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To Each the Same and to Each His Own. A Proposal to  
Measure Responsibility-Sensitive Income Inequality

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# To Each the Same And to Each His Own. A Proposal to Measure Responsibility-Sensitive Income Inequality.

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## Abstract

This paper deals with the question: How can one incorporate modern responsibility-catering egalitarianism into the economic theory on the measurement of income inequality? We first concisely describe what is meant by responsibility-sensitive egalitarianism and present a particular axiomatic expression of its main aspects, as advocated by Bossert and Fleurbaey [1996]. We defend that only inequality due to factors beyond the scope of one's responsibility is ethically offensive. Traditional income inequality measurement, however, takes all inequality to be offensive. To measure offensive inequality separately we propose to construct a norm or reference income distribution based upon the axiomatic model to replace the perfectly equal income distribution which is used as norm by all common inequality measures. We then defend the use of a particular measure of distributional change to determine the degree of offensive inequality. Finally, we demonstrate how the method works by applying it to Belgian income data.

**J.E.L.:** D31, D63.

**Keywords:** Inequality measurement, distributional change, income distribution.

## 1. Introduction.

Income equality is a modern ideal that appeals to many people including economists and policy-makers. Apart from some mavericks like Babeuf and Marechal [1794], modern egalitarians (such as Rawls [1971], Dworkin [1981a, 1981b], Arneson [1989], Cohen [1989, 1990], and Fleurbaey [1995c, 1995d, 1998] among others) never defend an absolute or perfect income equality where all citizens always have the same income. Theorists usually allow for income differences stemming from differences in needs, effort or preferences. It implies that someone wasting money, for instance, should not be compensated, whereas someone working hard is allowed to reap the fruits of his effort. It means that many contemporary egalitarians conceive egalitarianism with respect to personal responsibility, thus, generating (different versions of) responsibility-sensitive egalitarianism. Within an economic context this means, in short, that income inequality for which people cannot be held responsible is offensive, whereas inequality stemming from actions for which they could be held responsible is not offensive and should not appear in the statistics as offensive income inequality<sup>1</sup>.

Unfortunately, traditional income inequality measures (like the Gini-coefficient or Generalized Entropy measures) neglect completely these allowances for differences in responsibility since they take the perfectly equal income distribution as reference point for measurement. Therefore, the main aim of this paper is to develop a method to measure only the offensive inequality, i.e. inequality for which people cannot be held responsible, based upon an alternative (responsibility-sensitive) norm income distribution. More specifically, we will use the theoretical framework of Bossert-Fleurbaey [1996] to capture the main features of the mentioned ethic and to design some adequate reference income distributions. This framework of (re-)distribution seems very suitable for our task since it differentiates between characteristics for which people can be held responsible and characteristics for which people should not bear responsibility. This distinction is a prerequisite for our objective to measure only inequality for which people cannot

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<sup>1</sup>An early attempt to include the intuition that some income inequalities are "spurious", not "offensive", not "inequitable" or even "economically functional" can be found in the proposals of Morton Paglin [1975, 1977, 1979, 1989]. For critical comments, see Danziger et al. [1977], Johnson [1977], Kurien [1977], Minarik [1977], Nelson [1977], Wertz [1979], Formby and Seaks [1980], Formby, Seaks and Smith [1989].

be held responsible.

The relevance of this exercise stretches even beyond that of adapting economic measurement to contemporary egalitarian thinking. Since policy-makers only have limited resources to reduce income inequality, they obviously should give priority to reduce the "worst" or "most offensive" inequality and may therefore be helped by a measure which measures exactly that.

The paper follows the structure of the method. First of all, a model is presented and our interpretation of responsibility-sensitive egalitarianism explained. From here we take it further and develop a norm income distribution which deviates from the perfectly equal one taking into account personal responsibility (section 2). In the third section, then, we present a way to measure the distance between the actual income distribution and the newly designed norm income distribution. Section 4 contains an empirical application of our method employing Belgian data and section 5 concludes.

## 2. A Responsibility-Sensitive Egalitarian Model.

The responsibility-sensitive egalitarian ethics as it appears in the literature may be summarized as follows: it is unjust if there is inequality of the *equalisandum* among individuals due to factors which are beyond the personal responsibility of the individuals; whereas it is considered as just if inequality is due to the exercise of personal responsibility. The entire normative debate about how to discern the factors which are within and beyond the ambit of individual responsibility is beyond the scope of this paper. Therefore the actual allocation of the responsibility cut, i.e., the divide between responsibility and compensation variables is taken for granted (or already decided upon by the policy maker). We should also add at this stage that for the remainder we will restrict our discussion to income distributions, thus taking income as the relevant *equalisandum*. Bossert and Fleurbaey (Bossert [1995], Bossert and Fleurbaey [1996]) have given concrete content to some responsibility-sensitive ideas in a quasi-linear model of income redistribution. Although designed for other purposes, we believe that their model and more specifically the axioms incorporated in it, may provide a basis for the measurement of offensive and inoffensive inequality. Let us first describe the model.

We denote the set of real numbers by  $\mathfrak{R}$ , and  $N$  is the set of positive integers. The population in a given society is given by  $K = \{1, \dots, n\}$  where  $n \geq 2$ . There are  $r \in N$  individual characteristics that are considered "responsibility" variables

and  $c \in N$  "compensation" variables. Agent  $i$ 's ( $i \in K$ ) vector of responsibility variables is  $a_i^R \in \mathfrak{R}^r$ , and  $i$ 's vector of compensation variables is described by  $a_i^C \in \mathfrak{R}^c$ . The characteristics vector of  $i \in K$  is  $a_i = (a_i^R, a_i^C) \in \mathfrak{R}^{r+c}$ . A characteristics profile is  $\bar{a} = (a_1, \dots, a_n) \in \mathfrak{R}^{n(r+c)}$ , and can be partitioned into  $\bar{a}^R = (a_1^R, \dots, a_n^R) \in \mathfrak{R}^{nr}$  and  $\bar{a}^C = (a_1^C, \dots, a_n^C) \in \mathfrak{R}^{nc}$ . The set of all characteristics vectors is  $\Omega \subseteq \Omega_R \times \Omega_C$ , where  $\Omega_R \subseteq \mathfrak{R}^r$ ,  $\Omega_C \subseteq \mathfrak{R}^c$ , and  $\Omega_R, \Omega_C \neq \emptyset$ . Reference vectors of responsibility and compensation characteristics are denoted by  $\tilde{a}^R \in \Omega_R$  and  $\tilde{a}^C \in \Omega_C$ , respectively.

An income function assigns a pre-tax income to each possible characteristics vector. That is, an income function is a mapping  $f : \Omega \rightarrow \mathfrak{R}$ ,  $a = (a^R, a^C) \mapsto f(a)$ . Therefore, pre-tax income is determined by both responsibility and compensation characteristics. The income function  $f$  is additively separable in responsibility and compensation variables if and only if there exist functions  $g : \Omega_R \rightarrow \mathfrak{R}$  and  $h : \Omega_C \rightarrow \mathfrak{R}$  such that

$$f(a) = g(a^R) + h(a^C) \quad \forall a \in \Omega \quad (2.1)$$

A redistribution mechanism is a mapping  $F : \Omega^n \rightarrow \mathfrak{R}^n$ ,  $\bar{a} \mapsto F(\bar{a})$ , such that

$$\sum_{i=1}^n F_i(\bar{a}) = \sum_{i=1}^n f(a_i) \quad \forall \bar{a} \in \Omega^n \quad (2.2)$$

This budget constraint implies that the redistribution does not lead to an efficiency loss, i.e. that we are considering a first-best problem.

We restrict ourselves to anonymous redistribution mechanisms, that is, mechanisms satisfying

$$\forall \bar{a} \in \Omega^n, \forall i, j \in K, a_i = a_j \Rightarrow F_i(\bar{a}) = F_j(\bar{a}) \quad (2.3)$$

A responsibility-sensitive egalitarian, however, will point to the possibility that a subset of the characteristics  $a_i$  is within the responsibility of individual  $i$ . Her first problem -locating the responsibility cut- then becomes how to partition the vector  $a_i$  in  $(a_i^R, a_i^C)$ . We mentioned already that we suppose that this problem has been settled. The second problem is then how this partitioning can be exploited to give a concrete content to the idea of responsibility-sensitivity.

