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by

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Public Economics

Center for Economic Studies
Discussions Paper Series (DPS) 00.13
http://www.econ.kuleuven.be/ces/discussionpapers/default.htm

April 2000

DISCUSSION
PAPER

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Abstract

Economists providing policy advice often justify recommendations of government non-interference in the allocation of resources between production sectors and free trade with reference to the Diamond and Mirrlees efficiency theorem. However, such policy advice may be misleading when, as is in general the case, administrative costs effectively restrict the set of feasible tax instruments available to the government. Under plausible assumptions about the administrative costs of alternative tax structures optimal government policies may in fact be associated with significant production inefficiencies.

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Keywords:
Public economics, optimal taxation, support to low-skilled households, restrictions on tax instruments, production efficiency

JEL classification codes:
H2

Acknowledgements:
The work for this article has been made partly at CES, Catholic University of Leuven and partly at EPRU, University of Copenhagen. The resources provided at both places for this work are gratefully acknowledged. The author also wants to thank Jacques Drèze, Søren Bo Nielsen, Stef Proost, Agnar Sandmo and Peter Birch Sørensen for helpful comments and Mikolaj Jasiak for efficient research assistance.

1 Introduction

Diamond and Mirrlees (1971) have proved that when costless lump-sum transfers are not feasible production efficiency is still desirable, although the conditions for Pareto-efficiency are not attainable[1]. This theorem has had a considerable influence on decision-making at both national and international levels of government. The scope of potential application of the theorem should be gauged from the fact that it not only suggests that a government should not interfere in the allocation of resources between production sectors, but also, by considering the foreign sector as a production sector, that it should not interfere with external trade either. However, the assumptions that need to hold for the theorem to be valid, that tax rates on all market transactions and on pure profit can be fixed at their optimal levels at no cost, are very restrictive, as has been pointed out from the outset by Stiglitz and Dasgupta (1971).

The purpose of this paper is to demonstrate that the costs involved in tax administration are likely to invalidate the assumptions on which the efficiency theorem is based and may thus imply that optimal commodity taxation requires differentiation of tax rates between production sectors, i.e. that production efficiency is not desirable.

Existing tax structures reflect the importance of administrative costs. The widespread use of uniform tax rates for large groups of commodities reflects that the administrative costs of such tax structures are lower than when tax rates are differentiated. That governments in general[2] tax different types of labour income according to a common schedule undoubtedly reflect the significant administrative costs which are involved in differentiating the taxation of labour income according to differences in skill levels.

However, although it has been recognised as a serious weakness (see e.g. Sandmo 2000 p119), little work has been done to integrate administrative costs in optimal tax theory. Taking the administrative costs into account is therefore an endeavour with the potential to enhance considerably the relevance of optimal tax theory for policy analysis. The implication of government not being able to observe differences in skill levels has been extensively analysed in the context of optimal income taxation, but it seems that the implication for production efficiency within an optimal commodity tax framework has not been seriously investigated (see e.g. Boadway 1995).

The implications of restrictions on the set of feasible tax instruments for optimal commodity taxation have been investigated by Stiglitz and Dasgupta (1971), Dasgupta and Stiglitz (1972) and Munk (1980), adopting a framework with one representative

[1] Strictly speaking, the theorem only claims that production efficiency outside the household sector, i.e. in production sectors transforming marketed commodities, is desirable. Since the optimal tax system requires the rates of transformation within the household sector to be different from those within the production sectors transforming marketed commodities, when comparing household production with production in the sectors transforming marketed commodities, production efficiency is clearly not desirable.

[2] The progress in information technology in recent years may have reduced the costs of tax schemes to benefit low-skilled workers to the point where they are administratively feasible.

household and taking as given that certain tax instruments could not be used. We extend and deepen the analysis by providing an explanation in terms of administrative costs for the restrictions on the government's choice of tax rates, effectively making these restrictions endogenous. Furthermore, we consider a many household economy. This turns out to be crucial for bringing out the policy implications of taking administrative costs into account. We assume constant returns to scale in order to bypass the complications of having to interpret rules of normalisation in the presence of pure profit, and to show that untaxed profit is not the main reason why production efficiency may not be desirable[3].

We represent administrative costs as depending on tax structures, but not on the level of taxation, assuming that a differentiated tax structure is associated with higher administrative costs than a corresponding non-differentiated tax structure[4]. This involves a significant simplification, however a simplification that may be justified in a similar way as assuming that the use of lump-sum transfers to raise tax revenue and to redistribute income is not available to the government. The assumption that lump-sum transfers are not possible may be justified by essentially the same reasons that taxation of labour income according to differences in skill levels is not possible (see Atkinson and Stiglitz 1980, p357)[5]. As the first assumption invalidates the Second Theorem of welfare economics as a basis for providing policy advice, so do the second compromise the policy relevance of the Efficiency Theorem. Both assumptions are crude ways to represent the difficulties of observing the household characteristics required for income redistribution, but nevertheless serve as the basis for obtaining important insights.

The paper is organised as follows: In Section 2 the model is formulated. In Section 3 we consider the conditions for a tax system to be optimal, taking into account the administrative costs of alternative tax structures. In Section 4 we identify cases where administrative costs may provide a justification for differentiating the taxation of primary factors between production sectors and thus for production inefficiency, and provide a quantitative illustration of the theoretical analysis using a stylised CGE model. In a final section we highlight some implications of the results for policy analysis.

2 The model

The model considered involves the government choosing commodity and primary factor taxes and a uniform lump-sum transfer[6] to maximise a Pareto social welfare function

[3] Myles (1995) on the subject of production efficiency (with reference to Dasgupta and Stiglitz 1972) only mention that productive efficiency is not desirable when profit cannot be taxed at 100%.
[4] We thus make only a very limited contribution to explaining the mix of direct and indirect taxes (see Boadway 1995).
[5] They write: "The basic difficulty is [..] that the information on which we would like to base differential lump-sum taxes is not observable, or observable only at great cost, and individuals have an incentive not to reveal it. For these reasons, lump-sum taxes and transfers are widely assumed not to be available."
[6] This allows income taxation within the framework to be represented by a linear income tax.

consistent with egalitarian value judgements, subject to constraints representing the structure of the economy, including the administrative costs of taxation.

In the economy, there are N constant returns to scale production sectors, H households labelled h∈(1,..,H) and a government labelled G. The primary factors are labelled j∈(1,..,M), the produced commodities i∈(M+1,..,M+N) and the production sectors k∈(1,..,N). We denote the index set of households, *H*, of primary factors, *F*, of produced commodities, *C*, and of production sectors, *A*.

The h^{th} household's supply of primary factors and demand for final commodities is denoted $\mathbf{z^h} \equiv \left(z_1^h,..., z_M^h\right)$ and $\mathbf{x^h} \equiv \left(x_{M+1}^h,...,x_{M+N}^h\right)$, respectively. The government's resource requirements is $\left\{\mathbf{z}^G, \mathbf{x}^G\right\} \equiv \left(z_1^G,..z_M^G, x_{M+1}^G.,x_{M+N}^G\right)$,

The households face prices $\mathbf{w} \equiv \left(w_1,.., w_M\right)$ for primary factors, and $\mathbf{q} \equiv \left(q_{M+1},...,q_{M+N}\right)$ for produced commodities. The h^{th} household's indirect utility function is $V^h\left(\mathbf{q,w}, I^{\mathbf{h}}\right)$, and the corresponding expenditure function $E^h(\mathbf{q}, \mathbf{w}, u^h)$.

