KATHOLIEKE UNIVERSITEIT LEUVEN

Faculty of Economics and Applied Economics

Department of Economics

Does a Sudden Death Liven Up the Game?: Rules, Incentives, and Strategy in Football

by

Anurag BANERJEE
Johan F.M. SWINNEN

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DISCUSSION PAPER

Does A Sudden Death Liven Up The Game ? Rules, incentives, and strategy in football[1]

Anurag BANERJEE
Department of Economics
University of Southampton
Southampton,UK

Johan F.M. SWINNEN
Department of Economics
K. U. Leuven
Leuven, Belgium

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Abstract

In an effort to stimulate more attractive football, the international football association FIFA, has introduced several new rules over the past decades. One of the most recent rule change is the introduction of the "sudden death" or "golden goal" rule for games going into overtime. This paper analyses under which conditions, if any, the introduction of the sudden death rule improves the attractiveness of the football game. Our theoretical results indicate that the new rule will change the behavior of the teams, but not necessarily in the way intended. It may stimulate more offensive play during extra times, but only if a team considers itself having a "comparative advantage" in offensive play vis-a-vis playing defensively. In other cases it will induce more defensive playing. Empirical evidence suggests that both may occur, but that the latter may be the more common situation and that the introduction of the sudden death rule has induced more defensive soccer, i.e. the opposite of the intention.

Short Title : Sudden Death in Football

Corresponding Author: Anurag N Banerjee
Department of Economics
University of Southampton
Southampton
UK
Email : A.N.Banerjee@soton.ac.uk
JEL Codes: C72, L83

1 Introduction

Football (also referred to as "soccer") is the world's most popular sport. It is played around the globe and the final game of the world championship in France 1998 between France and Brazil was watched by hundreds of millions of people.

Despite (or because) of the popularity of the game and its increasing commercial success, officials of football associations have worried about its attractiveness. In an effort to stimulate more attractive football, associations, including the European football association UEFA and the international football association FIFA, have introduced several new rules over the past decades. Recent rule changes are the increase in the number of points (3 instead of 2) for a win and the rule forbidding the goalkeeper to pick up the ball with his hands if it is passed back to him by one of his teammates.

The most recent rule change is the introduction of the "sudden death" or "golden goal" rule for games going into overtime. The sudden death rule stipulates that in overtime, the team which scores a goal wins the game (and the game ends "suddenly" with the scoring of the goal).

The rule was first applied in international tournaments for national teams with players below 21 years. In 1996 the sudden death rule was applied for the first time in a European Championship for national teams (England '96) and in 1998 it was first introduced in a World Championship tournament (France '98). For club championships, the European football association first introduced it during the final game of its UEFA Cup in 1998 (Inter Milan - Lazio Rome in Paris).

The impact of these changes are not always clear. Intuitively, one would expect the increase in the number of points for games won and the prohibition of passing back to the goalie to stimulate offensive play and to complicate defensive play. However, analysing the number of goals per match in England between 1888 and 1996, Palacios-Huertas (1998) shows that these rule changes affected the variance of the number of goals, but not its mean.

In the case of the sudden death rule, even the intuitive benefit is not clear. First, while it obviously raises the benefits of scoring a goal in overtime (and thus presumably could stimulate more offensive play) it simultaneously increases the costs of receiving a goal (and thus presumably could stimulate more defensive play). Second, football fans will admit that the time left after one team scores a goal in overtime is often a very exciting part of the game, with one team forced to play very offensive football to try to even the score.

In this paper we analyse under which conditions, if any, the introduction of the sudden death rule improves the attractiveness of the football game. Our theoretical results indicate that the new rule will change the behavior of the teams, but not necessarily in the way intended. It may stimulate more offensive play during extra times, but only if a team considers itself having a "comparative advantage" in offensive play vis-a-vis playing defensively. In other cases it will induce more defensive playing. Empirical evidence suggests that both may occur, but that the latter may be the more common situation and that the introduction of the sudden death rule have, on average, induced more defensive soccer.

The paper is organized as follows. In section 2 we generate the theoretical setup of the model and state our assumptions. In section 3, we derive the ex-ante end of play utility under the old and the new rule. In section 4, we analyse the optimal strategies for the teams and how the change in rules has affected them. In section 5 we review empirical evidence.

2 Theoretical Setup of the model

Football games in knock-out tournaments go into extra times when the score is tied after 90 minutes of play. Under the old rule, the extra time given is 30 minutes. If the score is still tied after 30 minutes, the game is settled by a penalty shoot-out.

We divide the 30 minutes of extra time into an arbitrary T discrete time points (say every minute is one point). We assume that the teams can score only one goal at these time points (t) and $T > 1$.

We have two teams: team A and team B. We shall describe the game from the point of view of team A. At every t, the state of the game from for team A is described by the random state vector X_t, where

$$X_t = \begin{cases} 1 & \text{, if team A scores a goal, at time } t, \\ 0 & \text{, if neither team scores a goal, at time } t, \\ -1 & \text{, if team B scores a goal, at time } t. \end{cases}$$

The probabilities of this random variable are given by

$$P(X_t = 1) = p,\ P(X_t = -1) = q$$
$$P(X_t = 0) = r = 1 - p - q$$

The probability of team A scoring a goal is the same as the probability of team B conceding a goal. This implies that the state vector of team B is $-X_t$. At every instant t there is a zero sum game between team A and team B. The probabilities p and q are functions of the strategies of the teams.

