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Cournot Competition in the Electricity Market with Transmission Constraints.

by

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DISCUSSION
PAPER

Cournot Competition in the Electricity Market with Transmission Constraints

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Abstract

This paper studies the market power of generators in the electricity market when transmission capacity is scarce. We consider a simple world of two generators providing electricity to their consumers through a single transmission line.

In the literature, different Cournot equilibrium concepts have been developed. This paper applies these concepts and explains the implicit assumptions on the behavior of the System Operator made in those papers.

We show that these implicit assumptions are not realistic. For an alternative role of the System Operator, we solve the Cournot equilibrium and compare the outcome. Furthermore, we show that the axiomatic equilibrium concept of Smeers and Wei (1997) is linked with the model of Oren (1997) and can also be defined as a Nash Equilibrium.

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Contents

1 Introduction

Many countries are currently liberalizing their electricity industries. This process is usually viewed as a shift from tight regulation of vertically integrated monopolies to light regulation of functionally separated firms. This shift has been justified by changes in technology, such as diminished economies of scale in electricity generation.

To enhance competition, most countries separate the transmission services from the generation activities. The European directive foresees the unbundling of the electricity sector in generation, transmission, and distribution.

1. *Generation* is the production of electricity by a mixture of mostly large power plants. Although only a few large generators exist, contestability of the electricity market is viewed sufficient to limit market power.

2. *Transmission* is the transport of electricity over the high voltage grid. Most generators are directly connected to this grid, as are large industrial consumers. Since economies of scale in transmission services are large, transmission is a natural monopoly. Besides that, reliability concerns require a close coordination. Therefore, in all countries we have a single, regulated firm – the System Operator – operating the transmission grid.

3. *Distribution* is the local, low voltage provision of electricity for small users. The European directive does not oblige the member countries to organize competition in distribution.

In this paper we study the imperfect competition in transmission capacity and generation. The capacity of transmission lines is limited by technical constraints. It cannot be extended easily since building transmission lines takes a long time, and there is a strong opposition by environmentalists. Therefore, the opening of the electricity market will be limited by transmission capacity in the next ten to fifteen years.

We assume that the electricity market is centrally organized, this means that the System Operator owns and allocates all transmission capacity.[1] This setting is easier to model than the decentralized system, where we explicitly have to specify the market for transmission rights.

Many authors studied this problem by building Cournot type of models. But they do not obtain the same results nor do they specify their assumptions. In particular, they do not clarify the role of the System Operator.

The aim of this paper is to apply the different models to a simple electricity market with one transmission line. This allows us to classify these models, in function of the precise role of the

[1] Section 2 explains the difference between the decentralized and the centralized organization.

System Operator. Additionally, we formulate some alternative assumptions for the behavior of the System Operator and examine how this affects the results.

Structure of the paper In *section 2* we will summarize the relevant literature of industrial economics in the electricity market. In *section 3* we present a simple problem of a generation oligopoly on a grid with only one transmission line. *Section 4* studies two models where the System Operator does not set transmission prices, but allocates transmission capacity. In *section 5,* we look at models where the System Operator both sets transmission prices and allocates transmission capacity. *Section 6* concludes the paper.

2 Literature Review

Basically two alternative organizational structures exist in the electricity market: the *decentralized system* and the *centralized system.* In practice a mixture of both systems is commonly used.

Decentralized System[2] In the decentralized system, the market is responsible for determining electricity prices and transmission prices. Two markets exist: one for electricity, where electricity prices are set, and another for transmission capacity, where transmission prices are set. The System Operator is responsible for the safety on the grid. Examples of decentralized markets can be found in the Netherlands, the Nordic countries, and in California.[3]

Centralized System[4] In the centralized system, prices are not determined by the market. The generators and the consumers announce the System Operator their willingness to supply and to consume electricity. They do this by submitting a Supply Function or a Demand Function to the System Operator. This System Operator tries to achieve market equilibrium. By solving a complex optimization problem, thereby taking into account all constraints, he decides upon the quantities of consumption and generation. He sets the prices of electricity and transmission equal to the dual values of the constraints. In the centralized system, the System Operator plays the role of an auctioneer. Generators and Consumers trade with the System Operator and not directly with each other. Examples of centralized operation are the former England & Wales market, and the National Australian market.

We review only the literature of the centralized market organization. First we look at competition in generation, after that at competition in generation and transmission.

[2] Chao and Peck (1996) provide more information about the decentralized organization of the electricity market.
[3] Very few countries have effectively implemented a transmission market.
[4] Schweppe, Caramanis, Tabors and Bohn (1988) and Hogan (1992) explain the centralized market.

2.1 Competition in Generation

Because of the existence of the System Operator in his role of auctioneer, we can put some institutional detail in the models. Firms simultaneously bid a supply function – i.e. a price quantity relation – to the System Operator.

The models found in the literature differ in the type of supply function they allow for: differentiable supply functions or step supply functions.[5]

Differentiable Supply Functions The concept of differentiable supply functions is based on Klemperer and Meyer (1989). They show that when the demand function for electricity is known for certain and firms announce a differentiable supply function, an infinite number of Nash equilibria exists. However, when the demand function is uncertain, the supply function has to be appropriate for several situations, and the number of equilibria diminishes.[6] Under certain conditions, the differentiable supply function equilibrium becomes unique.

Green and Newberry (1992) apply the model of Klemperer and Meyer for the two largest generators in the English market.[7] By adding an output constraint for each generator they can reduce the set of equilibria. Furthermore, they assume that the generators will coordinate on that equilibrium which would give them the highest total profit. The model predicts that in the absence of threat of entry the two generators are able to sustain a non-collusive equilibrium in which prices are well above operating costs.

Step Supply Function Generators submit an offer price for each individual plant at which they are willing to supply their given capacities, i.e. they offer step-supply functions. Von der Fehr and Harbord (1993) argue that the "step length" is not small enough to consider the supply function as a differentiable supply function.

Bidders offering more than one unit have an incentive to increase their bids at high quantities. If a bid sets a high equilibrium price, it applies to all inframarginal units. Wolfram (1998) finds empirical evidence that this is the fact in the UK.

2.2 Competition in Generation and Transmission

One of the major drawbacks of the models mentioned above is that they do not incorporate the spatial structure of the market. Since the direct costs of transmission are small, introducing these

[5] In the literature the equilibrium with differentiable supply functions is called the 'supply function equilibrium' and the models that use a step supply functions are called 'multi-unit auctions'.

[6] Klemperer and Meyer consider horizontal shifts of the demand function.

[7] Other studies using this model are Bolle (1992), Newberry (1998), Green (1996) and Rudkevich, Duckworth and Rosen (1998).

costs would only lead to minor changes in the results.[8] On the other hand, when a line becomes congested it will have a large influence on the electricity market.[9] As the differentiable supply function and step supply function models are not (yet) applicable in a market with transmission constraints; most researchers opt for a sort of Cournot market. They drop some of the institutional complexity of the actual market.[10] Wolak and Patrick (1996) suggest in an empirical study that Cournot competition is an appropriate representation of the market. They argue that the market power of the dominant generators is manifested through those generators declaring certain plants unavailable to supply in certain periods. We will study the Cournot models of Oren (1997), and Smeers and Wei (1997) in more detail in this paper.[11]

3 Description and Classification of the Game.

In this section we discuss and classify the game used in this paper. The first subsection describes the set-up of our game. The next two subsections examine this game when the transmission capacity is infinite and when it is finite. In the last subsection we discuss the role of the System Operator.

3.1 Description of the Game

We will model the simplest transmission grid possible: a single transmission line connecting two generators in city North (N) with electricity consumers in city South (S).[12]

[8] The direct transmission cost is approximately 5 % of total electricity cost for large consumers.

[9] When a line becomes congested it will have an influence on the electricity market in two ways. (1) With congestion, the electricity market is no longer a single market where all generators compete with each other. Instead, the transmission constraint segments the market in local distinct regions, increasing the market power of the generators. (2) Since transmission capacity is a scarce good, there is a rivalry for the use of the constrained line. Generators who have the right to use the line can get a scarcity rent.

[10] The Cournot game could be seen as a game where the generators submit vertical supply functions to the System Operator.

[11] In a previous report (Willems 2000) also the model of Stoft (1998) has been studied.

Other studies using Cournot competition are Stoft (1997, forthcoming), Borenstein, Bushnell and Stoft (1998), Borenstein, Bushnell and Knittel (1999), Borenstein and Bushnell (1999), Hogan (1997), Cardell, Hitt and Hogan (1997).

[12] By choosing only two nodes we do not catch all effects of the transmission grid. Physically, an electricity flow can not be directed over a specific line, but distributes itself over all lines of the grid proportional to the admittance of the lines. Electricity is thus using all lines at the same time. This effect is called 'loop flow'. To study loop flow, we need a grid with at least three nodes, and three transmission lines. It is not yet clear if loop flow will fundamentally change the market power of the generators.

Also the location of generators and consumers has a great impact on the outcome of the game. (See appendix A.)

Consumers are price takers with a linear inverse demand function $\theta(q)$

$$\theta(q) = a - q, \tag{1}$$

which represents their willingness to pay (WTP) for electricity. This WTP covers delivered electricity, i.e. it includes transmission costs.

