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DISCUSSION
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ABSTRACT

The literature on non-parametric production analysis has formulated tests for profit maximizing behavior that do not require a parametric specification of technology. Negative test results have conventionally been interpreted as inefficiency, or have been attributed to data perturbations. In this paper, we exploit the possibility that negative test results reveal violations of the underlying neoclassical assumption that prices are exogenously fixed and perfectly certain. We propose non-parametric tests that do allow for endogenous price formation and price uncertainty. In addition, we investigate how to recover the technology and how to forecast behavior in new economic situations.

Key words: non-parametric production analysis, endogenous price formation, price uncertainty

1. INTRODUCTION

Within the neoclassical paradigm, firms are typically assumed to maximize profits. Given its crucial role in the neoclassical theory of the firm, it is interesting to test this assumption empirically. For a long time, the standard tests started from a functional form for the production frontier. Unfortunately, economic theory does not imply a particular functional form, and reliable empirical specification tests are not available in many cases.

Alternative, *non-parametric* testing tools for analyzing firm behavior have been introduced. These tools are non-parametric because they do not need a parametric specification of the production technology. Instead, only observed production plans are assumed to be feasible. Apart from testing for profit maximization, the non-parametric approach can derive empirical approximations for the production technology and forecast firm behavior. Afriat (1972) and Hanoch and Rothschild (1972) initiated this approach. Varian (1984) provides an insightful overview of the main results. The non-parametric approach has been applied to a multitude of problems (e.g. Lim and Shumway, 1992; and Chavas and Cox, 1995), and several theoretical extensions have been proposed (e.g. Chavas and Cox, 1990, 1992; and Silva and Stefanou, 1996).

The non-parametric tests frequently suggest violations of profit maximization. Such violations can be interpreted as non-optimizing behavior. In this respect, there is an intimate relationship between the non-parametric approach to production analysis and the efficiency measurement literature, which builds mainly on the classic articles by Debreu (1951), Koopmans (1951) and Farrell (1957). Banker and Maindiratta (1988) and Färe and Grosskopf (1995) have further explored this link. In general, however, economists have problems with the nature and interpretation of inefficiency. In fact, economic theory has adopted rationality as its most fundamental maintained assumption. See for example the sharp critique by Stigler (1976) on Leibenstein's X-inefficiency concept.

Inaccurate measurement of the firm data constitutes an alternative interpretation of violations. Varian (1985) has proposed tests that account for measurement error, and several alternative statistical testing procedures have been developed (see e.g. Matzkin, 1994, for an overview). In a similar vein, Varian (1990) presented a "goodness-of-fit" approach to the estimation of the *economic* significance of violations of the consistency tests.

In this paper, we explore a third possibility: negative test results may reveal violations of the underlying "neoclassical" assumptions that firms take prices as exogenously fixed and perfectly certain. Frequently, these assumptions are too stringent. In many cases, the production plans of individual firms affect the market prices. In addition, in many cases there is uncertainty about uncontrollable *ex post* prices when firms *ex ante* fix their production plans. Note that price endogeneity and price uncertainty often occur simultaneously. For example, under endogenous prices a particular production plan can be associated with different price equilibria (compare with Debreu, 1970; see also Grodal, 1996), which immediately implies ex ante price uncertainty. In addition, firm owners usually imperfectly observe the interaction between firm actions on the one hand and market prices on the other, which introduces further price uncertainty. If prices are endogenous and/or uncertain, the standard tests for profit maximizing firm behavior are no longer appropriate. Apparently inefficient firms may actually be efficient and, accordingly, profit-maximizing behavior may falsely be rejected.

A whole literature has emerged which centers on the derivation of equilibrium behavior starting from weaker versions of the basic neoclassical hypotheses. As for endogenous prices, the classic references are Chamberlin (1933) and Robinson (1933) who focused on partial equilibria, whereas Negishi (1961) has first proposed extensions to general equilibrium settings. Two classic contributions on price uncertainty are the articles by McCall (1967) and Sandmo (1971). However, the literature on non-parametric production analysis has largely ignored these alternative price conditions thus far. We complement the conventional testing tools with non-parametric tests for profit maximizing behavior under the more complicated but often more realistic conditions of endogenous and uncertain prices. In addition, we investigate the possibility to recover the technology, and to forecast firm behavior.