We assume that the teams can choose between two strategies only: a defensive strategy, denoted by L, and an offensive strategy, denoted by H. We define the strategy sets of the two teams at a given moment as S^A and S^B, where

$$S^i = \{H, L\},\ i = A, B.$$

The probability functions p and q are defined as

$$p : S^A \times S^B \to [0,1] \quad \text{and} \quad q : S^A \times S^B \to [0,1]\,.$$

Assumption (Increasing probability of scoring):

Throughout this paper we shall assume that the probability of scoring is higher with offensive football than with a defensive strategy. Formally:

$$p\,(H,.) > p\,(L,.) \ \text{ and } q\,(.,H) > q\,(.,L) \tag{1}$$

We define the ex-ante strategy sets of the two teams for the entire game as $\mathbf{S}^A$ and $\mathbf{S}^B$, where

$$\mathbf{S}^i = S^i_1 \times \ldots \times S^i_T,\ i = A, B. \tag{2}$$

An element of $\mathbf{S}^i$is the vector $\underline{s}^i$=$(s^i_1, \ldots, s^i_t, \ldots, s^i_T)$ where $s^i_t = H$ or L, $t = 1, \ldots, T$.

3 Comparing ex-ante utilities of the teams

As stated in the introduction our motivation is to study the behaviour/strategies of the teams under the two different policy regimes. We will do so by comparing the ex-ante pay-offs of the teams. Since in the new rule the game is played only when the scoreline is 0-0 it makes sense only to compare the expected payoff of the teams when the score is 0-0 (the state of the game). For the other states it makes no sense to compare the team behaviour / strategy since the game ends under the new rule when a goal is scored.

3.1 The Old Rule.

The old rule states that, after the specified time period of T , the team which has a positive goal difference is the winner. If there is no goal difference, the game goes to a penalty shoot-out. We define a random variable Z_t, the goal difference (from team A 's point of view) at time t. This stochastic process is a Markov chain (more precisely a random walk on integers),

$$Z_t = Z_{t-1} + X_t, \quad t = 1, \ldots, T.$$

The transition probabilities of the stochastic process are

$$P\left(Z_t = d' | Z_{t-1} = d\right) = \begin{cases} p & \text{if } d' = d+1 \\ r & \text{if } d' = d \\ q & \text{if } d' = d-1 \\ 0 & \text{otherwise} \end{cases}.$$

Definition 1 *The end of play utility U_z of team A is defined as*

$$U_z = I\left\{Z_T > 0\right\} - I\left\{Z_T < 0\right\}$$

where $I\{.\}$ is the indicator function.

Team B's end of play utility will be $-U_z$, which simply means that at the end of the game if team A scores more goals than team B, team A wins and otherwise team B wins.

Definition 2 *The ex-ante utility V_z of team A is defined as*

$$V_z\left(\underline{s}^A, s^B\right) \doteq E\left(U_z | Z_0 = 0\right), \tag{3}$$

where $\left(\underline{s}^A, \underline{s}^B\right)$ is the strategy pair decided by team A and B.

For team B the ex-ante utility is $-V_z\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B\right)$.

Using the following notation:

$$P\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B \middle| S\right) = \prod_{t \in S} p\left(s_t^A, s_t^B\right)$$
$$Q\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B \middle| S\right) = \prod_{t \in S} q\left(s_t^A, s_t^B\right)$$
$$R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B \middle| S\right) = \prod_{t \in S} r\left(s_t^A, s_t^B\right)$$

we can derive the following on the ex-ante utilities of the teams, summarised in theorem 1 (see Appendix for Proof).

Theorem 1 *Under the old rule*

a)

$$V_z\left(\underline{s}^A,\underline{s}^B\right)=E\left(U|Z_0=0\right)=$$
$$\sum_{d=1}^{T}\sum_{T_1,T_2}R\left(\underline{s}^A,\underline{s}^B\right|N/T_1\cup T_2\left)D\left(\underline{s}^A,\underline{s}^B\right|T_1,T_2\right)$$

where

$$D\left(\underline{s}^A,\underline{s}\right|T_1,T_2\left)=P\left(\underline{s}^A,\underline{s}\right|T_1\right)Q\left(\underline{s}^A,\underline{s}\right|T_2\left)-Q\left(\underline{s}^A,\underline{s}\right|T_1\right)P\left(\underline{s}^A,\underline{s}\right|T_2\right)$$

and T_1,T_2 , $N/T_1\cup T_2$ *is a partition of* $N=\{1,\ldots,T\}$ *such that* $|T_1|-|T_2|=d$.

b) In case of constant strategies, $\left(s_t^A,s_t^B\right)=\left(s^A,s^B\right)$ *for all* t,

$$\begin{aligned}V_z\left(\underline{s}^A,\underline{s}^B\right)&=E\left(U|Z_0=0\right)\\&=\sum_{d=1}^{T}\sum_{k=0}^{\left[\frac{T-d}{2}\right]}\frac{T!}{k!k+d!T-d-2k!}r^{T-d-2k}\left(pq\right)^k\left(p^d-q^d\right)\end{aligned}$$

where $p=p\left(s^A,s^B\right)$, $q=q\left(s^A,s^B\right)$ *and* $r=r\left(s^A,s^B\right)$.

Polominio (1999) analyses the game of football in general (similar to the extra-time game under the old rule) in a dynamic setup. They derive three properties, when attack is effective (same as our Assumption 7).

- Teams with equal score attack
- Winning team defends and the other attack
- A loosing team is more likely to attack

  We are not interested in the dynamic setup as the game under the new rule will end when a team will score also ex-ante we do not know when the teams will score. So the last to properties of the dynamic equilibrium will not happen under the new rule hence there is no point in comparing the dynamic game equlilibrium with the equlibrium of the "golden goal rule".

3.2 The New Rule

Under the new rule the team which first scores a goal within the 30 minutes of extra time wins the game. If neither scores the game goes on to penalty shoot-outs, as under the old rule. This rule does not give a chance for the other team to come back during the extra time. In terms of the stochastic process of goal difference this is similar to a random walk with absorbing barriers at 1 and -1.

