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DISCUSSION
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FACTOR ANALYSIS VS. FUZZY SETS THEORY: ASSESSING THE INFLUENCE OF DIFFERENT TECHNIQUES ON SEN'S FUNCTIONING APPROACH

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Abstract

This paper explores a couple of specific operational interpretations of Sen's approach in view of assessing the extent to which the results originated by the implementation of Sen's concepts are influenced by the choice of the specific technique. By means of a survey based on a representative sample of Belgian individuals, seven achieved functionings are identified via each technique and subsequently confronted. To structure the information and to facilitate comparisons, standard multivariate analysis is performed, while at the same time considering in more detail the sub-group of the most deprived individuals. In this way, a substantial accordance - yet no perfect equivalence - is uncovered in the general patterns of functionings' achievements.

1 Introduction

A major feature of Sen's theory, characterising well-being as a rich and multifaceted concept, unquestionably lies in its conforming to intuitive notions of what it means to be better off or worse off. Essentially, this account confronts anybody who is willing to assign some kind of concrete shape to functionings and capabilities with a frame of analysis involving, however, analogous issues if compared with the concepts of the living standard traditionally used in practical work. In spite of this, when it comes to working definitions and measurements, its informational richness is often regarded as a worrisome feature; a belief perhaps likely to account for the quite limited number of existing applications.

These few studies have nevertheless proven to be rather fruitful in terms of the array of suggested techniques. As Atkinson (1999, p. 185) has stressed, “there is more than one way in which an idea of this kind [Sen's framework] can be operationally effective”, and the truthfulness of such statement is, indeed, easily validated by the variety of options that have been explored up to now, in various fields (from poverty to inequality analysis) and starting from various points of view (micro vs. macro level). Focusing on micro applications, the literature has been basically ranging in between the use of simple descriptive statistics, multivariate methods and the “fuzziness” approach. The adoption of the two latter methodologies has been repeatedly endorsed by Sen himself (1990, 1994 and 1996), emphasising their suitability for his framework. Accordingly, most applications nowadays rely on either of the two techniques.

Meanwhile, increasing attention has been devoted to Sen's approach - at least in some countries - also by public institutions. At a more general level, the advent of the Human Development Index further substantiates the previous claim while stressing the importance of performing supplementary investigations aimed at assessing the reliability of the results emerging

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from the different methodological approaches. This represents, indeed, both the rationale and the purpose of this paper, *i.e.* ascertaining, by means of an illustrative empirical exercise, to what extent the results originated by the implementation of Sen's concepts are influenced by the choice of the specific technique, where the latter takes the form of either factor analysis or fuzzy sets theory. Such aim implies the fundamental idea that procedural decisions should, ideally, affect in no way the overall results, as any major discrepancy could bring about considerable normative implications, not to mention the potentially significant consequences it could entail in terms of political choices whenever the public institutions' interest towards this approach turned into actual endorsement.

The remainder of the paper, therefore, is organised as follows. After having sketched the basic notions underlying the capability approach in Section 2, the theoretical foundations of factor analysis and fuzzy sets theory will be reviewed in Section 3, while briefly discussing the existing literature on their use in association with well-being measurement. Section 4 will be devoted to the comparative analysis. Section 5 will conclude by summarising the findings.

2 The core concepts of the capability approach

The interpretation of the notion of well-being as not exclusively associated to affluence but to each one's abilities indeed represents a major feature characterising Amartya Sen's theory[1]. In Sen's view, the capacity of profiting from the available resources is greatly influenced by their utilisation, which in turn depends upon the specific circumstances experienced by the given individual. Hence, goods are not valued as a consequence of their being possessed; rather, they are evaluated on the basis of the effects that they may engender on the individual, effects that are likely to differ rather seriously according to the various circumstances under which those goods are exploited.

A couple of issues commonly disregarded by traditional welfare economics make up, in fact, the foundations of Sen's thought. First, the deep human diversity, which affects individual well-being through both personal characteristics' heterogeneity (*e.g.* intelligence, age, gender, physical conditions) and the specific socio-environmental conditions (*e.g.* economic infrastructures, social norms, public services). Next, the intrinsic value of one's freedom to choose and achieve. As a consequence of the alleged inadequacy of goods' availability as a measure of well-being, Sen maintains that although the provided goods could exactly coincide with the ones that would have been selected had the individual had the opportunity to choose, the quality of the life that receives the goods but has not chosen them will be reduced by the mere absence of such a right. "Acting freely and being able to choose are, in this view, directly conducive to well-being" (Sen, 1992, p.51).

Adopting Sen's particular terminology, the two above-mentioned issues may be translated by the notions of *functioning* and *capability*. Functionings are the various doings and beings of a person, the achievements of an individual determined by the particular way in which he is able to "let the available goods function". Formally, following Sen (1985), letting x_i be a vector of commodities possessed by person i and selected from the consumption set X_i, c be the function converting the commodity vector into a vector of characteristics of those commodities[2] (thus, the vector of characteristics consumed by person i will be given by $c(x_i)$) and u_i a utilisation function chosen by person i in the set U_i reflecting the specific use of the commodities that the person can make in generating a functioning vector out of the characteristics of the given commodities and in association with his actual abilities, person's i functioning b_i will be expressed as

$$b_i = u_i\,(c(x_i)) \qquad (1)$$

[1] The terms "well-being" and "standard of living" will be used interchangeably, in spite of the distinction made by Sen. For sake of accuracy, the focus in this work will be on what he defines as "standard of living", namely those elements relating to the nature of a person's life.

[2] Sen adopts, in fact, the Gorman-Lancaster's perspective, which enables him to see every commodity in terms of the vector of its characteristics.

Capabilities, instead, are intended to portray one's freedom to choose what kind of life to live and should, therefore, depict the actual autonomy in pursuing and achieving those doings and beings one deems valuable. Depending on two main factors - namely, the consumption set of the person (*i.e.* the set X_i) and the ability to convert commodities into achievements (*i.e.* the set U_i) - capabilities can be described as

$$Q_i = \{b_i | b_i = u_i(c(x_i)), \text{ for some } u_i(\cdot) \in U_i \text{ and some } x_i \in X_i\} \qquad (2)$$

In spite of its intuitive appeal, defining the value of the set Q_i has fairly significant and disturbing implications in that the notion of capability implies that hypothetical situations have to be taken into account when evaluating one's living standard. This makes, of course, the measurement of one's capabilities far more difficult than the measurement of one's actual functionings. As a consequence, in what follows Basu's (1987) suggestion will be taken on and the analysis will exclusively focus on functionings as indicators of one's living standard.

3 Some theoretical remarks on the adopted methodologies

Before turning to the empirical application, some remarks concerning the theoretical foundations of the two techniques I'm going to confront probably ought to be made. Accordingly, in what follows the major assumptions and procedures characterising factor analysis and fuzzy sets theory will be sketched, while briefly reviewing the existing literature on their use in association with well-being measurement.

3.1 Identifying functionings by factor analysis

Factor analysis represents a data reduction technique very often used in empirical research in the social sciences with the purpose of summarising the available data (usually test scores or questionnaire responses) in such a way that the researcher can easily grasp the empirical relationships among the variables under consideration. In doing so, factor analysis relies on the assumption that the observed variables are linear combinations of some common underlying dimensions, known as “factors”.

The factor analytic technique exploits the presumed correspondence between the system of latent factors and the set of observed variables in order to identify the separate dimensions for the given data and determine the extent to which each variable is explained by each dimension. Once this has been done, its two primary goals - summarisation and data reduction - can be achieved[3]. In assessing individual living standards, such methodology is thus considered as a helpful device for solving the problem of defining a limited number of well interpretable dimensions of well-being. The specific application of this technique to Sen's capability approach basically implies assessing the empirical validity of the following model and subsequently interpreting the different factors in terms of functionings

$$a_{ij} = l_{1j} f_{i1} + l_{2j} f_{i2} + \ldots + l_{pj} f_{ip} + u_{ij} \quad i = 1,\ldots,N; j = 1,\ldots,n. \qquad (3)$$

which, in matrix notation, reduces to

$$A = \Lambda F + U \qquad (4)$$

where A captures the matrix providing the answers of the i respondents on the j items, F represents the matrix of “factor scores” giving the position of each respondent on the p functionings, Λ captures the matrix of “factor loadings” showing the association between the answers on the items and the position of the respondents on the functionings and U is the matrix of

[3] In summarising the original data, in fact, factor analysis ascertains the minimum number of hypothetical dimensions that can account for the observed correlations among the variables, which - when interpreted and understood - can describe the data in a much smaller number of items than the original variables, namely by a composite variable for each dimension.

the residual terms. Each questionnaire response can, thus, be interpreted as a linear combination of a number of common factors contributing to the variance of different observed variables, and of one specific unique hypothetical factor including influences which affect only a specific variable and any other sources of bias.

The determination - carried out on the basis of the matrix A - of the factor loadings constitutes the first and major step of the whole procedure. The information provided by the matrix Λ is what makes the identification of the single functionings possible: since factors serve as predictors in deriving the observed variables and they are assumed - as it will be argued in the subsequent paragraph - to be uncorrelated with each other, the loadings may be interpreted as both regression weights and as correlation coefficients[4]. Hence, they can be seen as indicating the degree of correspondence between each item and the functioning, with higher loadings making the item representative of the functioning. This points to the fact that the derivation of such loadings - as well as of the subsequent factor scores - is performed in accordance with a number of preliminary assumptions whose fundamental implication is that the latent factor scores wholly explain the correlations among the observed variables and, thus, their estimation has to take place in such a way that they account for the maximum possible amount of the variance of the variables being considered[5]. Therefore, defining R_{AA} as the correlation matrix of the respondents' answers and trusting an estimate for the matrix Ψ,

$$R_{AA} = \Lambda R_{FF} \Lambda' + \hat{\Psi} \tag{5}$$

which, assuming that the common factors are orthogonal ($R_{FF} = I$), can be reexpressed as

$$R_{AA} - \hat{\Psi} = \Lambda \Lambda' \tag{6}$$

where Λ can be defined as

$$\Lambda = Q\Omega^{1/2} \tag{7}$$

Q being the matrix of the eigenvectors of $\left(R_{AA} - \hat{\Psi}\right)$ and Ω the matrix of the corresponding eigenvalues. However, once computed, the factors have to be interpreted and identified[6]. A rotation of the factors (*i.e.* a redistribution of the variance in between factors by applying an orthogonal transformation matrix T to the estimated loadings, so as to shift the original $\hat{\Lambda}$ into the new $\hat{\Lambda}^* = \hat{\Lambda}T$) providing a matrix in which each factor displays only a few high loadings is needed. The Varimax procedure - that was selected for the application - complies with such viewpoint by maximizing the variance of the squared loadings in each column of the matrix $\hat{\Lambda}^*$. This implies making the loadings on each factor either high (*i.e.* close to one) or near zero, so as to sharply indicate the association (or the lack of association) between the variables and the specific factor, thus facilitating the interpretation. Moreover, the choice of an orthogonal rotation method implies the theoretical assumption of absence of any correlation in between the rotated underlying

[4] Such coefficients basically play the same role in factor analysis as regression coefficients in regression analysis: the matrix Λ can be thought of as containing the optimal linear weights used in predicting the variables from the factors. Although in both cases the key variable is described as a linear weighted combination of another set of variables plus a residual, the traditional regression techniques postulate the observability of the set of independent variables, while factors just represent hypothetical constructs that can only be estimated from the data.

[5] Apart from the necessary hypothesis of some existing underlying structure in the system of observed variables, out of convenience all common factors F are supposed to be standardized and orthogonal; besides, the procedure requires that the residual terms U are uncorrelated with each other while at the same time not correlating with the common factors, have mean zero as well as a specific variance determined by the diagonal elements of a matrix Ψ.

[6] Initial solutions are not useful for such purpose, both because factors with many large loadings are difficult to identify, and because an almost infinite number of mathematically equivalent sets of factors exist, as the same correlation matrix may originate from many different latent configurations of loadings. This particular identification issue is known as the “rotation problem”.

dimensions, which allows to meet the well-known objections raised by Basu (1987) or Williams in Sen (1987) in what concerns the potential danger of functionings' overlapping[7].

Once the derivation and interpretation of loadings is complete, a last step usually relates to the data reduction goal. From Sen's perspective, the estimation of individual scores conveys an essential meaning, as they can be understood as describing the living standard of the respondents. The procedure (cf. Thomson, 1951) is quite straightforward: according to the basic equation (4) and, given both the observability of the matrix A and the availability of Λ, a least squares technique allows F to be estimated for each respondent. This way, each factor score will result in a linear composite of the optimally weighted variables under analysis[8].

3.1.1 Applications of factor analysis to well-being measurement

In the context of assessing "how well is a person's being", factor analysis has found increased use during the past decade. A common motivation shared by all the works making use of it seems to relate to what Sen himself (1990, p.484-485) has defined as the possibility of reducing a large list of potentially relevant elements "to arrive at a more focussed picture" of the individual's well-being. Indeed, it is common practice to try and be as exhaustive as possible when dealing with the identification of the elements affecting one's living standard from Sen's perspective; accordingly, a much too far-reaching enumeration of conceivable variables is likely to emerge - though in most cases not yet complete - to be actually workable and efficient, not to tell the risk of overlaps between variables. A major methodological rationale for the adoption of factor analysis relates, therefore, to some sort of rationalization of the doings and beings to be considered.

Such belief undoubtedly inspired Schokkaert and Van Ootegem (1990), who summarised the information contained in the 46 answers to a survey into 6 factors identified with as many specific functionings for a sample of Belgian unemployed. The relationship of these dimensions with a bundle of socio-economic features of the respondents was subsequently investigated, emphasizing the substantial irrelevance of monetary factors, while pointing to the importance of social integration as well as of companionship in determining the living conditions of the considered social group. Similarly, Nolan and Whelan (1991) used factor analysis in their study on the implications of adopting different measures of deprivation on a representative Irish sample. The technique was then adopted in studying the relationship between the various indicators as well as between the latter and standard monetary measures, thanks to an empirical exercise where elementary constituents were aggregated into three deprivation indices (oppositely to Schokkaert and Van Ootegem). More recent applications, though in the same line, are those by Balestrino and Sciclone (2000) and Delhausse (1995). The former employ factor analysis in building an index of well-being as functionings' achievement for the 20 Italian regions, the overall aim being the comparison of the resulting ranking with the ones stemming from two income-based measures. A different perspective is adopted by Delhausse, who basically applies factor analysis in the spirit of Schokkaert and Van Ootegem (1990) to discern five dimensions in judging the well-being of a sample of French individuals, and then regresses the obtained functionings on a number of personal characteristics so as to structure the factor values[9].

