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DISCUSSION
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The Effect of Monetary Unification on German Bond Markets

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Abstract

This paper uses reprojection to develop a benchmark to assess ECB monetary policy since January 1999, the start of EMU. We first estimate an essentially affine term structure model for the German SWAP yield curve between 1987:04-1998:12. The German monetary policy is then reprojected onto the EMU period (1999:01-2002:02). We find that the German real interest rate in place during the EMU period is significantly lower than it would have been in case the Bundesbank were still in charge of monetary policy. We also show the effect of EMU on the German SWAP yield curve. Short- and medium-term bonds seem to have been more affected than long-term bonds.

Keywords: EMU, ECB, Bundesbank, central bank monetary policy rule, essentially affine term structure model.

J.E.L.: E43, E44, E52, E58.

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1 Introduction

Since the beginning of European Economic and Monetary Union (EMU), January 1, 1999, European Central Bank (ECB) monetary policy has come under fierce attack from the financial press, the finance ministers of the participating countries, and investors in general. The critics blame the ECB for a lack of credibility and the quite remarkable drop in the euro-dollar exchange rate. However, this drop has not been unusual in a statistical sense and obviously any indicator can be (ab)used to criticize the ECB since there is no sensible benchmark to be used as a comparison. The main goal of this paper is to re-evaluate the ECB record by comparing it to a specific benchmark. We construct a model for the German yield curve and then project the model-based Bundesbank yield curve onto the EMU period. This filtered yield curve then serves as a benchmark to assess actual EMU yield curves.

In this paper we concentrate on the German bond market for two main reasons. First, bond markets are likely to be significantly affected by possible changes in the monetary policy since the main driving force in this market is the short-term interest rate. Moreover, yield curve changes also affect other markets as well. For instance, exchange rate markets are (at least in theory) partially linked to the bond markets.[1] Finally, given the dominance of Germany in the financial markets during the Exchange Rate Mechanism (ERM) period, it seems logical to concentrate on this market. More in particular, the German yield curve would form a benchmark yield curve if the ERM were still in place. The German yield curve can, therefore, be used to analyze the effects of the ECB policy on bond markets by comparing it to the one that would have prevailed if EMU were not there.

A second reason to turn to the German bond markets is motivated by the need of a good description of a financial market. This is obviously of crucial importance since the model is to serve as a benchmark in the comparison. There is by now evidence that bond markets (more specifically yield curves) can be modeled with reasonable accuracy within a no-arbitrage framework by means of parsimonious factor models. In fact, standard three factor models fit yields up to a measurement error of about ten basis points. This type of accuracy is not obtained by models describing other financial markets.

There exists already some research in this area. For instance, Faust et al. (2001), Alesina et al. (2001) and Gali (2001) use a policy rule approach to estimate the effects of EMU on the monetary policy adopted by the ECB. Typically, these studies use the Bundesbank Taylor rule and apply this Taylor rule to EMU-wide aggregates to see what interest rates would have been applied in EMU if the ECB had followed a German interest rate policy strategy. They tend to find that the interest rate target (under the assumption that the ECB would follow a Bundesbank Taylor rule) would typically be higher than the observed

[1] See, for instance, Backus et al. (2001), Brandt and Santa-Clara (2001), Dewachter and Maes (2001), and Clarida and Taylor (1997) for more details.

EMU interest rates. Our approach differs from these studies in many respects. First, we use a model that prices all bonds across the maturity spectrum and, therefore, the analysis is more informative regarding the effects of EMU than the analysis based on short rates only. As is well known, Taylor rules in observed output (gap) and inflation fail dramatically in modeling the long end of the term structure.[2] The model used in this paper is able to fit the entire term structure by encompassing variables that are typically left out in the standard analysis. These additional variables model the long-run expectations of output and inflation. By including these factors in the analysis, we believe we obtain a more detailed description of actual monetary policy rules. Second, we adopt a different perspective in comparison to the mentioned studies. Instead of answering the question of how EMU interest rates would look if the ECB were to follow the Bundesbank rule, we reverse the question: how different would the post-1999 yield curve have looked in Germany (or any other member state) if the Bundesbank (or any other central bank of the country under investigation) and not the ECB would set the monetary policy. Reversing the question has the important advantage that we do not need to speculate about which aggregates (output and inflation) the ECB uses in setting the interest rate policy.

The remainder of the paper is organized as follows. In section 2 we present the factor model including observable factors and explain in detail the link between these factors and the bond market using standard no-arbitrage arguments. Section 3 presents the estimation results of the model and the implied interest rate feedback rule of the Bundesbank. This feedback rule is used to extrapolate German monetary policy during the EMU period. Section 4 then proceeds by actually projecting German monetary policy in the EMU period and by testing whether or not we find significant differences between the observed and the projected yield curves. Section 5 concludes the paper.

2 The Model

In this section we present the continuous time model proposed by Dewachter et al. (2001) which links the dynamics of specific macroeconomic variables with the term structure of interest rates. The model includes a total of five factors. Two of them describe observable macroeconomic variables (output gap and inflation) while the other three represent non-observable latent factors. In contrast to the usual understanding of latent factor given in the literature, the ones included in this model do have an *a priori* clear interpretation in terms of macroeconomic aggregates. Two of them represent the long-run expectations of the output gap and inflation. As shown in Dewachter et al. (2001) and Kozicki and Tinsley (2001), the inclusion of such factors is crucial in modeling the link between financial markets and the macroeconomic variables. The third latent factor describes the real interest rate in the

[2] See, for instance, Dewachter et al. (2001) for an illustration on US data.

economy. The use of three latent factors to explain the yield curve is in accordance with most of the findings presented in the literature. In the following, we first specify the continuous time dynamics of the macroeconomic variables included in the model and then we analyze its implications for the term structure of interest rates.

2.1 Dynamics of macroeconomic and latent factors

The dynamics of the factors are described by a simple and stylized continuous time model. We start by presenting the dynamics of the observable macroeconomic variables, output gap $y(t)$ and inflation $\pi(t)$, and of their time-varying central tendencies $y^*(t)$ and $\pi^*(t)$:

$$\begin{aligned}
\mathrm{d}y(t) &= \left[\kappa_{yy}\left(y^*(t)-y(t)\right)+\kappa_{y\pi}\left(\pi^*(t)-\pi(t)\right)+\kappa_{y\rho}\left(\rho^*(t)-\rho(t)\right)\right]dt+\sigma_y dW_y(t) \\
\mathrm{d}\pi(t) &= \left[\kappa_{\pi y}\left(y^*(t)-y(t)\right)+\kappa_{\pi\pi}\left(\pi^*(t)-\pi(t)\right)+\kappa_{\pi\rho}\left(\rho^*(t)-\rho(t)\right)\right]dt+\sigma_\pi dW_\pi(t) \\
\mathrm{d}y^*(t) &= \kappa_{y^*y^*}\left(\theta_{y^*}-y^*(t)\right)\mathrm{d}t+\sigma_{y^*}dW_{y^*}(t) \\
\mathrm{d}\pi^*(t) &= \kappa_{\pi^*\pi^*}\left(\theta_{\pi^*}-\pi^*(t)\right)\mathrm{d}t+\sigma_{\pi^*}dW_{\pi^*}(t)
\end{aligned} \tag{1}$$

where $W_i(t)$, $i=\{y,\pi,y^*,\pi^*\}$, denote independent Wiener processes defined on the probability space $(\Omega,\mathcal{F},\mathbb{F},P)$, where $\mathbb{F}=\{\mathcal{F}_t\}_{0\le t\le T}$. Although it is assumed here that backward-looking models provide a good representation of the reality, the inclusion of factors representing long-run expectations of macroeconomic variables (i.e. the central tendencies) do give a forward-looking dimension to the model. The variable $\rho(t)$ denotes the real interest rate which is also used to specify the instantaneous interest rate $r(t)$:

$$r(t)\equiv\pi(t)+\rho(t) \tag{2}$$

It is implicitly assumed that the monetary authority uses a feedback rule for the real interest rate. In other words, changes in the (*ex post*) real interest rate $\rho(t)$ are a response to deviations of the output gap and/or inflation from their central tendencies and to a mean reverting (real interest rate smoothing) component relative to a stochastic long-run mean $\rho^*(t)$:

$$\begin{aligned}
\mathrm{d}\rho(t) &= \left[\kappa_{\rho y}\left(y^*(t)-y(t)\right)+\kappa_{\rho\pi}\left(\pi^*(t)-\pi(t)\right)+\kappa_{\rho\rho}\left(\rho^*(t)-\rho(t)\right)\right]\mathrm{d}t+\sigma_\rho \mathrm{d}W_\rho(t) \\
\rho^*(t) &= \gamma_0+\gamma_y y(t)+\gamma_\pi\pi(t)+\gamma_{y^*}y^*(t)+\gamma_{\pi^*}\pi^*(t).
\end{aligned} \tag{3}$$

with $W_\rho(t)$ also representing a Wiener process independent of the other Wieners as defined before. The long-run real interest rate, $\rho^*(t)$, represents, in fact, a long-run policy rule for the real interest rate which could depend both on the observed macroeconomic variables (y and π) and on their unobserved central tendencies (y^* and π^*). The central bank also has a short-run policy rule which can be inferred from the dynamics in (3). This short-run policy rule is discussed in detail in a later section.

As can be seen from equations (1) and (3), the dynamics of the system is modeled in terms of deviations of each factor from its long-run expectation, e.g. $(y^*(t) - y(t))$. This is also valid for the central tendencies of output gap and inflation which are assumed to have constant long-run expectations represented by the parameters θ_{y^*} and θ_{π^*}, respectively. In other words, only deviations from the long-run expectations (central tendencies) determine the short-run dynamics of each macroeconomic variable. This is, in fact, what guarantees that each central tendency acts as a long-run attractor for the respective factor once stability is imposed in the system.