An arbitrary generator is presented by the letter i and let $-i$ denote the other generator. The output q_i of each generator is not bounded by technical limitations of their generation plants ($q_i \in \mathbb{R}^+$). Each generator has a constant marginal cost c_i. The profit π of firm i equals

$$\pi_i = \gamma \cdot q_i - c_i\, q_i, \tag{2}$$

where γ is the net price that the generators receive for their electricity. It is the price at node N.

A single *transmission line* with capacity k connects city N with city S. By assumption transmission costs and transmission losses are zero. All electricity generated is thus consumed:

$$q = \sum_i q_i, \tag{3}$$

As transmission capacity is limited by technical constraints, it can become scarce, and have an opportunity cost. The transmission price p to transport electricity from N to S is then not necessary zero.

The net price that the generators receive (γ) is the consumers price θ minus the transmission price p:

$$\gamma = \theta - p \tag{4}$$

Defining the 'competitiveness' of a generator as[13]:

$$d_i \equiv a - c_i \tag{5}$$

and substituting eq. 4 his profit π_i becomes

$$\pi_i = (d_i - p - q)\, q_i. \tag{6}$$

3.2 Infinite Transmission Capacity

When the transmission capacity is infinite, it can not influence the outcome and transmission price p will be zero. In a Cournot game, each firm has one decision variable: the quantity q_i produced.

[13] A competitive firm has a low marginal cost c_i. For a monopolist d_i can be interpreted in two different ways: (1) It is the maximal output that the monopolist can generate without making losses. (2) It is twice the monopoly output of the firm.

Each player maximizes his profit π_i taking the output q_{-i} of the other player as given:

$$\max_{q_i \in \mathbb{R}^+} \pi_i(q_i; q_{-i}) \tag{7}$$

The firms have the following reaction function:

$$q_i(q_{\_i}) = \max\left\{\frac{d_i - q_{-i}}{2}, 0\right\} \tag{8}$$

The Nash-equilibrium is the intersection of the two reaction functions. The equilibrium falls apart in three distinct types:

$$q_i = \begin{cases} 0 & \text{if} \quad \frac{d_i}{d_{-i}} \in \left[0, \frac{1}{2}\right] \\ \frac{2\,d_i - d_{-i}}{3} & \text{if} \quad \frac{d_i}{d_{-i}} \in \left(\frac{1}{2}, 2\right) \\ \frac{d_i}{2} & \text{if} \quad \frac{d_i}{d_{-i}} \in [2, \infty] \end{cases} \tag{9}$$

- When $\frac{d_i}{d_{-i}} \leq \frac{1}{2}$, firm i has such a big cost disadvantage that it chooses not to produce.
- When $\frac{1}{2} < \frac{d_i}{d_{-i}} < 2$, the marginal costs of the firms are comparable, and we get the 'pure' duopoly outcome.
- When $2 \leq \frac{d_i}{d_{-i}}$, firm i is so competitive that when it produces the monopoly output $\frac{d_i}{2}$ the resulting price is lower than the marginal costs of firm $-i$. Firm i is a de-facto monopolist.

The solution type thus depends upon the relative competitiveness ($\frac{d_i}{d_{-i}}$) of the two firms. Figure 1 represents these different types in the space of the cost parameter $\vec{d} = (d_i, d_{-i})$.

The profit π_i associated with this equilibrium is:

$$\pi_i = \begin{cases} 0 & \text{if} \quad \frac{d_i}{d_{-i}} \in \left[0, \frac{1}{2}\right] \\ \frac{(d_i + d_{-i}) \cdot (2d_i - d_{-i})}{9} & \text{if} \quad \frac{d_i}{d_{-i}} \in \left(\frac{1}{2}, 2\right) \\ \frac{d_i^2}{4} & \text{if} \quad \frac{d_i}{d_{-i}} \in [2, \infty] \end{cases} \tag{10}$$

The Cournot game with infinite transmission capacity is our reference outcome, we will refer to it as the 'normal' game.

3.3 Finite Transmission Capacity

When the transmission line has a finite capacity, the transmission capacity may become scarce i.e. demand for transmission may exceed its supply. Allocation of this capacity thus becomes crucial.

The Cournot game with transmission constraint is defined by the vector of parameters $[d_i, d_{-i}, k]$. It is convenient to normalize this games by dividing quantities and prices by k, and profit by k^2. We will denote the normalized variables by capital letters. The normalized transmission capacity is thus equal to one ($K = 1$).

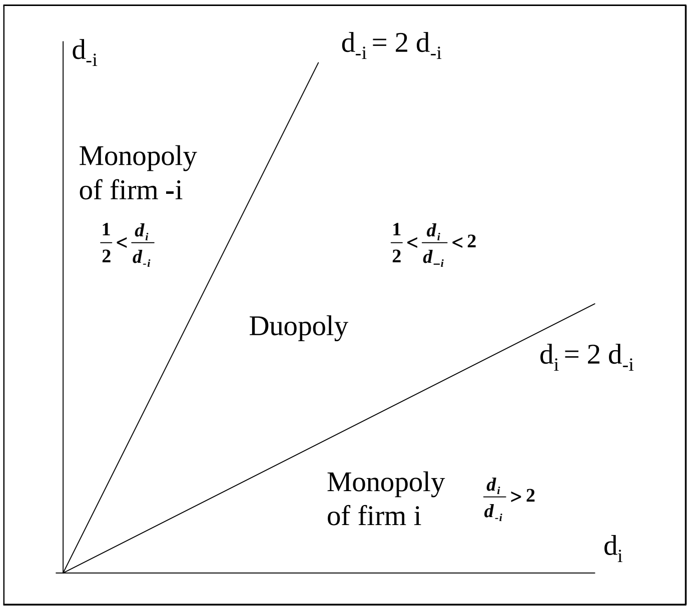

Figure 1: Representation of the different outcomes in the parameter space $\{(d_i, d_{-i})\}$. Firms with a large cost advantage obtain a de-facto monopoly. When the relative cost differences are small, both firms produce, and we obtain the duopoly outcome.

We already made a first classification of the Cournot game: by distinguishing the pure duopoly, and the de facto monopolist (See equation 9). We make a second classification between games where the normal outcome is physical feasible, (the *unconstrained* games) and where not (*constrained* games)[14]:

- When the transmission capacity k, and the production costs c_i are large, and the demand intercept a is small, ($D_i = \frac{a-c_i}{k}$ is small) the normal outcome with infinite transmission is possible. The game is *unconstrained.*
- When transmission capacity k, and production costs c_i are small, and demand intercept a is large, ($D_i = \frac{a-c_i}{k}$ is large) the normal Cournot equilibrium is physically no longer possible. The game is *constrained.*

By combining these two divisions we get 6 different outcomes (See Figure 2). To keep the derivations simple we will discuss only the *duopoly* case (shaded area in Figure 2).

[14]The term 'constrained' is only used to classify the games. It is premature to deduce from this classification the Nash equilibrium. When the game is unconstrained, the normal equilibrium is feasible. But this does not mean that it is the equilibrium of the game. This equilibrium depends upon the allocation rules for transmission. (See section 4)

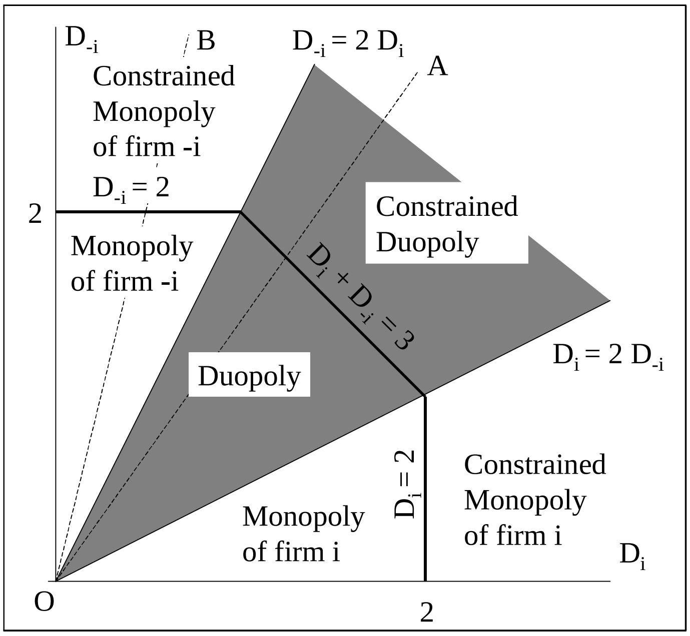

Figure 2: Classification of the parameter space $\vec{D}$. For small D_i the game is constrained, for large D_i the game is unconstrained.

