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# Physical and Financial Virtual Power Plants

by

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DISCUSSION
PAPER

# PHYSICAL AND FINANCIAL VIRTUAL POWER PLANTS

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**Abstract** Regulators in Belgium and the Netherlands use different mechanisms to mitigate generation market power. In Belgium, antitrust authorities oblige the incumbent to sell financial Virtual Power Plants, while in the Netherlands regulators have been discussing the use of physical Virtual Power Plants.

This paper uses a numerical game theoretic model to simulate the behavior of the generation firms and to compare the effects of both systems on the market power of the generators. It shows that financial Virtual Power Plants are better for society.

**Keywords:** Futures markets, Options markets, Cournot, Market power, Electricity, Arbitrage

**JEL: C72, D43, G13, L13, L50, L94**

## 1 INTRODUCTION

### 1.1 Market power and long run contracts

Several countries recently decided to liberalize their electricity markets and to organize competition in electricity generation. They assumed that economies of scale and entry barriers in the generation sector were sufficiently small to make competition viable.

In practice, the generation market is not always very competitive. Generators often succeed in driving up prices significantly above competitive levels. This also happens in markets with low levels of market concentration. Prices above marginal production costs have been shown to exist in several markets.[1]

Comparing different electricity markets in the US, Bushnell et al. (2004) show that California had a relatively unconcentrated generation market but that the lack of long term contracts led to high price-cost margins in the summer of 2000. With long term contracts, generators sell

[1] See Borenstein et al. (2002), Joskow and Kahn (2002), Wolak and Patrick (2001) and Wolfram (1999).

part of their electricity ex-ante, at a locked-in price. As a result, generators will behave more competitively in the spot market.

The intuition is that of the durable goods' monopolist in Coase's conjecture (Coase 1972). See Figure 1. Graph A shows the profit maximizing price $p^M$ for a monopolist who sells only in the spot market, has production costs $C(q)$ and faces an inverse demand function $P(q)$. The monopolist will set the price such that marginal revenue equals marginal cost. Graph B shows the same situation for a monopolist who signed long term contracts for $k$ units of electricity. In the spot market (the second stage), $k$ units will therefore disappear both at the demand and the supply side. The profit maximizing price then equals $\tilde{p}^M$ and is lower than $p^M$. In the first stage, the contracting stage, consumers will take into account that the price in the spot market will be equal to $\tilde{p}^M$. They will only buy long run contracts for electricity at the price $\tilde{p}^M$, as they would lose money otherwise. Hence, the monopolist will receive the price $\tilde{p}^M$ in the production as well as in the contracting stage. This fact is called the perfect arbitrage condition.

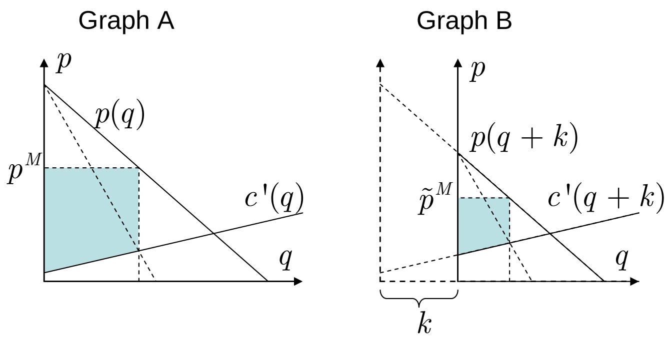


**Figure 1** Effect of an ex-ante contract on the behavior of a monopolist

The study of Bushnell et al. highlights the importance of long term contracts in electricity markets. There is, however, no consensus on the role of long term contracts in electricity markets.

Historically, policy makers have been opposed to long term contracting. They feared that long term contracts between incumbent generators and retailers might slow down entry in the generation market. They also assumed that long term contracting would decrease the transparency and the liquidity of the spot markets. Illiquid spot markets would lead to inefficient real time production decisions, and would also make entry more difficult. A small entrant will have to rely on the spot market to balance the difference between the energy sold and the energy produced.

Currently, policy makers are becoming more favorable towards long term contracts. They hope that long term contracts will ease entry in the generation market by reducing the risk for entrants and will reduce market power in the spot market. Long term contracts will also help retailers who sell electricity at fixed regulated prices to hedge their price risks.

