for the manufacturer to attract these locked-in consumers who would switch because the transportation cost is too high than to try to capture consumers who really preferred the manufacturer's brand.[8]. Given our assumption that there are always equal number of retailers for both manufacturers, one can show that indeed in equilibrium each manufacturers would like to assign a single retailer.[9]

Now let us look at the case when $t \leqslant 2s$.

Proposition 3 When per unit transportation cost is lower than per unit substitution cost, such that $t \leqslant 2s$, manufacturers can only be in the taste dominance case. The number of retailers will then be less important for manufacturers.

Proof. Lemma (2) shows that when $t < 2s$ prevails, manufacturers can only be in the taste dominance case ($n > \frac{t}{2s}$), and cannot switch to the transportation dominance case ($n < \frac{t}{2s}$). This can be easily shown. If $t < 2s$ we will have $\frac{t}{2s} < 1$. Since the smallest possible number of retailers is 1 and the condition for the transportation dominance case is $n < \frac{t}{2s}$, it is obvious that the transportation cost dominance cannot prevail. Proposition 1 states that in the taste dominance case the number of retailers is not important.

[7] Thus, we have a similar result as Rey and Stiglitz (1995). An exclusive territory restriction can be employed to dampen inter-brand competition. However, we have a different way of looking at how excusive territory can be used to reduce inter-brand competition. Rey and Stiglitz (1995) argue that exclusive territory alter the perceived demand curve, making each manufacturer believe she faces a less elastic demand curve which will increase the equilibrium price and producers' profit. In this paper we rely on the consumers' decision rather than on the manufacturers' perception on the demand curve.

[8] In order to capture consumers who really preferred a manufacturer's brand (say product A), the manufacturer can add additional retailers so that all consumers who would like to buy product A could buy and will not be deterred by the high transportation cost.

[9] We are aware that this assumption also acts as the limitation of this paper. If we relax this assumption, we may not be sure whether or not this is a subgame-perfect nash-equilibrium. We should check whether or not a manufacturer wants to deviate. The equilibrium depends on the magnitude of manufacturer' profits from a deviation. If the deviation increases profits, then we might have a classic prisonner's dilemma game. Each of manufacturers will be better off using a single retailer, but they could not credibly commit to do so. However, when a deviation decreases profits, then a situation where each of manufacturers assigns a single retailer could indeed be the sub-game perfect equilibrium.

This number will be determined by the zero profit condition at the retailer level. Thus, there will potentially be many retailers in the market.

Now, suppose that $\frac{t}{2s} = 1$. The only possible cases are the taste dominance case ($n > \frac{t}{2s}$) and the border case ($n = 1 = \frac{t}{2s}$) case. Since the number of retailers will not enter in the expressions for manufacturers' profits, manufacturers will not concern with the number of retailers in the market. If there is free-entry at the retailer level it will be more likely that n is large, and thus we will be in the taste dominance case rather than in the border case. ■

The case of $t \leqslant 2s$ depicts a situation in which consumers are willing to travel sufficiently far just to get their preferred brands. It implies that consumers are loyal enough to their preferred brands. This case is equivalent to the case in which there is a weak inter-brand competition. As the price competition is already soft enough there is no need to impose an exclusive territory restriction.

Our results clarify the result of Martinez-Goralt and Neven (1988) even further. We show that their result is true only if consumers are more averse to travel than to substitute their preferred brand. It is then better for manufacturers to establish only a single retail outlet to serve the whole market. In our interpretation this means that manufacturers will impose exclusive territory arrangement. However, if consumers are more averse to substitute their preferred brand than to travel, then we show that manufacturers will not concern about the number of their retail outlets. Market segmention might then prevail. It is because the benefit from market segmentation will outweigh the cost from price competition.

7 Concluding Remarks

In this paper we analyse the vertical distribution system between manufacturers and retailers using a two dimensional horizontal product differentiation model. Products are differentiated along consumers' taste and the location of retailers. We argue that the two dimensional specification is well grounded. Products are more often differentiated along several characteristics. There might be an interaction among these characteristics. These product characteristics and their interaction will influence consumers' buying decision. As a consequence the nature of product characteristics will bring important implication on manufacturers' decision.