Firms maximise profit under conditions of perfect competition and constant returns to scale. Y^k is the output of sector k, and $\mathbf{v}^k \equiv \left(v_1^k,..,v_M^k\right)$ the use of primary factors in that sector. Output prices are $p_{k+M} = c^k\left(\mathbf{p}^k\right)$, k∈*A*, where $c^k\left(\mathbf{p}^k\right)$ is the unit cost function in sector k, and $\mathbf{p}^k \equiv \left(p_1^k,.., p_M^k\right)$ the input prices. The output contingent primary factor input demands are $v_j^k = a_j^k\left(\mathbf{p}^k\right)Y^k$, j∈*F*, k∈*A*, where $a_j^k = a_j^k\left(\mathbf{p}^k\right)$, j∈*F*, k∈*A* are the technical coefficients.

Tax rates are defined relative to market prices, $\mathbf{p} \equiv \left(p_1,...,p_{M+N}\right)$. The market prices for primary factors, $\left(p_1,.., p_M\right)$, are defined to be equal to the corresponding producer prices in sector N. Market prices of produced commodities are defined to be equal to output producer prices in the corresponding production sectors, i.e. $\left(p_{M+1},...,p_{M+N}\right)$. We assume as a matter of normalisation that $p_1 = 1$.

The government can levy a number of different taxes: household taxes, $\left(t_{M+1},...,t_{M+N}\right)$, on the consumption of produced commodities, and, $\left(s_1,.., s_M\right)$, on the supply of primary factors; producer taxes on the use of primary factors in the different production sectors, $(t_1^k,...,t_M^k)$, $k \in A$; and a uniform lump-sum tax, *L*.[7] The level of taxation may for the consumption of produced commodities be expressed by ratios between household prices and market prices,

$$T_i \equiv q_i / p_i = (t_i + p_i) / p_i \qquad i \in C$$

[7] Thus, unearned income is for all households equal to the uniform lump-sum tax, i.e. I^h=-*L*. *L* is negative if it is interpreted as the fixed element in a progressive linear income tax schedule.

for supplies of primary factors by ratios between market prices and household prices,

$$S_j \equiv p_j / w_j = p_j /(p_j - s_j) \qquad j \in F$$

and for the use of primary factors in the different production sectors by ratios between producer prices and market prices,

$$T_j^k \equiv p_j^k / p_j = (t_j^k + p_j) / p_j \qquad k \in A \;\; j \in F$$

We define a tax system as $\boldsymbol{x} \equiv \left(\mathbf{S}, \mathbf{T}, (\mathbf{T}^k, k \in A), L\right)$, where $\mathbf{T} \equiv \left(T_{M+1}, \ldots, T_{M+N}\right)$, $\mathbf{S} \equiv \left(S_1, \ldots, S_M\right)$, and $\mathbf{T}^k \equiv (T_1^k, \ldots, T_M^k)$, $k \in A$, and assume that different tax systems may be associated with different administrative costs depending on their complexity. The tax revue required to finance the government's expenditures, $B \equiv \mathbf{p}\left\{\mathbf{z}^G, \mathbf{x}^G\right\}$, is therefore a function of the tax system, i.e. $B = \mathrm{B}(\boldsymbol{x})$. We assume that $\mathrm{B}(\boldsymbol{x})$ is homogeneous of degree 1 in $\mathbf{p}$ and L. A tax structure $\boldsymbol{x}^i \in \hat{\mathbf{I}}^i$ is a set of tax systems imposing the same restrictions on the feasible set of tax rates, and which are associated with the same administrative costs.

We assume that the government solves its maximisation problem in a two-step procedure. The government considers a finite number of different tax structures, $\hat{\mathbf{I}}^i$, i∈ □ . First, it calculates the optimal tax system, $\boldsymbol{x}^{i*}$, $i \in \hat{\mathbf{I}}^i$, for each tax structure considered. Then, in the second step, it chooses as the overall optimal tax system, $\boldsymbol{x}^*$, that of the solutions to the maximisation problems for the different tax structures considered, which is associated with the highest level of social welfare.

Using the expenditure function approach we specify the government's maximisation problem for a given tax structure as the maximisation of the social welfare function, $\mathrm{W}\left(u^1, u^2, \ldots, u^H\right)$, with respect to $u^1, u^2, \ldots, u^H$; L; $\mathbf{w} \equiv \left(w_1, \ldots, w_M\right)$; $\mathbf{q} \equiv \left(q_{M+1}, \ldots, q_{M+N}\right)$; and $\mathbf{p}^k \equiv \left(p_1^k, \ldots, p_M^k\right)$, $k \in A$, subject to the following constraints:

1) $E^h(\mathbf{q}, \mathbf{w}, u^h) = -L$, $h \in H$, which requires lump-sum taxation to be uniform and the levels of individual utilities to be consistent with the level of unearned income, $I^h = -L$, for each household.

2) $\sum_{h \in H} z_j^h\left(\mathbf{q}, \mathbf{w}, u^h\right) - \sum_{i \in C} a_j^{i-M}\left(\mathbf{p}^{k-M}\right)\left(\sum_{h \in H} x_i^h\left(\mathbf{q}, \mathbf{w}, u^h\right) + x_i^G\right) - z_j^G = 0$, $j \in F$, which represents the general equilibrium conditions of the economy.[8]

[8] For a price-tax vector to be consistent with an equilibrium situation it has to satisfy the conditions for profit maximisation, utility maximisation and the material balances. The conditions for profit maximisation and utility maximisation are substituted into the material balance conditions, and finally the material

3) According to the tax structure considered, a subset of $\{ p_j^k / w_j = T_j^k S_j ,\ j \in F,\ k \in A \}$, which for primary factors constrains the differences between sector specific producer prices, p_j^k, and household prices; w_j ; and $\{ q_i / c^{i-M}(\mathbf{p}^{i-M}) = T_i ,\ i \in C \}$, which for produced commodities constrain the differences between household prices, q_i , and producer prices (represented by unit costs, $c^i(\mathbf{p}^i)$).

3 The characterisation of the optimal tax structure

In this section we derive the conditions for the tax structure to be optimal when certain tax rates are constrained to be the same due to the administrative costs of differentiating the tax rates using a similar approach as in Munk (1980), but explicitly representing the administrative costs associated with different tax structures and considering a many household rather than a one household economy. We also briefly, as point of reference for the subsequent analysis, characterise the optimal tax structure in the absence of administrative costs.

As a matter of normalisation we fix the market price, $p_1 \equiv 1$, and in order to simplify the notation and facilitate interpretation we define for $h \in H$

$$E_u^h \equiv \frac{\partial E^h}{\partial u}; \qquad E_{j,u}^h \equiv \frac{\partial^2 E^h}{\partial q_j \partial u},\ j \in C; \qquad E_{j,u}^h \equiv \frac{\partial^2 E^h}{\partial w_j , \partial u},\ j \in F;$$

$$E_{ij}^h \equiv \frac{\partial^2 E^h}{\partial q_i \partial q_j},\ i,\ j \in C; \quad E_{is}^h \equiv \frac{\partial^2 E^h}{\partial q_i \partial w_s},\ i \in C,\ s \in F; \quad E_{rs}^h \equiv \frac{\partial^2 E^h}{\partial w_r \partial w_s},\ r,\ s \in F$$

$$E_{ij} \equiv \sum_{h \in H} E_{ij}^h \quad i, j \in C \cup F; \qquad E_{i,u} \equiv \sum_{h \in H} E_{i,u}^h$$

The Lagrangean expression corresponding to the government's maximisation problem for a given set of tax constraints, i.e. for a given tax structure, may then be formulated as

$$£ = W\left(u^1, u^2, .., u^H\right)$$

$$+ \sum_{h \in H} \psi^h \left(-E^h(\mathbf{q}, \mathbf{w}, u^h) - L \right)$$

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balance conditions for the produced commodities are substituted into the material balance conditions for the primary factors. Notice that since we consider more than one primary factor we cannot without imposing productive efficiency incorporate the material balance conditions into the government's budget constraint.