In the new rule the game is played according to a Markov stochastic process W_t, which has the following transition probabilities

$$P(W_t = d' | W_{t-1} = d) = \begin{cases} 1 & \text{if } d' = d \neq 0 \\ p & \text{if } d' = +1 \text{ and } d = 0 \\ r & \text{if } d' = d = 0 \\ q & \text{if } d' = -1 \text{ and } d = 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Definition 3 *The end of play the utility U_w of Team A is defined as,*

$$U_w = I\{W_T > 0\} - I\{W_T < 0\}$$

where $I\{.\}$ is the indicator function.

Definition 4 *Under the new rule, the ex-ante utility of team A is*

$$V_w\left(s^A, s^B\right) \doteq E\left(U_w | W_0 = 0\right), \tag{5}$$

We can then derive the ex-ante utility under the new rule as:

Theorem 2 *Under the new rule*

a)

$$\begin{aligned} V_w\left(\underline{s}^A, \underline{s}^B\right) &= E\left(W_T | W_0 = 0\right) \\ &= \sum_{t=1}^{T} R\left(\underline{s}^A, \underline{s}^B \middle| S_{t-1}\right)\left(p\left(s_t^A, s_t^B\right) - q\left(s_t^A, s_t^B\right)\right) \end{aligned}$$

where $S_{t-1} = \{1, \ldots, t-1\}$

b) In case of a constant strategy $\left(s_t^A, s_t^B\right) = \left(s^A, s^B\right)$ for all t,

$$E\left(W_T | W_0 = 0\right) = (p - q)\ \frac{\left(1 - r^T\right)}{(1 - r)}$$

where $p = p\left(s^A, s^B\right)$, $q = q\left(s^A, s^B\right)$ and $r = r\left(s^A, s^B\right)$.

3.3 Optimal team strategies.

Team A will try to maximise its own objective, that is to win the game, by choosing a strategy sequence that maximises the ex-ante payoff. Team B's optimal choice is a strategy sequence that minimises the ex-ante payoff. For both teams, the optimal choice of the strategy sequence will be a convex combination of the strategies in the strategy sets $\underline{s}^i$, $i = A, B$. To compare the equilibrium strategy sequences under the old and the new rule, we make an additional assumption.

Assumption (Equality): We assume that the teams which are playing are equally likely to score under similar situations and strategies. This is a reasonable assumption given the fact that a game between two teams is more likely to go to extra times if the teams are of similar qualities. (A similar assumption is made by Palomino et al. (1999)). Formally,

Definition 5 *Teams A and B are defined to be equal if and only if*

$$p\left(s^A, s^B\right) = q\left(s^B, s^A\right). \tag{6}$$

for all $\left(s^A, s^B\right) \in \mathbf{S}^A \times \mathbf{S}^B$.

Theorem 3 *Under the old rule, if team A maximises (team B minimises) end of play utility,*

a) the value of the game is 0, and

b) any symmetric pair of statigies $(\underline{s}, \underline{s})$ *is an equilibrium.*

This theorem 3 (proof: see appendix) implies that, under the old rule, in equilibrium the optimal strategy sequences chosen by the teams are any symmetric pair of strategy sequences. However this is no longer the case under the new rule. There the equilibrium strategy will depend on whether the teams have a "comparative advantage" in playing offensively or defensively.

This comparative advantage is defined as follows. A team has a comparative advantage in playing offensive football (i.e. "attack is effective") if the team is more likely to score playing an offensive strategy against a defending team of equal quality compared to when it plays defensively against an equal quality team playing offensively. Formally,

Definition 6 *Team A has a comparative advantage in playing offensive football if and only if*

$$p\left(H, L\right) > p\left(L, H\right) \tag{7}$$

Inversely, a team has a comparative advantage in playing defensive football if the team is less likely to score playing an offensive strategy against a defending team of equal quality compared to when it plays defensively against an equal quality team playing offensively. That is,

$$p(H, L) < p(L, H) \tag{8}$$

Empirically, an (extreme) example of teams with a comparative advantage in defensive football are teams specialized in playing "counter-attack". Such team strategies have most of the players to play defensively, preventing the other team from scoring, with just a few offensive players, specialized in scoring on fast outbreaks from the defensive strategy. However, also other teams may consider defense to be more effective than offense.

Theorem 4 *Under the new rule, if team A maximises (team B minimises) end of play utility,*

a) the value of the game is 0,

b) if teams have a comparative advantage in playing offensive football then $(\underset{\sim}{H}, \underset{\sim}{H})$ *is the only equilibrium,*

c) otherwise $(\underset{\sim}{L}, \underset{\sim}{L})$ *is the only equilibrium*

where $\underset{\sim}{H}$ *is a* $n-$*vector of* $H's$ *and* $\underset{\sim}{L}$ *is a n-vector of* $L's$

Theorem 4 (proof: see appendix) states that under the new rule the optimum strategy is still a symmetric equilibrium, but much more restricted than under the old rule. Depending on the comparative advantage of the teams there is only one equilibrium. Since both teams are assumed to have similar qualities, they have the same comparative advantage – by definition. Theorem 4 implies that teams will at all times opt for the strategy in which they have a comparative advantage. The reason is that since there will be no more "second chance", i.e. an opportunity to score after the other team has scored, that both teams will at all times specialize in the strategy in which they have a comparative advantage.

This implies that the impact of the change in the rule on the likelihood that both teams will play offensively, depends on the comparative advantage of the team. In case the teams have an advantage in offensive play, the result will be more offensive play under the new rule. However in case that the teams consider themselves as having an advantage in defensive play, they will play more defensively under the new rule. In the latter case the impact of the new rule is opposite to what it intended.