In order to facilitate the representation of the model, we rewrite the above dynamics in matrix notation. Denoting the number of factors by n (equal to five in our case), we define an n x 1 vector $\mathbf{f}(t)$ containing all the factors included in the model and an n x 1 vector $\mathrm{d}\mathbf{W}(t)$ containing the respective shocks to each of those factors as:

$$\mathbf{f}(t) \equiv \begin{pmatrix} y(t) \\ \pi(t) \\ \rho(t) \\ y^*(t) \\ \pi^*(t) \end{pmatrix}, \qquad \mathrm{d}\mathbf{W}(t) \equiv \begin{pmatrix} \mathrm{d}W_y(t) \\ \mathrm{d}W_\pi(t) \\ \mathrm{d}W_\rho(t) \\ \mathrm{d}W_{y^*}(t) \\ \mathrm{d}W_{\pi^*}(t) \end{pmatrix}$$

The volatilities of the factors are represented by an n x n diagonal matrix $\mathbf{S}$ as:

$$\mathbf{S} \equiv diag\left(\sigma_y, \sigma_\pi, \sigma_\rho, \sigma_{y^*}, \sigma_{\pi^*}\right)$$

In this way, the dynamics of the economy can be restated as follows:

$$\mathrm{d}\mathbf{f}(t) = \mathbf{K}\left(\boldsymbol{\psi} - \mathbf{f}(t)\right)\mathrm{d}t + \mathbf{S}\mathrm{d}\mathbf{W}(t), \tag{4}$$

where

$$\mathbf{K} = \begin{bmatrix} \kappa_{yy} - \kappa_{y\rho}\gamma_y & \kappa_{y\pi} - \kappa_{y\rho}\gamma_\pi & \kappa_{y\rho} & -\kappa_{yy} - \kappa_{y\rho}\gamma_{y^*} & -\kappa_{y\pi} - \kappa_{y\rho}\gamma_{\pi^*} \\ \kappa_{\pi y} - \kappa_{\pi\rho}\gamma_y & \kappa_{\pi\pi} - \kappa_{\pi\rho}\gamma_\pi & \kappa_{\pi\rho} & -\kappa_{\pi y} - \kappa_{\pi\rho}\gamma_{y^*} & -\kappa_{\pi\pi} - \kappa_{\pi\rho}\gamma_{\pi^*} \\ \kappa_{\rho y} - \kappa_{\rho\rho}\gamma_y & \kappa_{\rho\pi} - \kappa_{\rho\rho}\gamma_\pi & \kappa_{\rho\rho} & -\kappa_{\rho y} - \kappa_{\rho\rho}\gamma_{y^*} & -\kappa_{\rho\pi} - \kappa_{\rho\rho}\gamma_{\pi^*} \\ 0 & 0 & 0 & \kappa_{y^*y^*} & 0 \\ 0 & 0 & 0 & 0 & \kappa_{\pi^*\pi^*} \end{bmatrix}$$

and

$$\boldsymbol{\psi} = \mathbf{K}^{-1}\left(\kappa_{y\rho}\gamma_0, \kappa_{\pi\rho}\gamma_0, \kappa_{\rho\rho}\gamma_0, \kappa_{y^*y^*}\theta_{y^*}, \kappa_{\pi^*\pi^*}\theta_{\pi^*}\right)'.$$

Since the matrix $\mathbf{K}$ is in general not diagonal, it is not straightforward to obtain closed form equations for the expectation of the level and of the covariance matrix of the factors. These concepts are, nevertheless, of great importance in order to forecast the future evolution of the state of the economy. A procedure to generate the conditional means and the conditional covariance matrix of the factors is presented in Dewachter et al. (2001). A similar method can be found in Fackler (2000).

2.2 Implications for bond markets

Equations (2) and (4) completely specify the dynamics of the macroeconomic variables and of the instantaneous interest rate. As a consequence, this system must also determine the term structure of interest rates and its dynamics up to some risk premium component. In fact, the absence of arbitrage opportunities implies that zero-coupon default-free bond prices at time t maturing at T, $p(t,T)$, are the solution to the following conditional expectation:

$$p(t,T) = E_t^Q\left(\exp\left(-\int_t^T r(u)\,du\right)\right), \tag{5}$$

where Q denotes the risk-neutral probability measure. In general, this risk-neutral probability is unknown and can only be specified by assuming some specification for the prices of factor risk. Following Duffee (2001), time variability in the prices of risk can be captured by specifying prices of risk as an affine function of the factors. The vector containing the time-varying prices of risk $\boldsymbol{\xi}$ is then defined as:

$$\boldsymbol{\xi}(t) = \mathbf{S}\boldsymbol{\Lambda} + \mathbf{S}^{-1}\boldsymbol{\Xi}\mathbf{f}(t),$$

where $\boldsymbol{\Lambda} \equiv (\lambda_y, \lambda_\pi, \lambda_\rho, \lambda_{y^*}\lambda_{\pi^*})'$ and $\boldsymbol{\Xi}$ is an n x n matrix containing the sensitivities of the prices of risk to the levels of the state space factors. Changing probability measures is then easily done by means of the Girsanov theorem:

$$\mathrm{d}\mathbf{W}(t) = \mathrm{d}\tilde{\mathbf{W}}(t) - \boldsymbol{\xi}(t)\,\mathrm{d}t, \tag{6}$$

where $\tilde{W}_i(t)$ constitutes a martingale under measure Q. And the state space dynamics can be restated in terms of this risk-neutral metric Q as:

$$\begin{aligned} \mathrm{d}\mathbf{f}(t) &= \tilde{\mathbf{K}}\left(\tilde{\boldsymbol{\psi}} - \mathbf{f}(t)\right)\mathrm{d}t + \mathbf{S}\mathrm{d}\tilde{\mathbf{W}}(t) \\ \tilde{\mathbf{K}} &= \mathbf{K} + \boldsymbol{\Xi} \\ \tilde{\boldsymbol{\psi}} &= \tilde{\mathbf{K}}^{-1}\left(\mathbf{K}\boldsymbol{\psi} - \mathbf{S}^2\boldsymbol{\Lambda}\right) \end{aligned} \tag{7}$$

A functional form for bond prices can be obtained by assuming that bond prices are time homogeneous functions of the factors $\mathbf{f}(t)$ and the time to maturity, $\tau \equiv T - t$:

$$p(t,T) = p(\mathbf{f}(t), \tau) = \exp\left(-a(\tau) - \mathbf{b}(\tau)'\mathbf{f}(t)\right) \tag{8}$$

where $\mathbf{b}(\tau)$ is an n x 1 vector and by imposing the no-arbitrage condition in the bond markets:

$$\mathcal{D}^Q\left(p(\mathbf{f}(t), \tau)\right) = r(t)\,p(\mathbf{f}(t), \tau), \tag{9}$$

where $\mathcal{D}^Q$ denotes the Dynkin operator under the probability measure Q. The intuitive meaning of the latter condition is that, once transformed to a risk-neutral world, instantaneous holding returns for all bonds are equal to the instantaneous riskless interest rate. Obviously, using Girsanov's theorem we can immediately infer the implications for the real world by changing measure from the risk-neutral Q to the historical measure P.

Equations (8) and (9) determine the solution for the functions $a(\tau)$ and $\mathbf{b}(\tau)$ in terms of a system of coupled ordinary differential equations (ODEs) that, in the general case, can be solved numerically:

$$\begin{aligned} \frac{\partial a(\tau)}{\partial \tau} &= a_0 + \left(\tilde{\mathbf{K}}\tilde{\boldsymbol{\psi}}\right)' \mathbf{b}(\tau) - \frac{1}{2}\sum_{i=1}^{n} b_i^2(\tau) S_{ii}^2 \\ \frac{\partial \mathbf{b}(\tau)}{\partial \tau} &= \mathbf{b}_0 - \tilde{\mathbf{K}}'\mathbf{b}(\tau) \end{aligned} \tag{10}$$

A particular solution to this system of ODEs is obtained by specifying a set of initial conditions for a and $\mathbf{b}$. Inspection of equation (8) shows that the appropriate initial conditions are: $a(0) = 0$ and $\mathbf{b}(0) = \mathbf{0}$. The vectors of constants a_0 and $\mathbf{b}_0$ are determined by the definition of the nominal interest rate presented in equation (2). In our case, it implies that $a_0 = 0$ and $\mathbf{b}_0 = (0\ 1\ 1\ 0\ 0)'$.

The bond pricing solution derived here differs in important ways from the ones implied by standard independent multi-factor term structure models presented in the literature. First, allowing for interrelations among the factors (*i.e.* non-zero off-diagonal elements in $\tilde{\mathbf{K}}$) generates a coupled system of ODEs instead of a set of uncoupled ODEs. The bond pricing solution for the a and $\mathbf{b}$ functions, therefore, is not reduced to the standard multi-factor result presented in, for instance, de Jong (2000). Second, the factor loadings no longer start from unity at maturity $\tau = 0$. The introduction of stochastic central tendencies makes that all of the these factors have zero loadings in the determination of the short rate. They only influence the instantaneous rate indirectly by serving as a long-run (stochastic) attractor.

2.3 Estimation method

Given the Gaussian (discrete time) properties of the above model we can estimate all its parameters consistently by means of maximum likelihood estimation, using the Kalman filter algorithm to construct the loglikelihood function. The Kalman filter is crucial since we need to filter the central tendencies (long-run forecasts) of output gap and inflation. For the measurement equation, we construct a vector $\mathbf{z}(t)$ containing m zero-coupon bond yields ($\hat{y}_i$) for maturities τ_1 through τ_m and the output gap and inflation rate at time t. Based on the theoretical model, we can write this vector $\mathbf{z}(t)$ in terms of the factors $\mathbf{f}(t)$ using no-arbitrage relations and a perfect updating assumption for the output gap and inflation. We then have that:

$$\begin{pmatrix} \hat{y}_1\left(t,\tau_1\right) \\ \vdots \\ \hat{y}_m\left(t,\tau_m\right) \\ y(t) \\ \pi\left(t\right) \end{pmatrix} = \begin{pmatrix} \mathbf{a} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \mathbf{B} \\ \mathbf{e}_1' \\ \mathbf{e}_2' \end{pmatrix} \begin{pmatrix} y(t) \\ \pi(t) \\ \rho\left(t\right) \\ y^*(t) \\ \pi^*\left(t\right) \end{pmatrix} + \boldsymbol{\varepsilon}_t \ , \tag{11}$$

where $\mathbf{e}_i$ is a $(n \times 1)$ column vector of zeros with a one on the ith row, $\boldsymbol{\varepsilon}_t$ is an $(m{+}2)$ x 1 vector of measurement errors and

$$\mathbf{a} = \left(a\left(\tau_1\right)/\tau_1,\ ....,a\left(\tau_m\right)/\tau_m\right)'$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{b}\left(\tau_1\right)'/\tau_1 \\ \vdots \\ \mathbf{b}\left(\tau_m\right)'/\tau_m \end{pmatrix}. \tag{12}$$

We write equation (11) more concisely as:

$$\begin{aligned} &\mathbf{z}\left(t\right) = \mathbf{c}_z + \mathbf{Hf}\left(t\right) + \boldsymbol{\varepsilon}\left(t\right) \\ &E_t\left(\boldsymbol{\varepsilon}\left(t\right)\boldsymbol{\varepsilon}\left(t\right)'\right) = \mathbf{R}, \\ &\text{with } R_{i,j} = 0 \text{ for } i,j > m. \end{aligned} \tag{13}$$

The transition equation is derived as the discrete time representation of the continuous time model dynamics:

$$\begin{aligned} &\mathbf{f}_{t+\Delta t} = \mathbf{c} + \boldsymbol{\Phi}(\Delta t)\left(\mathbf{f}_t - \mathbf{c}\right) + \boldsymbol{\nu}(t), \\ &\text{with } E_t\left(\boldsymbol{\nu}(t)\boldsymbol{\nu}(t)'\right) = \mathbf{Q}(\Delta t) \end{aligned} \tag{14}$$

The state-forecasting and updating equations are given respectively by:

$$\begin{aligned} \hat{\mathbf{f}}_{t+\Delta t\,|\,t} &= \mathbf{c} + \boldsymbol{\Phi}(\Delta t)\left(\hat{\mathbf{f}}_{t\,|\,t} - \mathbf{c}\right) \\ \hat{\mathbf{P}}_{t+\Delta t\,|\,t} &= \boldsymbol{\Phi}(\Delta t)\hat{\mathbf{P}}_{t\,|\,t}\boldsymbol{\Phi}(\Delta t)' + \mathbf{Q}\left(\Delta t\right) \end{aligned} \tag{15}$$

and:

$$\begin{aligned} &\hat{\mathbf{f}}_{t+\Delta t\,|\,t+\Delta t} = \hat{\mathbf{f}}_{t+\Delta t\,|\,t} + \hat{\mathbf{P}}_{t+\Delta t\,|\,t}\mathbf{H}'\left(\mathbf{H}\hat{\mathbf{P}}_{t+\Delta t\,|\,t}\mathbf{H}' + \mathbf{R}\right)^{-1}\left(\mathbf{z}\left(t+\Delta t\right) - \mathbf{c}_z - \mathbf{H}\,\hat{\mathbf{f}}_{t+\Delta t\,|\,t}\right) \\ &\hat{\mathbf{P}}_{t+\Delta t\,|\,t+\Delta t} = \hat{\mathbf{P}}_{t+\Delta t\,|\,t} - \hat{\mathbf{P}}_{t+\Delta t\,|\,t}\mathbf{H}'\left(\mathbf{H}\hat{\mathbf{P}}_{t+\Delta t\,|\,t}\mathbf{H}' + \mathbf{R}\right)^{-1}\mathbf{H}\hat{\mathbf{P}}_{t+\Delta t\,|\,t}. \end{aligned} \tag{16}$$

The definitions and computations of the matrices $\boldsymbol{\Phi}(\Delta t)$ and $\mathbf{Q}(\Delta t)$ can be found in Dewachter et al. (2001) or Fackler (2000). The final goal of this procedure is to maximize the likelihood (multivariate normal) of the prediction errors of the model $(\mathbf{z}\left(t+\Delta t\right) - \mathbf{c}_z - \mathbf{H}\,\hat{\mathbf{f}}_{t+\Delta t\,|\,t})$.