$$+\sum_{j\in F} \lambda_j \left(\sum_{h\in H} z_j^h(\mathbf{q},\mathbf{w},u^h) - \sum_{i\in C} a_j^{i\text{-}M}(\mathbf{p}^{i\text{-}M})\left(\sum_{h\in H} x_i^h(\mathbf{q},\mathbf{w},u^h) + x_i^G \right) - z_j^G \right)$$

$$+\sum_{j\in F} \sum_{k\in A} \gamma_j^k \left(p_j^k / w_j - T_j^k S_j \right) + \sum_{i\in C} \gamma_i \left(q_i / c^{i-M}(\mathbf{p}^{i-M}) - T_i \right) \qquad (1)$$

where

ψ^h is the net social value of transferring income lump-sum wise from the government to household h

λ_j is the opportunity cost price of primary factor j[9]

γ_j^k is the social welfare cost price of the tax on the use of primary factor j in sector k, and equal to 0 if not constrained

γ_i is the social welfare cost price of the tax on the consumption of commodity i, and equal to 0 if not constrained

The first order conditions with respect to $u^1, u^2, .., u^H$; L; $\mathbf{w} \equiv (w_1, .., w_M)$; $\mathbf{q} \equiv (q_{M+1}, ..., q_{M+N})$; and $\mathbf{p}^k \equiv (p_1^k, .., p_M^k)$, $k \in A$, to be satisfied by an optimal tax system for a given tax structure are therefore:

$$\frac{\P\pounds}{\P u^h} = \frac{\partial W}{\partial u^h} - \psi^h E_u^h + \sum_{j\in F} \lambda_j \left(-E_{ju} - \sum_{k=C} a_j^{k-M} E_{ku} \right) = 0 \qquad h \in H \quad (2)$$

$$\frac{\P\pounds}{\P L} = -\sum_{h\in H} \psi^h = 0 \qquad (3)$$

$$\frac{\P\pounds}{\P w_s} = \sum_{h=H} \psi^h z_s^h + \sum_{j\in F} \lambda_j \left(-E_{js} - \sum_{k=C} a_j^{k\text{-}M} E_{is} \right) - \sum_{k\in A} \frac{g_s^k}{w_s} T_s^k S_s = 0 \qquad s \in F \quad (4)$$

$$\frac{\P\pounds}{\P q_i} - \sum_{h=H} \psi^h x_i^h + \sum_{j\in F} \lambda_j \left(-E_{ji} - \sum_{k=C} a_j^{k\text{-}M} E_{ki} \right) + \frac{\gamma_i}{p_i} = 0 \qquad i \in C \quad (5)$$

$$\frac{\P\pounds}{\P p_s^k} - \sum_{j\in F} \lambda_j a_{js}^k Y^k + \frac{\gamma_s^k}{w_s} - \frac{\gamma_{k+M}}{p_{k+M}} T_{k+M} a_s^k = 0 \qquad k \in A,\ s \in F \quad (6)$$

where some γ_i and γ_j^k are zero according to the tax structure considered.[10]

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[9] λ_1 may also be interpreted as the opportunity cost price of government funds.

[10] The opportunity cost price of a tax constraint is not only zero when the corresponding transaction can be taxed freely, but also when, according to tax equivalence theorems, a tax on the transaction is equivalent to taxes on other transactions which are feasible (see Munk 1980). For example, the fact that the consumption of only one produced commodity or the supply of only one primary factor cannot be taxed, does not in

From (2) we obtain that the *net marginal social value of a lump-sum transfer from the government to household h* may be expressed as

$$\psi^h = \mu^h - \lambda_1 \qquad \text{h} \in H \quad (7)$$

where (in analogy with the definition by Diamond 1975 for an economy without tax restrictions) the *net marginal social welfare of income for household h* we defined as

$$\mu^h \equiv \beta^h - \lambda_1 \sum_{j \in F} \left(w_j - \tilde{p}_j \right) \frac{\partial z_j^h}{\partial I^h} + \lambda_1 \sum_{i \in C} \left(q_i - \tilde{p}_i \right) \frac{\partial x_i^h}{\partial I^h} \qquad \text{h} \in H \quad (8)$$

and where $\beta^h \equiv \dfrac{\partial W}{\partial u^h} \dfrac{\partial v^h}{\partial I^h}$ is the *marginal social welfare of income for household h,* and

$$\tilde{p}_j \equiv \lambda_j / \lambda_1 \qquad \text{j} \in F \quad (9)$$

$$\tilde{p}_i \equiv \sum_{j \in F} \mathrm{a}_{\mathrm{j}}^{\mathrm{i-M}} \tilde{p}_j \qquad \text{i} \in \text{C} \quad (10)$$

the *opportunity cost prices of primary factors and produced commodities,* respectively. By definition therefore $\tilde{p}_1 = p_1$.

From (3) we have (when a uniform lump-sum tax is feasible) that

$$\tilde{R}_{\mathrm{L}} = \lambda_1 , \qquad (11)$$

where $\tilde{R}_{\mathrm{L}} = \dfrac{\sum_{\mathrm{h} \in H} \mu^{\mathrm{h}}}{\mathrm{H}}$

We transform (4) and (5) into

$$d_s = \frac{\sum_{i=C} \left(q_i - \tilde{p}_i \right) E_{is} + \sum_{j=F} \left(w_j - \tilde{p}_j \right) E_{js}}{-Z_s} = -\frac{\lambda_1 - \tilde{R}_{\mathrm{s}}}{\lambda_1} + \sum_{k \in A} \frac{\gamma_{\mathrm{s}}^k}{-Z_s w_{\mathrm{s}} \lambda_1} T_{\mathrm{s}}^{\,k} S_s \qquad \text{s} \in \text{F} \ (12)$$

$$d_k = \frac{\sum_{i=C} \left(q_i - \tilde{p}_i \right) E_{ik} + \sum_{j=F} \left(w_j - \tilde{p}_j \right) E_{jk}}{X_k} = -\frac{\lambda_1 - \tilde{R}_{\mathrm{k}}}{\lambda_1} - \frac{\gamma_k}{X_k p_k \lambda_1} \qquad \text{k} \in \text{C} \ (13)$$

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itself impose a binding constraint. Similarly, that one of the inputs in an industry cannot be taxed does not impose a constraint if both the other inputs and the consumption of the commodity produced in the sector can.

where

$$\tilde{R}_{\mathrm{s}} = \frac{\sum_{\mathrm{h}\in H} \mu^{\mathrm{h}} z_s^h}{Z_s} \quad \text{and} \quad \tilde{R}_{\mathrm{k}} = \frac{\sum_{\mathrm{h}\in H} \mu^{\mathrm{h}} x_k^h}{X_k}$$

are *net distributional characteristics of primary factors and produced commodities*, respectively, (defined in analogy with Feldstein's 1972 *distributional characteristics*). We interpreted d_i, $\mathrm{i} \in C \cup F$ as *indices of discouragement* (Mirrlees 1976).[11]

Rewriting (6) we get[12]

$$\frac{p_2^k}{p_1^k} - \tilde{p}_2 = -\frac{\gamma_{\mathrm{s}}^k}{w_{\mathrm{s}}} / \lambda_1 \mathrm{a}_{2\mathrm{s}}^{\mathrm{k}} Y^{\mathrm{k}} + \frac{\gamma_{\mathrm{k+M}}}{p_{\mathrm{)k+M}}} \; T_{\mathrm{k+M}} \; \mathrm{a}_{\mathrm{s}}^{\mathrm{k}} / \lambda_1 \mathrm{a}_{2\mathrm{s}}^{\mathrm{k}} Y^{\mathrm{k}} \qquad \mathrm{s}\in F,\ \mathrm{k}\in A \quad (14)$$

Equation (14) shows that production efficiency will in general not be desirable when it is not possible to tax all market transactions (cf. Stiglitz and Dasgupta 1971 and Munk 1980). It is difficult at the level of generality of the present model to provide an intuitive interpretation of (14), in particular because of the complexity of the problem of normalisation. For example, whether certain tax constraints are binding may depend on whether certain other tax rates are restricted or not (see Munk 1980). To facilitate the task

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[11] The *indices of discouragement* provide an approximation to the reduction in compensated demand from the first best situation where household prices are equal to producer prices to the second best situation. Notice that the indices of discouragement here are defined for differences between household prices and opportunity cost prices rather than the difference between household prices and producer prices.

