And the winner is...: an Empirical Evaluation of Two Competing
Approaches to Household Labour Supply.


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# DISCUSSION
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# And the winner is... An empirical evaluation of two competing approaches to household labour supply

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## Abstract

In this paper, an empirical evaluation is presented of two competing flexible labour supply models. The first is a standard unitary model, while the second is based on the collective approach to household behaviour. The evaluation focuses on the testing of the models' theoretical implications, on their ability to identify structural information, like preferences and on their empirical performance. Models are estimated on Belgian microdata from 1992 and 1997. The unitary model cannot be rejected for single person households, while it is rejected for a sample of two person households. The alternative collective model cannot be rejected for the same sample. However, since the crucial assumption of egoistic or Beckerian caring individual preferences is rejected, the comparative advantage of the collective model as basis for intrahousehold welfare evaluations cannot be fully exploited. Finally, the collective model has the best empirical fit.

Key words: collective household models, household bargaining, intrahousehold allocation, labour supply.
JEL classification: D12, J22.

# 1. Introduction

In the traditional approach to household behaviour it is assumed that households behave as if they were single decision making units. Consequently, household consumption and labour supply are considered to be the observable result of the maximization of unique rational preferences, constrained by a household budget restriction. Examples of this approach are Blundell and Walker (1986) for labour supply and Browning and Meghir (1991), Banks, Blundell and Lewbel (1997) and Blundell and Robin (2000) for consumption behaviour.

This unitary model however, suffers from methodological, empirical and welfare economic deficiencies. Methodologically, one can argue that the notion of subjective preferences is inseparable from methodological individualism, that asserts that social theories should run in terms of the behaviour of individuals (see, e.g., Blaug, 1980). At the empirical level, one can refer to Arrow's impossibility theorem, or more narrowly, to aggregate demand theory which state that an aggregate of individuals do not necessarily behave as a single individual with own rational preferences. Accordingly, the unitary model may act as an empirical strait-jacket for observable household behaviour. Perhaps not surprisingly, the theoretical implications of homogeneity, symmetry and negative semidefiniteness of the Slutsky matrix were repeatedly rejected when confronted with the data (see, e.g., Deaton and Muellbauer, 1980 and Blundell, 1988). Another potentially important restriction of the unitary model, is that individual nonlabour incomes of the household members do not play any role in the household's allocation of consumption and labour supply, once total nonlabour income is taken into account. Also this restriction has been strongly rejected in numerous empirical studies (see, e.g., Thomas, 1990, Bourguignon et alii, 1993, Browning et alii, 1994 and Fortin and Lacroix, 1997). Closely related to the positive (descriptive) modelling of household behaviour, is the normative welfare analysis of changes in the economic environment. As to the intrahousehold allocation of consumption and labour supply, and consequently of individual welfare, the unitary model is rather barren. Traditional welfare economic models, e.g., only consider the distribution of welfare over households. Apps and Rees (1988) and Brett (1998) have shown however, that when evaluating the welfare effects of tax changes, intrahousehold distributional effects can in general not be ignored. Moreover, Alderman et alii (1995) argue that accepting the unitary model, when it is inappropriate, has more serious consequences for policy prescriptions than rejecting the unitary model when it is appropriate. In programs that target individuals in certain groups (e.g., women or children), knowledge of the intrahousehold decision process may be especially important.

A valuable alternative to the unitary model is the collective approach to house-

hold behaviour. In the collective household model, as initially defined by Chiappori (1988, 1992), it is explicitly assumed that many person households consist of several individuals who may have different preferences. Among these individuals, a household bargaining process takes place that is assumed to be Pareto efficient. Chiappori (1988, 1992) showed that even this rather weak assumption is able to generate testable implications on labour supply that differ from those of the unitary model. Moreover, the repeated rejections of symmetry and negativity can be interpreted somewhat, since the latter are not to be satisfied in the collective household model (see Browning and Chiappori, 1998). Given its particular set-up, the collective household model incorporates a natural answer on the methodological, empirical and welfare economic disadvantages of the unitary model.

This paper has the objective of balancing the above two competing theories in a household labour supply context. The question can be asked however, when is a theory of household behaviour a good theory ? Answers on it are manifold. These (normative) answers can be more or less debatable (e.g., social theories should fit in with methodological individualism). Two more neutral basic requirements are the following (see Bourguignon and Chiappori, 1994). First, one should expect a theory of household behaviour to generate testable implications that are potentially rejectable. Apart from the ability to save on degrees of freedom, these restrictions can be used to test the adequacy of the theory. A second requirement is that a theory of household behaviour should be able to recover some structural information, like preferences. This allows the theory to be used as a formal basis for a normative welfare analysis. Apart from these basic requirements, no one will deny that some explanatory power is far from a disadvantage for a good theory.

As to these requirements, the unitary model passes with flying colours. It generates the well-known testable restrictions on demand and leisure of adding up, homogeneity, symmetry and negative semidefiniteness of the Slutsky matrix. On the other hand, demand and leisure (labour supply) that satisfy these four restrictions can be shown to be integrable to a rational preference ordering. Moreover, it can be argued that it gives a fairly adequate description of observable household behaviour. Also the collective household model meets the above requirements to some extent. In Chiappori (1988), Browning et alii (1994) and Browning and Chiappori (1998), to give only a few examples, testable and rejectable implications of the model are derived. As to the second requirement, integrability results are less strong than in the unitary model. Extra assumptions, apart from Pareto efficiency of household decisions, are needed to recover a great deal of the intrahousehold allocation process and individual preferences. More specifically, a necessary condition for such an identification result is that individual preferences of the household members are of the 'egoistic' or 'caring' type. Preferences that

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only depend on individual consumption and leisure (labour supply) are of the egoistic type. Beckerian caring preferences are characterized by the presence of the consumption and leisure bundle of the other household members in the own preference ordering, although that bundle only enters in a weakly separable way (see Becker, 1974a, 1974b).

We will evaluate the unitary and collective approaches by means of the above three requirements of good theories. Firstly, attention will be focused on the estimation and testing of both a unitary and a collective labour supply model. This will be done on three samples of households from the 1992 and 1997 Belgian Socio-Economic Panel (SEP). The first two samples consist of male and female singles with a positive labour supply, since for these groups of households, the unitary approach should be entirely applicable. The second sample consists of two-person households where both individuals have a positive labour supply. In general, the unitary and collective approaches should have different implications on observable behaviour for this sample. In order not to reject the theoretical implications of the models because of a too restrictive specification of labour supply, a flexible labour supply model that is a generalization of the Linear Expenditure System is opted for. Estimation and testing of the unitary model does not pose too many problems in the given set-up. As for its competitor, in the first instance account will be taken of general individual preferences. This is a different approach than Fortin and Lacroix (1997), who start from a collective labour supply model where egoistic and caring preferences are assumed from the outset. On the other hand, our approach is on a par with Chiappori, Fortin and Lacroix (forthcoming). To test the collective model, we will make use of the robust distribution factor proportionality test of Bourguignon et alii (1993) and Browning and Chiappori (1998). This test makes use of so-called 'distribution factors'. The latter are defined as variables that influence the intrahousehold distribution process, but that do not directly affect individual preferences or the household budget constraint (see Bourguignon, Browning and Chiappori, 1994). Examples of such distribution factors are laws on alimony, child benefits, tax laws that differ according to marital status and individual nonlabour incomes (that affect the household budget constraint only indirectly through a change in total household nonlabour income).

Secondly, as already mentioned, the identification of individual preferences and the intrahousehold allocation process requires egoistic or caring preferences in the collective approach. Therefore, the general collective labour supply model will be reformulated in terms of these types of preferences. Again useful testable restrictions can be derived (see Chiappori, 1988, 1992 and Chiappori, Fortin and Lacroix, forthcoming). Note however, that a rejection of these restrictions does not necessarily imply that the collective model is rejected. If the above distribution factor proportionality test is not rejected, a rejection of the additional restrictions

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may be considered as a rejection of the particular egoistic or caring preferences, rather than a rejection of the collective approach. Since separate estimations are done for male and female singles, this approach will also allow to test whether singles have the same preferences as individuals in two person households.

The third part of the analysis will focus on the empirical performance of both models. This will be done by means of a comparison of their labour supply elasticities and the mean squared deviations between observed and predicted labour supply. As a benchmark, the empirical performance of a very naive labour supply model is evaluated. As far as we know, no such performance analysis has been done on a collective household model.

The paper is structured as follows. In section 2, the unitary and collective approaches to household labour supply are formally discussed. Section 3 deals with the functional specifications that are chosen for the empirical evaluation of the models. Data and econometric issues are discussed in section 4. Section 5 gives empirical results, while section 6 concludes.

## 2. Household labour supply: two competing approaches

In what follows, we focus on households consisting of two working-age individuals who both participate in the labour market. The labour supply approaches that will be dealt with in this section are static within-period models in a cross-sectional context. Consequently, the only price variation that is assumed to be observable are the wages of both individuals. Moreover, we assume that these hourly wage rates do not depend on hours worked, which gives rise to simple linear budget constraints. Note that this implies a model without taxation or with a linear income tax.

### 2.1. The unitary labour supply model

In the traditional approach to household behaviour, it is assumed that households behave as if they were single decision making units. More specifically, each household's needs and desires are assumed to be captured by a rational preference ordering over alternative consumption and leisure bundles. These preferences are usually assumed to be representable by a well-behaved utility function that is unique up to a monotone increasing transformation. Formally, the utility function of a household with two working-age individuals $A$ and $B$ equals:

$$u = v \left( c, q_0^A, q_0^B, \mathbf{d} \right),$$

where $v$ is a strongly quasi-concave, increasing and twice continuously differentiable function. The arguments are the household's consumption of a Hicksian

aggregate commodity $c$ and the individuals' leisure amounts $q_0^A$ and $q_0^B$, all three $\in \mathbb{R}_+$. Finally, preferences also depend on a vector of demographic and taste shifter variables $\mathbf{d}$ (e.g., age and education level of household members).