3 Estimation of the pre-EMU Bundesbank Monetary Policy Rule

This section concentrates on the estimation of the above model using German data for the period going from 1987 until the beginning of EMU in January 1999. We begin by describing the data used in the estimation and the main empirical results of the model. We end this section with a more detailed analysis of the short-run real interest rate (a Taylor-type rule) applied by the Bundesbank during this period. In the next section, we use those estimates to filter the real interest rate rule that would have been in place in case the Bundesbank were still in charge of the monetary policy in Germany.

3.1 Data

For the estimation of the model, we use monthly data for Germany for the period 1987:04 to 1998:12. In order to get a significantly accurate representation of the yield curve we employ a total of eighteen maturities: 1 to 12 months and 2, 3, 4, 5, 7, and 10 years. We retrieved interbank market rates and swap rates from Datastream. The yields with maturities up to 1 year were constructed from interbank market rates and those with higher maturities were constructed from swap rates using the method presented in Piazzesi (2001). In the following, we refer to it as the SWAP yield curve[3]. The dynamics of a selection of these yields can be seen at the bottom panel of Figure 1.

Insert Figure 1

The output gap and inflation series were obtained from IMF International Financial Statistics (IFS). Since GDP is not available at a monthly frequency we use the industrial production series as a proxy for the output variable. Output is then transformed into output gap by subtracting a linear trend. Note that we do not use the standard Hodrick-Prescott (HP) filter to construct the output gap since this would generate a highly non-linear output trend and would leave part of the information regarding the production dynamics out of the analysis.[4] The inflation series is constructed using the German CPI index and is reported in p.a. terms. The output gap and inflation series are also depicted in Figure 1.

A statistics summary of pre-EMU and EMU data can be found in Table 1. The yield curve is on average monotonically increasing while its variance tends to decrease with maturity. Normality is strongly rejected for most time series.

Insert Table 1

[3]Note also that we will neglect any credit risk that might be present in the data. Credit risk is not the focus of this paper and it can be argued that the implied SWAP yields are only minimally affected by the credit risk due to special neting features (see Duffie and Huang (1996)).

[4]The model was also estimated using the standard HP-filtered output gap series. The main conclusions are not significantly altered.

3.2 Empirical Results

We now present the estimates of the model parameters for the pre-EMU period (1987:04-1998:12). These estimates are considered to give a good representation of the Bundesbank monetary policy rule. The parameter estimates can be found in Tables 2 and 3[5]:

Insert Tables 2 and 3

Our first observation concerns the significance of the attracting properties of the time-varying central tendencies in the model. This can be seen from the statistical significance of the first three elements of the diagonal of the matrix $\mathbf{K}$ (see Table 2). All the filtered factors can be seen in Figure 2. The dashed lines depict the long-run behavior of each macroeconomic series. The central tendency of output gap (y^*) gives us an idea of the German business cycle during our sample period. The central tendency of inflation (π^*) can be seen as the long-run expectation of the market regarding the level of inflation. One observes a significant drop in this variable at the end of 1998. Finally, the central tendency of the real interest rate tracks the long-run monetary policy of the Bundesbank. The bottom panel of this figure also presents the Bundesbank short-term real interest rate rule. This time series will be discussed in the next sub-section. The resulting fit of the three macroeconomic series (output gap, inflation and real interest rate) is presented in Figure 3. The 'data' for the real interest rate was computed based on the one-month yield and is, therefore, an approximation of the instantaneous real interest rate.

Insert Figures 2 and 3

In order to keep the model as parsimonious as possible, we assume the measurement error covariance matrix ($\mathbf{R}$) to be diagonal. Hence, the number of parameters to be estimated is restricted to m. The last two rows and columns of this matrix are equal to zero since we perfectly update the output gap and inflation (see eq.(13)). The maximum measurement error implied by these estimates is equal to approximately 10 basis points (see Table 3). The resulting fit of the term structure can be seen in Figures 4-6. By visual inspection, we consider the general fit for all maturities as reasonably good. The loadings for each maturity with respect to the various factors (see equation 12) are shown in Figures 7 and 8. We can draw a number of observations from Figure 8: i) we do not find a clear level effect as is commonly reported in the literature (see, for instance, Berardi (2001)); ii) almost all factors (except the output gap) exert a significant influence on the short-end of the yield curve (note that due to the specification of the interest rate in equation (2), both inflation and the real

[5] The model was estimated in a single-step procedure. The optimization was performed using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm with a convergence tolerance for the gradient of the estimated coefficients equal to 1E-6. As usual, the robustness of the reported 'optimum' was checked by evaluating the convergence of the model from an array of starting points.

interest rate start by definition at 1); iii) the long-end of the yield curve is mainly determined by the central tendency of inflation or, in other words, by long-run inflation forecasts. Ang and Piazzesi (2001) also point out the importance of similar factors in explaining this region of the yield curve; iv) both output gap and output gap central tendency have a hump-shaped effect on the yield curve with a maximum effect on yields with maturities of approximately 6 months and 2 years, respectively.

Insert Figures 4 to 8

Although one can see the relatively low effect of the output gap on the yield curve (Figure 8), one should notice that this factor is one of the important sources of variation in the risk premia, along with the real interest rate and the central tendency of inflation. This can be seen in Figure 9 which presents the decomposition of the risk premia among all the factors for a selection of the yields. The average risk premium level (not presented here) increases with maturity, reaching a maximum of 1.2% for long-term bonds.

Insert Figure 9

3.3 The Bundesbank monetary policy rule (1987:04-1998:12)

The estimates presented in the previous section allow us to quantitatively assess an extended type of Taylor rule which gives us a further interpretation on the Bundesbank monetary policy. The above model posits implicitly a real interest rate rule for the central bank of the following form:

$$E_t \mathrm{d}\rho\,(t) = \kappa_{\rho\rho}\,(\rho_s^* - \rho(t))\,\mathrm{d}t$$

$$\rho_s^* = \underbrace{\gamma_0 + \left(\gamma_y - \frac{\kappa_{\rho y}}{\kappa_{\rho\rho}}\right) y\,(t) + \left(\gamma_\pi - \frac{\kappa_{\rho\pi}}{\kappa_{\rho\rho}}\right) \pi\,(t) + \left(\gamma_{y^*} + \frac{\kappa_{\rho y}}{\kappa_{\rho\rho}}\right) y^*\,(t) + \left(\gamma_{\pi^*} + \frac{\kappa_{\rho\pi}}{\kappa_{\rho\rho}}\right) \pi^*\,(t)}_{\text{central bank short-term target}}\,. \tag{17}$$

This restatement of the dynamics of the real interest rate can be used to analyze the implicit monetary policy rule used by the Bundesbank. This policy rule can be decomposed into two parts: a target real interest rate value, ρ_s^*, and a speed of convergence (interest rate inertia) in terms of the adjustment parameter $\kappa_{\rho\rho}$. The estimated target rule can be rewritten as:

$$\rho_s^* = 0.0285 - 0.088(y\,(t) - y^*\,(t)) - 0.254(\pi\,(t) - \pi^*\,(t)) + 0.437 y^*\,(t) + 0.400 \pi^*\,(t) \tag{18}$$

This equation stands in contrast to the standard backward-looking Taylor rules which do not incorporate the long-run tendencies of output gap and inflation. The inclusion of the central tendencies in the policy rule is in our view indicative of the forward looking nature of the monetary policy. The described rule states that target real interest rate moves positively with the long-run central tendencies (expectations) of output gap and inflation. In this respect, we see that the real interest rate rule is strongly dependent on expected future output gap and inflation. Next to these long-run sources of variation, real interest rates are also determined by short-run expectations based on current economic conditions. These factors are modeled through deviations from their central tendencies. Here we find that the sensitivities are negative and smaller than the long-run ones. Note that the negative sign should not come as a surprise. The mean reverting properties of the model in fact imply that, for instance, a positive inflation differential ($y(t) > y^*(t)$) implies an expectation of a future decrease in observed inflation. In short, we find evidence of a monetary policy rule that reacts strongly to future expected output gap and inflation and adjusts to a lesser extent to short-run expectations (decreasing interest rates whenever the output gap and inflation are expected to decrease in the near future).

A second part of the actual real interest rate dynamics is related to the convergence properties towards this target. This is measured by the parameter $\kappa_{\rho\rho}$, estimated at 2.5 on a yearly basis. This estimate implies a halving time of the deviation of the current rate from the target of about three to four months, indicating that the monetary policy is such that real interest rates are adjusted quickly to this time-varying target. This strong and statistically significant mean reversion allows us to interpret the target rate as a relatively short-term target rate.

Finally, the model also allows for what could be called an equilibrium interest rate rule, which presents a rule that would apply if all macroeconomic variables were at their long-run expectations, i.e. $\pi(t) = \pi^*(t)$ and $y(t) = y^*(t)$ (implying that $\rho(t) = \rho^*(t)$). This rule can be interpreted as an equilibrium Taylor rule. The estimated equilibrium interest rate rule resembles very strongly the standard Taylor rule estimates. More in particular, the estimated equilibrium rule of our model gives:

$$\bar{\rho} = 0.0285 + 0.437y^*(t) + 0.4\pi^*(t) \tag{19}$$

which rewritten in terms of the nominal interest rate target gives:

$$\bar{\imath} = 0.0285 + 0.437y^*(t) + 1.4\pi^*(t) \tag{20}$$

The latter equation comes close to the target rules reported in the literature for the Bundesbank. For instance, Faust et al. (2001) find values for the constant real interest rate of 0.0258, an output sensitivity of 0.19 and an inflation sensitivity of 1.31. Our estimates are also reasonably close to those reported by Clarida et al. (1998).

To summarize, the estimates reported in this section do provide evidence that the proposed model does capture interest rate policy and term structure dynamics sufficiently accurately to use this model to generate the benchmark term structures for the german bond markets.

In the next section we use this model to generate, or filter, the term structure that would occur under the assumption that the Bundesbank were still in control of monetary policy.

4 Projecting German monetary policy onto the EMU period (1999:01-2002:02)

We now turn to the main objective of this paper which is the assessment of the effects of European monetary unification on the German monetary policy rule and on the German SWAP yield curve. We assume that EMU did not take place and that the Bundesbank remained in charge of German monetary policy since the beginning of 1999. We use two different methods to obtain the three unobserved series in our model: the real interest rate and the central tendencies of output gap and inflation for the period 1999:01 until 2002:02. These series are then used to compute the implied yield curve for the same period. We finally compare both the observed real interest rate and the yield curve under EMU with the filtered ones based on the Bundesbank behavior. This comparison allows us to assess period by period the existence of structural differences between the two possible states for the Germany monetary policy (under ECB or Bundesbank control).