The paper is further organized as follows. Section 2 develops non-parametric testing tools that allow for endogenous and uncertain prices. That section focuses on minimal assumptions with respect to the price distribution, the objectives of the firm, and the production technology. Obviously, such a non-parametric orientation can reduce discriminating power. However, our analysis can serve as a starting point for analyses that include additional hypotheses. In this respect, Section 3 discusses various kinds of additional (price, preference and technology) assumptions and how such assumptions can be included in the analysis. Section 4 subsequently considers how we could recover the technology and forecast firm behavior in new economic situations. Section 5 discusses some computational issues. The new tools are illustrated using a numerical example in Section 6. Finally, Section 7 provides a discussion of our results as well as directions for further research. For expositional convenience, we focus on the profit maximization hypothesis exclusively. However, a straightforwardly analogous treatment applies to the less restrictive cost minimization and revenue maximization assumptions.

2. TESTING FOR CONSISTENCY WITH PROFIT MAXIMIZING BEHAVIOR

INCLUDING PRICE ENDOGENEITY AND UNCERTAINTY: A THEORETICAL APPROACH

We analyze the optimizing behavior of n firms under restrictions for the production possibilities and the price formation process. Specifically, firms select a (non-zero) netput vector $y = (y^1 ... y^q) \in T$ from the production possibility set $T \subseteq \Re^q$. Positive components of y represent outputs and negative components represent inputs. We assume that the decision on the netputs to be produced must be taken prior to the sales date, at which the market prices become known. The beliefs of firm j about the prices are summarized by a subjective conditional distribution function $F_j(\cdot|y): \Re^q_+ \to [0,1]$, $j \in \{1,...,n\}$, which assigns a cumulative probability density to (non-zero) price vectors $p = (p^1 ... p^q)^T \in \Re^q_+$ conditional upon the selected netput vector $y \in \Re^q$. Note that we deviate from the traditional framework by using a conditional distribution function to represent the price formation process, hence allowing for uncertain and endogenous prices. Also note that we use a firm-specific distribution function, so as to allow for differences in the economic microenvironment.

Following McCall (1967) and Sandmo (1971), among others, we assume that firm preferences can be represented in expected utility form, with a Von Neumann-Morgenstern utility function $U_j(\cdot):\Re^1 \to \Re^1$ that is defined over profit *yp*. This may appear a strong assumption. In recent years, non-expected utility theories for individual decision making have become increasingly popular (see Starmer, 2000, for a review). Moreover, many firm decisions are taken by a group of individuals, and group preferences may not always satisfy the transitivity axiom required for the existence of a Von Neumann-Morgenstern utility function. Nevertheless, the expected utility framework remains a standard analytical tool, mainly because of its analytical tractability. Furthermore, there are many firms in which essentially one person makes the decisions, and there are presumably many firms in which preferences are sufficiently similar within the group of decision makers to guarantee the existence of a group preference function. Finally, we emphasize that the below tests also apply for a whole range of non-expected utility theories of choice behavior under uncertainty.

Under the above assumptions, the optimal netput vector for the *j*-th firm is obtained as the solution to the following constrained optimization problem:

$$(1) \qquad \max_{y \in T} \int_{p \in \Re_+^q} U_j(yp)\partial F_j(p|y).$$

Hence, the following statistic can test whether the observed behavior of the *j*-th firm, say y_j, is consistent with constrained optimizing behavior:

$$(2) \qquad \theta(y_j, F_j, U_j, T) = \max_{y \in T} \int_{p \in \Re_+^q} U_j(yp)\partial F_j(p|y) - \int_{p \in \Re_+^q} U_j(y_j p)\partial F_j(p|y_j),$$

Obviously, a necessary and sufficient condition for optimal firm behavior is $\theta(y_j, F_j, U_j, T) = 0$.

If complete information about the price distribution (F_j), the firm preferences (U_j) and the production possibilities (*T*) were available, we could readily compute $\theta(y_j, F_j, U_j, T)$. However, in practice such complete information is typically not available and only necessary tests for optimal behavior can be designed.

INCLUDING PRICE ENDOGENEITY AND UNCERTAINTY: AN EMPIRICAL APPROACH

We construct necessary tests for optimal firm behavior by gradually weakening the informational requirement. We focus on minimal assumptions with respect to the price distribution, the objectives of the firm, and the production technology. Section 3 discusses various kinds of additional (price, preference and technology) information and how that information can be included in the analysis, so as to increase the discriminating power of the tests.

First, to reduce the informational requirement for the price distribution (F_j), we assume that for each firm *j* a price domain D_j is observed that contains all price vectors that have a strictly positive probability at some feasible netput vector, i.e. $\Re^q_+ \supseteq D_j \supseteq \{p \in \Re^q_+ | \exists y \in T : \partial F_j(p|y) > 0\}$. It is often possible to construct $D_j \subset \Re^q_+$ by exploiting some minimal application-specific information. For example, economic theory suggests that the cost of equity capital exceeds that of debt because equity involves more risk for the capital suppliers than debt does (see e.g. Kuosmanen and Post, 1999a). Similarly, we could use the stylized fact that the wage rate for white-collar workers is higher than that for blue-collar workers.

