Optimal Monetary Policy Rules for the Euro Area in a DSGE Framework*

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Abstract

This paper evaluates optimal monetary policy rules within the context of a dynamic stochastic general equilibrium model estimated for the Euro Area. Under assumption of an ad hoc loss function for the central bank, we compute the unconditional losses both under discretion and commitment. We compare the performance of unrestricted optimal rules to the performance of optimal simple rules. The results indicate that there are considerable gains from commitment over discretion, probably due to the stabilization bias present under discretion. The lagged variant of the Taylor type of rule that allows for interest rate inertia does relatively well in approaching the performance of the unrestricted optimal rule derived under commitment. On the other hand, simple rules expressed in terms of forecasts to next period's inflation rate seem to perform relatively worse.

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1 Introduction

Estimated Dynamic Stochastic General Equilibrium (DSGE) models have recently become successful in replicating the dynamics observed in data and matching results from previous empirical VAR studies[1]. The logical next step is to use these models in monetary policy evaluation since this macroeconomic framework provides us with the important and favourable advantage of protection against the Lucas critique. Since aggregate relationships are derived explicitly from intertemporal optimizing behaviour by rational and forward-looking agents which are led by their preferences and technology, the structural relations become invariant to changes in the monetary policy regime.

The aim of this paper is to study optimal monetary policy within the context of such an Estimated New-Keynesian DSGE model for the Euro Area developed by Smets and Wouters (SW) (2003). We compare the performance of both unrestricted and simple optimal monetary policy rules under alternative monetary policy preferences over the target variables. We assume that the central bank's intertemporal loss function can be approximated by an ad hoc loss function as commonly applied in the literature, which is quadratic in the target variables (i.e. the model-consistent output gap, inflation and interest rate). We compute the fully optimal monetary policy rules under commitment and under discretion together with the corresponding values of the loss function. Accordingly, the gains from commitment are discussed and possible explanations for these gains are suggested. The results obtained under unrestricted optimal rules under commitment are compared to those under alternative specifications of optimal simple rules both with and without the possibility of interest rate smoothing.

The remainder of the paper is organized as follows. In the next section we briefly summarize the main features of the model and present the linearized version of the structural equations around the nonstochastic steady state. We then outline the general setup of the optimization problem faced by the central bank and present the assumed form of the loss function in section three. In section four we perform our monetary policy evaluation exercise by first considering the case of unrestricted optimal rules where particular attention is paid to gains from commitment over discretion. We then turn to the discussion of the performance of optimal simple rules where distinction is made between the baseline specification without interest rate smoothing and the smoothing variant where interest rate inertia is allowed for. We examine which type of simple rule performs best in approaching the performance of unrestricted optimal rules in yielding the lowest possible unconditional losses under commitment. Finally section five concludes.

[1] Leading examples are the models derived by Christiano, Eichenbaum and Evans (2001) and Smets and Wouters (2003).

2 The Model

We assume that the Euro Area economy is described by the DSGE model presented in SW (2003). We summarize the main features and present the linearized version of the model. For a more detailed description of the model and the underlying assumptions in deriving the linear rational expectations equations, we refer to the original paper SW (2003). We keep the same notation as in SW (2003), where all variables are expressed as log deviations from their steady state levels denoted by $\wedge$, i.e. $\hat{x} = \log\left(\frac{x}{x*}\right)$.

The model features nominal rigidities in the goods and labour markets, augmented by real rigidities like habit formation, costs of adjustment in capital accumulation and variable capital utilization. It shows many similarities with and builds further upon the model presented in Christiano, Eichenbaum and Evans (CEE) (2001) with the main distinction that it includes an additional number of structural shocks, (Bayesian) estimation method and partial indexation to past inflation in the labour and the goods markets. The main purpose of the inclusion of persistence is to provide a better empirical fit. A consequence of this persistence for monetary policy is that policy effects will come with a delay and therefore monetary policy should respond to current and future forecasts of inflation.

2.1 Household sector

The model consists of a household sector. Each household supplies a differentiated type of labour and maximizes an intertemporal utility function of the following type:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^{\tau} \tag{1}$$

where β is the discount factor. The utility function is separable in consumption and labour, defined as follows:

$$U_t^{\tau} = \varepsilon_t^b \left(\frac{1}{1-\sigma_c} \left(C_t^{\tau} - H_t\right)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(l_t^{\tau}\right)^{1+\sigma_l} \right) \tag{2}$$

This set up leads to the following consumption Euler equation:

$$\hat{C}_t = \frac{h}{1+h} \hat{C}_{t-1} + \frac{1}{1+h} E_t \hat{C}_{t+1} - \frac{1-h}{(1+h)\,\sigma_c} \left(\hat{R}_t - E_t \overset{\wedge}{\pi}_{t+1} \right) + \frac{1-h}{(1+h)\,\sigma_c} \left(\overset{\wedge}{\varepsilon}{}_t^b - E_t \overset{\wedge}{\varepsilon}{}_{t+1}^b \right) \tag{3}$$

where h is the degree of habit persistence in the external habit variable $H_t = hC_{t-1}$. The symbol σ_c represents the inverse of the intertemporal elasticity of substitution between consumption at different dates. $\hat{R}_t$ is the nominal interest rate, $\overset{\wedge}{\pi}_t$ the inflation rate and $\overset{\wedge}{\varepsilon}{}_t^b$ is an AR(1) preference shock to the discount rate with an i.i.d. normal error term. Since households supply a differentiated type of labour in a labour market that is characterized by nominal wage rigidities, they set their nominal wages according to a Calvo (1983) type of scheme. In this context, households are only able to adjust their nominal wages optimally according to a constant probability $1-\xi_w$ after receiving a wage change signal. Households that cannot reoptimize will adjust their nominal wages partially to past inflation with a degree $0 \leq \gamma_w \leq 1$, leading to the real wage equation:

$$\begin{aligned}\hat{w}_t &= \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} \\ &\quad -\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{(\frac{\lambda_w+(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w}\left[\hat{w}_t - \sigma_L\hat{L}_t - \frac{\sigma_c}{1-h}(\hat{C}_t - h\hat{C}_{t-1}) - \hat{\varepsilon}_t^L - \eta_t^w\right] \end{aligned} \tag{4}$$

where ξ_w is the probability that the wage will not be reoptimized in the next period. λ_w is a constant in $\lambda_{w,t} = \lambda_w + \eta_t^w$ with $\lambda_{w,t}$ a shock to the wage mark-up assumed to be i.i.d. normal around the constant term. σ_L is the inverse of the elasticity of work with respect to the real wage, $\hat{L}_t$ the aggregate labour and $\hat{\varepsilon}_t^L$ a shock to labour supply assumed to follow an AR(1) process with an i.i.d. normal error term. The final term in square brackets shows the effect of the discrepancy between the real wage prevailing in the labour market today and the wage that would be set if the labour market would be fully flexible. Households also make the investment and capital accumulation decision, they are owners of the capital stock and rent it out to the producers of intermediate goods at the rental rate of capital r_t^k. the supply of rental services can be adjusted by increasing investments I_t or by increasing the utilization rate z_t of installed capital. The linearized investment equation, which is characterized by adjustment costs depending on the size of investments, is given by the following equation:

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1} + \frac{\varphi}{1+\beta}\hat{Q}_t + \frac{\beta E_t\hat{\varepsilon}_{t+1}^I - \hat{\varepsilon}_t^I}{1+\beta} \tag{5}$$

with φ the inverse of adjustment costs which captures the investment dynamics and $\hat{Q}_t$ the real value of capital expressed in units of consumption goods. $\hat{\varepsilon}_t^I$ is an AR(1) shock to investment costs with an i.i.d. normal error term. The real value of capital is represented as:

$$\hat{Q}_t = -(\hat{R}_t - \hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_t\hat{Q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t\hat{r}_{t+1}^k + \eta_t^Q \tag{6}$$

with τ the depreciation rate, $\bar{r}^k$ the steady state real rental rate of capital and η_t^Q an i.i.d. normal shock that captures changes in the external finance premium due to informational frictions. However, this shock is not related to the structure of the economy and it is further assumed that distortions caused by informational frictions are zero in the steady state. By definition $\beta = 1/(1-\tau+\bar{r}^k)$. Finally, the capital accumulation equation fulfills:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\hat{I}_t \tag{7}$$

2.2 Technologies and Firms

There is a final goods sector and an intermediate goods sector. The final goods sector is perfectly competitive and produces one final good used for consumption and investment by the households. The intermediate goods sector is characterized by monopolistic competition where each firm is the only producer of the intermediate good j. In the intermediate goods sector each monopolist sets its price, given the total demand function for its differentiated good in line with Calvo (1983). As in the case of wage setting by households, each firm can only reoptimize its price after receiving a price change signal with a constant probability $1-\xi_p$. Firms that cannot reoptimize will adjust their price partially to past inflation with a degree $0 \leq \gamma_p \leq 1$. This leads to the following New-Keynesian Phillips curve:

$$\begin{aligned} \hat{\pi}_t &= \frac{\beta}{1+\beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \hat{\pi}_{t-1} \\ &+ \frac{1}{1+\beta\gamma_p} \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p} \left[\alpha \hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{\varepsilon}_t^a + \eta_t^p\right] \end{aligned} \tag{8}$$

with α the share of capital in total output in the Cobb-Douglas production function in the intermediate goods sector with constant returns to scale. $\hat{\varepsilon}_t^a$ is an AR(1) productivity shock with i.i.d. normal error term. The final term in square brackets represents the real marginal costs augmented with an i.i.d. (cost-push shock) η_t^p to the mark-up in the goods market, i.e. $\lambda_{p,t} = \lambda_p + \eta_t^p$. The labour demand equation for a given capital stock, which results from (the linearized version of) the cost minimization condition is given by:

$$\hat{L}_t = -\hat{w}_t + (1+\mathbf{\Psi})\hat{r}_t^k + \hat{K}_{t-1} \tag{9}$$

where $\mathbf{\Psi}$ is the inverse of the elasticity of the capital utilization cost function, which comes from the condition $\hat{r}_t^k = \frac{1}{\mathbf{\Psi}}\hat{z}_t$ and determines the dynamics.

2.3 Equilibrium

The goods market equilibrium condition is given by the following expression where total output equals total demand for output by households and the government (first line) and total supply of output by the firms (second line):

$$\begin{aligned} \hat{Y}_t &= c_y \hat{C}_t + g_y \varepsilon_t^G + \tau k_y \hat{I}_t + \bar{r}^k k_y \mathbf{\Psi} \hat{r}_t^k \\ &= \phi(\hat{\varepsilon}_t^a + \alpha \hat{K}_t + \alpha \mathbf{\Psi} \hat{r}_t^k + (1-\alpha)\hat{L}_t) \end{aligned} \tag{10}$$

where ε_t^G is an AR(1) government spending shock with i.i.d. normal error term. This introduction of government spending as an exogenous AR(1) process is justified e.g. in Woodford (2003) by the assumption of Ricardian equivalence of fiscal policy. Under this type of (locally Ricardian) policy rule, details of the fiscal policy rule are not relevant and have no effect on the aggregate economic variables (inflation, output, interest rate). c_y, $g_y$and k_y stand for the steady state ratio of consumption to output, government spending to output and capital to output, respectively. ϕ is equal to $1+\frac{\Phi}{\bar{Y}}$ where the second term is the ratio of fixed production cost to output in the steady state.

2.4 Monetary Policy

Finally the model in SW (2003) is closed by an empirical Taylor-type of rule including interest rate smoothing with smoothing parameter ρ and additional response to changes in the output gap and inflation:

$$\begin{aligned} \hat{R}_t &= \rho\hat{R}_{t-1} + (1-\rho)\left\{\bar{\pi}_t + r_\pi(\hat{\pi}_{t-1} - \bar{\pi}_t) + r_y(\hat{Y}_t - \hat{Y}_t^p)\right\} \\ &\quad + r_{\Delta\pi}(\hat{\pi}_t - \hat{\pi}_{t-1}) + r_{\Delta y}((\hat{Y}_t - \hat{Y}_t^p) - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^p)) + \eta_t^R \end{aligned} \quad (11)$$

The interest rate $\hat{R}_t$ is the monetary policy instrument, $\bar{\pi}_t$ is the inflation target that follows an AR(1) process with i.i.d. normal error term and is normalized to zero. η_t^R is an i.i.d. normal monetary policy shock and $(\hat{Y}_t - \hat{Y}_t^p)$ is the output gap defined as the difference between the actual output level and the level of potential output. The latter is assumed to be the output level under the absence of nominal rigidities (i.e. full price- and wage flexibility) and the three i.i.d. cost-push shocks $(\eta_t^Q, \eta_t^p, \eta_t^w)$. Since the potential output level is determined by the model, we extend the model including the set of linear equations (3)-(11) with their flexible wage and price versions in absence of the three cost-push shocks. For our purpose of the evaluation of alternative optimal monetary policy rules, the estimated Taylor rule in (11) will be either dropped out of the model in computing the unrestricted optimal reaction function or modified to a specific simple rule form with only a few free parameters to be optimized.

In summary $(\hat{\pi}_t, \hat{w}_t, \hat{K}_{t-1}, \hat{Q}_t, \hat{I}_t, \hat{C}_t, \hat{R}_t, \hat{r}_t^k, \hat{L}_t)$, i.e. the set of nine endogenous variables in the model, are described by the system of linear rational equations (3)-(11). The set of ten exogenous shocks consists of five AR(1) shocks with i.i.d. normal error term related to preferences and technology, $(\varepsilon_t^b, \varepsilon_t^L, \varepsilon_t^I, \varepsilon_t^a, \varepsilon_t^G)$, three i.i.d. cost-push shocks $(\eta_t^Q, \eta_t^p, \eta_t^w)$ and two monetary policy shocks $\left(\bar{\pi}_t, \eta_t^R\right)$ with the first term being AR(1) and the second i.i.d. As for the parameter values, we take the values given in SW (2003) that result from Bayesian estimation techniques. These parameter values are either estimated using Euro Area data, which makes it suitable for our framework, or calibrated following standard practice in the literature. The following parameters (and their respective values) are to be fixed: $\beta = 0.99$, $\tau = 0.025$ (per quarter), $\alpha = 0.30$, $c_y = 0.6$, $k_y = 2.2$, $k_I = 0.22$ and $\lambda_w = 0.5$. For the estimated values of the remaining parameters in the model and details concerning the estimation procedure we refer to SW (2003)[2].

3 General setup of the optimization problem

The aim of this paper is to evaluate the performance of alternative monetary policy rules in the context of the model for the Euro Area described in the previous section[3]. We will consider both unrestricted optimal rules as well as optimal simple rules that are restricted in the parameters they are allowed to respond to. The unrestricted optimization will be carried out under commitment and discretion in order to analyze the gains from commitment. We now proceed with the description of the general setup of the problem that applies throughout this paper.