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[7] Stephan Klasen kindly drew my attention to the fact that Sen's approach actually does not require the different functionings to be uncorrelated. This is correct; yet, I personally subscribe to Basu's warning and believe that, from a theoretical point of view, sorting out the various components of well-being constitutes a wise precautionary measure. Moreover, out of carefulness an oblique rotation on the specific data used in this empirical application was performed as well and it did not produce any modification in the results.

[8] For a thorough description of the technical details concerning factor analysis see, *e.g.*, Harman (1976).

[9] The use of a multivariate technique akin to factor analysis - namely, principal component analysis - characterizes several papers the finest of which is probably the one by Klasen (2000). The two techniques differ in that principal components are linear combinations of the observed variables (conversely to factors) and their scores are not estimates, but perfect representations of the components. Besides, principal components account for the total variance in the dataset, rather than only for the common one. Cf. Harman (1976).

This set of applications, though limited in its size, highlights in a meaningful way how the conceptual framework elaborated by Amartya Sen can be translated into practice by exploiting the possibilities offered by multivariate techniques. In spite of its potential usefulness, the adoption of such technique by economists still remains rather confined. However, such remarks apply even more to the alternative operational interpretation of the capability approach I'm going to consider, namely fuzzy sets theory.

3.2 Identifying functionings by fuzzy sets theory

The idea of a fuzzy set is quite simple in itself. A classical set is just a container that wholly includes or wholly excludes any given element. Thus, there is no in between, no thing - for instance - that is both a day of the week and not a day of the week. In fuzzy sets theory, on the contrary, an element is allowed to partially belong to a set. Fuzzy sets are, therefore, generalizations of classical sets, in that they are classes within which the transition from membership to non-membership takes place gradually rather then suddenly. As a result, each set is characterized by a function m assigning to each of its elements a real number in the interval [0,1] taken to represent its membership value[10]. More formally, let X be a set whose elements are denoted by x, then a fuzzy set A will be defined as a mapping $m: X \rightarrow [0,1]$. Following such definition, $m(x)=0$ implies non-membership, $m(x)=1$ stands for full membership, and intermediate values between 0 and 1 denote partial membership.

In order to provide an adequate representation of a fuzzy set, one basically needs to figure out the relationship among four different but related factors: a set of elements $x \in X$ (*e.g.* "day" in "days"); a linguistic variable V representing an attribute of an element (*e.g.* weekendness); a verbal term A of a linguistic variable (*e.g.* "days of the weekend"); a subjective numerical assignment $m(x)$, representing the degree to which an element x belongs to the set of elements identified by the verbal term A. In the light of my example, the latter could be the grade of membership assigned by an individual to a day in the set of weekdays - say, Friday - when classifying the days of the weekend, or - within the context of well-being evaluation - the membership of an individual to the set A of the population with a high standard of living with respect to a specific indicator (*e.g.* the frequency of social contacts).

It is evident how this theory rests on what could be thought of as a "tricky ground" while at the same time being suitable for the operationalisation of Sen's concepts, in that it takes individual perceptions and cultural backgrounds into account when defining what pertains to a set and to what extent. Fuzzy reasoning aims, in fact, at providing models that mirror people's intuitions and thinking processes when confronted with fuzzy categories in reality. Thus, according to Dubois and Prade (1980), membership values would just be meant to represent propensity indices subjectively assigned by an individual. What is actually relevant, therefore, would be not so much the exact membership values associated to the x elements; rather, the fact that they reflect an ordering of such elements, an ordering generated by the concept the fuzzy subset is intended to represent[11]. Probably as a consequence of this, several methods for assigning degrees of membership either on a mathematical or empirical basis have been used and others proposed in the literature.

3.2.1 Selection and estimation of membership functions

It seems to be nowadays widely accepted that the only condition a membership function must really satisfy is that it has to range in between 0 and 1. The function itself is, thus, allowed to

[10]Note that the degrees of membership are just conventionally fixed, and the choice of the unit interval is, in itself, arbitrary. Besides, the set of possible membership values need not necessarily consist of numerical values; it may well be just an ordered set of verbal hedges.

[11]Alternative views expressed in the literature come from Lakoff (1973), who stresses how the membership function's specification is just supposed to capture the general intuition of the researcher about the considered attribute, thus simply providing a "not too bad approximation" of it, or from Barrett and Pattanaik (1989, p. 234), who emphasize how fuzzy sets' advocates "assign what they feel are appropriate indicator values".

take various forms, according to one's prior beliefs. More specifically, the researcher can legitimately select any curve whose shape is defined according to what suits him best from the point of view of simplicity, convenience or efficiency. In addition to this and in line with what already argued, however, a membership function also depends on the specific context to which it refers, in that it is strictly related to its domain.

In spite of the fact that the literature hasn't found any binding agreement on how the operational evaluation of degrees of membership should take place, a number of techniques have been proposed. Two main measures - distance and frequency - have proved to be helpful. Advocating an essentially subjective interpretation, the distance concept allows to estimate membership by similarity judgements: assuming the existence of an ideal perfect element in the set, the degree of membership of any other element is taken to be a function of its relative distance with respect to it. Of course, the distance function has to be specified. A widespread monotonic version is the quadratic sigmoid curve (or, alternatively, its reflected image, namely the logistic function), defined by three parameters: the zero and full membership values (α and γ, respectively) and the crossover point ($\beta = \alpha + \gamma/2$) whose membership value equals 0.5 and, hence, represents the most doubtful point with respect to the specific attribute under analysis[12]. The key parameters are determined by identifying sharp extreme conditions (*i.e.* stipulating conditions of absolute deprivation and absolute fulfilment with respect to a given variable), while the derivation of the intermediate degrees of membership for the points falling in the intervals $[\alpha,\beta]$ and $[\beta,\gamma]$ is carried out thanks to a quadratic interpolation leading to the following specification (Sanchez, 1986, p.337)

$$\mu(x) = \begin{cases} 0 & if \quad x \le \alpha \\ 2[(x-\alpha)/(\gamma-\alpha)]^2 & if \quad \alpha < x \le \beta \\ 1 - 2[(x-\gamma)/(\gamma-\alpha)]^2 & if \quad \beta < x \le \gamma \\ 1 & if \quad x > \gamma \end{cases} \tag{8}$$

What makes this representation frequently used is its adaptability - thanks to the parameters α, β and γ - to altering conditions and concepts without essentially changing the general shape of the curve. Besides, an increasing curve such as the sigmoid relies on the claim that when the belief that an element x belongs to the set A strengthens, the value of $\mu(x)$ will increase. Hence, it is best fit for representing notions like "how often do you go to the cinema?", while its logistic counterpart looks more appropriate for depicting negative concepts like "financial difficulties".

Linear membership functions are very popular in common empirical analysis as well, as they are easy to specify, interpret and visualize. These representations presuppose the variables' modalities to be equidistant from one another and assume a direct proportionality between the elements of the domain and the membership grade; a very restrictive and not always appropriate assumption. Two main classes of linear representations are prevalent: first, the basic linear function depending exclusively on the extreme values of the variable x, whose increasing version takes the simple following form

$$m(x) = \begin{cases} 0 & if \quad x = x_{\min} \\ \dfrac{(x - x_{\min})}{(x_{\max} - x_{\min})} & if \quad x_{\min} < x < x_{\max} \\ 1 & if \quad x = x_{\max} \end{cases} \tag{9}$$

Alternatively, the trapezoidal specification postulates the choice of two threshold values a_1 and a_2 (larger than the minimum and smaller than the maximum) corresponding to the modalities of the variable x beneath which a condition of low living standard is evident and above which a situation of high living standard can undoubtedly be assumed, respectively

[12] This formulation, of course, strongly restricts the nature of the set X, implying that the latter be both ordered and equipped with a multiplication and an addition operator. The graphical representations of the μ functions are given in Appendix B.

$$m(x) = \begin{cases} 0 & if \quad x \le a_1 \\ \dfrac{(x - a_1)}{(a_2 - a_1)} & if \quad a_1 < x < a_2 \\ 1 & if \quad x \ge a_2 \end{cases} \tag{10}$$

Two plateaux thus characterize trapezoidal specifications: all the elements of the domain falling within a given plateau share equal membership in the fuzzy set. As argued, however, such a representation requires the preliminary definition of two critical values to separate the definitely deprived and the definitely non-deprived, hence lays open to an obvious critique in what concerns the grounds on which the choice of the thresholds takes place. Usually, the subjective beliefs of the researcher performing the analysis represent the rationale for discriminating among the given modalities, thus introducing precise normative assumptions in the whole procedure[13].

The notion of frequency, on the other hand, has been considered helpful in offering a way out from the issue of aprioristic choices. Basically starting from the intuition, put forth by Desai and Shah (1988), according to which the social environment plays an essential role in measuring deprivation, some authors have proposed to define the membership grades with reference to the distribution of the considered element in the specific society. Assuming a non-linear and monotonic relation between the indicator variable x and the degrees of membership, Cheli and Lemmi (1995) propose to order the modalities of x with respect to the risk of deprivation $k=1,...,K$ associated to them (the higher k, the lower deprivation, thus the higher one's standard of living) and subsequently make use of the following specification

$$m(x) = \begin{cases} 0 & if \quad x = x^1; k = 1 \\ m(x^{k-1}) + \dfrac{F(x^k) - F(x^{k-1})}{1 - F(x^1)} & if \quad x = x^k; k > 1 \\ 1 & if \quad x = x^K; k = K \end{cases} \tag{11}$$

where $F(x^k)$ denotes the cumulative distribution of x ranked according to k. From the perspective of the authors, such approach is intended to allow the membership function to be based exclusively on the empirical evidence; yet, I believe, such procedure entails specific normative implications as well. The rationale lying behind the choice of any specification by a researcher necessarily implies, in my reading, the implicit adoption of a particular standpoint, entailing precise opinions and/or judgements about the way a line has to be drawn between deprived and non-deprived individuals. Thus, also the idea of taking the distribution of an attribute in society to be the key determinant of one's degree of deprivation represents, in this view, some kind of stance.

It emerges quite clearly, therefore, that the process of definition of the building blocks of fuzzy sets theory makes allowance for a significant discretionary power on behalf of the researcher. However, this is just an aspect of the problem, the subsequent step being the combination of the m functions relating to the single variables by means of appropriate operators, so as to form fuzzy sets, *i.e.* so as to get a measure of functionings' achievement.

3.2.2 From membership degrees to fuzzy sets: how to aggregate fuzzy subsets

The membership function being the crucial element of a fuzzy set, it comes as no surprise that operations with fuzzy sets are defined via their membership functions. Likewise, fuzzy sets being strongly and directly related to classical sets theory, it is not amazing that a natural extension of the union and intersection operations of ordinary set theory applies to them. According to Zadeh (1965), the classical union and intersection can be extended to fuzzy sets simply by associating

[13] Moreover, the choice of a given specification itself could be interpreted in the light of the precise normative implications it entails: for instance, the trapezoidal specification could be understood as reflecting the idea of equal marginal contributions to the probability to be deprived with respect to a given element.

them with the maximum and the minimum operators, respectively[14]. Their simplicity, as well as the fact that they satisfy a large number of useful properties - from commutativity to associativity, symmetry, distributivity, continuity, monotonicity, among others - explain their predominant role in empirical problems. As a matter of fact, when measuring well-being, a different meaning will be attributed to the composite fuzzy set one gets according to the selected aggregator. Adopting the union operator an emphasis will be placed on the indicators expressing the most favourable position, *i.e*. the highest membership grade among the considered ones (as a consequence of its selecting the maximum membership function among the ones to be aggregated), thus producing a functioning that will only reflect the best achievements of each individual in depicting the considered dimension. Conversely, the intersection operator will offer the possibility to construct a functioning assessing the individuals' doings and beings exclusively on the basis of the indicator showing the lowest membership grade, while disregarding the remaining ones. Hence, standard intersection will yield some sort of a deprivation index, mirroring one's worst accomplishments[15].

An alternative and frequently used class of aggregation criteria is represented by the so-called *averaging operators*, allowing the membership grade of each element x in the resulting aggregated fuzzy set to lie between the minimum and the maximum[16]. The unweighted means belong to such class which, in terms of axiomatic conditions, satisfies the same properties as the union and intersection operators (in particular they are continuous and symmetrical)[17]. Formally, unweighted averaging operators amount to

$$UA[\mu_A(x),\ldots\mu_Z(x)] = \left[\left(\sum_{i=A}^{Z} \mu_i(x)\right)^{\alpha} \cdot \frac{1}{Z}\right]^{\frac{1}{\alpha}} \qquad (12)$$

with $\alpha=1$ denoting the arithmetic mean, $\alpha=-1$ denoting the harmonic mean and $\alpha=0$ denoting the geometric mean.

Yet, the unweighted averaging operators introduce an idea of compensation among the various items under consideration, but confine themselves to a fix compensation: a rather strict assumption in describing a variety of phenomena. Removing the symmetry axiom allows to derive a group of weighted averaging operators attributing a different importance to the indicators at issue. They are nothing else than a mapping having an associated vector W of weights such that $w_i \in [0,1]$ and $\sum w_i = 1$. Similarly to the previous class, also these operators provide the possibility, by appropriate choice of the weighing vector, to range in between the minimum and the maximum type of aggregation, taking the following general form

$$WA[\mu_A(x),\ldots, \mu_Z(x); w_A,\ldots, w_Z] = \left[\left(\sum_{i=A}^{Z} w_i(\mu_i(x))^{\alpha}\right) \cdot \frac{1}{Z}\right]^{\frac{1}{\alpha}} \qquad (13)$$

The main feature characterising such an aggregative procedure is the set of weights, whose selection - as for the membership function - essentially depends both on the context of the analysis

[14]The intuitive basis for such a definition relies on the observation that the intersection between sets is closely related to the logical conjunction "AND", which in turn claims that the degree to which an element x is both a member of the set A and of the set B should not be greater than the degree of membership this same x has in each of the two sets. The rationale for the association between the union and the max operator derives, instead, from the existing interrelations of the former with the logical disjunction "OR", that - via the 'De Morgan law' stating that *(A or B)= not (not-A) and (not-B)* - leads to the above mentioned conclusion.

