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Parental Preference, Heterogeneity, and Inequality Within the Household.

by

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Abstract: Given siblings are heterogeneous in their genetic endowment, we strive to model the implication of parental behaviour for inequality among siblings. Children are posited as passive recipients of human capital inputs from their parents. Unless there is a significant difference in genetic endowment among siblings, parents tend to reinforce the impact of genetic endowment by investing more in those who are likely to earn higher any ways. However, introducing uncertainty in earnings dilutes the above results. Parents' decision regarding the allocation of human capital resources among siblings takes a different tune when the process involves the distribution of physical capital as well.

Parental Preference, Heterogeneity, and Inequality Within the Household.

In this paper we attempt to model households' decision regarding investment in the human capital of children. Particularly, the focus is on the issue of the allocation of parental resources among children of different abilities. In this line, a number of questions are taken up, including; whether parents tend to invest more on a particular type of children? What will happen to the optimal level of investment in each child if the proportion of one type of children changes? What would be the effect of changes (temporary shocks or otherwise) in prices and income on the level of investment in children? Do these changes affect different types of children asymmetrically? How would the answers to the above questions be altered if the household were (un)able to make a direct transfer in terms of physical capital? Or able to borrow against their own or that of their children's future income?

While examining allocations among sibling, household size is considered as given. By doing so we follow the literature by implicitly assuming that unobserved heterogeneity affecting the number of children does not determine the distribution of resources among them (see Behrman, 1997 for the development of the literature towards integrating these two aspects of household decisions).

To focus attention on the distribution of resources among children, the total resource devoted to children is also considered as given. However, we note that imperfections in the capital market imply that these resources might vary over time.

Moreover the motivation of parents in investing in their children is not addressed. Parents might have a range of motivations for investing in their children. It could be out of their altruistic feelings so that it is up to the children to provide old age support for the parents. Given that children make transfers, the intention might be to increase the amount of transfer that parents expect from their children when the former gets old (see Berneim, Shleifer and Summers, 1985; Cox and Ranks, 1992; Raut and Tran, 1997). We rather prefer not to go in detail into this. However, we made an effort to see whether the motivation of parents has any significant say in the allocation decision.

Whether these decisions of allocation within the household are made co-operatively or through some kind of bargaining, though has an important policy implication, is not addressed as detail as the issue might call for. Particularly the interest is on the effect of endowment on the distribution of human capital resources within the household. That is to see whether the allocation respond to endowment differences, so that in allocating human capital resources among siblings parents reinforce or compensate the *exogenous* endowment differences among siblings.

The Literature: Though individual genetic endowments explain a good part of the variations in the human capital outcomes and thereby in the earnings of siblings, parental preference in distributing human capital resources among siblings is unlikely to leave these outcomes unaffected. Prices and endowments, together with the preference of parents, play their part in the determination of the level of human capital invested in a particular type of children (boy versus girl, first born versus the rest, etc.). Short of these general relationships, the specific shares of genetic versus acquired endowments in the human capital outcome of individuals is not that apparent.

Parents might be inclined to temper the inequality that might arise among their children due to their endowed and/or acquired human capital. In doing so they find themselves engaged in trading off productivity for equity. The trade-off refers to the fact that productivity calls for investing in those with high marginal returns while the concern to equity calls for compensating the less able, which in effect means reversing the course implied by productivity. However, it is also perfectly plausible that parents show unequal concern for their children, so that they give more weight to the outcome of one (e.g. male, older children, etc.) over the other. Behrman (1988) found that during the lean season where food is in short supply, Indian parents are less concerned about equity and allocate more to the more productive ones (see also Behrman and Taubman (1986) and Behrman *et al*, 1986).

Rosenzweig and Wolpin's (1988) work also reveals how the timing of births and allocation of resources to children vary with both child specific as well as family endowments. They presented their analysis in a dynamic model framework where child bearing is a sequential decision. The traits of a child is known only after its birth so that the endowments of all siblings born prior to it will affect parental decisions in allocating resources to the new born. However, since the parents do not know endowments of the yet unborn child, the endowment of the latter has no affect on the allocation of resources to the already existing children. They use this sequential nature of the process to identify their model. The authors were able to compute the endowment of each child rather than resorting to data on twins, which is widely used in the literature for controlling heterogeneity at the cost of employing a non-randomly selected sample.

Focusing on the access to human capital resources, Behrman *et al* (1989) studied the variation in the human capital outcomes of siblings in small and large families. They identified two sources of variations: the preference displacement effect and the price effect. They posited that in their allocation decision, parents are willing to trade off between productivity and equity only that amount which is above the *subsistence level.* Increase in the number of children tends to crowd household resources, which is a movement towards the *subsistence level.* Such movement towards the *subsistence level* makes parents less and less willing to trade off. This effect which works towards equalising the outcomes among siblings, they call this the preference displacement effect. The price effects exist because of the differences in the price of human capital among siblings. Increase in the number of children will increase the probability that the price of human capital vary substantially among siblings. This is because large number of children are likely to include children born to older parents which are likely to have birth defects. Moreover households with large number of children are likely to have children of varied quality where some, for example, get qualified for scholarship. Also with the presence of liquidity constraints, households with large number of children are likely to have them at very different points of the life cycle income path of the parents. As the result their children face more varied budget constraints than households with fewer children. Depending on the relative importance of the two effects, increase in the size of the household either tend to equalise or diverse the human capital outcome among its members.

Pitt *et al* (1990) examined the relationship between endowments and the allocation of food and work effort among household members. Depending on the sex and age of individuals, the allocations of food and work effort depend up on the endowment of members. Endowment is posited to positively affect the health of an individual, the quantity of food allocated to him and his work effort. However, work effort expended decreases the health of the individual, while the food resources increase his health. In general, the net effect of endowment is positive but less than one. They interpreted this result as households compensate for endowment differences among members.

Using the variance component method on a sample of twins (both identical and fraternal), Behrman *et al* (1994, 1995) reported that individual health as well as earnings endowments constitute significant parts of total variation in health (47% of the total variance in measured BMI) and earnings (27% of the total variance in log earnings). And parents reinforce endowment difference in their allocation of both schooling and health resources. Such reinforcement in the allocation of schooling resources resulted in 80% increase from what would the difference in earnings of siblings' have been in the absence of the reinforcement.

In section 2, we motivate our formulation by presenting a brief overview of the models of human capital investment. Section 3 contains the basic models where two regimes are considered. The first is a case where investment in children is made in order to increase the amount of transfer that children make to their parents when the latter retire. The second case assumes that parents invest in their children's human as well as physical capital solely on altruistic grounds. In section 4, uncertainty and reverse altruism are introduced. The last section concludes.

Section 2. Competing models of investment in children and intrahousehold allocation

While no attempt was made to exhaust models of parental investment in children, two broad categories, which are relevant to the issues raised in the paper, are presented. In all cases we assume inactive children who are passive recipients in the sense that they do not have an independent decision making role (for cases where children assume an active role, see Becker 1974, 1991; Hirshleifer, 1977,1985; Bergstrom, 1989 and Bruce and Waldman, 1990).

2.1 The preference models

Generally, in this class of models the preference of the parents' play an important role in determining the level of investment in children. What characterises these models is the explicit consideration of parents' concern about the distributional consequences of their decision. In what follows two branches of the preference model are presented.