As mentioned, two methods are employed in order to obtain the three unobserved macroeconomic series for the period 1999:01-2002:02 which are used to construct the hypothetical German yield curves under the Bundesbank. The first method consists of generating the prediction densities for the unobserved series during the EMU era and computing the implied yield curves and their confidence bounds based on the estimates presented in the previous section. We then verify whether the observed yield curves fall within the constructed confidence bounds. Within the context of the above Gaussian model, the prediction of the factors and the consequent computation of the yield curves can be done easily. Denoting the end day of the Bundesbank period by T and the prediction horizon by τ, the probability density of the yield curve is given by:

$$\mathcal{N}\left(\mathbf{a}+\mathbf{B}\left(\mathbf{c}+\Phi(\tau)\left(\hat{\mathbf{f}}\left(T\right)-\mathbf{c}\right)\right),\mathbf{B}\hat{\mathbf{P}}_{\mathbf{T}+\tau|\mathbf{T}}\mathbf{B}^{'}+\mathbf{R}\right), \tag{21}$$

where $\hat{\mathbf{P}}_{\mathbf{T}+\tau|\mathbf{T}}$ denotes the variance-covariance matrix of the cumulative factor shocks over the prediction horizon τ. From this multivariate normality assumption, yield curve forecasts and confidence bounds are easily constructed for any relevant prediction horizon τ. The results for the 18 maturities used before are depicted in Figures 10-12.

Insert Figures 10-12

The main conclusion drawn from those graphs is clearly that predicting the yield curve some months into the future does not yield a lot of information regarding the evolution of future interest rates. Due to the very high inertia in the latent factors, we find that the prediction densities are dominated by the near unit root component in the interest rate series: optimal predictors are close to the last observed value and confidence intervals increase quasi-linearly in the prediction horizon.

In order to obtain more precise estimates of the german term structure, we employ a second method that makes use of additional information regarding the german economic conditions (inflation and output) during the EMU period. In the above forecasting exercise, we did not use this additional information to generate the implied yield curves. Forecasted values were used for all the factors including output gap and inflation. Now, instead of forecasting we filter the economic state (real interest rate and central tendencies of output gap and inflation) conditional on the observed output gap and inflation levels in Germany. This filtering procedure can be easily performed given the estimated state dynamics presented in the previous section.

Filtering of the output gap and inflation central tendencies can be done by adapting the measurement and the updating equation as follows:

$$\begin{pmatrix} y(t) \\ \pi(t) \end{pmatrix} = \begin{pmatrix} \mathbf{e}_1' \\ \mathbf{e}_2' \end{pmatrix} \begin{pmatrix} y(t) \\ \pi(t) \\ \rho(t) \\ y^*(t) \\ \pi^*(t) \end{pmatrix} + \boldsymbol{\varepsilon}_t \tag{22}$$

which in vector notation becomes:

$$\tilde{\mathbf{z}}(t) = \tilde{\mathbf{H}}\mathbf{f}(t) + \tilde{\varepsilon}(t) \tag{23}$$

yielding the state vector and variance updating dynamics:

$$\begin{aligned} &\tilde{\mathbf{f}}_{t+\Delta t\,|\,t+\Delta t} = \tilde{\mathbf{f}}_{t+\Delta t\,|\,t} + \tilde{\mathbf{P}}'_{t+\Delta t\,|\,t}\tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}\tilde{\mathbf{P}}'_{t+\Delta t\,|\,t}\tilde{\mathbf{H}}\right)^{-1}\left(\tilde{\mathbf{z}}(t+\Delta t) - \tilde{\mathbf{H}}\tilde{\mathbf{f}}_{t+\Delta t\,|\,t}\right) \\ &\text{with: } \tilde{\mathbf{f}}_{t+\Delta t\,|\,t} = \mathbf{c} + \boldsymbol{\Phi}\left(\tilde{\mathbf{f}}_{t\,|\,t} - \mathbf{c}\right) \\ &\tilde{\mathbf{P}}_{t+\Delta t\,|\,t+\Delta t} = \tilde{\mathbf{P}}_{t+\Delta t\,|\,t} - \tilde{\mathbf{P}}'_{t+\Delta t\,|\,t}\tilde{\mathbf{H}}\left(\tilde{\mathbf{H}}\tilde{\mathbf{P}}'_{t+\Delta t\,|\,t}\tilde{\mathbf{H}}\right)^{-1}\tilde{\mathbf{H}}\tilde{\mathbf{P}}_{t+\Delta t\,|\,t} \\ &\text{with } \tilde{\mathbf{P}}_{t+\Delta t\,|\,t} = \tilde{\mathbf{H}}\left(\boldsymbol{\Phi}\tilde{\mathbf{P}}_{t\,|\,t}\boldsymbol{\Phi}' + \tilde{\mathbf{Q}}(\Delta t)\right)\tilde{\mathbf{H}}'. \end{aligned} \tag{24}$$

Adapting the Kalman filter in this way implies that we filter for the entire economic state variable $\mathbf{f}$ taking into account the observations on y and π during the EMU period. Moreover, the filter will perfectly update the actual output gap and inflation series and generate *optimal* filtered values for the unobservable real interest rate and output gap and inflation central tendencies. Note, however, that in generating the filtered variables we take into account all information of the state space dynamics, i.e. we use the matrices $\boldsymbol{\Phi}$ and $\tilde{\mathbf{Q}}$ in this filtering procedure. This filtering is moreover very likely to reduce the uncertainty around the predicted filtered Bundesbank yield curve, as new information concerning the at that time prevailing inflation and output gap are incorporated into the information set. The probability density of the yield curve can then be described as:

$$\mathcal{N}\left(\mathbf{a}+\mathbf{B}\left(\mathbf{c}+\Phi(\tau)\left(\tilde{\mathbf{f}}_{t+\Delta t\,|\,t}-\mathbf{c}\right)\right),\mathbf{B}\tilde{\mathbf{P}}_{\mathbf{T}+\boldsymbol{\tau}|\mathbf{T}+\boldsymbol{\tau}}\mathbf{B}^{'}+\tilde{\mathbf{R}}\right), \tag{25}$$

The confidence bounds for the unobserved factors and for the constructed yield curves can be seen in Figure 13. Figure 14 shows the filtered real interest rate (dashed line) in comparison with the observed German real interest rate (solid line). One can see clearly that after mid-1999 the observed monetary policy under EMU (solid line) starts to differ substantially from the one which would have been applied by the Bundesbank in case it were still in charge of the German monetary policy. Around the third quarter of 2000 this difference becomes then statistically significant. Most importantly, it seems, as expected, that the Bundesbank would have preferred higher real interest rates than the ones in place since 1999.

Insert Figures 13 and 14

Figures 15-17 plot the filtered yields for the 18 maturities used so far, ranging from 1 month to 10 years. We see that the difference between the observed and the filtered yields is more pronounced for the short-term maturities. For the long-term maturities the effects of the different monetary policies (under EMU and under the Bundesbank) become then less significant.

Insert Figures 15-17

To summarize, using this second filtering method, we obtain relatively precise signals that allow us to make a distinction between what the Bundesbank would have done, given the observed inflation rate and output gap, and what the ECB imposes as a yield curve upon Germany. We find that the Bundesbank would have chosen higher interest rates, especially in the short run. Moreover, it seems that the long-run yields do not differ to the same extent, implying that not only the level of interest rates is affected by the ECB but also the spread of the yield curve. In fact, the ECB basically imposed lower short-run interest rates on the EMU area but increased the spread of the term structure. This suggests a larger risk premia component during the ECB period than during the Bundesbank period.

5 Conclusions

In this paper we analyzed the implications of the monetary unification for German bond markets by reprojecting the Bundesbank monetary policy over the EMU period. By extension, and assuming that Germany would have remained the core of the ERM, this Bundesbank yield curve would also serve as a benchmark in the other ERM countries. In line with many financial analysts, we find that the actually observed ECB interest rates (in German markets) are significantly lower than they would have been under Bundesbank policy. Moreover, it seems that, next to the lower interest rate level, ECB spreads tend also to be larger than they would have been under the Bundesbank regime.

Next to providing estimates of the benchmark Bundesbank monetary policy, this paper also provides an easy-to-use method to evaluate the ECB policy relative to the Bundesbank benchmark for some time into the future. By incorporating the current economic situation (instead of generating prediction densities) we are able to use filtering procedures to generate the unobserved latent factors (real interest rate and central tendencies of output gap and inflation). This filtering technique has the desirable property that the filtering uncertainty remains limited and, in fact, (based on our estimates) results in a relatively precise asymptotic variance covariances matrix. As such, the main contribution of this paper consists in providing a method to generate benchmarks to evaluate the ECB policy for some time into the future which in turn may help in the vigorous ECB debate.

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Figures

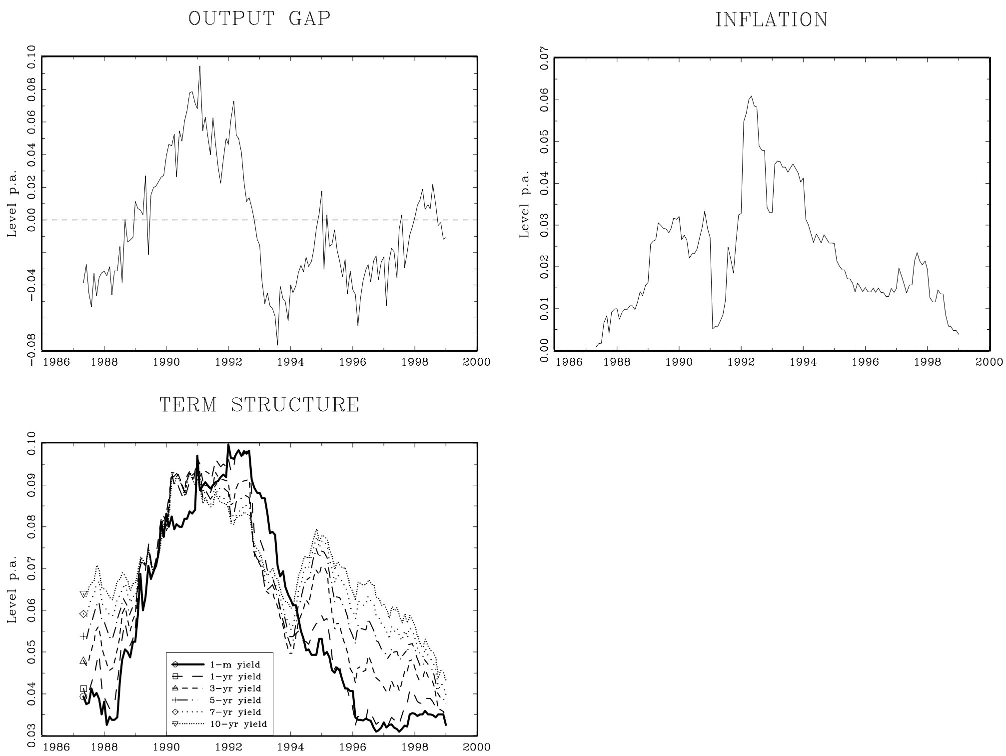

Figure 1: Data on output gap, inflation and the term structure of interest rates (1987:04-1998:12).