[12] $$- l_1 \mathrm{a}_{1\mathrm{s}}^{k} Y^k - l_2 a_{2s}^k Y^k + \frac{g_s^k}{w_s} - \frac{g_{k+M}}{p_{k+M}} \; T_{k+M} \; a_s^k = 0 \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

$$-l_1 \left[\frac{a_{1s}^k}{a_{2s}^k} + \frac{l_2}{l_1} \right] a_{2s}^k Y^k + \frac{g_s^k}{w_s} - \frac{g_{k+M}}{p_{k+M}} \; T_{k+M} \; a_s^k = 0 \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

By the assumption of constant returns to scale production functions and cost minimisation

$$1 = f^k \left(a_1^k \left(p^k \right), a_2^k \left(p^k \right) \right) \qquad \mathrm{k}\in A$$

By differentiation with respect to p_s^k and by implications of cost minimization we have

$$\frac{a_{1s}^k}{a_{2s}^k} = -\frac{f_2^k}{f_1^k} = -\frac{p_2^k}{p_1^k} \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

where $a_{js}^k \equiv \frac{\partial^2 c^k \left(p^k \right)}{\partial p_j^k \partial p_s^k}$

Substituting for $\frac{a_{1s}^k}{a_{2s}^k}$ and $\frac{l_2}{l_1}$ we therefore get

$$\left[\frac{p_2^k}{p_1^k} - \tilde{p}_2 \right] = -\left(\frac{g_s^k}{w_s} - \frac{g_{k+M}}{p_{k+M}} \; T_{k+M} \; a_s^k \right) / \; l_1 a_{2s}^k Y^k \qquad \mathrm{s}\in F,\ \mathrm{k}\in A$$

of interpretation we therefore have chosen to base the analysis of the optimal tax structure when certain tax constraints are binding based on a number of specific assumptions. However, to provide a point of reference for the subsequent analysis we at this stage briefly consider the optimal tax theory in the case where all market transactions may be taxed at their optimal level.

When the government can tax all market transaction, (14) becomes

$$\frac{p_2^k}{p_1^k} - \tilde{p}_2 = 0 \qquad \mathrm{k} \in A \quad (15)$$

Production efficiency is thus desirable, as may be inferred from the Diamond and Mirrlees' efficiency theorem.

When all market transactions may be taxed, we may further, as a matter of normalisation, assume the supply of factor 1, and its use in all production sectors to be untaxed, i.e. $w_1 = p_1$, $p_1^k = p_1$, $\mathrm{k} \in A$. Furthermore, we may choose market prices so that one production sector is untaxed. By (9) and (10) all market prices are therefore equal to the corresponding opportunity cost prices, i.e.

$$p_i = \tilde{p}_i \qquad \mathrm{i} \in C \cup F \quad (16)$$

In (12) and (13) we may therefore replace opportunity cost prices with market prices. The indices of discouragement[13] thus become

$$d_k = \frac{\sum_{i=C} (\mathrm{q}_i - \mathrm{p}_i) E_{ik} + \sum_{j=F} (\mathrm{w}_j - \mathrm{p}_j) E_{jk}}{E_k} = -\frac{\lambda_1 - \tilde{R}_\mathrm{k}}{\lambda_1} \qquad \mathrm{k} \in F \cup \mathrm{C} \quad (17)$$

The interpretation of this equation is well known (see for example Myles 1995). If the government is *inequality-neutral*, the indices of discouragement are the same for all commodities (the Ramsey result), and commodities, which are complementary to leisure, are in general taxed at relatively high rates (Corlett and Hague 1953). If both the different types of labour and the consumption of produced commodities are separable from leisure in utility, and homothetic in commodities and the different types of labour, respectively, the optimal tax structure involves that incomes of different types of labour are taxed according to the same schedule and that the consumption of the two commodities are taxed at the same rate. If the government is *inequality-averse*, commodities consumed by high-income households disproportionately relative to their income will in general be discouraged relatively more than commodities which are consumed disproportionately by low-income households.

[13] For more details see for example Myles (1995).

4 The optimal tax structure with administrative costs

In this section we attempt to provide intuitive explanations of how administrative costs may influence the optimal tax structure, and more specifically when such costs may justify production inefficiency. To facilitate the exposition we limit the number of different cases to consider by making a number of specific assumptions. Therefore, these assumptions do not really limit the generality of the analysis.

We assume, that the economy has two types of labour, *high-skilled labour*, labelled 1, and *low-skilled labour,* labelled 2. Both types of labour are supplied by all households, but in various proportions. Some households therefore have higher income than others.[14]

Furthermore, we assume that each household demands two types of produced commodities, *services*, labelled 3, and *manufactured goods*, labelled 4. Services are less complementary to leisure than manufacturing goods, and demanded relatively more by high-skilled households.

Finally we assume that the *service industry*, sector 1, is more intensive in the use of low-skilled labour than the *manufacturing industry*, sector 2.

With respect to the administrative costs of different tax structures we assume,

1) that taxation of all types of labour income according to the same schedule, i.e. for

$$S_j \equiv \frac{p_j}{w_j} = S \ , \qquad j \in F \quad (18)$$

involves lover administrative costs than taxing households at different rates;

2) that taxation of all commodities at the same rate, i.e. for

$$T_j \equiv \frac{q_j}{p_j} = T \ , \qquad j \in C \quad (19)$$

involves lower administrative costs than differentiating the tax rates between commodities; and

3) that no taxation of the use of primary factors in the production sectors, i.e.

$$T_i^j \equiv \frac{q_i^j}{p_i^j} = 1, \qquad j \in F\ , \ j \in A \quad (20)$$

[14] The specification allows substitution between the two types of labour. The model may therefore be used to represent the disincentive effects on the supply of high-skilled labour of subsidizing low-skilled labour.

involves lower administrative costs than when inputs are taxed at different rates in the different production sectors.

We assume that in the process of identifying the overall optimal tax system, the government considers a tax structure, $\hat{\imath}_0$, where there are no restrictions on the government's choice of tax rates, and the following four tax structures,

$$\hat{\imath}_1 \equiv \{(\mathbf{S}{=}S), \mathbf{T}, (\mathbf{T}^1 = (1, T_2^1)), (\mathbf{T}^2 = (1,1)), 0\}^{15}$$
$$\hat{\imath}_2 \equiv \{(\mathbf{S}{=}S), \mathbf{T}, (\mathbf{T}^1 = (1,1)), (\mathbf{T}^2 = (1,1)), 0\}$$
$$\hat{\imath}_3 \equiv \{(\mathbf{S}{=}S), (\mathbf{T}{=}T, (\mathbf{T}^1 = (1, T_2^1)), (\mathbf{T}^2 = (1,1)), 0\}$$
$$\hat{\imath}_4 \equiv \{(\mathbf{S}{=}S), (\mathbf{T}{=}T), (\mathbf{T}^1 = (1,1)), (\mathbf{T}^2 = (1,1)), 0\}.$$

Notice that the tax structures, $\hat{\imath}_1$ and $\hat{\imath}_3$, allows the taxation of inputs to be differentiated between production sectors, whereas under $\hat{\imath}_2$ and $\hat{\imath}_4$ factor inputs cannot be taxed, and production efficiency is therefore imposed, although it may not be desirable. We assume that all tax systems $\boldsymbol{x}^i$, which belong to the same tax structure, are associated with the same administrative costs. We therefore have

$$\mathrm{B}(\boldsymbol{x}^i \in \hat{\imath}_0) > \mathrm{B}(\boldsymbol{x}^i \in \hat{\imath}_1) > \begin{Bmatrix} B\left(\boldsymbol{x}^i \in \hat{\imath}_2\right) \\ B\left(\boldsymbol{x}^i \in \hat{\imath}_3\right) \end{Bmatrix} > \mathrm{B}(\boldsymbol{x}^i \in \hat{\imath}_4)$$

Finally, we make two alternative assumptions about the income distributional preferences of the government. We assume that the government either attaches the same weight to the changes in real income for all households, i.e. is *inequality-neutral*, or higher weights to the changes in real income for low-income households than for high-income households, i.e. is *inequality-averse*.