3.1 The Loss function

The objective of the central bank is assumed to be the minimization of a standard discounted, quadratic intertemporal loss function expressed in terms of target variables of the following type:

$$E_t \sum_{i=0}^{\infty} \delta^i [y'_{t+i} W y_{t+i}], \qquad 0 < \delta < 1 \tag{12}$$

with δ the discount factor and E_t the expectations operator conditional on information available at time t. The vector $y_t = [x'_t \;\; u'_t]'$ contains the $n \times 1$ endogenous variables and AR(1) exogenous variables in the model included in x_t[4] and the $p \times 1$ vector of control variables included in u_t. W is a time-invariant symmetric, positive semi-definite matrix of given policy weights[5].

Although within the context of the model it is possible and even preferable to derive a consumer welfare based approximation to the loss function consistent with the microfoundations of the model, we will leave this for future work. For the time being we will focus on the standard ad hoc loss function, following a remarkable stream in the literature on monetary policy analysis as in Rudebusch

[2] A similar exercise in estimating the parameters in the SW (2003) model has been performed by Onatski and Williams (2004). These authors find different parameter estimates in using different prior specifications, but get to the same conclusions concerning the qualitative features of the model.

[3] A similar exercise is carried out by Rotemberg and Woodford (1998) for a structural model of the US economy under a welfare based approximation to the loss function.

[4] Note that, for practical convenience we include the four i.i.d. shocks $\left(\eta_t^Q, \eta_t^p, \eta_t^w, \eta_t^R\right)$ in the x_t vector by writing them in AR(1) form and assigning a zero value to the AR(1) coefficient. This enables us to include all ten shocks in the x-vector.

[5] As noted by Dennis (2004), with this type of loss function formulation, possible penalties on variations in the policy instruments and on interactions between variables in the x-vector and policy variables in the u-vector are easily incorporated in W.

and Svensson (1998) and Svensson (2003). As shown by Woodford (2003) in more simple models this type of loss function can be obtained from the microfoundations[6].

The one period loss function in terms of the target variables for (12) is:

$$L_t = \hat{\pi}_t^2 + q_y(\hat{Y}_t - \hat{Y}_t^p)^2 + q_r \hat{R}_t^2 \tag{13}$$

where $q_y \geq 0$ and $q_r \geq 0$. Since we normalized the inflation target $\bar{\pi}_t$ to zero, the inflation rate enters in the loss function, together with the output gap and the variability of the interest rate. This makes our loss function to have a commonly adopted form as e.g. in Giannoni and Woodford (2002). The inclusion of the latter term can be justified for reasons such as ensuring financial stability or to explain the high inertia observed in the policy instrument. Note that it is also common to include interest rate smoothing instead of the interest rate level as a target variable, in which case the final term would become $(\hat{R}_t - \hat{R}_{t-1})^2$ for the same reasons[7],[8]. The weights assigned to the output gap and the interest rate in the loss function are determined by the preferences of the central bank. In the case that $q_y = q_r = 0$, such that the central bank only takes into account the variability of the inflation rate, we can define the monetary policy strategy as one of "strict" inflation targeting as defined in Svensson (1998). A monetary policy strategy that allows for nonzero weights on other variables than the inflation rate can be considered as one of "flexible" inflation targeting. In our optimal monetary policy evaluation exercise, we will experiment with alternative combinations of $[q_y, q_r]$ and compare the corresponding losses from the different possible formulations.

In our application below we will assume the limiting case of (12) where $\delta = 1$. This will allow us to write the intertemporal loss function as the infinite sum of the unconditional means of the period's loss function (13). Therefore the loss function to be minimized can be expressed as follows[9]:

$$E[L_t] = Var[\hat{\pi}_t] + q_y Var[(\hat{Y}_t - \hat{Y}_t^p)] + q_r Var[\hat{R}_t] \tag{14}$$

3.2 Representation of the structural model

Most of the literature on solving for optimal monetary policy rules within the context of rational expectations models is based on the state space representation of the structural model where the variables are

[6] For a welfare based approximation to the loss function in the SW (2003) model, see Onatski and Williams (2004). See also Rotemberg and Woodford (1998), Giannoni and Woodford (2003) and Amato and Laubach (2002), among others, for the same exercise in other DSGE models.

[7] See, for example, Rudebusch and Svensson (1998) and Giannoni and Woodford (2003) for this version of the loss function.

[8] Since the results do not change qualitatively when we replace the term $\hat{R}_t^2$ by $(\hat{R}_t - \hat{R}_{t-1})^2$, we will focus here only on the representation of the loss function as in equation (13).

[9] See for example Svensson(1998) and Dennis (2005) and the Appendices therein for a detailed description of the steps undertaken to come to this expression.

ordered according to their nature of being either "predetermined" or "forward-looking". Algorithms based on this tradition are presented e.g. in Söderlind (1999) and Backus and Driffil (1986). Applications based on this distinction can be found in Rudebusch and Svensson (1998), Svensson and Woodford (2000) and Giannoni and Woodford (2002). However, in this work we will apply the algorithms suggested in Dennis (2004) and Dennis (2005) that do not require such distinction of variables. Instead, we write the structural representation of the model under study in the following second-order form:

$$Ax_t = BE_t x_{t+1} + Fx_{t-1} + Gu_t + Dz_t, \qquad z_t \sim iid[0, \Sigma_{zz}] \tag{15}$$

where z_t is an $n \times 1$ vector of stochastic innovations to the variables in x_t with mean zero and variance-covariance matrix Σ_{zz}.

4 Optimal Monetary Policy Evaluation

In this section we perform the optimization exercise. We start the discussion by considering first the fully optimal rules and continue afterwards with the analysis of some alternative forms of simple rules. The rules are evaluated according to their corresponding losses computed under alternative assumptions of policy preferences $[q_y, q_r]$.

4.1 Performance of Unrestricted Optimal Rules

The optimization problem outlined in the previous section is the same both under discretion and under commitment. The standard framework of linear constraints (15) to the quadratic objective function (12) enables us to solve for the explicit optimal reaction functions that are linear with respect to the state variables. The optimization procedure, however, depends on the assumptions we make with respect to the degree of commitment made by the monetary authorities. We first consider the commitment case and proceed afterwards with the assumption that the optimization problem is faced under discretion.

4.1.1 Commitment

In the case of full commitment the central bank optimizes only once at time t_0 and promises to stick to the resulting policy rule forever, respecting past commitments in the future periods to come after t_0. This is represented by the presence of the Lagrange multipliers in the optimal reaction function, which makes policy history dependent. Only at t_0 when the optimization occurs, past commitments are ignored and initial values of the Lagrange multipliers are set to zero[10]. Therefore the equilibrium

[10] Woodford (2003) shows the case of optimality from a "timeless perspective" in which the Lagrange multipliers are set to nonzero even in the initial period, leading to a time consistent equilibrium.

is not time consistent and expectations of the private sector are exploited in the initial period, but this is promised to happen only once.

Following Sims (2002) and Dennis (2005), we use the definition of rational expectations,

$$E_t x_{t+1} + \eta^x_{t+1} = x_{t+1} \tag{16}$$

to substitute for the expectational term in the structural equation (15) and partition the weight matrix W into Q, U and R[11] in order to express the loss function (12) in terms of the variables x_t and u_t,

$$E_t \sum_{i=0}^{\infty} \delta^i [x'_{t+i} Q x_{t+i} + 2x'_{t+i} U u_t + u'_t R u_t], \qquad 0 < \delta < 1 \tag{17}$$

which simplifies the optimization procedure. This results into the following Euler equations that can be represented in the second-order form:

$$A1^* Y_t = B1^* E_t Y_{t+1} + C1^* Y_{t-1} + D1^* z_t \tag{18}$$

where

$$A1^* = \begin{bmatrix} Q & U & A' \\ U' & R & -G' \\ A & -G & 0 \end{bmatrix} \qquad B1^* = \begin{bmatrix} 0 & 0 & \delta F' \\ 0 & 0 & 0 \\ B & 0 & 0 \end{bmatrix}$$

$$C1^* = \begin{bmatrix} 0 & 0 & \frac{1}{\delta}B' \\ 0 & 0 & 0 \\ F & 0 & 0 \end{bmatrix} \qquad D1^* = \begin{bmatrix} 0 \\ 0 \\ D \end{bmatrix} \qquad \text{and } Y_t = \begin{bmatrix} x_t \\ u_t \\ \rho_t \end{bmatrix} = \begin{bmatrix} y_t \\ \rho_t \end{bmatrix}$$

and the final term in Y_t, ρ_t, is the vector of Lagrange multipliers. After transforming equation (18) into the first-order form, we solve the system using the generalized Schur decomposition method as suggested in Sims (2002)[12].