2.2.1 The wealth model

According to this model parents are concerned only with the total income (earned plus bequest) that their children receive, irrespective of its sources (Adams, 1980; Tomes, 1981; Behrman, Pollak and Taubman, 1982, 1995; Tuabman, 1996; Behrman, 1997). Thus, the concern is on the sum of bequests[1], which is physical capital, and earnings, attributed to human capital, that ultimately goes to their children. To illustrate, consider a parental utility function that is weakly separable in its arguments:

$U= u \{ a(f^1(k_1),b_1; f^2(k_2),b_2;\ldots\ldots;f^n(k_n),b_n), c\}$ where i= 1…….n, represent number of children and k_i, is expenditures on inputs affecting the human capital of child i, and $f^i(k)$ is earnings function of child i. b_i is bequest to child i , a(.) represent the substitutability and c is parental consumption. The assumption of weak separability enables one to examine the allocation of resources to children independently from parental consumption. The implication is that if a given distribution of earnings (bequests) is preferred at a certain level of parental consumption, it is preferred at all levels of parental consumption.

Since the concern is on the sum of bequests and earnings that goes to children, in the wealth model the a (.) function will take the following form:

$a(f^1(k_1)+b_1; f^2(k_2)+b_2;\ldots\ldots;f^n(k_n)+b_n)$.

An important feature of this representation is that parents can and are willing to substitute bequests for expenditures on human capital affecting earnings and vice versa,

[1] Bequest refers to bequests plus inter vivo gifts.

in order to achieve a certain level of wealth. Looking at the sequence of the budgetary decisions further illuminates this point. Parents first decide how much resource to allocate for each child, then for each child its share is divided between human and physical capital (bequests) investments. Denoting Y to be an all purpose commodity that parents are endowed with, it is split between parental (e.g. consumption) (Y^p) and children (Y^c) use. The first step involves dividing Y^c among children so that each child gets Y^{c_i}. The second step is allocating each child's share (i.e. Y^{c_i}) between k_i, and b_i ; where $k_i+b_i = Y^{c_i}$, $\sum_{i=1}^{n} Y^{c_i} = Y^c$ and $Y = Y^c + Y^p$.

The division of Y^{c_i} between b_i and k_i is done in such a way that the wealth of the child is maximised. It is important to note that when all b_i's are positive (which occurs when parents provide enough Y^{c_i} s) , the level of k_i is independent of the preference of parents over the distribution of wealth among children. This is because the distributional (equity) concern of parents is addressed through the allocation of bequests. Hence the latter can be used to compensate for lower level of human capital so that the allocation of k is solely made on efficiency grounds which requires the equalisation of marginal returns across children.

A testable implication of this model is that given equal concern and positive b_i's for all children, in a typical household where children differ in their endowment[2], human capitals as well as bequests are divided unequally among siblings. Also, when bequests are positive for all siblings, return from human capital is equal to returns from other assets.

2.2.2 The separable earnings-bequest model (EB model)

The basic assumption here is the separability of earnings and bequests in the utility function so that the a(.) function in the wealth model will now take the following form: $a(.) = a\{ a^k (k_1, \dots, k_n), a^b (b_1, \dots, b_n)\}$ so that once resources are grossly divided between bequests and human capital investments, preference over the distribution of bequests among children is independent from the distribution of human capital among children (Behrman, Pollak and Taubman 1982, 1995). Put differently, it means that physical capital can not be used to compensate for shortfalls in human capital investments. Such separation of earnings and transfers in the parental utility function is rationalised on the grounds that parents might value a unit of transfer differently from a unit of earned income.

Here the sequence of the budgetary decision is that, first parents decide on the amount allocated for bequests and for expenditure on human capital. The second stage involves two allocation decisions: allocation of bequests and allocation of expenditures affecting human capital among children. Instead of dividing Y^c into Y^{c_i} 's, the former is divided into Y^b (resources devoted to bequests) and Y^h (resources devoted to expenditures on human capital). Then Y^b is divided among children (i.e. b_i) and similarly Y^h is also split among children (i.e. k_i) so that $\sum_{i=1}^{n} b_i = Y^b$ and $\sum_{i=1}^{n} k_i = Y^h$.

<u>Inequality among children</u>

Whenever there are differences in marginal returns among children emanating from either their inherited endowments or environmentally acquired features (henceforth we use child types to refer to these differences), depending on their preference, parents are bound to pursue either a compensatory or a reinforcing or a neutral strategy. In the former case parents allocate more resource to the low return type children. A reinforcing strategy occurs when more human capital is invested in the high return type children. Symmetric treatment of all children irrespective of their type implies a neutral strategy.

In the EB model, equal concern for all children implies that investment in physical capital are divided equally among all children. However, investment in human capital depends up on the price of human capital and child specific endowments in addition to the preference of parents towards inequality among siblings, which put different types of children in asymmetric position. Thus, if child type and returns from human capital investment are positively related, and if parents are more concerned about productivity and less about equity, then better endowed children gets more human capital than the rest. It follows that in this model both preference of parents and the earnings function (through the relationship between endowment and rate of return) spell whether parents

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[2] We take up the concept of endowments later and discuss it in some concrete sense.

pursue a reinforcing, neutral or a compensatory strategy. Therefore, generally determining the type of strategy pursued, is an empirically issue.

However in the wealth model, given that all children receive positive bequests and parents have equal concern to all children, only the earnings function has a part to take in the determination of the type of strategy parents follow in their investment decision[3]. That is only the relationship between child type and rate of return from human capital in the earnings function determines the type of strategy parents follow[4]. This is due to the two-stage nature of the budgetary decision. The first stage involves a decision about dividing the total into each child's share. The second stage is about splitting the child's share into physical (bequest) and human capital investments. It is straightforward to see that it is efficient to equalise the rates of returns across these options. For a given child, increasing the investment on the type of capital whose marginal return is higher increases total wealth. It's only when the high return type child has both higher marginal as well as average return, that she gets more of human capital and less of bequest; while the low return child gets the reverse. Otherwise the general case is that it is possible, even with the wealth model, not to have a reinforcing strategy (see result 4 in section 3).

2.2 The pure investment model

Unlike the preference model, this one fails to provide a full theoretical framework to examine households' decisions. For instance, issues like the distribution of bequests among children can not be addressed using this model. Unlike the preference model, investment in children is made without parents making reference to the distributional consequences it might entail (Becker, 1979, 1991, 1993; Behrman, Pollak and Taubman, 1982; Ashenfelter and Rouse, 1998). In the words of Becker (1993), 'essentially all that is involved *in the analysis of human capital investment* (emphasis added) is the application to human capital of a framework traditionally used to analyse investment on other capital'. Here, similar to investment on other assets, investment in human capital solely depends on the net returns obtained from such investment.

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[3] Specifically, if the sign of the derivative of the marginal return to endowment with respect to human capital investment is positive , then parents follow a reinforcing strategy. A negative sign implies a compensating strategy.

[4] Unlike the EB model, here parents do not face the productivity-equity trade off.

The marginal benefit and cost determine the level of investment in human capital. The former is negatively sloped indicating the presence of diminishing returns[5]. The latter is equal to the market interest rate in the case of perfect capital market or the marginal borrowing rate when markets are segmented/imperfect, which is the likely scenario in the case of human capital[6]. As a result the marginal cost curve is either horizontal or positively slopped. The intersection of the two curves determines the level of human capital invested in each child. By providing genetic endowments and through the supply of funds (see footnote 7), parents play a role in determining the shape (which depends on the borrowing rate) and the specific position of the marginal cost and the marginal benefit (which depends on endowment) curves (see Taubman, 1996).

Similar to the wealth model, here also the type of strategy parents pursue is determined by the earnings function (i.e. the relationship between child type and the returns from human capital). However, the preferences of parents do not enter the analysis here.