Nowadays, the policy debate is whether one should *impose* the usage of long term contracts or whether generators and retailers will sign the right amount of long term contracts on their own. See for example Creti and Fabra (2004).

## 1.2 Virtual Power Plants

In this new philosophy of imposing long term contracts on incumbent generators to mitigate market power, several European regulators have relied upon a specific type of contracts: the Virtual Power Plants (VPPs). Such a system is currently used in for example Belgium and France, and is also being discussed in the Netherlands. With a VPP, the incumbent generator sells part of its production capacity to market entrants. This sale of generation capacity remains virtual as no production capacity changes hand. Legally, the incumbent generator remains the owner of all its generation plants.

Regulators often prefer VPPs above a divestiture, because the latter is irreversible, might be more costly and is politically difficult to implement. Moreover, as European markets become more integrated, the need for mechanisms to reduce market power might diminish.

VPPs can be implemented in two different ways. In Belgium, antitrust authorities oblige the incumbent to sell *financial VPPs*, while in the Netherlands the regulator has been discussing *physical VPPs*. The main difference between financial and physical VPPs is that a physical VPP is associated with a specific generation plant while a financial VPP is not.

If a retailer buys a physical VPP, he reserves generation capacity of a generation plant i.e. he buys the right produce one MW of production in that generation plant.

If a retailer buys a financial VPP, then the retailer receives a pure financial insurance contract. The contract is not linked to any generation plant. If the spot price increases above a certain level, then the retailer will receive a payment from the generator.[2]

Virtual Power Plants are characterized by a *virtual* production cost $S$ and a maximal production capacity $k$. If a retailer owns a physical VPP with a production cost $S$ and capacity $k$, then he can decide freely to produce electricity up to the production capacity $k$, as long as he refunds the generator the production costs $S$. If the retailer owns a financial VPP, he will receive money from the generator when the spot price $p$ is above the virtual production costs $S$. He receives the amount: $k \cdot \max\{0, p - S\}$. Essentially, physical and financial VPPs are thus physical and financial call options with a strike price $S$.

Options might have some advantages compared with futures contracts, which are contracts in which retailers are obliged to use the VPP always at full capacity:

- Options allow generators and retailers to *hedge* quantity risks, while futures can only be used to hedge price risks. Given that electricity cannot be stored very easily, quantity risks are very important in the electricity market, and options therefore play an important role.
- *Market power* is most pronounced during periods of peak demand and is characterized by high spot prices. Retailers might sign option contracts to counter the market power of generators during these periods[3].

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[2] The advantage of physical VPPs is that because specific production plants are assigned to the contracts, the probability that electricity is physically delivered increases. It might also induce generators to invest more in generation capacity. The disadvantage is that the counterparty risk is larger with physical options and that the production efficiency decreases when generation plants are scheduled in a more decentralized way.

- The electricity sector is characterized by a lot of missing markets. Often options are used to correct these problems.

### 1.3 This paper

In this paper we look at the strategic effects of physical and financial VPPs in a Cournot game. We assume that there are only two markets: a VPP market, where retailers buy virtual generation capacity and a spot market. In our set-up, firms decide themselves how many VPPs they sell. There is thus no regulation on the amount of VPPs that has to be sold. In the model there is no uncertainty, so hedging is not an issue. The number of generators is assumed to be fixed. Hence, we do not look at the entry decision of new generation firms.

The paper is an extension of Allaz and Vila (1993). They showed that, in a Cournot game, firms have a strategic reason to sell futures contracts, because futures contracts serve as a commitment device for the firms to obtain a larger market share in the spot market. Selling futures leads to a prisoners' dilemma type of problem. All firms sell futures, and as a result the spot price will decrease. We will use a similar framework as Allaz and Vila to analyze VPPs instead of futures contracts[4]

### 1.4 Relation with Chao and Wilson

The paper is closely related to recent work of Chao and Wilson (2004). They argued that generators should be obliged to sell physical VPPs to retailers. They see several reasons for this. (1) In the long run electricity markets are contestable and thus more competitive. (2) Physical call options might have better strategic effects than futures contracts. (3) The regulation of market power might be easier with physical call options than with futures. And (4) physical delivery makes sure that generation is effectively built.

Our paper looks at a similar problem as Chao and Wilson but makes different assumptions.