Consumption and leisure bundles are financed by means of limited household resources, which is represented by the (full) household budget constraint:

$$c + w^A q_0^A + w^B q_0^B \leq y^A + y^B + y^H + w^A T + w^B T,$$

where $w^A$ and $w^B$ are the hourly wage rates of both individuals, $T$ is total time available, $y^A$ and $y^B$ are personal nonlabour incomes and $y^H$ is the household's nonlabour income that cannot be assigned to one of its members. Note that, without losing generality, the price of the Hicksian aggregate commodity has been set equal to 1. The household's choice problem then, is reduced to the maximization of the utility function $v$ subject to the household budget constraint, and gives rise to two differentiable Marshallian labour supply functions (where labour supply $\ell^I = T - q_0^I$, $I = A, B$) and a consumption function:

$$
\begin{aligned}
\ell^A &= h^A \left( y^S, \mathbf{w}, \mathbf{d} \right) \\
\ell^B &= h^B \left( y^S, \mathbf{w}, \mathbf{d} \right) \\
c &= g \left( y^S, \mathbf{w}, \mathbf{d} \right),
\end{aligned}
\tag{2.1}
$$

where $y^S = y^A + y^B + y^H$ is the household's total nonlabour income and $\mathbf{w} = \left( w^A, w^B \right)'$. Denoting the Slutsky effects on *labour supply* by $s_{IJ} = \frac{\partial h^I}{\partial w^J} - \frac{\partial h^I}{\partial y^S} \ell^J$, $I, J = A, B$, the utility maximization problem implies the following restrictions on observable labour supply (note that adding up and homogeneity are automatically satisfied in the given setting):

Slutsky symmetry: $s_{AB} = s_{BA}$

positivity: $s_{AA} \geq 0$, $s_{BB} \geq 0$ and $s_{AA} \cdot s_{BB} - s_{AB} \cdot s_{BA} \geq 0$.

These restrictions have been tested and rejected on numerous occasions. See Blundell and Meghir (1986) and Blundell and Walker (1986) for some evidence and interpretation in labour supply models, or Deaton and Muellbauer (1980) and Blundell (1988) with regard to consumption allocation models.

Apart from the above properties of labour supply, the unitary model implies the so-called 'income pooling hypothesis'. This asserts that the source of the household's nonlabour income $y^S$ does not affect the household's allocation. In other words, marginal changes of the different nonlabour incomes have the same effect on labour supply: $\frac{\partial h^I}{\partial y^A} = \frac{\partial h^I}{\partial y^B} = \frac{\partial h^I}{\partial y^H} = \frac{\partial h^I}{\partial y^S}$, $I = A, B$. Also this restriction has been repeatedly rejected when confronted with the data (see, e.g., Lundberg, 1988, Thomas, 1990 and Fortin and Lacroix, 1997). An alternative approach, that does not put observable household behaviour in such an empirical strait-jacket, is the collective approach to household behaviour.

## 2.2. The collective model

### 2.2.1. A general model

In this section, a labour supply model is presented that is based on Chiappori's (1988, 1992) collective approach to household behaviour. Contrary to the unitary model, the collective approach explicitly takes into account that *many person* households consist of several household members with potentially different preferences. Among these household members, an intrahousehold bargaining process is assumed to take place that results in observed household consumption and labour supply. Unlike Manser and Brown (1980) and McElroy and Horney (1981), who assume explicit bargaining concepts like the Nash or the Kalai-Smorodinsky solution, Chiappori (1988, 1992) only assumes that household decisions are Pareto efficient. I.e., household consumption and labour supply are such that no household member can be made better off, without making the other one worse off. In contrast to explicit bargaining rules, no restriction is imposed a priori on which allocation on the Pareto frontier will be chosen by the household. To derive an empirical test for this collective approach, the individual preferences may be very general (see Chiappori, Fortin and Lacroix, forthcoming). However, if one wants to derive some identification results on individual preferences or the intrahousehold bargaining process by means of observable variables, more restrictions are to be made on the form these preferences may take.

In the collective approach to household behaviour, both household members $A$ and $B$ have their own rational preferences over consumption and leisure bundles. These preferences can be very general in that they may be defined over both one's own consumption and leisure and the consumption and leisure bundle of the other household member. Individual preferences are assumed to be representable by the following utility functions ($I = A, B$):

$$u^I = v^I \left( c^A, c^B, q_0^A, q_0^B, \mathbf{d} \right),$$

where $v^I$ is a twice continuously differentiable, strongly concave utility function with individual consumptions of the Hicksian aggregate commodity $c^A$ and $c^B$ and the leisure amounts $q_0^A$ and $q_0^B$ as arguments. Individual preferences moreover depend on demographic and taste shifter variables $\mathbf{d}$. The utility function $v^I$ is assumed to be strictly increasing in $c^I$ and $q_0^I$, $I = A, B$. Externalities in consumption and leisure can be both positive and negative. Therefore, $v^I$ is not necessarily increasing in $c^J$ and $q_0^J$, for $J \neq I$. The (full) household budget constraint now equals:

$$c^A + c^B + w^A q_0^A + w^B q_0^B \leq y^A + y^B + y^H + w^A T + w^B T,$$

where the price of the Hicksian aggregate commodity is normalized to 1.

Since individual preferences are assumed to be represented by strongly concave utility functions and the household budget constraint defines a convex set, the utility possibility set is strictly convex. It is a standard result in welfare economics that each Pareto efficient allocation on the Pareto frontier of a strictly convex utility possibility set can be obtained by means of the maximization of a linear social welfare function for some positive weights for both individuals (see, e.g., Mas-Colell et alii, 1995). More concretely, the household allocation problem can be defined as the unique solution to the following maximization problem:

$$\max_{c^A, c^B, q_0^A, q_0^B} \mu \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right) v^A \left( c^A, c^B, q_0^A, q_0^B, \mathbf{d} \right) + \left( 1 - \mu \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right) \right) v^B \left( c^A, c^B, q_0^A, q_0^B, \mathbf{d} \right)$$

(2.2)

subject to

$$c^A + c^B + w^A q_0^A + w^B q_0^B \leq y^S + w^A T + w^B T,$$

where the weights $\mu$ and $(1 - \mu)$ are assumed to be continuously differentiable and homogeneous of degree zero in $y^S$, $\mathbf{y}$ and $\mathbf{w}$. Other arguments of these weights are the vectors $\mathbf{d}$ and $\mathbf{z} = (z_1, ..., z_m)'$. The latter is a vector of distribution factors, different from demographic variables $\mathbf{d}$ and nonlabour incomes $\mathbf{y}$ (see Bourguignon, Browning and Chiappori, 1994). Distribution factors are defined as variables that affect the welfare weight attached to household members, but that do not have any *direct* influence on the household members' preferences or the household budget constraint. Examples of such distribution factors are laws on alimony or child benefits. In Chiappori, Fortin and Lacroix (forthcoming), the sex ratio (defined as the number of males over the number of males and females for several sociological groups) and an index capturing divorce laws are used as distribution factors.

An interpretation of the welfare weights $\mu$ and $(1 - \mu)$ is that they represent the bargaining power of the household members in the household allocation process[1]. For example, changes in relative wages or individual nonlabour incomes may shift bargaining power from one individual to the other. This will then be reflected in individual labour supplies and consumption. Also changes in other distribution factors may affect outside opportunities for the household members and may thus have consequences on their bargaining power and on the household's allocation.

Household preferences, as represented by the household utility function (2.2) amount to Kalman's (1968) and Pollak's (1977) price dependent preferences. They have shown (in a consumption context) that the usual 'Slutsky effects', defined as

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[1]Note that these welfare weights are normalized Lagrangian multipliers associated with the Pareto-problem of maximizing the utility of household member $A$ subject to a pre-allocated utility for individual $B$ and the household budget constraint. Therefore, welfare weights $\mu$ and $(1 - \mu)$ will generally depend on the exogenous variables of the above maximization problem.

the sum of the 'standard' uncompensated price effect and the 'standard' income effect are not to be symmetric any more. Moreover, the matrix consisting of these 'Slutsky effects' is not necessarily negative semidefinite. It will also become apparent that the income pooling hypothesis needs not to be satisfied in the collective approach. Such household preferences are thus not necessarily 'rational' any more. Intransitivities can easily occur in the collective setting.

In Browning and Chiappori (1998), a symmetry test of the collective model is derived which makes use of these results. However, they have shown that one needs at least five different commodities to apply their test. Consequently, in the given labour supply context with no information available on the allocation of total expenditures to different commodities, the above collective model cannot be tested by means of the Browning and Chiappori (1998) symmetry test.

An alternative test for the collective labour supply model can be derived. The test is driven by the properties of the distribution factors $\mathbf{z}$ and the individual nonlabour incomes $\mathbf{y}$. As is clear from equation (2.2) both vectors only occur in the bargaining weights $\mu$ and $(1 - \mu)$ and do not enter individual preferences or the household budget constraint (apart from an indirect effect via $y^S$). This imposes a specific structure on labour supply. Start from the maximization of the household utility function (2.2) subject to the full household budget constraint. This results in the following set of differentiable consumption and labour supply equations:

$$
\begin{aligned}
\ell^A &= h^A \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right) \\
\ell^B &= h^B \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right) \\
c^A &= g^A \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right) \\
c^B &= g^B \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right),
\end{aligned}
\tag{2.3}
$$

where $c^A + c^B = c$, of which only total household consumption of the Hicksian aggregate commodity $c$ is observed. Since, apart from the usual dependence on exogenous variables, the household allocation depends on the bargaining weights $\mu$ and $(1 - \mu)$, labour supply can be written as:

$$
\begin{aligned}
\ell^A &= l^A \left( \mu \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right), y^S, \mathbf{w}, \mathbf{d} \right) \\
\ell^B &= l^B \left( \mu \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right), y^S, \mathbf{w}, \mathbf{d} \right).
\end{aligned}
\tag{2.4}
$$

Note that these functions are never directly observed. We can only observe the labour supply equations of (2.3), without independent variation in the bargaining weights. However, it is clear from (2.4) that nonlabour incomes $\mathbf{y}$ and distribution factors $\mathbf{z}$ only enter labour supply via $\mu$. This allows to derive the following test for the collective labour supply model (see also Bourguignon et alii, 1993 and Browning and Chiappori, 1998):

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**Proposition 2.1.** Distribution factor proportionality: *If observed labour supplies $\mathbf{h} = \left( h^A, h^B \right)'$ fit into the collective household approach and distribution factors have a nonzero effect on $\mu$, then $\frac{\partial \mathbf{h}}{\partial \mathbf{r}'} = \frac{\partial \mathbf{h}}{\partial z_1} \boldsymbol{\theta}'$ for $\mathbf{r} = (z_2, ..., z_m, \mathbf{y}')'$ and $\boldsymbol{\theta}$ a vector of dimension $(m + 1)$. The latter vector captures the marginal substitution effects between $z_1$ and every element of $\mathbf{r}$ in the function $\mu$. Marginal effects associated with nonlabour incomes are direct effects, that do not run through total nonlabour income.*

**Proof**: See Appendix A. The result says that the ratios of marginal effects of nonlabour incomes and the distribution factors on both individual labour supplies are equal. This general restriction on the given collective model is easily tested. Note however, that the result is fundamentally driven by the fact that at least two separate nonlabour incomes and/or distribution factors $\mathbf{z}$ can be observed.