The extreme example being teams specialised in "counter attack". Since both teams are specialised in counter attack, they wait so that their opponents attacks them, whereby they can go into counter attack mode and score the winning goal. The game in the mean time gets boring since neither of the team would give the other the advantage of mounting a counter attack , the spectators only see the two playing the defensive waiting game.

In addition, all the offensive play and excitement which normally results after a goal was scored in extra-times is lost under the new rule – independent of the strategy choice before one team scored. This is independent of the comparative advantage of the teams since each team is more likely to score playing offensively, hence it will play offensively after the other team has scored. This "dynamic equilibrium" is shown in a regular game framework by Palomino (1999).

4 Empirical evidence

Since the impact of the rule change is conditional in the theory, let us take a look at what the empirically evidence suggests. The empirical evidence so far is limited for drawing any strong conclusions. However, at the very least, it does not support the case that the sudden death rule has stimulated more attractive soccer.

After the introduction of the golden goal, two major international tournaments were played. During the 1996 European Championship (EC) tournament in England, 5 games went into extra times and 4 ended scoreless, still requiring penalty kicks to decide. Only in the final game a goal by Bierhoff, 5 minutes into extra times, secured the game (and the title) for Germany against the Czech Republic. Similarly, during the 1998 World Championship tournament in France, only in 1 of 4 games that went into extra times a goal was scored. However in the most recent EURO 2000 tournament two out of three games that went into extra-time were decided by a golden goal.

The data in Table 1 suggests that the introduction of the golden goal rule has, on average, reduced the likelihood of a goal being scored in extra times (34% compared to 47% before the introduction of Golden goal rule). During the 3 World Championship (WC) tournaments before the golden goal rule was introduced, in 40-50% of the extra times at least one goal was scored, almost double the percentage of the WC 1998 and the EC 1996 tournament. However, this percentage was considerably higher (67%) during Euro 2000, a tournament generally characterised by more offensive soccer. This observation may be consistant with our result that the golden goal rule will be more likely to lead to more offensive play, and hence presumeably more goals, with teams holding a comparative advantage in offensive play.

Finally, as mentioned before, the sudden death rule cancels the exciting part of the extra time after one goal is scored. Table 1 indicates that the likelihood that a second goal was scored in extra time under the old rule was almost as high on average (26 %) than a single goal being scored under the new rule (34%).

$[Table 1]$

5 Conclusion

In this article we have analysed the effect of the change of the football rule in the extra time play. Under the old regime the rule was that the two teams will play 30 minutes during the extra time and the team which scores most goals wins. With the introduction of the "golden goal" or "sudden death" rule, the team which scores first wins. To compare the change of behaviour of the teams, we looked at the ex-ante pay-offs of the teams when the score line is 0-0. This is done since the game would stop under the new rule if either of the teams score a goal.

Our theoretical analysis shows that under the new rule the optimum strategy for both teams is to opt for the strategy in which they have a 'comparative advantage'. The reason is that since there will be no more "second chance", i.e. an opportunity to score after the other team has scored, so both teams will at all times specialize in their comparative advantage strategy. The impact of the change in the rule on the likelihood that both teams will play offensively, therefore depends on the characteristics and qualities of the teams in the game. In case the teams have an advantage in offensive play, the result will be more offensive play under the new rule. However in case that the teams consider themselves as having an advantage in defensive play, they will play more defensively under the new rule. In the latter case the impact of the new rule is opposite to what it intended.

The empirical evidence is limited because only few tournaments were played since the introduction of the rule. The evidence suggests that both cases may have occured, although more defensive play seems to be the more common case. The data do not support the case that on average the sudden death rule has stimulated more attractive soccer. To the contrary, comparing the most important international football tournaments after the introducton of the sudden death rule with the most important tournaments before, yields that both the number of goals scored in extra times as the likelihood that a goal is scored during extra-times has decreased. These data suggest that the introduction of the golden goal rule has reduced the likelihood of a goal being scored in extra times, although the opposite may occur with teams having stronger offensive qualities.

In addition, all the offensive play and excitement which resulted under the old rule after a goal was scored in extra-times is lost under the new rule. This is independent of the comparative advantage of the teams since any team will play offensively after the other team has scored. In fact, the data indicate that the likelihood that a second goal was scored in extra time under the old rule was almost as high than a single goal being scored under the new rule.

6 References

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7 Appendix: Proof of Results

Proof of Theorem 1 a) By definition it follows that

$$\begin{aligned} E\left(U_z|Z_0=0\right) &= \Pr\left(Z_T>0|Z_0=0\right)-\Pr\left(Z_T<0|Z_0=0\right) \\ &= \sum_{d=1}^{T}\Pr\left(Z_T=d|Z_0=0\right)-\Pr\left(Z_T=-d|Z_0=0\right) \end{aligned}$$

Define a partition of $N=\{1,\ldots,T\}$ as $T_1=\{t:X_t=1\}, T_2=\{t:X_t=-1\}$ and $N/T_1\cup T_2=\{t:X_t=0\}$. Then $|T_1|$ is the number of goals scored by team A and $|T_2|$ is the number of goals scored by team B. So if $|T_1|-|T_2|=d$, team A wins by d goals. Notice that T_1,T_2 and $N/T_1\cup T_2$, is an arbitary partition of N, so team A can win by d goals with any such parition as long as $|T_1|-|T_2|=d$.

