Cartel damages claims and the passing-on defense

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Abstract

We develop a general economic framework for computing cartel damages claims by purchaser plaintiffs. We decompose the lost profits from the cartel in three parts: the direct cost effect (or anticompetitive price overcharge), the pass-on effect and the usually neglected output effect. The pass-on effect is the extent to which the plaintiff passes on the price overcharge by raising its own price, and the output effect is the lost business resulting from this passing-on. We subsequently introduce various models of imperfect competition for the plaintiff's industry. This enables us to evaluate the relative importance of the cost, pass-on and output effects. We show that an adjusted passing-on defense (i.e. accounting for the output effect) is justified under a wide variety of circumstances, provided that sufficiently many firms in the plaintiff's market are affected by the cartel. We derive exact discounts to the direct cost effect, which depend on relatively easy-to-observe variables, such as the pass-on rate, the number of firms, the number of firms affected by the cartel, and/or the market shares. We finally extend our framework to assess the cartel's total harm, further demonstrating the crucial importance of the output effect. Our results are particularly relevant in light of the recent developments by U.S. and European antitrust authorities to make cartel damages claims more in line with actually lost profits.

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1 Introduction

The anticompetitive price overcharge has been commonly used as a basis for computing damages claims in price-fixing cartels. There is, however, an ongoing debate as to whether the cartel members may resort to a passing-on defense. Such a defense entails the argument that the purchaser plaintiff may have passed on part of the cartel's price overcharge to its own customers and correspondingly suffered lower losses than the overcharge. Both U.S. and European antitrust authorities have recently shown a renewed interest in properly assessing cartel damages, including the possible consideration of the passing-on defense.[1]

Against this background we develop a general economic framework for computing cartel damages. We decompose the purchaser plaintiff's lost profits from the cartel into a direct and two indirect effects. First, the direct cost effect is the anticompetitive price overcharge suffered by the plaintiff, multiplied by the number of units purchased at that price. Second, the pass-on effect reflects the extent to which the purchaser can shift the burden of the price overcharge to its own customers. Third, the usually neglected output effect refers to the sales that may be lost when part of the price overcharge is passed on to the customers.

We then introduce various models of imperfect competition to describe the industry in which the purchaser plaintiff operates. This enables us to evaluate the relative importance of the cost, pass-on and output effects from the cartel. Consistent with common practice we take the direct cost effect (price overcharge) as the basis for computing damages and show how to compute a discount to this direct cost effect. This discount captures the pass-on effect but suitably adjusted for the output effect. We first consider the case of a common cost increase, in which all competitors in the plaintiff's industry are affected by the cartel. We show that in this case the discount to the cartel's direct cost effect is generally positive, unless the plaintiff operates itself in a fully cartelized industry. This motivates an adjusted passing-on defense, where the adjustment factor reflects the output effect and depends on the intensity of competition, as illustrated for Bertrand or Cournot industries.

We next consider the case of a firm-specific cost increase, in which not necessarily all of the plaintiff's competitors are affected by the cartel. This is relevant in several circumstances, for example when some of the cartel members are vertically integrated and do not overcharge their own downstream units. In this case the output effect becomes more important, because the plaintiff may now also lose business to its unaffected competitors within the industry, in addition to losing business outside the industry. We show that an adjusted passing-on defense

[1] The U.S. Antitrust Modernization Commission (2007) has recently recommended to make cartel damages claims in line with lost profits, implying the possibility of a passing-on defense. The European Commission (2005) issued a Green Paper on private cartel damages and the possibility of a passing on defense. We review these developments against the history of previous case law in section 2.

remains economically justified in a Bertrand industry. In a Cournot industry, however, the passing-on defense becomes invalid if there are sufficiently many unaffected competitors, since these respond expansively to the output contractions of the plaintiff and other affected firms. For both the Bertrand and the Cournot models we derive exact discounts to the cartel's direct cost effect when the adjusted passing-on defense is justified. These discounts are easy to interpret and depend on observable variables for the plaintiff's industry, such as the pass-on rate, the total number of firms, the number of firms affected by the cartel, the plaintiff's and/or the other firms' market shares.