[15]Besides these, however, alternative forms of the AND and OR operations are available in the fuzzy literature as a consequence of the fact that, when applied to fuzzy variables, many operators lead to different results in that the degrees of membership are no longer restricted to the values 0 and 1. Cf. Zimmermann (1991) for a careful examination.

[16] The above-discussed union and intersection operators, in fact, generate composite sets containing *exclusively* the maximum and the minimum degree of membership of the sets involved in the aggregation process.

[17] Also the median and the symmetric summation operators belong to this class, but they do not satisfy associativity, *i.e.* $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$.

and on the beliefs of the researcher. An often chosen alternative is the frequency-based weighing, relying on the implicit assumption - echoing Desai and Shah - that the smaller the proportion of people exhibiting a low achievement on a specific item, the larger the weight which should be attributed to such an item in the aggregate set so as to express the idea of some kind of a relief induced by the sharing of a negative experience with other persons. Two formulations have been proposed during the last decade which have become rather widespread in applied fuzzy theory: Cerioli and Zani (1990) suggest taking weights to be equal to the inverse of the proportion of individuals who are deprived with respect to a given item; Cheli and Lemmi (1995) opt for a generalisation of the latter specification, *i.e.*

$$w_i = -\ln\left[\frac{1}{n}\sum_i m_i(x)\right] \tag{14}$$

In spite of the different normative assumptions, these specifications do not seem to provide contrasting evidence with respect to either the traditional equal weighing or subjective judgements. Indeed, the Cerioli and Zani's (1990) formula has been empirically tested against the above mentioned ones, without necessarily yielding considerably dissimilar results (cf. Brandolini and D'Alessio, 1998, p. 40-41).

3.2.3 Applications of fuzzy sets theory to well-being measurement

While the use of fuzzy sets theory has been widespread in the physical sciences, mathematics or engineering, the application of this tool in the social sciences appears to have been mainly limited to psychology. Applications in economics are few, some exceptions being Barrett *et al.* (1986), Dasgupta and Deb (1996) or Sengupta (1999) who approached the issue of social welfare, consumer choice and consumer preferences in microeconomics using fuzzy relations. However, the fuzzy methodology has been recently gaining some attention in the areas of inequality and well-being analysis. In both cases, in fact, Sen's claim according to which “well-being and inequality are broad and partly opaque concepts” (Sen, 1992, p. 48) implicitly offers a justification for drawing on mathematical tools that account for ambiguity and complexity. With the purpose of measuring individual well-being and deprivation from a multidimensional perspective, Chiappero Martinetti (1994, 1996, 2000), Cerioli and Zani (1990), Cheli and Lemmi (1995) among others offered notable contributions.

Cerioli and Zani (1990), in particular, have been the first arguing the potential fruitfulness of fuzzy sets theory for poverty analysis. Specifically, they suggested to represent the individual's global deprivation as a weighted aggregate of the membership degrees to the fuzzy set of the deprived people, where the membership function takes the trapezoidal form of equation (10) and the weighing structure equals the inverse of the proportion of individuals who are deprived with respect to the given item. The average across the population of such measure is then interpreted as a general index of deprivation. As argued in the previous sections, specific assumptions - which can be met with as many objections - are necessary in order to build a model like the latter. The approach advised by Cheli and Lemmi (1995) represents, indeed, an attempt at overcoming the limits inherent to the particular membership function adopted by their precursors. They propose what they name as a “totally fuzzy and totally relative” procedure to the measurement of deprivation, defining the membership to the fuzzy set of the deprived people on the basis of the distribution functions of the considered variables (see equation 11) and coupling it with a weighted averaging operator where the weighing system corresponds to the one illustrated in expression (14), which in case of simple dichotomous variables coincides with the Cerioli and Zani's one. The reference to sampling distributions makes possible, according to the authors, to take both the social context (by looking at the individual's position in the distribution of a given item within the society) and the relevance of each item (by observing its frequency) into account when assessing deprivation.

Several empirical applications of these methodologies have been recently proposed (see, *e.g.*, Cheli *et al.*, 1994; Cheli, 1995; Lemmi *et al.*, 1997; Cheli and Betti, 1999; Betti *et al.*, 2000), but as far as their use in explicit association with Sen's approach is concerned, Chiappero Martinetti

(2000) unquestionably constitutes the reference work. Making use of the 1994 Italian household survey and mostly drawing on the Cheli and Lemmi's (1995) approach, she aggregates a number of indicators into five doings and beings, then subsequently elaborates an overall index of well-being. Such an exercise, when brought together with the examination of a number of individual socio-economic attributes, allows her to depict the Italian situation as characterised by a relatively low fulfilment in education as well as in social interactions, together with a slight gender inequality, worse achievements for the elderly, for Southern people and, above all, for housewives and blue-collar workers. For my purposes, this contribution represents the main source of inspiration; moreover, being the only existing application of fuzziness to Sen's theory, it constitutes an exclusive yardstick for drawing comparisons with my results. Bearing this in mind, I move now therefore to the effective application of the described methodologies.

4 A comparative empirical application

4.1 The data

The data used in this study are taken from the Panel Study of Belgian Households (PSBH), *i.e*. the survey whose questions make up the Belgian section of the European Community Household Panel (ECHP). The specific questionnaire was submitted in 1998 to a sample of about 3800 households (corresponding to 7021 individuals) in view of collecting information on many socio-economic and demographic variables both at the household and individual level. It represents the most recent - and probably also the richest - currently available body of data for the purpose of assessing multidimensional well-being in Belgium.

From the overall set of variables (more than eight hundred), a small number of indicators (namely, 54) were selected. Such indicators were classified into 7 categories: social interactions, cultural activities, economic conditions, health, psychological distress, working conditions and shelter[18]. This procedure has clear implications for the analysis: first, it means that when operationalizing fuzzy sets theory each category has been implicitly taken as denoting a functioning; next, that the factor analytic exercise can be thought of as an attempt to validate the postulated seven-factors structure. It can be remarked that most of the variables are qualitative, either categorical or just dichotomous; moreover, they are rather heterogeneous in that they include both subjective and objective indicators, both direct and indirect measures of the given functioning[19].

Social interactions have been appraised by means of both a direct indicator - *i.e.* the frequency of contact and meeting with friends - and a group of variables relating to the incidence of a number of leisure activities likely to imply some kind of social relations. Though being perfectly aware of the conceptual weakness inherent to the way this dimension has been depicted (unfortunately, any other specific question on the issue was absent in the survey), it was included to stress the relevance of a functioning accounting for one's social contacts. Similarly, *cultural activities* have been evaluated on the basis of the answers to some questions pertaining both to one's membership in any socio-cultural association and to the regularity in visiting musea or attending conferences, concerts, plays, movies or in participating to creative activities. The *economic status* has been taken to constitute a specific functioning in spite of the debatable nature the latter could be charged with. In the spirit of what argued by Brandolini and D'Alessio (1998, p. 33), the available economic resources within the household are interpreted both as a measure of the social status of its members and as reflecting the ease of one's life in terms of available goods and/or services as well

[18] A complete list of the variables used can be found in Appendix A.

[19] It is well known that the use of subjective information is likely to raise several objections. Hence, being aware that much caution is required when adopting such opinions as indicators of well-being, the potentially distortive effect of each of them was (hopefully) offset by supplementing it with a number of objective variables.

as in terms of subjective perception of safety relative to the occurrence of unexpected events[20]. Several items have been considered, among which the regularity in saving money, individual perceptions concerning both the household's present economic conditions and its successfulness in making ends meet, the existence of difficulties in facing payments or loans' reimbursements or the lack of a number of commodities due to unaffordability[21].

The information concerning *health* has been partly directly captured by a self-assessed ordinal variable and partly indirectly extrapolated through dichotomous indicators referring to the presence of illnesses or to recent admission to a hospital. Besides, three continuous variables quantifying the frequency by which a physician is consulted provide additional evidence on the subject. A very interesting negative type of functioning somehow strongly related to one's health, namely *psychological distress*, has been investigated thanks to a group of 12 categorical indicators denoting the occurrence of symptoms as feeling depressed, sleeping badly, losing appetite or feeling irritable, which play an extremely significant role in determining the mental well-being of a person.

The indicators grouped under the label *working conditions* aim at disentangling some non-economic aspects related to one's activity, in the light of the idea that work satisfaction does not exclusively stem from salary, but rather from several psychic advantages as well. Accordingly, a number of non-monetary features such as the person's judgement (expressed on an ordinal scale articulated in six modalities) of either his work schedule or his working environment, the specific position he holds, the actual number of hours devoted to the job, etc. have been explored, along with two variables sizing up one's possible feeling of overqualification as well as one's potential intention to look for an alternative job. Finally, the *sheltering* conditions are to be interpreted as the result of three main types of indicators: a crowding index, taking note of the household size as well as of possible economies of scale, and a heating availability variable [22]; a subjective ranking depicting the level of satisfaction about one's housing situation; two summated rating scales corresponding to the occurrence of problems related to the dwelling and/or to its specific location[23].

4.2 Functionings' assessment according to factor analysis

The first and basic step in designing a factor analysis focuses on ascertaining that the fundamental statistical requirements associated to the use of such methodology are fulfilled. More precisely, one should make sure that the data matrix displays sufficient correlations to justify the application of this technique[24]. Once this has been done, the factor analytic procedure moves on to

[20] Although fully acknowledging the instrumental nature attributed by Sen to one's economic resources, I interpret their command as assuming an independent value.

[21] Each of the last two variables pulls together a number of indicators whose summated rating scale has been considered in the analysis: 4 dichotomous variables for financial difficulties and 9 dichotomous variables for non-affordability. Scale reliability was assessed by computing coefficients alpha. The estimates were 0.77 and 0.84 for the financial difficulties and the non-affordability scales, respectively.

[22] The crowding index has been computed as the ratio between the total number of rooms and an equivalence coefficient determined on the basis of the OECD scale. Accordingly, the following weights have been used: 1.0 for the first adult; 0.7 for any other adult (18 and over); 0.5 for children (under 18). Only heating has been considered as a result of the extremely poor variation exhibited by other indicators of basic utilities, which made them not appropriate for factor analysis.

[23] Both the dwelling's and the area's scale bring together 7 dichotomous variables. The reliability coefficients for these scales are 0.60 and 0.67, respectively. The values are slightly low when compared to the widely used rule of thumb of 0.70; however, I consider them still acceptable.

[24] For this reason, the partial correlations between each pair of variables controlling for the effects of all other variables were first inspected. Only in a very few cases they appeared to be larger than .30, thus providing some preliminary evidence in favour of the aptness of the selected indicators. Out of caution, Kaiser's measure of sampling adequacy was computed as well. This index, ranging from zero to one, summarises how small the partial correlations are relative to their ordinary counterpart. The overall measure for the sample amounted to 0.89, which according to the commonly used guidelines undoubtedly falls within the range of meritorious values. The inspection of the indices for the individual variables as well revealed a substantial correctness of the data selection, the lowest value being 0.66.

the initial computation of the correlation matrix, which is then transformed through estimation of a factor model to obtain a factor matrix. The initial extraction provided 54 factors, in line with what was expected but definitely too large a number for any meaningful analysis. Yet, the visual inspection of both the scree plot and the eigenvalues made obvious that 7 of them could have been already considered sufficient for best representing the data[25]. As a result, only the latter restricted number of factors were retained for subsequent rotation[26]. Conceptual and practical considerations led to the selection of an orthogonal rotation method, namely varimax: from a conceptual point of view, it was best suited for making sure that functionings do not overlap; from a practical point of view, it made the interpretative task easier *vis-à-vis* an oblique solution.

The results stemming from the varimax rotation are presented in Table 4.1. All values have been multiplied by 100 and rounded to the nearest integer, so as to improve readability. Besides, the values equal to or exceeding 0.30 have been flagged with an asterisk[27]. As immediately appears, except for a limited number of cases, the loadings fall indeed substantially above or below the selected threshold and their interpretation does not seem to be too problematic.

Factor 1 exhibits extremely high loadings on the twelve items related to the mental attitudes of the respondents, hence confirming the existence of a latent negative type of functioning likely to be named "psychological distress". *Factor 2*, instead, visibly collects a number of items suggesting a clear orientation towards a number of leisure activities implying the contact with other individuals: from regularly going to cafes or discotheques to generally hanging out in the evening or playing games with some friends. In spite of its poor and debatable nature, I interpret this functioning as referring to some kind of "social interactions"[28]. *Factor 3* obviously reflects the good "economic conditions" of the respondents, with large positive weights for saving behaviour, being able to make ends meet, being content with one's financial resources, and almost as large but negative weights for cash shortage and enforced lack of commodities. Moving to *Factor 4*, highly scoring respondents often go to the theatre or to the cinema, visit exhibitions, attend concerts and/or conferences, thus evoking the idea of a rich "cultural life" and inducing to label this latent construct accordingly. The collection of items on which *Factor 5* exhibits significant loadings visibly echoes people's judgements in relation to their job and may accordingly be thought of as a "working conditions" functioning. *Factor 6* undoubtedly relates to one's "health" dimension. However, the signs are such that a high score on this health functioning should be seen as revealing a bad condition. In the subsequent part of the analysis, this construct will thus be reversed in order to facilitate the comparison with its counterpart stemming from the application of fuzziness. Finally, the pattern of loadings characterising *Factor 7* stands out for its focus on the problematic aspects - both in terms of location and of dwelling's features - related to the housing situation of the respondents, as well as for the relevance attributed to personal dissatisfaction in relation to it. Exactly the same issue as for the fourth factor will have to be faced also for this "shelter" functioning[29].