### 2.2.2. Identification results: the sharing rule interpretation

It is a well-known result of the unitary model that if demand and individual leisure amounts add up, are homogeneous of degree zero and have a symmetric and negative semidefinite Slutsky matrix, then these are integrable to a rational preference ordering. In the collective approach, more assumptions are needed to obtain a similar result. More specifically, individual preferences have to be of the egoistic or caring type.

Household members have *egoistic* preferences if their preferences only depend on their own consumption and leisure:

$$u^I = v^I \left( c^I, q_0^I, \mathbf{d} \right), \ I = A, B.$$

Preferences are of the *caring* type if they can be represented as (see Becker, 1974a, 1974b):

$$u^I = f^I \left( v^A \left( c^A, q_0^A, \mathbf{d} \right), v^B \left( c^B, q_0^B, \mathbf{d} \right) \right), \ I = A, B,$$

where $f^I$ is an increasing function in its arguments. Caring preferences can thus be seen as coming from household members who positively value increases in the other one's welfare, but who are not primarily interested in how this welfare is obtained. Note that both types of preferences imply weak separability of the own consumption and leisure bundle from that of the other household member. This allows a favourable re-interpretation of the collective labour supply model (see Chiappori, 1988, 1992).

**Proposition 2.2.** Sharing rule result. *If individual preferences are of the egoistic or caring type, then the household allocation problem (2.2) is equivalent to the maximization problems ($I = A, B$):*

$$\max_{c^I, q_0^I} v^I \left( c^I, q_0^I, \mathbf{d} \right), \tag{2.5}$$

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*subject to*

$$c^I + w^I q_0^I \leq \phi^I \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right) + w^I T,$$

*for some function $\phi$ such that $\phi^A = \phi$ and $\phi^B = y^S - \phi^A$.*

**Proof**: This result is nothing else than an application of the second fundamental theorem of welfare economics (see, e.g., Mas-Colell et alii, 1995). The Pareto optimal allocation $\left( c^A, c^B, q_0^A, q_0^B \right)'$ can be represented as a price equilibrium with lump-sum transfers between household members. Note that caring preferences incorporate Pareto irrelevant externalities. Consequently, a competitive economy with individuals with caring preferences also achieves Pareto optimality (see Parks, 1991). □

The result can be interpreted as a two-stage budgeting process. Firstly, household members allocate the total nonlabour income $y^S$ among each other according to the *sharing rule $\phi$*. Following earlier results, $\phi$ will in general depend on nonlabour incomes, wages and distribution factors $\mathbf{z}$. In a second stage, both individuals allocate their share in the household's means to their own consumption and leisure in a way that maximizes their individual welfare.

Since individual preferences are assumed to be weakly separable in $\left( c^A, q_0^A \right)$ and $\left( c^B, q_0^B \right)$, individual labour supplies can be written as:

$$\begin{aligned} \ell^A &= m^A \left( \phi \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right), w^A, \mathbf{d} \right) \\ \ell^B &= m^B \left( y^S - \phi \left( y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d} \right), w^B, \mathbf{d} \right). \end{aligned} \tag{2.6}$$

This particular representation of observable labour supplies produces the following result (see Chiappori, Fortin and Lacroix, forthcoming, for the result without individually observable nonlabour incomes):

**Proposition 2.3.** Testability and identification results. *Assume that the partial derivatives of the labour supply functions are not equal to zero. Then the following conditions are necessary for $\ell^A$ and $\ell^B$ to be compatible with the individual maximization problems (2.5) for some sharing rule $\phi$:*

$$\frac{\partial \boldsymbol{\psi}}{\partial \mathbf{f}'} = \left( \frac{\partial \boldsymbol{\psi}}{\partial \mathbf{f}'} \right)' \tag{2.7}$$

$$\frac{\partial m^A}{\partial w^A} \Big|_{\bar{\phi}} - \frac{\partial m^A}{\partial \phi} \ell^A \geq 0 \tag{2.8}$$

$$\frac{\partial m^B}{\partial w^B} \Big|_{y^S - \phi} - \frac{\partial m^B}{\partial \left( y^S - \phi \right)} \ell^B \geq 0,$$

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where $\mathbf{f} = \left( y^S, \mathbf{w}', \mathbf{y}', \mathbf{z}' \right)'$. *The vector $\boldsymbol{\psi}$ is of dimension $(m+5)$ and consists of a set of partial differential equations involving derivatives of individual labour supplies. Moreover, if the conditions (2.7) and (2.8) are satisfied, then the sharing rule is identified up to an additive constant $k(\mathbf{d})$. Its partial derivatives with respect to the variables in $\mathbf{f}$ are equal to the elements of the vector $\boldsymbol{\psi}$. For a given constant $k(\mathbf{d})$, individual preferences are uniquely defined.*

**Proof**: See Appendix A. The intuition, behind which these conditions are derived is the following. The obtained results are entirely driven through the applicability of the sharing rule result when preferences are egoistic or of the caring type. Firstly, as is clear from (2.6) marginal changes in the nonlabour incomes $y^S$ and $\mathbf{y}$ and distribution factors $\mathbf{z}$ only affect individual labour supplies $\ell^A$ and $\ell^B$ via the individuals' shares in total nonlabour income. Secondly, a marginal change of a household member's wage has only an income effect on the other one's labour supply. This effect runs again through the individuals' shares of total nonlabour income. Taken together, these results allow one to derive the marginal rates of substitution, between each couple of variables in $\mathbf{f}$, of the sharing rule $\phi$ in terms of observable labour supplies $\ell^A$ and $\ell^B$. By means of this set of marginal substitution rates, the partial derivatives of the sharing rule $\phi$ (i.e., the elements in $\boldsymbol{\psi}$) can be derived. In order to make this set of partial differential equations integrable to $\phi$, a set of cross-equation restrictions has to be satisfied. This is captured by (2.7). Since labour supplies $\ell^A$ and $\ell^B$ can also be considered as resulting from utility maximization problems (2.5), standard integrability conditions on the associated Slutsky matrices have to be satisfied. This is given by (2.8).

Thus, if the above conditions are satisfied, then the sharing rule $\phi$ is identified up to an additive constant. Consequently, the individual consumptions of the Hicksian aggregate commodity $c^A$ and $c^B$ are also identified up to the same additive constant. Moreover, keeping in mind the sharing rule result, individual indirect utility functions can also be defined for the given additive constant, via observable labour supply behaviour. As for the sharing rule, this implies that it will in general be impossible to predict that, say, 60% of total household income will be allocated to individual $A$ and 40% to individual $B$ in a certain wage and nonlabour income regime. On the other hand, the given set-up allows statements as "a one percent increase of individual $A$'s wage will change her share in total income by $x$ percent". These are important identification results, since next to a restriction on individual preferences, only Pareto efficiency of household behaviour is assumed. In particular, the collective approach allows to analyse the effects of policy reforms upon individual household members, both in terms of individual welfare and in terms of derived individual consumption. Note that such identification results will in general not exist in the unitary model. Consequently,

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with regard to normative welfare analyses, the collective model has a decisive advantage over the unitary one.

## 3. Functional specifications

Apart from the choice of an appropriate theory to model household labour supply, a functional form for the labour supply functions has to be chosen. This choice may depend on a number of criteria (see, e.g., Stern, 1986). Given this paper's objective, the functional specifications should satisfy at least three criteria. Firstly, labour supply should be consistent with utility maximization (both in the unitary model and in the collective approach for singles) or with the collective setting (for many person households). Secondly, there should be some flexibility in the functional form, even if (most of) the unitary or collective restrictions are imposed upon it. With regard to changes in the wage rates, a variety of responses should be allowed. Finally, a normative welfare analysis requires judgements on the effect of policy reforms on the household's or the individual's welfare. Therefore, structural information like utility functions should be recoverable by means of the specification. In what follows, we opt for functional forms for the unitary and collective model that only coincide for single individual households. The approach followed here is different from that of Fortin and Lacroix (1997), who derived a labour supply model that nests both a unitary and collective model. This approach has the advantage that both models are easily tested against each other. On the other hand, the model does not allow much flexibility in behavioural responses. Our approach does not encompass a unitary and collective model in the same functional specification for many person households. The gain in flexibility comes at the cost of both models being less easily tested against each other.