4.1 Unrestricted tax structure ($\hat{\imath}_0$)

If the government is *inequality-neutral*, the indices of discouragement are the same for all commodities, and, generally speaking, commodities that are complementary to leisure are taxed at relatively high rates (cf. Corlett and Hague 1953). One would therefore expect services to be taxed at a lower rate than manufactured goods, since services have been assumed to be less complementary to leisure. Notice that if both labour and consumption were separable from leisure, to tax income according to the same tax schedule and to tax the consumption of the two commodities at the same rate would be the optimal solution.

[15] Notice that if the government were able to tax low-skill labour in both sectors, then the restriction that the two types of labour must be taxed at the same rate would not be binding. We would therefore obtain the same optimal allocation as without this restriction, although with different tax rates.

Under the assumption that the government is *inequality-averse*, the net distributional characteristics for low-skilled labour, $\tilde{R}_2$, will in general be higher than for high-skilled labour, $\tilde{R}_1$. The optimal tax structure will therefore involve a lower level of discouragement for low-skilled labour than for high-skilled labour. The tax on the supply of low-skilled labour will therefore, in general, be lower than on the supply of high-skilled labour, or in other words, an optimal tax system will, involve low-skilled labour being subsidised, or, if taxed, taxed at a lower rate than high-skilled labour.

The net distributional characteristics for services, $\tilde{R}_3$, are likely, since they are consumed disproportionately by the high-skilled households, to be lower than for manufacturing goods, $\tilde{R}_4$. However, whether services will be taxed at a lower or a higher rate than manufacturing products depend on the balancing of efficiency considerations (the objective of discouraging the consumption of leisure, which suggest a relatively low tax rate on services) with distributional considerations (which suggest a relatively high rate of tax on services).

4.2 General income tax (î$_1$ and î$_2$)

Under the two tax structures, î$_1$ and î$_2$, it is possible to tax the consumption of services and manufactured goods at different rates, and therefore $g_3 = g_4 = 0$. Under î$_1$, where the use of low-skilled labour in the service industry also may be taxed, $\gamma_2^1 = 0$, and also $\gamma_1^1 = 0$, because, although the use of high-skilled labour in the service industry cannot be taxed, this constraint is not binding, since there are no restrictions on the tax on the consumption of services.

The optimal levels of discouragement are therefore under î$_1$

$$d_s^1 = -\frac{\lambda_1 - \tilde{R}_s}{\lambda_1} + \frac{\gamma_s^2}{-Z_s w_s} S \qquad s \in F \quad (20)$$

$$d_k^1 = -\frac{\lambda_1 - \tilde{R}_k}{\lambda_1} \qquad k \in C \quad (21)$$

and under î^2 where factor inputs cannot be taxed

$$d_s^2 = -\frac{\lambda_1 - \tilde{R}_s}{\lambda_1} + \sum_{k \in A} \frac{\gamma_s^k}{-Z_s w_s} S \qquad s \in F \quad (22)$$

$$d_k^2 = -\frac{\lambda_1 - \tilde{R}_k}{\lambda_1} \qquad k \in C \quad (23)$$

The optimal levels of discouragement for low-skilled and high-skilled labour are thus likely to be different than under $\hat{\imath}_0$.

From (14), under $\hat{\imath}_1$ the optimal tax on the use of low-skilled labour in the service industry is[16]

$$t_2^1 = p_2^1 - p_2 = -\gamma_2^2 S / \lambda_1 \, v_2^2 \, \alpha_1^2 \, \sigma^2 \qquad s \in F \quad (24)$$

where σ^2 is the elasticity of substitution between the two types of labour in the manufacturing industry, and α_1^2 the share of high-skilled labour. Under $\hat{\imath}_1$ by properties of Lagrange multipliers we thus have

$$\frac{dW}{ds_2} = -\frac{\gamma_2^2}{w_2} T_2^2 \qquad s \in \mathrm{F} \quad (25)$$

Were it desirable (although it is not possible) to tax the supply of low -skilled labour at a lower rate than high-skilled labour, and therefore $\gamma_2^2 > 0$, then (see 24) it is desirable to tax the use of low-skilled labour in the service industry, i.e. $t_2^1 < 0$. If the opposite were the case, then $\gamma_2^2 < 0$ and $t_2^1 > 0$. In absolute terms t_2^1 will be the greater the smaller the use of low-skilled labour, the smaller the cost share of high-skilled labour, α_1^2, and the smaller the elasticity of substitution between high and low-skilled labour, σ^2, in the manufacturing industry.

For an *inequality-neutral* government, based on the assumptions made, we may expect $\gamma_2^2 \approx 0$, and hence (under $\hat{\imath}_1$) that $t_2^1 \approx 0$; whereas for an *inequality-averse* government, where the supply of low-skilled labour under a unrestricted tax structure $\hat{\imath}_0$ is likely to be taxed at a lower rate than high-skilled labour, we would expect $t_2^1 < 0$.

Since $\hat{\imath}_1$ imposes fewer restrictions on the government's choice of tax instruments than $\hat{\imath}_2$, the optimal solution under $\hat{\imath}_1$, is superior to that under $\hat{\imath}_2$, if the benefits of the tax (subsidy) on the use of low-skilled labour in the service industry is greater than the opportunity cost value of the larger administrative costs of $\hat{\imath}_1$ compared to $\hat{\imath}_2$. Notice however the optimal solution under $\hat{\imath}_1$ involves production inefficiency, which is not the case under $\hat{\imath}_2$.

[16] See Analytical Appendix for details on the derivation.

4.3 A general income tax and a uniform VAT ($\hat{\imath}_3$ and $\hat{\imath}_4$)

Under $\hat{\imath}_3$ and $\hat{\imath}_4$, a proportional tax structure is imposed not only on the taxation of labour income, but also on taxation of the produced commodities (in the form of a uniform VAT), such that $S_j = S$, j$\in F$, and $T_i = T$, i$\in C$.[17]

The indices of discouragement under $\hat{\imath}_3$ are

$$d_1^3 = -\frac{\lambda_1 - \tilde{R}_1}{\lambda_1} + \sum_{k\in A} \frac{\gamma_1^k}{-Z_1 w_1} S \qquad (26)$$

$$d_2^3 = -\frac{\lambda_1 - \tilde{R}_2}{\lambda_1} + \frac{\gamma_2^2}{-Z_2 w_2} S \qquad (27)$$

$$d_k^3 = -\frac{\lambda_1 - \tilde{R}_k}{\lambda_1} - \frac{\gamma_k}{X_k p_k} \qquad k\in C \quad (28)$$

and under $\hat{\imath}_4$ where factor inputs cannot be taxed

$$d_s^4 = -\frac{\lambda_1 - \tilde{R}_s}{\lambda_1} + \sum_{k\in A} \frac{\gamma_s^k}{-Z_s w_s} S \qquad s\in F \quad (29)$$

$$d_k^4 = -\frac{\lambda_1 - \tilde{R}_k}{\lambda_1} - \frac{\gamma_k}{X_k p_k} \qquad k\in C \quad (30)$$

The optimal levels of discouragement are therefore likely to be different from those under $\hat{\imath}_0$ not only for the supply of low-skilled and high-skilled labour, but also for the consumption of services and manufactured goods.