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[25] The scree plot graphs the value of the characteristic roots against the roots' factor number in their order of extraction. The shape of the resulting curve is used to evaluate the number of factors to extract. Specifically, the point at which the curve first begins to turn into an approximately horizontal line is considered to indicate the maximum number of factors to extract.

[26] Out of carefulness, factor analysis was performed also for the hypothesis that the economic variables were excluded from the sample. This choice turned out not to affect the results at all.

[27] The choice of 0.30 as the threshold value for identifying significant factor loadings is in line with the use of a 5% significance level and the assumption of standard errors being twice those of conventional correlation coefficients.

[28] Of course, an obvious drawback of the way this dimension has been measured lies in the fact that, for instance, an alcoholic individual would presumably score pretty well on the social interactions functioning though not being necessarily fully integrated in society. Hence, any subsequent interpretation based on this sphere will have to be carefully weighted up.

[29] I feel necessary to stress that all functionings will be positively measured in the remainder of this paper.

Table 4.1 - Rotated factor pattern

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7
Friends	-5	15	-18	5	-1	4	-12
Sport match	-2	45*	7	0	1	-5	-5
Café	1	57*	2	13	0	-5	4
Restaurant	0	34*	24	37*	1	-9	-6
Disco	5	54*	-10	3	-1	-8	3
Games	3	52*	0	0	1	-7	4
Going out	3	54*	8	21	2	-11	7
Sport practice	-3	38*	13	13	1	-8	-5
Meet ends	-13	5	73*	11	4	-5	-13
Savings	-3	12	61*	9	2	-10	1
Perceived sit.	-6	10	46*	11	8	-8	0
Fin. difficulties	9	0	-30*	-7	-1	0	13
Non affordab.	8	1	-46*	-10	-1	2	24
Economic satisf.	-19	-1	61*	10	20	-2	-23
Health status	-24	26	15	9	8	-61*	-1
Chronic illness	10	-12	-7	-2	-1	54*	-1
Recent illness	10	-4	-3	-2	-3	47*	2
Hospital	7	0	0	-5	2	45*	-1
Generalist	8	-12	-9	-9	1	57*	-7
Specialist	15	-2	-1	2	0	47*	5
Other physician	7	-2	2	11	0	10	1
Psychol. problems	24	-5	-4	0	0	29	3
Depressed	71*	-8	-18	-2	-9	19	4
No appetite	44*	6	-6	-5	-5	22	8
Insomnia	49*	-10	-8	1	-6	22	4
Unrested	63*	-3	-7	1	-10	20	9
Nervous	42*	4	-2	-5	-1	3	5
Feel guilty	70*	9	-2	7	-5	-1	9
No concentration	67*	2	2	4	-9	6	8
Weeping	57*	-4	-9	1	2	12	-2
Pessimistic	71*	-4	-9	1	-9	8	3
Irritable	63*	6	0	3	-5	-1	9
Need reassurance	70*	-7	-3	7	-2	0	3
Out of sorts	75*	-2	-6	0	-7	18	5
Housing satisf.	-17	-7	19	2	16	5	-53*
Area	13	-2	-15	4	-4	4	46*
Dwelling	11	3	-13	0	-3	2	55*
Heating	4	5	20	17	1	-11	-21
Crowding	-7	-13	-4	16	-3	12	-16
Theatre	2	13	6	63*	3	-6	-2
Cinema	12	45*	-2	34*	-3	-17	6
Concert	1	26	8	60*	3	-5	1
Museum	1	5	10	69*	0	-5	1
Conference	-2	1	8	57*	-1	-6	-2
Creative activity	3	6	4	28	3	0	0
Association	-6	22	12	27	1	1	-6
Work certitude	-6	1	9	1	43*	2	-1
Work type	-10	0	2	3	65*	-2	-3
Number of hours	-7	4	1	-2	61*	3	1
Work schedule	-5	4	3	6	65*	3	3
Working environm.	-9	4	1	2	65*	-2	-3
Work distance	-2	-1	0	1	36*	-2	-5
Job search	-4	16	-1	2	-14	2	2
Overqualified	3	14	-7	0	-23	4	6

The rotated factors substantially confirm, then, the picture that was postulated. Nevertheless, loadings are nothing else than weights, providing no information about the specific standing of each subject on the retained functionings, thus about the individuals' functioning levels. The latter can only be extrapolated by computing factor scores. This final step accomplished, seven variables will be available that provide a description of the living standard of the respondents and that are likely to be used, therefore, for subsequent analyses.

4.3 Functionings' assessment according to the fuzzy methodology

As previously argued, within the context of the fuzzy procedure each category of selected indicators is implicitly assumed to denote a functioning, thus the first step to be taken concerns the selection and the computation of the most appropriate membership functions to represent each of the 54 observed variables[30]. A preliminary action is represented by the identification - for each variable - of two modalities below and above which the extreme conditions of full membership and non-membership can be ascertained. Besides, achieved functionings being the underlying concepts to be represented, full membership will always be associated with the threshold stipulating conditions of absolute fulfilment with respect to a given indicator[31]. This being accomplished, membership grades will have to be specified for all the intermediate modalities.

In the light of the fact that membership functions are likely to bring a substantial element of subjectivity into the analysis, I feel it is important to conduct a range of sensitivity tests to determine the robustness of the results to the choice of a particular specification for $m(x)$. Accordingly, three different widely common models have been applied to the intermediate values of the indicators' list. At the outset, the frequency-based approach taken by Chiappero Martinetti (2000) has been followed, representing most of the membership functions by the above specification (11) or - whenever modalities appeared to be equidistant - by the simple linear one depicted in expression (9). Alternatively, both sigmoid/logistic functions (8) and trapezoidal ones (10) have been defined for exactly the same indicators[32].

The membership functions so defined will provide a description of each individual's position in relation to each elementary indicator. What is needed for a complete assessment, however, is each one's degree of achievement relative to the aggregated groups of indicators, *i.e*. the given functionings. The next step resides, therefore, in combining the variables pertaining to the same category by means of adequate operators. In the first model (the "Cheli and Lemmi"), the indicators have been aggregated by means of compensatory operators whose weighing structure was illustrated in expression (14). The reason for such choice lies specifically in the possibility of comparing these results with the ones by Chiappero Martinetti (2000). In addition, an aggregation of the same membership functions (the "Cheli and Lemmi") as well as of the ones arising from the trapezoidal and the S-shaped specifications was performed using the unweighted averaging operators, so as to get three cumulative representations treating all attributes equally. The latter will then be compared with each other for the purpose of emphasizing the effect of the selected specifications. Although being perfectly aware of the obvious drawbacks of the symmetric assumption, I'd like to avoid any possible influence stemming from the particular weighing system when confronting the various operationalisations of $m(x)$. Ultimately, four different series

[30] To be more precise, one should only talk about 43 variables owing to the binary nature of a number of them for whom, obviously, no transformation is needed.

[31] This implies, for instance, that when dealing with the "frequency of contact with friends" indicator, a membership grade of one will be attributed to the modality "everyday" and a zero value will be reasonably ascribed to "never".

[32] Being conscious of the highly debatable appropriateness of the linearity assumption for most of the variables in the dataset, I wish to stress that the comparative goal constitutes the only rationale driving this speculative procedure and no further use will be made in the remainder of this study of the so derived trapezoidal membership functions. It is also worth reiterating that, in determining the threshold values for the latter and the S-shaped functions, a remarkable subjective component has unavoidably been introduced. Appendix C presents a detailed list of the adopted membership functions.

comprising seven fuzzy sets each will have been derived from the outlined fuzzy procedure, thus bringing about the possibility of drawing significant comparisons both among fuzzy aggregates and between the latter and the previously obtained factor scores, which is precisely the major aim of this paper.

4.4 To what extent do the results differ?

4.4.1 A first glance at correlations and some summary characteristics of the derived functionings

There are no formal connections, in my reading of the methods, between fuzzy sets theory and traditional parametric techniques such as factor analysis. Hence, one cannot easily draw comparisons in quantitative terms between them, relying on either the explained variance or the accuracy of the estimates[33]. As a matter of fact, the obvious measure providing an assessment of the nature and the strength of the relationship between the two methodologies as well as among the various specifications of the membership functions is the Pearson correlation coefficient. Table 4.2 summarises the computed pairwise correlations for the latter curves, all significant at the 1% level. Immediately, a remarkably strong association emerges, the lowest coefficient amounting to 0.82. Furthermore, though not being reported in the table, an almost perfect relationship between the doings and beings measured on the basis of either the trapezoidal or the S-shaped definitions appears: no matter the considered functioning, the absolute value of the correlation coefficient relating these two specifications permanently equals 0.99.

Table 4.2 - Pairwise correlations among fuzzy specifications

	Cheli & Lemmi equal weighing						
	Psychol. distress	Social interac.	Econ. cond.	Cultur. activ.	Working cond.	Health	Shelter
S-shaped	0.93	0.98	0.96	0.95	0.82	0.97	0.85
Trapezoidal	0.88	0.95	0.97	0.91	0.93	0.97	0.88

A first glance at these results seems to suggest the existence of extensive evidence in favour of those fuzzy researchers thinking of membership grades as exclusively offering an indication, a tendency index on an object assigned by an individual[34]. Accordingly, on account of the resemblance of the various outcomes, one should not be too much concerned about the specifically selected functions denoting an element's belonging to a set. Yet, this conclusion leaves a bit puzzled. If one is not keen on considering membership functions as mere “rescaling devices”, but rather believes that they entail some normative implications as a result of their reflecting specific ethical viewpoints, the outcome of this analysis has to be interpreted as stating that different normative assumptions are likely not to affect the results: a somewhat curious conclusion! More thoughtful considerations, however, reveal that these outcomes come, in a way, as no surprise: the trapezoidal and the S-shaped specifications, for instance, can well be thought of as proxies, while the frequency-based formulation shares several common traits with the sigmoid/logistic curves. Hence, some additional and more radical specifications (*i.e.* more oriented towards a clear

[33] Actually, fuzzy sets theory provides also a data reduction technique which, in a sense, evokes the rationale underlying factor analysis. It aims, in fact, at reducing a collection of fuzzy sets to a smaller collection of derived fuzzy sets exhibiting certain properties (namely, the latter should highly overlap with their respective original sets and they should be as few as possible). Thus, while factor analysis essentially depends on correlations, the fuzzy reduction technique mainly relies on overlap coefficients. It could perhaps represent a fruitful direction for future research.

[34] This interpretation of membership uncovers a clear link with scaling theory. In many cases, actually, membership functions have even been elicited thanks to a suitable method for scaling the subject's perception of an attribute (cf. Norwich and Turksen, 1984). More generally, however, remarkable similarities can be found in between unidimensional scaling and the fuzzy procedure.

dichotomic partition of the state values) should have probably been investigated in order to further the usefulness of the comparisons, though even from those analyses I would not expect to uncover any major discrepancies, owing to both the presence of several dichotomous variables and the limited number of modalities exhibited by the remaining elementary indicators which are being used. What is more, a recent and very interesting work by Filippone *et al.* (2001) somehow confirms the above mentioned findings: within the context of a thorough analysis of some drawbacks inherent the "totally fuzzy and relative" poverty measures, the authors use BHPS data from 1991 to 1997 in order to compare the results obtained by applying both two different membership specifications and three weighing systems to the same set of elementary indicators, uncovering the irrelevance of both choices and, thus, reaching essentially analogous conclusions with respect to the ones here presented[35].

Similar results, however, also emerge from the comparison between the outcomes produced by the two key methodologies: the estimated factor scores (rescaled so as to range, them too, in between zero and one) exhibit an impressively high degree of association with any possible kind of fuzzy aggregate, as depicted by Table 4.3 (also in this case they are all significant at 1%)[36].

Table 4.3 - Pairwise correlations: factor scores vs. fuzzy sets

	Factor scores						
	Psychol. distress	Social interac.	Econ. cond.	Cultur. activ.	Working cond.	Health	Shelter
S-shaped	0.95	0.91	0.93	0.85	0.84	0.91	0.74
Cheli &Lemmi EW	0.96	0.93	0.93	0.91	0.89	0.91	0.83
Trapezoidal	0.92	0.89	0.94	0.83	0.88	0.91	0.76

In line with these findings, therefore, the picture of the Belgians' living standard emerging from the two techniques bears impressive similitudes, as a look at the summary statistics given in Table 4.4 confirms. Here, only factor scores and the specific model of fuzzy sets' representation that, in the remainder of the analysis, will be considered as the exclusive term of comparison - namely the "Cheli and Lemmi" - are depicted. The other models display, in my view, a number of clear limitations owing, on the one side, to the postulated - and rather unrealistic - equal importance assigned to the various elementary indicators and, on the other side, to the excessive rigidity of the linearity assumption underlying the trapezoidal specification. In the light of this, the Cheli and Lemmi's approach appears after all more convincing. As the table illustrates, both methodologies seem to suggest the same general picture in what concerns the degree of achievement exhibited by the Belgian sample in a number of doings and beings.

Table 4.4 - Summary characteristics of the selected aggregates

	Factor scores		Fuzzy Cheli & Lemmi	
	Mean	*Std. Dev.*	*Mean*	*Std. Dev.*
Psychological distress	0.338	0.169	0.321	0.195
Social interactions	0.313	0.164	0.311	0.217
Economic conditions	0.597	0.166	0.582	0.266
Cultural activities	0.286	0.163	0.276	0.233
Working conditions	0.677	0.127	0.589	0.161
Health	0.832	0.090	0.689	0.226
Shelter	0.734	0.123	0.683	0.186

Bearing in mind that any distribution takes on values in the interval [0,1] where zero denotes the minimum level of functioning's fulfilment and one the complete attainment, a relatively high degree of accomplishment can be said to emerge in the material dimensions (shelter, economic

[35] The membership function specifications are the original "Cheli and Lemmi" of expression (11) and a transformed one, always in line with the "totally fuzzy and relative" approach. The three weighing systems are the one of expression (14), a linear and an exponential one.