We focus the discussion on the interaction between two different product characteristics, namely consumers' taste and retailers' location. Depending on some parameters of product characteristics, a manufacturer might or might not be tempted to impose an exclusive territory restriction.

We assume that there is a duopoly competition at the manufacturer level and multi-firms competition at the retailer level. Manufacturers have all the bargaining power. We construct a four-stage game. In the first stage, manufacturers decide the optimal number of retailers. In the second stage manufacturers decide the wholesale prices. In the third stage retailers decide the retail prices. Finally, on the fourth stage consumers purchase goods.

We distinguish between two possible cases, namely, the case of transportation cost dominance and the case of taste dominance. The first case refers to a situation in which consumers' decision is influenced more by transportation cost rather than by the taste. The second case is the reverse. For each of these cases we derive three different segments of the aggregate demand. We show that the Nash equilibrium in pure strategy in retail and wholesale prices exist only on the second segment of the demand function for both cases.

Manufacturers can influence one dimension of the product differentiation, namely the distance between retailers. When a manufacturer adds new retailers, the distance between retailers will get closer. If the distance becomes sufficiently close, the manufacturer can switch from the case of transportation cost dominance to the case of taste dominance. The reverse switching is also possible when the manufacturer decides to decrease the number of retailers. Which case is preferable depends on the nature of the product characteristics.

Our results show that when the transportation cost dominates the substitution cost, it is better for manufacturers to make the distance between retailers sufficiently far. In other words, manufacturers have to make the transportation cost even more dominant. This can be done by appointing a single retailer to serve the whole city. However, when the substitution cost dominates the transportation cost, then the number of retailers in the market is not important for manufacturers. The number will be determined by the free entry equilibrium at the retailer level. Hence, there could potentially be many retailers in the market.

The analysis could be extended into several directions. In this paper we assume that manufacturers can only influence one aspect of product differentiation, namely the distance between retailers. It could be interesting to incorporate endogeneous decision of manufacturers to influence the consumers' taste, for instance by using advertisement or by increasing brand quality. We also consider only the case of exclusive dealing in which a retailer carries only a manufacturer's brand, and reatailers do not reside at the same location. We could extend the model to consider the case when retailers carry the products of two manufacturers, or the case when two retailers, each carries a different brand, reside at the same location. With this kind of analysis, we will be able to discuss the issue of retail stores location. How should manufactures design and locate their retail stores? Should manufacturers design a back-to-back retail competition, or should they not?.

Appendix 1

We will show the derivations of the demand functions. We start from the case of transportation cost dominance. The first segment of the demand for product A can be found by calculating the following definite integral,

$$D_A^{'}\left(p_A,p_B\right)=\int_0^{x(0)}\int_0^{y(x)}dydx=\frac{1}{32n^2st}\left(2p_Bn-2p_An+t+2sn\right)^2$$

The second segment of the demand function can be calculated as follows,

$$D_A^{''}\left(p_A,p_B\right)=\int_0^{1}\int_0^{x(y)}dxdy=\frac{p_B-p_A}{2t}+\frac{1}{4n}$$

The third segment of the demand function is the can be calculated as follows,

$$D_A^{'''}\left(p_A,p_B\right)=\frac{1}{2n}-\int_{y\left(\frac{1}{2n}\right)}^{1}\int_{x(y)}^{\frac{1}{2n}}dxdy=\frac{1}{2n}-\frac{1}{32n^2st}\left(2np_A-2np_B+2ns+t\right)^2$$

For the case of taste dominance, the first and the third segments of the demand function are the same as in the case of transportation cost dominance. Only the second segment of the demand function is different. This can be calculated by solving the following integration.