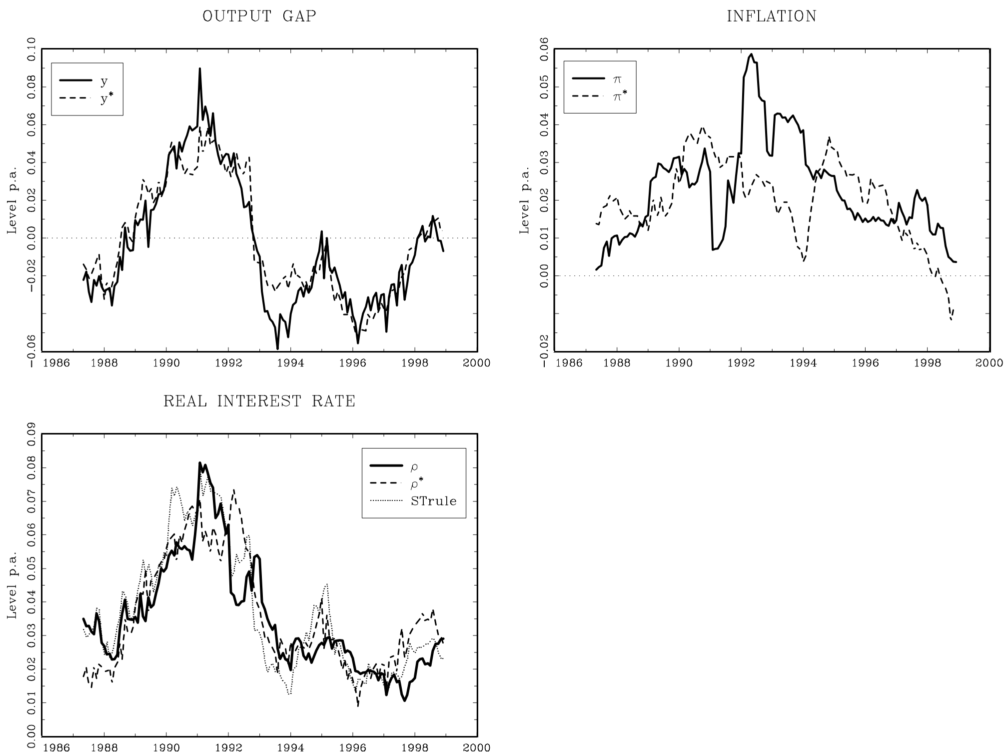

Figure 2: Macroeconomic variables, their respective central tendencies and the short-term real interest rate rule (1987:04-1998:12).

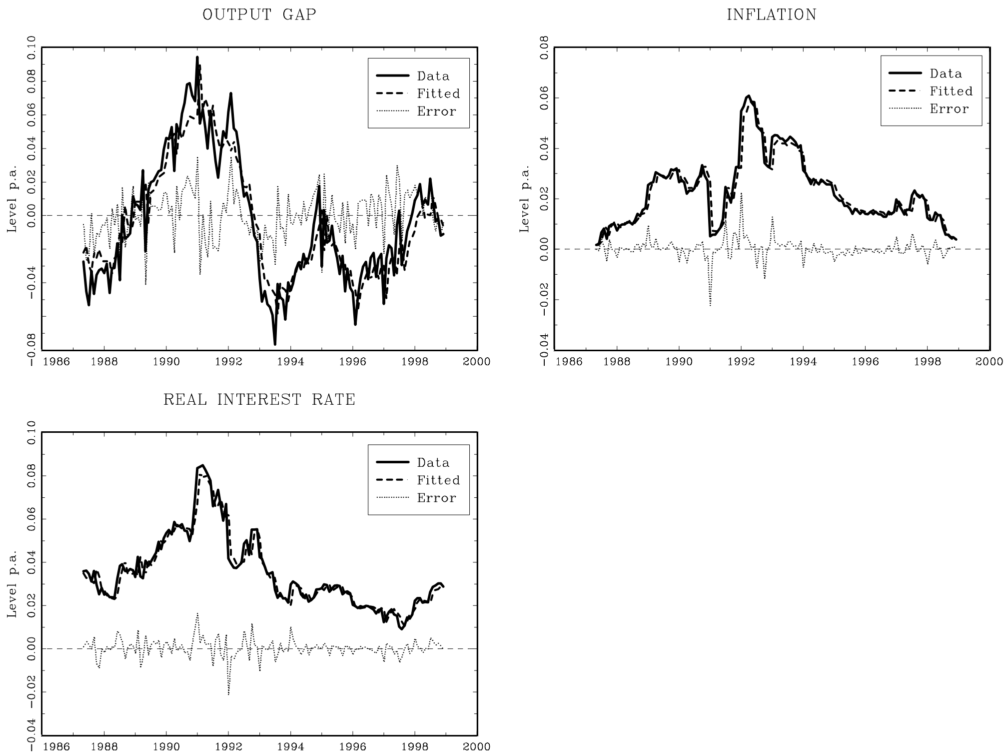

Figure 3: Fit of output gap, inflation, and the real interest rate (1987:04-1998:12).

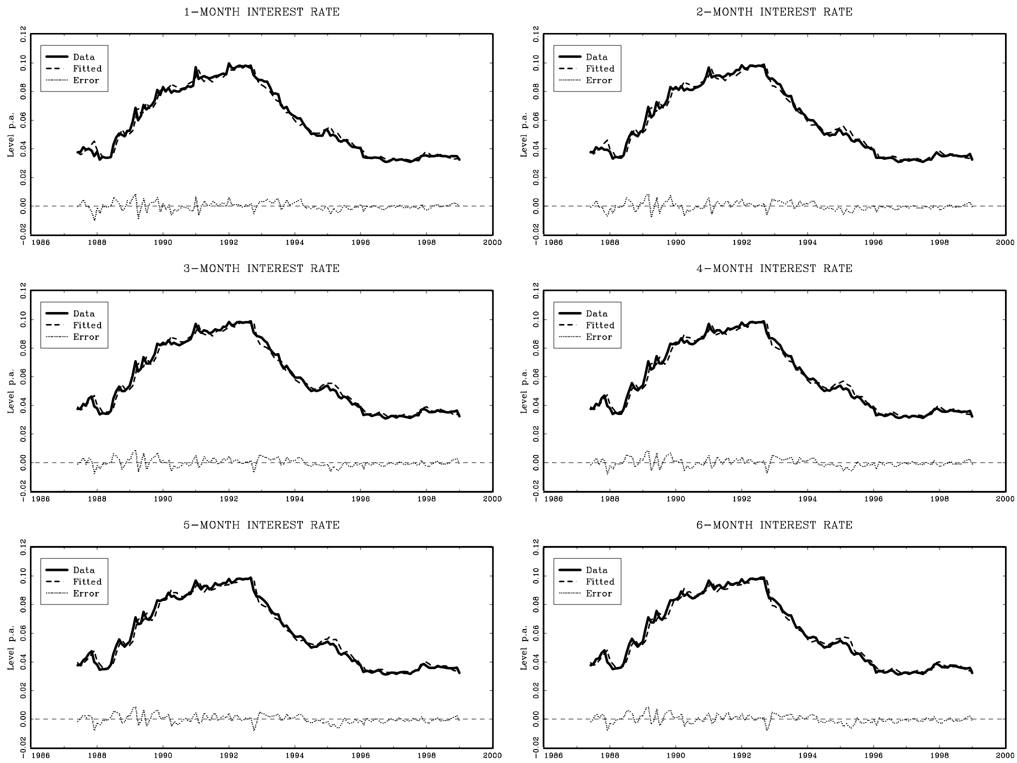

Figure 4: Fit of the term structure of interest rates, 1-6 months (1987:04-1998:12).

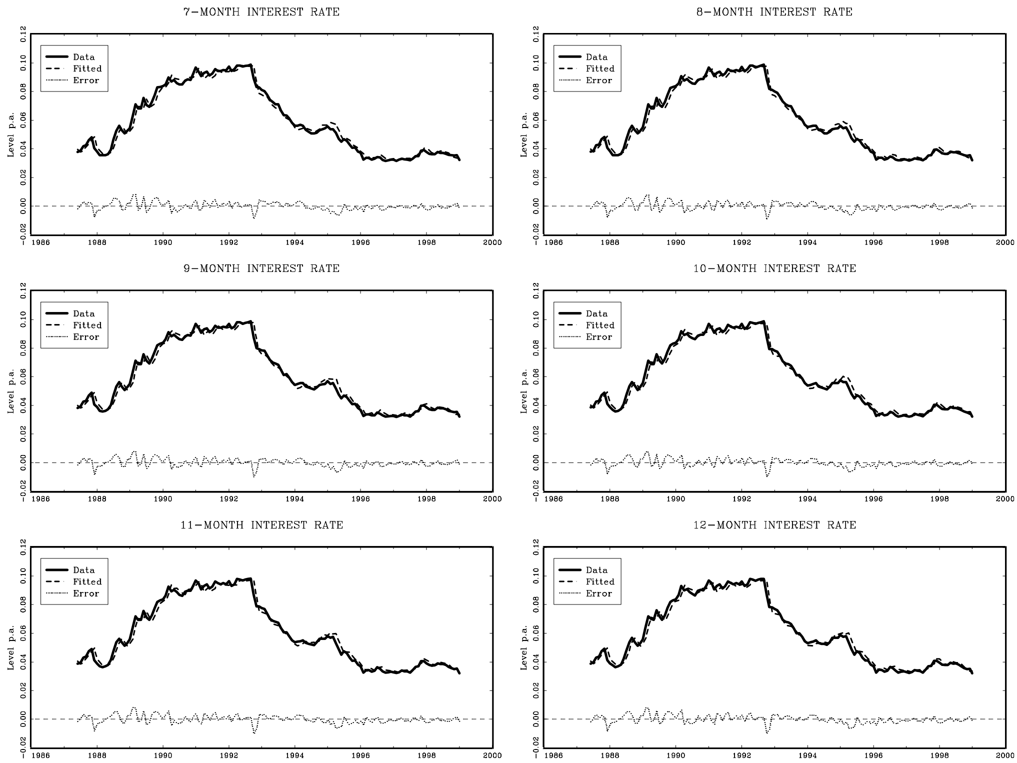

Figure 5: Fit of the term structure of interest rates, 7-12 months (1987:04-1998:12).