They assume perfect regulation of the number of options that generators have to sell and free entry in the contracting stage. In our paper we assume a fixed number of firms, and that generators decide themselves about the number of options they sell.

Chao and Wilson assume that generators bid linear supply functions in the spot market while we assume that they behave à la Cournot.

## 2 BENCHMARK: COURNOT GAME WITH FUTURES CONTRACTS

This section explains the standard Cournot game and the Cournot game with futures contracts (i.e. the Allaz and Vila model). It presents the set up of the model, and the definition of the main variables. The next section then continues with the Cournot game with VPPs.

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[3] Also the regulator can use options to aim its regulation more precisely at periods of high demand, minimizing its intervention in the market.

[4] Tthe results of Allaz and Vila depend critically on the assumptions of Cournot competition, perfect intertemporal arbitrage and observability of the contract positions. See Willems 2004 for references.

Our paper considers an oligopoly with two identical firms $i, j \in \{1,2\}$. Firm $i$ produces $q_i$ units at a production cost $C(q_i) = cq_i$. Total production of both firms is equal to $q_1 + q_2$, and the spot price $p$ is given by the inverse demand function:

$$p = P(q_1 + q_2). \quad (1)$$

### 2.1 Standard Cournot game

We start with the standard Cournot game without futures contracts, for which we will use the superscript 'C'.

The profit of a firm $i$ is equal to its revenue minus production costs:

$$\pi_i^C = (p - c) \cdot q_i. \quad (2)$$

In a Cournot game, firm *i* maximizes its profit (2), by setting its production quantity $q_i$, taking into account that the spot price depends on the joint production of the firms (1). All firms set their production level $q_i$ simultaneously.

The Nash equilibrium of this game is the intersection of the best response functions of the players.

To illustrate our paper, we will use a numerical example in which the inverse demand function is linear, and normalized to $P(q) = 1 - q$. We will write all solutions as function of the cost parameter $c$.

The equilibrium production quantities and spot price $\vec{q}^{\,C}(c)$ and $P^C(c)$ are given by

$$q_i^C = \frac{1-c}{3} \quad (3)$$

$$p^C = \frac{1+2c}{3} \quad (4)$$

where $\vec{q}$ is shorthand for the vector $(q_1, q_2)$.

### 2.2 Cournot game with futures contracts

If there are futures contracts, then we need to model the game with two stages: a contracting stage and a production stage. Figure 2 shows the timing of the game.

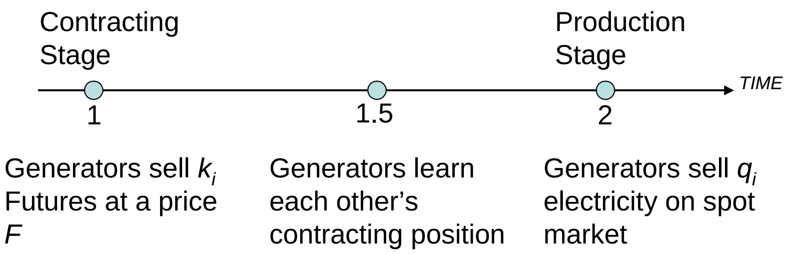


**Figure 2** Timing of the Cournot game with futures

In the first stage, the *contracting stage*, generators decide simultaneously about the number of futures contracts $k_i$ they sell to retailers.

Each futures contract is a two-sided insurance contract which insures the price of one unit of electricity. If the spot price $p$ is above the futures price $f$, then the generator will refund the retailer the difference of the spot price $p$ and the futures price $f$. If the spot price is below the futures price, then the retailer will pay the generator the difference between the futures price and the spot price. The total payment of generator $i$ is thus $k_i(p-f)$.[5]

After the first stage and before the second stage, each firm learns the contract position of the other firms. In Figure 2 this happens at time = 1.5. There is therefore perfect information at the beginning of the second stage.

In the second stage, the *production stage*, the firms simultaneously set their production level $q_i$. Each firm will take the contracting positions $\vec{k}$ as given.

Firm $i$'s profit is equal to revenue in the spot market, minus production costs and payments related to the futures contracts. The superscript *'F'* denotes the game with futures contracts.

$$\pi_i^F = (p-c)\cdot q_i - k_i(p-f) \qquad (5)$$

We will solve the game by backward induction and derive first the Nash Equilibrium in the second stage of the game as a function of the number of futures sold in the first stage $\vec{q}_{II}^F(\vec{k},c)$. After deriving the second stage equilibrium, we will solve the equilibrium of the first stage $\vec{k}_I^F(c)$.