Oren's (1997) model This model illustrates the problems you can expect when the role of the System Operator is not explicitly incorporated in the description of the game. In Oren's model, each generator maximizes his profit function π_i, taking the output of the other player as given.[15]

$$\max_{Q_i} \Pi_i = Q_i \cdot \Theta(Q) - C_i \cdot Q_i \tag{11}$$

subject to

$$\sum Q_i \leq 1 \tag{12}$$

The constraint specifies that total output must be smaller than the transmission capacity. In his formulation a transmission price is not included and the System Operator is of no importance. Oren solves these equations for the symmetric duopoly ($D_i = D_{-i} = D_{sym}$). He argues that because of symmetry the players choose the same output $Q_i = Q_{-i} = Q_{sym}$. When the game is not constrained ($D_{sym} < \frac{3}{2}$) they play the normal Cournot outcome. When the game is constrained the generators divide the transmission capacity k equally. Each generator thus chooses the following strategy:

$$Q_{sym} = \begin{cases} \frac{D_{sym}}{3} & \text{if } D_{sym} < \frac{3}{2} \\ \frac{1}{2} & \text{otherwise} \end{cases} \tag{13}$$

[15] We will restrict Oren's model to a case with two players with constant marginal costs and a linear demand function.

The price for the consumers is

$$\Theta = \Theta(2\, Q_{sym}) \tag{14}$$

In equilibrium the generators restrict their output to the transmission capacity k. Therefore the equilibrium transmission price is equal to zero

$$P = 0 \tag{15}$$

Oren states that this conclusion is consistent with the Coase Theorem:

> ...which supports the argument that in the absence of transaction costs ... bargaining will capture all the congestion rents.

Problems in Oren's model The equations 11 and 12 do not define a single-stage game in the classical sense, because the pay-off function Π_i is not defined on the whole strategy space of the players. Each player chooses his production $Q_i \in \mathbb{R}^+$. In a simultaneous game a player does not know the action taken by the other. Therefore, a player can not be sure that he restricts his output such that it satisfies equation 12.

The profit-function used by Oren is only defined on the triangle:

$$\{(Q_i, Q_{-i})\,|\sum Q_i\, \leq 1\} \tag{16}$$

It is however necessary to define the profit function Π_i upon the whole strategy space $\mathbb{R}^{+2}$. We have to define the behavior of the System Operator when generators want to use more than the available capacity.

The conclusion that all rent is captured by the generators is basically due to the fact that the System Operator is not present in the game. In reality, the System Operator is also a player and depending upon the specific market structure, the scarcity rent is captured by the generators or the System Operator. In the next subsection we describe the role of the System Operator. This allows us to construct a more complete set-up of the game.

3.4 The Role of the System Operator

From now on, we will make a distinction between the actual quantity of electricity produced Q_i and the quantity of electricity a generator would like to produce, his bid B_i. A generator is not allowed to bid more than the available capacity:

$$0 \leq B_i \leq 1 \tag{17}$$

Revelation game We can distinguish two roles for the System Operator: *allocating transmission capacity* (setting Q_i) and *setting transmission prices* (setting P). In principle, the price P and the quantity Q_i of each generator are set by the System Operator based upon the bids of the generators. These bids should include price information B_i^p and quantity information B_i. The quantity Q_i is allocated using an *allocation rule*

$$Q_i = Q_i(B_i, B_{-i}, B_i^p, B_{-i}^p) \tag{18}$$

and the transmission price P using a *price rule*

$$P = P(B_i, B_{-i}, B_i^p, B_{-i}^p) \tag{19}$$

See Figure 3.A.

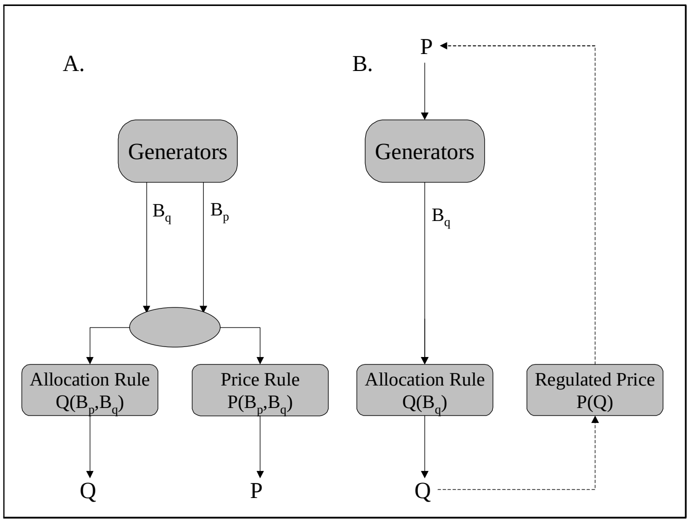

Figure 3: (A.) Normal revelation game. The generators give price and quantity information to the System Operator. Using this information the quantities Q_i are allocated, and the transmission price P is set. (B.) Exogenous transmission price. Generators bid quantity information to the System Operator, who allocates the transmission capacity.

In a Cournot game the players have one strategic value, the amount of electricity they want to produce (B_i). Each generator submits only a quantity bid B_i, but no price bid B_i^p. Because no price information is available inside the game, we suppose that the transmission price is set by the System Operator before the game starts. Transmission price is exogenous to the game, i.e. generators can not influence this transmission price.

$$P = P_{exog} \tag{20}$$

See Figure 3.B. Once the transmission price is set, the generators supply quantity bids B_i to the System Operator. He will allocate the transmission to the different generators according to an allocation rule:

$$Q_i = Q_i(B_i, B_{-i}). \tag{21}$$

In the remaining of this paper, we study two different allocation rules and two different price settings. In *section 4* the transmission price P is zero and in *section 5* the transmission price is regulated by the System Operator.

4 No Transmission Price

In this section the System Operator sets the transmission price $P = 0$. We will discuss a model with unfriendly allocation (a reformulation of Oren, 1997), and one with proportional allocation. The models are solved for the pure duopoly game ($2 > \frac{D_i}{D_{-i}} > \frac{1}{2}$).

4.1 Unfriendly Allocation

The System Operator allocates capacity according to the *unfriendly allocation rule*:

$$Q_i(B_i, B_{-i}) = \begin{cases} B_i & \text{if } B \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

where $B = \sum B_i$. When the sum of the bids is smaller than the transmission capacity, generators receive their bid. However, when demand exceeds the available capacity, the System Operator forbids the use of the line. We will proof that this game is equivalent to the formulation of Oren.

Generators make the following profit:

$$\Pi_i(B_i, B_{-i}) = \begin{cases} \Pi_i^{\infty}(B_i, B_{-i}) & \text{if } B \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

where $\Pi_i^{\infty}(B_i, B_{-i})$ is the profit of the normal game.[16]

Reaction Function Each generator restricts its output in order not to break the transmission constraint because this gives him zero profit. Generator i will play the normal reaction curve $B_i^{\infty}(B_{-i})$ as long as $B_i^{\infty}(B_{-i}) + B_{-i} \leq 1$. Otherwise, he will adjust its bid in order not to break the constraint $B_i(B_{-i}) = 1 - B_{-i}$. The reaction function becomes thus:

$$B_i(B_{-i}) = \begin{cases} \text{if } B_{-i} \leq 2 - D_i & \begin{cases} \text{if } B_{-i} > D_i & 0 \\ \text{if } B_{-i} < D_i & \frac{D_i - B_{-i}}{2} \end{cases} \\ \text{if } B_{-i} > 2 - D_i & 1 - B_{-i} \end{cases} \tag{24}$$

[16] The 'normal' game is the game with infinite transmission capacity. The profit of the normal game is $\Pi_i^{\infty}(B_i, B_{-i}) = (D_i - B)\, B_i$. The reaction function of generator i is $B_i^{\infty}(B_{-i}) = \begin{cases} 0 & \text{if } B_{-i} \geq D_i \\ \frac{D_i - B_{-i}}{2} & \text{if } B_{-i} < D_i \end{cases}$.

Figure 4 and Figure 5 show the reaction functions for two different set of parameters $\vec{D}$.

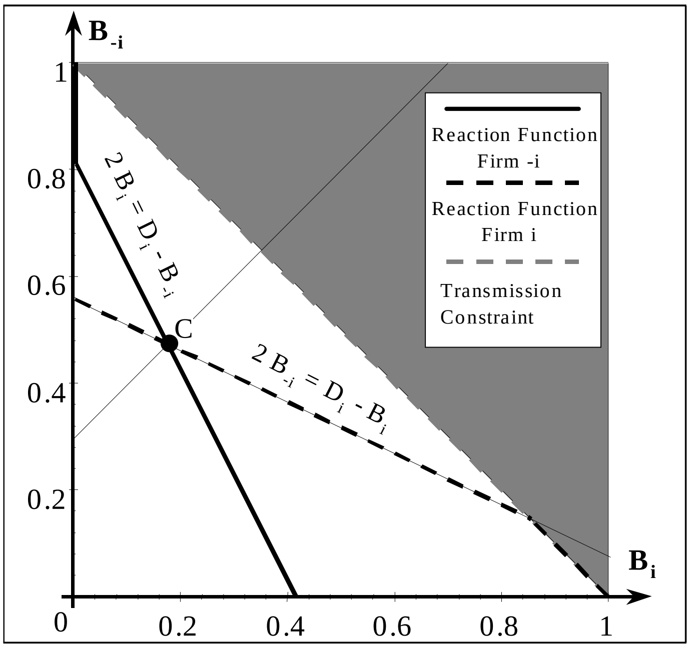

Figure 4: Case A: When the transmission capacity is big (Small D_i and D_{-i}) the normal Cournot Equilibrium is the equilibrium of the game. (Shown: $D_i = 0.84$; $D_{-i} = 1.12$)

Nash equilibrium The intersection of both reaction curves gives the set of Nash equilibria. We obtain the following types of Nash equilibria:

- For the unconstrained duopoly $(D_i + D_{-i} \leq 3)$ the players will play the 'normal' Cournot outcome. (See equation 9.)

$$B_i = \frac{2\,D_i - D_{-i}}{3} \tag{25}$$

  This outcome is represented in Figure 4

- For the constrained duopoly $(D_i + D_{-i} > 3)$ the following set is a Nash equilibrium:

$$\left\{(B_i, B_{-i}) \middle| \sum B_i = 1 \text{ and } \forall i : B_i \leq \tfrac{D_i - B_{-i}}{2}\right\} \tag{26}$$

  The first condition specifies that all transmission is used. The second condition states that each equilibrium must lie below the two normal reaction functions. (See Figure 5.)