Most previous research has focused on the law and economics of the passing-on defense, providing informational and incentive arguments for or against a passing-on defense in cartel damages cases.[2] In contrast, we focus on the economic effects and stress the essential importance of the output effect whenever the passing-on defense is considered. To our knowledge there are only a few recent papers which explicitly elaborate on the output (or "lost business") effect, in particular Kosicki and Cahill (2006) and Hellwig (2006). These papers focus on the case in which the plaintiff's industry is itself fully cartelized (so that the passing-on defense becomes invalid), or consider some other special cases with graphical or numerical analysis. In contrast, our framework shows how it is possible to incorporate the output effect in a wide variety of oligopoly industries, with and without the possibility that some of the plaintiff's competitors are not affected by the cartel. We show that the informational requirements in implementing a passing-on defense increase only moderately relative to the case in which the output effect is ignored.

Our paper focuses on the cartel damages to a purchaser plaintiff. In a recent paper Basso and Ross (2007) assess the total harm of the cartel, i.e. the effects on all purchaser plaintiffs and their (final) consumers. They show that this total harm is generally higher than what we call the direct cost effect. At the end of the paper, we show how to extend our own framework to assess the cartel's total harm. We show that the output effect is the crucial explanation for why the cartel's total harm exceeds the direct cost effect. Hence, an adjusted passing-on defense may actually turn against the defendant, since the evidence required to adjust for the output effect may also be used to demonstrate by how much the total harm exceeds the direct cost effect.

The paper is organized as follows. Section 2 reviews previous practices towards cartel

[2] Landes and Posner (1979) advance several arguments why damages claims by direct purchasers without a passing-on defence are more reliable and hence have a better deterrence effect. Harris and Sullivan (1979), in contrast, argue in favour of the passing-on defense to ensure a correct damages compensation. In an interesting recent paper Schinkel et al. show how the cartel may have an incentive to "bribe" direct purchasers not to bring damages claims if indirect purchasers cannot bring damages claims. See van Dijk and Verboven (2007) for a more detailed overview of these arguments.

damages claims and the passing-on defense. Section 3 presents the general economic framework, decomposing the plaintiff's lost profits in a cost, pass-on and output effect. Section 4 considers the case of a common cost increase (to all of the plaintiff's competitors), and section 5 the case of a firm-specific cost increase (to a subset of the plaintiff's competitors). Section 6 considers total harm. Section 7 concludes.

2 State of play in the U.S. and Europe

We briefly review the history and logic of cartel damages claims, with a focus on the passing-on defense. For a more detailed discussion, we refer to the references in this section, and the large literature they cite.

United States The current situation in the U.S. is the result of three major Supreme Court decisions, *Hanover Shoe*, *Illinois Brick*, and *ARC America*, and subsequent legislations by the states.[3] In the *Hanover Shoe* decision of 1968 the defendant argued that the overcharged purchaser plaintiff does not suffer losses if the overcharge is imposed equally on all of the purchaser's competitors and if the demand is so inelastic that the purchasers can pass on the overcharge without suffering a decline in sales. The Supreme Court rejected this argument on the grounds of insurmountable practical difficulties in proving that the purchaser indeed passed on the price overcharge and how this passing-on would affect sales. Furthermore, it considered that the indirect purchasers tend to be too dispersed and too weak to subsequently recover any damages resulting from the passing-on by the direct purchasers, implying that the cartel offenders might get off too lightly. In the *Illinois Brick* decision of 1977 the Court continued this logic and denied indirect purchasers the right to claim damages, since the *Hanover Shoe* decision already made the cartel liable for the full damages to direct purchasers.[4]

There was considerable opposition against *Illinois Brick* in the decade following the decision. Congress was not able to pass any bills to overturn the decision, but in the *ARC*

[3] The three cases are Hanover Shoe Inc. v. United Shoe Machinery Corp., 392 U.S. 481 (1968) and Illinois Brick Co. v. Illinois, 431 U.S. 720 (1977), and California v. ARC America Corp., 490 U.S. 93 (1989).