### 3.1. The unitary model

The functional specification for the unitary model is based on Blundell and Meghir's (1986) generalization of the Linear Expenditure System. It allows a wide variety of wage responses, and demographic variables are easily incorporated. The indirect utility function for two income earners, underlying the chosen specification, is of the Gorman polar form:

$$u = \frac{y^S + a\left(p, \mathbf{w}, \mathbf{d}\right)}{b\left(p, \mathbf{w}, \mathbf{d}\right)}, \tag{3.1}$$

where $p$ is the price of the Hicksian aggregate commodity and $a$ and $b$ are linearly homogeneous and concave functions in $\mathbf{w}$ and $p$. The following forms are chosen

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for these functions:

$$a\left(p,\mathbf{w},\mathbf{d}\right) = \alpha_1\left(\mathbf{d}\right)w^A + \alpha_2\left(\mathbf{d}\right)w^B - \alpha_3\left(\mathbf{d}\right)p - \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\gamma_{ij}^*\ln v_i\ln v_j$$

$$b\left(p,\mathbf{w},\mathbf{d}\right) = \left(w^A\right)^{\beta_1}\left(w^B\right)^{\beta_2}p^{\beta_3},$$

where $v_i$ is the $i$th element of the vector $\mathbf{v} = \left(w^A, w^B, p\right)'$. These functions are linearly homogeneous if $\sum_i \gamma_{ij}^* = \sum_j \gamma_{ij}^* = 0$ and $\sum_i \beta_i = 1$. Demographic variables are only taken up in $a$ in order not to complicate matters too much. We obtain the following labour supply functions (where the price of consumption is normalized to one) via Roy's identity:

$$\ell^A = \alpha_1\left(\mathbf{d}\right) - \frac{1}{w^A}\left(\gamma_{11}\ln w^A + \gamma_{12}\ln w^B\right) - \frac{\beta_1}{w^A}\left(y^S + a^*\left(\mathbf{w},\mathbf{d}\right)\right) \qquad (3.2)$$

$$\ell^B = \alpha_2\left(\mathbf{d}\right) - \frac{1}{w^B}\left(\gamma_{21}\ln w^A + \gamma_{22}\ln w^B\right) - \frac{\beta_2}{w^B}\left(y^S + a^*\left(\mathbf{w},\mathbf{d}\right)\right), \qquad (3.3)$$

where $\gamma_{ij} = \frac{\gamma_{ij}^* + \gamma_{ji}^*}{2}$ and $a^*\left(\mathbf{w},\mathbf{d}\right) = a(1,\mathbf{w},\mathbf{d})$. Symmetry is satisfied if $\gamma_{12} = \gamma_{21}$. As is well-known for flexible functional forms, concavity cannot be imposed globally without losing flexibility. Therefore, we opt to estimate the above labour supplies without imposing concavity restrictions and then check whether these are satisfied for the given data. Note that if $\gamma_{ij}$ $(i, j = 1, 2)$ are equal to zero, then the above labour supply system reduces to that corresponding to the Linear Expenditure System coming from a Stone-Geary utility function. Note finally that the above specification does not include distribution factors, since these cannot play any role in the unitary approach by definition.

## 3.2. The collective model

We choose the following flexible functional specification for individual labour supplies in the collective model (where the price of consumption has been normalized to one):

$$\ell^A = \gamma_1\left(\mathbf{d}\right) - \frac{\gamma_2}{w^A}\ln w^A - \frac{\gamma_3}{w^A}[\frac{1}{2}y^S + \gamma_1\left(\mathbf{d}\right)w^A - \frac{1}{2}\gamma_2\ln^2 w^A \qquad (3.4)$$
$$+\gamma_4 y^A + \gamma_5 y^B + \boldsymbol{\gamma}_6'\mathbf{z} + \gamma_7\left(\ln w^A - \ln w^B\right) + \gamma_8]$$

$$\ell^B = \delta_1\left(\mathbf{d}\right) - \frac{\delta_2}{w^B}\ln w^B - \frac{\delta_3}{w^B}[\frac{1}{2}y^S + \delta_1\left(\mathbf{d}\right)w^B - \frac{1}{2}\delta_2\ln^2 w^B \qquad (3.5)$$
$$+\delta_4 y^A + \delta_5 y^B + \boldsymbol{\delta}_6'\mathbf{z} + \delta_7\left(\ln w^A - \ln w^B\right) +\delta_8],$$

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where $\boldsymbol{\gamma}_6$ and $\boldsymbol{\delta}_6$ are two vectors of dimension $m$.

In the empirical section, only two distribution factors (apart from individual nonlabour incomes) will be used (i.e., $m = 2$). A general test for this collective labour supply model with distribution factors and individual nonlabour incomes, is the distribution factor proportionality result of proposition (2.1). It is easily checked that *distribution factor proportionality* is satisfied if:

$$\frac{\gamma_{6;2}}{\gamma_{6;1}} = \frac{\delta_{6;2}}{\delta_{6;1}}; \frac{\gamma_4}{\gamma_{6;1}} = \frac{\delta_4}{\delta_{6;1}}; \frac{\gamma_5}{\gamma_{6;1}} = \frac{\delta_5}{\delta_{6;1}} \tag{3.6}$$

where $\gamma_{6;i}$ and $\delta_{6;i}$ are the $i$th elements of respectively $\boldsymbol{\gamma}_6$ and $\boldsymbol{\delta}_6$. These are easily testable restrictions.

Let us now consider the more specific setting of the sharing rule interpretation, where individual labour supplies can be considered as the observable result of a two-stage budgeting process. The first stage consists of the allocation of total nonlabour income $y^S$ to both individuals according to some sharing rule $\phi$. In the second stage, each individual independently allocates her means to own consumption and leisure. Via the partial derivatives of the individual labour supply functions, the following set of partial derivatives of the sharing rule can be derived (cf. proposition (2.3)):

$$\frac{\partial \phi}{\partial y^S} = \frac{\delta_{6;1}}{\delta_{6;1} - \gamma_{6;1}}$$

$$\frac{\partial \phi}{\partial w^A} = \frac{2\gamma_{6;1}\delta_7}{w^A \left( \delta_{6;1} - \gamma_{6;1} \right)}$$

$$\frac{\partial \phi}{\partial w^B} = \frac{-2\delta_{6;1}\gamma_7}{w^B \left( \delta_{6;1} - \gamma_{6;1} \right)}$$

$$\frac{\partial \phi}{\partial y^A} \mid_{\overline{y^S}} = \frac{2\gamma_4\delta_{6;1}}{\delta_{6;1} - \gamma_{6;1}} \tag{3.7}$$

$$\frac{\partial \phi}{\partial y^B} \mid_{\overline{y^S}} = \frac{2\gamma_5\delta_{6;1}}{\delta_{6;1} - \gamma_{6;1}}$$

$$\frac{\partial \phi}{\partial z_i} = \frac{2\gamma_{6;i}\delta_{6;1}}{\delta_{6;1} - \gamma_{6;1}} \text{ for } i = 1, 2.$$

Note that the denominator of the above equations has to be different from zero in the collective setting. This can be easily tested.

A necessary and sufficient condition to integrate this set of partial differential equations back to the sharing rule, is that the matrix $\frac{\partial \psi}{\partial \mathbf{f}'}$ is symmetric. This is always satisfied for the chosen specification. The *sharing rule* $\phi$ is identified, up to an additive constant $k(\mathbf{d})$, by the solution of the set of partial differential equations in (3.7):

$$
\begin{aligned}
\phi \;=\; & \frac{1}{\delta_{6;1} - \gamma_{6;1}} (\delta_{6;1} y^S + 2\gamma_{6;1}\delta_7 \ln w^A - 2\gamma_7\delta_{6;1} \ln w^B \\
& + 2\gamma_4\delta_{6;1} y^A + 2\gamma_5\delta_{6;1} y^B + 2\gamma_{6;1}\delta_{6;1} z_1 + 2\gamma_{6;2}\delta_{6;1} z_2) + k(\mathbf{d}). \quad (3.8)
\end{aligned}
$$

Second stage individual labour supply functions and individual indirect utility functions for the chosen specification can be derived by means of the next proposition:

**Proposition 3.1.** Second stage individual labour supply functions. *If the restrictions $\gamma_4 = -\delta_4, \gamma_5 = -\delta_5$, $\gamma_{6;1} = -\delta_{6;1}, \gamma_{6;2} = -\delta_{6;2}$ and $\gamma_7 = -\delta_7$ are satisfied, then second stage individual labour supplies, corresponding to (2.6), are derived from the indirect utility functions (with normalized price of the Hicksian aggregate commodity)*

$$
u^A = \frac{\phi + a^A(w^A, \mathbf{d})}{b^A(w^A)} \quad (3.9)
$$

*and*

$$
u^B = \frac{y^S - \phi + a^B(w^B, \mathbf{d})}{b^B(w^B)} \quad (3.10)
$$

*where the price index functions $a^I$ and $b^I$ $(I = A, B)$ are of the form:*

$$
\begin{aligned}
a^A(w^A, \mathbf{d}) &= \alpha_1^A(\mathbf{d}) w^A - \alpha_2^A - \frac{1}{2}\gamma_{11}^A \ln^2 w^A \\
b^A(w^A) &= (w^A)^{\beta_1^A} \\
a^B(w^B, \mathbf{d}) &= \alpha_1^B(\mathbf{d}) w^B - \alpha_2^B - \frac{1}{2}\gamma_{11}^B \ln^2 w^B \\
b^B(w^B) &= (w^B)^{\beta_1^B}.
\end{aligned}
$$

*Coefficients of these indirect utility functions are identified as follows: $\alpha_1^A(\mathbf{d}) = \gamma_1(\mathbf{d}), \gamma_{11}^A = \gamma_2, \beta_1^A = \gamma_3, \alpha_1^B(\mathbf{d}) = \delta_1(\mathbf{d}), \gamma_{11}^B = \delta_2$ and $\beta_1^B = \delta_3$. The coefficients $\alpha_2^A$ and $\alpha_2^B$ cannot be identified by means of observable labour supply, since they depend on the additive constant $k(\mathbf{d})$.*

**Proof**: See Appendix A. Second stage labour supply functions are thus of the same form as the unitary labour supply functions for singles (see (3.2) or (3.3)). Consequently, welfare evaluations on the intrahousehold level of changes in wages or other exogenous variables can be done by means of the Gorman polar form individual utility functions (3.9) and (3.10) which are identified for any given $k(\mathbf{d})$. Note that the functional specification does not allow to test for negativity since the Slutsky effects are a function of $\phi$, which is unobservable.

## 4. Data and econometric issues

The above unitary and collective labour supply models are estimated and tested on the 1992 and 1997 waves of the Belgian Socio-Economic Panel (SEP)[2]. From this dataset, two samples of households are kept back. Both labour supply approaches are applied on a sample of two person households of which both individuals have a positive labour supply[3]. The sample is further restricted to households of which both members are between 25 and 55 years old. This avoids problems due to part-time students and partly early-retirement. Also self-employed are excluded. Since the unitary model should be applicable on single person households in both approaches, two samples of male and female singles with a positive labour supply, between 25 and 55 years old and who are not self-employed act as a type of benchmark. Summary statistics of these samples are given in tables 6 and 7 of Appendix B.

Wage rates are obtained by dividing the individual labour incomes by the number of hours worked. In order to avoid a division bias, wages are treated as endogenous variables and are instrumented by work experience (years employed and the square of years employed), the square of age and dummies capturing the individuals' education level.