We derive the optimal explicit[13] monetary policy rules for seven alternative specifications of policy preferences, i.e. the pairs of weights $[q_y, q_r]$ assigned to the output gap and the interest rate in the objective function, respectively. The first one is the combination $[0, 0]$, the case of "strict" inflation targeting where only inflation variability is allowed to enter the loss function. For computational reasons, however, we will assign values to $[q_y, q_r]$ that are not exactly equal to zero, but slightly positive[14].

[11] Note that this matrix is not the same as the interest rate $\hat{R}_t$.

[12] An alternative based on the same principle is suggested by Klein (2000), which relies on the separation of variables into "predetermined" and "non-predetermined" blocks. Anderson and Moore (1985) provide another alternative.

[13] We adopt here the same definition as in Svensson (1998) and Svensson and Woodford (2003) for the "explicit instrument rule". By the latter we mean an instrument rule that is solved as an explicit function of the predetermined and exogenous variables.

[14] This was necessary in order to obtain a determinate solution. Therefore we assign the combination $[0.01, 0.01]$ instead of $[0, 0]$, which yields a stable and unique solution. Since these values are almost zero, their effect will be negligable. The

The second case involves the pair $[0.5, 0.5]$, i.e. flexible inflation targeting with equal concern for variability in output and interest rate where inflation variability is still of prior concern. We also consider the combination $[1, 1]$, where equal weight is assigned to all variables in the loss function, $[1, 0.5]$ the case where output and inflation variability are equally distasteful and are assigned a larger weight than interest rate variability. The combination $[1, 0.01]$ is a similar strategy to the previous one but here the central bank is even less concerned about interest rate variability. The final two strategies involve $[0.2, 0.5]$ and $[2, 0.5]$, the latter being the case where output gap variability becomes more costly than inflation variability. The minimized losses based on the variability of the target variables in terms of unconditional variances in the limiting case $\delta = 1$ for all seven cases are reported in Table 1. Before we discuss these losses obtained from optimization under commitment, however, it is interesting to contrast the results with their discretionary counterparts that will be computed in the next section where we will adopt the standard linear dynamic programming approach. This will allow us to examine how much the degree of commitment affects the results within the context of the particular structural model we are using in the current study. We now turn to a short description of the optimization procedure we apply under discretion and calculate the corresponding unconditional variances and losses for the same seven cases considered under commitment.

4.1.2 Discretion

Under discretion the central bank reoptimizes every period, taking expectations of the private sector as given. This leads to a time consistent Markov-perfect Nash equilibrium. In other words, in discretionary equilibrium reoptimization will lead to the same interest rate rule, which is optimal for any given range of expectations that cannot be affected by the current actions of the central bank. Accordingly, today's actions will not affect the future behaviour of the economy. Following Dennis (2005), the discretionary solution to be obtained through backward iteration is assumed to be given by the following linear relations:

$$\begin{aligned} x_t &= H_1 x_{t-1} + H_2 z_t \\ u_t &= F_1 x_{t-1} + F_2 z_t \end{aligned} \tag{19}$$

problem of indeterminacy is probably due to the fact that a zero weight is assigned to the third term, q_r. It is not uncommon that a nonzero weight has to be assigned to the interest rate variability in order to resolve the issue of indeterminacy. Apart from this technical justification, the reason why this weight should be nonzero in the objective function is that it might take account for the observed inertial behaviour of the interest rates. Although the latter reason is admittedly not a "micro-founded" justification either. See e.g. Woodford (1999) for more on this issue.

Where in equilibrium the reaction function becomes a function of the state only. The solution procedure simply boils down to finding the time invariant matrices H_1, H_2, F_1 and F_2. Rewriting (15) by replacing the expectational term $E_t x_{t+1}$ by $H_1 x_t$ as suggested in (19) results in the following expression for the structural form:

$$Ax_t = BH_1 x_t + Fx_{t-1} + Gu_t + Dz_t \tag{20}$$

$$Mx_t = Fx_{t-1} + Gu_t + Dz_t \tag{21}$$

where

$$M = (A - BH_1)$$

After writing the loss function (12) in terms of H_1, H_2, F_1, F_2, x_t, and u_t, we substitute the constraint (21) into the objective function and transform the problem into one of unconstrained optimization. The resulting first order condition with respect to the instrument is then solved for the explicit optimal monetary policy rule, which has the form as in (19), with

$$\begin{aligned} F_1 &= -(R + G'M'^{-1}PM^{-1}G)^{-1}G'M'^{-1}PM^{-1}F \\ F_2 &= -(R + G'M'^{-1}PM^{-1}G)^{-1}G'M'^{-1}PM^{-1}D \end{aligned} \tag{22}$$

and implying the following forms for H_1 and H_2 by substitution in (21):

$$\begin{aligned} H_1 &= M^{-1}(F + GF_1) \\ H_2 &= M^{-1}(GF_2 + D) \end{aligned} \tag{23}$$

where

$$P = Q + \delta F_1' R F_1 + \delta H_1' P H_1 \tag{24}$$

is the matrix Sylvester equation[15]. Analogously with the previous section on commitment, we solve for the explicit optimal monetary policy rules under discretion, for seven alternative pairs of policy preferences $[0.01, 0.01]$, $[0.5, 0.5]$, $[1, 1]$, $[1, 0.5]$ $[1, 0.01]$, $[0.2, 0.5]$ and $[2, 0.5]$. After computing the unconditional variances of the target variables the corresponding minimized values of the loss function are calculated, which are shown in Table 1 next to the results obtained from the previous section under commitment. We now proceed with the discussion of our findings.

[15] We apply the "doubling algorithm" to solve (24) for P, as explained in Hansen, McGrattan and Sargent (1994). This method is more efficient by reaching faster convergence than the standard iteration on the matrix Riccati equation. The same doubling algorithm is applied to compute the unconditional covariance matrix of the target variables, this is especially useful in our case under commitment since we have the initial vector of variables extended by the Lagrange multipliers.

Table 1: Unconditional Losses Fully Optimal Rules

$[q_y, q_r]$	$E[L_t]^C$	$E[L_t]^D$	π'
$[0.01, 0.01]$	0.0017	0.2272	0.475
$[0.5, 0.5]$	0.0120	1.9535	1.393
$[1, 1]$	0.0195	3.7066	1.92
$[1, 0.5]$	0.0138	3.6101	1.896
$[1, 0.01]$	0.0022	2.2360	1.495
$[0.2, 0.5]$	0.0104	0.9042	0.945
$[2, 0.5]$	0.0164	6.6634	2.578

4.1.3 Gains from commitment

The minimized unconditional losses resulting from optimization under commitment and discretion, $E[L_t]^C$ and $E[L_t]^D$, respectively, are shown in Table 1[16].