Fleurbaey [1994, 1995a, 1995b, 1995c] and Bossert [1995] have modelled two basic intuitions in this respect. The first intuition basically reflects the egalitarian aspect of the approach. We call it *Equal Income For Equal r-variables* (EIER) or *Full Compensation* and it states that for all possible  $\bar{a} \in \Omega^n$ , for any two individuals, one should have

$$a_i^R = a_j^R \Rightarrow F_i(\bar{a}) = F_j(\bar{a}) \quad (2.4)$$

If two persons are identical on all characteristics for which they can be held responsible -if they only differ with respect to characteristics for which they must be compensated- then the redistribution mechanism must assign these two persons the same post-tax income.

The second intuition captures the idea of responsibility, i.e. of the limits to be imposed on egalitarianism. We call it *Equal Transfer for Equal c-variables* (ETEC) or *Strict Compensation* and it says that for all possible  $\bar{a} \in \Omega^n$ , for any two individuals, the redistribution mechanism must satisfy

$$a_i^C = a_j^C \Rightarrow F_i(\bar{a}) - f(a_i) = F_j(\bar{a}) - f(a_j) \quad (2.5)$$

If two persons have identical compensation characteristics, the differences in their pre-tax income will only reflect differences in their responsibility characteristics, and hence there is no reason why these differences should diminish through the redistribution process. Equation (2.5) formalizes this by imposing that these two persons should pay the same tax or receive the same transfer.

The main result of Fleurbaey is that the two intuitions of full compensation and strict compensation are in general incompatible if  $n \geq 4$ . In the context of the quasi-linear income redistribution model, there will only be a redistribution rule satisfying (2.4) and (2.5) for all possible  $\bar{a} \in \Omega^n$ , if  $f$  is additively separable in responsibility and compensation variables, i.e. if (2.1) obtains, in which case a natural redistribution mechanism is  $F^0$ , assigning to individual  $k$  the post-tax income

$$F_k^0(\bar{a}) = g(a_k^R) + \frac{1}{n} \sum_{i=1}^n h(a_i^C) \quad \forall \bar{a} \in \Omega^n, \quad \forall k \in K \quad (2.6)$$

If  $f$  is not additively separable, it is impossible to satisfy full and strict compensation at the same time. Bossert and Fleurbaey [1996] characterize several distribution mechanisms that satisfy a combination of one axiom with a weakened version of the other. If one wants to obtain full compensation (EIER) and is willing to loosen strict compensation<sup>2</sup> one may select a distribution scheme within

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<sup>2</sup>Fleurbaey and Bossert [1996] show very precisely which families of redistribution mechanisms remain for combinations of EIER with various weaker versions of strict compensation. A different 'weakening' of ETEC will lead to a different class of redistribution mechanisms. They do the same for combinations of ETEC and weaker versions of full compensation (see below).

the family of *egalitarian-equivalent mechanisms*  $F^{EE}$ , defined in general as:

$$F_k^{EE}(\bar{a}) = f(a_k^R, \tilde{a}^C) - \frac{1}{n} \sum_{i=1}^n [f(a_i^R, \tilde{a}^C) - f(a_i)] \quad \forall \bar{a} \in \Omega^n, \quad \forall k \in K \quad (2.7)$$

where  $\tilde{a}^C$  is a (freely chosen) benchmark vector. With this mechanism, every agent has a post-tax income equal to the pre-tax income she would earn if her compensation characteristics were  $\tilde{a}^C$ , plus a uniform transfer. If, on the other hand, one gives priority to strict compensation (ETEC) and is willing to deviate from full compensation, one may select within the family of *conditionally egalitarian mechanisms*  $F^{CE}$ , given by

$$F_k^{CE}(\bar{a}) = f(a_k) - f(\tilde{a}^R, a_k^C) + \frac{1}{n} \sum_{i=1}^n f(\tilde{a}^R, a_i^C) \quad \forall \bar{a} \in \Omega^n, \quad \forall k \in K \quad (2.8)$$

Every agent  $k$  is guaranteed the average income of a hypothetical economy in which all agents have relevant characteristics equal to  $\tilde{a}^R$  (the third term at the right hand side), provided that  $a_k^R = \tilde{a}^R$  (in which case the first term equals the second one). If the agent deviates from this reference level ( $a_k^R \neq \tilde{a}^R$ ), then, she has to bear the consequences herself.

A word of caution is needed. Both intuitions definitely do not account for the entire ethics of (responsibility-sensitive) egalitarianism. It is obvious that the principles take the actual income distribution for granted without questioning how the income function  $f$  (or  $g$ ) is determined. Although EIER and ETEC embody ethical qualities, a (re-)distribution that satisfies both requirements could lead to ethically offensive situations and even perverse situations might not be disallowed. Suppose that effort is the only responsibility variable. Imagine now a situation with only two effort levels where all those who perform a high level of effort have the same income, which is however lower than the income received by each one of those with a low level of effort. Loosely speaking, lazy ones earn more than their more industrious colleagues. The EIER-axiom is not violated since those with equal  $a^R$  have equal income but the outcome may still be questioned on justice grounds: is it just that hard workers earn less than lazy ones, *ceteris paribus*? We believe that many contemporary people will view this as a breach of justice. Or a more realistic example: is it acceptable that those who work twice as much earn more than three times as much? This ultimately questions the ETEC axiom since a more accurate transfer or redistribution scheme should have prevented this offensive situation.

One might wonder whether there are really no other things than differences in compensation variables one wants to compensate for? Some responsibility variables are extremely well remunerated - maybe too well. Indeed, some effort is not remunerated at all (charity work, child-rearing and work at home by housewives, ...), other work only to a very limited extent (teaching, nursing, social development work, ...), whereas the same amount of effort in other cases gives rise to huge amounts of income (business managers, sportsmen, ...). These differences could not be reduced entirely to differences in personal responsibility and/or compensation variables. A business manager leaving his job to become a teacher will face a considerable pay-cut for the same amount of effort (and all other things equal). Such remuneration schemes based on past political decisions, the social fabric and cultural habits should be subject to some kind of compensation as well. One might think about redefining  $a^C$  to include other factors besides personal characteristics or at some adapted version of ETEC.

For the time being, we will restrict our attention to the basic axioms, keeping in mind that some specific additions and or adjustments would give a more accurate representation of the responsibility-sensitive egalitarian ethic.

How can the Fleurbaey-Bossert model contribute to measure both the offensive and the inoffensive parts of inequality in a more appropriate manner? The described (families of) redistribution mechanisms (2.6), (2.7), and (2.8) suggest themselves as alternative *reference income distributions*. Usually, the perfectly equal distribution is used as reference income, implying that there is perfect equality if everyone has the same income. The responsibility-sensitive approach does not support this view. Some inequality may be acceptable if it is the result of the exercise of personal responsibility. The reference distribution is not necessarily equalitarian. The income distributions which evolve from the application of the mentioned redistribution mechanisms effectively describe how ideal distributions should look like if one caters for responsibility. It is then only a matter of using the alternative reference distribution, instead of the equally distributed one, to get an idea of the degree of offensive inequality. This, however, requires adequate measures, as has been studied in the literature on the reference income approach and the measurement of distributional change. The choice of an appropriate measure is the topic of the next section.



### 3. Measuring the Deviation from the Norm.

Basically, the norm or reference income approach incorporates information about differences in non-income characteristics of the income unit in the construction of a reference income distribution  $z$ . In our setting, information on personal responsibility is the relevant information to be included. The social planner then determines how much income each individual is entitled to from a responsibility-sensitive point of view, consistent with the given amount of total income available. The answer is the norm or reference income distribution  $z$ .

In the literature, a lot of attention has been paid to summary methods which capture distributional change, i.e. the distance apart of two distributions, and possess desirable properties<sup>3</sup>. This research was absolutely necessary because inequality measures could not be used to measure distances between distributions for at least one important reason.

Indeed, measuring distances between a norm and an observed income distribution has to depart from absolute or complete anonymity<sup>4</sup>. This implies that there might be distributional change without any change in inequality as can be shown with a simple example. Suppose person A has 5 but ought to have 8 and exactly the opposite applies for person B: he has 8 but should have 5. If one compares the two income vectors with respect to inequality, the observed one  $f$  (5, 8) and the norm distribution  $z$  (8, 5), then it is clear that both distributions face the same inequality and  $I(f) - I(z) = 0$ . However, the observed and the norm income of the **same** income unit should be compared to obtain the distributional change which clearly should deviate from zero. This example shows that a measure of distributional change should not measure inequality nor increments or decrements in inequality. Cowell emphasises this essential distinction between the concept of distributional change and the concept of a change in inequality. *"In the latter case, anonymity is complete. We can re-label the components of vectors  $f$  and  $z$  independently and leave  $I(f) - I(z)$  unaltered. In the former case anonymity is assured only in a qualified sense: we may reorder the pairs  $(f_1, z_1)$ ,  $(f_2, z_2)$ ,  $(f_3, z_3)$ , ... and leave [the distributional change] unaltered but not permute components of  $f$  and  $z$  independently."* (Cowell [1980, p 150-151]; adjusted to own notation).