Two problems emerged with regard to total household nonlabour income. Firstly, the SEP does not capture any information on savings (in the sense of changes in assets) of households. Since labour supply behaviour is part of a lifetime decision-making process, the failure to take into account the intertemporal adjustment in assets when deriving nonlabour income, may impose myopic behaviour on economic agents. Secondly, there are indications that the household's

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[2] In order to make nominal data comparable, account is taken of an inflation rate of 10.71% between 1992 and 1997.

[3] At the cost of sacrificing degrees of freedom, we opted to exclude households with children. At least if the latter have a certain age, it can be argued that children also have some bargaining power in the household which should be reflected in observed household behaviour. See Dauphin and Fortin (2001) for a test of the number of decision makers in the household in a consumption context.

nonlabour income, that cannot be assigned to individuals, is very badly measured in the dataset. A good remedy for these problems is the use of the consumption based nonlabour income (see Blundell and Walker, 1986 and Blundell and MaCurdy, 1999). This unearned income concept (measured as total household consumption minus labour income) fits perfectly into an intertemporal two-stage budgeting framework, where economic agents first allocate lifetime wealth across periods and then maximize each period's utility subject to the corresponding budget constraint. Since the SEP does not contain any information on total consumption either, we opted to impute consumption based nonlabour incomes by means of an auxiliary dataset: the household budget survey of 1997-98 of the Belgian National Statistics Institute. To do this, consumption based nonlabour income is regressed on socio-economic household characteristics and labour incomes as explanatory variables. The obtained parameters are used to estimate the consumption based nonlabour income for the households in the SEP.

Apart from individually assignable nonlabour incomes $\mathbf{y}$, two distribution factors were taken up in the collective labour supply model. These are the age difference of both household members and a dummy variable indicating whether the individuals are married or cohabiting. Together with individual nonlabour incomes, these variables are used to test the collective restrictions of equation (3.6) and of proposition (3.1).

Age, dummies for social status and region are taken up as demographic and taste shifter variables in the labour supply equations (3.2), (3.3), (3.4) and (3.5). Linear specifications for the functions $\alpha_1(\mathbf{d})$, $\alpha_2(\mathbf{d})$, $\gamma_1(\mathbf{d})$ and $\delta_1(\mathbf{d})$ were chosen.

Estimations are done with the generalized method of moments (GMM), which does not require distributional assumptions (apart from the validity of the moment conditions) and allows for heteroskedasticity of unknown form.

## 5. Empirical results

### 5.1. The unitary model

#### 5.1.1. Single person households

Table 1 gives parameter estimates and standard errors of the estimation of the unitary model for single males and females (cf. singles' version of (3.2)). Parameters are somewhat imprecisely estimated[4]. At a 5 percent significance level, only the constant and the parameter associated with social status is statistically

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[4]Note that for estimation reasons, the parameter associated with the price of the Hicksian aggregate commodity has been assigned a value of -50. Filled in into the labour supply equation, this can be interpreted as committed expenditure on household consumption of 50 euro.

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significant for males. As for females, the constant and the nonlabour income parameter are significantly estimated at the 5% significance level. An increase in single women's nonlabour income decreases their labour supply, while the effect of being a blue-collar worker on the labour supply of men is positive. Since the $\gamma_{11}$-parameters are not significantly estimated at conventional significance levels, the data cannot reject the LES labour supply specification. A test for the functional specification is provided by Hansen's overidentifying restrictions test. By means of this test the joint validity of the moment conditions cannot be rejected for single men and women (test statistics of 3.92 and 6.92 $< \chi_{0.05}^2 (5)$=11.07). The sign of the compensated wage effects was checked for each observation in both samples. For women, all observations have the expected positive sign so that Slutsky conditions are at least locally satisfied for the sample. As for men, only one observation did not have the required positive sign.

| Table 1: GMM parameter estimates singles | | |
|---|---|---|
| | Males | Females |
| Constant | 37.64 (3.79) | 52.84 (9.14) |
| $\gamma_{11}$ | -77.19 (56.40) | -34.21 (32.17) |
| $\beta_1$ | 0.33 (0.19) | 0.29 (0.13) |
| Age | 0.14 (0.15) | -0.03 (0.12) |
| Dummy for social status | -2.89 (0.99) | -4.92 (3.19) |
| Dummy 1 for region | -0.31 (1.00) | -4.38 (2.45) |
| Dummy 2 for region | 4.25 (4.74) | 0.41 (2.46) |
| Objective function | 3.9168 | 6.9245 |

Note: Asymptotic standard errors between brackets.

### 5.1.2. Two person households

GMM estimation results for the unitary model applied on the sample of two person households are given in table 2 (see equations (3.2) and (3.3))[5]. In the symmetry restricted version of the equation for men, the constant and the parameters associated with the household's nonlabour income, the partner's wage, the partner's age, the partner's social status and both regional dummies are significant at the 5% significance level. Statistically significant parameters in the equation for women, are the constant and the parameter associated with nonlabour income. An increase in the household's nonlabour income significantly decreases the labour supply of both men and women. Living in the Walloon or Brussels Capital region has a positive effect on the men's labour supply. Also the age of the

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[5] Note that the parameter associated with the Hicksian aggregate commodity has been set equal to -100 to make the committed expenditure comparable to that of single person households.

partner positively affects the men's labour supply. On the other hand, an increase in the partner's wage or having a white-collar partner decrease the labour supply of men.

Hansen's overidentifying restrictions test does not reject the validity of the moment conditions in both the unrestricted and restricted versions of the household labour supply model (critical values are respectively $\chi^2_{0.05}(20) = 31.41$ and $\chi^2_{0.05}(21) = 32.67$). It is important for the theoretical consistency of the model that the Newey-West counterpart of the Wald test (see Greene, 1997) cannot reject symmetry (test statistic of $0.24 < \chi^2_{0.05}(1) = 3.84$). The LES labour supply specification ($\gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = 0$) is strongly rejected by means of a Wald test in both unrestricted and restricted versions (test statistics of respectively 18.24 and $16.92 > \chi^2_{0.05}(4)=9.49$). Finally, concavity conditions are not satisfied for 288 of the 299 households in the sample for the unrestricted labour supply version. In the symmetry restricted version, Slutsky conditions are not satisfied for 291 households. Note the important difference between these results and the estimation results for singles where Slutsky conditions are satisfied for all observations but one.

Table 2: GMM parameter estimates unitary model two person households

|  | Unrestricted estimates | | Symmetry restricted estimates | |
|---|---|---|---|---|
|  | Males | Females | Males | Females |
| Constant | 37.13 (4.68) | 54.94 (10.06) | 34.86 (4.66) | 54.03 (9.46) |
| $\gamma_{11}$ | 40.61 (28.31) | | 22.60 (22.23) | |
| $\gamma_{12}$ | -97.59 (48.30) | | -72.11 (19.56) | |
| $\beta_1$ | 0.12 (0.06) | | 0.10 (0.03) | |
| $\gamma_{21}$ | | -72.13 (21.51) | | |
| $\gamma_{22}$ | | 22.57 (16.53) | | 21.74 (15.88) |
| $\beta_2$ | | 0.26 (0.08) | | 0.25 (0.07) |
| Age men | -0.13 (0.10) | -0.25 (0.22) | -0.13 (0.10) | -0.22 (0.21) |
| Age women | 0.29 (0.12) | 0.33 (0.28) | 0.27 (0.12) | 0.29 (0.26) |
| Dummy social status men | 0.36 (1.03) | -0.75 (1.42) | 0.28 (0.91) | -0.63 (1.37) |
| Dummy social status women | -2.64 (0.85) | -0.55 (1.51) | -2.26 (0.85) | -0.33 (1.45) |
| Dummy 1 region | 2.19 (0.88) | 3.71 (1.97) | 2.13 (0.83) | 3.54 (1.90) |
| Dummy 2 region | 6.07 (2.19) | 2.74 (3.13) | 6.28 (2.28) | 2.57 (2.88) |
| Objective function | 21.5844 | | 21.7633 | |

Note: Asymptotic standard errors between brackets.

## 5.2. The collective model

In table 3 GMM estimation results are shown for three versions of the collective labour supply model: an unrestricted version of the model (see (3.4) and (3.5)), the

collective labour supply model with distribution factor proportionality restrictions (3.6) imposed and the collective model with the restrictions of proposition (3.1) imposed[6]. Statistically significant parameters in the most restricted version of the collective labour supply model for the men's equation are the constant, both regional dummies and the parameters associated with total household nonlabour income and the men's own wage. In the women's equation, the constant and the parameters associated with total household nonlabour income, the difference in logarithmic wages, the age difference and the dummy capturing whether the couple is married or cohabiting are statistically significant at the 5% significance level.

In all three versions of the collective labour supply model, the joint validity of the moment conditions cannot be rejected by means of Hansen's overidentifying restrictions test. Critical values are respectively $\chi^2_{0.05}(18) = 28.87$, $\chi^2_{0.05}(21) = 32.67$ and $\chi^2_{0.05}(23) = 35.17$ and are to be compared with the values of the objective function.

Let us now turn attention to the testing of the collective restrictions. By means of the Wald test, distribution factor proportionality (3.6) cannot be rejected in the unrestricted version of the collective labour supply model at the 5% significance level (test statistic of $1.85 < \chi^2_{0.05}(3) = 7.82$). Consequently, the theoretical implications of the general collective household model cannot be rejected for the given sample. However, in both the unrestricted version and the collective model with distribution factor proportionality imposed, the hypothesis of second stage individual labour supplies derived from Gorman polar form preferences (see proposition (3.1)) is rejected. Test statistics are respectively 16.20 and 30.91 and are to be compared to $\chi^2_{0.05}(5) = 11.07$ and $\chi^2_{0.05}(2) = 5.99$). It can be argued that this result is an indication of the rejection of egoistic preferences of individuals in two person households. Therefore, it remains a question whether intrahousehold welfare evaluations of changes in the explanatory variables can be done by means of the implied individual indirect utility functions (3.9) and (3.10)[7].

Since there are separate parameter estimates for single men and single women, a test whether their preferences equal preferences of individuals in two person households can be carried out. If we test for this after imposing the restrictions for individual Gorman polar form preferences, it turns out that parameter equality is strongly rejected (test statistic of 199.46 that is to be compared to

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[6]Parameters with an asterisk are not estimated and are calculated by means of collective restrictions.