$$D_A^{''}\left(p_A,p_B\right)=\int_0^{\frac{1}{2n}}\int_0^{y(x)}dydx=\frac{1}{4n}\left(1+\frac{p_B-p_A}{2s}\right)$$

The demand for product B can be found by substracting the demand for product A from the total aggregate demand.

$$D_B\left(p_A,p_B\right)=\frac{1}{2n}-D_A\left(p_A,p_B\right)$$

Appendix 2

In what follows we will show the derivation of the equilibrium retail prices occuring on the first segment of the demand function. The demand functions for product A and product B on this segment are,

$$D_A^{'}\left(p_A,p_B\right)=\frac{1}{32n^2st}\left(2p_Bn-2p_An+t+2sn\right)^2$$

$$D_B^{'}\left(p_A,p_B\right)=\frac{1}{2n}-\frac{1}{32n^2st}\left(2p_Bn-2p_An+t+2sn\right)^2$$

Profit functions for retailers can be expressed as,

$$\pi_A\left(p_A,p_B\right)=2D_A\left(p_A,p_B\right)\left(p_A-w_A\right)$$

$$\pi_B\left(p_A,p_B\right)=2\left(\frac{1}{2n}-D_A\left(p_A,p_B\right)\right)\left(p_B-w_B\right)$$

Taking f.o.c's for profit maximisation we obtain,

$\frac{\partial\pi_A(p_A,p_B)}{\partial p_A}=-\frac{1}{8nts}\left(2n\left(p_B-p_A\right)+t+2sn\right)\left(p_A-w_A\right)+$
$\frac{1}{32n^2ts}\left(2n\left(p_B-p_A\right)+t+2sn\right)^2=0$
$\frac{\partial\pi_B(p_A,p_B)}{\partial p_B}=\frac{1}{2n}-\frac{1}{8nts}\left(2n\left(p_B-p_A\right)+t+2sn\right)\left(p_B-w_B\right)-$
$\frac{1}{32n^2ts}\left(2n\left(p_B-p_A\right)+t+2sn\right)^2=0$

Summing the two f.o.c.'s and multiplying by $\frac{2n}{p_A+p_B-w_A-w_B}$ we obtain,

$$\frac{1}{p_A+p_B-w_A-w_B}=\frac{1}{4ts}\left(2n\left(p_B-p_A\right)+t+2sn\right)$$

Multiplying the first f.o.c. by $\frac{32n^2t_1t_2}{2n(p_B-p_A)+t_1+2t_2n}$ we get,

$$4n\left(p_A-w_A\right)=2n\left(p_B-p_A\right)+2t+4sn$$

Hence, the two f.o.c's can be simplified into,

$$\left(p_A-w_A\right)\left(p_A+p_B-w_A-w_B\right)=\tfrac{ts}{n}$$
$$4n\left(p_A-w_A\right)=2n\left(p_B-p_A\right)+t+2sn$$

Solving for the equilibrium retail prices we obtain two pair of retail prices equilibria,

$$p_A^1=\frac{w_B+7w_A}{8}+\frac{1}{16n}\left(2sn+t+\sqrt{Z}\right)$$

$$p_B^1=\frac{3w_B+5w_A}{8}-\frac{10sn+5t-3\sqrt{Z}}{16n}$$

$$p_A^2 = \frac{w_B + 7w_A}{8} + \frac{1}{16n}\left(2sn + t - \sqrt{Z}\right)$$

$$p_B^2 = \frac{3w_B + 5w_A}{8} - \frac{10sn + 5t + 3\sqrt{Z}}{16n}$$

In which $Z = \sqrt{\left(2n\left(w_A - w_B\right) - 2ns - t\right)^2 + 64nts}$.

Appendix 3

The proof that the wholesale price Nash-equilibrium does not exist when the equilibrium retail prices are on the first and the third segments of the demand function is outlined as follows.