According to (14) under $\hat{\imath}_3$ the optimal tax on the use of low-skilled labour in the service industry is[18]

$$t_2^1 = -\frac{\gamma_3}{p_3} T \mathrm{p}_2^2 / \lambda_1 \alpha_1^1 \sigma^1 Y^1 - \gamma_2^2 S / \lambda_1 \mathrm{a}_2^2 \alpha_1^2 \sigma^2 Y^2 + \gamma_4 T \mathrm{p}_2^2 / \lambda_1 \alpha_1^2 \sigma^2 Y^2 \qquad (31)$$

Under $\hat{\imath}^3$, by properties of Lagrange multipliers we have

[17] In the case of $\hat{\imath}_4$, as mentioned above, only one tax system is feasible. The optimal tax system under $\hat{\imath}_4$ is therefore the same whether the government is inequality-neutral or inequality-averse.
[18] See the Analytical Appendix for details on the derivation.

$$\frac{dW}{ds_1} = -\sum_{k \in A} \frac{\gamma_1^k}{w_1} T_1^k \tag{32}$$

$$\frac{dW}{ds_2} = -\frac{\gamma_2^2}{w_2} T_2^2 \tag{33}$$

$$\frac{dW}{dt_k} = -\frac{\gamma_k}{p_k} \qquad k \in C \tag{34}$$

Were it desirable (although it is not possible) to tax the supply of low-skilled labour at a lower rate than high-skilled labour then γ_2^2 >0, as under $\hat{\boldsymbol{\imath}}_1$, and if the opposite is the case, γ_2^2 <0. Furthermore, were it desirable (although it is not possible) to tax services at a lower rate than manufactured goods, then γ_3 >0 and γ_4 <0; and if the opposite were the case, then γ_3 <0 and γ_4 >0.

For an *inequality-neutral* government since services have been assumed to be less complementary to leisure than manufactured goods, it is likely that γ_3 >0, γ_4 <0, and $\gamma_2^2 \approx 0$. Disregarding administrative costs, the optimal solution under $\hat{\boldsymbol{\imath}}^3$ is therefore likely to involve a subsidy to the use of low-skilled labour in the service industry, i.e. t_2^1<0, in order to encourage the consumption services and hence leisure and thus decrease the distortionary costs of taxation (cf. 31).

For an *inequality-averse* government γ_2^2 >0, and if efficiency considerations dominate distributional considerations with respect to the tax on the consumption of services, then γ_3 >0 and γ_4 <0. Under these assumptions a subsidy to the use of low-skilled labour is justified both on efficiency grounds and on distributional grounds, i.e. t_2^1<0 (cf. 31). However, if γ_3 <0 and γ_4 >0 (i.e. if efficiency considerations dominate with respect to the a tax on services), then whether it will be desirable to subsidise the use of low-skilled labour in the service industry will depend on the relative strength of a number of factors drawing in opposite directions. These in turn depend on the share of low-skilled labour and the elasticities of substitution in the two industries and which draw in different directions (cf. 31). The objective of discouraging the consumption of services for distributional reasons, conflicts in this case with the objective of encouraging it for efficiency reasons and with the objective of supporting the income of low-skilled households.

Finally, repeating the point made above, since $\hat{\boldsymbol{\imath}}_3$ imposes fewer restrictions than $\hat{\boldsymbol{\imath}}_4$, the optimal tax system under $\hat{\boldsymbol{\imath}}_3$, is superior to that under $\hat{\boldsymbol{\imath}}_4$ if the benefits of the tax (subsidy) on the use of low-skilled labour in the service industry are more important than the social value of the larger administrative costs. In this case the optimal tax system under $\hat{\boldsymbol{\imath}}_3$ dominates the optimal tax system under $\hat{\boldsymbol{\imath}}_4$, although the former involves production inefficiency and the latter does not.

5 Ranking of the tax-transfer systems considered

5.1 Theoretical analysis

Disregarding administrative costs, the level of social welfare associated with an optimised tax-transfer system cannot by definition decrease with the addition of an additional tax instrument. The (partial) ranking of the tax structures considered is therefore when administrative costs are not taken into account $\hat{\mathbf{i}}_0 \succsim \hat{\mathbf{i}}_1 \succsim \left\{\begin{matrix} \hat{\mathbf{i}}_2 \\ \hat{\mathbf{i}}_3 \end{matrix}\right\} \succsim \hat{\mathbf{i}}_4$, whether the government is inequality neutral or inequality averse. However, in terms of falling administrative costs, the (partial) ranking of the tax structures is $\hat{\mathbf{i}}_4 \succsim \left\{\begin{matrix} \hat{\mathbf{i}}_2 \\ \hat{\mathbf{i}}_3 \end{matrix}\right\} \succsim \hat{\mathbf{i}}_1 \succsim \hat{\mathbf{i}}_0$. The ranking is in other words the opposite of the ranking according to social welfare when administrative costs are disregarded. The tax structure, which is associated with the highest level of social welfare, cannot therefore be identified on the basis only of theoretical considerations, but requires empirical information about the economy and the preferences of the government. More specifically, it is thus not possible based on theoretical considerations to say *a priori* whether the optimal tax system involves production efficiency or not.

5.2 Quantitative example

To support the insight as to when production inefficiency is desirable, we have constructed a stylised CGE model according the assumptions on which the theoretical analysis has been based and used the model to calculate the optimal tax system for each of the five tax structures considered. Details on the data, the functional forms, the parameter values and the simulation results, are available in the Annex

Using the CGE model it is possible to establish a complete ranking of the tax structures considered and to determine whether or not the optimal tax structure involves production efficiency or not.

When the government is *inequality-neutral* we get the following ranking of the tax systems (see Column 4 in the Table)

$$\hat{\mathbf{i}}_4 \succ \hat{\mathbf{i}}_3 \succ \hat{\mathbf{i}}_2 \succ \hat{\mathbf{i}}_1 \succ \hat{\mathbf{i}}_0$$

and when the government is *inequality-averse* (see Table, Column 5)

$$\hat{\mathbf{i}}_4 \succ \hat{\mathbf{i}}_3 \succ \hat{\mathbf{i}}_2 \succ \hat{\mathbf{i}}_1 \succ \hat{\mathbf{i}}_0 .$$

Table Administrative costs and social welfare under different tax structures

	Administrative costs relative to an un-differentiated tax structure	**Optimal tax rates on the use of low-skilled labour in the service industry**		**Change in social welfare relative to an un-differentiated tax structure**	
	(1)	Neutral (2)	Adverse (3)	Neutral (4)	Adverse (5)
All market transactions taxable, x_0^*	1.500	0.000	0.000	-1.466	-0.341
Proportional income tax and sector specific tax/subsidy , x_1^*	0.500	0.100	-0,500	-0.471	0.340
Proportional income tax, x_2^*	0.250	0,000	0,000	-0.220	-0.051
Proportional income tax, VAT and sector specific tax/subsidy, x_3^*	0.100	-0,001	-0,500	-0.095	0.734
Proportional income tax and VAT, x_4^*	0.000	0.000	0.000	0.000	0.000

Source: Stylised CGE model (see Annex).

When the government is inequality-neutral, the overall optimal tax system therefore involves production efficiency, but under the more realistic assumption that it is inequality-averse production efficiency is **not** desirable.

The simulation results thus highlight that which tax structure is the optimal depends on both the administrative costs of the different tax structures and on the government's distributional preferences. Not only the optimal level of the tax rates, but also which tax instruments it will be desirable to use, depends crucially on the government's distributional value judgements.

6 Summary and concluding remarks

We have considered an economy where different types of labour are taxed at the same rate reflecting an almost universal feature of existing income tax systems, at least until recently. We have analysed how both the objective of reducing the distortionary costs of the tax system (efficiency considerations) and the objective of improving the income distribution (equity considerations) may make it desirable to design a tax structure that creates production inefficiency.