Section 3. The model

Two regimes are considered. The first one is where parents leave no bequest but are supported by children during old age. In the second regime parents are allowed to leave bequests. The two cases are compatible with the view that both children and parents are selfish and share the risk that parents live longer. If parents live long, they deplete

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[5] Factors contributing for the existence of diminishing returns include limited physical capacity of human being, increased cost or decreased benefit of later investments and high opportunity cost of increasing the stock of human capital. As investments are made on human being, limited physical capacity implies that diminishing returns will eventually operate. Moreover, since the considerable part of the inputs to human capital formation is own time and all previously accumulated stock of human capital enters as an input when additional investment is made, the forgone earnings of time spent in acquiring additional capital is a cost to the additional human capital. Thus, as more is invested the higher would be the earnings forgone so that the less would be the net benefit of additional investment at higher level of human capital stock. Although the productivity (in the production of human capital) of ones time increases at higher level of human capital stock, such increase is far below the cost incurred in terms of the forgone earnings (Becker, 1993).

[6] As human capital is not a good collateral for loans, investments in this form of capital are 'self-financed' in a sense that parents/relatives etc. serve as a creditor or as a provider of collateral. Consequently, markets are segmented, which imply that costs of investing in human capital are not uniform for an individual across time, at different level of human capital stock and for all agents at a given time.

all their wealth, leave no bequest and have to be supported by their children. If they die earlier they leave bequest.

Regime A: Altruistic parents with old age-support investment motive and zero bequests

The allocation of resources within the household members is posited in the context of overlapping generations framework. Adults allocate resources among themselves, their children and the deserving old, where the latter deserves to get his share only if he confirm to the norm (i.e. had done the same during his/her adulthood). In this regard, consumption transfer to children is an investment from the perspective of parents as it represents forgone current consumption and guarantees future consumption (as in Dasgupta, 1993).

More specifically, we consider a world where parents live for two periods. During the first period they spend their income on first period consumption, on investment in their children and transfer to the aged family members. In the second period they live on transfers from their children. Formally parents are assumed to have a utility function of the following form.

$$U(C_1) + \beta U(C_2, C_i, C_j) \qquad [1]$$

Where C_1 and C_2 are parental consumption during the first and the second period, and C_i and C_j are consumption of child i and j respectively, realised in period two. Indexes i and j represent child type[7]. In the absence of bequest, consumption of a child is a function of its earnings, which in turn depends up on the amount invested in the child as well as its type. Furthermore, the utility function is assumed to have the following properties: $U_s > 0 \text{ and } U_{ss} < 0 \text{ where } U_s = \frac{\partial U}{\partial s} \text{ and } s = C_1, C_2, C_i \text{ and } C_j$.

Resources devoted to investment in children and consumption during the first period can be augmented by borrowing, which has to be paid back during the second period. Thus, the above utility maximisation is subject to the following budget constraints:

[7] Since every child has his own specific endowment (type), analytically it doesn't make any difference if there are as many child types as the number of children. The only difference is in the representation of the utility function; which will now be $U=u(C_1)+\beta u(C_2,C_i,\ldots,C_n)$ and the budget constraints will be $Y=C_1+p^k\Sigma k_i+b$ and $C_2=q\,\Sigma f_i(.)-b(1+m)$.

$C_1 + p^k(k_i + k_j) - b = Y$; [2]

p^k is the price of a unit of human capital, assumed to be same across children. k's are amount of human capital invested in children; b represents the amount that can be borrowed and Y[8] is parents' initial income.

Second period consumption is totally financed through remittances from children. The latter is assumed to be proportional to the earnings of the children.

Thus, using q as a rate of remittance[9] from children and m as the borrowing rate and $f_i\ (k_i)\, and\, f_j\ (k_j)$ expected earnings of type i and j respectively, second period consumption of the parent is given by

$$C_2 = \left(f_i\ (k_i) + f_j\ (k_j) \right) q - b(1+m) \geq 0 . \qquad [3]$$

Differences in child types are captured through the return functions that allow for differences in rates of returns from human capital. The high return type is assumed to have higher average as well as marginal returns $\forall\ k_i = k_j$. Furthermore the earnings function is assumed to be concave (i.e. $f_i^{'}(k_i), f_j^{'}(k_j) > 0$ and $f_i^{''}(k_i), f_j^{''}(k_j) < 0$).

In the absence of bequests, children's consumption is financed through earnings only:

$$C_i = f_i\ (k_i) - qf_i\ (k_i) - I_i \qquad [4]$$

$$C_j = f_j\ (k_j) - qf_j\ (k_j) - I_j \qquad [5]$$

where I's are amount saved for other activities (e.g. for investing in children and consumption in period three).

Maximising [1] with respect to the k's and b, subject to [2],[3],[4] and[5] yields the following first order conditions

$$\frac{\P U}{\P C_1} p^k = b \left\{ \frac{\P U}{\P C_2} q + \frac{\P U}{\P C_i} - q \frac{\P U}{\P C_i} \right\} f_i^{'}(.) \qquad [6]$$

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[8] This does not include transfers made to the old -aged members of the household.

[9] For the sake of simplicity we assumed equal rate of remittance by all children.

$$\frac{\partial U}{\partial C_1} p^k = \beta \left\{ \frac{\partial U}{\partial C_2} q + \frac{\partial U}{\partial C_j} - q \frac{\partial U}{\partial C_j} \right\} f_j^{'}(.) \qquad [7]$$

$$\frac{\partial U}{\partial C_1} = \beta \frac{\partial U}{\partial C_2}(1+\mu) \qquad [8]$$

If parents express equal concern towards all types of children, returns determine investment in children. Returns, however, have double effects on the utility of the parents as it affects both their own consumption and the consumption of their children. Expression [6] and [7] show the equality of the costs and the benefits of investing in children, at the margin. The left-hand side (LHS) expression is the marginal cost of investing in children expressed in terms of foregone consumption[10]. The returns from this investment accrue in two forms. First, to the extent children transfer part of their earnings to the parents, investment in children will increase second period consumption. Second, by increasing the earning capability of children it will increase their consumption, which itself is an argument in the parental utility function.

Let $\bar{b}$ be the maximum amount that can be retrieved through borrowing. If the amount of borrowing required is less than $\bar{b}$ (i.e. the constraint is not binding) the above result still holds. But when the constraint is binding equation [8] will now become

$$\frac{\partial U}{\partial C_1} = \beta \frac{\partial U}{\partial C_2}(1+\mu) + \lambda \qquad [8']$$

(for an interior solution) where λ is the lagrangean multiplier associated with the constraint b $\leq \bar{b}$. Because the cost of investing in children is much higher than the case where the constraint is not binding, unlike the non-binding case, here investment in all types of children will be less than otherwise (compare expressions [8] and [8']).

Result 1: *Given that parents are not extremely inequality avert, equal concern implies* $k_i > k_j$ i.e. *high return type children will receive more human capital investment than the low return type.*

[10] These are similar to the FOCs for optimal consumption and portfolio formulation in consumption based asset-pricing models.

However, if parents are extremely inequality avert (i.e. the parental preference curve is L-shaped) equal concern implies both types of children will have equal outcome.

Before proceeding with the proof, we make a graphical illustration to explain why the concern of the parents as well as their attitude towards inequality among siblings, are used to qualify our result.

Generally the distributions of human capital resources among siblings depend on the preference of parents towards averting inequality, the concerns they have for different type of children and the genetic endowments that each type of children are endowed with.

Similar to the social planner preference in social choice theories, the preference of parents towards averting inequality among siblings is represented by the curvature of the indifference curves in the siblings' consumption/earnings space. It ranges from no inequality aversion depicted by a straight line, to an extreme desire to avert inequality represented by a right-angled indifference curve.

Parental concern on the other hand is about the symmetry of the indifference curve. This determines the position of the indifference curve with respect to a 45° line from the origin. When parents have equal concern for all types of children, the indifference curve will be symmetric around the 45° ray from the origin. The movement of the indifference curve towards the axis representing that particular child show more concern for a particular type of child so that the curve ceases to be symmetric.