Using D_j we obtain the following conservative test statistic

(3) $$\vartheta(y_j, D_j, U_j, T) = \max_{y \in T} \min_{p \in D_j} U_j(y\ p) - \max_{p \in D_j} U_j(y_j\, p).$$

This statistic bounds $\theta(y_j, F_j, U_j, T)$ from below, i.e. $\vartheta(y_j, D_j, U_j, T) \leq \theta(y_j, F_j, U_j, T)$. Hence, a necessary condition for optimal behavior is $\vartheta(y_j, D_j, U_j, T) \leq 0$.

Usually, the specification of firm preferences (U_j) is also problematic. Let us only assume that firm utility is monotonically increasing in profit, i.e. $U_j(z) \geq U_j(z') \quad \forall z, z' \in \Re : z \geq z'$. Then

(4) $$\rho(y_j, D_j, T) = \max_{y \in T} \min_{p \in D_j} y\ p - \max_{p \in D_j} y_j\, p > 0$$

implies $\vartheta(y_j, D_j, U_j, T) > 0$, and we get $\rho(y_j, D_j, T) \leq 0$ as a necessary condition for optimal firm behavior. Since U_j is assumed to be increasing in profit, we can also refer to such optimal firm behavior as (ex ante) “profit maximizing” behavior.

Finally, a full specification of the production set T is normally not available. However, an empirical approximation can be obtained from observed firm behavior, say $S=\{y_1,...,y_n\}$. We will adhere to the standard assumption that the observed netput vectors are feasible, i.e. $S\subseteq T$. (Note, however, that our approach can be extended to include measurement error along the lines of e.g. Varian (1985).) Since $S\subseteq T$, we can use S as an empirical production set. Specifically, $\rho(y_j,D_j,S)\leq 0$ gives a necessary condition for optimal behavior, because $S\subseteq T$ implies $\rho(y_j,D_j,S)\leq \rho(y_j,D_j,T)$. We will mainly use this minimal condition in the remainder of this paper.

Interestingly, this condition has a first-order stochastic dominance interpretation, because $\rho(y_j,D_j,S)>0$ directly implies that for some specification of F_j

(5) $$\exists y\in T: \int_{\{p\in\Re^q_+ | yp\leq w\}} \partial F_j(p|y) \leq \int_{\{p\in\Re^q_+ | y_j p\leq w\}} \partial F_j(p|y_j) \quad \forall w\in\Re^1,$$

with strict inequality for at least one $w\in\Re^1$, i.e. the evaluated netput vector y_j is first-order stochastically dominated by another feasible vector. Consistency of choice behavior with the first-order stochastic dominance criterion is in fact equivalent to monotonically increasing U in the expected utility framework (Hadar and Russell, 1969). However, it is also widely accepted as a choice criterion in non-expected utility theories. Moreover, it is supported well by empirical evidence. As Starmer (2000) summarizes, first order stochastically dominated options are usually not selected when stochastic dominance is transparent. This holds a fortiori for the even weaker decision criterion we end up with in our empirical tests, i.e. $\rho(y_j,D_j,S)\leq 0$.

3. ADDITIONAL PRICE, PREFERENCE AND TECHNOLOGY ASSUMPTIONS

The test statistic $\rho(y_j, D_j, S)$ as defined above only involves very weak assumptions concerning the price distribution (specifying the price domain D_j), firm preferences (monotonicity, or alternatively consistency with the first-order stochastic dominance criterion) and technology (observed netput choices $S \subseteq T$). Actually, the above optimality test simultaneously tests for optimizing firm behavior and these weak assumptions. That is, violations of the minimal profit maximization conditions can reflect inappropriate assumptions about price domains, preferences and/or technology. Still, we emphasize that the profit maximization conditions derived above are much weaker than the conditions traditionally considered in non-parametric production analysis. Sometimes it is possible to formulate stronger price, preference and technology assumptions, which entail more stringent tests with more discriminating power.

Conversely, we could employ the above-described general framework for testing more stringent hypotheses. Specifically, if we assume optimizing behavior, we can test a particular set of assumptions by comparing the test results for a model that imposes these assumptions with a model that does not impose these assumptions. If the test results do not change (i.e. all observations pass the optimality tests in both instances), then the set of assumptions cannot be rejected. In addition, given that more stringent assumptions imply more powerful tests, one could also specify price, preference and technology assumptions taking into account the power of the concomitant optimality tests with respect to (arbitrary) alternative hypotheses of non-optimal behavior, in the spirit of Bronars (1987).