Let

$$\pi\left(T_1,T_2\right)=\Pr\left(\begin{array}{l}\{X_t:t\in T_1\}, \\ \{X_t:t\in T_2\}, \\ \{X_t:t\in N/T_1\cup T_2\}\end{array}\right)$$

be the probability that team A scores at times $t\in T_1$ and team B scores (team A conceeds a goal) at times $t\in T_2$. Hence

$$\begin{aligned} \Pr\left(Z_T=d|Z_0=0\right) &= \sum_{k=0}^{\left[\frac{T-d}{2}\right]}\sum_{T_1:|T_1|=k+d}\sum_{T_2:|T_2|=k}\pi\left(T_1,T_2\right) \\ &= \sum_{T_1,T_2:|T_1|-|T_2|=d}\pi\left(T_1,T_2\right) \end{aligned}$$

where T_1,T_2 and $N/T_1\cup T_2$ is a partition of N. Also notice that

$$\pi\left(T_1,T_2\right)=P\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)R\left(\underline{s}^A,\underline{s}^B\middle|\,N/T_1\cup T_2\right)$$

Therefore

$$\begin{aligned} &\Pr\left(Z_T=d|Z_0=0\right) \\ =& \sum_{T_1,T_2:|T_1|-|T_2|=d}R\left(\underline{s}^A,\underline{s}^B\middle|\,N/T_1\cup T_2\right)P\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right) \end{aligned}$$

Similarly (notice that team A now loses by d goals)

$$\begin{aligned} &\Pr\left(Z_T=-d|Z_0=0\right) \\ =& \sum_{T_1,T_2:|T_1|-|T_2|=d}R\left(\underline{s}^A,\underline{s}^B\middle|\,N/T_1\cup T_2\right)P\left(\underline{s}^A,\underline{s}^B\middle|\,T_2\right)Q\left(\underline{s}^A,\underline{s}^B\middle|\,T_1\right) \end{aligned}$$

Therefore

$$\sum_{d=1}^{T} \Pr\left(Z_T = d | Z_0 = 0\right) - \Pr\left(Z_T = -d | Z_0 = 0\right)$$
$$= \sum_{T_1, T_2 : |T_1| - |T_2| = d} R\left(\underline{s}^A, \underline{s}^B \middle| N / T_1 \cup T_2\right) D\left(\underline{s}^A, \underline{s}^B \middle| T_1, T_2\right)$$

b) Notice that given integers d and $k = 0, \dots, \left[\frac{T-d}{2}\right]$ we have $\frac{T!}{k!k+d!T-d-2k!}$ possible partitions. Therefore,

$$\Pr\left(Z_T = d | Z_0 = 0\right)$$
$$= \sum_{k=0}^{\left[\frac{T-d}{2}\right]} \frac{T!}{k!k+d!T-d-2k!} r\left(s^A, s^B\right)^{T-2k+d} q\left(s^A, s^B\right)^{k} p\left(s^A, s^B\right)^{k+d}$$
$$\Pr\left(Z_T = -d | Z_0 = 0\right)$$
$$= \sum_{k=0}^{\left[\frac{T-d}{2}\right]} \frac{T!}{k!k+d!T-d-2k!} r\left(s^A, s^B\right)^{T-2k+d} q\left(s^A, s^B\right)^{k+d} p\left(s^A, s^B\right)^{k}$$

This implies that in case of constant strategies, $\left(s_t^A, s_t^B\right) = \left(s^A, s^B\right)$ for all t,

$$E\left(U | Z_0 = 0\right) = \sum_{d=1}^{T} \sum_{k=0}^{\left[\frac{T-d}{2}\right]} \frac{T!}{k!k+d!T-d-2k!} r^{T-d-2k} (pq)^k \left(p^d - q^d\right)$$

where $p = p\left(s^A, s^B\right)$, $q = q\left(s^A, s^B\right)$ and $r = r\left(s^A, s^B\right)$.

Proof of theorem 2: a) By definition it implies that,

$$E\left(U_w | W_0 = 0\right) = \Pr\left(W_T = 1 | W_0 = 0\right) - \Pr\left(W_T = -1 | W_0 = 0\right)$$

Notice that,

$$\Pr\left(W_T = 1 | W_0 = 0\right) = \sum_{t=1}^{T} \Pr\left(X_t = 1,\ X_{t'} = 0\ \forall\ t' < t\right)$$
$$= \sum_{t=1}^{T} \Pr\left(X_t = 1\right) \prod_{t' \in S_{t-1}} \Pr\left(\ X_{t'} = 0\right)$$
$$= \sum_{t=1}^{T} p\left(s_t^A, s_t^B\right) R\left(\underline{s}^A, \underline{s}^B \middle| S_{t-1}\right) \tag{9}$$

Similarly for

$$\Pr\left(W_T=-1|W_0=0\right)=\sum_{t=1}^{T} q\left(s_t^A,s_t^B\right) R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{s}}^B\middle| S_{t-1}\right). \tag{10}$$

Combining (9) and (10) we get

$$E\left(U_w|W_0=0\right)=\sum_{t=1}^{T}\left(p\left(s_t^A,s_t^B\right)-q\left(s_t^A,s_t^B\right)\right) R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{s}}^B\middle| S_{t-1}\right).$$

b) If $\left(s_t^A,s_t^B\right)=\left(s^A,s^B\right)$ then

$$\begin{aligned}
&\sum_{t=1}^{T}\left(p\left(s_t^A,s_t^B\right)-q\left(s_t^A,s_t^B\right)\right) R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{s}}^B\middle| S_{t-1}\right)\\
&=\left(p\left(s^A,s^B\right)-q\left(s^A,s^B\right)\right)\sum_{t=1}^{T} r\left(s^A,s^B\right)^{t-1}\\
&=\left(p\left(s^A,s^B\right)-q\left(s^A,s^B\right)\right)\frac{1-r\left(s^A,s^B\right)^T}{1-r\left(s^A,s^B\right)}.
\end{aligned}$$