The first remarkable thing from the table is that in overall the difference between the unconditional losses under the alternative approaches towards optimal policy is very large. For any given combination of policy preferences, the percentage gains from commitment compared to discretion are considerably high. Although the mere fact that commitment outperforms the discretionary case does not come as a surprise, for the reasons discussed next, the magnitude of this advantage of commitment is mainly determined by the estimated parameters of the model. Given that the parameters adopted in the SW model are based on estimations using Euro Area data, we can state that monetary policy in the Euro Area can achieve a much better social outcome in terms of minimized unconditional losses, if it is conducted under commitment rather than under discretion.

Since the minimized loss under commitment is almost zero, this leads to huge percentage gains over discretion. Therefore it seems more appropriate in this case that we use an additional and alternative measure to calculate the welfare differential between discretion and commitment. Following Dennis and Söderström (2005) we calculate the 'inflation equivalent' to measure the welfare gain from commitment. This inflation equivalent,

$$\pi' = \sqrt{E[L_t]^D - E[L_t]^C} \tag{25}$$

depends on the difference in loss under commitment and discretion and represents the permanent deviation of inflation from target. This measure of welfare gain from commitment is reported for each pair of policy preferences in the final column in Table 1. The table shows that the inflation equivalent

[16]Note that, as we have already mentioned, throughout the study we consider the limiting case where $\delta = 1$. Given that δ is usually considered to be high and close to 1, this is a reasonable assumption. The same computations were also done for $\delta = 0.9$, but the results do not differ much and therefore we prefer to focus on the limiting case.

varies from 0.475 to 2.578. In other words, the welfare loss in performing discretionary policy rather than policy under commitment, can be expressed as a permanent deviation of inflation from target of up to about 2.6 percentage points for the pair of policy preferences $[2, 0.5]$.

Following the early work of Kydland and Prescott (1977) and Barro and Gordon (1983), the advantages of commitment over discretion have been broadly discussed in the literature pointing to the familiar "inflation bias" problem that comes along with discretion. The fact that the central bank has the incentive to boost output above its potential level which is taken into account by the private sector in forming expectations creates the inflation bias under discretion. This leads to an inefficiently high steady state inflation rate and no gains in output[17]. However, inflation bias is not present in our current model set up. Therefore this can not provide an explanation for the high discrepancy in the results in Table 1 since the objective function (12) does not allow for such an incentive for the central bank[18]. An alternative explanation to the credibility problem faced under discretion that cannot be assigned to the inflation bias is the so called "stabilization bias" discussed in e.g. Clarida et al. (1999), Woodford (1999) and Dennis and Söderström (2005). Stabilization bias occurs when the model features forward looking behaviour. In this case where private agents have forward looking expectations, there is an additional advantage to a credible commitment on the side of the central bank over discretion. Since private sector expectations affect the future evolution of the economy[19], stabilizing these expectations when the economy is hit by a supply shock, will eventually stabilize the economy as well and decrease the output/inflation tradeoff. This improvement in the form of lower social welfare costs requires a history dependent monetary policy strategy, which is provided by the commitment framework.

Since the structural model in this study is characterised by forward looking behaviour, the stabilization bias could be an important explanation for the high gains from commitment observed in Table 1. However, the fact that the gains from commitment are high suggests that there is a high degree of forward lookingness in the estimated model. Therefore the coefficient on expected future inflation in the Phillips curve in (8), namely the coefficient $\frac{\beta}{1+\beta\gamma_p}$, is of particular interest. Under the current parameter setting, based on the estimations in SW (2003), with $\beta = 0.99$ and $\gamma_p = 0.469$ this coefficient is approximately equal to 0.676, implying a considerable degree of forward lookingness in price setting. Setting $\gamma_p = 0.1$, i.e. assuming a low degree of indexation to past inflation, increases this

[17] Among the popular solutions for this inflation bias is the one suggested by Rogoff (1985). Here the appointment of a 'conservative' central banker with a greater aversion for inflation than society may take away part of the bias. In our context if inflation bias were an issue, this could boil down to increasing the value of weight assigned to inflation in the loss function or decreasing the value of q_y.

[18] An insightful critique to this "unrealistic" inflation bias of discretionary policy is given by Blinder (1998).

[19] For example, price setting by the intermediate goods producers in the monopolistically competitive market is forward looking, leading to the inflation equation (8) that depends also on expected future inflation, next to the past inflation.

coefficient on expected inflation to 0.9 approximately. The corresponding unconditional losses both under commitment and discretion for the weight pairs $[0.5, 0.5]$, for example, are 0.0074 and 0.8215, respectively. The inflation equivalent is 0.902. The gain from commitment in this case is lower than the gain under the original parameter setting (0.0120 vs. 1.9535 in terms of the unconditional losses, or 1.393 in terms of the inflation equivalent). Increasing the degree of indexation γ_p up to 0.90, leads to a relatively lower value for $\frac{\beta}{1+\beta\gamma_p}$ of approximately 0.524. The corresponding unconditional losses equal 0.0281 under commitment and 17.4737 under discretion, implying an inflation equivalent of 4.177 and hence a much higher gain from commitment than the previous two examples of higher degrees of forward lookingness. This result at first glance seems to contradict the claim that a higher gain from commitment would be expected at a higher level of forward lookingness, since the stabilization bias becomes more important when γ_p is lowered and a higher difference in unconditional losses should be observed. A similar observation is found by Dennis and Söderström (2005), they show that the relation between the degree of forward lookingness and the importance of stabilization bias is not monotonic. Instead, they state that at relatively low degrees of forward looking behaviour (up to 0.6), an increase in forward lookingness increases the gains from commitment. However, for relatively high levels of forward looking behaviour (above 0.6), the stabilization bias problem becomes less important since under a high degree of forward looking behaviour the output/inflation tradeoff improves. Our findings seem to support this hump shaped relation between the importance of forward looking expectations and the magnitude of the stabilization bias problem.

It can further be noted from the table that the advantages of commitment over discretion increase along with the weight assigned to output in the objective function, when we hold the weight of interest rates constant at 0.5 (and 0.01). This suggests that the stabilization bias becomes more severe whenever monetary policy cares relatively more about output variability. This leads us to the conclusion that the same cures suggested to overcome the inflation bias problem under discretion are valid in the context of the stabilization bias problem. Thus either decreasing the weight q_y on output in the loss function or appointing a 'conservative' central banker (in the sense of Rogoff (1985)) with a higher inflation aversion (or a lower q_y) than society[20].

In line with the findings in Dennis and Söderström (2005), the percentage gains from commitment increase also when the weight assigned to interest rate variability q_r is lowered, holding q_y constant at 1 in the table above. When the concern for interest rate variability under discretion increases, monetary policy becomes less responsive to shocks and interest rates will tend to show more inertial behaviour.

[20] The same point is also made by Clarida et al. (1999).

This gives the central bank less incentive to stabilize output and therefore the stabilization bias becomes less important. However, the inflation equivalent measure does not seem to support this view in this case.

We turn next to the analysis of various optimal simple (i.e. restricted) rules and compare their performance to that of the 'benchmark' unrestricted optimal rules under commitment discussed above.

4.2 Performance of Optimal Simple Rules

In the previous section we discussed and compared explicit optimal monetary policy rules that were not restricted in their feedback coefficients. Monetary policy responds to all observable variables in these cases (and, in addition, to the Lagrange multipliers under commitment). However, our analysis of monetary policy rules would be incomplete if we would not consider optimal simple rules, where monetary policy is restricted to react to a few variables only. Following Taylor (1993), simple rules have received a lot of attention in the analysis of monetary policy. Mostly it is shown that they are able to compete with the fully unrestricted optimal rules. For example in terms of the minimzed loss that seems to approach the loss obtained under the fully optimal rules as in the case of Rudebusch and Svensson (1998), or robustness to alternative specifications concerning the underlying structural model[21] or for practical reasons, as it is often stated that monetary policy is not able to observe all the variables that are supposed to be reacted upon according to the fully optimal rules at the time monetary policy actions are undertaken[22]. The main purpose of our analysis of optimal simple rules in this section is to draw conclusions on their performance based on unconditional variances and minimized loss functions. We will look whether these type of rules perform relatively well for the Euro Area when one considers a large-scale DSGE model as it is in these cases that the simplification in the optimal rule will be more prominent[23]. We will proceed by first explaining briefly the optimization procedure under commitment when the monetary policy feedback rule is restricted to contain only a limited set of (two or three) variables and continue with the discussion of the results for alternative forms of commonly discussed simple rules.