The endowments of children determine their earning/consumption possibilities that are represented by a possibility frontier curve. This curve is not symmetric around the 45° ray from the origin unless both types of children are equally endowed.

In what follows three possible shapes of the consumption/earnings frontier, each for cases where siblings are equally endowed and one or the other child is better endowed, are shown.

Three classes of parental preference curves are considered; each representing the degree of parental preference towards inequality among children. Combining this with the three possibilities of the distribution of genetic endowment, we get nine scenarios. In what follow these scenarios are summarised into three figures.

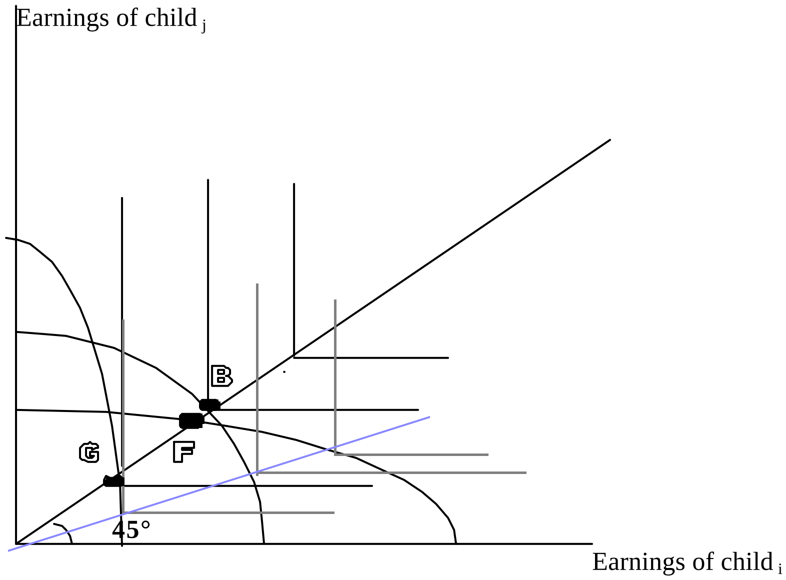

Fig. 1a. Extremely inequality-avert parents.

When parents are highly concerned about inequality among their children, equal concern implies the outcome of the distribution always lies on the 45° ray from the origin, which is the intersection point of the indifference curve and the possibility frontier. Points B,F and G, in figure 1a, show allocation outcomes when the children are equally endowed, child i is the better endowed and child j is the better endowed, respectively. Even when parents are extremely concerned about equity, equal earnings for all siblings are not guaranteed as long as siblings are heterogeneous in their endowment. When parents are more concerned for child i, the indifference curves shift to the right, as shown by the broken-lined curves.

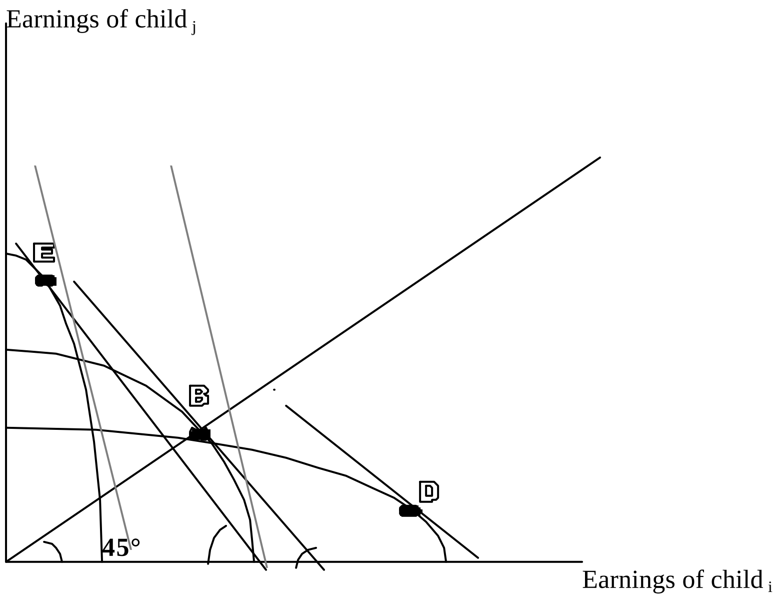

Fig. 1b. Extremely inequality-tolerant parents.

When parents do not favour a particular type of children and the consideration for equity is eliminated, the distribution of resources trails the distribution of endowments. Point D, in figure 1b, shows that more human capital resource is given to type i who possesses higher endowments. On the other hand point E would be the outcome if type j were the better-endowed child. Point B is where equal parental concern coincides with equal distribution of endowment among children. However, when parents favour type i, even though endowments are distributed equally, favoured child receive more resources. This is shown by the movement of the indifference curve to the right as shown by broken-lined curves.

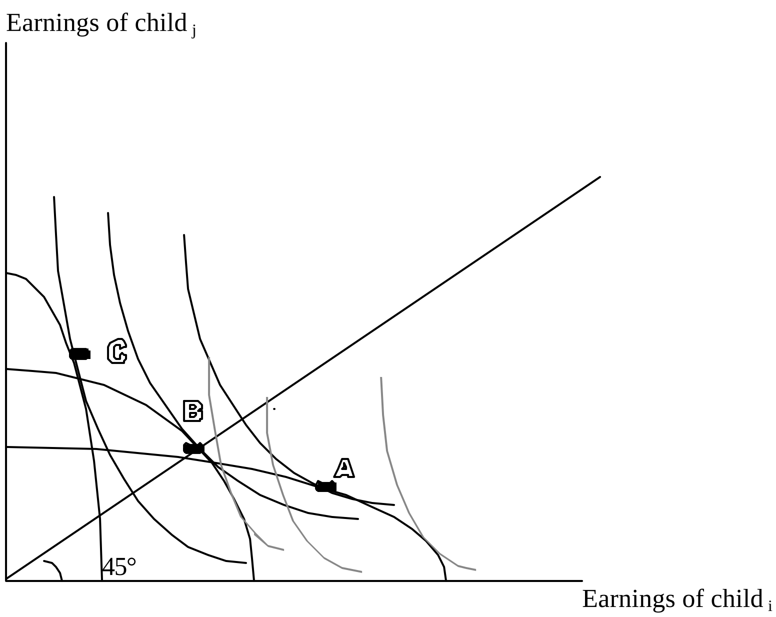

Fig. 1c. An intermediate case.

The above two figures represent cases that are extreme in their treatment of parental attitude towards inequality aversion. An intermediate scenario is a situation where parents are still concerned about equity but not as their only target, so that the distribution of human capital resources depends up on both the preferences of parents as well as the distribution of endowment among siblings as shown in figure 1c.

We now turn to the proof of result 1.

Proof:

Let i represent the high return type child. Ceteris paribus, the RHS of equation [6] will be higher than the RHS of equation [7] evaluated at same level of k. Under increasing and concave benefits function, the equality of the two equations imply $k_i > k_j$.

Graphically, by bringing all the above figures together we see that, when parents show equal concern for all children, the high return type child gets more human capital when parents have less consideration for equity (compare points E,C and G or points D, A and F in fig. 1).