### 2.3 Second Stage

Firm $i$ maximizes its profit (5) by setting its production $q_i$, taking the contracting position $\vec{k}$ as given and taking into account that the spot price is determined by (1).

In our small numerical example, the equilibrium quantity of the generators in the second stage is:

$$q_{II,i}^F(\vec{k},c) = \frac{1-c-2k_i+k_j}{3}\,. \qquad (6)$$

The fact that firm $i$ owns futures contracts changes its incentives to produce in the second stage. Firm $i$ needs to refund buyers of the futures contract for high strike prices. It has therefore less interest in high spot prices, and produces more in the second stage of the game.

Hence, owning futures contracts makes a firm more aggressive in the second stage, i.e. it produces more, and its reaction function moves outwards. This effect is based on exactly the same intuition as Coase's conjecture as explained in Figure 1.

### 2.4 Perfect arbitrage

Allaz and Vila, assume perfect arbitrage between the contracting and the production stage. This means that there is no profit to be made by arbitraging between the spot market and the futures market, i.e.

$$f = P(q_{II,1}^F(\vec{k},c) + q_{II,2}^F(\vec{k},c))\,. \qquad (7)$$

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[5] This discussion explains the futures contract as a financial insurance contract. An alternative explanation considers the futures contract as a physical contract. It can be shown that both approaches are equivalent.

Note that (7) implies that arbitrageurs correctly anticipate the strategic effects of the futures contracts on the spot price $p$.

### 2.5 First Stage

In the first stage the firms maximize their profit (5), taking into account that $q_i$ is determined by the second stage behavior of the firms (6), the price $p$ by the inverse demand function (1) and the forward price $f$ by arbitrage condition (7).

By selling more futures in the first stage, a generator can change the second stage equilibrium. By selling futures, total production in the second stage is increased, leading to a lower price. This influences firm 1's profit negatively. However, selling futures increases the market share of firm 1, which increases profit.

At the optimal number of futures both effects are balanced. This trade-off defines the first stage reaction functions of both firms.

In equilibrium, the generators will sell

$$k_{I,1}^F(c) = \frac{1-c}{5} \tag{8}$$

futures contracts in the first stage. The equilibrium price will be equal to

$$p_I^F(c) = \frac{1+4c}{5}. \tag{9}$$

## 3 VIRTUAL POWER PLANTS

VPPs will not be modeled as power plants with a constant virtual marginal production cost $S$. Instead we assume that the virtual production costs are quadratic.

$$\text{Virtual Cost} = \frac{q^2}{2\gamma} \tag{10}$$

The parameter $\gamma$ is a measure for the size of the Virtual Power Plant. As the VPP becomes larger (growing $\gamma$) the virtual marginal production costs will have a lower slope. The unit of $\gamma$ is $MW^2/\$$

We choose this approach in order to reduce problems which are related with corner solutions in the model. (See Willems 2004 for constant virtual marginal costs.)

Another interpretation of VPPs with increasing marginal costs is that of a linear bundle of call options with different strike prices. The generators sell $\gamma$ bundles of call options to retailers. Each bundle contains $dS$ options with a strike price between $S$ and $S+dS$.

## 4 FINANCIAL VIRTUAL POWER PLANTS

A Financial VPP is a linear bundle of financial call options. A financial call option is a one-sided insurance contract which insures retailers against price increases above the strike price $S'$. If the spot price $p$ is above the strike price, then the generator will refund the retailer the difference between the spot price and the strike price: $p-S'$. When the spot price is below strike price, then there is no payment. In short, the generator pays the retailer the amount $\max\{P-S',0\}$.

If a retailer buys $\gamma$ bundles, it owns $\gamma dS'$ financial call options with a strike price $S'$. The generator pays the retailer the amount $(\gamma dS')\cdot\max\{p-S',0\}$.

The total payment by the generator to the retailer is equal to

$$\int_0^\infty \gamma\max\{0,p-S'\}dS' = \gamma\frac{p^2}{2}. \tag{11}$$

The profit of the generation firm is the sum of the profit in the spot market and profit in the financial market. In the spot market the generator sells $q_1$ units of electricity at a price $p$. The production cost is $c$. In the financial market, the firm sells $\gamma$ VPPs at a price $f$ but it will need to refund the retailers the amount (11).