We have moved the analysis of the subject forward by considering a many, rather than a one household economy identifying distributional considerations as an important reason for why production efficiency may not be desirable. Furthermore, we have provided an explanation of why certain tax rates are restricted by taking into account administrative costs. How this is done may be considered as an overly simplistic way to capture the importance of transaction cost for the design of an optimal tax structure, but so may the assumption that lump-sum taxes are not feasible. However, in the absence of alternative assumptions, which are both manageable in a general equilibrium framework and more realistic, we suggest that the assumptions made concerning administrative costs, as the assumption that lump-sum taxes are not feasible, may serve to make optimal tax theory a more useful framework for policy analysis.

The theoretical analysis, and the simulation results generated to illustrate the theoretical results suggest that it is likely that the optimal system of taxes under reasonable assumptions will involve production inefficiencies, and that production efficiency, and hence free trade should be taken as a normative benchmark for the assessment of economic policies with more caution than is often the case.

The main lessons for policy analysis to be drawn from this paper may be summarised in the following three points:

First, the administrative costs associated with different tax systems and with individual tax instruments may be crucial for the design of an optimal tax structure. The introduction of a new tax instrument may dramatically change the optimal values of other tax instruments. Changes in administrative costs associated with a given tax instrument may therefore not only justify its suppression or its introduction, but may also significantly change the overall tax structure.

Second, a tax system should not be characterised as inefficient without reference to the value judgements on which the design of the system has been based. A government with strong redistributional preferences is likely to implement a more differentiated tax system and one that is more likely to create production inefficiencies than a government without redistributional objectives.

Third, social and technological developments changing the administrative costs of taxation and the distribution of income may justify the introduction of new tax instruments, including tax instruments that create production inefficiency. Technological progress, in particular in information technology, and the globalisation of the economy, have, in industrial countries, decreased the costs of tax administration and increased the differences in income between low-skilled and high-skilled workers. This may justify a shift towards more differentiated tax-transfer systems in favour of the low-skilled workers, even if a general income tax and a uniform VAT at earlier times have been the optimal solution. The approach adopted in this paper may thus potentially serve as a framework to assess proposals for reforms of the Welfare State.

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Analytical Appendix

We may define the elasticity of substitution between factor 1 and 2 as

$$\sigma^k = a_{12}^k c^k(\mathbf{p}^k)/a_1^k\ a_2^k \qquad k \in A$$

and the share of factor 1 in total costs in sector k as

$$\alpha_1^k = p_1^k\ a_1^k / c^k(\mathbf{p}^k) \qquad k \in A$$

hence

$$a_{12}^k = a_1^k\ a_2^k\ \sigma^k / c^k(\mathbf{p}^k) \qquad k \in A$$

$$a_{12}^k = \alpha_1^k\ \sigma^k\ a_2^k / p_1^k \qquad k \in A$$

Since $a_{22}^k = -a_{12}^k\ p_1^k / p_2^k$

$$a_{22}^k = -\alpha_1^k\ \sigma^k\ a_2^k / p_2^k \qquad k \in A$$

When the supplies of the primary factors are taxed at the same rate as well as the consumption of the produced commodities, and the use of primary factors in sector 2 cannot be taxed, but a tax on the use of factor 2 in sector 1 is feasible, such that $\gamma_2^1 = 0$, then for k=1 and s=2

$$p_2^1 - \tilde{p}_2 = \frac{\gamma_3}{p_3}\ T a_2^1 / \lambda_1 a_{22}^1 Y^1$$

$$p_2^1 - \tilde{p}_2 = -\frac{\gamma_3}{p_3}\ T p_2^1 / \lambda_1\ \alpha_1^1\ \sigma^1\ Y^1$$

and for k=2 and s=2

$$p_2 - \tilde{p}_2 = -(\frac{\gamma_2^2}{w_2} - \frac{\gamma_4}{p_4}\ T\ a_2^2) / \lambda_1 a_{22}^2 Y^2$$

$$p_2 - \tilde{p}_2 = -(\frac{\gamma_2^2}{w_2} - \frac{\gamma_4}{p_4}\ T\ a_2^2) / \lambda_1 a_{22}^2 Y^2$$

$$p_2 - \tilde{p}_2 = (\frac{\gamma_2^2}{w_2} - \frac{\gamma_4}{p_4}\ T\ a_2^2) p_2^2 / \lambda_1 Y^2\ \alpha_1^2\ \sigma^2\ a_2^2$$

$$p_2 - \tilde{p}_2 = \gamma_2^2 S / \lambda_1\ a_2^2\ \alpha_1^2\ \sigma^2\ Y^2 \ - \ \frac{\gamma_4}{p_4}\ T\ p_2^2 / \lambda_1\ \alpha_1^2\ \sigma^2\ Y^2$$

Therefore

$$p_2^1 - p_2 = -\frac{\gamma_3}{p_3} T\mathrm{p}_2^2/\lambda_1\, \alpha_1^1\, \sigma^1\, Y^1 - \gamma_2^2 S/\lambda_1\, \mathrm{a}_2^2\, \alpha_1^2\, \sigma^2\, Y^2 + \gamma_4\, T\, \mathrm{p}_2^2/\lambda_1\, \alpha_1^2\, \sigma^2\, Y^2$$

and under the same assumptions as above, except that the consumption of the produced commodities may be taxed at separate rates such that $\gamma_3 = 0$ and $\gamma_4 = 0$,

$$p_2^1 - p_2 = -\gamma_2^2 S/\lambda_1\, \mathrm{a}_2^2\, \alpha_1^2\, \sigma^2\, Y^2$$
$$p_2^1 - p_2 = -\gamma_2^2 S/\lambda_1\, \mathrm{v}_2^2\, \alpha_1^2\, \sigma^2$$

Annex: *The CGE model. The specification and simulation results*

Specification

A) Functional forms

The following supplementary assumptions have been made:

The constant returns to scale *production functions,* $f^k(v_1^k, v_2^k; \sigma^k)$, are CES functions.

$U^h\left(C^h, c_0^h; \sigma^{L.h}\right)$, express for each household utility as a function of the aggregate consumption and pure leisure. $C^h = C^h\left(C_1^h, C_2^h; \sigma^{D.h}\right)$ express the degree of substitution between the two composite goods, and $C_i^h = C_i^h(x_i^h, c_0^{i.h}; \sigma^{Ci.h})$ indicates for each composite good the substitution possibilities between the amount purchased of the commodity, x_i^h, and the time used for its consumption, $c_0^{i.h}$.

The *utility functions,* $u^h\left(\mathbf{x^h}, \mathbf{z^h}\right)$ are assumed to take the form[19], $U^h\left(C^h\left(C_1^h(C_1^h(x_1^h, c_0^{1.h}; \sigma^{C1.h}), C_2^h(x_1^h, c_0^{2.h}; \sigma^{C2.h}); \sigma^{D.h}\right), \left(\omega_0 - \sum_{i \in C} c_0^{i.h} - z_0^h(z_1^h, z_2^h; \sigma^{Z.h})\right); \sigma^{L.h}\right)$
The quantities of the two composite commodities, combining services and time and the manufactured good and time, are indicated by C_i^h, i=1,2, respectively. For each of the two composite commodities, a function $C_i^h = C_i^h(x_i^h, c_0^{i.h}; \sigma^{Ci.h})$ indicates how the quantity purchased of the corresponding marketed commodity, x_i^h, is combined with the time used for its consumption, $c_0^{\mathrm{i..h}}$. Each household has the same time endowment, indicated by ω_0. The total time used to supply labour to the market is $z_0^h = z_0^h(z_1^h, z_2^h; \sigma^{Z.h})$, "Leisure" or "non-market use of time" is therefore ω_0 - $z_0^h(z_1^h, z_2^h; \sigma^{Z.h})$. The total time used for the consumption of purchased commodities is $\sum_{i \in C} c_0^{i.h}$. "Pure leisure", c_0^h, is the amount of time used on activities that do not involve the consumption of purchased commodities; therefore $c_0^h = \omega_0 - \sum_{i \in C} c_0^{ih}$ - $z_0^h(z_1^h, z_2^h; \sigma^{Z.h})$.
The functions, $U^h\left(C^h, c_0^h; \sigma^{L.h}\right)$, expresses for each household utility as a function of aggregate consumption and pure leisure. The functions $U^h\left(C^h, c_0^h; \sigma^{L.h}\right)$,

[19] It is important to emphasise that utility function constitutes a special case of the utility function defined on $u^h\left(\mathbf{x^h}, \mathbf{z^h}\right)$, on which the theoretical analysis is based. Any results derived for the theoretical model must therefore apply to the specific CGE model.