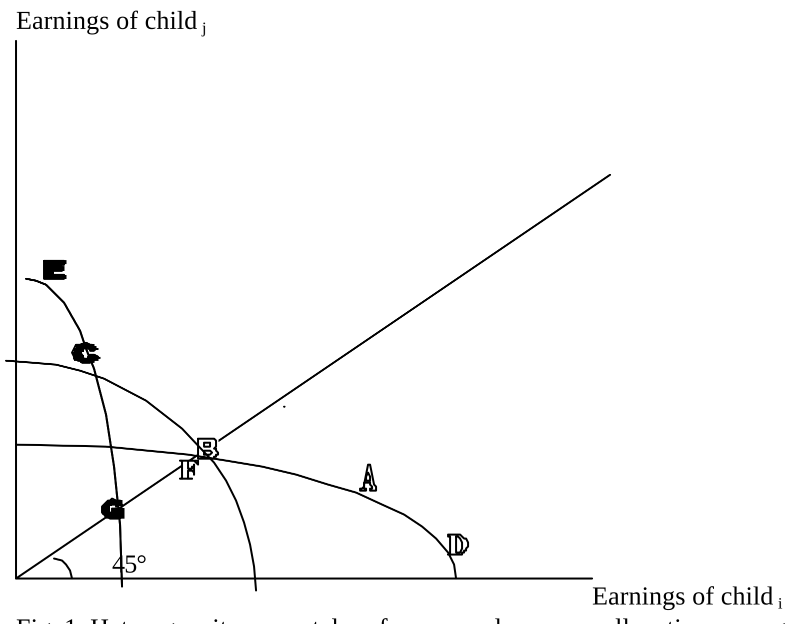

Fig. 1. Heterogeneity, parental preference and resource allocation among children.

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3.1. Competition among siblings

Efficient level of investment is implied by equations [6]-[8]. Lack of resources or constraints in obtaining credit mean the level of investment would be less than the efficient level. In this case all children will get less than the efficient level and competition among siblings becomes fierce.

A pure investment model stipulates that investment in each child will continue to be made until marginal return from such investment is equal to the marginal cost of borrowing, the one with higher return receiving more investment. However, insufficient income/wealth or lack of access to credit opens the door for competition among different types of children (additional child will always lowers the resources available and make children get less than what they otherwise would get).[11]

In a pure investment framework, even when the total number of children is kept constant, competition among siblings means that a change in the type composition of children makes a difference in terms of the level of investment received by all types of children. Notably, an increase in the proportion of the low return type benefits all. To illustrate this point, let x be the proportion of the low return type, say type j, children. Increase in x, while keeping the total number of children constant, is expected to increase the level of human capital invested in both types. Because, in (a pure) investment sense the low return children will gain when their proportion increases as this shrinks the proportion of the high return children in a given household thereby reducing the number of stronger rivals. The high return type would have taken more resources, which otherwise be shared by the low return, if a high return type replaces a low return type. The high return children will also gain from an increase in the proportion of low return children, because by competing with the low return type they will be able to secure more resource than they could when competing with the fellow high return type.
Thus we have the following result.

[11] Moreover increase in the quantity of children will raise the shadow price of their quality thereby decrease the amount of child quality demanded. However our interest is not on such interactions (see Becker and Tomes, 1976 for detail on the interaction between quantity and quality).

Result 2: *An increase in the proportion of the low return type, keeping the total number of children constant, will increase investment in both types of children. On the contrary, an increase in the proportion of the high return type will lower investment in both types of children.*

Proof:

Taking the derivative of equations [6] and [7] with respect to x and solving for $\frac{\partial k_i}{\partial x}$ and $\frac{\partial k_j}{\partial x}$ we get (see appendix for the derivation),

$$\frac{\partial k_i}{\partial x} = \frac{\left(U_1''(k_i - k_j)H - bU_2''Hqf_i'(f_j - f_i)\right)\left(R - xf_j' bU_2''q^2 f_i'\right)}{QR - \left((1-x)bU_2'' f_i' f_j' q\right)\left(xf_j' bU_2'' q^2 f_i'\right)} > 0 \qquad (R_{2.1})$$

And

$$\frac{\partial k_j}{\partial x} = \frac{\left(U_1''(k_i - k_j)H - bU_2''Hqf_i'(f_j - f_i)\right)\left(Q - (1-x) f_j' bU_2''q\, f_i'\right)}{QR - \left((1-x)bU_2'' f_j' f_i' q\right)\left(xf_j' bU_2''q^2 f_i'\right)} > 0 \qquad (R_{2.2})$$

Where $R = bf_j''\left(U_2' + U_j'(1+q)\right) + bU_j''\left((f_j' - qf_j')(f_j' - qf_j')\right) + bU_2''qxf_j' f_j'$, and

$Q = bf_i''\left(U_2' + U_i'(1+q)\right) + bU_i''\left((f_i' - qf_i')(f_i' - qf_i')\right) + bU_2''q(1-x) f_i' f_i'$,

both of which are negative due to the concavity of the utility and the earnings functions.z

In fact increase in the proportion of the high return children will increase resources that can be transferred to parents during the second period. However, due to the time lag involved in making human capital investments and realising the transfers from children, in an environment where capital markets are imperfect, this has no effect in augmenting the overall resource that is available for investing in children during the first period.

3.2. The response of investment in children to shocks

There are evidences (see Shea, 1997 and references there in) suggesting that as far as parents are able to borrow against their own income or that of their children, current income (as opposed to permanent income) does little in determining human capital investment in children.

When shocks are transitory in nature, in a sense that the long term behaviour of income is not that affected, households who are not credit constrained are expected to show a negligible change in their investment behaviour. If there is any change, it should be to the extent that the shock alters the marginal benefit and/or cost of investing in children. However, credit constrained households will respond to such shocks. Our model predicts that the response to be higher for the high return children. Also the response is expected to be higher among poorer households.

Result 3: *Those who receive more human capital investment show at least equal response to changes in price as well as income, compared to those who get less human capital investment. And the gap in the price responsiveness of investment in the two types of children is higher among poor households.*

Proof:

Graphically consider the effect of a change in the price of human capital. The same reasoning applies to changes in income except that a decrease in price will have similar consequences to an increase in income.

A decrease (an increase) in the price of human capital lowers (raises) the LHS of [6] and [7], so that a new equilibrium is obtained. The movement is along a path where $f_i'(k_i) = f_j'(k_j)$ as shown in figure 2. Since the slope of the line where $f_i'(k_i) = f_j'(k_j)$ is less than one, any movements along that line entails more change in k_i than k_j. Therefore, the one who receives more human capital investments will end up having at least equal response to changes in price and income.

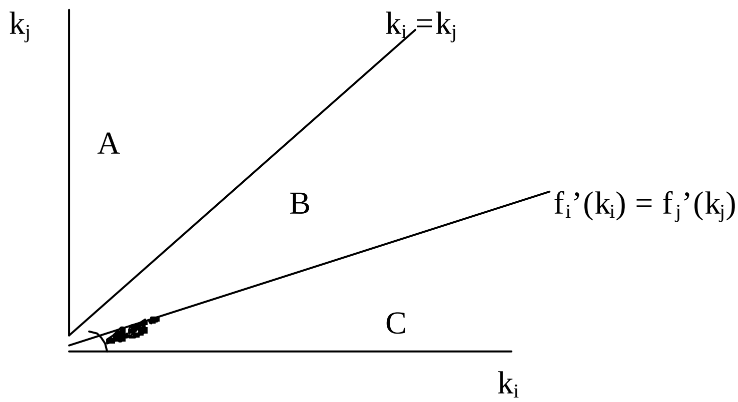

Fig. 2. Optimal path of human capital investments:

$k_i < k_j$, in region A, while the reverse is true in regions B and C.

$f_i'(k_i) < f_j'(k_j)$ in region C, while the reverse is true in regions A and B.

Moreover equations [6] and [7] also tell that increase in the price of human capital entails a larger decrease in human capital investment when income is low (i.e. when marginal utility of consumption is large). That is due to the concavity of the utility function; increases in the price of human capital investments imply a higher marginal cost when income is low. The implication of these inverse relationship between price elasticity of human capital investment and income is that, as parental income increases the gap in the responsiveness (to prices) of human capital investment between different types of children diminishes.z

Regime B: Purely altruistic parents with bequest

As in regime A, we consider two generations: parents and their children. And the former makes all investments in children. Moreover, parents bequeath B_i and B_j amounts to type i and j respectively. Two cases suggest themselves. The first is where parents are allowed to transfer even a negative amount of bequests, for example, by making their children pay for the debt that parents incur; which effectively means that parents can borrow against their children's future income. The second case is where parents are constrained to transfer only non-negative amounts i.e. $B_i \geq 0$ and $B_j \geq 0$.