Consequently, since we are working with a heterogeneous population the trans-

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<sup>3</sup>On measuring distances between income distributions see Cowell [1980, 1985], Ebert [1984], Jenkins [1994], Jenkins and O'Higgins [1989], Shorrocks [1982b], Silber [1995].

<sup>4</sup>Absolute anonymity means that reordering the incomes of the observed income distribution and/or reordering the incomes of the norm income distribution does not change the measured distance between both distributions.

fer principle which we are used to in conventional inequality measurement is no longer attractive either. A valuable alternative, however, is to require that measured inequality only falls due to a transfer of a small amount of income,  $\delta$ , from a richer income unit  $k$  to a poorer one  $j$  if  $f_j/f_k < z_j/z_k$ . This means that the effect of a transfer depends on whether the ratio of the observed incomes of income unit  $j$  and  $k$  is smaller than the ratios of their reference incomes. It may even lead to situations in which a transfer from a rich to a poor person increases measured inequality if the income unit from which income is transferred away has a high  $f$  and a high  $z$ , more specifically if  $f_j/f_k > z_j/z_k$ . Other specific cases lead to specific conditions. Always considering a transfer from a richer income unit  $k$  to a poorer one  $j$  which does not change the norm income distribution in any way, inequality decreases: for  $z_j = z_k$  if  $f_j < f_k$ ; for  $f_k = z_k$  if  $f_j < z_j$ ; for  $f_j = z_j$  if  $f_k > z_k$ ; for  $f_k > z_k$  if  $f_j \leq z_j$ ; if both  $f_k > z_k$  and  $f_j > z_j$  if  $\frac{f_j - z_j}{z_j} < \frac{f_k - z_k}{z_k}$ . This can be checked easily from the general condition written as proportional distances  $\frac{f_j}{z_j} < \frac{f_k}{z_k}$ .

Now, Cowell [1985] has shown that an interesting class of distance measures stands at our disposal, to measure the distance apart of two income distributions and of which the well-known family of Generalized Entropy (GE) inequality measures is a subclass. Cowell's measures of distributional change assure that the observed income and reference income of the **same** income unit are compared, they satisfy the alternative interpretation of the transfer principle discussed above and also possess the desirable properties of symmetry, mean independence, additive decomposability by population subgroups and independence of replications of the population (among other). Measures of distributional change summarise the distance apart of the actual distribution and the norm distribution and could be seen as an indication of the redistribution needed to obtain the norm distribution.

For any two distributions  $f$  and  $z$  (and both distributions have an equal mean ( $\bar{f} = \bar{z}$ )) the class of distance measures can be written in the form (Cowell [1985], Jenkins and O'Higgins [1989]):

$$J_\alpha(f, z) = \frac{1}{n\alpha(\alpha - 1)} \sum_{i=1}^n \left[ \left( \frac{(f_i)^\alpha (z_i)^{1-\alpha}}{\bar{f}} \right) - 1 \right], \quad \alpha \neq 0, 1 \quad (3.1)$$

$$J_1(f, z) = \frac{1}{n} \sum_{i=1}^n \left( \frac{f_i}{\bar{f}} \right) \log \left( \frac{f_i}{z_i} \right)$$

$$J_0(f, z) = \frac{1}{n} \sum_{i=1}^n \left( \frac{z_i}{\bar{z}} \right) \log \left( \frac{z_i}{f_i} \right)$$

In this context, distribution  $f$  is the observed income distribution, and  $z$  the norm income distribution. Parameter  $\alpha$  summarises the sensitivity to changes in the distribution in different parts of the distribution: for a large and positive  $\alpha$  the index is particularly sensitive to changes that affect the upper tail, whereas for  $\alpha$  negative the index is sensitive to changes that affect the lower tail<sup>5</sup>.

It is our intention to apply this general class of distance measures to quantify the distance apart of the observed income distribution and the responsibility-sensitive reference or comparative distribution. Bossert-Fleurbaey specify redistributive mechanisms (2.6), (2.7) and (2.8) of which the outcome may serve as fully specified norm income distributions. In case of an additively separable income function the 'natural' redistribution mechanism is  $F_k^0(\bar{a})$ , designated as  $F^0$  (see equation 2.6).  $F^0$  assigns to each individual the income he or she should have had in a perfect responsibility-sensitive egalitarian society, i.e. where both EIER and ETEC prevail. It seems obvious to determine the 'fair' income for each individual according to  $F^0$ , of which the distribution then could be compared with the actual income distribution. The distance could then be interpreted as an indication of the inequality, which is offensive to EIER and ETEC, since the 'natural' redistribution mechanism satisfies both axioms. In case the income function is not additively separable, either an egalitarian-equivalent mechanism,  $F^{EE}$  (see equation 2.7), or a conditionally egalitarian mechanism,  $F^{CE}$  (see equation 2.8), could be selected depending on the aspect of the responsibility-sensitive egalitarian ethic that one wants to stress.

Now the tools are ready to be used in the empirical application. The next steps will be as follows. From the data we will construct the fully specified reference incomes:  $F^0$  in the separable case, some (for different specifications of reference persons) from the classes  $F^{EE}$  and  $F^{CE}$  in the non-separable case. Thereafter, we calculate the offensive distance between the observed income distribution and the norm income distribution using (3.1) for different values of  $\alpha$ .

## 4. Empirical Application.

All data are taken from the seventh wave of the Panel Study on Belgian Households (PSBH). The data were gathered during spring 1998, which implies that

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<sup>5</sup>In traditional inequality measurement the perfectly equal distribution is used as reference income implying that perfect equality obtains if everyone has the same income. In this particular case, this class of distance measures reduces to the class of Generalized Entropy inequality measures ( $z_i = \bar{z} = \bar{f}$ ).

the income data refer to income earned during 1997. The PSBH98 dataset contains information on 7021 adults (aged 16 or more). Of these adults, 3618 had a regular job - defined as a job which demands more than 15 working hours a week. Individuals who work less are not taken into account in order to avoid the results being biased by data of students working part-time. Moreover, self-employed people (525) are left out, which reduces the dataset to 3093 relevant observations. Self-employed people are not useful to our exercise since we want to estimate a labour income equation and earnings from labour cannot be separated from earnings from investment in the case of the self-employed. Due to missing variables another 1247 observations had to be dropped, resulting in a dataset containing 1846 individuals for whom all needed information is available.

The jobless (and all those working less than 15 hours a week), the self-employed and the pensioners are thus not taken into account. Obviously, these groups are not randomly drawn from the population. This calls for a correction so that the results of the estimation do not suffer from sample selection bias. This problem can be solved by using Heckman's two-step estimation (see Heckman [1979]).

Yearly pre-tax income (in Belgian francs) has been computed as the monthly wage times the number of months the respondent has received that amount. Since pre-tax data on extra premiums (such as sales premiums), allowances (13th and 14th month, holiday money, ...), and other transfers are lacking, these transfers are not included in the income variable.

First, in order to calculate the reference incomes, we need to know how (much) each personal characteristic contributes to the total income. Those variables are then labelled either responsibility variable or compensation variable and used to calculate  $F^{EE}$ ,  $F^{CE}$  or  $F^0$  where applicable. Since we do not have knowledge of the income function one could estimate income as a function of the personal characteristics using regression analysis. In the Bossert-Fleurbaey setting it does make a difference whether the income function is additively separable or not. For that reason we deal with the two cases separately, starting with the former.

#### 4.1. The case of an additively separable labour income function.

In the case of an additively separable specification of the income function, one should be able to write:

$$f(a_i) = g(a_i^R) + h(a_i^C). \quad (4.1)$$

The estimated equation however will look like

$$f(a_i) = \hat{\beta}_0 + \hat{\beta}' a_i^R + \hat{\gamma}' a_i^C + \hat{\delta} \lambda_i + e_i \quad (4.2)$$

where  $a^R$  and  $a^C$  are vectors of explanatory variables (hereby taking the responsibility cut for granted),  $\lambda$  the sample selection variable, the vectors  $\hat{\beta}$  and  $\hat{\gamma}$  contain estimated coefficients,  $\hat{\beta}_0$  and  $\hat{\delta}$  are estimated as well and  $e_i$  is the residual or error term. All the effects of the unspecified variables are thus collected in this residual. One should ultimately come up with a clear-cut specification like (4.1) implying that  $\hat{\beta}_0$ ,  $\lambda_i$  and  $e_i$  are attributed either as part of  $g(a_i^R)$  or as part of  $h(a_i^C)$ . The normative choices of the researcher or policy-maker hereby become prominent.