$C^h\left(C_1^h, C_2^h; \sigma^{D.h}\right)$, $C_1^h(x_1^h, c_0^{1.h}; \sigma^{C1.h})$ and $C_2^h(x_1^h, c_0^{2.h}; \sigma^{C2.h})$ are all CES functions.[20] This specification allows household preferences defined over marketed commodities, $u^{\mathrm{h}}\left(\mathbf{x^h}, \mathbf{z^h}\right)$, to represent different degrees of complementarity with "leisure" in the meaning non-market use of time. The supplies of high-skilled and low-skilled labour by household h are, in efficiency units, z_1^h and z_2^h, respectively. The functions, $z_0^h(z_1^h, z_2^h; \sigma^{Z.h})$, are household specific CET transformation functions. These transformation functions reflect differences in inherent abilities and in education between the households, but not differences in taste. In supplying labour to the market the households are only concerned about the time involved, not whether the employment is high-skilled or low-skilled.[21]

B) Parameters

The parameters of the two production functions and of the utility functions are provided in Annex Table 1. The time used per unit of expenditures is for both households assumed to be higher for manufactured goods than for services (see Annex Table 1). The manufactured goods are therefore more complementary with "leisure" (i.e. non-market use of time) than the services, (think of childcare).

The government is assumed to be either *inequality-neutral* (maximising aggregate real income) or *inequality-adverse* (maximising a weighted sum of the real incomes of the households). In the case of an inequality-adverse government the social welfare weight for the low-income household has been fixed at twice that for the high-income household.

C) Benchmark data

The benchmark data set on which the CGE model is calibrated is provided in Annex Table 2. The tax system in the benchmark consists of a 40% income tax and a VAT at 0%.

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[20] Often CGE models, which are based on such assumptions, are unsuited for optimal tax analysis. Often an economy with only one type of households is considered, and labour supply is assumed either to be fixed, or if endogenous, preferences are assumed to be homothetic with the consumption of produced goods being separable from the consumption of leisure. These assumptions imply that a proportional tax structure is optimal. They may for some purposes constitute a convenient simplification of reality, but may also give rise to misleading conclusions with respect to what constitutes desirable directions of policy reform. The specification of household preferences adopted here overcomes this problem.

[21] Assuming that the supply to the market of the two types of labour (z_1^h, z_2^h) is separable from composite consumption, C^h, and from leisure, $\omega_0 - z_0^h$, is restrictive. It implies, in the absence of distributional considerations, that the optimal tax structure for the two types of labour will be proportional. Notice, however, that since $z_0^h = z_0^h(z_1^h, z_2^h)$ the model is able to represent the distortion of the supply of high skilled compared to low skilled labour.

D) Computer program

The model has been programmed in GAMS/MPSGE (see Tom Rutherford 1994, "Applied General Equilibrium Modelling with MPSGE as a GAMS subsystem", *Memo*). The program can be obtained from the author.

E) Simulation results

The results of the calculation of the optimal solutions to the government's maximisation problem for five different tax structures considered are provided in Annex Table 3

Annex Table 1 ***Model parameters***

	Households	
	Low-skilled	**High-skilled**
$\sigma^{D.h}$ 1)	0.50	0.50
$\sigma^{L.h}$ 2)	0.50	0.50
$\sigma^{Ci.h}$ 3)	0.00	0.00
$\sigma^{z.h}$ 4)	1.00	2.00
$z_0^h/(\omega_0 - z_0^h)$ 5)	1.67	1.00
$c_0^{1.h}/x_1^h$ 6)	0.001	0.010
$c_0^{2.h}/x_2^h$ 7)	0.500	0.600
	Production sectors	
	Service	**Manufacturing**
σ^k 8)	1.50	0.50

1) Elasticity of substitution between leisure and aggregate consumption in $U^h\left(C^h, c_0^h; \sigma^{L.h}\right)$

2) Elasticity of substitution between household produced consumption services in $C^h\left(C_1^h, C_2^h; \sigma^{D.h}\right)$

3) Elasticity of substitution between the purchase and time used in $C_i^h(x_i^h, c_0^{i.h}; \sigma^{Ci.h})$

4) Elasticity of transformation between high-skilled and low-skilled labour in $z_0^h(z_1^h, z_2^h; \sigma^{Z.h})$

5) Time for non-market use as share of time used to supply labour to the market

6) Share of time for non-market use used to consume the service commodity

7) Share of time for non-market use used to consume other commodities

8) Elasticity of substitution between high-skilled and low-skilled labour in $f^k(v_1^k, v_2^k; \sigma^k)$

Benchmark values of prices, of quantities demanded and supplied, of real incomes and of social welfare

Annex Table 2.1 Prices

	Households prices	Producer prices		Market prices
		Services	Manufacturing	
High-skilled labour	0,60	1,00	1,00	1,00
Low-skilled labour	0,60	1,00	1,00	1,00
Services	1,00	1,00	1,00	1,00
Manufacturing	1,00	1,00	1,00	1,00

Annex Table 2.2 Quantities

	Households types		Production sectors	
	Low-skilled	High-skilled	Services	Manufacturing
High-skilled labour	3,52	152,47	2,00	90,00
Low-skilled labour	5,00	1,00	3,00	2,00
Services	0,10	4,90		
Manufacturing	5,00	87,00		

Annex Table 2.3 Shares

	Household types		Production sectors	
	Low-skilled	High-skilled	Services	Manufacturing
High-skilled labour	0,41	0,99	0,40	0,98
Low-skilled labour	0,59	0,01	0,60	0,02
Services	0,02	0,05		
Manufacturing	0,98	0,95		

Annex Table 2.4 Normative indicators

	Household types	
	Low-skilled	High-skilled
Real income	5,11	92,08
Welfare	10,22	92,08

Simulation results

Annex Table 3.1: Optimal tax system when all market transaction can be taxed

$\hat{\imath}^{0}$	Neutral	Averse
Household taxes		
High-skilled labour,	40%	40%
Low-skilled labour,	44%	-26%
Services,	-15%	2%
Manufacturing,	1%	5%
Producer taxes		
Low-skilled labour in service industry	0%	0%

Table 3.2: Optimal tax system with a general income tax

	$\hat{\imath}^{1}$		$\hat{\imath}^{2}$	
	Neutral	Averse	Neutral	Averse
Household taxes				
High-skilled labour,	40%	40%	40%	40%
Low-skilled labour,	40%	40%	40%	40%
Services,	-16%	-10%	-13%	-32%
Manufacturing,	1%	3%	1%	2%
Producer taxes				
Low-skilled labour in service industry	10%	-50%		

Table 3.3: Optimal tax system with a general income tax and a VAT

	$\hat{\imath}^{3}$		$\hat{\imath}^{4}$	
	Neutral	Averse	Neutral	Averse
Household taxes				
High-skilled labour,	40%	40%	40%	40%
Low-skilled labour,	40%	40%	40%	40%
Services,	0%	3%	0%	0%
Manufacturing,	0%	3%	0%	0%
Producer taxes				
Low-skilled labour in service industry	-0.1%	-50%		