[7]Note that the concavity conditions on second stage individual labour supplies cannot be directly checked since these depend on the integration constant $k(\mathbf{d})$. Some indication of the validity of these restrictions might be obtained by testing the sign of compensated wage responses for $k(\mathbf{d})$ fixed to zero. In this case, concavity is not satisfied for only one man and only one woman in the sample.

$\chi^2_{0.05}(14){=}23.69)$.

If individual preferences are of the egoistic or caring type, then the observed household allocation can be seen as the implicit result of a two-stage budgeting process. First household members allocate total household nonlabour income among each other according to some sharing rule, while in a second stage, each individual maximizes her utility subject to her share in total nonlabour income. Partial derivatives of the sharing rule (3.8), associated with the chosen parameterization, are found in the estimation results of the third version of the collective labour supply model in table 3. These partials can be interpreted as follows. First of all, being married implies a significant increase of 81 euro per week in the implicit share going to women in comparison with cohabitation. In terms of household bargaining, one might argue that marriage improves the bargaining position of women in comparison with cohabitation. On the other hand, if the age difference between men and women increases, then the share going to women decreases. There is a significantly estimated negative effect on the weekly transfer to women of 4.57 euro per year. Also an increase in the individual nonlabour income of men has a less favourable effect on the share accruing to women, although this effect is not significantly estimated. An increase of the individual nonlabour income of men of 1 euro per week implies an increase of the women's share of 0.27 euro, while the men's share increases by 0.73 euro. The same increase of the women's nonlabour income has a positive effect of 0.61 euro on her share in total nonlabour income, while the men's share increases by only 0.39 euro. Finally, at mean wage rates, an increase of the hourly wage rate of men by 1 euro, implies a significantly estimated decrease of the share going to women of 7.30 euro per week.

Table 3: GMM parameter estimates collective model two person households

| | Unrestricted estimates | | Distribution factor proportionality | | GPF second stage labour supply | |
|---|---|---|---|---|---|---|
| | Males | Females | Males | Females | Males | Females |
| Constant | 34.90 | 56.48 | 32.82 | 56.29 | 37.12 | 45.34 |
| | (8.77) | (12.40) | (6.51) | (10.85) | (7.05) | (7.55) |
| $\gamma_2$ | -92.04 | | -90.26 | | -74.87 | |
| | (35.92) | | (34.17) | | (25.24) | |
| $\gamma_3$ | 0.46 | | 0.40 | | 0.42 | |
| | (0.15) | | (0.13) | | (0.12) | |
| $\delta_2$ | | -53.67 | | -33.31 | | -18.79 |
| | | (23.57) | | (17.00) | | (18.15) |
| $\delta_3$ | | 0.40 | | 0.37 | | 0.18 |
| | | (0.08) | | (0.07) | | (0.04) |
| Cohabiting or married | -61.92 | 41.91 | -29.73* | -16.01 | -80.84* | 80.84 |
| | (32.97) | (44.46) | | (18.45) | | (36.87) |
| Age difference | 5.92 | 3.14 | 6.67 | 3.59 | 4.57* | -4.57 |
| | (2.16) | (2.35) | (2.17) | (1.91) | | (2.08) |
| Age men | 0.32 | | 0.29 | | 0.22 | |
| | (0.19) | | (0.15) | | (0.13) | |
| Age women | | -0.18 | | -0.21 | | -0.11 |
| | | (0.11) | | (0.10) | | (0.08) |
| Nonlabour income men | 0.26 | 0.23 | 0.28* | 0.15 | 0.23* | -0.23 |
| | (0.19) | (0.16) | | (0.10) | | (0.17) |
| Nonlabour income women | 0.29 | 0.72 | 0.73* | 0.39 | -0.11* | 0.11 |
| | (0.69) | (0.39) | | (0.30) | | (0.52) |
| Difference in log wages | 40.13 | -91.48 | 23.36 | -62.74 | 67.65* | -67.65 |
| | (35.11) | (24.26) | (32.44) | (11.94) | | (28.63) |
| Dum. soc. stat. men | 1.33 | | 0.77 | | 1.15 | |
| | (1.96) | | (1.65) | | (1.54) | |
| Dum. soc. stat. women | | -0.79 | | -0.82 | | -0.94 |
| | | (1.89) | | (1.82) | | (1.31) |
| Dummy 1 region | 4.90 | 0.99 | 3.74 | 2.77 | 3.88 | 0.46 |
| | (2.20) | (2.05) | (1.50) | (1.84) | (1.61) | (1.41) |
| Dummy 2 region | 13.29 | 0.62 | 12.73 | -0.16 | 10.36 | -1.69 |
| | (7.37) | (3.35) | (6.04) | (2.67) | (4.81) | (2.49) |
| Objective function | 21.0725 | | 23.1751 | | 26.2360 | |

Note: Asymptotic standard errors between brackets.

## 5.3. Elasticities and a goodness-of-fit measure

Table 4 provides elasticities for the different labour supply models with the above theoretical restrictions imposed. As is clear from the table, own wage elasticities of women differ both in sign and magnitude for the unitary and collective models for two person households. While the mean elasticity is positive in the unitary model, female household members in the collective model seem to be on the backward bending part of the labour supply curve. Also the cross-wage elasticities differ for females in both models. On the other hand, income elasticities are more or less the same across models and negative for the majority of individuals. Males' elasticities are very similar (both in sign and in magnitude) in the unitary and collective models. Regarding singles, both males and females seem to be on the backward bending part of the labour supply curve. Also for most of them leisure is a normal commodity.

The elasticities between brackets in table 4 are calculated by means of the second stage labour supply functions (see proposition 3.1). They can be interpreted as elasticities with the sharing rule $\phi$ kept constant. Of course, this sharing rule is only observed up to an additive constant $k(\mathbf{d})$. Therefore, the elasticities are calculated with this constant equal to zero. Total elasticities can in this way be disentangled in a part stemming from a marginal change in wages or nonlabour income with the sharing rule held constant, and a part originating from a shift in nonlabour income from one individual to the other (i.e., due to a change in bargaining power). As is clear from the table, the bargaining power does not entail sign reversals between both sets of elasticities.

Table 4: Elasticities

| | Mean | First quartile | Median | Third quartile |
|---|---|---|---|---|
| **UNITARY MODEL MALE SINGLES** | | | | |
| Own wage | -0.17 | -0.21 | -0.18 | -0.15 |
| Nonlabour income | -0.04 | -0.07 | -0.04 | 0.004 |
| **UNITARY MODEL FEMALE SINGLES** | | | | |
| Own wage | -0.00 | -0.06 | -0.03 | 0.01 |
| Nonlabour income | -0.07 | -0.09 | -0.05 | -0.02 |
| **UNITARY MODEL COUPLES** | | | | |
| Males | | | | |
| Own wage | -0.18 | -0.22 | -0.18 | -0.14 |
| Partner's wage | 0.08 | 0.04 | 0.08 | 0.11 |
| Nonlabour income | -0.01 | -0.02 | -0.00 | 0.01 |
| Females | | | | |
| Own wage | 0.01 | -0.06 | 0.00 | 0.07 |
| Partner's wage | -0.20 | -0.20 | -0.11 | -0.07 |
| Nonlabour income | -0.04 | -0.06 | -0.01 | 0.04 |
| **COLLECTIVE MODEL COUPLES*** | | | | |
| Males | | | | |
| Own wage | -0.33 (-0.23) | -0.38 (-0.28) | -0.33 (-0.24) | -0.28 (-0.19) |
| Partner's wage | 0.10 (0) | 0.08 (0) | 0.10 (0) | 0.11 (0) |
| Nonlabour income | -0.02 (-0.03) | -0.04 (-0.09) | -0.01 (-0.02) | 0.03 (0.05) |
| Females | | | | |
| Own wage | -0.04 (-0.10) | -0.07 (-0.12) | -0.04 (-0.10) | -0.02 (-0.07) |
| Partner's wage | 0.06 (0) | 0.04 (0) | 0.05 (0) | 0.06 (0) |
| Nonlabour income | -0.02 (-0.03) | -0.02 (-0.05) | -0.00 (-0.01) | 0.01 (0.03) |

\* Elasticities between brackets are calculated with the sharing rule kept constant.

Let us now draw attention to the empirical performance of the labour supply models for couples. Therefore, we will make use of the mean squared error that captures the squared deviations between observed and predicted labour supply of the household members:

$$
MSE = \frac{1}{n} \sum_{i=1}^{n} \left[ \left( \ell_i^A - \widehat{\ell}_i^A \right)^2 + \left( \ell_i^B - \widehat{\ell}_i^B \right)^2 \right],
$$

where $\ell_i^I$ and $\widehat{\ell}_i^I$ denote observed and predicted labour supply respectively of household member $I$ $(I = A, B)$ in household $i$ $(i = 1, ..., n)$ and $n$ is the number of observations in the sample.

In table 5 the mean squared errors are given for the unitary and the collective labour supply models. For each model the measure is calculated for the different

sets of parameter estimates. Apart from the unitary and collective models, the measure is given for a naive model that acts as a type of benchmark. Estimated labour supplies in this naive model are simply the average labour supplies in the sample. As can be seen from the table, the collective labour supply model has the smallest mean squared error for each set of estimates. Consequently, it seems to give the best fit of the data. The naive model has a mean squared error that is relatively high in comparison with the theoretical models (and especially with regard to the collective model). The gain in explanatory power of behavioural models seems to be substantial.

Table 5: Mean squared errors for two person households

|  | Unitary model | Collective model | Naive model |
|---|---|---|---|
| No restrictions imposed | 133.7 | 113.3 | 151.4 |
| Symmetry | 135.5 | | |
| Distribution factor proportionality | | 115.2 | |
| Gorman polar form restrictions | | 117.4 | |

## 6. Conclusion

In this paper, the objective was to empirically evaluate two competing approaches to household labour supply. The first approach is the standard unitary model where households are assumed to behave as single rational decision making units. An alternative to this traditional model is the collective approach. In this approach, as initially defined by Chiappori (1988, 1992), it is explicitly taken into account that many person households consist of several individuals with own rational preferences. Among these individuals, a Pareto efficient intrahousehold bargaining process is assumed to take place.