The profit of the generator is equal to

$$\pi_i^{fO} = (p-c)q_i + \gamma(f-\frac{p^2}{2}) \tag{12}$$

where we use the superscript "*fO*" for financial options.

### 4.1 Second Stage

In the second stage generators set their production quantity $q_1$ and maximize their profit (12), taking into account that the spot price is determined by (1), and assuming that $f$ and $\gamma$ are fixed.

In our example, the Nash equilibrium of second stage is

$$q_{II,i}^{fO}(\vec{\gamma},c) = \frac{1+\gamma_i+c(\gamma_i-\gamma_j-1)}{3+\gamma_i+\gamma_j}. \tag{13}$$

### 4.2 Perfect arbitrage

We assume perfect arbitrage between the first stage and the second stage of the game. The price for buying a financial VPP needs to be equal to the expected pay-out (11).

$$f = \frac{p^2}{2} \tag{14}$$

### 4.3 First Stage

In the first stage of the game, generators will sell bundles of financial VPPs, maximizing their profit function (12), taking into account that the spot price $p$, the production quantities $q$ and the price of the bundle $f$ are determined by equations (1) (13) and (14). The Nash Equilibrium in the first stage of the game is equal to:

$$\vec{\gamma}_I^{fO}(c) = \frac{8+\sqrt{17c}}{4} - \frac{5}{4}. \tag{15}$$

The equilibrium price for electricity is then equal to

$$p_I^{fO} = \frac{2\sqrt{c}(1+2c)}{\sqrt{c}+\sqrt{8+17c}}. \tag{16}$$

## 5 PHYSICAL VIRTUAL POWER PLANTS

Physical options are more difficult to model than financial options. The reason for this is that we now also have to model the production decisions of the retailers who reserved production capacity.

If a retailer bought $\gamma$ bundles of VPPs, then it reserved $\gamma dS'$ MW of a virtual plant with a virtual production cost equal to $S'$. This bundle of infinitesimal small production plants with increasing production costs is equivalent to a virtual production plant with a virtual production cost

$$VC(q_V) = \frac{{q_V}^2}{2\gamma}. \tag{17}$$

In the second stage the retailer will minimize his procurement costs for electricity. It can buy electricity on the spot market at a price $p$, or can use its virtual power plant to produce electricity $q_V$ at a cost $\frac{{q_V}^2}{2\gamma}$. In the optimum, the retailer will use its virtual power plant up to the point where the virtual marginal production costs is equal to the spot price $VC'(q_V) = p$, or:

$$q_V = \gamma p. \tag{18}$$

Equation (18) describes the behavior of the retailers. We can now derive the spot price when generators produce $\vec{q} = (q_1, q_2)$ with the generation plants which were not reserved, and sold $\vec{\gamma} = (\gamma_1, \gamma_2)$ VPPs in the first period of the game.

The equilibrium price $p$ depends on $\vec{\gamma}$ and $\vec{q}$, and is determined by the following two equations:

$$\begin{aligned} &P(q_1 + q_2 + q_V) = p \\ &q_V = (\gamma_1 + \gamma_2)p \end{aligned}. \tag{19}$$

The profit of a generator firm is equal to

$$\pi_i^{pO} = q_i p + \gamma_i f + \gamma_i \frac{p^2}{2} - (q_i + \gamma_i p)c. \tag{20}$$

The first term reflects the revenue from selling production with unreserved production capacity in the spot market, the second term is the revenue from selling physical VPPs in the contracting stage of the game. The third term is the revenue received from retailers when they use their VPPs, and the last term reflects the total production costs of the firm. Total production of firm $i$ is equal to $q_i + \gamma_i p$.

### 5.1 Second Stage Equilibrium

In the second stage, generators will simultaneously set their production quantities $q_i$ in order to maximize their profit function (20), taking into account that the amount $\vec{\gamma}$ of VPPs sold in the first stage are fixed and that the spot price is determined by equations (19).

The second stage Nash Equilibrium in the numerical application is:

$$\vec{q}^{\,pO}_{II,i}(\vec{\gamma},c) = \frac{(1+\gamma_j)(1-c(1+\gamma_i+\gamma_j))}{3+2\gamma_i+2\gamma_j} \quad (21)$$

The equation describes how the generators will produce in the second stage of the game.