Lemma 1 *Under the assumption that teams are equal we have*

i) $R\left(\underline{s}^A,\underline{s}^B\middle| T\right)=R\left(\underline{s}^B,\underline{s}^A\middle| T\right)$ *for all* $T\subset N$

ii) $Q\left(\underline{s}^A,\underline{s}^B\middle| T\right)=P\left(\underline{s}^B,\underline{s}^A\middle| T\right)$ *for all* $T\subset N$

iii) $p\left(s^A,s^B\right)-q\left(s^A,s^B\right)=-\left(p\left(s^B,s^A\right)-q\left(s^B,s^A\right)\right)$

Proof of Lemma 1 :

i)

$$\begin{aligned}
R\left(\underline{\mathrm{s}}^A,\underline{\mathrm{s}}^B\middle| T\right) &= \prod_{t\in T} r\left(s_t^A,s_t^B\right)=\prod_{t\in T}\left(1-p\left(s_t^A,s_t^B\right)-q\left(s_t^A,s_t^B\right)\right)\\
&= \prod_{t\in T}\left(1-q\left(s_t^B,s_t^A\right)-p\left(s_t^B,s_t^A\right)\right)\ \ (by\ (6))\\
&= R\left(\underline{\mathrm{s}}^B,\underline{\mathrm{s}}^A\middle| T\right)
\end{aligned}$$

ii)

$$\begin{aligned}
Q\left(\underline{\mathrm{s}}^A,\underline{\mathrm{s}}^B\middle| T\right) &= \prod_{t\in T} q\left(s_t^A,s_t^B\right)=\prod_{t\in T} p\left(s_t^A,s_t^B\right),\ \ (by\ (6))\\
&= P\left(\underline{\mathrm{s}}^B,\underline{\mathrm{s}}^A\middle| T\right)
\end{aligned}$$

iii)

$$p\left(s^A,s^B\right)-q\left(s^A,s^B\right)=\left(q\left(s^B,s^A\right)-p\left(s^B,s^A\right)\right)\ \ (by\ (6))$$

Proof of Theorem 3: Notice that using theorem (1) and Lemma (1), we have

$$V_z\left(\underline{s}^A, \underline{s}^B\right) = -V_z\left(\underline{s}^B, \underline{s}^A\right) \tag{11}$$

The payoff matrix is skew-symmetric, where the vertices are all the sequences of the H and L as defined in (2)

a) Since the payoff matrix is skew-symmetric the value of the game is 0 (See Owen 1995, page 29) and any equilibrium strategy will be of the form $\underline{s}^B = \underline{s}^A = \underline{s}$

b) We shall show that $(\underline{s}, \underline{s})$ is a pure strategy equilibrium for any $\underline{s}$. Let $s_{t_0} = H$ without loss of generality. Let $\underline{s}^A$ be the strategy vector with $s^A_{t_0} = L$ and $s^A_t = s_t$ $\forall\ t \neq t_0$ Team A will then deviate at time t_0, if

$$V_z\left(\underline{s}^A, \underline{s}\right) - V_z\left(\underline{s}, \underline{s}\right) > 0 \tag{12}$$

From (11) we know $V_z\left(\underline{s}, \underline{s}\right) = 0$, for any $\underline{s}$. So team A deviates if

$$V_z\left(\underline{s}^A, \underline{s}\right) > 0$$

Notice that for any given partition of N, T_1, T_2 and $N/T_1 \cup T_2$,

$$\begin{aligned}
P\left(\underline{s}^A, \underline{s}\middle| T_1\right) &= P\left(\underline{s}, \underline{s}\middle| T_1 - \{t_0\}\right) p\left(L, H\right) \text{ if } t_0 \in T_1 && (13)\\
&= P\left(\underline{s}, \underline{s}\middle| T_1\right) \text{ if } t_0 \notin T_1 \\
Q\left(\underline{s}^A, \underline{s}\middle| T_2\right) &= P\left(\underline{s}, \underline{s}\middle| T_2 - \{t_0\}\right) q\left(L, H\right) \text{if } t_0 \in T_2 && (14)\\
&= P\left(\underline{s}, \underline{s}\middle| T_2\right) \text{ if } t_0 \notin T_2 \\
R\left(\underline{s}^A, \underline{s}\middle| N/T_1 \cup T_2\right) &= R\left(\underline{s}, \underline{s}\middle| N/T_1 \cup T_2\right) \text{ if } t_0 \in T_1 \cup T_2 && (15)\\
&= R\left(\underline{s}, \underline{s}\middle| N/T_1 \cup T_2 - \{t_0\}\right) r\left(L, H\right) \text{ if } t_0 \in N/T_1 \cup T_2
\end{aligned}$$

Therefore using (15), (13) and (14) we have

$$\begin{aligned}
D\left(\underline{s}^A, \underline{s}\middle| T_1, T_2\right) &= P\left(\underline{s}^A, \underline{s}\middle| T_1\right) Q\left(\underline{s}^A, \underline{s}\middle| T_2\right) - Q\left(\underline{s}^A, \underline{s}\middle| T_1\right) P\left(\underline{s}^A, \underline{s}\middle| T_2\right) \\
D\left(\underline{s}^A, \underline{s}\middle| T_1, T_2\right) &= \begin{cases}
\begin{array}{c} P\left(\underline{s}, \underline{s}\middle| T_1 - \{t_0\}\right) p\left(L, H\right) Q\left(\underline{s}, \underline{s}\middle| T_2\right) \\ -Q\left(\underline{s}, \underline{s}\middle| T_1 - \{t_0\}\right) q\left(L, H\right) P\left(\underline{s}, \underline{s}\middle| T_2\right) \end{array} & \text{if } t_0 \in T_1 \\
\\
\begin{array}{c} P\left(\underline{s}, \underline{s}\middle| T_1\right) Q\left(\underline{s}, \underline{s}\middle| T_2 - \{t_0\}\right) q\left(L, H\right) \\ -Q\left(\underline{s}, \underline{s}\middle| T_1\right) P\left(\underline{s}, \underline{s}\middle| T_2 - \{t_0\}\right) p\left(L, H\right) \end{array} & \text{if } t_0 \in T_2
\end{cases}
\end{aligned}$$