Everything remains the same, except bequests are introduced into equations [3], [4] and [5].

$C_1 = Y_1 - p^k(k_i + k_j)$ [2']

$C_2 = Y_2 - B_i - B_j$ [3']

$C_i = f_i(k_i) + (1+m) B_i - I_i$ [4']

$C_j = f_j(k_j) + (1+m) B_j - I_j$ [5']

$B_i \geq 0$ and $B_j \geq 0$

$Y = Y_1 + Y_2$. Substituting for C_1, C_2, C_i, and C_j, and denoting λ_1 and λ_2 as the Lagrange multipliers attached to $B_i \geq 0$ and $B_j \geq 0$, respectively; the maximisation yields the following first order conditions:

$$-p^k \frac{\partial u}{\partial C_1} + \beta \frac{\partial u}{\partial C_i} f_i'(.) = 0 \quad [9]$$

$$-p^k \frac{\partial u}{\partial C_1} + \beta \frac{\partial u}{\partial C_j} f_j'(.) = 0 \quad [10]$$

$$-\beta \frac{\partial u}{\partial C_2} + \beta \frac{\partial u}{\partial C_i}(1+m) + \lambda_1 = 0 \quad [11]$$

$$-\beta \frac{\partial u}{\partial C_2} + \beta \frac{\partial u}{\partial C_j}(1+m) + \lambda_2 = 0 \quad [12]$$

If bequest constraints are not binding, i.e. $\lambda_1 = \lambda_2 = 0$ then from the first order conditions we get

$f_i'(k_j) = f_j'(k_j)$ [13]

Which convey that when parents are able to provide sufficient resources (sufficient enough to satisfy both human and physical capital investment requirements), investment in children will be efficient[12] in a sense that returns from such investment will be equalised across children. Also

$$\frac{\partial u}{\partial C_i} = \frac{\partial u}{\partial C_j} \quad [14].$$

Expression [14] implies that at equilibrium the marginal utility of children's consumption, to the parents, is equal. Combining [13] and [14] we obtain what is reminiscent of result 1. However, this one is somewhat stronger in a sense that here the parental preference towards inequality aversion does not affect the level of human

[12] In the framework of the wealth model discussed in section 2, this is a wealth maximising level of human capital.

capital invested in either type of children. Any difference in the consumption of different types of children arising from differences in the level of human capital is offset by means of bequests so that investment in human capital is determined solely on efficiency ground. Sheshinsky and weiss (1982) also provide similar results.

An interesting case emerges when bequest is constrained in that only non negative transfers transfer is possible.

Result 4: *when the high return type gets no bequest (i.e. $B_i=0$) and the difference in the rate of return between the two types of children is considerably high, it is possible that the high return type (i.e. type i) gets less human capital investment as well.*

Proof:

In the context of the wealth model discussed in section 2, the constraint is binding for either one or both of the two types of children, depending on their earning opportunities and the level of parental resources devoted to them. Since investment in human capital partly depends upon the rate of return from such investment relative to returns from alternative assets, it is possible that the high return type have a very high rate of return from human capital investment such that resources allocated for its human as well as physical capital are all spent on his human capital. This may push bequests to the level below which it is impossible to go (i.e. B=0). So, assume $B_i=0$ so that $\lambda_1>0$ and $\lambda_2=0$. From [11] and [12] we get

$$\left(\frac{\partial u}{\partial C_i} - \frac{\partial u}{\partial C_j}\right)(1+m) + \lambda_1 = 0 \qquad [15]$$

From [9], [10] and [15] we have $\left(\frac{\partial u}{\partial C_i}\right)/\left(\frac{\partial u}{\partial C_j}\right) = \frac{f_j'(k_j)}{f_i'(k_i)} < 1 \qquad [16]$

The inequality follows from [15].

Actually no conclusive relationship between k_i and k_j can be established from [16].

Their relationship is indeterminate. However, when the difference in earnings capacity between the two types is considerably high (i.e. the $f_i'(k_j)=f_j'(k_j)$ path in fig. 2 is more flat), and also parents are concerned about the inequality among children, narrowing the gap among siblings require parents to allocate more of both human as well as physical capital to the low return type. The need to transfer bequest to the low return type comes due to the fact that after a certain point investing more in the human capital of these children marginally yields less than what would have been obtained from other options of investment. ∎

This result is similar to what Behrman *et al* (1995) presented as a supporting evidence for their argument that the Becker and Tomes (1979) result is not always true. That it is not necessarily true that more human capital and less physical capital is invested in high return type children; and vice versa for low return type children.

However, if the constraint is binding for both types[13], the inequality in [16] will be replaced by equality implying, unlike result 4, the high return type gets more human capital investment.

Section 4. Extensions

Regimes A and B are two polar cases in their treatment of parental consumption in period two. In the former, second period consumption is totally dependent on remittances from children, while in regime B it is rather totally financed by parents themselves. In the latter parents, instead, can afford to bequeath to their children.

What is missing is an intermediate case where C_2 is partially dependent on remittances from children. In what follows this issue is addressed in an environment where outcomes are uncertain. But before venturing into such exercise, we try to merge the two regimes and show that the results obtained in the previous paragraphs still remain intact.

4.1. Altruistic parents, old age-support motive and non zero bequest

By bringing the two regimes together we get the consumption of parents and children as

[13] Which means either the resources allocated to children is not sufficient enough to allow for bequests or the return from investing in children's human capital is still higher than the returns from other assets.

$$C_1 = Y_1 - p^k(k_i + k_j) + b$$

$$C_2 = Y_2 - B_i - B_j + \left(f_i(k) + f_j(k)\right)q - b(1+m)$$

$$C_i = f_i(k) - qf_i(k) + B_i(1+m) \text{ -I}_i$$

$$C_j = f_j(k) - qf_j(k) + B_j(1+m) \text{ -I}_j$$

Maximising the utility function in [1] with respect to k_i, k_j,B_i,B_j and b, given the above consumption constraints, gives first order conditions as

$$\frac{\P U}{\P C_1} p^k = \left(\frac{\P U}{\P C_2} q + \frac{\P U}{\P C_i}(1-q)\right) \mathbf{b} f'_i(k) \qquad [17]$$

$$\frac{\P U}{\P C_1} p^k = \left(\frac{\P U}{\P C_2} q + \frac{\P U}{\P C_j}(1-q)\right) \mathbf{b} f'_j(k) \qquad [18]$$

$$\frac{\P U}{\P C_2} = \frac{\P U}{\P C_i}(1+m) + \boldsymbol{l}_1 \qquad [19]$$

$$\frac{\P U}{\P C_2} = \frac{\P U}{\P C_j}(1+m) + \boldsymbol{l}_2 \qquad [20]$$

$$\frac{\P U}{\P C_1} = \mathbf{b} \frac{\P U}{\P C_2}(1+m) \qquad [21]$$

In our model bequests are physical capital transferred from parents to children. If the former can afford to make transfer to their children, it means that they can also afford to finance their consumption in period two so that consumption is independent of remittances from children (i.e. giving out bequests while at the same time taking in remittance makes no sense). Therefore, if B>0 it must be the case that q=0. In this case the above first order conditions are exactly same as those obtained in regime B (i.e. expressions [17]=[9] and [18]=10]). However if parents are unable to leave bequests, they are likely to need remittance to partly finance their consumption. This is the case where B=0 and q>0, so that we get regime A (expressions [17]= [6] and [18]= [10]).