Since the Bossert-Fleurbay model is modelled upon pre-tax incomes, we have opted for the yearly pre-tax labour income to serve as dependent variable  $f(a_i)$ . Note that an attractive specification of the income function in log or semi-log (because it creates approximate normality) is, by the nature of logarithms, not additively separable. As sample selection biases are not implausible, we use the coefficients of the regression of the selection equation to calculate Heckman's lambda, which will be included in the regression of the labour income equation as an independent variable<sup>6</sup>.

The variables we include in the labour income equation are rather straightforward:

$$\begin{aligned}
 f(a) = & \beta_0 + \beta_1 \text{hourly} + \beta_2 \text{gender} + \beta_3 \text{age} + \beta_4 \text{age2} & (4.3) \\
 & + \beta_5 \text{marry} + \beta_6 \text{nation} + \beta_7 \text{edu1} + \beta_8 \text{edu2} + \beta_9 \text{edu3} \\
 & + \beta_{10} \text{edu4} + \beta_{11} \text{edu5} + \delta \text{hecklam} + \varepsilon
 \end{aligned}$$

We incorporate age as a proxy for experience, and age squared (*age2*) because empirical studies show that wage -the main part of labour income- is a decreasing positive function of age. To capture the effect of education, five dummy variables are included. Not included is the lowest level of education, primary schooling or less, which serves as a benchmark. The choice of the control variables capturing personal characteristics is set by the available data and by checks on whether their contribution is significant. Regional and sectorial (private/public) differences seem to be insignificant and are thus not included.

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<sup>6</sup>In the first stage of Heckman's two step estimation procedure we estimate whether someone holds a job or not. This is a binary variable and hence probit is an appropriate model. Our selection equation looks as follows:  $\Pr(\text{job}=1) = \alpha + \beta_1 \text{gender} + \beta_2 \text{ver} + \beta_3 \text{age} + \beta_4 \text{marry} + \beta_5 \text{child} + \beta_6 \text{nation} + \beta_7 \text{region1} + \beta_8 \text{region2} + \beta_9 \text{edu1} + \beta_{10} \text{edu2} + \beta_{11} \text{edu3} + \beta_{12} \text{edu4} + \beta_{13} \text{edu5} + \varepsilon$ . For a description of the included variables and the results of the estimation of the probit model, see Appendix. To evaluate the statistical performance a goodness-of-fit test has been performed. McFadden's  $R^2$  amounts to 0.1854 indicating that the fit is reasonable. Complete estimation results are available upon request.

Source	SS	df	MS			
Model	2.4130e+14	12	2.0108e+13	Number of obs = 1846		
Residual	3.5006e+14	1833	1.9098e+11	F( 12, 1833) = 105.29		
				Prob > F = 0.0000		
				R-squared = 0.4080		
				Adj R-squared = 0.4042		
Total	5.9136e+14	1845	3.2052e+11	Root MSE = 4.4e+05		

  

bruty	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
hecklam	83569.62	137973.2	0.606	0.545	-187031.5	354170.7
houry	300.669	26.54008	11.329	0.000	248.6171	352.721
gender	-234313.1	45661.11	-5.132	0.000	-323866.4	-144759.8
age	48790.72	8416.34	5.797	0.000	32284.09	65297.34
age2	-341.5121	106.1983	-3.216	0.001	-549.7944	-133.2297
marry	87721.87	52757.11	1.663	0.097	-15748.49	191192.2
nation	-102004.8	61481.77	-1.659	0.097	-222586.5	18576.9
edu1	188838.2	67238.81	2.808	0.005	56965.49	320710.9
edu2	354704.1	81194.83	4.369	0.000	195460	513948.2
edu3	568858.6	121593.7	4.678	0.000	330381.8	807335.3
edu4	595767.7	118720.1	5.018	0.000	362926.8	828608.6
edu5	828779.9	112806	7.347	0.000	607538.1	1050022
_cons	-1442769	246233.2	-5.859	0.000	-1925696	-959842.1

**Table 1: Regression results: additively separable case.**

Let us consider the estimation results (Table 1). All parameters are significant at a 10% level except the Heckman lambda. This might indicate that not being at work is not linked with unobservable personal characteristics or individual preferences. We opted to keep the Heckman lambda in the regression because performing a regression without it did not lead to significantly different results. Age seems to be in a linear relation to the dependent variable (since labour income starts decreasing not before the age of 71.4). The sign of gender is as expected: being a women is on average not favourable for one's labour income. That the non-Belgians earn less might be due to 'poor' immigrants (even outweighing the 'rich' Brussels' Eurocrats). Married people on average earn more and the additional income due to schooling increases with the level of education. All as expected. The  $R^2$  is 0.41, implying that it is impossible to assign 60 % of the inequality to either component. The Breusch-Pagan test statistic ( $N = 1846$  times  $R^2 = 6.8302$ ;  $Chi^2$  with 12 degrees of freedom) indicates that the null hypothesis of homoskedasticity cannot be rejected.

Now we can take the next step. We can no longer escape choices about which variables are to be considered as responsibility variables and which as compensation ones. Indeed, each element from (4.3) should be allocated to either  $a^R$  or  $a^C$  to calculate  $F^0$  (2.6).

The easiest assignment is that of the 'equally distributed' regression constant: it does not really matter for  $F^0$  whereto one allocates it. The most difficult problem is the treatment of the error term. The Bossert-Fleurbaey framework assumes that entirely identical persons have the same pre-tax income. This implies that every part of every income exhaustively should be assigned as caused by either a responsibility or a compensation variable. If otherwise identical persons *de facto* have a different pre-tax income then this difference should be assigned to either  $a^R$  or  $a^C$ , even if this means that we have to introduce new variables. The same is true in the case where the difference is due to brute (good or bad) luck. The error term effectively acts as a collector of unassignable income differences. It guarantees that persons who are identical in terms of included control variables are 'able' to earn different pre-tax incomes.

The error or disturbance term should be suitably dealt with. If one has good normative reasons to suppose that this term summarises effects for which one should be held responsible it could be included in the first term of equation (4.1). Alternatively, if one considers the disturbance term as a combination of unobserved effects for which one should be compensated it could be added to the second term of that equation. If one doesn't know, one could assign half of it (or any other proportion) to each component.

Giving the benefit of the doubt, one might prefer to consider the error term as a compensation variable. Then one certainly does not disadvantage anyone by wrongly assigning inequality to characteristics for which one is responsible. Indeed, many will be opposed to being held responsible for unknowns. But again, this is mainly a normative question and alternative arguments may emerge<sup>7</sup>. For the remainder of this section we have opted for this solution. Note, however, that this particular choice has a significant impact on the results as is clear from Table 2. The measure of distributional change (for  $\alpha = 0, 1$ ) is given for three different allocations of the error term. In all cases the number of hours worked (labelled

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<sup>7</sup>This problem might be tackled in a technical, i.e. statistical way as well. The problem then could be considered as decomposing a standard linear regression model  $y = X\beta + u$  into two constituting parts  $y_1 = X_1\beta_1 + u_1$  and  $y_2 = X_2\beta_2 + u_2$  where  $u$  is known and  $y = y_1 + y_2$ . We also know that  $u = u_1 + u_2$  but are ignorant about the specific value of  $u_1$  and  $u_2$ . Attempts have been made to statistically decompose  $u$  into  $u_1$  and  $u_2$ , see Schokkaert, Dhaene and Van de Voorde [1998] and Dhaene, Schokkaert and Van de Voorde [2003].