In the following, we discuss a number of price, preference and technology assumptions that are frequently maintained in applied and theoretical work. While this list is far from exhaustive, it clearly illustrates the generality of our framework.

PRICE ASSUMPTIONS

In the extreme case no information is available about the price formation process and $D_j = \Re^q_+$. It is immediate that the condition $\rho(y_j, D_j, S) \leq 0$ would always be met. Profit maximizing firm behavior cannot be rejected, which is intuitive precisely because of the absence of information. At a minimal level we can specify the "generic" price domain $D_j \subset \Re^q_+$, as we have considered in the previous section. At a maximum level, the conditional distribution function F_j is fully specified. This immediately allows us to perform the first-order stochastic dominance test (see (7)). A less stringent informational requirement is to specify non-generic price domains that depend on the specific netput values, i.e. at each *y* the price domain can be characterized as $D_j(y)$. In addition, under price endogeneity but price certainty these sets $D_j(y)$ will be singletons (as in Varian, 1984, section 10). A straightforward extension of the analysis above gives the appropriate test statistic. Of course, the specification of the endogenous price system, and thus of these non-generic price domains, is typically an application-specific matter.

Alternatively, we can postulate that prices are exogenous but uncertain at the time of decision making. Every two netput vectors are then to be compared at the same price vector, and we get

(6) $$\rho^X(y_j, D_j, S) = \max_{y \in S} \min_{p \in D_j} \left(y \; p - y_j \, p \right).$$

See also Kuosmanen and Post (1999b). Furthermore, firms may face price endogeneity on some input or output markets while prices are exogenous on other markets. The corresponding test statistics are straightforward combinations of those in (4) and (6).

Finally, when prices are exogenous and certain, the price domain $D_j = \{p_j\}$, and we obtain the conventional test statistic for profit maximizing firm behavior (see e.g. Varian, 1984)

(7) $$\rho^X(y_j, D_j, S) = \max_{y \in S} \left(y \; p_j - y_j \, p_j \right).$$

PREFERENCE ASSUMPTIONS

A full characterization of *U* immediately allows us to compute $\vartheta\left(y_j, D_j, U_j, S\right)$ or, under a fully specified F_j, even $\theta\left(y_j, F_j, U_j, S\right)$. Weaker preference assumptions can equally well be implemented. For example, a frequently maintained assumption is that firms are *risk neutral* so that only expected profit matters. In that case firm *j*'s netput selection is always evaluated at the same (expected) price vector when comparing it to other possible netput choices. Under price endogeneity this does not change the formal structure of the optimality test (see (4)). However, (when D_j is convex) combination with price exogeneity yields instead of (6)

(8) $$\rho^{XN}(y_j, D_j, S) = \min_{p \in D_j} \max_{y \in S} \left(y \ p - y_j p \right).$$

Evidently, when complete information about F is available, we can determine the (unique) expected price vector, and we get a construction that is formally similar to (7). Clearly, while we will not explore this in detail in this paper, other restrictions regarding risk preferences can be included through specification of the Arrow-Pratt measure of absolute risk aversion or the measure of relative risk aversion.

TECHNOLOGY ASSUMPTIONS

Additional technological assumptions can also be imposed and tested. For example, monotonicity of *T* (or free disposability of inputs and outputs) is frequently assumed. We can impose this assumption by replacing the set of observations *S* by its monotone hull (i.e. $m(S) = S - \Re_+^q$), so as to increase discriminating power. In addition, convexity of *T* can be imposed by using $c(S)$, the convex hull of *S*, and monotonicity and convexity can be imposed simultaneously by using the convex monotone hull $c(m(S))$. In addition, other technology properties (e.g. returns-to-scale specifications) can be investigated. (See e.g. Färe *et al.* (1994) for an overview of possible technology representations that build on the set *S* and some additional technology postulates.)

Finally, homogeneity (constant returns-to-scale), homotheticity and separability conditions (and tests) can be implemented, like for example in Varian (1984).[1]

It is worth to emphasize that the role of monotonicity and convexity assumptions under conditions of endogenous and uncertain prices is very different from that in the neoclassical setting with exogenous and certain prices. Varian (1984; following Samuelson, 1947, and Hanoch and Rothschild, 1972) demonstrated that monotonicity and convexity assumptions are *harmless* (i.e. do not interfere with the test results) if prices are exogenous and certain. In that case, the objective (profit) is an increasing and linear function of the netputs.