The type of solution of this game in function of $\vec{D}$ is represented in Figure 6. For $\vec{D} \in A$ the normal equilibrium is played; in region B a line of equilibria exists.

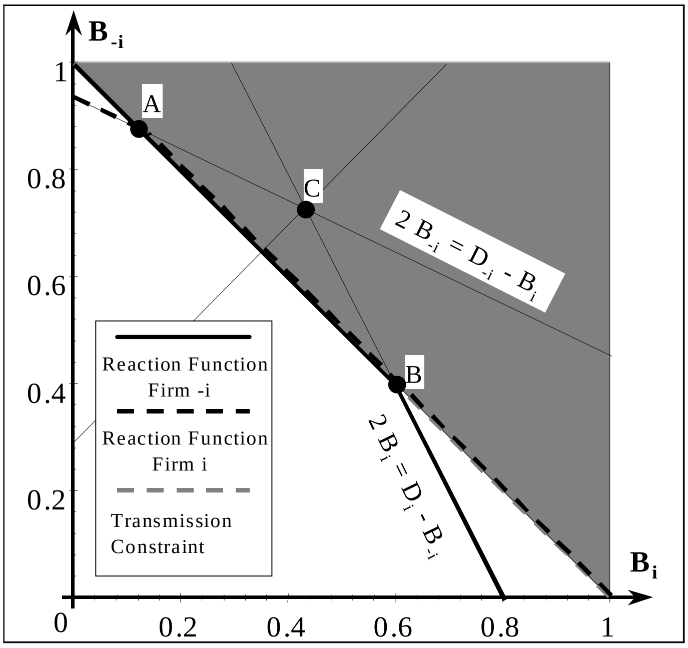

Figure 5: Case B: When the transmission capacity is small (big D_i and D_{-i}) a line of equilibria is present (the line A-B). Point C represents the normal Cournot equilibrium, which would have been played with a larger transmission capacity. (Shown: $D_i = 1.60$; $D_{-i} = 1.88$)

Proposition 1 *The equilibrium concept of Oren (defined by equations 11 and 12) implicitly assumes a System Operator who sets the transmission price $P = 0$, and uses the unfriendly allocation rule. (equation 22).*

The Nash equilibrium we found in this game can be specified as:

$$B_i^* = \arg \max_{0 \leq B_i \leq 1} \Pi_i(B_i, B_{-i}^*) \tag{27}$$

where the profit function $\Pi_i(B_i, B_{-i})$ is defined by equation 23. Oren's equilibrium concept (equations 11 and 12) can be written as:

$$B_i^{**} = \arg \max_{0 \leq B_i \leq 1 - B_{-i}^{**}} \Pi_i(B_i, B_{-i}^{**}) \tag{28}$$

Because all equilibria of equation 27 satisfy:

$$B_i^* + B_{-i}^* \leq 1 \tag{29}$$

they must also be a solution of equation 28. Since both concepts have the same equilibrium, they are equivalent formulations. □

Note 1 *Oren incorrectly assumes that the equilibrium is symmetric when the generators have the same costs and finds only one solution.*

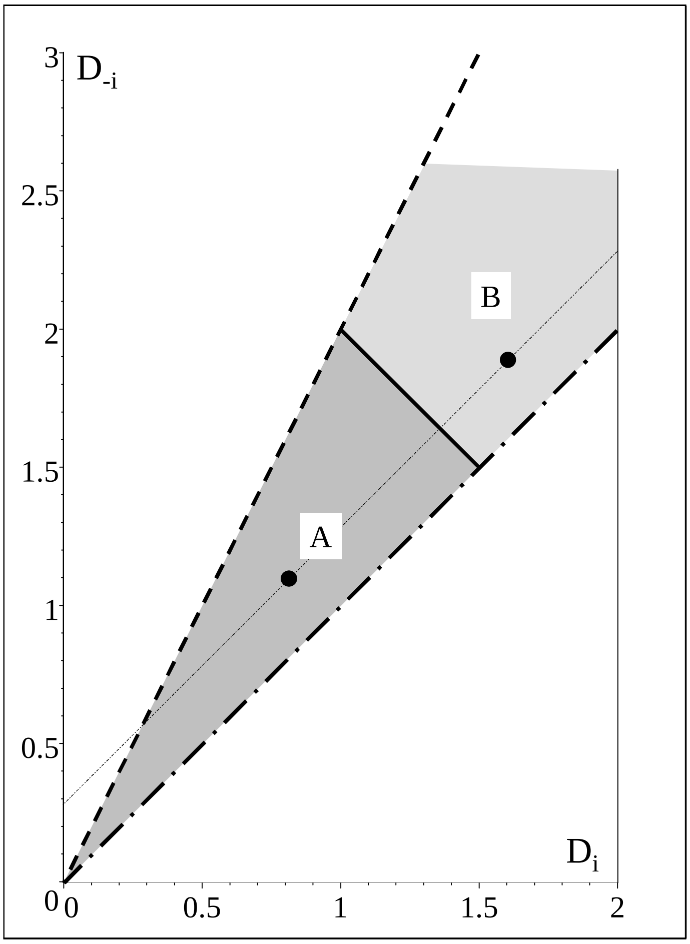

Figure 6: In region A, the normal Cournot outcome is the only Nash equilibrium. In region B a line of Nash Equilibria is present. The points in the figure refere to the parameters in figures 4 and 5. (Since the figure is symmetrical arround the 45 degrees line, only half of the figure is presented.)

4.2 Proportional Allocation

The unfriendly allocation rule of the previous subsection is not realistic, since it is political not implementable.[17] Therefore, we look for an alternative model.

Consider the proportional allocation rule:

$$Q_i(B_i, B_{-i}) = \begin{cases} B_i & \text{if } B < 1 \\ \frac{B_i}{B} & \text{otherwise} \end{cases} \tag{30}$$

where $B = \sum B_i$. Each generator receives his bid when there is sufficient transmission capacity. When demand exceeds the capacity available, the System Operator divides the quantity proportionally to the bids. Transmission price is still equal to zero ($P = 0$).

[17]Furthermore, if we would model the strategic behavior of the System Operator, forbidding the grid access would not be a credible threat as it would decrease welfare.

This gives each generator the following profit:

$$\Pi_i(B_i, B_{-i}) = \begin{cases} (D_i - B) \cdot B_i & \text{if } \; B < 1 \\ (D_i - 1) \cdot \frac{B_i}{B} & \text{otherwise} \end{cases} \tag{31}$$

Reaction function Basically, each firm decides to break (b) or not to break (nb) the constraint. We will derive the optimal bids (B_i^{nb} and B_i^b) in both cases. The firm will choose to break or not to break whichever gives him the highest profit.

- *Not Breaking* If firm i chooses not to break ($B_i < 1 - B_{-i}$) he maximizes Π_i by playing

$$B_i^{nb}(B_{-i}) = \begin{cases} \text{if } B_{-i} < 2 - D_i & \begin{cases} \text{if } B_{-i} > D_i & 0 \\ \text{if } B_{-i} < D_i & \frac{D_i - B_{-i}}{2} \end{cases} \\ \text{if } B_{-i} > 2 - D_i & \qquad 1 - B_{-i} \end{cases} \tag{32}$$

  and receives the following profit:

$$\Pi_i^{nb}(B_{-i}) = \begin{cases} \text{if } B_{-i} < 2 - D_i & \begin{cases} \text{if } B_{-i} > D_i & 0 \\ \text{if } B_{-i} < D_i & \frac{1}{4}(D_i - B_{-i})^2 \end{cases} \\ \text{if } B_{-i} > 2 - D_i & \qquad (D_i - 1) \cdot (1 - B_{-i}) \end{cases} \tag{33}$$

- *Breaking* If he chooses to break he maximizes his profit by playing $B_i^b(B_{-i}) = 1$ and he receives the following profit:

$$\Pi_i^b(B_i) = \frac{D_i - 1}{1 + B_{-i}} \tag{34}$$

A generator chooses breaking when this gives him the largest profit.

$$B_i(B_{-i}) = \begin{cases} B_i^b(B_{-i}) & \text{if } \; \Pi_i^b(B_i) > \Pi_i^{nb}(B_i) \\ B_i^{nb}(B_{-i}) & \text{otherwise} \end{cases} \tag{35}$$