[36] Appendix D reports the graphic representations of the distributions as well.

and working conditions) as well as in health and in psychological status, while relatively poor realizations concern social interactions and cultural activities[37]. Although already informative, this simple comparison looks rather unsatisfactory as both the matrix of factor scores extrapolated via factor analysis and the fuzzy evaluations of functionings do not lend themselves - as such - to straightforward interpretations. To structure the information contained in both of them so as to make possible to get an insight into each individual functioning, while at the same time facilitating comparisons based on each technique's capacity to recover data structures that plausibly and intelligibly capture individual well-being, standard multivariate analysis is used.

4.4.2 Multivariate analysis of functionings

Let individual's i achievements on the p dimensions be expressed as a function of his income y_i and of some personal factors π_i

$$f_{ip} = f(y_i, p_i) \tag{15}$$

Postulating a non-linear relationship between individual achievements and income, and representing personal factors by a number of socio-economic characteristics, the following regression model can be estimated for both types of functionings (*i.e.* factor scores and fuzzy aggregates)

$$f_{ip} = a_{0p} + a_{1p} y_i + a_{2p} (y_i)^2 + \sum_{j=3}^{m} a_{jp} x_j + u_p \quad p = 1,...,7. \tag{16}$$

where x_j denotes a vector containing the observations of all individuals about the socio-economic characteristics j, u_p is the vector of disturbance terms and the a's are coefficients to be estimated. The following characteristics were incorporated in the model[38]: (a) gender, via a female dummy; (b) age, expressed through seven dummies denoting as many age brackets (reference category: age 56-65); (c) civil status, articulated in married, single, widowed or divorced (reference category: single); (d) federal region (reference category: Wallonia); (e) level of education, represented by five dummies (reference category: senior secondary school); (f) number of children; (g) working status (reference category: employee). The results are displayed in Tables 4.5 to 4.7[39].

A foremost feature relates to the coefficients of determination that, at an absolute level, look rather low. The *R*-squares range, in fact, in between 2 and 40 percent. Yet, in the light of the fact that we are dealing with cross-section data, this finding is not at all surprising. Quite the contrary, the proportion of the functionings' variance explained by the selected characteristics appears to be relatively large in relation to the usual results for this kind of analyses[40]. Accordingly, I feel that an interesting picture of the living standard can probably be drawn on the basis of the examined specifications and, above all, that they make possible to compare and contrast evidence for the two methodologies on the role of the same set of attributes.

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[37] As already stressed, a note of caution is needed in the interpretation of these results: social interactions and cultural activities are, by far, the conceptually weakest functionings among the derived ones.

[38] The complete table summarising the main features of the sample can be found in Appendix E, together with a description of the least obvious variables.

[39] ***, ** and * denote significance at 1%, 5% and 10% level respectively. Whenever necessary, heteroskedasticity-consistent standard errors have been computed according to White's covariance matrix. Standard errors in parenthesis.

[40] As can easily be remarked, a slight difference in the sample size on which the estimations have been performed characterises the results. Such gap originates from the specificity of the procedures: as the factor analytic technique computes the loadings and, subsequently, estimates the scores for all factors simultaneously, using only complete observations, any individual displaying a missing value on some item is automatically dropped from the analysis. The fuzzy procedure, conversely, computes each functioning separately; this also explains the different sample sizes exhibited by the various fuzzy aggregates.

Table 4.5	Psychological distress		Social interactions		Economic conditions	
Variable	*Factor score*	*Fuzzy eval.*	*Factor score*	*Fuzzy eval.*	*Factor score*	*Fuzzy eval.*
Intercept	0.251 ***	0.217 ***	0.289 ***	0.293 ***	0.517 ***	0.475 ***
	(0.014)	(0.014)	(0.011)	(0.013)	(0.013)	(0.019)
Income	0.033 ***	0.044 ***	0.032 ***	0.048 ***	0.122 ***	0.185 ***
	(0.009)	(0.010)	(0.007)	(0.009)	(0.010)	(0.013)
Income sqr	-4.45e-09 ***	-6.76e-09 ***	-4.21e-09 ***	-6.16e-09 ***	-1.14e-08 ***	-1.54e-08 ***
	(1.16e-09)	(1.81e-09)	(1.09e-09)	(1.06e-09)	(2.41e-09)	(2.61e-09)
Effect of children	0.002	0.001	-0.008 ***	-0.010 ***	0.003	0.007 ***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)
Female	0.075 ***	0.088 ***	-0.059 ***	-0.071 ***	0.006	0.004
	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.006)
Age 16-25	0.068 ***	0.077 ***	0.237 ***	0.248 ***	-0.058 ***	-0.071 ***
	(0.014)	(0.015)	(0.012)	(0.014)	(0.013)	(0.018)
Age 26-35	0.058 ***	0.065 ***	0.144 ***	0.152 ***	-0.046 ***	-0.073 ***
	(0.010)	(0.011)	(0.007)	(0.009)	(0.009)	(0.014)
Age 36-45	0.066 ***	0.077 ***	0.084 ***	0.094 ***	-0.048 ***	-0.081 ***
	(0.010)	(0.010)	(0.007)	(0.008)	(0.009)	(0.013)
Age 46-55	0.046 ***	0.053 ***	0.035 ***	0.035 ***	-0.027 ***	-0.045 ***
	(0.010)	(0.010)	(0.007)	(0.008)	(0.009)	(0.013)
Age 66-70	0.008	0.008	-0.007	-0.006	0.003	0.006
	(0.011)	(0.011)	(0.006)	(0.007)	(0.010)	(0.014)
Age 71-75	-0.012	-0.006	-0.015 **	-0.029 ***	-0.002	-0.013
	(0.011)	(0.012)	(0.006)	(0.007)	(0.011)	(0.016)
Flanders	-0.027 ***	-0.014 ***	0.067 ***	0.067 ***	0.073 ***	0.131 ***
	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.006)
Brussels	-0.035 ***	-0.034 ***	0.009	-0.003	-0.013 *	-0.006
	(0.009)	(0.008)	(0.006)	(0.007)	(0.008)	(0.011)
Married	0.002	-0.005	-0.060 ***	-0.052 ***	0.054 ***	0.036 ***
	(0.007)	(0.008)	(0.007)	(0.008)	(0.007)	(0.010)
Divorced	0.020 *	0.020 *	-0.024 ***	-0.013	-0.060 ***	-0.108 ***
	(0.011)	(0.011)	(0.009)	(0.010)	(0.009)	(0.013)
Widowed	-0.008	-0.010	-0.059 ***	-0.055 ***	0.001	-0.020
	(0.012)	(0.012)	(0.008)	(0.010)	(0.011)	(0.016)
Primary school	-0.006	-0.004	-0.016 ***	-0.038 ***	-0.058 ***	-0.095 ***
	(0.008)	(0.008)	(0.005)	(0.006)	(0.007)	(0.010)
Junior secondary	0.005	-0.002	-0.001	-0.008	-0.036 ***	-0.058 ***
	(0.006)	(0.006)	(0.005)	(0.006)	(0.006)	(0.008)
Third level non-uni.	0.002	-0.002	-0.005	-0.004	0.022 ***	0.041 ***
	(0.006)	(0.007)	(0.005)	(0.006)	(0.005)	(0.008)
University and over	-0.008	-0.019 **	-0.024 ***	-0.019 **	0.038 ***	0.068 ***
	(0.008)	(0.009)	(0.007)	(0.008)	(0.007)	(0.010)
Self employed	-0.012	-0.022 **	-0.018 **	-0.019 **	-0.014 *	-0.006
	(0.008)	(0.009)	(0.007)	(0.008)	(0.008)	(0.011)
Student	-0.025	0.014	-0.038	0.053 ***	-0.056 *	-0.019
	(0.033)	(0.012)	(0.029)	(0.013)	(0.032)	(0.015)
Unemployed	0.020 **	0.022 **	-0.038 ***	-0.063 ***	-0.111 ***	-0.168 ***
	(0.010)	(0.010)	(0.008)	(0.010)	(0.009)	(0.013)
Retired	-0.014	-0.001	-0.033 ***	-0.054 ***	-0.017 *	-0.036 ***
	(0.010)	(0.011)	(0.007)	(0.008)	(0.010)	(0.014)
Home duties	-0.008	-0.006	-0.015 **	-0.027 ***	-0.028 ***	-0.035 ***
	(0.010)	(0.010)	(0.006)	(0.008)	(0.009)	(0.013)
Other	0.043 ***	0.069 ***	-0.023 **	-0.080 ***	-0.103 ***	-0.164 ***
	(0.017)	(0.014)	(0.011)	(0.012)	(0.013)	(0.018)
Adj. R^2	0.1061	0.0919	0.4045	0.3985	0.2645	0.2373
Sample size	5227	6570	5227	6776	5227	6675

Table 4.6	Cultural activities		Working conditions		Health	
Variable	*Factor score*	*Fuzzy eval.*	*Factor score*	*Fuzzy eval.*	*Factor score*	*Fuzzy eval.*
Intercept	0.309 ***	0.221 ***	0.664 ***	0.616 ***	0.819 ***	0.669 ***
	(0.012)	(0.016)	(0.011)	(0.012)	(0.007)	(0.016)
Income	0.062 ***	0.092 ***	-0.010	-0.004	-0.005	-0.007
	(0.010)	(0.012)	(0.007)	(0.008)	(0.004)	(0.010)
Income sqr	-7.04e-09 ***	-1.11e-08 ***	3.23e-09 ***	2.11e-09	7.64e-10	2.79e-10
	(2.44e-09)	(2.97e-09)	(1.00e-09)	(1.45e-09)	(5.91e-10)	(1.43e-09)
Effect of children	-0.007 ***	0.001	0.005 ***	0.005 ***	0.005 ***	0.012 ***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)
Female	0.026 ***	0.018 ***	0.019 ***	0.025 ***	0.001	-0.047 ***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.002)	(0.005)
Age 16-25	-0.094 ***	-0.043 ***	-0.035 ***	-0.075 ***	0.025 ***	0.070 ***
	(0.012)	(0.015)	(0.012)	(0.013)	(0.007)	(0.016)
Age 26-35	-0.073 ***	-0.046 ***	-0.030 ***	-0.066 ***	0.023 ***	0.067 ***
	(0.009)	(0.011)	(0.008)	(0.009)	(0.005)	(0.012)
Age 36-45	-0.037 ***	-0.019 *	-0.037 ***	-0.057 ***	0.013 **	0.042 ***
	(0.009)	(0.011)	(0.007)	(0.008)	(0.005)	(0.012)
Age 46-55	-0.011	0.003	-0.015 **	-0.023 ***	0.013 **	0.033 ***
	(0.009)	(0.011)	(0.007)	(0.007)	(0.005)	(0.012)
Age 66-70	0.009	0.026 **	0.001	-0.001	-0.005	-0.005
	(0.009)	(0.013)	(0.002)	(0.003)	(0.006)	(0.013)
Age 71-75	-0.013	-0.010	-0.001	-0.001	-0.028 ***	-0.041 ***
	(0.010)	(0.012)	(0.003)	(0.002)	(0.008)	(0.015)
Flanders	0.032 ***	0.084 ***	0.027 ***	0.016 ***	0.003	0.022 ***
	(0.004)	(0.005)	(0.004)	(0.004)	(0.002)	(0.005)
Brussels	0.043 ***	0.045 ***	0.005	0.004	0.005	0.019 **
	(0.008)	(0.009)	(0.006)	(0.007)	(0.004)	(0.008)
Married	-0.032 ***	-0.045 ***	-0.004	0.007	0.002	-0.008
	(0.007)	(0.009)	(0.007)	(0.008)	(0.004)	(0.008)
Divorced	-0.007	-0.012	0.006	0.005	0.001	-0.012
	(0.009)	(0.012)	(0.008)	(0.010)	(0.005)	(0.012)
Widowed	-0.049 ***	-0.070 ***	-0.001	0.007	-0.020 ***	-0.050 ***
	(0.010)	(0.013)	(0.007)	(0.008)	(0.007)	(0.014)
Primary school	-0.097 ***	-0.138 ***	0.002	0.004	-0.023 ***	-0.051 ***
	(0.006)	(0.008)	(0.004)	(0.005)	(0.004)	(0.009)
Junior secondary	-0.039 ***	-0.058 ***	0.003	0.007	-0.006 **	-0.015 **
	(0.005)	(0.007)	(0.005)	(0.005)	(0.003)	(0.007)
Third level non-uni.	0.070 ***	0.096 ***	0.003	0.003	0.006 **	0.023 ***
	(0.006)	(0.008)	(0.005)	(0.006)	(0.003)	(0.007)
University and over	0.122 ***	0.126 ***	-0.003	-0.008	0.007 *	0.034 ***
	(0.008)	(0.010)	(0.007)	(0.009)	(0.004)	(0.008)
Self employed	0.022 ***	0.011	0.023 **	0.042 ***	0.018 ***	0.050 ***
	(0.008)	(0.009)	(0.009)	(0.010)	(0.003)	(0.008)
Student	0.073 **	0.140 ***	0.006	0.027	0.006	-0.003
	(0.030)	(0.014)	(0.036)	(0.047)	(0.013)	(0.012)
Unemployed	-0.022 ***	-0.050 ***	0.005	-0.057 ***	-0.008 *	-0.042 ***
	(0.008)	(0.010)	(0.004)	(0.005)	(0.005)	(0.012)
Retired	-0.008	-0.023 *	-0.019 ***	-0.097 ***	-0.027 ***	-0.083 ***
	(0.009)	(0.012)	(0.006)	(0.007)	(0.006)	(0.013)
Home duties	0.003	0.002	-0.011 **	-0.093 ***	-0.014 ***	-0.027 **
	(0.008)	(0.011)	(0.005)	(0.006)	(0.005)	(0.011)
Other	-0.023 *	-0.065 ***	0.005	-0.066 ***	-0.095 ***	-0.218 ***
	(0.013)	(0.015)	(0.005)	(0.005)	(0.012)	(0.021)
Adj. R^2	0.2208	0.2277	0.0204	0.0550	0.1786	0.2064
Sample size	5227	6768	5227	6164	5227	6806

Confining the analysis to the statistically significant coefficients, one remarks that monetary resources remain in most cases quite relevant, offering some support to the position according to which a multidimensional assessment of the living standard should ideally supplement the income information with other elements, rather than completely neglecting it. Three unusual outcomes, however, stand out: the absence of any income effect on one's health, which is probably to be attributed to the presence of schooling and age variables; its impact on one's psychological conditions, suggesting that extra household resources - at least up to some point - tend to worsen one's mental distress; finally, the irrelevance of additional earnings for one's job satisfaction, stressing how a greater work contentment may well derive from aspects such as a stimulating environment or challenging tasks rather than from the salary one receives.