[4] Interestingly, Justice White, who delivered the opinion of the Court in *Illinois Brick*, points at two complicating factors in practice that are assumed away in "the economist's hypothetical model" (original wording from the *Hanover Shoe* decision): "Overcharged direct purchasers often sell in imperfectly competitive markets. They often compete with other sellers that have not been subject to the overcharge . . . " (see http://www.ripon.edu/Faculty/bowenj/antitrust/ilbrvill.htm). Our paper deals with precisely these two factors: section 4 focuses on imperfect competition, and section 5 adds complications relating to the fact that not all competitors may be subject to the overcharge.

America decision of 1989 the Supreme Court legitimized indirect purchaser suits in state courts. Furthermore, various states have passed *Illinois Brick* repealer laws or used existing consumer protection statutes to permit indirect purchasers to bring damages claims against cartels; see Hussain, Garrett and Howell (2001). As discussed in Kosicki and Cahill (2006) several of these states also entitled the defendant to resort to a passing-on defense against these indirect purchasers.

Under the current situation the direct purchasers can thus in principle claim the full amount of the cartel damages (whether or not passed on to indirect purchasers), and the indirect purchasers may obtain a duplicate part of that amount, i.e. their own lost profits. While there do not appear to be cases in which this has lead to an overcompensation of all parties affected by the cartel, the Antitrust Modernization Commission (2007) has recently recommended to make direct and indirect purchaser damages claims more in line with their actually lost profits from the cartel. If these recommendations will be followed, this opens the door for a passing-on defense, not just against the indirect but also against the direct purchasers of a cartel.

Europe The experience with private antitrust damages cases in Europe has been rather limited. There is, however, extensive case law in other areas, for example in cases where undertakings claim restitution of illegal duties and levies from the state. As discussed in Norberg (2005), in the notable *Comateb* decision of 1997, the European Court of Justice accepted that a passing-on defense was compatible with Community Law, but also clarified that the plaintiff "may have suffered damage as a result of the very fact that he passed on the charge ... because the increase in price has led to a decrease in sales".[5]

Only very recently, in the important *Courage* decision of 2001, the European Court of Justice confirmed that infringements of Art. 81 and 82 of the EC Treaty provide a legal basis for bringing damages claims in antitrust infringements.[6] Following this decision the European Commission put private cartel damages claims and the possible consideration of a passing-on defense as a high priority on the agenda. It requested the Ashurst study on private enforcement of competition policy in 2004.[7] This lead to the European Commission's (2005) Green Paper on damages actions for breach of antitrust rules, which includes a discussion on the role of the passing-on defense. Interestingly, the Commission writes: "It can be said that there is no passing on defense in Community law; rather, there is an unjust enrichment defense which requires (1) proof of passing on ... and (2) proof of no reduction in sales

[5] Joined cases C-192/95 to C-218/95 *Société Comateb*, 1997 ECR I 165, CJ (http://europa.eu.int/eur-lex/lex/LexUriServ/LexUriServ.do?uri=CELEX:61995J0192:EN:HTML).

[6] C-453/99 Courage v. Crehan 2001 ECR I-6297 CJ.

[7] The legal part of the Ashurst study was written by Waelbroeck, Slater and Even-Shoshan (2004).

or other reduction to income" (European Commission, 2005, p. 48). One may interpret this unjust enrichment defense as an adjusted version of the passing-on defense, which also accounts for the additional output effect following pass-on. Our own analysis will show how to implement such an adjusted passing-on defense in a wide variety of competitive circumstances.

3 General economic framework

This section decomposes the purchaser plaintiff's lost profits from the cartel into three effects: the cost, pass-on and output effects. At this point our only assumptions are that the plaintiff takes its input prices as given, including the price of the input purchased from the cartel, and chooses its input mix to minimize its total costs. We do not yet make specific assumptions on the nature of competition in the plaintiff's industry.