More generally, monotonicity and convexity of the constraint set are harmlessly imposed if the objective function is monotonically increasing and quasi-convex. Unfortunately, this condition is rather stringent for the alternative price conditions discussed in this paper. For example, risk aversion can violate quasi-convexity. However, many elementary facts of economic life seem to indicate a prevalence of risk aversion (the standard assumption for utility functions is (quasi-)concavity rather than quasi-convexity!). Therefore, in general we cannot safely replace the set S with $m(S)$, $c(S)$ or $c(m(S))$ in the tests discussed in this paper.[2]

[1] To keep our discussion focused, we will not explore these technology properties in detail here. The extensions of the Varian results are obvious for the homogeneity and homotheticity properties. Extending the separability results is more complicated (e.g. we would need an appropriate redefinition of the "Generalized Axiom of Revealed Preferences" (see also Varian, 1982)).

[2] Note that monotonicity becomes a harmless regularity property if a generic price domain is used for all firms. However, in the case where non-generic price domains are employed, monotonicity cannot be imposed without harm in general.

Interestingly, there is no a priori reason why technologies should be monotone or convex, although such properties are frequently imposed. For example, Farrell (1959) stresses indivisible netputs and economies of scale and specialization as possible sources of non-convexities. In addition, Färe and Grosskopf (1983) stress congestion of netputs as possible violation of monotonicity. McFadden (1978, pp. 9) explicitly states that the appeal of monotonicity and convexity assumptions in microeconomic production theory "lies in their analytical convenience rather than in their economic realism."

To conclude, as for price and preference assumptions, including additional production assumptions can increase the discriminating power of the tests. However, it also introduces the risk of specification error, i.e. the profit maximization hypothesis may be wrongly rejected because of erroneous production assumptions. Since it is difficult to verify a priori frequently imposed technology properties such as monotonicity and convexity, we see the possibility to expose these technology properties to (non-parametric) empirical tests as an attractive by-product of our approach.

4. Recoverability and Forecasting

Varian (1984) emphasized alternative uses of the non-parametric approach in addition to testing for profit maximizing firm behavior, viz. recovering the production set and forecasting firm behavior under alternative price scenarios. Chavas and Cox (1995), for example, applied the non-parametric approach for these uses to a real-life data set. Recoverability and forecasting questions can also be addressed within the general framework that is discussed in this paper. To ease the exposition, our discussion will concentrate on the minimal test obtained in section 2, but straightforward extensions apply to the refinements discussed in section 3.

RATIONALIZATION

Varian (1984) started from a concept of data rationalization. We generalize that concept towards settings that are possibly characterized by price uncertainty and endogeneity. Using $\Delta = \{D_1,..., D_n\}$, a production set $X \subseteq \Re^q$ *rationalizes* the data set *S* if and only if all observations pass the optimality test, i.e.

(9) $$\rho(y_j, D_j, X) \leq 0 \quad \forall (y_j, D_j) \in S \times \Delta .$$

Since we adhere to the standard assumption $S \subseteq T$, our following discussion will center on cases where the data set 'rationalizes itself', i.e. $\rho(y_j, D_j, S) \leq 0 \quad \forall (y_j, D_j) \in S \times \Delta$.

RECOVERING TECHNOLOGY

Generally, multiple empirical production sets can rationalize the data set. It is therefore interesting to "bound" these sets. By assuming $S \subseteq T$, we directly find the observed netput vectors *S* as an inner bound. The outer bound should include all production vectors $y \in \Re^q$ so that $\{y\} \cup S$ still rationalizes *S*. We can define it as

(10) $$\Gamma(S, \Delta) = \{ y \in \Re^q \,|\, \rho(y_j, D_j, \{y\} \cup S) \leq 0 \quad \forall (y_j, D_j) \in S \times \Delta \}.$$

In effect, any production set $S \subseteq X \subseteq \Re^q$ that rationalizes the observed set of firms must satisfy $X \subseteq \Gamma(S,\Delta)$.

FORECASTING

The consistency tests as developed above can also be employed to non-parametrically forecast firm behavior. Like in Varian (1984) an exact prediction cannot be obtained, but rather the widest range of choices that is consistent with the previously observed (ex ante profit maximizing) behavior is derived. Specifically, for a given price domain $D \subseteq \Re^q_+$, this range is represented by the set

(11) $$P(D,S,\Delta) = \Gamma(S,\Delta) \cap \{ y \in \Re^q \,|\, \rho(y,D,S) \le 0 \}.$$

The set $P(S,D,\Delta)$ contains all vectors $y \in \Re^q$ that belong to $\Gamma(S,\Delta)$ and that are consistent with the profit maximization hypothesis for the given price domain *D* when compared to the observed sample *S*.