The empirical evaluation focused on the testing of implied theoretical restrictions of both models, on the ability to identify structural information like preferences and on the empirical performance. Therefore, flexible functional specifications were derived for both approaches. For the unitary model, labour supply functions come from preferences that can be represented by an indirect utility function of the Gorman polar form. Testable implications of this unitary labour supply model are standard restrictions on the Slutsky matrix. As to the collective model, we followed the approach of Chiappori, Fortin and Lacroix (forthcoming) who derived a collective labour supply model with distribution factors. The latter are defined as variables that affect the intrahousehold distribution process, but that do not have any direct effect on individual preferences or the joint household budget constraint. By means of these distribution factors, testable implications of the collective approach can be derived. If some additional restrictions on the chosen parameterization of the collective model are satisfied, then preferences of

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individuals in a many person household and preferences of singles in the unitary model coincide.

Both models were applied to a sample coming from the 1992 and 1997 waves of the Belgian Socio-Economic Panel of two person households where both members have a positive labour supply. Since the unitary model should be entirely applicable to singles, the former was also tested on samples of male and female singles. In the collective labour supply model, individually assignable nonlabour incomes, a variable capturing whether the household members are married or cohabiting and the age difference between individuals were taken up as distribution factors. Estimations were done by means of the generalized method of moments.

Since symmetry is not a theoretical restriction for singles, the only restriction incorporated by the unitary model is the requirement of positive compensated wage effects. These were of the correct sign for all but one singles in the sample. Consequently, at least locally, the unitary model could not be rejected for single person households. As to the theoretical implications of the unitary model for two person households, symmetry could not be rejected at conventional significance levels. However, Slutsky sign restrictions were not satisfied for 97% of the sample. The fact that the unitary model is not rejected for singles, but rejected when applied to couples data, is really important since it may indicate that there is something wrong with the usual aggregation assumptions in the unitary model. Note that the same conclusion was obtained by Browning and Chiappori (1998) in a consumption context.

As for the collective model, the general collective restriction of distribution factor proportionality could not be rejected for the given sample. On the other hand, the more specific restrictions associated with egoistic preferences of the individual household members were rejected.

A second criterion that was focused on to evaluate the unitary and the collective models is the ability to identify structural information. If observed labour supply satisfies the usual unitary restrictions, then it can be integrated back to rational household preferences. This is important for welfare economic evaluations of policy reforms on the household level. A similar integration result applies to the collective model. Structural information that can be identified is a great deal of the intrahousehold allocation process and of the individual preferences. More specifically, if the assumption of egoistic or Beckerian caring individual preferences cannot be rejected, household labour supply behaviour can be modelled as a two stage budgeting process. Firstly, household members allocate total household nonlabour income among each other according to some sharing rule. In a second stage, both individuals maximize individual preferences subject to the implied individual budget constraints. Some main results connected with this sharing rule interpretation of the collective model are the following. Being married in

comparison to cohabiting implies a substantial increase of the implicit share going to women. On the other hand, increases in age difference between partners and in the males' nonlabour income have a less favourable effect on the share of total nonlabour income going to women. Also an increase in the wage difference between men and women decreases the implicit share accruing to women. Note that the ability to gain information on the intrahousehold allocation and the implications of it on individual welfare, is an important comparative advantage of the collective approach. Welfare analyses cannot only be done on the household level, but also in terms of individual welfare levels. But keep in mind that the crucial assumption of egoistic or caring preferences was rejected for the sample in question.

The last criterion that was focused on to discriminate between the unitary and collective labour supply models is the empirical performance. On the basis of the mean squared error, the collective labour supply model seems to provide the best fit. Both the unitary and the collective model had a better fit than a naive model that served as a benchmark. Finally, elasticities obtained by both theoretical models do not differ very much. Males' elasticities in particular are very similar (both in sign and magnitude) across models.

Future research may test whether the above results are robust with respect to other functional forms that can describe the same variety in observed labour supply behaviour. Another research avenue is the incorporation of nonparticipation into the collective model (see, e.g., Blundell et alii, 2001 and Donni, forthcoming). Note that this option has the positive by-product of generating extra observations for the sample, which might benefit the precision of estimated parameters.

# 7. Appendix A: Proofs

*Proof of proposition* (2.1). The distribution factors $\mathbf{z}$ will be concentrated on first. From equations (2.3) and (2.4), and assuming that all distribution factors have an effect different from zero on the bargaining power, we can derive that $\frac{\partial h^I}{\partial \mathbf{z}} = \frac{\partial l^I}{\partial \mu} \frac{\partial \mu}{\partial \mathbf{z}}$ $(I = A, B)$. It follows that $\frac{\partial l^I}{\partial \mu} = \frac{\partial h^I}{\partial z_1} \left( \frac{\partial \mu}{\partial z_1} \right)^{-1}$. Let $\widetilde{\mathbf{z}} = (z_2, ..., z_m)'$ and $\widetilde{\boldsymbol{\theta}}$ an $(m-1)$ vector. It is now easily seen that $\frac{\partial h^I}{\partial \widetilde{\mathbf{z}}} = \widetilde{\boldsymbol{\theta}} \frac{\partial h^I}{\partial z_1}$. The vector $\widetilde{\boldsymbol{\theta}}$ has typical element $\theta_i = \frac{\partial \mu}{\partial z_i} \left( \frac{\partial \mu}{\partial z_1} \right)^{-1}$, that is the marginal rate of substitution between $z_i$ and $z_1$ in the bargaining weight $\mu$.

Secondly, from equations (2.3) and (2.4), it can be derived that $(I = A, B, J = A, B)$:

$$\frac{\partial h^I}{\partial y^S} \frac{\partial y^S}{\partial y^J} + \frac{\partial h^I}{\partial y^J} \mid_{\overline{y^S}} = \frac{\partial l^I}{\partial \mu} \frac{\partial \mu}{\partial y^S} \frac{\partial y^S}{\partial y^J} + \frac{\partial l^I}{\partial \mu} \frac{\partial \mu}{\partial y^J} \mid_{\overline{y^S}} + \frac{\partial l^I}{\partial y^S} \frac{\partial y^S}{\partial y^J} \mid_{\overline{\mu}} .$$

Note that $\frac{\partial h^I}{\partial y^S}\frac{\partial y^S}{\partial y^J} = \frac{\partial l^I}{\partial \mu}\frac{\partial \mu}{\partial y^S}\frac{\partial y^S}{\partial y^J} + \frac{\partial l^I}{\partial y^S}\frac{\partial y^S}{\partial y^J} \mid_{\overline{\mu}}$ (the effect of a marginal change in $y^J$ that runs through $y^S$). Therefore, $\frac{\partial h^I}{\partial y^J} \mid_{\overline{y^S}} = \frac{\partial l^I}{\partial \mu}\frac{\partial \mu}{\partial y^J} \mid_{\overline{y^S}}$. Since $\frac{\partial l^I}{\partial \mu} = \frac{\partial h^I}{\partial z_1}\left(\frac{\partial \mu}{\partial z_1}\right)^{-1}$, it follows that $\frac{\partial h^I}{\partial \mathbf{y}} \mid_{\overline{y^S}} = \widehat{\boldsymbol{\theta}}\frac{\partial h^I}{\partial z_1}$, $\widehat{\boldsymbol{\theta}}$ being a vector of dimension 2 with marginal rates of substitution of $\mu$.

Putting things together, we obtain $\frac{\partial h^I}{\partial \mathbf{r}} = \boldsymbol{\theta}\frac{\partial h^I}{\partial z_1}$ for $I = A, B$, where $\mathbf{r} = (\widetilde{\mathbf{z}}', \mathbf{y}')'$ and $\boldsymbol{\theta} = \left(\widetilde{\boldsymbol{\theta}}', \widehat{\boldsymbol{\theta}}'\right)'$. This can be rewritten as $\frac{\partial \mathbf{h}}{\partial \mathbf{r}'} = \frac{\partial \mathbf{h}}{\partial z_1}\boldsymbol{\theta}'$.

$\square$

*Proof of proposition* (2.3). We will first derive the $m + 5$ partial derivatives of the sharing rule $\phi$. Start from labour supplies (2.6) associated with the sharing rule interpretation:

$$\ell^A = m^A\left(\phi\left(y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d}\right), w^A, \mathbf{d}\right)$$
$$\ell^B = m^B\left(y^S - \phi\left(y^S, \mathbf{w}, \mathbf{y}, \mathbf{z}, \mathbf{d}\right), w^B, \mathbf{d}\right).$$

Assuming that the partial derivatives of $\phi$ are not equal to zero, the following $2m + 6$ (observable) equations can be derived:

$$A_1 = \frac{\frac{\partial m^A}{\partial w^B}}{\frac{\partial m^A}{\partial y^S}} = \frac{\frac{\partial \phi}{\partial w^B}}{\frac{\partial \phi}{\partial y^S}}$$

$$A_{i+1} = \frac{\frac{\partial m^A}{\partial z_i}}{\frac{\partial m^A}{\partial y^S}} = \frac{\frac{\partial \phi}{\partial z_i}}{\frac{\partial \phi}{\partial y^S}}, \text{ for } i = 1, ..., m$$

$$A_{m+2} = \frac{\frac{\partial m^A}{\partial y^A}}{\frac{\partial m^A}{\partial y^S}} = 1 + \frac{\frac{\partial \phi}{\partial y^A}\mid_{\overline{y^S}}}{\frac{\partial \phi}{\partial y^S}}$$

$$A_{m+3} = \frac{\frac{\partial m^A}{\partial y^B}}{\frac{\partial m^A}{\partial y^S}} = 1 + \frac{\frac{\partial \phi}{\partial y^B}\mid_{\overline{y^S}}}{\frac{\partial \phi}{\partial y^S}}$$

$$B_1 = \frac{\frac{\partial m^B}{\partial w^A}}{\frac{\partial m^B}{\partial y^S}} = \frac{-\frac{\partial \phi}{\partial w^A}}{1 - \frac{\partial \phi}{\partial y^S}}$$

$$B_{i+1} = \frac{\frac{\partial m^B}{\partial z_i}}{\frac{\partial m^B}{\partial y^S}} = \frac{-\frac{\partial \phi}{\partial z_i}}{1 - \frac{\partial \phi}{\partial y^S}}, \text{ for } i = 1, ..., m$$

$$B_{m+2} = \frac{\frac{\partial m^B}{\partial y^A}}{\frac{\partial m^B}{\partial y^S}} = 1 - \frac{\frac{\partial \phi}{\partial y^A}\mid_{\overline{y^S}}}{1 - \frac{\partial \phi}{\partial y^S}}$$

$$B_{m+3} = \frac{\frac{\partial m^B}{\partial y^B}}{\frac{\partial m^B}{\partial y^S}} = 1 - \frac{\frac{\partial \phi}{\partial y^B}\mid_{\overline{y^S}}}{1 - \frac{\partial \phi}{\partial y^S}}.$$