The reaction function becomes (See appendix)

$$B_i(B_{-i}) = \begin{cases} \text{if } \;\; B_{-i} \leq B_{-i}^{cr} & \begin{cases} \text{if } B_{-i} > D_i & 0 \\ \text{if } B_{-i} < D_i & \frac{D_i - B_{-i}}{2} \end{cases} \\ \text{if } \;\; B_{-i} > B_{-i}^{cr} & \qquad 1 \end{cases} \tag{36}$$

with $B_{-i}^{cr}(D_i)$ the critical value for B_{-i} (See Figure 7).

$$B_{-i}^{cr} = \begin{cases} \text{if } D_i > 1 & \frac{1}{2}\left(D_i + 1 - \sqrt{D_i^2 + 6D_i - 7}\right) \\ \text{if } D_i < 1 & 1 \end{cases} \tag{37}$$

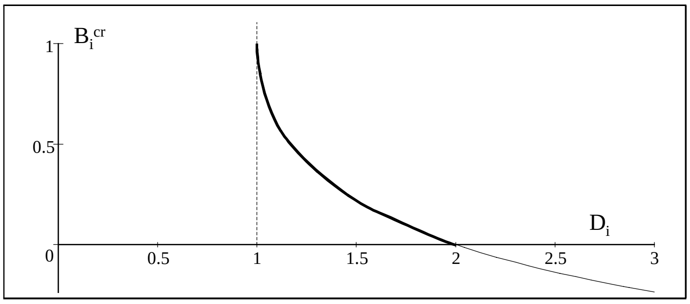

Figure 7: Critical values $B_i^{cr}(D_i)$ as function of the parameter D_i. For $D_i > 2$ the generator will always break. If $D_i < 1$ the generator will never break.

We know that B_{-i}^{cr} lies upon the normal reaction curve $B_i = \frac{D_i - B_{-i}^{cr}}{2}$. Using equation 37 and eliminating D_i we get:

$$B_i B_{-i}^{cr} + B_i + B_{-i}^{cr} = 1 \tag{38}$$

This equation tells us the size of B_i on the normal reaction curve when B_{-i}^{cr} reaches his critical value. This relation is symmetric in B_i and B_{-i} and does not depend upon D_i and D_{-i}. It is a locus of critical points in the $\vec{B}$-space. (See also Figures 8, 9 and 10.) The reaction functions 'jump' from breaking to non breaking when they cross this curve.

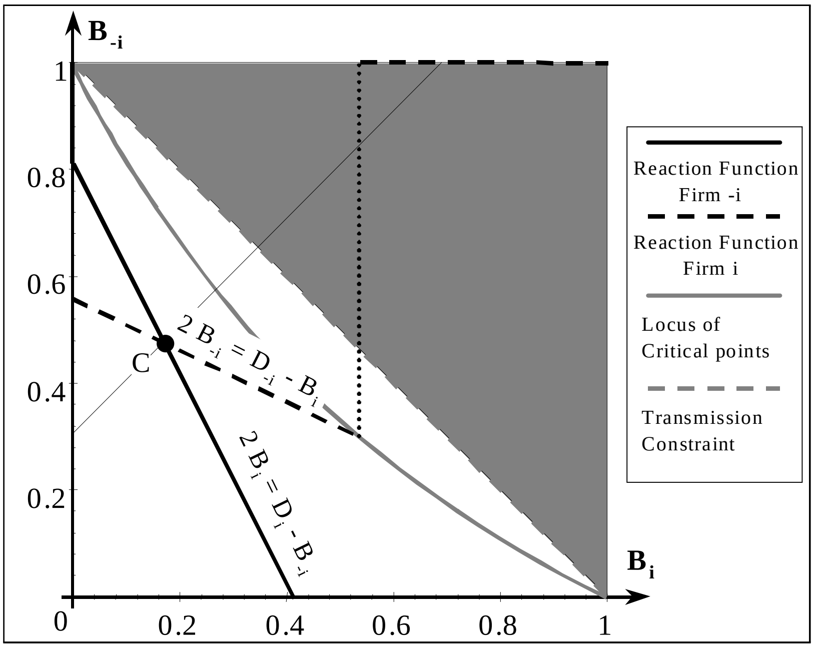

Figure 8: Case A: For large transmission capacities the normal Cournot outomce is the only equilibrium. (Shown: $D_i = 0.84$; $D_{-i} = 1.12$)

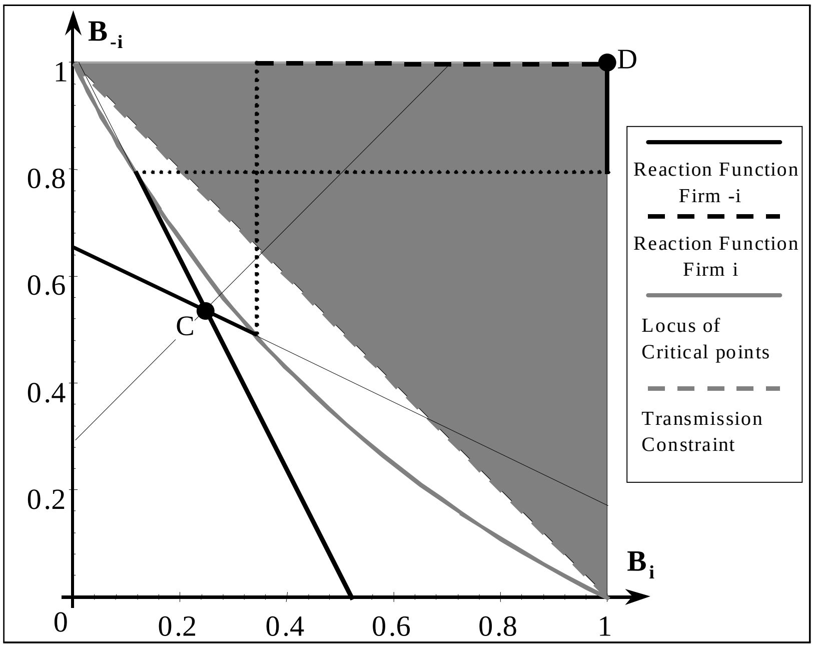

Figure 9: Case B: For intermediate capacities two equilibria are present: the normal cournot equilibrium, and the (break,break) equilibrium (Shown: $D_i = 1.04$; $D_{-i} = 1.32$).

Nash Equilibrium The Nash equilibrium can be found as the intersection of the two reaction functions. In the appendix we derive the number and the type of the equilibria as function of the parameter $\vec{D}$. The results are summarized in Figure 11. In region A (large transmission capacity) the normal Cournot equilibrium is the only equilibrium. (See Figure 8.) In region B (intermediate transmission capacity) two equilibria exists: the normal Cournot equilibrium and the (break, break) equilibrium where both players bid maximal capacity. (See Figure 9.) In region C (small transmission capacity) only the (break, break) equilibrium exists. (See Figure 10.)

Welfare analysis We will compare the welfare of the unfriendly allocation method, with the proportional allocation method.

For region A of Figure 11 both allocation rules are equally good, as they both result into the same equilibrium (the normal Cournot outcome). For region C_3 a line of equilibria exists under the unfriendly allocation rule. Dependent on which equilibrium the generators coordinate, welfare can be higher or lower.

In the regions C_1, C_2, and B the generators can play the (break,break) equilibrium under the proportional allocation, but they would play the normal Cournot equilibrium under the unfriendly allocation method.[18] This has an double effect on welfare: With proportional allocation, the

[18] Under the proportional allocation players can also co-ordinate on the normal Cournot outcome in region B, in that case proportional and unfriendly allocation are equally good.

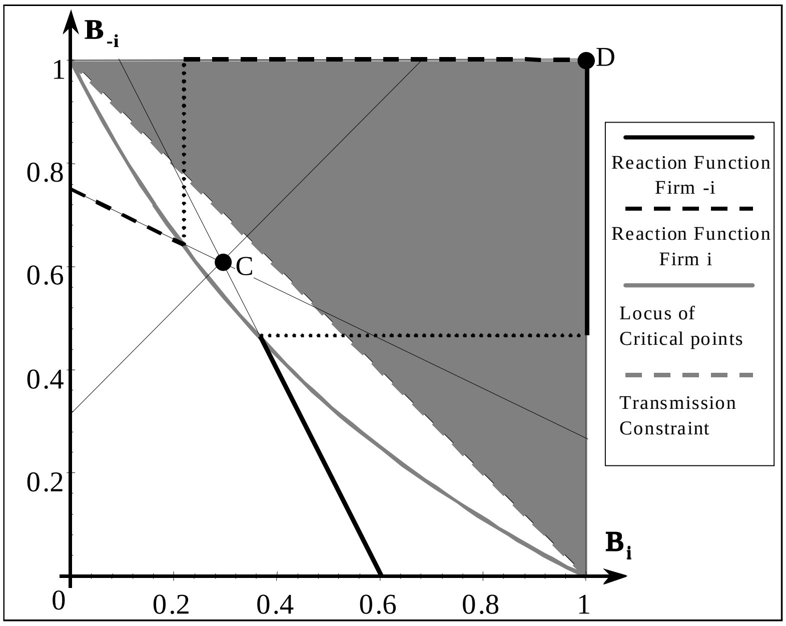

Figure 10: Case C: For small transmission capacities (break, break) is the only equilibrium. See point D. The normal Cournot outcome (point C) is technically feasible, but it is *not* an equilibrium. (Shown: $D_i = 1.22$; $D_{-i} = 1.50$)

generators end up producing more because they have an incentive to bid higher. As a consequence the electricity price is lower and consumers' surplus is larger. On the other hand, the production of electricity is less efficient as in the normal Cournot outcome the low cost generator produces more than the high cost generator, but produce the same quantity in the (break,break) equilibrium.