By the lemma (1) we have

$$D\left(\underline{s}^A,\underline{s}\middle|T_1,T_2\right) = \begin{cases} \begin{array}{c} P\left(\underline{s},\underline{s}\middle|T_1-\{t_0\}\right)P\left(\underline{s},\underline{s}\middle|T_2\right)p\left(L,H\right) \\ -P\left(\underline{s},\underline{s}\middle|T_1-\{t_0\}\right)P\left(\underline{s},\underline{s}\middle|T_2\right)q\left(L,H\right) \end{array} & \text{if } t_0\in T_1 \\ \\ \begin{array}{c} P\left(\underline{s},\underline{s}\middle|T_1\right)P\left(\underline{s},\underline{s}\middle|T_2-\{t_0\}\right)q\left(L,H\right) \\ -P\left(\underline{s},\underline{s}\middle|T_1\right)P\left(\underline{s},\underline{s}\middle|T_2-\{t_0\}\right)p\left(L,H\right) \end{array} & \text{if } t_0\in T_2 \end{cases}$$

$$= \begin{cases} P\left(\underline{s},\underline{s}\middle|T_1\cup T_2-\{t_0\}\right)\left(p\left(L,H\right)-q\left(L,H\right)\right) \text{ if } t_0\in T_1 \\ P\left(\underline{s},\underline{s}\middle|T_1\cup T_2-\{t_0\}\right)\left(q\left(L,H\right)-p\left(L,H\right)\right) \text{ if } t_0\in T_2 \end{cases}$$

Also notice that if $t_0\in N/T_1\cup T_2$, using lemma (1) and (15), (13)

$$D\left(\underline{s}^A,\underline{s}\middle|T_1,T_2\right)=0$$

Let us now calculate $V_z\left(\underline{s}^A,\underline{s}\right)$.

$$V_z\left(\underline{s}^A,\underline{s}\right)=\sum_{d=1}^{T}\sum_{T_1,T_2:|T_1|-|T_2|=d} R\left(\underline{s}^A,\underline{s}\middle|N/T_1\cup T_2\right)D\left(\underline{s}^A,\underline{s}\middle|T_1,T_2\right)$$

We split the the inner sum (the sum over partitions) into three parts depending on which part t_0 belongs (since $T_1,T_2$and $N/T_1\cup T_2$is a partition t_0 has to be only one of the three parts)

$$\begin{aligned} V_z\left(\underline{s}^A,\underline{s}\right) &= \sum_{d=1}^{T}\sum_{t_0\in T_1} R\left(\underline{s}^A,\underline{s}\middle|N/T_1\cup T_2\right)D\left(\underline{s}^A,\underline{s}\middle|T_1,T_2\right) \\ &\quad+\sum_{t_0\in T_2} R\left(\underline{s}^A,\underline{s}\middle|N/T_1\cup T_2\right)D\left(\underline{s}^A,\underline{s}\middle|T_1,T_2\right) \\ &\quad+\sum_{t_0\in N/T_1\cup T_2} R\left(\underline{s}^A,\underline{s}\middle|N/T_1\cup T_2\right)D\left(\underline{s}^A,\underline{s}\middle|T_1,T_2\right) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{d=1}^{T} \sum_{t_0 \in T_1} R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| N/T_1 \cup T_2\right) D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}} \middle| T_1, T_2\right) \\
&\quad + \sum_{t_0 \in T_2} R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| N/T_1 \cup T_2\right) D\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}} \middle| T_1, T_2\right) \\
&= \sum_{d=1}^{T} \left\{ \sum_{t_0 \in T_1} R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| N/T_1 \cup T_2\right) P\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| T_1 \cup T_2 - \{t_0\}\right) \left(p\left(L,H\right) - q\left(L,H\right)\right) \right. \\
&\quad \left. + \sum_{t_0 \in T_2} R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| N/T_1 \cup T_2\right) P\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| T_1 \cup T_2 - \{t_0\}\right) \left(q\left(L,H\right) - p\left(L,H\right)\right) \right\} \\
&= \sum_{d=1}^{T} \sum_{t_0 \in T_1 \cup T_2} R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| N/T_1 \cup T_2\right) P\left(\underline{\mathrm{s}}, \underline{\mathrm{s}} \middle| T_1 \cup T_2 - \{t_0\}\right) \left\{ \begin{array}{c} \left(p\left(L,H\right) - q\left(L,H\right)\right) \\ + \left(q\left(L,H\right) - p\left(L,H\right)\right) \end{array} \right\} \\
&= 0
\end{aligned}
$$

Since there is no positive payoff due to deviation and therefore team A does not deviate. As this is a zero-sum game team B also does not deviate $(\underline{\mathrm{s}}, \underline{\mathrm{s}})$ is

If the teams want to deviate at multiple time points $t_1, , t_k$,there is also no positive payoff, the proof of which is similar. Therefore $(\underline{\mathrm{s}}, \underline{\mathrm{s}})$ are optimal strategies.