Section 6 illustrates these alternative uses of the consistency tests using a numerical example.

5. COMPUTATIONAL ISSUES

To conclude our formal exposition, we briefly elaborate on the computation of the test statistic $\rho(y_j, D_j, S)$, which is also relevant for the construction of the sets $\Gamma(S,\Delta)$ and $P(S, D, \Delta)$. Linear Programming tools suffice if the price domain is formulated in terms of linear inequalities. For example, the following convex cone

(12) $$D_j = \left\{p \in \Re_+^q \middle| A_j p \geq b_j \right\},$$

represents the price domain in terms of *l* linear inequalities, with A_j a $l \times m$ matrix and b_j a $l \times 1$ vector. Such a cone can often be constructed in practice. For example, a single linear inequality can represent the assumption that the cost of equity capital exceeds that debt capital, and that the wage rate of white-collar workers exceeds that of blue-collar workers. Interestingly, such convex cones are applied extensively in Operations Research/Management Science applications of the non-parametric productivity and efficiency analysis (sometimes dubbed Data Envelopment Analysis), starting with Charnes *et al.* (1990) (see e.g. Allen *et al.* (1997) for an insightful survey). Kuosmanen and Post (1999a) discuss how to use the convex cones for measuring economic efficiency in the traditional setting with exogenous and certain prices.

When the price domain is formulated in terms of linear inequalities, we can compute $\rho(y_j, D_j, S)$ (if defined) from the maximum of the solutions to the following *n* linear programming problems:

(13) $\min_{p\in D_j} yp = \min_{p\in \Re^q_+}\left\{yp \middle| A_j p \geq b_j\right\} \quad \forall y \in S$,

and the solution to the following Linear Programming problem:

(14) $\max_{p\in D_j} y_j p = \max_{p\in \Re^q_+}\left\{y_j p \middle| A_j p \geq b_j\right\}$.

Consequently, the outer bound approximation

(15) $\Gamma(S,\Delta) = \left\{ y \in \Re^q \middle| \min_{p\in D_j} yp \leq \max_{p\in D_j} y_j p \quad \forall (y_j, D_j) \in S \times \Delta \right\}$

can be characterized by a set of linear constraints. Finally, for given *D*, the specification of the set $P(S, D, \Delta)$ is analogous to that of $\Gamma(S,\Delta)$.

6. NUMERICAL EXAMPLE

To illustrate the above concepts and tools, consider a sample of three firms $S = \{y_1, y_2, y_3\}$ that operate under a single input-single output technology. For expositional convenience, we concentrate on the minimal test outlined in section 2. Table 1 displays the input and output data and the price domains for the three firms. Clearly, $\rho(y_j, D_j, S) \leq 0$ for $j = 1,2,3$ and thus the data set at least rationalizes itself.

The largest production set that rationalizes the data set (i.e. the outer bound technology approximation) is formally defined as

(16) $$\Gamma(S,\Delta)=\left\{y\in\Re^2 \middle| y_1+y_2\leq 1; y_1+2y_2\leq 6; y_1+3y_2\leq 12.5; y_1\leq 0; y_2\geq 0\right\}.$$

This set is displayed in figure 1. This figure also clearly reveals that (ex ante) profit maximizing production vectors need not lie on the technically efficient boundary of the technology approximation if one departs from the neoclassical price conditions. Essentially, this occurs because profit may increase when netput amounts decrease under endogenous prices.

j	(y_j^1, y_j^2)	D_j
1	(-4,2)	$\left\{p\in\Re_+^2 \middle| p^1=1;1\leq p^2\leq 2.5\right\}$
2	(-3,3)	$\left\{p\in\Re_+^2 \middle| p^1=1;2\leq p^2\leq 3\right\}$
3	(-5,5)	$\left\{p\in\Re_+^2 \middle| p^1=1;3\leq p^2\leq 3.5\right\}$