The net welfare effect depends thus on the cost difference between the two generators. If the two generators have similar costs ($\vec{D}$ close to the 45° line) the loss of efficiency in production is small. On the contrary, when the production costs of the two generators are very different, efficiency losses are high. In the Appendix we show that in region B_1 and in C_1 the proportional allocation is preferred, whereas in C_2 and B_2 the unfriendly allocation outperforms..

5 Regulated Transmission Price

In the previous section we looked at games where the transmission price P was set equal to zero. In this section we will look for an alternative assumption for the transmission price.

How will an exogenous transmission price influence the outcome of the game? In the presence of a transmission price, the profit of the generators becomes:

$$\Pi_i = (D_i - P - Q)\, Q_i. \tag{39}$$

The impact of the transmission price P is that of an apparent increase of the marginal cost of

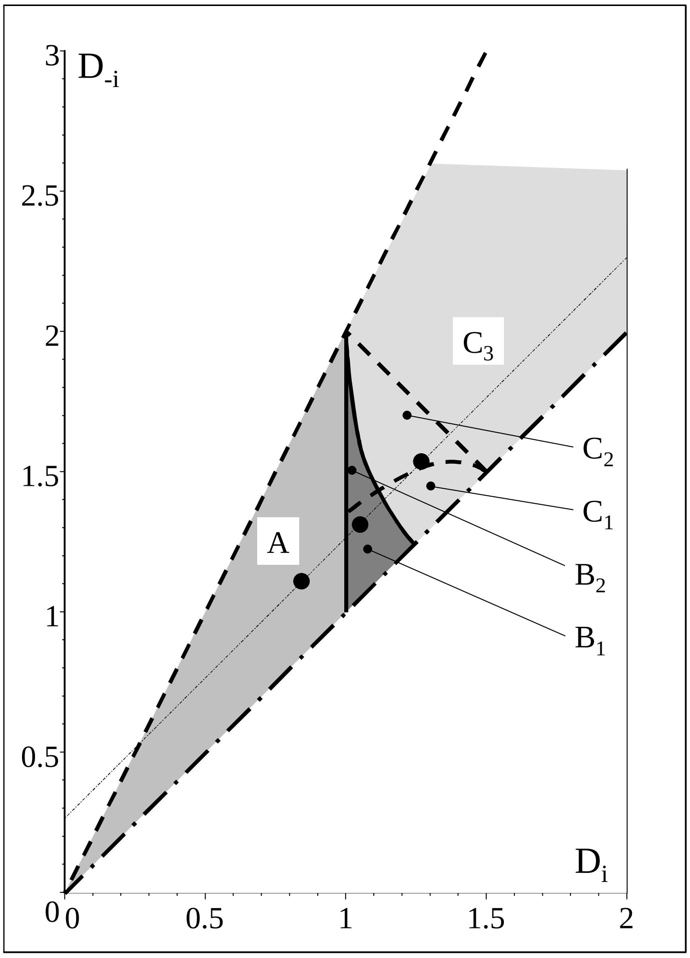

Figure 11: In region A, the normal Cournot game is the only Nash equilibrium. In region B both the normal, and the (break, break) equilibrium are possible. In region C only (break, break) is an equilibrium. (Since the figure is symmetrical arround the 45 degrees line, only half of the figure is presented.) The points in the figure refere to the examples in figures 8,9 and 10.

each generator. That is, the game with competitiveness parameter $\vec{D}$ and transmission price P is identical with a game with cost parameter $\vec{D}^* = \vec{D} - (P, P)$ without transmission price.[19] Therefore, we can apply the theory of the previous section on $\vec{D}^*$.

Still we did not define how we set the transmission price P. We choose the lowest possible transmission price P for which the generators play the normal Cournot equilibrium.

In the next two subsections we discuss the unfriendly and the proportional allocation rules.

[19] In the $\vec{D}$-space, the vector $\vec{D}$ shifts parallel with the 45° line, because the generators are charged the same transmission price. (This is exactly what hapened in the examples of the previous section. The different figures can thus also represent games with the same competitiveness parameter $\vec{D}$ but with different transmission prices P.)

5.1 Unfriendly Allocation

Let us return to the unfriendly allocation rule:

$$Q_i(B_i, B_{-i}) = \begin{cases} B_i & \text{if } B \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

The solution of this game with zero transmission price is presented in Figure 12. In region A we have one unique equilibrium: the normal Cournot outcome, in region B we find an interval of Nash equilibria.

Assume that we choose the lowest possible transmission price P for which the generators play the normal Cournot equilibrium. We shift down from an equilibrium D in the region B until we reach region A (See equilibrium D^* in the figure). The size of the horizontal and the vertical shift is equal to the transmission price P.

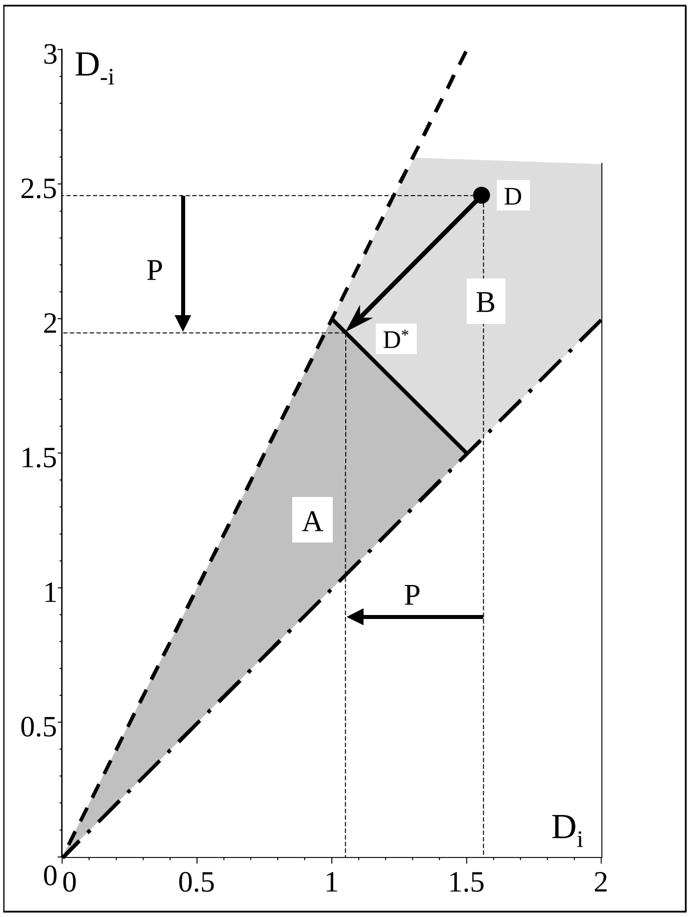

Figure 12: Equilibrium type of the game with the unfriendly System Operator. The competitiveness parameter of the generators is $\vec{D}$. The transmission price P is increased until the normal Cournot game becomes possible. The apparent competitiveness becomes $\vec{D}^*$.

Numerically, we look for a P such that $D_i^* + D_{-i}^* = 3$.[20] The transmission price than becomes:

$$P = \frac{1}{2}\left(D_i + D_{-i} - 3\right) \tag{41}$$

And in equilibrium the generators bid:

$$B_i = \frac{2\, D_i^* - D_{-i}^*}{3} \tag{42}$$

Using the definition of D_i^* and eliminating P we get:

$$B_i = \frac{1}{2}\left(D_i - D_{-i} + 1\right) \tag{43}$$

Link with the model of Smeers and Wei 1997. In this subsection we find the same equilibrium as Smeers and Wei (1997). However, they use a different formulation. In their model the generators are 'ignorant' and do not realize that there is a transmission constraint.

We will sketch the model of Smeers and Wei. Each generator maximizes his profit, taking the output of the other generator Q_{-i} and the transmission price P as given

$$\max_{Q_i} \Pi_i = \left(D_i - P - Q\right) Q_i. \tag{44}$$

Each firm then has a reaction function $Q_i = Q_i(D_i, P, Q_{-i})$. The intersection of these reaction functions is the Nash equilibrium: $Q_i^*(D_i, D_{-i}, P)$. The transmission price is set by the System Operator such that the output chosen by these 'ignorant' generators is equal to the constraint:

$$\sum Q_i^*(D_i, D_{-i}, P) = 1 \tag{45}$$

As the generators do not break the constraint, the System Operator does not need an allocation rule.