Proof of Theorem 4: Notice that using theorem (1) and Lemma (1), we have

$$
V_w\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}^B\right) = -V_w\left(\underline{\mathrm{s}}^B, \underline{\mathrm{s}}^A\right) \tag{16}
$$

The payoff matrix is a skew-symmetric, with the vertices being all the sequences of H and L as defined in (2)

a) Since the payoff matrix is a skew-symmetric matrix of the game is 0.(See Owen 1995, page 29) and any equilibrium strategy will be of the form $\underline{\mathrm{s}}^B = \underline{\mathrm{s}}^A = \underline{\mathrm{s}}$

b) Let $p(H,L) - p(L,H) = \alpha > 0$ and $\underline{\mathrm{H}}$=$(H, \ldots, H)$.We prove the statement in two parts

1) $(\underline{\mathrm{H}}, \underline{\mathrm{H}})$ is an equilibrium and

2) Any other pair of strategies $(\underline{\mathrm{s}}, \underline{\mathrm{s}})$ are not equilibrium strategies.

Let $\underline{\mathrm{s}}^A$ be the strategy vector with $s_{t_0}^A = L$ and $s_t^A = H \ \forall\ t \neq t_0$ Team A will then deviate at time t_0, if

$$
V_w\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right) - V_w\left(\underline{\mathrm{s}}, \underline{\mathrm{H}}\right) > 0 \tag{17}
$$

From (11) we know that $V_w(\underline{\mathrm{H}}, \underline{\mathrm{H}}) = 0$, for any $\underline{\mathrm{s}}$.. So team A deviates if

$$
V_w\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\right) > 0
$$

Notice that

$$\begin{aligned}
V_w\left(s^A, \underline{\mathrm{H}}\right) &= \sum_{t=1}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle| S_{t-1}\right)\left(p\left(s_t^A, H\right) - q\left(s_t^A, H\right)\right) \\
&= \sum_{t=1, t\neq t_0}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle| S_{t-1}\right)\left(p\left(H, H\right) - q\left(H, H\right)\right) \\
&+ R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle| S_{t_0-1}\right)\left(p\left(L, H\right) - q\left(L, H\right)\right) \\
&= R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle| S_{t_0-1}\right)\left(p\left(L, H\right) - q\left(L, H\right)\right) \text{ (by Symmetry)} \\
&= R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle| S_{t_0-1}\right)\left(p\left(L, H\right) - p\left(H, L\right)\right) \text{ (by Symmetry)} \\
&= -\alpha R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{H}}\middle| S_{t_0-1}\right)
\end{aligned}$$

So there is a negative payoff for deviation and therefore team A does not deviate. The same holds for team B. If the teams want to deviate at multiple time points $t_1, , t_k$,there is also a negative payoff, the proof of which is similar as above.

Hence ($\underline{\mathrm{H}}$,$\underline{\mathrm{H}}$) is an equilibrium.

Secondly, we need to prove the uniqueness of the equilibrium. As the payoff matrix is skew symmetric the there is asymmetric equilibrium. So any equilibrium must be of the form $(\underline{\mathrm{s}}, \underline{\mathrm{s}})$. We shall show that if $\underline{\mathrm{s}} \neq \underline{\mathrm{H}}$, there is a positive payoff for team A to deviate.

If $\underline{\mathrm{s}} \neq \underline{\mathrm{H}}$, then $\exists\ t_0\ s.t\ s_{t_0} = L$.

Let team A deviate at t_0, and plays H. Let $\underline{\mathrm{s}}^A$ be the strategy vector with $s_{t_0}^A = L$ and $s_t^A = s_t \forall\ t \neq t_0$. Then

$$\begin{aligned}
&V_w\left(s^A, \underline{\mathrm{s}}\right) \\
&= \sum_{t=1}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\middle| S_{t-1}\right)\left(p\left(s_t^A, s_t\right) - q\left(s_t^A, s_t\right)\right) \\
&= \sum_{t=1, t\neq t_0}^{T} R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\middle| S_{t-1}\right)\left(p\left(s_t, s_t\right) - q\left(s_t, s_t\right)\right) \\
&+ R\left(\underline{\mathrm{s}}^A, \underline{\mathrm{s}}\middle| S_{t_0-1}\right)\left(p\left(H, L\right) - q\left(H, L\right)\right) \\
&= 0 + R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}}\middle| S_{t_0-1}\right)\left(p\left(H, L\right) - p\left(L, H\right)\right) \text{ (by Lemma (1))} \\
&= \alpha R\left(\underline{\mathrm{s}}, \underline{\mathrm{s}}\middle| S_{t_0-1}\right) > 0
\end{aligned}$$

Hence team A will deviate. The same holds for team B. Therefore $(\underline{\mathrm{s}}, \underline{\mathrm{s}})$ is not an equilibrium strategy pair. Hence ($\underline{\mathrm{H}}$,$\underline{\mathrm{H}}$) is an unique equilibrium.

c) If $\alpha < 0$, then $(\underline{\mathrm{L}}, \underline{\mathrm{L}})$ will be the only equilibrium. Proof is similar as b)

8 Tables

Table 1: Scoring in extra-time (ET) games in recent World Championship (WC) and European Championship (EC) tournaments (*)

Tournament	Games in ET	Goals scored in ET	Games with goals scored in ET	Average number of goals per ET-games	ET-games with goals scored (% of total)	ET-games with more than 1 goal scored (% of total)
EC 2000	3	2	2	0.67	67	-
WC 1998	4	1	1	0.25	25	-
EC 1996	5	1	1	0.20	20	-
WC 1994	4	3	2	0.50	50	25
WC 1990	8	6	4	0.75	50	12.5
WC 1986	5	5	2	1.00	40	40

(*) WC tournaments had 16 knock-out games, the EC 1996, 2000 tournament 7; EC tournaments in 1992 and 1988 had only 3.