Table 1 Example data set

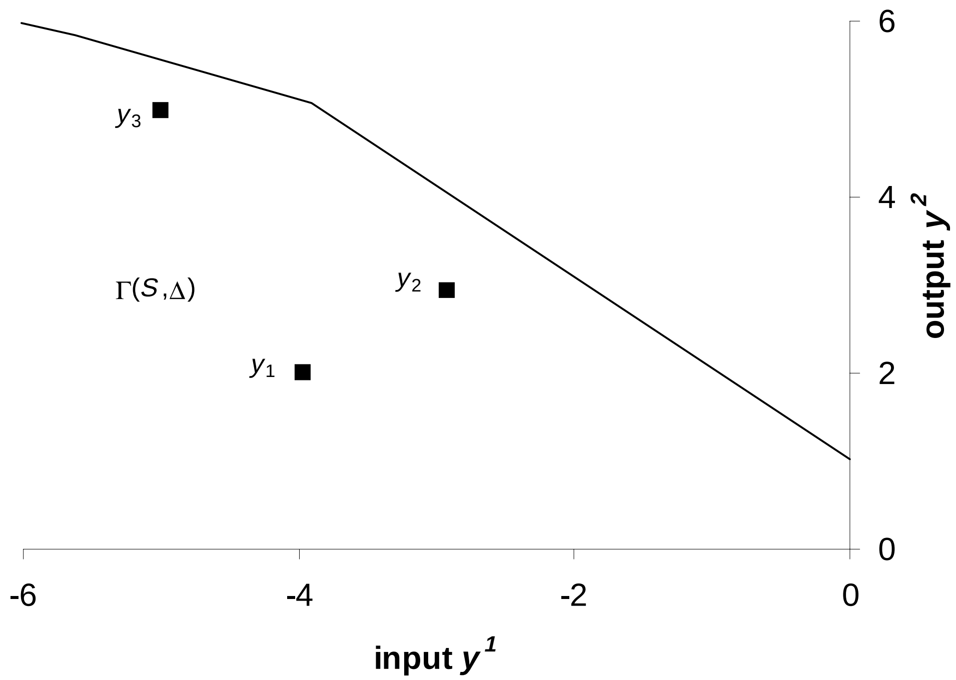

Fig. 1 Recovering technology

Next, suppose that we want to forecast firm behavior for the price domain $D_A = \{p \in \Re^2_+ | p_1 = 1; 0.25 \leq p_2 \leq 0.5\}$. All netput choices that are consistent with observed past behavior are contained in:

(17) $$P(D_A, S, \Delta) = \Gamma(S, \Delta) \cap \{y \in \Re^q | y_1 + 0.5y_2 \geq -2.25\},$$

Figure 2 displays this set as the dark shaded area. As the price domain shrinks, the predictions will become more accurate. For example, for $D_B = \{(1, 0.5)\}$ we have

(18) $$P(D_B, S, \Delta) = \Gamma(S, \Delta) \cap \{y \in \Re^q | y_1 + 0.5y_2 \geq -1.5\},$$

which is displayed in Figure 2 as the light shaded area. Obviously, $P(D_B, S, \Delta) \subset P(D_A, S, \Delta)$.

To conclude, we note that the outer bound approximation will generally become smaller when the number of observations in the sample increases, as the number of inequalities that determine the outer bound will increase. As a result, more precise predictions of firm actions can be made.

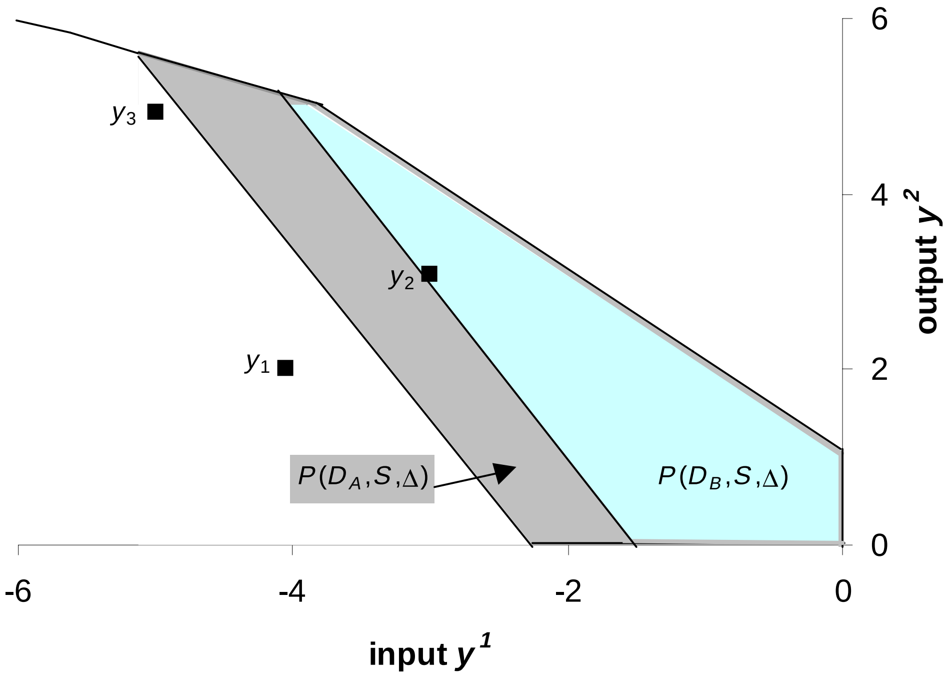

Fig. 2 Forecasting firm behavior

7. DISCUSSION

We have introduced non-parametric tests for the hypothesis that firms (ex ante) seek to maximize profits, which also apply when prices are endogenous and/or uncertain. In addition, we have demonstrated how technology information can be recovered and how firm behavior can be forecasted non-parametrically. Thus, even when we account for the possibility of endogenous and uncertain prices, non-parametric analysis of the production process remains feasible and essentially the same kind of questions can be addressed as under the (standard) neoclassical price conditions.