Since $B_i = \frac{B_2 A_i}{A_2}$, $i = 3, ..., m+1$ and $B_i = 1 + \frac{B_2}{A_2}(A_i - 1)$, $i = m+2, m+3$, there are $m + 5$ linearly independent equations in the above set. Combining $A_2$ and $B_2$ and assuming that $A_2 \neq B_2$ results in:

$$\frac{\partial \phi}{\partial y^S} = \frac{B_2}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial z_1} = \frac{A_2 B_2}{B_2 - A_2}.$$

Making use of the above result, the rest of the partial derivatives of $\phi$ can easily be derived. From $A_1$ and $B_1$ we respectively have:

$$\frac{\partial \phi}{\partial w^B} = \frac{A_1 B_2}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial w^A} = \frac{A_2 B_1}{B_2 - A_2}.$$

From $A_{i+1}$, for $i = 2, ..., m$ we can derive:

$$\frac{\partial \phi}{\partial z_i} = \frac{A_{i+1} B_2}{B_2 - A_2}.$$

Finally, from $A_{m+2}$ and $A_{m+3}$ we have:

$$\frac{\partial \phi}{\partial y^A} \ \Big|\ _{\overline{y^S}} = (A_{m+2} - 1) \frac{B_2}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial y^B} \ \Big|\ _{\overline{y^S}} = (A_{m+3} - 1) \frac{B_2}{B_2 - A_2}.$$

Summarizing, we have the following set of $m + 5$ partial differential equations:

$$\frac{\partial \phi}{\partial y^S} = \frac{B_2}{B_2 - A_2} \tag{7.1}$$
$$\frac{\partial \phi}{\partial w^A} = \frac{A_2 B_1}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial w^B} = \frac{A_1 B_2}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial y^A} \ \Big|\ _{\overline{y^S}} = (A_{m+2} - 1) \frac{B_2}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial y^B} \ \Big|\ _{\overline{y^S}} = (A_{m+3} - 1) \frac{B_2}{B_2 - A_2}$$
$$\frac{\partial \phi}{\partial z_i} = \frac{A_{i+1} B_2}{B_2 - A_2}, \text{ for } i = 1, ..., m.$$

A necessary and sufficient condition for the existence of a solution to this set of partial differential equations, is that the $(m + 5) \times (m + 5)$ derivative matrix of the system (7.1) is symmetric at all points of its domain (see Frobenius' theorem). Let $\mathbf{f} = \left( y^S, \mathbf{w}', \mathbf{y}', \mathbf{z}' \right)'$ and $\boldsymbol{\psi}$ be an $(m + 5)$ vector consisting of the right-hand sides of (7.1). Then a solution for this set of partial differential equations exists if and only if:

$$\frac{\partial \boldsymbol{\psi}}{\partial \mathbf{f}'} = \left( \frac{\partial \boldsymbol{\psi}}{\partial \mathbf{f}'} \right)'. \tag{7.2}$$

If condition (7.2) is satisfied, then the sharing rule $\phi$ is identified up to an additive integration constant $k(\mathbf{d})$.

Since individual labour supplies $\ell^A$ and $\ell^B$ are the result of standard utility maximization problems (2.5) under the assumption of egoistic or caring preferences, the following two conditions also have to be satisfied:

$$\frac{\partial m^A}{\partial w^A} \Big|_{\overline{\phi}} - \frac{\partial m^A}{\partial \phi} \ell^A \geq 0 \tag{7.3}$$

$$\frac{\partial m^B}{\partial w^B} \Big|_{\overline{y^S - \phi}} - \frac{\partial m^B}{\partial (y^S - \phi)} \ell^B \geq 0.$$

Finally, following standard integrability results, individual preferences, represented by the utility functions $v^A$ and $v^B$ are uniquely defined for a given choice of the additive constant $k(\mathbf{d})$.

□

*Proof of proposition* (3.1). Firstly, taking account of the restrictions $\gamma_{6;1} = -\delta_{6;1}$ and $\gamma_7 = -\delta_7$ and rewriting the sharing rule (3.8) results in:

$$\frac{1}{2} y^S = \phi - \gamma_4 y^A - \gamma_5 y^B - \gamma_{6;1} z_1 - \gamma_{6;2} z_2 - \gamma_7 \left( \ln w^A - \ln w^B \right) - k(\mathbf{d}). \tag{7.4}$$

Substituting the right-hand side of equation (7.4) for $\frac{1}{2} y^S$ in individual $A$'s collective labour supply function (3.4) results in

$$\ell^A = \gamma_1(\mathbf{d}) - \frac{\gamma_2}{w^A} \ln w^A - \frac{\gamma_3}{w^A} (\phi + \gamma_1(\mathbf{d}) w^A - \frac{1}{2} \gamma_2 \ln^2 w^A + \gamma_8 - k(\mathbf{d})),$$

which is easily seen to be derived from $A$'s indirect utility function (3.9) with parameters $\alpha_1^A(\mathbf{d}) = \gamma_1(\mathbf{d})$, $\gamma_{11}^A = \gamma_2$, $\beta_1^A = \gamma_3$ and $\alpha_2^A = \gamma_8 - k(\mathbf{d})$. Second, taking into account $\gamma_4 = -\delta_4, \gamma_5 = -\delta_5, \gamma_{6;1} = -\delta_{6;1}, \gamma_{6;2} = -\delta_{6;2}$ and $\gamma_7 = -\delta_7$ and manipulating the sharing rule (3.8) somewhat, results in the implicit equation:

$$\frac{1}{2} y^S = y^S - \phi - \delta_4 y^A - \delta_5 y^B - \delta_{6;1} z_1 - \delta_{6;2} z_2 - \delta_7 \left( \ln w^A - \ln w^B \right) + k(\mathbf{d}). \tag{7.5}$$

Substitution of the right-hand side of (7.5) for $\frac{1}{2}y^S$ in the collective labour supply function of individual $B$ (3.5) gives

$$\ell^B = \delta_1\left(\mathbf{d}\right) - \frac{\delta_2}{w^B}\ln w^B - \frac{\delta_3}{w^B}(y^S - \phi + \delta_1\left(\mathbf{d}\right)w^B - \frac{1}{2}\delta_2\ln^2 w^B + \delta_8 + k(\mathbf{d})),$$

which corresponds to individual $B$'s labour supply derived from the indirect utility function (3.10) with parameters $\alpha_1^B\left(\mathbf{d}\right) = \delta_1\left(\mathbf{d}\right)$, $\gamma_{11}^B = \delta_2, \beta_1^B = \delta_3$ and $\alpha_2^B = \delta_8 + k\left(\mathbf{d}\right)$.

$\square$

## 8. Appendix B: Data and empirical results

Table 6: Sample statistics male and female singles

| Variable | Males (182 observations) | | Females (128 observations) | |
| --- | --- | --- | --- | --- |
| | Mean | Standard deviation | Mean | Standard deviation |
| Weekly hours of work | 36.73 | 8.63 | 34.66 | 9.56 |
| Net hourly wage rate in euro | 9.17 | 5.44 | 8.88 | 4.48 |
| Age | 36.63 | 10.17 | 37.40 | 12.16 |
| Years employed | 14.76 | 11.64 | 15.54 | 11.95 |
| Dummy 1 for schooling (1 for secondary school)* | 0.59 | | 0.39 | |
| Dummy 2 for schooling (1 for higher non university education) | 0.26 | | 0.39 | |
| Dummy 3 for schooling (1 for university education) | 0.07 | | 0.15 | |
| Dummy for social status (1 for white collar worker)** | 0.55 | | 0.82 | |
| Dummy 1 for region (1 for Walloon Region)*** | 0.33 | | 0.28 | |
| Dummy 2 for region (1 for Brussels-Capital Region) | 0.14 | | 0.24 | |
| Consumption based weekly nonlabour income in euro**** | 16.97 | 90.22 | 45.17 | 61.87 |

\* Benchmark is primary school.

\*\* Benchmark is blue collar worker.

\*\*\* Benchmark is Flemish Region.

\*\*\*\* Imputed from the National Statistics Institute household budget survey of 1997-98.

Table 7: Sample statistics two person households (number of observations: 299)

| Variable | Mean | Standard deviation |
|---|---|---|
| Weekly hours of work males | 38.52 | 9.19 |
| Weekly hours of work females | 34.30 | 8.23 |
| Net hourly wage rate males in euro | 8.78 | 4.69 |
| Net hourly wage rate females in euro | 7.22 | 2.25 |
| Age males | 36.57 | 8.76 |
| Age females | 34.62 | 8.64 |
| Years employed males | 15.88 | 9.66 |
| Years employed females | 13.79 | 9.28 |
| Dummy 1 for schooling (1 for secondary school) males* | 0.55 | |
| Dummy 2 for schooling (1 for higher non-university education) males | 0.29 | |
| Dummy 3 for schooling (1 for university education) males | 0.12 | |
| Dummy 1 for schooling (1 for secondary school) females* | 0.55 | |
| Dummy 2 for schooling (1 for higher non-university education) females | 0.32 | |
| Dummy 3 for schooling (1 for university education) females | 0.08 | |
| Dummy for social status (1 for white collar) females** | 0.59 | |
| Dummy for social status (1 for white collar) females** | 0.69 | |
| Dummy 1 for region (1 for Walloon Region)*** | 0.23 | |
| Dummy 2 for region (1 for Brussels-Capital Region) | 0.09 | |
| Consumption based weekly household nonlabour income in euro**** | 7.80 | 94.12 |
| Individually assignable weekly nonlabour income males in euro | 3.34 | 31.43 |
| Individually assignable weekly nonlabour income females in euro | 3.10 | 16.40 |
| Cohabiting or married (1 for married) | 0.81 | |

* Benchmark is primary school.

** Benchmark is blue collar.

*** Benchmark is Flemish Region.

**** Imputed from the National Statistics Institute household budget survey of 1997-98.

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