*hourly*) is taken to be a responsibility variable. In the first case, the residual is neglected and thus implicitly treated as a compensation variable; the second situation divides the error term evenly among both parts and, in the third case, the error term is allocated completely to the responsibility part and added to the already included variable. Making the disturbance term part of vector  $a^R$  reduces the distance between the actual distribution and the norm distribution by about 50 %.

	g(hourly)	g(hourly+0.5error)	g(hourly+error)
$J_0$	0.1310	0.0801	0.0661
$J_1$	0.1183	0.0656	0.0540

**Table 2: Different allocations of the error term.**

The responsibility cut cannot be taken for granted. We strongly feel that this cut has to be made by societal consensus or at least by people's representatives in democratic institutions. Subject to the particular normative choices a society might make, various scenarios are conceivable. One might think of the **status quo** w.r.t. the egalitarian ethic: one assigns all variables to the compensation part, implying that the equal distribution is the norm income distribution - each should have mean income. We showed that the measures of distributional change (3.1) in that case collapse into the class of generalized entropy measures whereof the results are given in the first line of Table 3 for different values of  $\alpha$  ( $\alpha = 0, 1, 2$ ). All inequality is considered offensive as is common practice. It is equally straightforward to deal with a society which agrees upon some kind of '**maximal responsibility**': all variables (including the error term) are considered as responsibility variables implying that the actual distribution coincides with the reference one.

We might, alternatively, think of a society which reaches consensus that hours worked is one -although the only one- variable for which people should be held responsible. Such agreement upon '**minimal responsibility**' might reflect a paternalistic society or one with a strong inclination to social intervention that takes a first step towards the responsabilisation of inequality. The result of this case is given on the second line and in comparison with the first line should be read as follows: the distance between the actual distribution and the norm distribution  $F^0$  (built upon *hourly*) is smaller than the distance between the actual distribution and the distribution where everyone gets mean income. One could also discuss societies where consensus upon some sort of '**intermediate responsibility**' prevails: some but not all variables are assigned to the responsibility part. One



might for instance add the estimated income resulting from being married and one's level of education to the responsibility part (hereby neglecting the ethical (im-)plausibility of this shift).

$g(a^R)$ includes ...	$\mathbf{J}_0$	$\mathbf{J}_1$	$\mathbf{J}_2$
no variables ( $J_\alpha = GE_\alpha$ )	0.15420	0.13836	0.16923
hourly	0.13103	0.11827	0.15078
hourly, marry	0.12687	0.11535	0.14654
hourly, marry, edu	0.11679	0.10336	0.12697
hourly, marry, edu, hecklam	0.11546	0.10183	0.12517
hourly, marry, hecklam	0.12995	0.11810	0.14975
hecklam	0.15844	0.14220	0.17394

**Table 3: Distributional change with  $F^0$ -norm**  
(with error term included in  $h(a^C)$ ).

Finally, taking the Heckman lambda as a responsibility variable, which again is ethically rather unattractive since it collects unknowns, hints at a remarkable aspect of this procedure. Adding a variable to the responsibility part does not necessarily mean that the distance to the norm reduces as is the case in the last but one row. Taking Heckman's lambda as the only element of  $g(a^R)$  even shows that the situation where no variables belong to  $g(a^R)$  is not the upper bound of distributional change (compare first and last row). Intuitively speaking, this occurs when the distribution of the variable in question is negatively correlated with the actual income distribution. There is thus no monotone (decreasing) relationship between the number of variables included in  $g(a^R)$  and the distance between the actual and the norm distribution.

#### 4.2. The case of a non-additively separable labour income function.

Non-additively separable income functions are more likely to be appropriate in representing real-world situations since the influences of responsibility variables and compensation variables are often entangled. In our model it can be interpreted as the situation in which income due to responsible factors is dependent upon the level of compensation characteristics, or vice versa. An attractive non-additively separable specification of the income function is the semi-logarithmic functional form. Since our theory does not suggest anything specific, for the remainder we take the natural logarithm of pre-tax incomes as left-hand side variable. This

means that we assume multiplicative effects as we propose to work with the following equation:

$$f(a) = e^{(\beta_0)} \cdot e^{(\beta_1 \text{houry})} \cdot e^{(\beta_2 \text{gender})} \cdot e^{(\beta_3 \text{age})} \cdot e^{(\beta_4 \text{age2})} \cdot e^{(\beta_5 \text{marry})} \cdot e^{(\beta_6 \text{nation})} \cdot e^{(\beta_7 \text{edu1})} \cdot e^{(\beta_8 \text{edu2})} \cdot e^{(\beta_9 \text{edu3})} \cdot e^{(\beta_{10} \text{edu4})} \cdot e^{(\beta_{11} \text{edu5})} \cdot e^{(\delta \text{hecklam})} \quad (4.4)$$

This could be log-transformed to be estimated linearly by OLS which gives<sup>8</sup>:

$$\begin{aligned} \log(f(a)) = & \hat{\beta}_0 + \hat{\beta}_1 \text{houry} + \hat{\beta}_2 \text{gender} + \hat{\beta}_3 \text{age} + \hat{\beta}_4 \text{age2} \\ & + \hat{\beta}_5 \text{marry} + \hat{\beta}_6 \text{nation} + \hat{\beta}_7 \text{edu1} + \hat{\beta}_8 \text{edu2} + \hat{\beta}_9 \text{edu3} \\ & + \hat{\beta}_{10} \text{edu4} + \hat{\beta}_{11} \text{edu5} + \delta \text{hecklam} + \hat{\varepsilon} \end{aligned} \quad (4.5)$$

Source	SS	df	MS	Number of obs = 1846		
Model	318.888567	12	26.5740473	F( 12, 1833) = 134.75		
Residual	361.476027	1833	.197204597	Prob > F = 0.0000		
				R-squared = 0.4687		
				Adj R-squared = 0.4652		
				Root MSE = .44408		
Total	680.364594	1845	.368761297			

  

logbruty	Coef.	Std. Err.	t	P> t	[95% Conf. Intervall]	
hecklam	.1810966	.1402045	1.292	0.197	-.0938806	.4560739
houry	.000384	.000027	14.237	0.000	.0003311	.0004369
gender	-.2976156	.0463995	-6.414	0.000	-.3886171	-.2066141
age	.1079085	.0085524	12.617	0.000	.0911349	.1246821
age2	-.0010621	.0001079	-9.842	0.000	-.0012737	-.0008504
marry	.120512	.0536103	2.248	0.025	.0153683	.2256556
nation	-.2156308	.062476	-3.451	0.001	-.3381625	-.093099
edu1	.1916196	.0683262	2.804	0.005	.0576143	.325625
edu2	.3958726	.0825079	4.798	0.000	.2340533	.557692
edu3	.6648619	.1235601	5.381	0.000	.4225285	.9071952
edu4	.6839793	.1206401	5.670	0.000	.4473729	.9205857
edu5	.8151532	.1146303	7.111	0.000	.5903335	1.039973
_cons	9.781276	.2502153	39.091	0.000	9.290539	10.27201

**Table 4: Regression results: non additively separable case.**

<sup>8</sup>Such semi-logarithmic specification with the natural logarithm of income as left-hand side variable produces approximate normality (see Maddala [1991, 33]) and therefore leads to better estimation results (see Cramer [1991, 31]). It is a standard specification in the human capital literature since Mincer [1974].

For reasons of comparability (and because it yields the best results), we stick to the same variables to be included in the regression analysis, the results of which are in Table 4. All is as expected. The signs of *gender*, *nation* and *age squared* are negative as in the additively separable case. Evidently, the selection equation is exactly the same as well. Note that the determination coefficient  $R^2$  is 0.47 and tests show that heteroskedasticity is not an issue either.

However, the construction of the norm income is far from straightforward. Recall that the Bossert-Fleurbaey axioms EIER and ETEC are in general incompatible in the non-additively separable case and thus the natural redistribution scheme  $F^0$  is no option. Introducing benchmark or reference vectors to calculate hypothetical incomes from which norm incomes could be derived are the proposed solutions to satisfy either axiom while relaxing the other.