[21] See, for example Williams (2003) and Onatski and Williams (2004) for studies in favour of this statement.

[22] More on this information issue can be found in McCallum and Nelson (1999).

[23] Similar approach has been undertaken by Williams (2003) for the Federal Reserve Board's FRB/US model. The case of optimal simple rule for one Taylor type of simple rule form for the SW (2003) model is also discussed in Onatski and Williams (2004). Examples of studies of simple rules in relatively "simple" models can be found in e.g. Rudenbusch and Svensson (1998).

4.2.1 Commitment

As treated in Dennis (2004), under commitment the structural model (15) is augmented by the following general form for the monetary policy instrument:

$$u_t = \varphi_1 x_{t-1} + \varphi_2 x_t + \varphi_3 E_t x_{t+1} + \varphi_4 u_{t-1} + \varphi_5 z_t \tag{26}$$

where the matrices φ_1, φ_2, φ_3, φ_4 and φ_5 are restricted conform to the specific form of the rule under consideration. The system of equations (15) and (26) can then be written in the following second-order form:

$$A2^* y_t = B2^* E_t y_{t+1} + C2^* y_{t-1} + D2^* z_t \tag{27}$$

where

$$\begin{aligned} A2^* &= \begin{bmatrix} A & -G \\ -\varphi_2 & I \end{bmatrix} \quad B2^* = \begin{bmatrix} B & 0 \\ \varphi_3 & 0 \end{bmatrix} \\ C2^* &= \begin{bmatrix} F & 0 \\ \varphi_1 & \varphi_4 \end{bmatrix} \quad D2^* = \begin{bmatrix} D \\ \varphi_5 \end{bmatrix} \end{aligned}$$

After solving this equation using the generalized Schur decomposition method of Sims (2002), we obtain a solution in the first-order form for given initial values of the free parameters in the feedback rule that have to be optimized later on under the requirement that these initial values yield a determinate rational expectations solution. Using numerical methods for unconstrained optimization, we minimize the unconditional loss function (14) since we assume the limiting case where $\delta = 1$ with respect to the free parameters in φ_1, φ_2, φ_3, φ_4 and φ_5 in order to obtain the optimal simple rules. In the next section the alternative rules under consideration are presented, followed by the results from optimization with respect to their free parameters.

4.2.2 Alternative froms of simple rules

We consider the following four types of linear feedback rules[24];

1. Taylor-type of rule:

   $$R_t^T = r_\pi \pi_t + r_y (Y_t - Y_t^p) \tag{28}$$

   where the parameters r_π and r_y are determined by optimization. When these parameters are set to 1.5 and 0.5, respectively, this rule becomes the familiar Taylor (1993) rule.

[24] The choice of the forms is largely based on the ones discussed in Rudebusch and Svensson (1998). However, most of these rules are standard and variants are adopted in most studies on optimal simple rules. Williams (2003), Woodford (2003), Woodford (1999), McCallum (1999) and Dennis (2004) are just a few among a large list of examples.

2. Inflation Forecast rule:

$$R_t^I = r_\pi E_t \pi_{t+1} \tag{29}$$

where monetary policy is assumed to respond only to next period's rational inflation forecast[25]. Although this one argument case is admittedly an oversimplification of reality, the main purpose is for comparison of the resulting unconditional variances and losses under this rule with that obtained under the other rules.

3. Forecast rule with additional response to the current output gap:

$$R_t^F = r_\pi E_t \pi_{t+1} + r_y (Y_t - Y_t^p) \tag{30}$$

this is the Taylor-type of rule where π_t is replaced by $E_t \pi_{t+1}$.

4. Lagged Taylor rule:

$$R_t^L = r_\pi \pi_{t-1} + r_y (Y_{t-1} - Y_{t-1}^p) \tag{31}$$

This is the lagged variant of the Taylor-type of rule. The main justification for this type is the potential information problem faced by monetary policy at the time that the interest rate is set due to lags in data collection, as pointed e.g. by McCallum (1997).

Since generally a high degree of interest rate smoothing is found, we also wish to study each type of these rules described above with smoothing by including the one-period lag of the interest rate, which we represent as follows:

$$R_t^{S,j} = \rho R_{t-1}^{,j} + (1-\rho) R_t^{,j} \qquad j = T, I, F \text{ or } L \tag{32}$$

where the superscript S denotes the smoothing variant and j refers to the specific type of rule. By the inclusion of the smoothing variant we end up with eight alternative rules. These parameters are optimized under the same seven combinations of $[q_y, q_r]$ considered in the previous section. The resulting values for the unconditional losses $E[L_t]^C$ and the corresponding optimal feedback parameters P for each combination $[q_y, q_r]$ for the baseline case under commitment are reported in Table 2. The values for the smoothing variant under commitment are given in Table 3. Both results are discussed in detail in the next part. We will compare the welfare costs between the different types of rules and their performance with respect to the 'benchmark' unrestricted optimal commitment rule.

Table 2: Unconditional Losses Baseline Case (without smoothing) : Commitment

	Taylor-type of rule	R_t^T	Inflation forecast rule	R_t^I
$[q_y, q_r]$	$P = [r_y, r_\pi]$	$E[L_t]^C$	$P = [r_\pi]$	$E[L_t]^C$
$[0.01, 0.01]$	Indeterminate		$[2.0291]$	0.0355
$[0.5, 0.5]$	$[0.4189, 3.3837]$	0.1163	$[2.0291]$	0.4335
$[1, 1]$	$[0.4038, 3.3280]$	0.2245	$[2.0291]$	0.8395
$[1, 0.5]$	$[0.5978, 3.9930]$	0.1402	$[2.0291]$	0.7868
$[1, 0.01]$	$[5.2109, 12.6480]$	0.0128	$[2.0291]$	0.7352
$[0.2, 0.5]$	$[0.2689, 2.7874]$	0.0944	$[2.0291]$	0.2215
$[2, 0.5]$	$[0.8596, 4.7671]$	0.1725	$[2.0298]$	1.4923
	Forecast rule with output gap	R_t^F	Lagged Taylor rule	R_t^L
$[q_y, q_r]$	$P = [r_y, r_\pi]$	$E[L_t]^C$	$P = [r_y, r_\pi]$	$E[L_t]^C$
$[0.01, 0.01]$	$[1.4211, 8.2723]$	0.0076	$[2.6798, 6.1070]$	0.0057
$[0.5, 0.5]$	$[0.3285, 4.0092]$	0.1317	$[0.5485, 2.9882]$	0.0937
$[1, 1]$	$[0.3165, 3.9433]$	0.2549	$[0.5250, 2.9338]$	0.1803
$[1, 0.5]$	$[0.4760, 4.7558]$	0.1586	$[0.7830, 3.4842]$	0.1118
$[1, 0.01]$	$[4.4618, 15.7844]$	0.0143	$[7.3758, 9.8266]$	0.0096
$[0.2, 0.5]$	$[0.2059, 3.2817]$	0.1071	$[0.3516, 2.4901]$	0.0769
$[2, 0.5]$	$[0.6943, 5.7120]$	0.1950	$[1.1341, 4.1097]$	0.1360

4.2.3 Results for the Baseline specification

All optimized parameters reported in Table 2 correspond to a stable and unique equilibrium, except for the first class of Taylor-type of rules where the central bank is assumed to take only inflation variability into account (i.e. with combination $[0.01, 0.01]$)[26]. Because of this reason we do not wish to consider this indeterminate case in our analysis. It is noticable that the inflation feedback coefficient in all remaining cases always exceeds unity, as expected since determinacy is present in agreement with the Taylor principle[27]. A first remarkable observation is that the optimal value for r_π is the same under the inflation forecast rule where monetary policy is allowed to react to expected inflation only, throughout all alternative policy preferences considered here. Concerning the optimized feedback coefficients for the other three rules, the highest responsiveness to both output and inflation are obtained when monetary policy cares equally about output and inflation variability and places no weight on interest rate variability. Moreover, in general when we abstract from changes in the weight assigned to the output gap, the responsiveness to both variables increases whenever a lower weight is

[25] The presence of the inflation forecast makes the instrument rule, together with the rule discussed next, a forward looking instrument rule.