4.2 An intermediate case: Introducing uncertainty

The previous analysis is confined to the world of certainty. It takes that parents face no uncertainty about their consumption, because both future earnings of children as well as parents own income is known for certain. However, as far as consumption in period two is concerned there are at least two sources of uncertainty, namely; One is related to the earnings of children and the other is concerning the subsequent remittances they give to their parents. By assuming that a fixed proportion of children's earnings is transferred to parents, we focus only on uncertainties involved with the earnings of children.

The second period consumption is treated as partially dependent on remittances form children. Particular attention was given to the effect of introducing uncertainty on the relative magnitude of investment in the two types of children. Formally, the problem is maximising the following parental utility function with respect to k_i, k_j ,and b.

$$u\ (C_1) + Ebu(C_2, C_i, C_j) given$$

$$C_i = \tilde{y}_i - q\tilde{y}_i - I_i$$
$$C_j = \tilde{y}_j - q\tilde{y}_j - I_j$$
$$C_1 = Y_1 - p^k (k_i + k_j) + b$$
$$C_2 = Y_2 + q(\tilde{y}_i + \tilde{y}_j) - b(1+m)$$

Where E is expectation operator and $\tilde{y}$'s are expected earnings of children. All the rest as defined earlier.

The first order conditions, yield

$$p^k u'_1 = bEq\frac{\partial\tilde{y}_i}{\partial k_i}u'_2 + Eb\,(1-q)\frac{\partial\tilde{y}_i}{\partial k_i}u'_i \qquad [22]$$
$$p^k u'_1 = Ebq\frac{\partial\tilde{y}_j}{\partial k_j}u'_2 + Eb\,(1-q)\frac{\partial\tilde{y}_j}{\partial k_j}u'_j \qquad [23]$$
$$u'_1 = bu'_2(1+m) \qquad [24]$$

Though bequest is allowed for, the relative magnitude of k_i and k_j is indeterminate. The relationship is determined, among other things, by the covariance between the earnings of children and the marginal utilities to the parents of children's as well as own consumption, and by the extent to which change in the human capital investment affect the variability of children's earnings.

When returns from investments co-vary with consumption (or discount factor) than such ventures need risk correction in terms of either higher return or lower price. The conventional wisdom is that those investments that make consumption more volatile need to have higher returns/lower prices in order to compensate for the volatility.

Using the definition of variance the RHS expressions of [22] and [23], keeping the constant q and ß aside, can be rewritten as :

$$E\left(\frac{\partial \tilde{y}_i}{\partial k_i}\right)E\left(U_2^{'}\right)+\operatorname{cov}\left(\frac{\partial \tilde{y}_i}{\partial k_i},U_2^{'}\right)+E\left(\frac{\partial \tilde{y}_i}{\partial k_i}\right)E\left(U_i^{'}\right)+\operatorname{cov}\left(\frac{\partial \tilde{y}_i}{\partial k_i},U_i^{'}\right) \quad [22']$$

$$E\left(\frac{\partial \tilde{y}_j}{\partial k_j}\right)E\left(U_2^{'}\right)+\operatorname{cov}\left(\frac{\partial \tilde{y}_j}{\partial k_j},U_2^{'}\right)+E\left(\frac{\partial \tilde{y}_j}{\partial k_j}\right)E\left(U_j^{'}\right)+\operatorname{cov}\left(\frac{\partial \tilde{y}_j}{\partial k_j},U_j^{'}\right) \quad [23']$$

when $k_i=k_j$, we know that

$$E\left(\frac{\partial \tilde{y}_i}{k_i}\right)>E\left(\frac{\partial \tilde{y}_j}{k_j}\right) \text{ and}$$

$$\operatorname{cov}\left(\frac{\partial \tilde{y}_i}{\partial k_i},U_2^{'}\right)>\operatorname{cov}\left(\frac{\partial \tilde{y}_j}{\partial k_j},U_2^{'}\right) \quad \textit{in absolute terms.}$$

$$\textit{Assuming},\quad \operatorname{cov}\left(\frac{\partial \tilde{y}_i}{k_i},U_i^{'}\right)=\operatorname{cov}\left(\frac{\partial \tilde{y}_j}{k_j},U_j^{'}\right)$$

which are both negative, in order $k_i>k_j$, we need to have

$$\left(E\left(\frac{\partial \tilde{y}_i}{\partial k_i}\right)-E\left(\frac{\partial \tilde{y}_j}{\partial k_j}\right)\right)EU_2^{'}+E\left(\frac{\partial \tilde{y}_i}{\partial k_i}\right)EU_i^{'}-E\left(\frac{\partial \tilde{y}_j}{\partial k_j}\right)EU_j^{'}>$$

$$\operatorname{cov}\left(\frac{\partial \tilde{y}_i}{\partial k_i},U_2^{'}\right)-\operatorname{cov}\left(\frac{\partial \tilde{y}_j}{\partial k_j},U_2^{'}\right)$$

In words it means that, the difference in the expected marginal earnings weighted by the marginal utility of consumption of the parents plus the difference in the weighted expected marginal earnings (where the weights are the marginal utility to the parents of children's consumption) must be greater than the difference in the covariance between marginal earnings and second period consumption of the parents.

This is in contrast to the case where no uncertainties were involved. In the latter case when parents are able to leave bequests we know that $k_i > k_j$.

4.3. <u>An intermediate case: non-passive children with reverse altruism</u>

In the previous sections children were posited as having no control over the remittance they transfer during the second period. What if they are given all the freedom to decide on transfers to their parents. What will happen to the level of human capital invested in children during period one. What is address here is the effect on the relative magnitude of the k's, if giving remittance is totally up to the children in a sense that whatever remittances made by children is out of their altruistic feeling towards their parents. And parents also know that this is the case. We model this in a Nash equilibrium framework.

Only with a slight modification, the same formulation used in regime A is also employed here. Instead of the direct consumption of children, what enters as an argument in the utility of parents is the utility that children drive from their own consumption (i.e. w(c)). The parental utility function will be

$$u(C_1) + bu(C_2, w^i(C_i), w^j(C_j)) \qquad [1']$$

Finally in order to emphasise the role of altruism we assumed that remittance (denoted by R) that children made does not depend on their earnings. Thus, in addition to constraint [2] we get the following budget constraints.

$$C_2 = Y_2 + R_i + R_j - b(1 + m) \qquad [3'']$$

$$C_i = f_i(k_i) - R_i - I_i \qquad [4'']$$

$$C_j = f_j(k_j) - R_j - I_j \qquad [5'']$$

Taking R's as given, parents maximise [1'] with respect to k_i , k_i and b subject to [2], [3''] , [4''] and [5''].

Since R's are choices of children, the decision involved in the determination of the R is posited as follows. Taking k as given child of type h solves

$$w(C_h, u(C_2)) \quad where\ h = i\ and\ j,$$

subject to constraints [3''] and [4''] or [3''] and [5''], as the case may be.

The first order conditions for the parents and for the children, respectively, are given as:

$$u'_1 = b \frac{\partial u}{\partial w_i} \frac{\partial w_i}{\partial C_i} f_i^{'} \qquad [25]$$

$$u'_1 = b \frac{\partial u}{\partial w_j} \frac{\partial w_j}{\partial C_j} f_j^{'} \qquad [26]$$

$$u'_1 = b u'_2 (1+m) \qquad [27]$$

$$\frac{\partial w_i}{\partial C_i} = \frac{\partial w_i}{\partial u} u'_2 \qquad [28]$$

$$\frac{\partial w_j}{\partial C_j} = \frac{\partial w_j}{\partial u} u'_2 \qquad [29]$$

After eliminating the price of human capital by taking the k's as the monetary values. Combining [25], [27] and [28] on the one hand and [26], [27] and [29] on the other, we get $1+m = f_i^{'}(k_i) \frac{\partial u}{\partial w_i} \frac{\partial w_i}{\partial u} = f_j^{'}(k_j) \frac{\partial u}{\partial w_j} \frac{\partial w_j}{\partial u}$, which amounts to the result obtained at the outset. However if either the parents have no altruistic feelings towards their children or if they knew that their children have no altruistic feelings towards them, , no human capital investment will be made in any type of children. It is the presence of altruism and perfect information about the reverse altruism that saves the intergenerational transfer of human capital.