Suppose one wants to retain EIER, then one could take a norm income distribution belonging to the class of egalitarian equivalent redistributions  $F^{EE}$ , which has been defined as

$$F_k^{EE}(\bar{a}) = f(a_k^R, \tilde{a}^C) - \frac{1}{n} \sum_{i=1}^n [f(a_i^R, \tilde{a}^C) - f(a_i)]. \quad (4.6)$$

Apart from the choices we also faced in the additive separable case, we here have an additional degree of freedom, namely the benchmark vector  $\tilde{a}^C$ . The choices made necessary to calculate  $f(a_k^R, \tilde{a}^C) \forall k$ , which is the crucial bit here, are discussed next. The **regression constant** has been included but it does not matter for the outcome of any  $F^{EE}$  whether you put it in  $a^R$  or  $\tilde{a}^C$ . One could of course choose to consider the constant as part of the  $\tilde{a}^C$  vector and assign a benchmark value to it that differs from the estimated value. Since the regression constant has a specific meaning in the regression as reference for the contribution of the other control variables, we have opted to keep the estimated value - although this particular function of the constant could be the reason for others to come up with an alternative benchmark value. Further, we have taken the fundamental option to treat the **error term** as a compensation variable. We think it is not unreasonable to include it in the benchmark vector with the value of the log of the mean of  $e^{\hat{\varepsilon}}$  ( $\log(\overline{e^{\hat{\varepsilon}}}) = 0.0836$ ). Next, the **cut** between  $R$ - and  $C$ -variables has to be supplemented by a choice of a fixed level of the  $C$ -variables. We started with the same responsibility cut as in Table 3 and added **benchmark levels** for the  $C$ -variables which in our case resulted in seven different ways to calculate  $f(a_k^R, \tilde{a}^C)$  as summarised in Table 5. The columns give the values used to calculate the hypothetical income  $f(a_k^R, \tilde{a}^C)$ . A *number* in the table means that that particular

variable is fixed at that level and thus belongs to the benchmark compensation vector. The *name* of the variable in the table means that the variable as such is included in the calculation of  $f(a_k^R, \tilde{a}^C)$  as a responsibility variable.

$f(a_k^R, \tilde{a}^C)$	GE	A	B	C	D	E	F	G
houry	1900.7	houry	houry	houry	houry	houry	1900.7	houry
gender	1	1	1	1	1	1	1	0
age	16	16	16	16	16	16	16	60
marry	0	0	marry	marry	marry	marry	0	1
nation	1	1	1	1	1	1	1	0
edu1	0	0	0	edu1	edu1	0	0	0
edu2	0	0	0	edu2	edu2	0	0	0
edu3	0	0	0	edu3	edu3	0	0	0
edu4	0	0	0	edu4	edu4	0	0	0
edu5	0	0	0	edu5	edu5	0	0	1
heckl	0	0	0	0	heckl	heckl	heckl	1

**Table 5: Normative choices for  $F^{EE}$ -norm.**

Could our responsibility-sensitive theory give information on how the levels of the variables included in  $\tilde{a}^C$  could be fixed? We suggest that one plausible view is to grant people responsibility for the part of the income that is independent of the compensation factors, but not for the part that depends upon compensation factors<sup>9</sup>. This is the spirit of the responsibility-sensitive egalitarian ethics, but now we have to internalise this famous cut into the effects of responsibility variables themselves. We intend to do the following: anyone who works one hour extra ( $\in a^R$ ) is entitled to keep the fruits of his additional effort as far as his extra income is independent of his compensation variables ( $a^C$ ). The part of the income that is independent of the compensation variables is taken to be the income one could earn if one were the most disadvantaged in terms of earning power or the least marginally productive member of the economy. Earning more than the least advantaged, for the same level of the responsibility-variables, is due to personal characteristics of which you are the lucky owner and for that very reason is open for redistribution. This is what Tungodden [2004] hints at in the context of first best taxation.

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<sup>9</sup>Another plausible view could be to choose the reference vector  $\tilde{a}^C$  such that the resulting norm income distribution minimizes the deviation from the other axiom ETEC.

In our Belgian sample, the least advantaged reference person is taken to be an unmarried, non-Belgian, 16-year old, uneducated woman with no psychological characteristics captured by Heckman's lambda ( $\lambda = 0$ ). For instance, reference GE calculates  $f(a_k^R, \tilde{a}^C)$  using the estimated coefficients for such a person working the average number of hours (1900.7 hours a year). All variables are considered compensation variables and fixed at a particular level in a non-arbitrary way. Whatever the fixed levels in this case, the reference income will be the average income and the measures of distributional change will reduce to the generalized entropy measures. Reference A then keeps the compensation variables fixed at the same level, but considers hours worked as the only responsibility variable. We gradually move variables from  $\tilde{a}^C$  to  $a^R$  (reference B, C, D, respectively). Reference E introduces *hecklam* at another stage in the sequence (after B) and reference F takes *hecklam* as the only responsibility variable. This is exactly the same sequence as in Table 3. For illustrative reasons, the benchmark level of the most advantaged person has been calculated as reference G.

The results are given in Table 6. Evidently, the overall inequality as measured by GE remains the same as with the  $F^0$ -norm. Sequentially adding more variables to the responsibility part, keeping benchmarks constant, decreases distances (e.g. a 10% decrease in going from GE to D). This result though partly depends on the order in which the variables are sequentially added, as becomes evident from reference E. The decrease of the distances is quite small compared to the decrease encountered in Table 3. The choice of the benchmark vector  $\tilde{a}^C$  is the determining element here. Recall equation (4.6). We have chosen the benchmark vector such that it represents the least advantaged member of the society under scrutiny. That means that  $f(a_k^R, \tilde{a}^C) \forall k$  will generate quite low reference incomes. The second term of (4.6) implies that much of the actual income will disappear in a basket which will be divided equally among all. Thus, the chosen benchmark has a huge redistributive effect which makes that the norm income distribution is approaching the case where all should get equal (average) income (the GE-case). Note that adopting the benchmark vector  $\tilde{a}^C$  of the most advantaged individual (reference G) leads to the smallest  $J_0$  of all.

$F^{EE}$	$J_0$	$J_1$	$J_2$
Reference GE	0.15420	0.13836	0.16923
Reference A	0.15030	0.13485	0.16547
Reference B	0.14919	0.13394	0.16432
Reference C	0.13919	0.12370	0.15072
Reference D	0.13806	0.12257	0.14929
Reference E	0.14936	0.13416	0.16473
Reference F	0.15526	0.13932	0.17042
Reference G	0.13189	0.11867	0.15947

**Table 6: Distributional change with  $F^{EE}$ -norm.**  
(with error term included in  $\tilde{a}^C$ )

Alternatively, one could prefer to relax EI<sub>ER</sub> and require that ETEC should be satisfied. Then one is restricted to a norm income belonging to the conditionally egalitarian class  $F^{CE}$ , defined as follows

$$F_k^{CE}(\bar{a}) = f(a_k) - f(\tilde{a}^R, a_k^C) + \frac{1}{n} \sum_{i=1}^n f(\tilde{a}^R, a_i^C) \quad (4.7)$$

To determine  $f(\tilde{a}^R, a_k^C) \forall k$  attention has been paid to make similar choices to those in the egalitarian equivalent case: the constant has been included (although it does not change anything whether one considers it a compensation or responsibility variable), the responsibility cut has been repeated (although what is fixed and what is variable is reversed of course) and the error term is added as a compensation variable.

Table 7 summarises the choices. We begin with the GE-reference -all variables are compensation ones- implying that  $f(\tilde{a}^R, a_k^C)$  reduces to  $f(a_k)$  and  $F_k^{CE}(\bar{a})$  to average income for each  $k$ . Reference A is such that *hourly* is the only responsibility variable and one thus has to fix a constant reference value for it. What value should one take? For comparative reasons we have opted for the same fixed benchmark values as in Table 5. Choosing average levels as benchmarks could be a quite neutral choice. On the other hand, one could opt for the levels of the responsibility-variables of the person with the highest earning capacity i.e. those levels which in that particular society give the highest income (e.g. working maximum hours per year - as in reference G, highly educated, married). In that case, the last term of equation (4.7) takes the average of the incomes if everyone were to have chosen to act at this most profitable  $\tilde{a}^R$ . If you earned, under these conditions, more than that average (see last two terms of (4.7)) then the difference

will be deduced from your actual income. Given the functional form, one might expect the value of  $f(\tilde{a}^R, a_k^C), \forall k$  to be higher on average for this highest earning  $\tilde{a}^R$  than for a less earning or profitable  $\tilde{a}^R$ , ceteris paribus<sup>10</sup>. This will lead to higher transfers and taxes (again the last two terms of (4.7)) and might be seen as compensating for compensation variables most heavily. This does not necessarily imply that it is more equalizing since all depends on  $f(a)$ : one might be taxed heavily because one has a high education, but is quite poor since one has chosen to work only part-time. The scheme  $F^{CE}$  is less compensating if one chooses the least profitable  $\tilde{a}^R$  as the fixed benchmark vector<sup>11</sup>.