### 5.2 Arbitrage

As in the Allaz and Vila model we assume that there is perfect arbitrage between the first period and the second period of the game. The perfect arbitrage condition requires that

$$\gamma f + \gamma \frac{p^2}{2} = \gamma p^2 . \quad (22)$$

The left side is the cost of buying $\gamma$ bundles of physical options for a price $f$ and paying $\gamma \frac{p^2}{2}$ for a total production of $\gamma p$ units of electricity. The right hand side is the cost of buying $\gamma p$ units of electricity on the spot market at a price $p$. Hence we obtain that the price $f$ of a physical VPP is

$$f = \frac{p^2}{2} . \quad (23)$$

### 5.3 First Stage Equilibrium

In the first stage of the game, generators will sell a bundle of physical options, maximizing their profit function (20), taking into account that production quantities $q$, the spot price $p$ and the price of VPPs $f$ are given by (21), (19) and (23). Generators can only sell a positive amount of physical VPPs $\gamma \geq 0$.

The generators will sell their VPPs in a non-cooperative way. In our example, the equilibrium in the first stage is

$$\gamma^{pO}_{II}(c) = \max(0, \gamma^*(c)) \quad (24)$$

where $\gamma^*(c)$ is the root of a polynomial $A$ of the third order:

$$A = (1+5c-4c^2) - (4-10c+21c^2)\gamma$$
$$-8c(-1+4c)\gamma^2 - 16c^2\gamma^3 \quad (25)$$

According to equation (24) generators will sell no physical VPPs when the production cost is below $1/4$. For production costs above $1/4$ generators will sell a positive amount.

The equilibrium price is given by

$$p^{pO}_I(c) = \frac{1+2c(1+\gamma^{pO}_I(c))}{3+4\gamma^{pO}_I(c)} . \quad (26)$$

## 6 COMPARISON

Figure 3 shows the equilibrium amount of VPPs as function of the cost $c$. The dotted and continuous line present the financial and physical VPPs respectively. Note that generators do not sell physical VPPs if the production cost is below $1/4$.

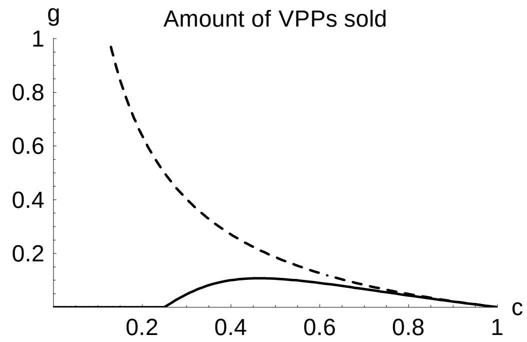


**Figure 3** Amount of VPPs sold in equilibrium.

Figure 4 shows the final price as function of the production cost of the generators for four different cases. The thick dashed and thick continuous lines are the equilibrium prices of the Cournot game with financial and physical VPPs respectively. The thin dotted and thin continuous lines are equilibrium prices of the Cournot game with forward contracts, and the standard Cournot game.

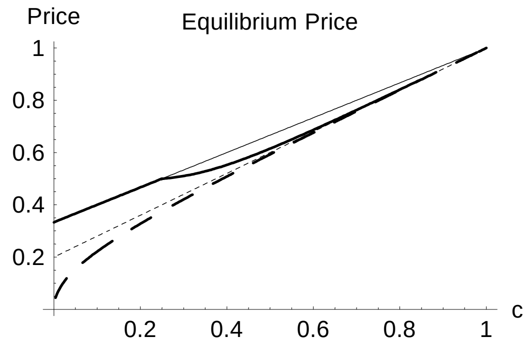


**Figure 4** Equilibrium prices

Figure 4 shows that prices with financial VPPs are lower than those with physical VPPs. Prices with forward contracts lie somewhere in between.

In the example total welfare is uniquely determined by the equilibrium price. A lower price corresponds with a higher welfare. Financial VPPs are thus preferable from a societal viewpoint.

## 7 CONCLUSION

This paper shows that there are large differences in the strategic effects of physical and financial VPPs. It also shows that financial Virtual Power Plants might be preferred to physical Virtual Power Plants.

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