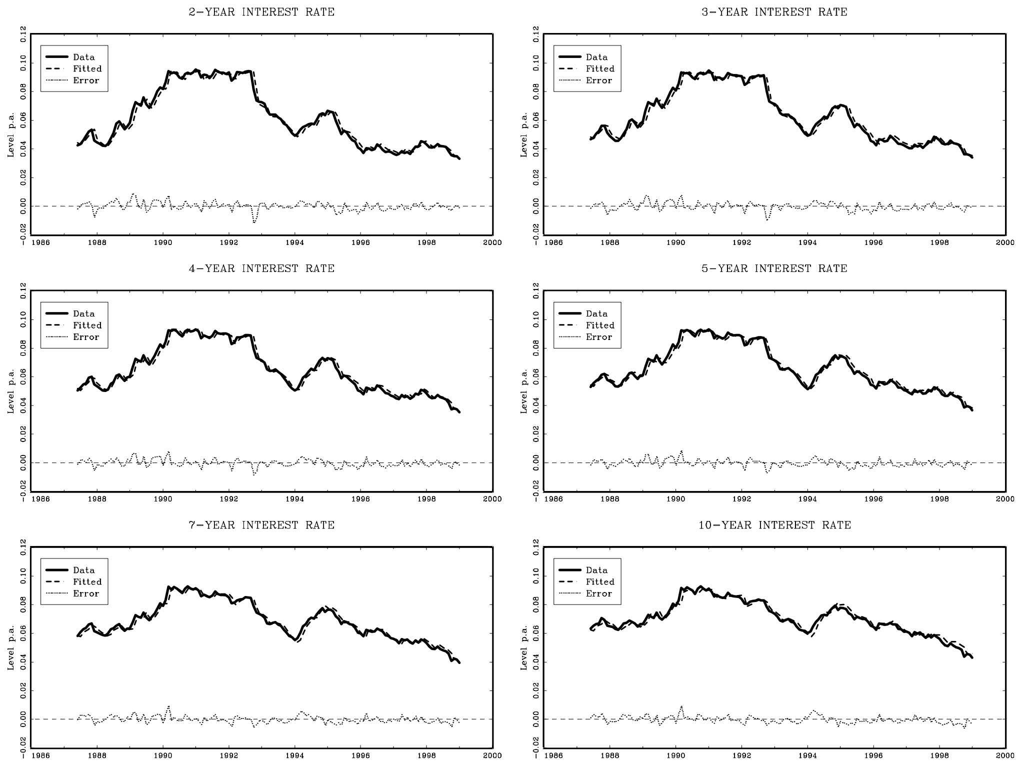

Figure 6: Fit of the term structure of interest rates, 2, 3, 4, 5, 7, and 10 years (1987:04-1998:12).

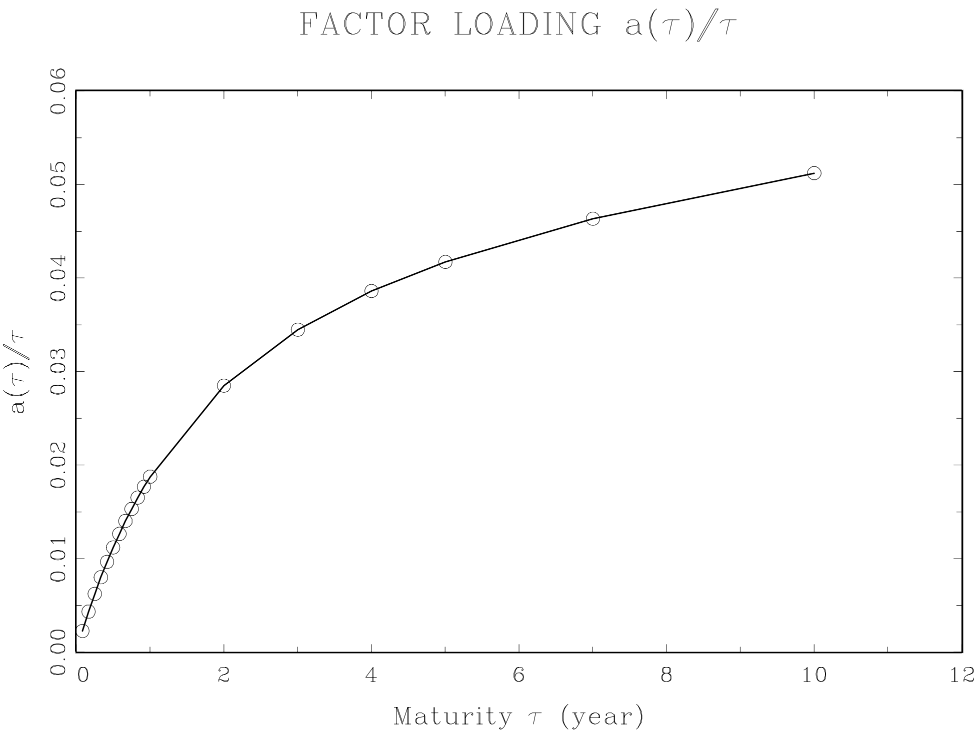

Figure 7: Estimated constant factor loading.

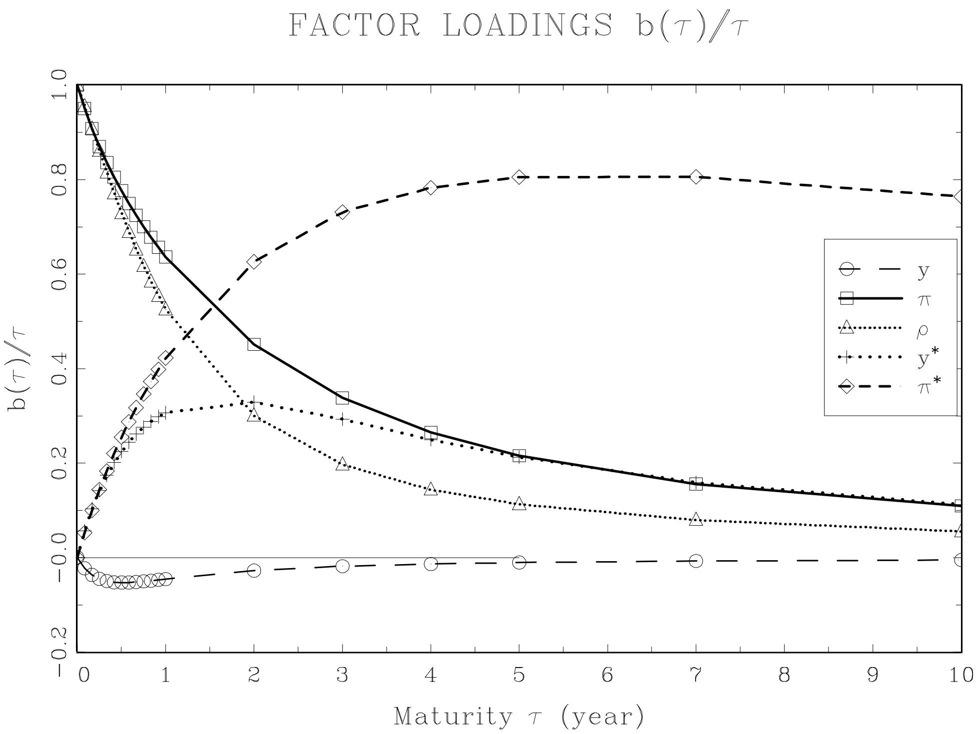

Figure 8: Estimated factor loadings.

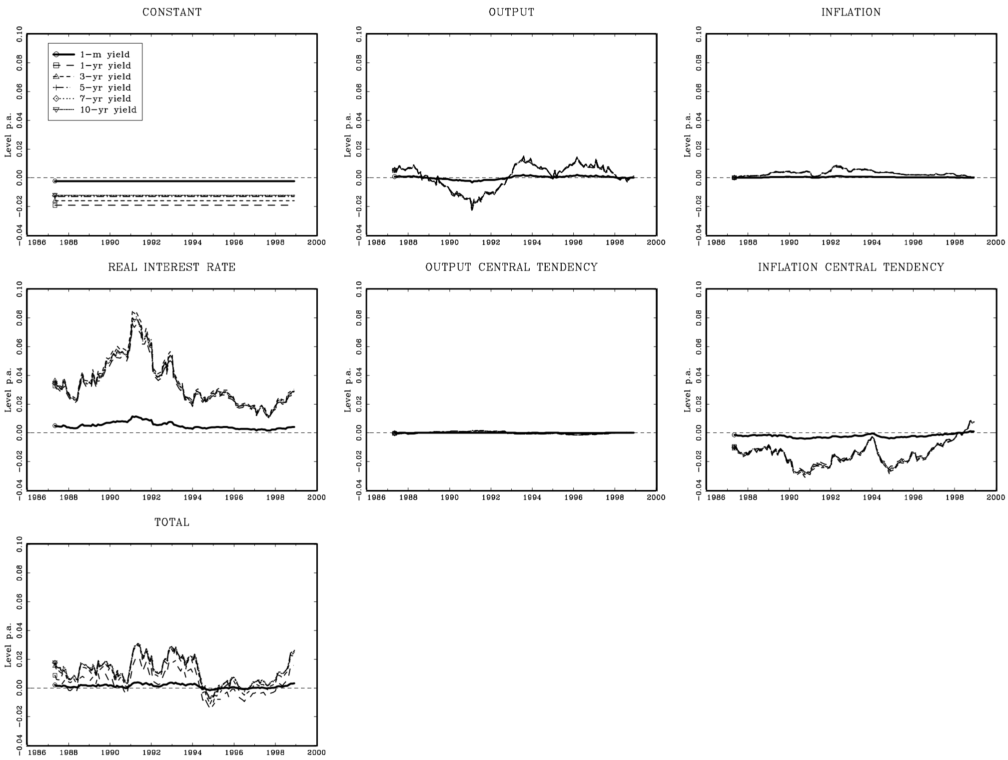

Figure 9: Factor decomposition of risk premium (1987:04-1998:12).

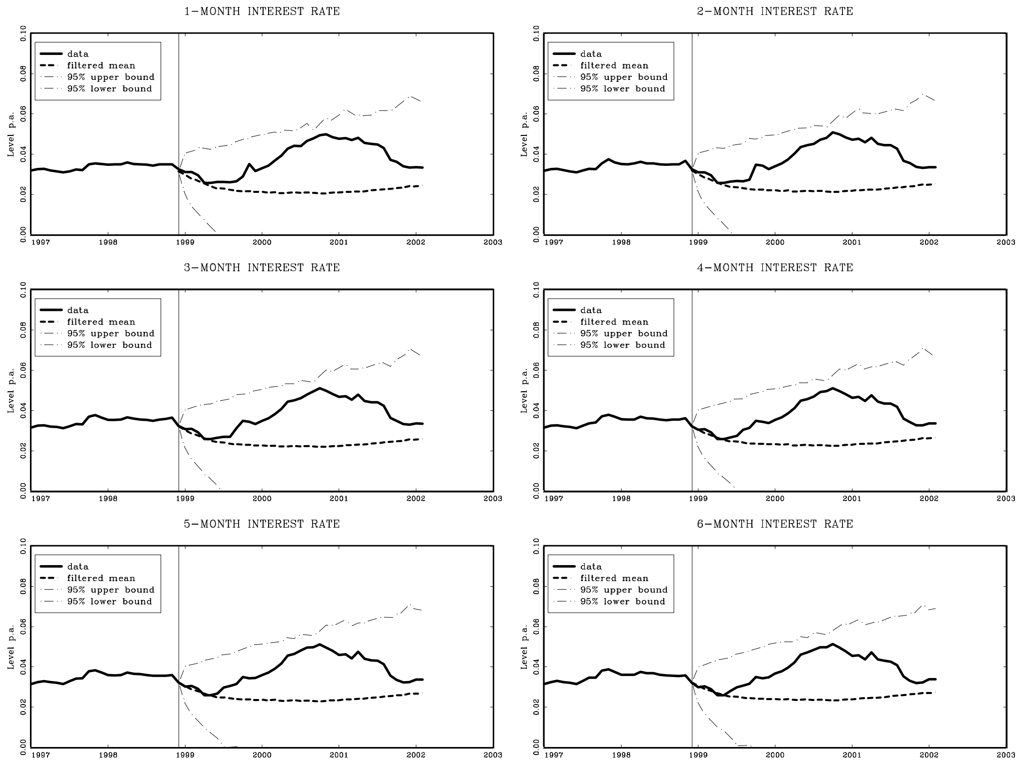

Figure 10: Simulated term structure, 1-6 months (1999:01-2002:02).