$f(\tilde{a}^R, a_k^C)$	GE	A	B	C	D	E	F	G
houry	houry	1900.7	1900.7	1900.7	1900.7	1900.7	houry	2800
gender	gender	gender	gender	gender	gender	gender	gender	gender
age	age	age	age	age	age	age	age	age
marry	marry	marry	0	0	0	0	marry	marry
nation	nation	nation	nation	nation	nation	nation	nation	nation
edu1	edu1	edu1	edu1	0	0	edu1	edu1	edu1
edu2	edu2	edu2	edu2	0	0	edu2	edu2	edu2
edu3	edu3	edu3	edu3	0	0	edu3	edu3	edu3
edu4	edu4	edu4	edu4	0	0	edu4	edu4	edu4
edu5	edu5	edu5	edu5	0	0	edu5	edu5	edu5
heckl	heckl	heckl	heckl	heckl	0	0	0	heckl

**Table 7: Normative choices for  $F^{CE}$ -norm.**

The results are given in Table 8. Again, adding *hecklam* moves things in the other direction. One also immediately sees that by sequentially moving variables

<sup>10</sup>Reference A has a mean of 944,859 and a standard deviation of 533,236 while reference G has a mean of 1,333,612 and a standard deviation of 753,174.

<sup>11</sup>An interesting aspect of this redistributive mechanism should be mentioned. Suppose that the benchmark vector is the  $\tilde{a}^R$  of the person with the highest earning capacity of the whole population. The person(s) with  $a^R = \tilde{a}^R$  will according to (4.7) end up with a post-tax income equal to  $\frac{1}{n} \sum_{i=1}^n f(\tilde{a}^R, a_i^C)$  (i.e. the third term of (4.7)). For the others ( $a^R \neq \tilde{a}^R$ ) it holds that the first term will be smaller than the second one and, consequently, that their post-tax income will be lower than  $\frac{1}{n} \sum_{i=1}^n f(\tilde{a}^R, a_i^C)$ . In other words, there is an upper bound equal to the third term of (4.7). Loosely speaking, everyone gets the average expressed by the third term of (4.7), only the person(s) with the highest earning capacity are allowed to keep that average, and all the others bear the consequences of not reaching  $\tilde{a}^R$  and have to give in on that average. Similarly, if the  $\tilde{a}^R$  of the person with the lowest earning capacity of the population is chosen as benchmark, the third term now becomes a lower bound. Everyone gets that average income and all those with  $a^R \neq \tilde{a}^R$  receive an extra.

from  $a^C$  to  $\tilde{a}^R$  the distances decrease quite considerably, which is a kind of change we did not encounter with  $F^{EE}$  as norm (e.g. an 80% decrease in going from reference A to D). Why is this? There are two separate factors interacting here. On the one hand, each time one factor is moved from  $a^C$  to  $\tilde{a}^R$ . On the other hand, the moved factor is fixed at a particular level. The latter is of utmost importance. Take for instance the situation where only *hourly* is considered a responsibility variable. Fixing hourly at 1900.7 hours worked per year (which is the average) results in a distance between the actual distribution and the appropriately calculated  $F^{CE}$  of 0.128750 as measured by  $J_0$  (reference A). However, if one pins down hourly at 2800 hours, then the distance is almost twice as large (0.240623, reference G). Intuitively, one is compensated for the (non-additively separable) effects one's compensation variables have on income in the situation where these effects are expected to be the highest. Alternatively, if one takes only 720 hours as the benchmark level, the distance is 0.05587, less than half of the initial distance<sup>12</sup>. This illustrates the earlier conclusion that the more profitable the fixed value for elements belonging to the benchmark vector is, the more compensating the redistribution is. Moving more variables to the  $\tilde{a}^R$  vector which we fix at the least profitable level (e.g. unmarried and lowest education level) does decrease the measured distances further. Again this is due to the benchmark vector rather than to the fact that variables are shifted from  $a^C$  to  $\tilde{a}^R$ .

$F^{CE}$	$J_0$	$J_1$	$J_2$
Reference GE	0.154204	0.138360	0.169235
Reference A	0.128750	0.091810	0.057620
Reference B	0.108381	0.100902	0.200473
Reference C	0.041123	0.033850	0.031526
Reference D	0.028907	0.023366	0.020979
Reference E	0.081838	0.069202	0.071410
Reference F	0.118540	0.102440	0.106674
Reference G	0.240623	0.199201	0.287623

**Table 8: Distributional change with  $F^{CE}$ -norm.**  
(with error term included in  $a^C$ )

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<sup>12</sup>The complete results for reference A (or G) with hourly = 720 instead are:  $J_0$ = 0.05587;  $J_1$ = 0.04598;  $J_2$ = 0.04373.



## 5. Conclusion.

The norm income approach to income inequality measurement proves to be a particularly valuable tool in the hands of the responsibility-sensitive egalitarian. His ethics concentrate on individualised normative incomes which each citizen in the best of circumstances should have. However, reality often does not come even near to the constructed ideal world of the kind-hearted theorist. Summary methods may capture such ethically 'offensive' divergences in a single index and subsequently, distributions could be ordered according to the degree of ethical 'offensiveness'. The set of attractive properties, including a qualified reading of anonymity and the transfer principle and the presence of a sensitivity parameter, prompted us to use the measures of distributional change as described by Cowell [1985].

The crucial part however is the construction of the responsibility-sensitive norm income for each individual of the society under scrutiny. We opted to interpret readily available and attractive redistributive schemes -  $F^0$ ,  $F^{EE}$  and  $F^{CE}$  - introduced in the literature by Bossert and Fleurbaey. First, we decomposed total income of each person into income resulting from responsibility variables and income due to compensation variables. We performed a regression analysis in order to do so. A decision had to be made about the responsibility cut, the assignment of the 'equally distributed' regression constant  $\alpha$  and the person-specific disturbance term. Next, we calculated the reference income. In the additively separable case, we took 'natural solution'  $F^0$  as the reference (the other mechanisms reduce to this one). In the non-additively separable case we used  $F^{CE}$  and  $F^{EE}$  as reference incomes (for different benchmarks). Finally, we calculated the distributional change  $J_\alpha$  between the observed income distribution and the reference income distribution for different values of  $\alpha$ . The results can be seen as an indication of how far away incomes actually are from the norm distribution.

Interesting results have been derived from Belgian pre-tax labour income data. For a reasonable responsibility cut, we derived that the distance between the actual distribution and the  $F^0$  norm distribution is about 80 % of the distance between the actual distribution and the equal distribution. This proportion is about 97% with the  $F^{EE}$  norm and 70% with the  $F^{CE}$  norm (for benchmark reference B). However, one should not attach too much importance to this specific percentages since another distance measure might give a totally different percentage.

This brings us to some of the shortcomings of our approach. First, a sensitivity

analysis which examines the impact on the results of the use of different distance measures has been postponed to future research from the outset. We are also well aware that the analysis focuses more on summary indices rather than on the ordering of distributions in general. Again, this might be a topic for future discussion.

Second, the main inconvenience of the norm income approach is that in the case of a non-additively separable income function the 'ideal' or the 'norm' distribution cannot be defined (unequivocally). In that case benchmark values have to be fixed and we have made an attempt to do so in a non *ad hoc* way. We agree that more work could be done on this. Many other choices have to be made as well. On some technical issues about whereto assign the effects of some variables ( $\alpha$ ,  $\lambda$ ,  $e$ ), the economist could make valuable suggestions whereas on the responsibility cut within vector  $a$  societal consensus should be sought. In general, such flexibility is not a bad thing at all. The approach is able to incorporate the many convictions policy-makers or politicians might have and would like to have implemented.

Other specifications of non-additively separable income functions should be applied. Especially, functions incorporating multiplicative effects seem to be promising. For instance, one's hourly wage (given *hourly* is an R-variable) is higher the more educated one is (given education is a C-variable). Consequently, hard working but less educated people earn less than equally hard working but higher educated ones, all other things equal. Although both groups have the same level of effort, or more generally the same vector  $a^R$ , the income following their effort is different due to differences in  $a^C$ .

Finally, one may wonder how the responsibility-sensitive egalitarian approach deals with differences in needs. One might lead a relatively normal life (and earn a normal income) but only at the cost of expensive treatment or drugs. If society wants to compensate for these and similar medical costs and account for it in inequality measurement, the reference incomes should be suitably adjusted. Further research might lead to such 'responsibility and needs'- sensitive egalitarian approach.

## References.

- Arneson, R.J., *Equality and Equal Opportunity for Welfare*, in *Philosophical Studies* 56 (1989), 77-93.
- Babeuf, F.N. & Marechal, S., *Manifeste des Egaux*, 1794.
- Bossert, W., *Redistribution Mechanisms Based on Individual Characteristics*, in *Mathematical Social Sciences* 29 (1995), 1-17.