Table 4.7 - Shelter

Variable	*Factor score*		*Fuzzy eval.*	
	Coeff.	*Std. Err.*	*Coeff.*	*Std. Err.*
Intercept	0.757 ***	(0.011)	0.783 ***	(0.014)
Income	0.015 **	(0.007)	0.051 ***	(0.008)
Income sqr	5.01e-10	(8.04e-10)	-2.06e-09 **	(1.01e-09)
Effect of children	-0.003	(0.002)	-0.023 ***	(0.002)
Female	0.010 ***	(0.003)	0.008 *	(0.005)
Age 16-25	-0.055 ***	(0.011)	-0.098 ***	(0.015)
Age 26-35	-0.066 ***	(0.008)	-0.099 ***	(0.010)
Age 36-45	-0.052 ***	(0.008)	-0.066 ***	(0.010)
Age 46-55	-0.033 ***	(0.007)	-0.043 ***	(0.010)
Age 66-70	0.001	(0.007)	0.001	(0.010)
Age 71-75	-0.002	(0.007)	-0.004	(0.011)
Flanders	-0.010 ***	(0.003)	-0.017 ***	(0.005)
Brussels	-0.041 ***	(0.007)	-0.049 ***	(0.009)
Married	0.026 ***	(0.006)	0.032 ***	(0.008)
Divorced	-0.022 ***	(0.009)	-0.036 ***	(0.011)
Widowed	0.027 ***	(0.008)	0.052 ***	(0.011)
Primary school	-0.019 ***	(0.006)	-0.049 ***	(0.007)
Junior secondary	-0.010 **	(0.005)	-0.024 ***	(0.006)
Third level non-uni.	-0.002	(0.004)	0.017 ***	(0.006)
University and over	-0.006	(0.006)	0.024 ***	(0.008)
Self employed	0.017 ***	(0.006)	0.023 ***	(0.008)
Student	-0.051 *	(0.030)	0.039 ***	(0.012)
Unemployed	-0.019 **	(0.008)	-0.034 ***	(0.011)
Retired	-0.001	(0.007)	-0.019 *	(0.010)
Home duties	-0.010	(0.008)	-0.021 **	(0.010)
Other	-0.025 *	(0.013)	-0.043 ***	(0.015)
Adj. R^2	0.0817		0.1192	
Sample size	5229		6694	

Some slight gender bias in favour of men can be remarked as far as social interactions are concerned, while women seem to be far more subject to psychological distress though more involved into culturally-oriented activities and enjoying slightly better working and housing conditions. The latter findings look rather curious. Nevertheless, the peculiar character of the first result has to be reappraised in the light of the fact that the “working conditions” functioning should probably better be seen as a job satisfaction indicator. Along these lines and besides the possible selection bias in the female work force, it is not difficult to interpret women's higher satisfaction with their job as related to their lower expectations, which - as Clark (1997) suggests - originate in turn from the worse position historically held within the labour market by women in comparison with men. A possible explanation for the connection between housing circumstances and the feminine gender may instead come from the high proportion of women living alone (over 60

percent) whose sheltering situation exhibits clear improvements with respect to both their male counterpart and their "colleagues" living in larger households.

The social group of the Belgian elderly, contrary to the quite common contention according to which they would be psychologically vulnerable, turns out to perform pretty well at the emotional level: both the factor score and the fuzzy evaluation suggest that only people up to 55 years old seem to experience depression, insomnia, loss of concentration and analogous symptoms far more frequently than their older "colleagues", reaching a maximum for the age brackets 16-25 and 36-45. Furthermore and probably as a consequence of their higher economic attainments, the over-55 also enjoy the best shelter and working conditions if compared with any other age group, while devoting themselves more often to cultural activities. More importantly, the regressions could be interpreted as suggesting that senility (in particular, after the seventieth year of age) makes one more socially isolated. However, due to the structure of the "social relations" functioning (mainly comprising items related to leisure activities), this result comes as no surprise and, accordingly, it should not be taken as a too rigorous indication of social isolation.

From a geographical point of view, a number of considerable discrepancies in functionings' achievements sharply characterise the country. Flanders appears, in fact, to outperform the remaining regions in what concerns participation in the social life, working and economic conditions[41]. Its inhabitants also score rather well on culture, but the lack of any statistical significance of regional differences for the estimates based on the factor score induce one to rely exclusively on the indications provided by the fuzzy aggregate, according to which Flemish people would experience a more intense cultural involvement. Notwithstanding its various negative primacies, Wallonia can be said to be unquestionably associated to the best housing conditions, whereas the capital holds the opposite position in the rank.

Not astonishingly, the marital condition significantly affects (in a negative way) the achievement of a satisfactory relational life, both in its social and cultural dimensions. The latter deprivation amplifies even more, however, when turning to widowed individuals, in spite of controlling for the presumably probable age effect. Yet, widowhood is robustly associated to the highest achievements in shelter; an unusual outcome that is probably to be ascribed to the adaptive preferences of the elderly constituting this social group. Even worse, however, appears the situation of divorced or separated individuals, who are found to experience several forms of deprivation: from large income losses or fairly modest housing conditions to restricted social contacts (at least according to the factorized measure). Besides, the mentally distressing impact of divorce is also robustly captured, even if its significance level is quite large.

Education plays the expected role, improving health, shelter, economic means and, needless to say, increasing the participation in cultural activities. Not too surprisingly in the light of the items defining one's social interactions, the highest level of schooling seems to exert a negative influence on leisure activities involving contacts with other people. The lack of contacts with friends or other people in general, nonetheless, plays a considerable role also with respect to the own working condition. The results suggest, indeed, that unemployed, retired and people entirely devoted to home duties would be particularly concerned by such risk[42]. They would also be all confronted with

[41] The significance of the differences between the coefficients for Brussels and Flanders has been tested, revealing a lack of statistical significance for regional differences on the functionings "health" (for the fuzzy aggregate), "cultural activities" (only the factor score) and "psychological distress" (both for the factor score and the fuzzy aggregate).

[42] In the light of the already mentioned feature of the factor procedure according to which only complete observations are used, a problem is likely to arise for those respondents having had no type of paid activity during 1998, such as housewives, unemployed or retired. Otherwise stated, normally these individuals won't have any score on the "working conditions" factor. In order to avoid such an event, a statistical solution was adopted in this paper. Specifically, for the variables making up the "working conditions" functioning, prior to performing factor analysis the individuals belonging to the above mentioned categories were attributed a value equal to the mean of each variable (*i.e.* zero, since the items are standardized). This transformation basically does not affect the overall distribution while allowing to compute scores also for the respondents

low fulfilments in both the economic and - of course - the working dimensions of life, while not enjoying a good health status. In addition, jobless and irregular workers would face poor shelter circumstances and reveal sporadic involvement in cultural occupations. What's more, corroborating the findings by Sweeney (1998), a clear harmful effect of unemployment on mental health is suggested. Self-employed, on the contrary, exhibit better achievements, no matter the adopted methodology: higher work satisfaction, superior housing conditions, enhanced overall health than traditional employees. Finally, as for students an undefined influence on shelter conditions would require more careful investigation. However, it is important to stress that this represents the only circumstance where a clear conflict emerges between the two considered methodologies. Otherwise stated, the main finding of the direct observation of the contributions of several personal attributes to individual living standards lies in the extensive resemblance of the resulting snapshots.

From a general point of view, I interpret this as a valuable and recomforting result. However, it is likely to be even more plainly revealed if tackling the issue from a slightly different perspective, *i.e.* by focusing on the specific portray of the most deprived fraction of the entire population according to each functioning. In view of such exploratory exercise, the bottom 10% individuals were arbitrarily selected, bearing in mind that this is a purely illustrative choice implying in no way a judgement concerning the degree of actual deprivation displayed by the data. Tables 4.8 and 4.9 show the results for the subsamples.

As already claimed, women are significantly more distressed than men. Most frequently, these women are divorced, either unemployed or housewives (the latter only according to the fuzzy aggregate) and have a fairly low level of education. Confirming the previous findings, psychological distress seems to be an event that is most likely to occur earlier than one's seventies, with a worrisome presence among the youngsters. In the same line, financial limitations mostly tend to involve women between 36 and 55, especially if Waloons, living alone as a consequence of their being divorced or widowed. Next to them, a sensible presence of unemployed, of individuals having occasional jobs and people devoted to home duties can be remarked. Keeping to material dimensions, shelter deprivation is robustly associated with youth as well as with Brussels, while once again the widowed and the self-employed stand out among the best performing categories. Unhealthiness and social isolation primarily hit elderly women[43], thus the bottom share of the corresponding functionings logically display a connection with retirement as well as with widowhood and restricted family size. More significantly, however, a sensible presence of divorced individuals has to be remarked, as well as an obvious (due to the inclusion of some disabled) considerable number of sporadic workers among the unhealthiest. Similarly, the unemployed look rather concerned by the lack of social activities.

Deprived workers mostly belong to the masculine gender, are quite young, live in large families and have a fairly high level of education. The latter finding probably has to be interpreted in the light of the well-known higher job expectations commonly cherished by graduates, which are likely to impinge on their judgements. Finally, as for the absence of cultural involvement some disagreement between the methodologies appears. The picture offered by factor scores mainly points at either young unemployed Flemish men or retired people in their seventies. The fuzzy aggregate, instead, reveals that a low value for cultural activities is mostly shared by Waloon women, belonging to the same working categories indicated by the factor score, but mostly exhibiting a different marital status (*i.e.* either divorced or widowed instead of married).

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without any paid job. Nevertheless, much caution is required in the use of the obtained factor scores for the considered social groups, as for them the resulting "working conditions" functioning amounts to nothing else than a statistical construct, and, therefore, the indications stemming from it do not have much empirical value. Moreover, the adopted solution implies some obvious *a priori* normative assumptions that can be highly debatable.

[43] Also in this case, some caution is needed in evaluating the relationship of social interactions with senility.

Table 4.8 - Distribution of the 10% most deprived individuals (factor scores)								
Characteristics	Sample	Psych. distr.	Social interac.	Econ. cond.	Cultur. activ.	Work cond.	Health	Shelter
Male	47.0	43.6	30.8	42.5	53.5	60.8	42.5	45.3
Female	53.0	56.4	69.2	57.5	46.5	39.2	57.5	54.7
Age 16-25	13.9	26.7	0.6	9.0	10.7	9.6	2.3	10.9
Age 26-35	18.5	12.4	4.8	16.9	21.8	29.9	9.2	25.5
Age 36-45	22.9	24.3	11.9	25.8	23.5	30.3	16.9	28.6
Age 46-55	16.5	13.6	18.2	19.8	13.0	20.6	15.3	13.5
Age 56-65	11.4	8.8	19.8	11.3	12.6	3.3	18.7	8.1
Age 66-70	5.9	4.1	15.5	6.0	6.9	-	16.0	4.2
Age 71-75	4.6	3.8	22.9	4.9	5.2	-	15.3	2.9
Flanders	56.0	55.8	29.7	32.4	63.1	47.0	51.6	56.4
Brussels	9.0	8.9	15.7	12.1	5.9	9.7	8.8	14.1
Wallonia	35.0	35.3	54.6	55.5	31.0	43.3	39.6	29.5
Married	60.0	59.4	60.0	41.6	70.5	65.6	57.5	53.9
Single	24.3	24.0	7.8	22.6	17.2	24.7	11.3	24.7
Widowed	7.3	5.9	22.2	9.4	7.5	0.9	20.5	4.8
Divorced/Sep.	8.4	10.7	10.0	26.4	4.8	8.8	10.7	16.6
Primary school	14.3	12.4	30.3	29.1	31.3	11.8	30.9	18.9
Junior secondary	25.4	26.4	23.9	36.5	32.3	18.9	28.8	28.8
Senior secondary	32.3	32.6	22.5	23.6	27.3	31.9	25.6	26.1
Third level	19.4	19.7	13.7	8.1	8.0	25.4	11.5	18.0
University	8.6	8.9	9.6	2.7	1.1	12.0	3.2	8.2
Employee	45.0	35.0	18.9	26.8	43.4	-[44]	23.7	47.8
Self-employed	7.0	7.0	3.8	4.4	3.6	-	0.8	5.3
Unemployed	6.0	25.8	6.9	25.1	13.4	-	5.7	14.7
Studies	9.0	7.1	0.2	1.3	0.4	-	0.4	1.3
Retirement	21.0	14.9	50.0	21.3	23.9	-	46.6	14.1
Home duties	9.0	7.2	16.9	11.1	12.8	-	11.5	10.7
Other	3.0	3.0	3.3	10.0	2.5	-	11.3	6.1
Household size								
1	13.0	10.1	24.3	22.9	7.3	10.3	25.0	12.7
2	29.0	25.9	41.0	31.0	29.4	21.2	43.6	25.6
3	20.0	19.9	13.9	21.3	22.4	25.0	12.6	27.5
4 or more	38.0	55.9	20.8	24.8	40.9	43.5	18.8	34.2

Some resemblance between these sketched out images and the one proposed by Chiappero Martinetti (2000) for housewives can be clearly identified, as well as some similitudes in between the categories of the socially and the culturally deprived. The latter phenomenon may lead one to presume that some interaction between these two dimensions is actually going on, especially for fuzzy sets. Factor scores, in fact, by construction are orthogonal to each other, thus uncorrelated. In order to dispel such a doubt, the Pearson correlation coefficients between the various fuzzy evaluations have been computed and reported in Table 4.10. They do not only seem to confirm the suspicions (with a 0.39 coefficient for the two alleged parties), but in addition point to a not

[44] Due to the statistical expedient that has been used in the computation of both the factor score and the fuzzy aggregate concerning "working conditions", it is not possible to depict the portray of the most deprived according to their occupation. Such figures would lack, in fact, any empirical meaning as a result of the artificial nature of the construct for jobless individuals.

surprising significant relationship between health and mental distress, as well as to a positive influence of leisure activities on one's physical condition.