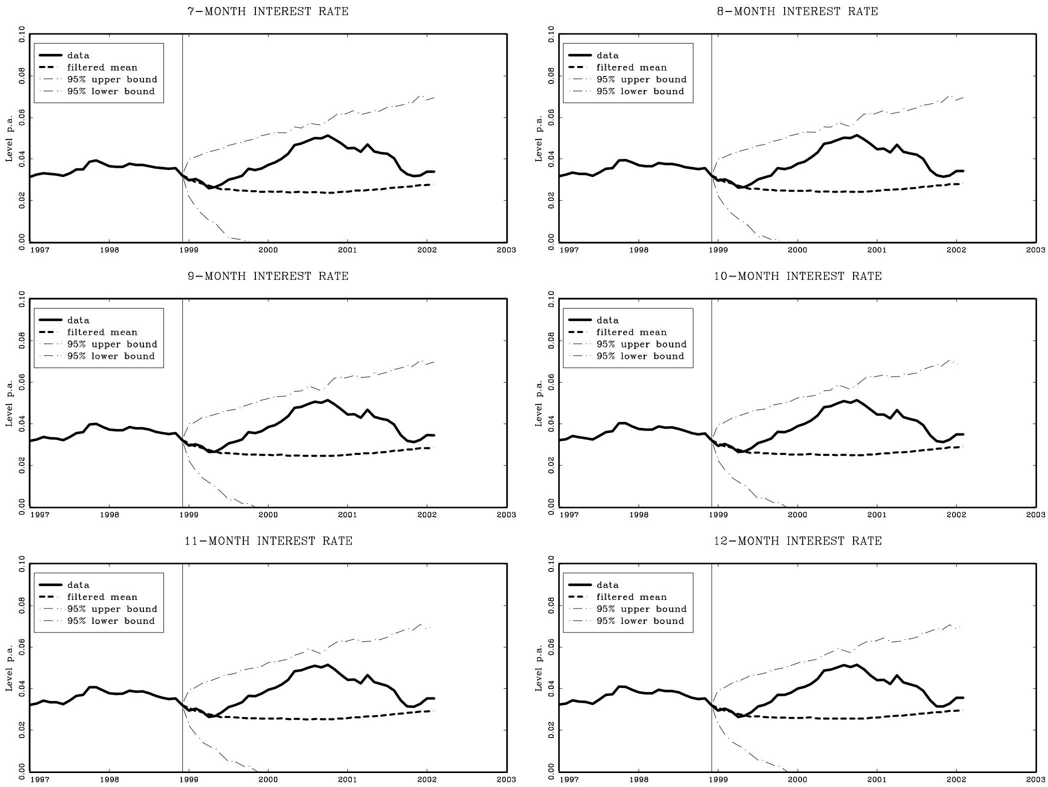

Figure 11: Simulated term structure, 7-12 months (1999:01-2002:02).

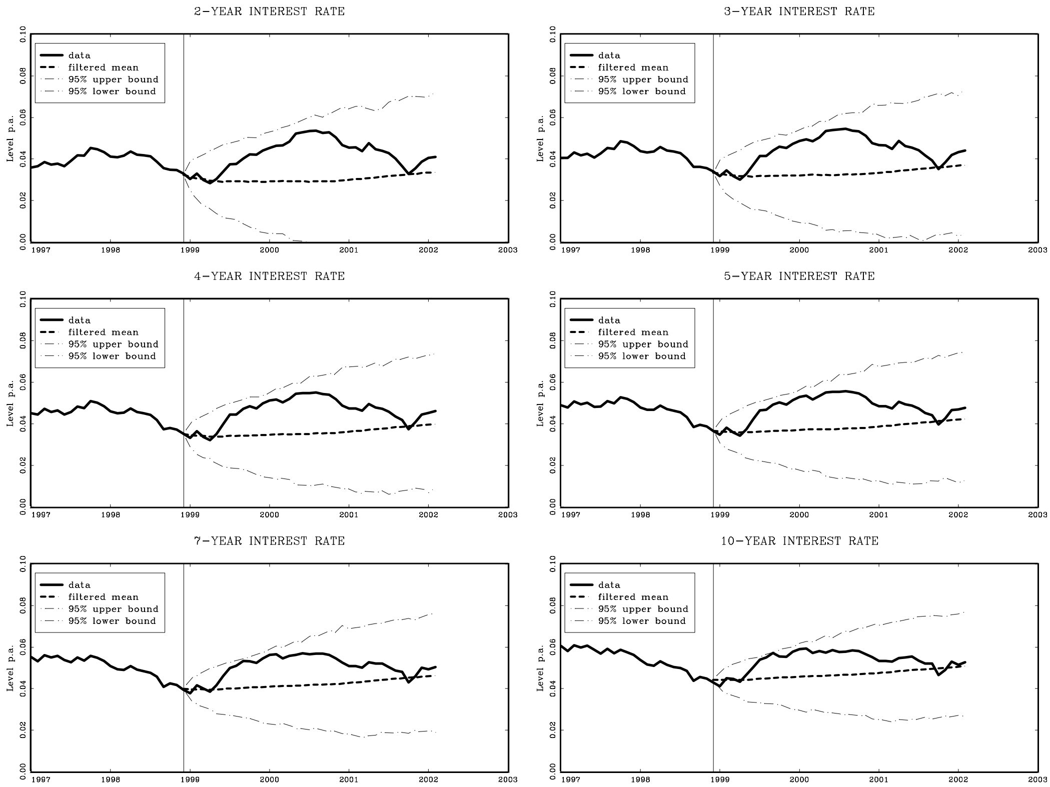

Figure 12: Simulated term structure, 2, 3, 4, 5, 7, and 10 years (1999:01-2002:02).

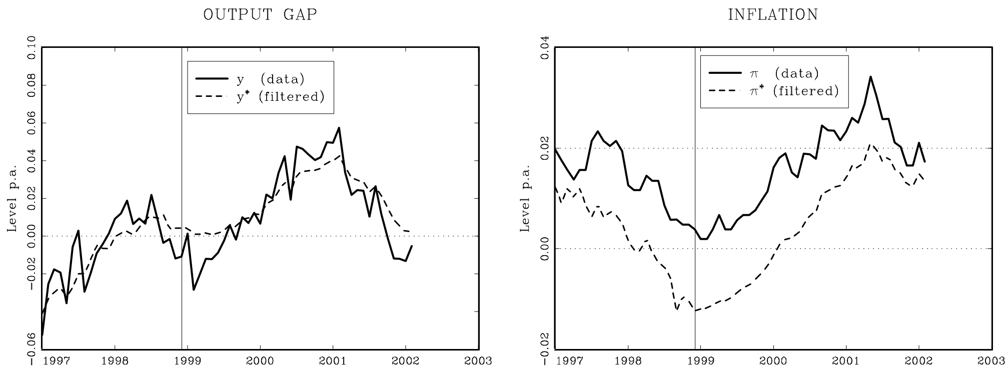

Figure 13: Filtered output gap and inflation (1999:01-2002:02).

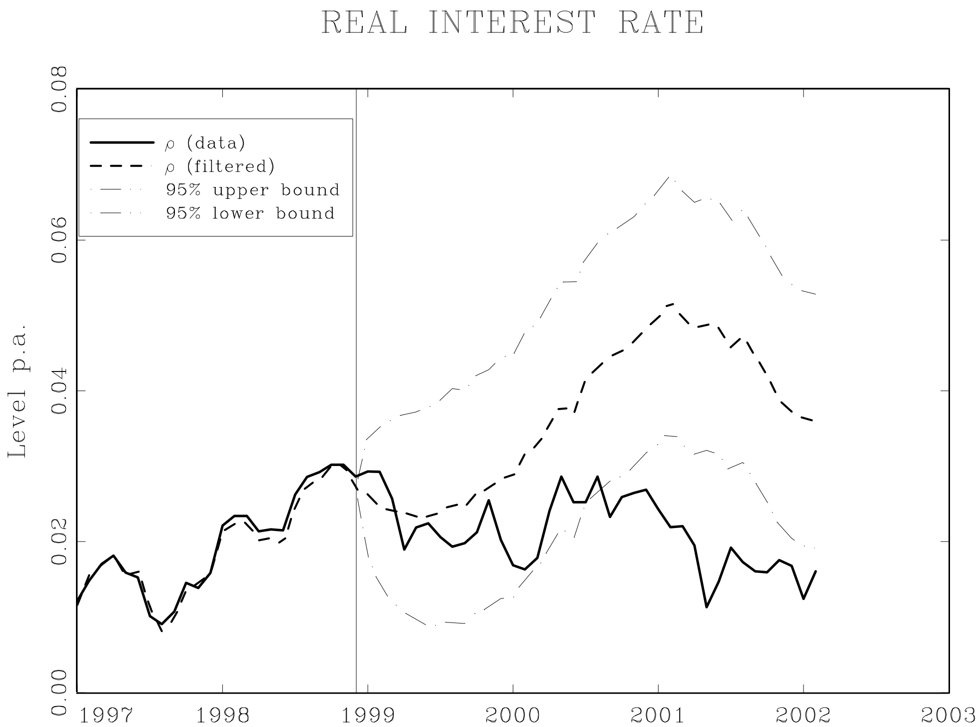

Figure 14: Filtered real interest rate (1999:01-2002:02).

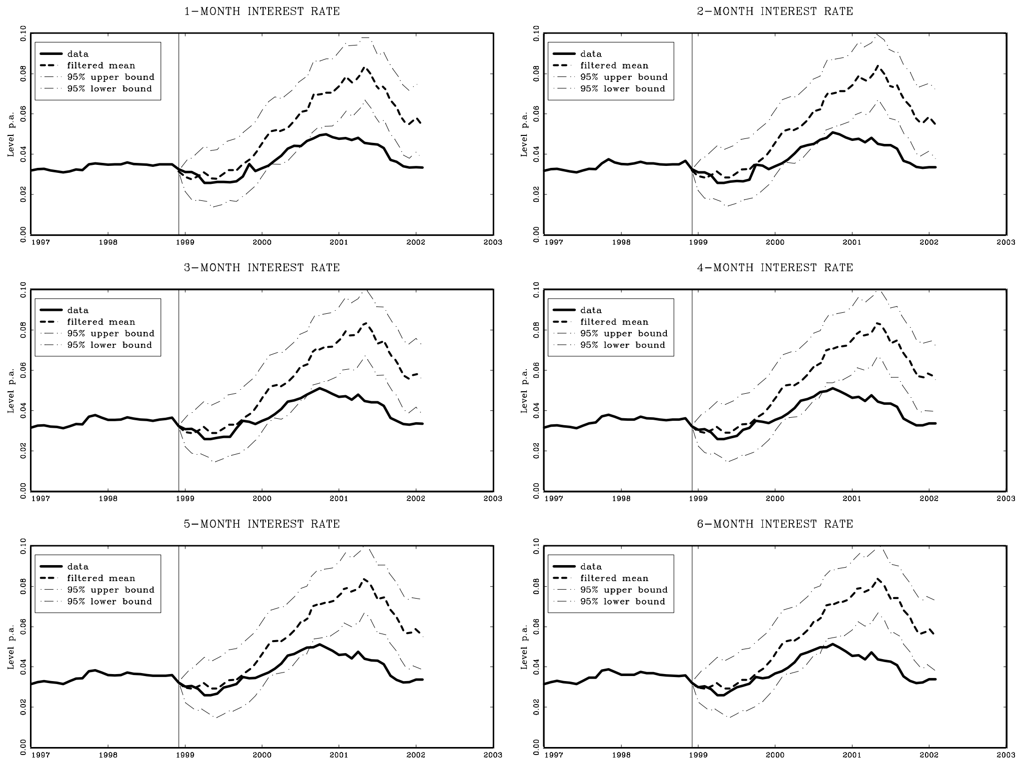

Figure 15: Filtered term structure, 1-6 months (1999:01-2002:02).