- Bossert, W. & Fleurbaey, M., *Redistribution and Compensation*, in *Social Choice and Welfare* 13 (1996), 343-355.
- Cohen, G.A., *On the Currency of Egalitarian Justice*, in *Ethics* 99 (1989), 906-944.
- Cohen, G.A., *Equality of What? On Welfare, Goods and Capabilities*, in *Recherches Economiques de Louvain* 56 (1990), 357-382.
- Cowell, F.A., *Generalized Entropy and the Measurement of Distributional Change*, in *European Economic Review* 13 (1980), 147-159.
- Cowell, F.A., *The Measurement of Distributional Change: An Axiomatic Approach*, in *Review of Economic Studies* 52 (1985), 135-151.
- Cramer, J.S., *The Logit Model*, Edward Arnold, London, 1991.
- Danzinger, S., Haveman, R. & Smolensky, E., *Comment on Paglin (1975)*, in *American Economic Review* 67 (1977), 505-512.
- Dhaene, G., Schokkaert, E. & Van De Voorde, C., *Best Affine Unbiased Response Decomposition*, in *Journal of Multivariate Analysis* 86 (2003), 242-253.
- Dworkin, R., *What is Equality? Part 1: Equality of Welfare*, in *Philosophy and Public Affairs* 10 (1981a), 185-246.
- Dworkin, R., *What is Equality? Part 2: Equality of Resources*, in *Philosophy and Public Affairs* 10 (1981b), 283-345.
- Ebert, U., *Measures of Distance between Income Distributions*, in *Journal of Economic Theory* 32 (1984), 266-274.
- Fleurbaey, M., *On Fair Compensation*, in *Theory and Decision* 36 (1994), 277-307.
- Fleurbaey, M., *The Requisites of Equal Opportunity*. In Barnett, W.A. (ed.). *Social Choice, Welfare, Ethics*, Cambridge U.P., 1995a, 37-53.
- Fleurbaey, M., *Three Solutions for the Compensation Problem*, in *Journal of Economic Theory* 65 (1995b), 505-521.
- Fleurbaey, M., *Equality and Responsibility*, in *European Economic Review* 39 (1995c), 683-689.
- Fleurbaey, M., *Equal Opportunity or Equal Social Outcome?*, in *Economics and Philosophy* 11 (1995d), 25-55.
- Fleurbaey, M., *Equality Among Responsible Individuals*. In Gravel, N., Fleurbaey, M., Laslier, J.F. & Trannoy, A. (eds.), *Freedom in Economics*, Routledge, London, 1998, 206-234.
- Formby, J. & Seaks, T., *Paglin's Gini Measurement of Inequality: A Modification*, in *American Economic Review* 70 (1980), 479-482.

- Formby, J., Seaks, T. & Smith, W., *On the Measurement and Trend of Inequality: A Reconsideration*, in *American Economic Review* 79 (1989), 256-264.
- Jenkins, S.P., *Social Welfare Function Measures of Horizontal Inequity*. In Eichhorn, W. (ed), *Models and Measurement of Welfare and Inequality*, Springer Verlag, Berlin, 1994, 725-751.
- Jenkins, S.P. & O'Higgins, M., *Inequality Measurement Using 'Norm Incomes': Were Garvy and Paglin Onto Something After All?*, in *Review of Income and Wealth* 35 (1989), 265-282.
- Johnson, W.R., *Comment on Paglin (1975)*, in *American Economic Review* 67 (1977), 502-504.
- Kurien, C.J., *Comment on Paglin (1975)*, in *American Economic Review* 67 (1977), 517-519.
- Maddala, G.S., *Limited-dependent and Qualitative Variables in Econometrics*, *Econometric Society Monographs* 3, Cambridge U.P., 1991.
- Minarik, J., *Comment on Paglin (1975)*, in *American Economic Review* 67 (1977), 513-516.
- Mincer, J., *Schooling, Experience and Earnings*, Columbia U.P., 1974.
- Nelson, E.R., *Comment on Paglin (1975)*, in *American Economic Review* 67 (1977), 497-501.
- Paglin, M., *The Measurement and Trend of Inequality: A Basic Revision*, in *American Economic Review* 65 (1975), 598-609.
- Paglin, M., *Reply*, in *American Economic Review* 67 (1977), 520-531.
- Paglin, M., *Reply to Wertz (1979)*, in *American Economic Review* 69 (1979), 673-677.
- Paglin, M., *On the Measurement and Trend of Inequality: Reply*, in *American Economic Review* 79 (1989), 265-266.
- Rawls, J., *A Theory of Justice*, Oxford U.P., 1971.
- Schokkaert, E., Dhaene, G. & Van De Voorde, C., *Risk Adjustment and the Trade-Off between Efficiency and Risk Selection: An Application of the Theory of Fair Compensation*, in *Health Economics* 7 (1998), 465-480.
- Shorrocks, A.F., *On the Distance between Income Distributions*, in *Econometrica* 50 (1982b), 1337-1339.
- Silber, J., *Horizontal Inequity, the Gini Index and the Measurement of Distributional Change*, in *Research on Economic Inequality* 6 (1995), 379-392.
- Tungodden, B., *Responsibility and Redistribution: The Case of First Best Taxation*, in *Social Choice and Welfare* (2004), forthcoming.

Wertz, K.L., *Comment on Paglin (1975)*, in *American Economic Review* 69 (1979), 670-672.

## Appendix A: List of Control Variables.

- **HOURLY**: This variable gives the total amount of hours worked per year. It has been computed as the average number of hours actually worked per week (thus not the number of hours stated in the contract and ranging from 15 to 90 hours/week) times 49 weeks.
- **AGE**: The age of the respondent.
- **AGE2**: The square of age to capture the concavity of the age earnings (and thus allows for a certain degree of non-linearity).
- **EDU0**: Dummy which takes value 1 if highest education of the respondent is primary schooling or less.
- **EDU1**: Dummy which takes value 1 if highest education of the respondent is lower secondary schooling.
- **EDU2**: Dummy which takes value 1 if highest education of the respondent is higher secondary schooling.
- **EDU3**: Dummy which takes value 1 if highest education of the respondent is higher education outside university (short type = less than three years).
- **EDU4**: Dummy which takes value 1 if highest education of the respondent is higher education outside university (long type = more than three years).
- **EDU5**: Dummy which takes value 1 if the respondent holds a university degree.
- **GENDER**: Dummy which takes value 1 for women.
- **NATION**: Dummy which takes value 0 for respondents with the Belgian nationality.
- **REGION1**: Dummy which takes value 1 for Brussels.
- **REGION2**: Dummy which takes value 1 for the Walloon provinces.

- **REGION3:** Dummy which takes value 1 for the Flemish provinces.
- **SECTOR:** Dummy which takes value 1 if the respondent is working in the public sector.
- **VER:** Dummy which takes value 1 for those being member of a socio-cultural society or club.
- **CHILD:** number of children for which one receives child allowances (children from birth until the moment that they are self-supporting and limited to the age of 25).
- **MARRY:** Dummy which takes value 1 if married.
- **HECKLAM:** Heckman's lambda, see text.

## Appendix B: Results of Estimation of Probit Model.

Probit Estimates		Number of obs = 6166	
		chi2(13) = 1171.67	
		Prob > chi2 = 0.0000	
Log Likelihood = -3177.5268		Pseudo R2 = 0.1557	

  

_____0000EL	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
gender	-.4737988	.0389227	-12.173	0.000	-.5500859	-.3975118
ver	-.0104355	.0381374	-0.274	0.784	-.0851834	.0643124
age	-.0234361	.0013929	-16.825	0.000	-.0261661	-.020706
marry	.4842969	.0426124	11.365	0.000	.4007781	.5678157
child	.0954256	.0211989	4.501	0.000	.0538766	.1369747
nation	-.3152162	.0886634	-3.555	0.000	-.4889933	-.141439
region1	.145092	.0661661	2.193	0.028	.0154088	.2747751
region2	.1138813	.0404967	2.812	0.005	.0345093	.1932533
edu1	.3452872	.0760274	4.542	0.000	.1962762	.4942981
edu2	.5713543	.0742427	7.696	0.000	.4258413	.7168673
edu3	1.083145	.0801231	13.519	0.000	.9261069	1.240184
edu4	1.008828	.0960759	10.500	0.000	.820523	1.197134
edu5	.9471183	.0868566	10.904	0.000	.7768826	1.117354
_cons	-.2707348	.0971327	-2.787	0.005	-.4611115	-.0803582

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