Proposition 2 *When the System Operator sets the transmission price optimal such that the transmission constraint is not broken, the model with ignorant generators, who do not recognize this constraint, is equivalent to the model with rational generators and an unfriendly allocation rule.*

A proof is trivial and would follow the same lines as proposition 1. □

The formulation of Smeers and Wei of the equilibrium relies on the assumption of the generators behaving 'ignorant'. Their equilibrium concept is not the Nash equilibrium of a game. By adding the unfriendly allocation rule their concept becomes the Nash equilibrium of a game.

[20] We assume that with the optimal transmission price P, we will not arrive in the de-facto monopoly case.

5.2 Proportional allocation

We repeat the same procedure for the proportional allocation rule. We increase the transmission price until the normal Nash equilibrium is feasible. The equilibrium shifts from D to the point D^*. See figure 13.

The transmission price P is equal to (See appendix):

$$P = \frac{1}{2}\left(D_{-i} + D_i + 6 - 3\sqrt{(D_{-i} - D_i)^2 + 8}\right) \tag{46}$$

Special Case: When the cost difference between the generators is small ($D_i \simeq D_{-i}$) the transmission price can be approximated by $P = -1.24 + \frac{D_i + D_{-1}}{2} + \frac{3}{16}(D_i - D_{-i})^2$ (Taylor Expansion).

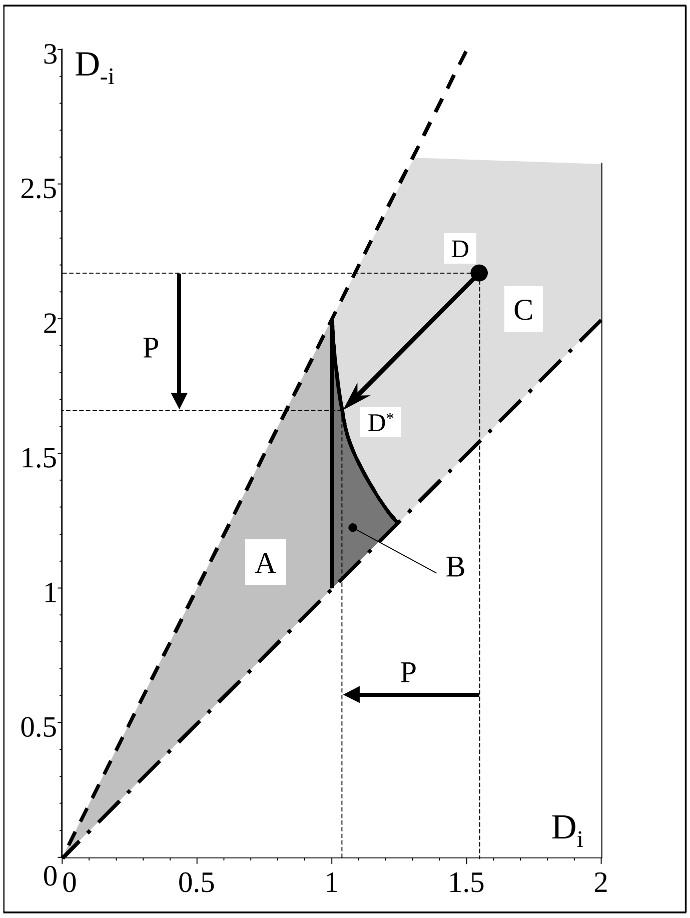

Figure 13: Equilibrium type of the game with proportional allocation. The competitiveness parameter of the generators is $\vec{D}$. The transmission price P is increased until the normal Cournot game becomes possible. The apparent competitiveness is then $\vec{D}^*$.

Comparing the two models with regulated transmission prices. In equilibrium, the transmission price is higher with proportional allocation than with unfriendly allocation. Under proportional allocation the transmission price equals

$$P \simeq -1.24 + \frac{D_i + D_{-1}}{2} + \frac{3}{16}(D_i - D_{-i})^2 \tag{47}$$

and with the unfriendly allocation rule

$$P = -1.5 + \frac{D_i + D_{-i}}{2} \tag{48}$$

A consequence of this higher transmission price is that in equilibrium, we do not use all the available transmission capacity with the proportional allocation rule. Furthermore, it is easy to show that welfare is lower under the proportional allocation rule.[21]

6 Conclusion

6.1 Conclusions of the paper

This paper models Cournot competition in an electricity market with transmission constraints.

One of the major difficulties in the electricity market is to assure that generators who independently decide about their output, will not produce more than the available transmission capacity. In practice, this problem has been solved by assigning certain decisions to a central authority, the System Operator. Identifying this structure, we explicitly model the role of the System Operator. The System Operator rations the transmission capacity available for the generators using two different instruments. In the long run, he can set the transmission price. In the short run, he can impose quantity restrictions on the output of the generators using an allocation rule.

By specifying the double role of the System Operator we develop a single framework that can be applied to the different models found in the literature. In this framework we study a duopoly of two generators with constant marginal costs. A transmission line with a fixed capacity connects these generators with consumers, who have a linear demand function. We normalize the game such that the transmission capacity equals one.

The first model we investigate is Oren (1997). As we do not agree with the assumptions he makes, we develop an alternative model. This model sheds some light on the controversy whether the generators receive or not receive all the congestion rent. In the last part of the paper we impose a regulated transmission price and look how the equilibrium changes.

[21] Countrary effects work: with proportional allocation the tax revenue inceases but consumers surplus and producers surplus decrease (despite the fact that the production mix becomes more efficient because the low cost firm produces relatively more).

Controversy: do the generators receive all congestion rent. Joskow and Tirole (1998) argue that the transmission price is equal to zero as long as not all the capacity is used by the generators.[22] Building upon the models of Stoft (1997, 1999) and Oren (1997), where the generators restrict their output to the available capacity, they conclude that the transmission price is zero and that generators receive all the scarcity rent. We show that these results are due to the specific and implausible role that Oren and Stoft assign to the System Operator.

1. It is not true that the generators will always restrict their generation to the available transmission capacity. In a variation of Oren's model, the System Operator sets transmission price equal to zero and uses a proportional allocation rule. In this game, the generators do not restrict their bids for the use of the transmission line.

2. In our framework we disentangle the price setting and the allocation of the transmission capacity. As a consequence, there is no direct relation between the use of transmission capacity and the resulting transmission price. Even when the whole capacity is used, the transmission price can be zero.

3. We reject the conclusion that the congestion rent is always captured by the generators. In our framework this is only valid when the System Operator sets a transmission price equal to zero.

6.2 Further Research

We see three major topics for further research: endogenizing the transmission price, the study of the decentralized market, and the investigation of the optimal allocation and price rule.

Price Setting The transmission prices are exogenous to our model. Further research should try to endogenize this transmission price in the model.

A first option is to consider a repeated game where the System Operator and the generators interact regularly. The System Operator learns the costs of the generators by observing their actions in the past. When the full capacity is demanded by the generators, they signal that the transmission capacity is scarce. The System Operator will then increase the transmission prices in the next round.

The generators will signal when the benefit of using more of the transmission line, exceeds the disadvantage of a higher future transmission price. Under certain conditions it is possible that

[22] Joslow and Tirole study the value of financial transmission rights. The value of such a right is proportional to the transmission price.

the players are not willing to signal the scarcity of the line. In that case the transmission price is zero, and the generators receive all the scarcity rent.

An interesting question is whether the proportional allocation rule and the unfriendly allocation rule would give a similar result. Under the unfriendly allocation rule the generators get only a small benefit when they signal. Transmission prices could become very low. Under the proportional allocation rule a player gets a larger benefit when he signals. Transmission prices can become higher.

Another option is to include price bids. A realistic model of the electricity market should include both price and quantity bids by the players. The System Operator then sets the transmission price based upon this price and quantity bids.

Decentralized Market For this issue, we would consider a three stage game. In the first stage, players submit to the System Operator how much they would like to produce. In the second stage, the System Operator allocates the available capacity to the generators, using an allocation rule. In the centralized market, the generators are obliged to use the quantity they obtained from the System Operator in the final stage.

In a decentralized market, the generators decide in the final stage how much they will use of the transmission capacity they received, but they have to pay the transmission price for the full quantity they obtained.

Optimal Rules A complete welfare analysis of the different allocation rules was not conducted in this paper, but remains an option for further research. Which rule outperforms another, depends upon the cost parameters of the two firms. In order to formulate a policy recommendation, we need to know the distribution of the cost structure of the firms.

We do not use the theory of mechanism design to define the optimal allocation rule that maximizes welfare. This as well, could be an area of further research.

A Externalities in electricity networks.

Transmission constraints influence the market power in two different ways: by splitting up the market into two sub markets, and by the creation of competition for a scarce transmission capacity. (See also Footnote 12). These two effects can be recognized in a model with only one transmission line and two generators.

Splitting the market When we place the generators on different sides of the line, transmission of electricity creates *a positive externality* for the generators. A generator transporting electricity, increases the available capacity for the other generator, because only the net transmission flow determines if a line is congested or not.[23] This game has been studied by Stoft (1997) using Cournot behavior. He demonstrates that for small transmission capacities, generators will not enter the other market, as they would "open the door" for their competitor. For intermediate capacities generators play a mixed strategy, and for large transmission capacities the transmission constraint does not influence the market equilibrium and they play the normal Cournot outcome.