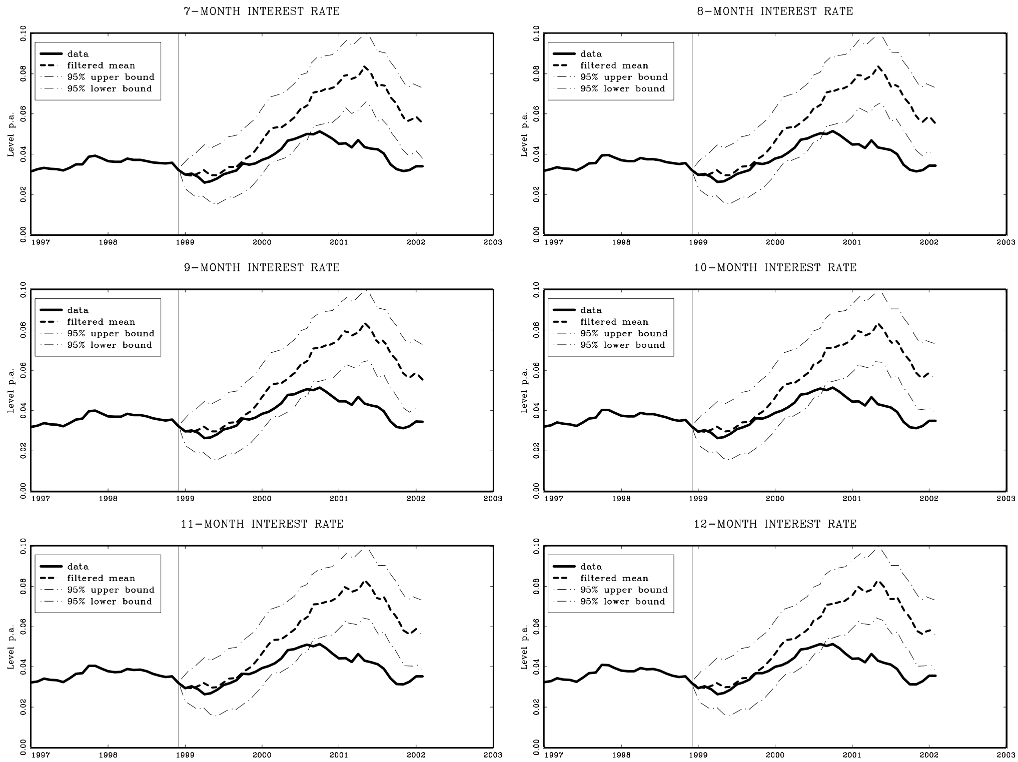

Figure 16: Filtered term structure, 7-12 months (1999:01-2002:02).

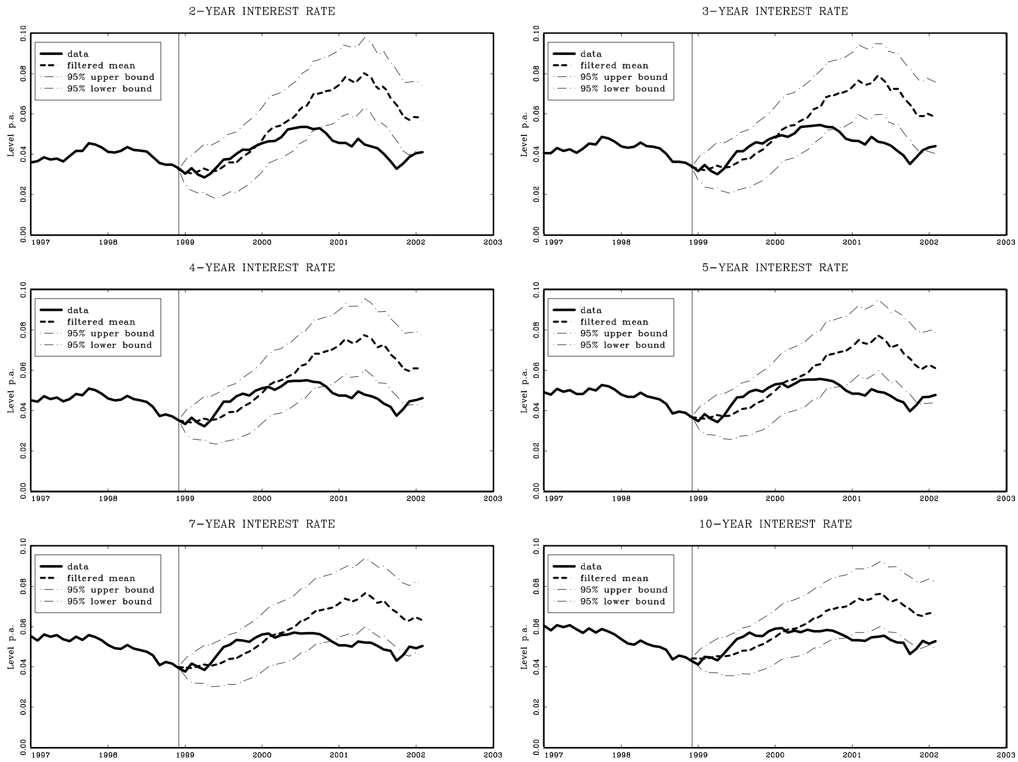

Figure 17: Filtered term structure, 2, 3, 4, 5, 7, and 10 years (1999:01-2002:02).

Tables

Table 1: Summary statistics for the complete data set used (1987:04-2002:02)

	Mean	**Std**	**Min**	**Max**	**Auto**	**Skew**	**Kurt**	**JB**
yield_{1m}	5.392	2.250	2.565	9.969	0.993	0.668	1.980	21.061*
yield_{2m}	5.424	2.265	2.569	9.875	0.994	0.678	1.982	21.449*
yield_{3m}	5.451	2.264	2.574	9.875	0.994	0.675	1.978	21.374*
yield_{4m}	5.462	2.260	2.579	9.875	0.994	0.677	1.982	21.426*
yield_{5m}	5.475	2.258	2.585	9.875	0.995	0.683	1.993	21.492*
yield_{6m}	5.486	2.258	2.588	9.875	0.994	0.689	2.007	21.524*
yield_{7m}	5.486	2.245	2.595	9.875	0.994	0.695	2.024	21.517*
yield_{8m}	5.490	2.233	2.631	9.875	0.994	0.703	2.045	21.550*
yield_{9m}	5.494	2.221	2.636	9.875	0.994	0.712	2.068	21.586*
yield_{10m}	5.505	2.215	2.638	9.875	0.994	0.716	2.080	21.622*
yield_{11m}	5.517	2.207	2.643	9.828	0.994	0.718	2.089	21.576*
yield_{12m}	5.528	2.202	2.644	9.813	0.993	0.723	2.099	21.657*
yield_{2yr}	5.730	1.995	2.833	9.560	0.991	0.721	2.170	20.631*
yield_{3yr}	5.906	1.819	3.005	9.490	0.990	0.676	2.185	18.579*
yield_{4yr}	6.073	1.673	3.215	9.330	0.990	0.605	2.199	15.685*
yield_{5yr}	6.217	1.570	3.429	9.320	0.989	0.546	2.235	13.272*
yield_{7yr}	6.455	1.392	3.764	9.290	0.990	0.375	2.267	8.201*
yield_{10yr}	6.684	1.250	4.126	9.285	0.990	0.236	2.316	5.154
output gap	0.029	3.570	-7.680	9.438	0.914	0.323	2.288	6.892*
inflation	2.148	1.299	0.087	6.096	0.949	0.899	3.612	26.882*

The yield series are constructed from interbank market rates for maturities below 1 year, and from SWAP rates for maturities above one year. Output gap and inflation are constructed as mentioned in the text. The data series cover the period from 1987:04 until 2002:02, totalling 179 monthly observations. *Mean* denotes the sample arithmetic average, expressed as a p.a. percentage, *Std.* standard deviation, *Min* minimum, *Max* maximum, *Auto* the first order autocorrelation, *Skew* and *Kurt* stand for skewness and kurtosis, and *JB* stands for the Jarque-Bera normality test statistic. The star superscripts in the last column denote those variables for which respectively the null of normality may be rejected at the 95% confidence interval.

Table 2: Maximum likelihood estimates (1987:04-1998:12)

	$\mathbf{y}$	$\boldsymbol{\pi}$	$\boldsymbol{\rho}$	$\mathbf{y}^*$	$\boldsymbol{\pi}^*$
$\boldsymbol{\kappa}_{y,\cdot}$	4.7908	1.4444	-5.0488		
	(1.2623)	(2.1955)	(2.4222)		
$\boldsymbol{\kappa}_{\pi,\cdot}$	0.1716	1.0559	0.3016		
	(0.1974)	(0.2510)	(0.2368)		
$\boldsymbol{\kappa}_{\rho,\cdot}$	1.5145	1.8628	2.5053		
	(0.5387)	(0.6909)	(0.5638)		
$\boldsymbol{\kappa}_{y^*,\cdot}$				0.5746	
				(0.1058)	
$\boldsymbol{\kappa}_{\pi^*,\cdot}$					0.0796
					(0.0111)
$\boldsymbol{\theta}_\cdot$				-0.0002	0.0392
				(0.0113)	(0.0071)
$\boldsymbol{\gamma}_0$			0.02853		
			(0.0049)		
$\boldsymbol{\gamma}_\cdot$	0.5163	0.4895		-0.0790	-0.0898
	(0.1214)	(0.1172)		(0.1160)	(0.0872)
$\boldsymbol{\lambda}_\cdot$	-46.1438	-100.9254	178.6729		
	(11.4791)	(83.4807)	(102.1964)		
$\Xi_{\rho,\cdot}$	0.4462	-0.2498	-1.7742	-0.0502	1.3195
	(0.2216)	(0.4617)	(0.5849)	(0.1054)	(0.5146)
$\boldsymbol{\sigma}^2_\cdot$	0.00454	0.00022	0.00025	0.00064	0.00009
	(0.00065)	(0.00007)	(0.00006)	(0.00020)	(0.00003)

ML estimates with robust standard errors underneath (see e.g. Duan and Simonato (1999)). Total loglikelihood amounts to 19164.6826 or 135.9197 on average (excluding constant in the loglikelihood).

Table 3: Measurement error covariance matrix

$\mathbf{R}_{1m}$	1.6516	$\mathbf{R}_{7m}$	0.0269	$\mathbf{R}_{2yr}$	0.3796
$\mathbf{R}_{2m}$	0.6816	$\mathbf{R}_{8m}$	0.0264	$\mathbf{R}_{3yr}$	0.2833
$\mathbf{R}_{3m}$	0.1596	$\mathbf{R}_{9m}$	0.0493	$\mathbf{R}_{4yr}$	0.1211
$\mathbf{R}_{4m}$	0.0570	$\mathbf{R}_{10m}$	0.0214	$\mathbf{R}_{5yr}$	0.0575
$\mathbf{R}_{5m}$	0.0205	$\mathbf{R}_{11m}$	0.0221	$\mathbf{R}_{7yr}$	0.4720
$\mathbf{R}_{6m}$	0.0439	$\mathbf{R}_{12m}$	0.0413	$\mathbf{R}_{10yr}$	1.3495

All values are multiplied by 10^6. Note that the entries are variances and not standard deviations, as most often reported.