The profit functions for manufacturers can be expressed as,

$$\Pi_A\left(p_A, p_B\right) = 2nD_A\left(p_A, p_B\right)w_A - nc = 2nf\left(p_B - p_A\right)w_A - nc$$

$$\Pi_B\left(p_A, p_B\right) = 2n\left(\frac{1}{2n} - f\left(p_B - p_A\right)\right)w_B - nc$$

In which $f\left(p_B - p_A\right)$ is the demand function expressed as a function of the spread between the two retail prices. The proof consists of three parts. Firstly, we will prove that $p_B - p_A < 0$ for this case. Let us start by stating conditions for the existence of this case, which are, $y(x) = y(1) < 0$, and $0 < y(x) = y(0) < \frac{1}{2n}$ (see figure 6).

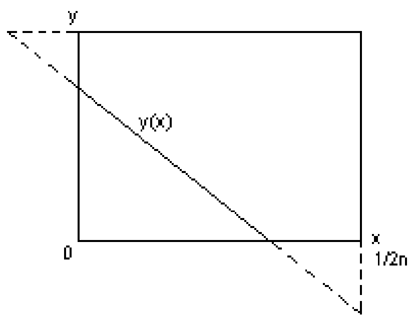


Figure 6: Conditions for the Existence of the First Segment of the Demand Function

Solving these conditions we obtain,

$$0 > 2n(p_B - p_A) - 2ns + t$$

$$0 < 2n(p_B - p_A) + 2ns + t < 2t$$

Therefore, the spread between the two retail prices is constrained either by,

$$0 > \frac{t}{2n} - s > p_B - p_A$$

or by this condition,

$$0 > s - \frac{t}{2n} > p_B - p_A$$

The two of these together imply,

$$p_B - p_A < 0$$

Secondly, we will prove that the spread between these two retail prices is monotonically increasing function of the spread between the two wholesale prices. We will use the equilibrium retail prices derived before (expressions (11), (12), (13), and (14).) to calculate $p_B^1 - p_A^1$ and $p_B^2 - p_A^2$. We will express them as a function k of the spread between the two wholesale prices.

$$p_B^1 - p_A^1 = k_1(w_B - w_A) = k(v) = \frac{1}{8}\frac{2nv - 3t - 6sn + \sqrt{(2nv + 2ns + t)^2 + 64nts}}{n}$$

$$p_B^2 - p_A^2 = k_2(w_B - w_A) = k(v) = \frac{1}{8}\frac{2nv - 3t - 6ns - \sqrt{(2nv + 2ns + t)^2 + 64nts}}{n}$$

Taking the derivatives of the above functions with respect to v, we obtain,

$$\frac{\partial k_1(v)}{\partial v} = \frac{1}{4}\frac{\sqrt{(2nv + 2ns + t)^2 + 64nts} + 2nv + 2ns + t}{\sqrt{(2nv + 2ns + t)^2 + 64nts}} > 0$$

$$\frac{\partial k_2(v)}{\partial v} = \frac{1}{4}\frac{\sqrt{(2nv + 2ns + t)^2 + 64nts} - 2nv - 2ns - t}{\sqrt{(2nv + 2ns + t)^2 + 64nts}} > 0$$

Both derivatives have positive sign. We will use these results later on.

Let us go back for a while to the results in the first part of the proof. We know that there are two possible spreads between the two retail prices, namely,

$$0 > s - \frac{t}{2n} > p_B - p_A$$

$$0 > \frac{t}{2n} - s > p_B - p_A$$

Substituting the results that we derived for $p_B^1 - p_A^1$, we obtain,

$$\frac{t}{2n} - s = \frac{1}{8}\frac{2nv + \sqrt{(2nv + 2ns + t)^2 + 64nts} - 3t - 6sn}{n}$$

$$s - \frac{t}{2n} = \frac{1}{8}\frac{2nv - \sqrt{(2nv + 2ns + t)^2 + 64nts} - 6ns - 3t}{n}$$