Section 5. Summary

Aiming at explaining why individual human capital outcomes vary among siblings, we started with an extremely simplified formulation where parents invest in the human capital of their children by anticipating that the latter will reciprocate such action by providing support during old age. Due to differences in their genetic endowments, the yield from these investments varies among siblings. Unless parents are extremely averse to inequalities among their children, better-endowed children (high return type) will always receive more human capital investment. In this case the type composition of children in a given household will affect the level of human capital investment in children. As competing with the low return type is relatively easier, all types of children within the household will gain when the proportion of the low return type increases.

There are instances where parents show more concern to avert inequality among siblings; In such case more human capital is invested in the low return type.

In the subsequent sections where bequests are introduced there by eliminating the need for old age support, the only reason to invest on the human capital of their children is because parents drive utility from the welfare of the former. When bequests are constrained to be greater or equal to zero, the outcome depends on the attitude of the parents towards inequality among siblings. However, if bequests are allowed to be negative, parents invest efficiently with no consideration for equity, as a result they follow a reinforcing strategy. This is because by transferring a positive bequest to the low return type and zero or negative to the high return, parents can always address their concern for equity.

Introducing uncertainty dilutes the previous results. With parents' facing risk in their consumption, there will be a consumption-smoothing motive for investing in children. The outcome depends on the relative magnitude of the covariance between on the one hand the earnings of children and on the other the marginal utility of parent's consumption and that of children's consumption.

In most cases, unequal-starting ground due to nature is exacerbated by parental action. Even when in our hypothetical community the composition of families in terms of income/wealth is uniform and, to make the case sharp, every family has equal number of children, the next generation of this community will inevitably show inequality in their income/wealth. The source of this inequality being the unequal earning capacity among children of the current generation that is induced/accentuated by the way human capital inputs are allocated within the household.

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Appendix

The derivation of expressions $R_{2.1}$ and $R_{2.2}$

Let x be the proportion of type j (low return) children. Thus $C_1 = Y - \{(1-x)\, k_i + x\, k_j\}H$ and $C_2 = \{q(1-x) f_i\,(.) + xqf_j\,(.).\}H$ And

$C_i = f_i\,(.) - qf_i\,(.)$,

$.C_j = f_j\,(.) - qf_j\,(.)$

Taking the derivative of [6] with respect to x, we have

$$U_1^{"}(k_i - k_j)H - U_1^{"}H\left((1-x)\frac{\partial k_i}{\partial x} + x\frac{\partial k_j}{\partial x}\right) =$$

$$b f_i^{'}\left[U_2^{"}Hq^2(f_j^{'} - f_i^{'}) + U_2^{"}Hq\left((1-x)f_i^{'}\frac{\partial k_i}{\partial x} + xf_j^{'}\frac{\partial k_j}{\partial x}\right) + bU_i^{"}\left(f_i^{'}\frac{\partial k_i}{\partial x} - qf_i^{'}\frac{\partial k_i}{\partial x}\right) - U_i^{"}q\left(f_i^{'}\frac{\partial k_i}{\partial x} - qf_i^{'}\frac{\partial k_i}{\partial x}\right)\right] +$$

$$b f_i^{"}\frac{\partial k_i}{\partial x}\left(qU_2^{'} + U_i^{'} - qU_i^{'}.\right)$$

Rearranging terms,

$$U_1^{"}(k_i - k_j)H - bU_2^{"}q^2 f_i^{'}Hq(f_j^{'} - f_i^{'}) =$$

$$\frac{\partial k_i}{\partial x}\left[bU_2^{"}qf_i^{'}f_i^{'}(1-x)H + bU_i^{"}f_i^{'}f_i^{'}(1-q)^2 + bf_i^{"}\left(qU_2^{'} + U_i^{'} - qU_i^{'}\right) + U_1^{"}H(1-x)\right] +$$

$$\frac{\partial k_j}{\partial x}\left[bHxf_j^{'}U_2^{"}q\ \ f_i^{'} + U_1^{"}xH\right] \qquad [A].$$

Similarly taking the derivative of equation [7] with respect to x we get

$$U_1^{"}(k_i - k_j)H - U_1^{"}H\left((1-x)\frac{\partial k_i}{\partial x} + x\frac{\partial k_j}{\partial x}\right) =$$

$$b f_j^{'}\left[U_2^{"}Hq^2(f_j^{'} - f_i^{'}) + U_2^{"}Hq\left((1-x)\frac{\partial k_i}{\partial x} + x\frac{\partial k_j}{\partial x}\right) + U_j^{"}\left(f_j^{'}\frac{\partial k_j}{\partial x} - qf_j^{'}\frac{\partial k_j}{\partial x}\right) - U_j^{"}q\left(f_j^{'}\frac{\partial k_j}{\partial x} - qf_j^{'}\frac{\partial k_j}{\partial x}\right)\right] +$$

$$b f_j^{"}\frac{\partial k_j}{\partial x}\left(U_2^{'}q + U_j^{'} + qU_j^{'}\right).$$

Rearranging terms,

$$U_1''(k_i - k_j)H - bU_2''q^2 f_j' Hq(f_j' - f_i') =$$

$$\frac{\partial k_j}{\partial x}\left[bU_2''qf_j'f_j'xH + bU_j''f_j'f_j'(1-q)^2 + bf_j''\left(qU_2' + U_j' - qU_j'\right) + U_1''Hx\right] +$$

$$\frac{\partial k_i}{\partial x}\left[(1-x) f_i' U_2'' q \; f_j' H b + U_1'' H(1-x)\right]. \qquad [B]$$

solving for $\frac{\partial k_j}{\partial x}$ from [B] and substituting it into [A] we get

$$\frac{\partial k_i}{\partial x} = \frac{\left[U_1''(k_i - k_j)H - bU_2''q^2 f_i' Hq(f_j' - f_i')\right]\left(R - xf_j'U_2''q^2 f_i' - U_1''Hx\right)}{QR - \left(U_1''(1-x)H + (1-x) f_i' f_j' U_2'' q\right)\left(xf_j' U_2'' qf_i' + U_1'' Hx\right)} > 0. \quad [C]$$

u sin *g* [C] *in* [B]

$$\frac{\partial k_j}{\partial x} = \frac{\left[U_1''(k_i - k_j)H - bU_2''qf_j' Hq(f_j' - f_i')\right]\left(Q - (1-x) f_j' U_2'' q \; f_i' - U_1'' H(1-x)\right)}{QR - \left(U_1''(1-x)H + (1-x) f_i' f_j' U_2'' q\right)\left(xf_j' U_2'' q^2 f_i'\right)} > 0. \quad [D]$$

where

$$Q = b\,U_2''qf_i'f_i'(1-x)H + b\,U_i''f_i'f_i'(1-q)^2 + b\,f_i''\left(qU_2' + U_i' - qU_i'\right) + U_1''H(1-x)$$

$$R = b\,U_2''qf_j'f_j'xH + b\,U_j''f_j'f_j'(1-q)^2 + b\,f_j''\left(qU_2' + U_j' - qU_j'\right) + U_1''Hx$$

□