Table 4.9 - Distribution of the 10% most deprived individuals (fuzzy aggregates)

Characteristics	Sample	Psych. distr.	Social interac.	Econ. cond.	Cultur. activ.	Work cond.	Health	Shelter
Male	47.0	34.3	29.7	44.1	44.0	61.5	42.2	48.2
Female	53.0	65.7	70.3	55.9	56.0	38.5	57.8	51.8
Age 16-25	13.9	15.3	1.8	13.1	12.6	12.7	3.5	16.8
Age 26-35	18.5	15.6	5.2	16.0	18.0	29.2	7.3	23.9
Age 36-45	22.9	24.7	12.9	28.1	23.1	36.2	14.6	26.5
Age 46-55	16.5	16.7	21.1	18.2	15.9	17.1	16.3	13.2
Age 56-65	11.4	12.2	17.9	9.2	10.7	2.1	19.7	6.5
Age 66-70	5.9	5.3	18.9	4.4	6.3	-	16.6	3.8
Age 71-75	4.6	3.9	15.9	4.7	7.1	-	15.7	3.0
Flanders	56.0	57.3	42.5	28.3	46.3	55.2	54.4	50.1
Brussels	9.0	7.8	11.4	12.7	9.4	9.6	7.1	13.1
Wallonia	35.0	34.9	46.1	59.0	44.3	35.2	38.5	36.8
Married	60.0	53.3	58.4	44.1	62.6	61.8	55.1	52.2
Single	24.3	24.2	9.5	24.0	10.1	27.9	12.4	31.8
Widowed	7.3	6.1	22.9	8.8	17.3	0.7	20.1	2.4
Divorced/Sep.	8.4	16.4	9.2	23.0	10.0	9.6	12.4	13.6
Primary school	14.3	15.9	40.3	24.2	39.4	10.7	33.1	18.8
Junior secondary	25.4	25.5	26.2	36.4	32.9	18.2	30.2	33.4
Senior secondary	32.3	29.8	18.6	27.1	19.8	32.6	22.6	26.6
Third level	19.4	20.5	10.4	9.4	5.9	27.3	10.0	15.7
University	8.6	8.3	4.5	2.9	1.1	11.2	4.1	5.5
Employee	45.0	43.0	14.7	27.4	24.3	-	20.8	45.5
Self-employed	7.0	5.4	2.8	4.4	4.2	-	1.6	5.1
Unemployed	6.0	7.7	8.1	21.9	10.2	-	5.9	10.9
Studies	9.0	9.4	0.7	8.0	0.8	-	2.7	11.9
Retirement	21.0	18.9	50.4	18.9	40.6	-	47.4	11.8
Home duties	9.0	10.2	18.2	10.3	14.4	-	10.5	9.7
Other	3.0	5.4	5.1	9.1	5.5	-	11.2	5.1
Household size								
1	13.0	12.4	24.2	19.3	20.2	11.0	26.8	8.7
2	29.0	31.8	38.6	29.3	36.4	22.1	42.6	20.0
3	20.0	20.1	14.1	21.1	19.4	23.9	13.0	26.3
4 or more	38.0	35.7	23.1	30.3	24.0	43.0	17.6	45.0

What is more notable although not innovative is the claim which seems to be conveyed by the remaining sizeable values, namely the inappropriateness of totally disregarding any financial information, due to its being so intrinsically related to most of the considered aspects of well-being. Still, as both methodologies have clearly been ascertaining, income accounts only for a very limited part of the story and this should definitively be seen as a reason to follow multidimensional approaches like Sen's one.

In the light of the analysis that has been undertaken, it is clear that the individual functionings' measures constructed via both factor analysis and fuzzy sets theory exhibit a remarkable similarity and, through the performed exploration, have proven to offer substantially equivalent pictures of the Belgians' living standard, emphasizing in particular the sensible

deprivation of some social subgroups (unemployed, housewives, retired, divorced) on most dimensions as well as the existence of notable regional disparities.

Table 4.10 - Pearsons correlations among functionings (fuzzy evaluations)[45]

	Psychol. distress	Social interac.	Econ cond.	Cultur. activ.	Work cond.	Health	Shelter
Psychol. distress	1.00						
Social interactions	0.05	1.00					
Economic conditions	-0.16	0.13	1.00				
Cultural activities	0.07	0.39	0.22	1.00			
Working conditions	-0.13	0.02	0.14	0.03	1.00		
Health	-0.29	0.26	0.17	0.16	0.09	1.00	
Shelter	-0.17	-0.06	0.22	0.06	0.07	-	1.00

But where do such similarities stem from? At the end of this exploration, no clear-cut answer to this question can be offered; rather, an intuition, suggesting to systematically explore the formal relationship between the various fuzzy aggregative operators and the factor scores' least squares estimation procedure, as a consequence of the standardization practice in factor analysis substantially producing analogous effects on the items as the attribution of membership values. My exclusive focus, in this paper, has been on the empirical consequences of the adoption of a particular procedure on the overall results. I believe, however, that further contributions in this area should, on the one hand, perform sensitivity analyses with the purpose of testing the dependency of the conclusions on the particular dataset which is adopted, and, on the other, try to structure the different approaches more explicitly along the lines of their normative implications.

5 Concluding remarks

In this paper a couple of operational interpretations of Sen's functioning approach have been explored in view of assessing the role played by methodological choices in the determination of individual's well-being profiles. Some scepticism towards the fuzzy methodology initially triggered this study. Nevertheless, such mistrust had to be reappraised in the light of the encouraging preliminary results that were reported. Clearly, much remains to be done to refine the procedures - particularly the fuzzy one - outlined here. However, an impressive resemblance seems to be borne by the general patterns of functionings' achievements arising from the application of the confronted techniques. The presence of a few discrepancies in the actual shapes of the derived functionings does not seem to significantly affect the indications stemming from the multivariate analysis. Hence, this overall outcome can be seen as a promising preliminary step in deepening our understanding of the reliability and practicability of the capability approach.

Moreover, the results do not look especially sensitive to the various subjective prior judgements that had to be made. Having examined the robustness of the fuzzy aggregates to changes in the various assumptions that were necessary for their construction, they were found to be rather insensitive to the selection of the specific form taken by the membership function. I feel, however, that further analyses should possibly incorporate a more comprehensive array of tests concerning the choice of the specific parameters on which the sigmoid and trapezoidal specifications are based or, alternatively, consider other membership functions.

The picture of the standard of living resulting from the empirical application does not differ too much from the ones illustrated by some previous studies, namely a relatively high degree of accomplishment in the material dimensions (shelter, working and economic conditions) as well as in health and psychological status, contrasted with relatively poor realizations in social interactions and cultural activities. The degree of reliability of the indications stemming from the latter two dimensions, however, remains quite doubtful owing to their particular definition. A closer look at the apparent contributions of a number of personal characteristics to raising or lowering individual

[45] One coefficient was omitted because not significant. Any other, on the contrary, is significant at 1%.

achievements reveals a slight gender bias together with the precarious physical, mental, economic and housing situation experienced by divorced or separated individuals, not to mention the sharp regional disparities in the Belgians' living standard. Poor social life for the retired, for women taking care of the housework as well as for the unemployed, bearing not only a financial but also a remarkable psychological burden consequent upon their condition, are further evident.

I believe an unquestionable merit of both the investigated approaches lies in having contributed to stress that the reality we are often confronted with cannot be fully grasped by the traditional methodological instruments and that, sometimes, the use of essentially equivalent methods borrowed from other areas may allow welfare economists to move further in their investigations. Hopefully, this very preliminary and unambitious exercise was able to convey such messages, while in the mean time stressing that applied work on Sen's approach nowadays probably constitutes the field embodying the main challenges for future research.

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Appendixes

Appendix A - Basic indicators

Functioning's components	Type of indicator	Description of the indicator
Social interactions		
Friends	Categorical (5 mod.)	Frequency of contact with friends
Sport match	Categorical (5 mod.)	Frequency of attending matches
Café	Categorical (5 mod.)	Frequency of going to cafés
Restaurant	Categorical (5 mod.)	Frequency of going to restaurants
Disco	Categorical (5 mod.)	Frequency of going to discos
Games	Categorical (5 mod.)	Frequency of playing games with friends (bowling, billiard, snooker, etc.)
Going out	Categorical (5 mod.)	Frequency of going out
Sport practice	Dichotomous	Participation at least once a week to a sporting activity
Economic status		
Meet ends	Categorical (6 mod.)	Possibility of making ends meet
Savings	Dichotomous	Regularity in saving
Perceived situation	Categorical (3 mod.)	Perception of the household present economic situation
Economic satisfaction	Categorical (6 mod.)	Degree of satisfaction about one's economic situation
Financial difficulties	Summated scale	Various economic difficulties [46]
Non-affordability	Summated scale	Lack of a number of commodities due to unaffordability[47]
Health		
Health status	Categorical (5 mod.)	Self-assessed health status
Chronic illness	Dichotomous	Presence of chronic illness, handicap or disability
Recent illness	Dichotomous	Interruption of one's activities due to recent illness or accident
Hospital	Dichotomous	Hospitalised during last year
Generalist	Continuous	Number of visits to a generalist in last year
Specialist	Continuous	Number of visits to a specialist in last year
Alternative medicine	Continuous	Number of visits to an homeopath, an osteologist, etc. in last year
Cultural activities		
Theatre	Categorical (5 mod.)	Frequency of going to the theatre
Cinema	Categorical (5 mod.)	Frequency of going to the cinema
Concert	Categorical (5 mod.)	Frequency of attending concerts
Museum	Categorical (5 mod.)	Frequency of visiting musea
Conference	Categorical (5 mod.)	Frequency of attending conferences
Creative activity	Dichotomous	Participation to a creative activity (dance, painting, singing, theatre, etc.)
Association	Dichotomous	Membership to a socio-cultural association

[46] The indicators whose summated rating has been considered are: difficulties in paying the rent; difficulties in paying invoices (water, electricity, etc.); difficulties in paying loans back; difficulties in paying instalments for consumer goods.

[47] The indicators whose summated rating has been considered relate to the impossibility to afford the following items: a car; a tv; a video recorder; a microwaves oven; a telephone; a dishwashing machine; a computer; a country house; an alarm system.

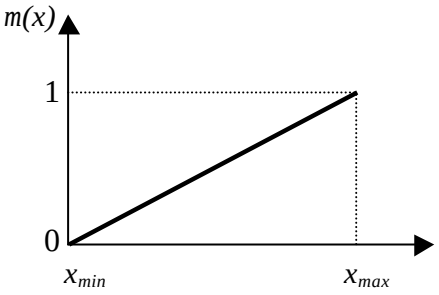
Functioning's components	Type of indicator	Description of the indicator
Psychological distress		
Depressed	Categorical (5 mod.)	Frequency of feeling depressed
No appetite	Categorical (5 mod.)	Frequency of losing appetite
Insomnia	Categorical (5 mod.)	Frequency of difficulty sleeping
Unrested	Categorical (5 mod.)	Frequency of feeling without energy
Nervous	Categorical (5 mod.)	Frequency of being unable to sit quietly
Feel guilty	Categorical (5 mod.)	Frequency of feeling guilty, self-doubting
No concentration	Categorical (5 mod.)	Frequency of being unable to concentrate
Weeping	Categorical (5 mod.)	Frequency of weeping easily
Pessimistic	Categorical (5 mod.)	Frequency of being pessimistic
Irritable	Categorical (5 mod.)	Frequency of being irritable
Need reassurance	Categorical (5 mod.)	Frequency of needing reassurance
Out of sorts	Categorical (5 mod.)	Frequency of feeling out of sorts
Psychological problems	Dichotomous	Interruption of one's activities due to psychological problems
Shelter		
Crowding index	Continuous	N. of rooms/equivalence scale
Heating	Dichotomous	Heating availability
Housing satisfaction	Categorical (6 mod.)	Degree of satisfaction about one's housing
Dwelling's problems	Summated scale	Presence of structural problems in the house[48]
Area's problems	Summated scale	Presence of problems due to the location[49]
Working conditions		
Work certitude	Categorical (6 mod.)	Degree of satisfaction about the certitude of one's work
Work type	Categorical (6 mod.)	Degree of satisfaction about one's type of activity
Number of hours	Categorical (6 mod.)	Degree of satisfaction about the number of hours spent at work
Work schedule	Categorical (6 mod.)	Degree of satisfaction about one's schedule
Working environment	Categorical (6 mod.)	Degree of satisfaction about one's working conditions and environment
Work distance	Categorical (6 mod.)	Degree of satisfaction about the distance of one's workplace from home
Job search	Dichotomous	Currently looking for an alternative job
Overqualified	Dichotomous	Feeling overqualified for the position currently held

[48] The indicators whose summated rating has been considered are: insufficient space; lack of brightness; heating problems; mould or humidity; damaged roof; cracks in the walls; damaged coatings.

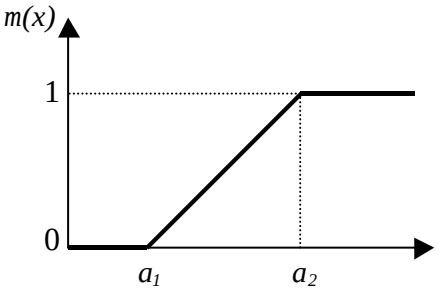
[49] The indicators whose summated rating has been considered are: noise from neighbours; noise from outside (street, factories, etc.); environmental pollution; criminality in the area; bad acoustic insulation; slum district; lack of privacy with respect to neighbours.