Solving for v from the first and the second expressions we obtain respectively,

$$w_B - w_A = v = \frac{3}{2}\frac{2ns - t}{n} < 0$$

$$w_B - w_A = v = -\frac{3}{2}\frac{2ns - t}{n} < 0$$

The inequality is derived from the condition that $0 > s - \frac{t}{2n}$. Repeating the same procedure for $p_B^2 - p_A^2$ we obtain,

$$s - \frac{t}{2n} = \frac{1}{8}\frac{2nv - \sqrt{(2nv + 2ns + t)^2 + 64nts} - 6ns - 3t}{n}$$

$$\frac{t_1}{2n} - s = \frac{1}{8}\frac{2nv - \sqrt{(2nv + 2ns + t)^2 + 64nts} - 6ns - 3t}{n}$$

$$w_B - w_A = v = \frac{3}{2}\frac{2ns - t}{n} < 0$$

$$w_B - w_A = v = -\frac{3}{2}\frac{2ns-t}{n} < 0$$

Previous derivation shows that $\frac{\partial k_1(v)}{\partial v} > 0$ and $\frac{\partial k_2(v)}{\partial v} > 0$. This means that the higher the spread between the two wholesale prices is, the higher the spread between the two retail prices is. Thus, *we are sure that for $p_B - p_A < 0$, we should have $w_B - w_A < 0$.*

Finally, we will prove the non-existence of the wholesale price Nash equilibrium in pure strategy by a contradiction. First we will express the profit function of manufacturers as a function of the spread between the two wholesale prices.

$$\Pi_A(w_A, w_B) = 2nf(k(w_B - w_A))\, w_A - nc$$

$$\Pi_B(w_A, w_B) = 2n\left(\frac{1}{2n} - f(k(w_B - w_A))\right) w_B - nc$$

Let us suppose that there exists a Nash equilibrium in pure strategy in this case. This Nash equilibrium is obtained from solving simultaneously first order conditions for wholesale profit maximisation.

$$\frac{\partial \Pi_A}{\partial w_A} = -2n\frac{\partial k(w_B - w_A)}{\partial(w_B - w_A)}\frac{\partial f(k(w_B - w_A))}{\partial k(w_B - w_A)} w_A + 2nf(k(w_B - w_A)) = 0$$

$$\frac{\partial \Pi_B}{\partial w_B} = -2n\frac{\partial k(w_B - w_A)}{\partial(w_B - w_A)}\frac{\partial f(k(w_B - w_A))}{\partial k(w_B - w_A)} w_B + 1 - 2nf(k(w_B - w_A)) = 0$$

Summing them together we obtain,

$$\frac{\partial \Pi_A}{\partial w_A} + \frac{\partial \Pi_B}{\partial w_B} = -2n\frac{\partial k(w_B - w_A)}{\partial(w_B - w_A)}\frac{\partial f(k(w_B - w_A))}{\partial k(w_B - w_A)}(w_A + w_B) + 1 = 0$$

$$\frac{\partial k(w_B - w_A)}{\partial(w_B - w_A)}\frac{\partial f(k(w_B - w_A))}{\partial k(w_B - w_A)} = \frac{1}{w_A + w_B}$$

This implies that the demand for product A and the demand for product B at the equilibrium are,

$$D_A(w_A^*, w_B^*) = 2nf(k(w_B^* - w_A^*)) = 2n\frac{\partial k(w_B^* - w_A^*)}{\partial(w_B^* - w_A^*)}\frac{\partial f(k(w_B^* - w_A^*))}{\partial k(w_B^* - w_A^*)} w_A = \frac{w_A^*}{w_A^* + w_B^*}$$

$$D_B(w_A^*, w_B^*) = 1 - 2nD_A(w_A^*, w_B^*) = \frac{w_B^*}{w_A^* + w_B^*}$$

From those demand functions, it can easily be seen that if we have $D_B(w_A^*, w_B^*) > D_A(w_A^*, w_B^*)$, then it must be that $w_B^* > w_A^*$. We proved before that retail prices are as such that $p_B < p_A$, which implies that $w_B^* < w_A^*$. This is clearly a contradiction, thus we complete our proof.

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