[26] In this latter case, the optimized feedback parameters could not yield a unique solution, i.e. they correspond to multiple equilibria or 'sunspots' where the economy can start moving after shocks that are not related to the fundamentals. See Levin et al. (2003) for a detailed illustration of the conditions under which indeterminacy is likely to occur in the study of monetary policy rules.

[27] However it should be noted that this is not a necessary condition for determinacy.

assigned to interest rate variability. This does not come as a surprise since monetary policy will become more aggressive towards shocks in order to stabilize both inflation and output. This in spite of the higher interest rate volatility that it brings along if output becomes less of a concern. The lowest feedback coefficients under all types of rules considered in Table 2, again except the inflation forecast rule, are obtained when the policy preferences $[q_y, q_r]$ are set equal to $[0.2, 0.5]$. This is probably due to the fact that the central bank is less concerned about output variability. An increasing concern for output gap variability, while keeping the concern for interest rate variability constant, leads to a higher responsiveness to both inflation and output gap. Hence we can state that the responsiveness to both variables increases with the weight assigned to the output gap in the loss function.

It should also be noted that for every alternative combination of policy preferences $[q_y, q_r]$, Table 2 shows that the lowest loss is always obtained under the lagged Taylor rule, followed by the Taylor-type of rule, the forecast rule including the output gap and finally the worst performing inflation forecast rule. This finding suggests that a simple feedback rule in function of the lagged variables only performs best, slightly better than a Taylor-type of rule expressed in terms of the contemporaneous variables. The worst outcome is obtained under a rule that is a function of forecast variables only. Moreover, the increase in unconditional loss is the highest when switching from the forecast rule that still includes the current output gap in addition to the inflation forecast rule to the case where monetary policy responds only to the inflation forecast. This is especially the case when there is a high concern for output gap variability; the loss under the inflation forecast rule increases with the weight assigned to the output gap in the loss function. It seems that the inflation forecast only is not a sufficient variable to react upon and the welfare costs increase with the concern for volatility in the output gap. This finding is in contrast to the argument made by Batini and Haldane (1999) that output gap can be left out of the rule specification if interest rate smoothing is taken into account to ensure stability. Although interest rate smoothing is not present in Table 2, we will see below that even when interest rate smoothing is allowed for in Table 3 this conclusion does not change. The result that a feedback rule expressed in function of the inflation forecast only performs badly is also indicated by Levin et al. (2003) and Rudebusch and Svensson (1998), despite the fact that the model assumptions in the latter differ substantially from the one we use in this study[28]. However, we might a priori expect that forward looking feedback rules where the output gap is included in some form, like the third type of rule in our analysis, could perform better than rules expressed in terms of current and/or lagged variables (e.g. the Taylor-type

[28] They assume backward looking expectations within the context of a standard two equation model that consists of a Phillips curve and an IS curve.

of rule or the lagged Taylor rule in our example). This assumption is simply based on the fact that in the latter case monetary policy reacts with a delay to shocks, whereas under the forward looking rule immediate response to these shocks is possible because their future effects are incalculated, which in turn prevents the shock from taking full effect[29]. In addition, as discussed in detail in Batini and Haldane (1999), the argument that monetary policy effects come with a delay, due to lags in the transmission mechanism, and reacting today to expected conditions is another reason why forward looking rules may be preferred[30]. Despite these arguments in favour of forward looking rules, our conclusions based on Table 2 where interest rate smoothing is absent, suggest that these kind of rules perform the worst. However, the welfare improvement in terms of losses in moving from the forecast rule including the output gap to either the Taylor-type of rule or the lagged Taylor rule is moderate[31].

Finally, comparing the losses under the lagged Taylor rule to the losses obtained under the 'benchmark' unrestricted optimal rule for every combination of preferences $[q_y, q_r]$ indicates that the welfare advantage of this benchmark rule over the most favourable optimal simple rule is relatively high. This leads us to the conclusion that, given the current specification of the four alternative simple rules (i.e. no allowance for interest rate inertia in the feedback rule), these rules in general do not perform very well in approaching the benchmark rule in terms of minimized unconditional losses. This is not surprising since interest rates are observed with a highly inertial pattern in forward looking rational expectations models. As will become clear in the next section, monetary policy can therefore achieve a better social outcome by including the lagged interest rate (i.e. a smoothing term) in all four alternative types of simple rules considered in the previous section. We now turn to the discussion of the results under the smoothing specification.

4.2.4 Results for the Smoothing specification

There are two cases in Table 3 under the inflation forecast rule that do not correspond to a determinate equilibrium which we prefer not to consider in our analysis for this reason. This is most likely due to the fact that output is not included explicitly in the feedback rule. As shown in Levin et al. (2003) and in the table, allowing for a response to the output gap solves the problem of indeterminacy. It is also noticable that, as in the baseline specification, the optimal response to expected inflation in the

[29] This argument is also put forward by Levin et al. (2003) and Batini and Haldane (1999).

[30] This argument is also used by Svensson (1997) and Svensson and Woodford (1999) in favour of inflation forecast targeting.

[31] In line with this conclusion, Levin et al. (2003) find that in rational expectations models, the gains from forward looking rules over outcome based rules (i.e. rules in terms of current and lagged variables) are limited. Similarly, Rotemberg and Woodford (1998) find that there is only a slight deterioration in performance when lagged variables are employed.

Table 3: Unconditional Losses Smoothing Variant : Commitment

	Taylor-type of rule	$R_t^{S,T}$	Inflation forecast rule	$R_t^{S,I}$
$[q_y, q_r]$	$P = [\rho, r_y, r_\pi]$	$E[L_t]^C$	$P = [\rho, r_\pi]$	$E[L_t]^C$
$[0.01, 0.01]$	na	0.0022	$[0.9766, 5.2218]$	0.0077
$[0.5, 0.5]$	$[0.9592, 0.0979, 5.6139]$	0.0175	$[0.9671, 1.9357]$	0.0290
$[1, 1]$	$[0.9592, 0.0797, 4.8021]$	0.0295	Indeterminate	
$[1, 0.5]$	$[0.9540, 0.1213, 6.6263]$	0.0219	Indeterminate	
$[1, 0.01]$	na	0.0033	$[0.9662, 1.9318]$	0.0417
$[0.2, 0.5]$	$[0.9664, 0.0872, 5.1866]$	0.0140	$[0.9672, 1.9620]$	0.0200
$[2, 0.5]$	na	0.0277	$[0.9665, 1.9335]$	0.0729
	Forecast rule with output gap	$R_t^{S,F}$	Lagged Taylor rule	$R_t^{S,L}$
$[q_y, q_r]$	$P = [\rho, r_y, r_\pi]$	$E[L_t]^C$	$P = [\rho, r_y, r_\pi]$	$E[L_t]^C$
$[0.01, 0.01]$	na	0.0024	na	0.0020
$[0.5, 0.5]$	$[0.9601, 0, 5.9041]$	0.0178	$[0.9590, 0, 5.9756]$	0.0164
$[1, 1]$	$[0.9617, 0.0977, 5.5656]$	0.0302	$[0.9584, 0.0787, 4.7631]$	0.0278
$[1, 0.5]$	$[0.9574, 0.1486, 7.7847]$	0.0222	$[0.9530, 0.1220, 6.6667]$	0.0205
$[1, 0.01]$	na	0.0034	na	0.0030
$[0.2, 0.5]$	$[0.9678, 0, 6.0114]$	0.0143	$[0.9661, 0.0870, 5.1904]$	0.0133
$[2, 0.5]$	$[0.9525, 0.1905, 9.6111]$	0.0286	na	0.0255