Competition of scarce capacity When we place the generators on the same side of the line they are rivals for the transmission capacity and for the electricity market. When a generator uses the transmission line, he creates *a negative externality* because he decreases the available capacity for his competitor. This is the setting used in this paper.

B Proofs

In this subsection we discuss some of the longer derivations. First we derive the reaction function and the Nash equilibrium and the regulated transmission price in a game with proportional allocation rule. Then we compare the welfare of the (Break, Break) equilibrium with the normal Cournot equilibrium.

B.1 Reaction function

The generator will play non-breaking when it is profitable for him:

$$F(B_{-i}) \equiv \Pi_i^{nb}(B_{-i}) - \Pi_i^{b}(B_{-i}) > 0.$$

with $F(B_{-i})$ the following function defined on three regions R_1, R_2, and R_3:

$$F(B_{-i}) = \begin{cases} \frac{1-D_i}{1+B_{-i}} & \text{if} \quad (D_i, B_{-i}) \in R_1 \\ \frac{1}{4}(D_i - B_{-i})^2 + \frac{1-D_i}{1+B_{-i}} & \text{if} \quad (D_i, B_{-i}) \in R_2 \\ (D_i - 1)\cdot(1-B_{-i})^2 + \frac{1-D_i}{1+B_{-i}} & \text{if} \quad (D_i, B_{-i}) \in R_3 \end{cases} \tag{49}$$

with $R_1, R_2, R_3 \subset \mathbb{R}^+ \times [0,1]$:

$$R_1 = \{(D_i, B_{-i}) \mid B_{-i} < 2 - D_i \text{ and } B_{-i} > D_i\} \tag{50}$$

$$R_2 = \{(D_i, B_{-i}) \mid B_{-i} < 2 - D_i \text{ and } B_{-i} < D_i\} \tag{51}$$

$$R_3 = \{(D_i, B_{-i}) \mid B_{-i} > 2 - D_i \ \} \tag{52}$$

[23] Two electricity flows in opposite directions cancel each other out. Physically only the net flow of electricity is transported. Suppose for example that a generator would like to send 100 MW from A to B, another generator 50 MW from B to A and that the capacity of the line is 70 MW. As the net electricity flow is only 50 MW from A to B, this flow is physical feasible.

Solving for the sign of $F(B_{-i};D_i)$ the following reaction function results:

$$B_i(B_{-i}) = \begin{cases} B_i^{nb}(B_{-i}) & \text{for } (D_i, B_{-i}) \in R_1 \\ \begin{cases} B_i^{nb}(B_{-i}) & \text{if } B_{-i} < B_{-i}^3 \\ B_i^{b}(B_{-i}) & \text{if } B_{-i} > B_{-i}^3 \end{cases} & \text{for } (D_i, B_{-i}) \in R_2 \\ B_i^{b}(B_{-i}) & \text{for } (D_i, B_{-i}) \in R_3 \end{cases} \tag{53}$$

with[24]:

$$B_{-i}^3 = \frac{1}{2}\left(D_i + 1 - \sqrt{D_i^2 + 6D_i - 7}\right) \tag{54}$$

This is equivalent with equation 36 and 37.

B.2 Nash Equilibrium

As a first step we search Nash Equilibria where one of the players breaks. Suppose that player $-i$ breaks ($B_{-i} = 1$). This is a Nash Equilibrium if $B_{-i}(B_i(1)) = 1$. The optimal reaction of player i to $B_{-i} = 1$:

$$B_i(1) = \begin{cases} 1 & \text{if } D_i > 1 \\ 0 & \text{if } D_i < 1 \end{cases} \tag{55}$$

The optimal action of player $-i$ to this reaction is:

$$B_{-i}(B_i(1)) = \begin{cases} 1 & \text{if } D_i > 1 \text{and } D_{-i} > 1 \\ 0 & \text{if } D_i > 1 \text{ and } D\_i < 1 \\ 1 & \text{if } D_i < 1 \text{and } D_{-i} > 2 \\ \frac{D_i - B_{-i}}{2} & \text{if } D_i < 1 \text{and } D\_i < 2 \end{cases} \tag{56}$$

Define $S_1 = \left\{\vec{D}\,|D_i > 1 \text{ and } D_{-i} > 1\right\}$.Observing equation 56 and noting that we study the pure monopoly case $\left(\frac{1}{2} < \frac{D_i}{D_{-i}} < 2\right)$ we find the following Corollaries:

Corollary 3 *When $\vec{D} \in S_1$ playing $\vec{B}^* = (1,1)$ is a Nash Equilibrium..*

Corollary 4 *When (B_i^*, B_{-i}^*) is a Nash Equilibrium and $B_i^* = 1 \Rightarrow \vec{D} \in S$ and $\vec{B}^* = (1,1)$.*

The last corollary implies that for all remaining Nash Equilibria both players do not break ($B_i^* < B_i^{cr}$ and $B_{-i}^* < B_{-i}^{cr}$). But as long as they do not break, the reactions functions of the proportional game and the normal Cournot game are the same. It is thus sufficient to inspect whether the

[24] The function $\frac{1}{4}(D_i - B_{-i})^2 + \frac{1-D_i}{1+B_{-i}}$ has three roots: $B_{-i}^1 = 2 - D_i$, $B_{-i}^2 = \frac{1}{2}\left(D_i + 1 + \sqrt{D_i^2 + 6D_i - 7}\right)$ and $B_{-i}^3 = \frac{1}{2}\left(D_i + 1 - \sqrt{D_i^2 + 6D_i - 7}\right)$.Only the third root is relevant (i.e. $(D_i, B_{-i}^3) \in R_3$) .

intersection of the reaction functions of the normal Cournot game $(B_i^*, B_{-i}^*) = (\frac{2\,D_i - D_{-i}}{3}, \frac{2\,D_{-i} - D_i}{3})$ satisfies $B_i^* < B_i^{cr}$ and $B_i^* < B_i^{cr}$.

These last two inequalities can be combined to (See eq. 38):

$$B_i^* B_{-i}^* + B_i^* + B_{-i}^* < 1 \tag{57}$$

Substituting D_i and D_{-i} for B_i^* and B_{-i}^*, we can conclude that for all parameters $\vec{D} \in S_2$ with

$$S_2 = \left\{ \vec{D} \,|\, 5 D_i D_{-i} - 2(D_i^2 + D_{-i}^2) + 3(D_i + D_{-i}) - 9 < 0 \right\} \tag{58}$$

that the equilibrium of the normal Cournot game is an equilibrium of the game with proportional allocation.

Summarizing we find three different regions:

- Region $A = S_1 \backslash S_2$: with the normal Cournot equilibrium,
- Region $B = S_1 \cap S_2$: with both normal Cournot outcome and the (Break,Break) equilibrium,
- and region $C = S_2 \backslash S_1$: with the (Break,Break) equilibrium.

B.3 Exogenous Transmission price.

The transmission price P is set such that (See Equation 58):

$$5 D_i^* D_{-i}^* - 2(D_i^{*2} + D_{-i}^{*2}) + 3(D_i^* + D_{-i}^*) - 9 = 0 \tag{59}$$

Substituting the definition of D_i^*, we find that P is the root of:

$$P^2 - (D_{-i} + D_i + 6)\, P - 2(D_{-i}^2 + D_i^2) + 5 D_{-i} D_i + 3(D_i + D_{-i}) - 9 \tag{60}$$

and thus is equal to:

$$P = \frac{1}{2}\left(D_{-i} + D_i + 6 - 3\sqrt{(D_{-i} - D_i)^2 + 8} \right) \tag{61}$$

B.4 Welfare Comparison

We compare the welfare of the normal Cournot Equilibrium with the (Break,Break) equilibrium Total welfare (w) is the sum of consumers surplus $\frac{1}{2}q^2$ and producers surplus $(a-q)q - c_i q_i - c_{-i} q_{-i}$. Normalized welfare ($W = \frac{w}{k^2}$) becomes:

$$W = D_i Q_i + D_{-i} Q_{-i} - \frac{1}{2} Q^2 \tag{62}$$

Total welfare in the normal Cournot equilibrium ($Q_i = \frac{2\,D_i - D_{-i}}{3}$ and $Q_{-i} = \frac{2\,D_{-i} - D_i}{3}$) is thus:

$$W^n(D_i, D_{-i}) = \frac{1}{18}\left[11 D_i^2 - 14 D_i D_{-i} + 11 D_{-i}^2 \right] \tag{63}$$

and in the proportional game ($Q_i = Q_{-i} = \frac{1}{2}$):

$$W^p(D_i; D_{-i}) = \frac{1}{2}(D_1 + D_2 - 1) \tag{64}$$

In the set $S_3 = \left\{ \vec{D} \,\middle|\, W^n(D_i, D_{-i}) - W^P(D_i, D_{-i}) > 0 \right\}$ the normal Cournot outcome gives a higher welfare than the proportional allocation. The regions $B_{1,}B_2, C_1$ and C_2 drawn in Figure 11 are defined as $B_1 = B \cap S_3$; $C_1 = C \cap S_3$; $B_2 = B \backslash S_3$ and $C_2 = C \backslash S_3$.

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