Appendix B - Graphic representations of membership functions

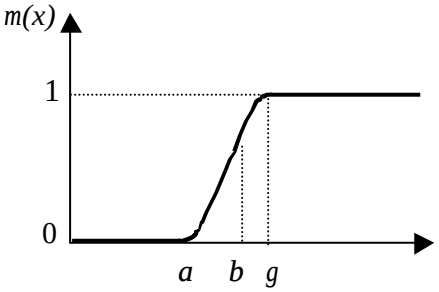
a) Linear function



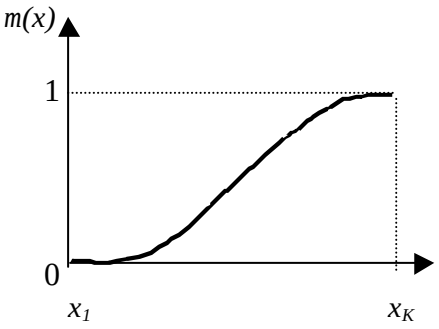
b) Trapezoidal function



c) Sigmoid function



d) Cheli and Lemmi's function



Appendix C - Membership functions for the different models

Functioning	Model 1	Model 2	Model 3
Social interactions			
Friends	Cheli-Lemmi	Sigmoid	Trapezoidal
Sport match	Cheli-Lemmi	Sigmoid	Trapezoidal
Café	Cheli-Lemmi	Sigmoid	Trapezoidal
Restaurant	Cheli-Lemmi	Sigmoid	Trapezoidal
Disco	Cheli-Lemmi	Sigmoid	Trapezoidal
Games	Cheli-Lemmi	Sigmoid	Trapezoidal
Going out	Cheli-Lemmi	Sigmoid	Trapezoidal
Economic status			
Meet ends	Cheli-Lemmi	Sigmoid	Trapezoidal
Perceived situation	Cheli-Lemmi	Linear[50]	Linear[50]
Economic satisfaction	Linear	Linear	Trapezoidal
Financial difficulties	Cheli-Lemmi	Logistic	Trapezoidal
Non-affordability	Cheli-Lemmi	Logistic	Trapezoidal
Health			
Health status	Linear	Linear	Trapezoidal
Doctor[51]	Cheli-Lemmi	Logistic	Trapezoidal
Psychological distress			
Depressed	Cheli-Lemmi	Sigmoid	Trapezoidal
No appetite	Cheli-Lemmi	Sigmoid	Trapezoidal
Insomnia	Cheli-Lemmi	Sigmoid	Trapezoidal
Unrested	Cheli-Lemmi	Sigmoid	Trapezoidal
Nervous	Cheli-Lemmi	Sigmoid	Trapezoidal
Feel guilty	Cheli-Lemmi	Sigmoid	Trapezoidal
No concentration	Cheli-Lemmi	Sigmoid	Trapezoidal
Weeping	Cheli-Lemmi	Sigmoid	Trapezoidal
Pessimistic	Cheli-Lemmi	Sigmoid	Trapezoidal
Irritable	Cheli-Lemmi	Sigmoid	Trapezoidal
Need reassurance	Cheli-Lemmi	Sigmoid	Trapezoidal
Out of sorts	Cheli-Lemmi	Sigmoid	Trapezoidal
Cultural activities			
Theatre	Cheli-Lemmi	Sigmoid	Trapezoidal
Cinema	Cheli-Lemmi	Sigmoid	Trapezoidal
Concert	Cheli-Lemmi	Sigmoid	Trapezoidal
Museum	Cheli-Lemmi	Sigmoid	Trapezoidal
Conference	Cheli-Lemmi	Sigmoid	Trapezoidal
Shelter			
Crowding index	Cheli-Lemmi	Sigmoid	Trapezoidal
Housing satisfaction	Linear	Linear	Trapezoidal
Dwelling's problems	Cheli-Lemmi	Logistic	Trapezoidal
Area's problems	Cheli-Lemmi	Logistic	Trapezoidal

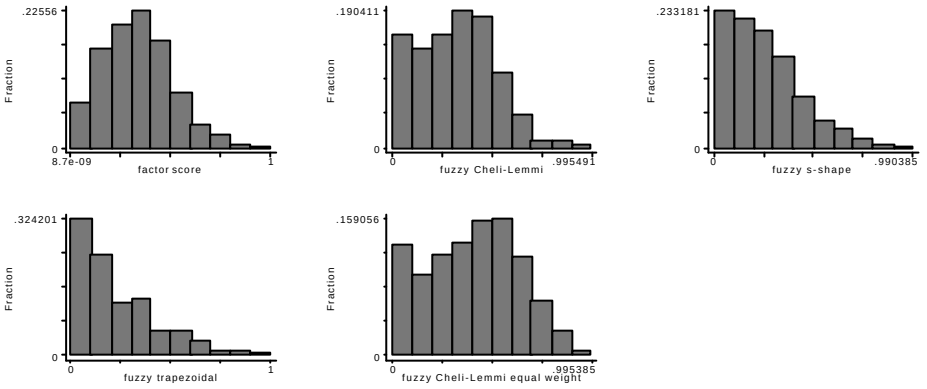
[50] As a result of this indicator being characterised by three modalities only, neither the sigmoid nor the trapezoidal functions could be specified.

[51] Three indicators were merged so as to facilitate their treatment: Generalist, Specialist and Alternative medicine were aggregated into an overall measure (named Doctor) by means of the standard union operator, so as to reflect the position of the least favourite individuals (from the point of view of the necessity of contacts with a physician) in the determination of the corresponding functioning.

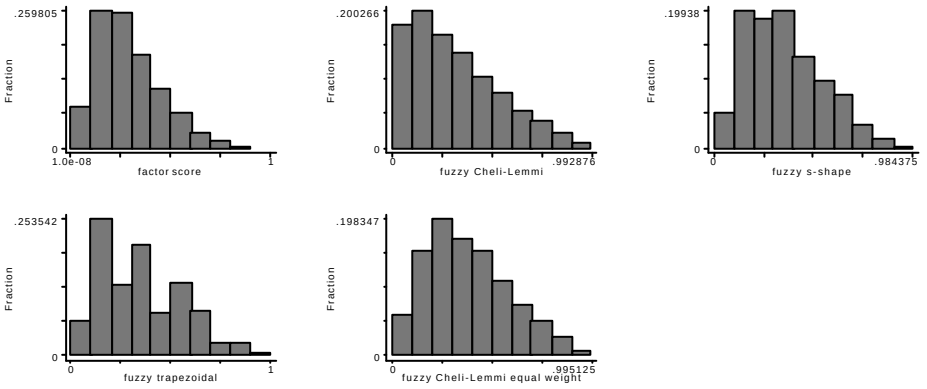
Functioning	Model 1	Model 2	Model 3
Working conditions			
Work certitude	Cheli-Lemmi	Sigmoid	Trapezoidal
Work type	Cheli-Lemmi	Sigmoid	Trapezoidal
Number of hours	Cheli-Lemmi	Sigmoid	Trapezoidal
Work schedule	Cheli-Lemmi	Sigmoid	Trapezoidal
Working environment	Cheli-Lemmi	Sigmoid	Trapezoidal
Work distance	Cheli-Lemmi	Sigmoid	Trapezoidal

Appendix D - Graphic comparison of distributions

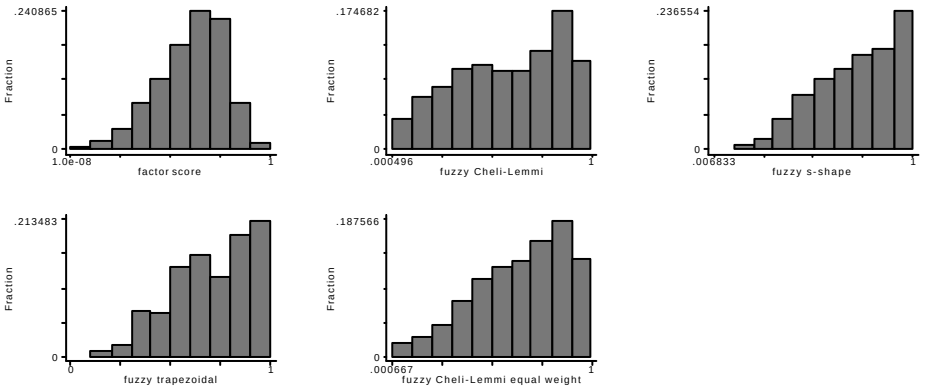
a) Psychological distress



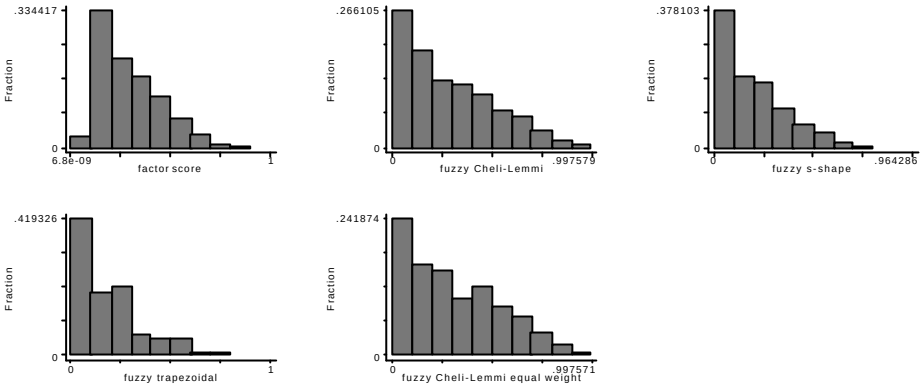
b) Social interactions



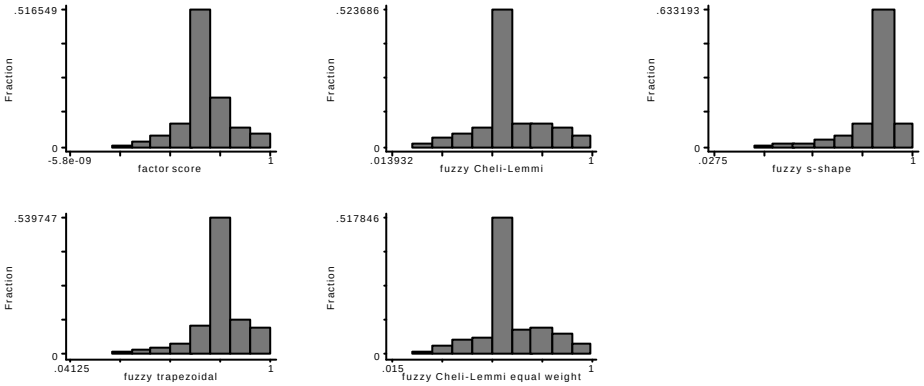
c) Economic conditions



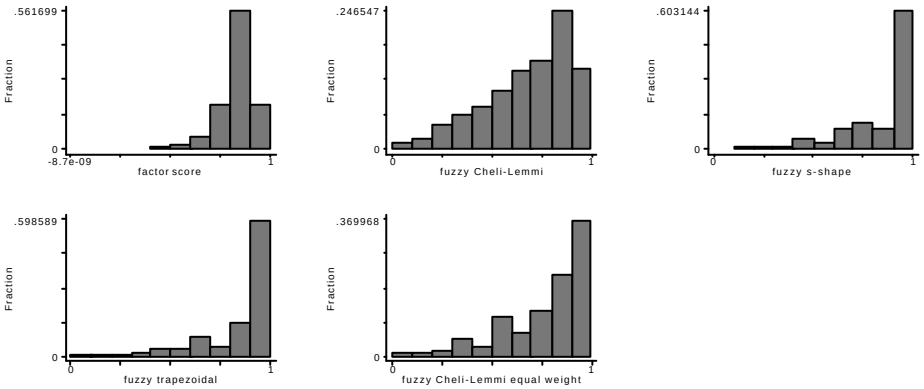
d) Cultural activities



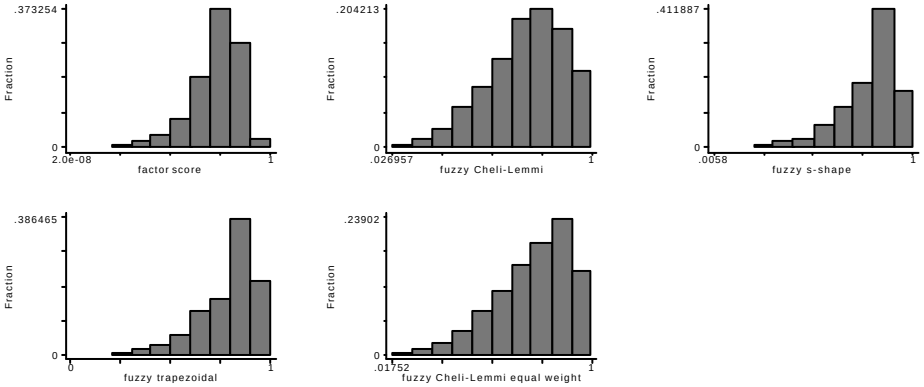
e) Working conditions



f) Health



g) Shelter



Appendix E - Sample means and variable definitions

Summary characteristics of the sample	
Characteristics	*Sample means (n=7021)*
Male	0.470
Female	0.530
Age 16-25	0.139
Age 26-35	0.185
Age 36-45	0.229
Age 46-55	0.165
Age 56-65	0.114
Age 66-70	0.059
Age 71-75	0.046
Flanders	0.560
Brussels	0.090
Wallonia	0.350
Married	0.600
Single	0.243
Widowed	0.073
Divorced/Separated	0.084
Primary school	0.143
Junior secondary	0.254
Senior secondary	0.323
Third level non univ.	0.194
University and over	0.086
Employee	0.450
Self-employed	0.070
Unemployed	0.060
Studies	0.090
Retirement	0.210
Home duties	0.090
Other	0.030
Household size	3
Mean household income	28148 Euros
1^{st} decile (% mean)	0.320
9^{th} decile (% mean)	1.750

Income: aggregate household disposable income per year corrected on the basis of the OECD equivalence scale. The resulting amount was further divided by 1 million.
Income sqr: aggregate household disposable income per year corrected by the OECD scale, divided by 1 million and squared.
Other: this category includes occasional jobs, career breaks, disabled, workers on half-pay.