inflation forecast rule is relatively restricted compared to the equivalent forecast rule that includes the output gap. This may, as pointed by e.g. Rotemberg and Woodford (1998) and Clarida et al. (1999), in part also explain the indeterminacy that occurs in the inflation forecast rule. When the response to (a shock that leads to) an increase in expected inflation is "too low", actual inflation will increase due to lower real interest rates and the expectations of higher inflation will be enforced. This self-fulfilling effect leads to multiple equilibria and hence the optimal parameters fail to ensure stability.

In line with the results for the baseline specification, we observe that mainly in the absence of concern for interest rate variability, the optimal feedback parameter for inflation r_π either becomes very high as is the case for the inflation forecast rule, or even too high (na) in all other cases. Although stability was ensured also in the baseline specification by high inflation coefficients when there is no weight assigned to interest rate variability, this effect is even reinforced under the smoothing specification. Since these values are very high, we cannot assign them an economically plausible meaning and hence we prefer to leave them out from the analysis.

In general the optimal values for the smoothing parameter ρ are very high, never below 0.95, under every alternative rule. This is in line with the common finding that models with rational expectations assumptions are typically characterized by high inertial behaviour in the interest rates[32]. The response

[32] Rotemberg and Woodford (1998) even find values larger than one, pointing to superinertial behaviour.

coefficients to the output gap in the smoothing specification under every rule that includes the output gap explicitly are lower than under the baseline specification for each combination of policy preferences $[q_y, q_r]$. On the contrary, the responses to inflation are substantially higher in the smoothing specification as compared to the baseline case, except for the inflation forecast rule where the responses to expected inflations are similar and low under the two specifications. This is not surprising because the degree of smoothing is high and therefore a high response coefficient is needed in order to induce the required change in the interest rates. The lowest response to inflation under the Taylor-type of rule and the lagged Taylor rule is obtained when there is equal concern to all target variables in the objective function. Because monetary policy considers all three target variables as equally important in this case, the response to inflation is moderated. Analogously with the baseline specification, the responsiveness to inflation increases with a decreasing concern for interest rate variability (abstracting from the concern for output gap variability), which leads to an explosive coefficient when there is no concern for interest rate variability at all. An increasing response to inflation is also observed, similarly to the baseline specification case, whenever the concern for output gap variability is increased under an unchanged concern for interest rate variability.

The conclusions based on the obtained losses under the alternative rules when interest rate smoothing is not present in the baseline specification remain qualitatively valid when smoothing is allowed for in Table 3. Similarly to the baseline case, for every possible combination of policy preferences $[q_y, q_r]$ in Table 3, the lagged Taylor rule with the lagged variables always performs best by yielding the lowest loss, followed by the Taylor-type of rule in terms of current variables, the forecast rule including the current output gap and finally the inflation forecast rule which performs worst. Although it should be noted that each time the improvement in welfare is not remarkably high at all. It is also here the case that the welfare improvement in switching from the inflation forecast rule to the forecast rule that responds to the output gap is relatively higher than when one moves from the Taylor-type of rule to the lagged Taylor rule. It should also be noted that, although the inflation forecast rule performs worst, an improvement occurs when a lower weight is assigned to the output gap.

A general conclusion from the results in Table 3 is that the smoothing variant of every type of rule considered in this study consistently corresponds to a lower unconditional loss than the alternative baseline specification. This holds for every set of policy preferences. This is a common observation in the context of rational expectations models[33] and corresponds to findings by Rotemberg in Woodford

[33] The reverse conclusion holds in the context of a backward looking model, as shown e.g. in Rudebusch and Svensson (1998).

(1998), who find that most efficient rules show superinertial behaviour (smoothing parameter larger than one). Other examples can be found e.g. in Woodford (1999), Williams (2003), Levin et al. (2003) and Angeloni et al. (2003).

Hence the lagged Taylor rule that allows for interest rate smoothing performs best in approaching the unrestricted optimal rule from the point of view of efficiency among all classes of optimal simple rules analysed in this study. Moreover, this rule yields a cost that is closer to the cost incurred under the fully optimal rule than its baseline counterpart to such an extent that the difference between this optimal simple rule and the fully optimal rule in terms of efficiency is quite small. It becomes clear from this conclusion that monetary policy that responds to only lagged values of the interest rate, inflation and the output gap, succeeds in approaching the efficiency provided by the fully optimal rule which includes a larger set of information variables. Hence this suggests that the information needed for optimal monetary policy in achieving the most favourable social outcome, can be mainly captured by a simple rule that includes lags of three variables only, i.e. the lagged instrument, lagged inflation and lagged output gap. Since the replacement of the current inflation and output gap by their lagged values in the policy rule brings only a slight improvement, the addition of the smoothing term takes account for the largest part of the welfare improvement in approaching the loss under the unrestricted optimal rule. Given the forward looking nature of the expectations in the model we are considering, when monetary policy is committed to a simple rule that is credible, future responses to shocks are anticipated by the private sector. Even if monetary policy does not immediately respond to these shocks because of high inertia in the instrument rule, the credibility issue ensures that rational agents can forecast the future response to the shocks. This immediate adjustment of expectations affects the current behaviour of private agents and therefore brings along part of the stabilization effects needed by policy that has not responded yet. This in turn will lead to the need of a more moderate response from part of the monetary authorities in order to stabilize the economy (which leads to the inertia) than would be necessary in the case of backward looking expectations. This argument is also put forward by Rotemberg and Woodford (1998), as Clarida et al. (1999) and Williams (2003).

5 Conclusion

The main purpose of this study was to evaluate the performance of fully optimal rules and optimal simple rules restricted in their parameter setting within the framework of an estimated New-Keynesian general equilibrium model for the Euro Area developed by Smets and Wouters (2003). We find that under the current parameter setting and the resulting degree of forward lookingness in the model, there are considerable gains from commitment with respect to discretion under the fully optimal rules. These gains might be for a large part due to the stabilization bias that is present under discretion. Within the class of simple rules, the lagged variant of the Taylor type of rule including a smoothing term for the interest rate performs relatively well in approaching the performance of the unrestricted optimal rule derived under commitment. However, the welfare improvement in replacing the specification of the Taylor type of rule in terms of current variables by their lags is limited. Simple rules expressed in terms of forecasts to next period's inflation rate, however, perform relatively worse.

The study performed in this paper can be improved on several aspects. An important shortcoming is that we did not test the robustness of the rules when there is a considerable degree of uncertainty present concerning the underlying structural model. We also limited our study to explicit monetary policy rules, whereas comparison to targeting rules is also an interesting exercise that we wish to perform in future work. Also, given the structural model considered, it would be interesting and more favourable to evaluate the alternative rules under a welfare based loss function which we intend to do in the future.

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