By building on a minimal set of maintained assumptions, we minimize Type I errors, i.e. the probability that the profit maximization hypothesis is wrongly rejected. However, violations of the optimality conditions may still be observed. We see two alternative interpretations for such violations:

1. Our maintained assumptions may be wrong. We have focused on a minimal set of maintained assumptions, and (when compared to the conventional approach) we excluded imperfect competition or price uncertainty as possible sources of Type I error. Nonetheless, some of our assumptions may still lead to erroneous rejections of the profit maximization hypothesis. We see at least the following four different Type I errors:

   A. We adhered to the standard assumption that the observed data set represents feasible production vectors (i.e. $S \subseteq T$). Measurement error and omission of input or output variables can violate this assumption. We could therefore extend to this new setting established tools that account for statistical significance (Varian, 1985; and Matzkin, 1994) and economic significance (Varian, 1990) of violations of the test results.

B. We may doubt whether firms in endogenous price settings are really interested in maximizing profits. While in a neoclassical framework maximum profit is usually accepted without question as the right objective for a firm, matters become more complicated under imperfect competition, as already pointed out by Marshall (1922, p. 402). The main argument is that firm owners are not interested in monetary profit as such but rather in its purchasing power. Owner preferences as consumer may interfere with owner preferences as producer. Grodal (1996) has more recently emphasized this point. Nevertheless, profit maximization is generally maintained as a behavioral assumption when modeling firm behavior under endogenous prices (see e.g. Hart, 1985). Also for this reason, it is interesting to expose the profit maximization hypothesis under price endogeneity to empirical tests.

C. We have assumed throughout that a price domain can be specified that reflects the possibly endogenous and uncertain price formation process. If no specific price information is included, our tests lose discriminatory power. However, an erroneously specified price domain may lead to erroneously rejecting the profit maximization hypothesis. Hence, it is important to formulate the maintained price assumptions (as reflected in the price domain) with sufficient caution. Alternatively, as we have illustrated, we could use our testing tools to reconstruct the price mechanisms faced by the firms under evaluation.

D. Finally, although we accounted for price uncertainty, we have held on to a deterministic technology. That is, we have assumed throughout that the output amounts produced and the input quantities consumed by firms were perfectly certain. For some industries (e.g. agriculture), this is not a very realistic assumption. This calls for developing testing tools that take such quantity uncertainty explicitly into account. Such tools could for example be constructed along similar lines as those followed in this paper. For example, the argument would be straightforwardly reversed if prices were the perfectly controllable decision variables and netput quantities the uncertain random variables.

2. On the other hand, when we could reasonably conjecture that our maintained assumptions hold, remaining violations of the profit maximization conditions we have set out can be interpreted as truly profit inefficient behavior (compare with Banker and Maindiratta, 1988). Such firm-level inefficiency could for example be rationalized by relating it to agency problems within the firm. That is, the firm may not act in accordance with profit maximization because the firm managers, who pursue different goals, are not fully controlled by the firm owners. In effect, negative test results do not immediately indicate “irrational” behavior, as the firm managers may act rational. Rather, they suggest inconsistency of firm behavior with the owner preferences. In this respect, it seems worthwhile to explore whether and to what extent the testing tools could be employed as monitoring instruments by the firm owners (compare with Bogetoft, 1994).

A second issue concerns the discriminating power of our tools. Clearly, the use of minimal assumptions can reduce such discriminating power. Still, our analysis can serve as a starting point for models that include additional information. In that respect, we have shown that our tools are very flexible in that they allow for implementing a whole range of alternative price, preference and technology assumptions. In fact, our general framework can be employed for obtaining evidence in favor of or against certain price, preference and technology specifications.

To conclude, we point at the analogy between producer and consumer behavior. For example, both the Afriat (1972) and Varian (1984) contributions have twin papers that focus on non-parametric demand analysis (respectively Afriat, 1967, and Varian, 1982). This might then again result in less stringent but possibly more realistic tests for "rational" consumer behavior, and consequently contribute to a better understanding of demand behavior.

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