Consider a plaintiff firm selling q units of total output at a price p in the but-for world, i.e. without the cartel. The plaintiff chooses its inputs to minimize its total costs, subject to a standard production function technology. One of its inputs is the cartelized input, of which it purchases x units at an input price w in the absense of the cartel. Write the plaintiff's cost function $C(w, q)$ as a function of w and q, and omit the other input prices as arguments. The plaintiff's profits π in the but-for world, i.e. without the cartel, are simply total revenues minus total costs:

$$\pi = pq - C(w, q).$$

The change in the plaintiff's profits due to the cartel is

$$\mathrm{d}\pi = -\frac{\partial C(w, q)}{\partial w}\mathrm{d}w + q\mathrm{d}p + \left(p - \frac{\partial C(w, q)}{\partial q}\right)\mathrm{d}q.$$

According to Shepard's Lemma, the plaintiff's demand for the cartelized input is $x = \frac{\partial C(w,q)}{\partial w}$, so that

$$\mathrm{d}\pi = -x\mathrm{d}w + q\mathrm{d}p + \left(p - \frac{\partial C(w, q)}{\partial q}\right)\mathrm{d}q. \tag{1}$$

Equation (1) shows that the change in the plaintiff's profits due to the cartel can be decomposed into three components.

Direct cost effect The first term ($-x\mathrm{d}w$) is the direct cost effect. It is the *price overcharge* $\mathrm{d}w$ (the cartel input price minus the but-for input price), multiplied by the total inputs x purchased from the cartel. This cost effect is obviously negative.

Pass-on effect The second term ($q\mathrm{d}p$) is the pass-on effect. It is the increase in revenue that follows if the plaintiff passes part of the input price increase on to its customers in the form of a higher output price $\mathrm{d}p$. The pass-on is typically positive ($\mathrm{d}p > 0$), thus counteracting at least part the direct damages from the price overcharge $\mathrm{d}w$.

Output effect The third term $\left(p - \frac{\partial C(w,q)}{\partial q}\right)\mathrm{d}q$ is the output effect. It refers to the lost profits associated with any lost sales $\mathrm{d}q$ following the higher output price set by the plaintiff. This effect is typically negative ($\mathrm{d}q < 0$), especially if the plaintiff earns a high profit margin $p - \frac{\partial C(w,q)}{\partial q}$, as in imperfectly competitive markets. The output effect can only be ignored if the plaintiff is active in a perfectly competitive market, since then $p = \frac{\partial C(w,q)}{\partial q}$.

The direct cost effect forms the basis for the plaintiff's cartel damages claims in both the U.S. and Europe. The defendant may subsequently attempt to resort to the pass-on effect to obtain a discount from the cost effect, at least if this has a legal basis in the jurisdiction. However, our framework shows that the pass-on effect also implies an output effect. Hence, if a passing-on defense is allowed the output effect should also be incorporated.

While our framework stresses the role of three key effects from the cartel in general terms, it does not say anything about their relative magnitudes. In the next sections, we will introduce additional structure on the competitive conditions in the plaintiff's downstream market to quantify the relative importance of the three effects. We identify conditions under which the pass-on effect dominates the output effect. This motivates an adjusted passing-on defense in the form of easy-to-interpret discounts to the cost effect. We begin with the simpler case of a common cost increase, where all firms in the plaintiff's downstream market are symmetrically affected by the price overcharge $\mathrm{d}w$, and subsequently move to the more complicated setting in which some of the plaintiff's rivals are not affected.

Constant returns to scale

To simplify the exposition, the rest of the paper assumes that the plaintiff has a constant returns to scale cost function, $C(w,q) = c(w)q = cq$.[8] This implies that marginal cost is independent of output, $\frac{\partial C(w,q)}{\partial q} = c(w) = c$, and that input demand is proportional to total output, $x = \frac{\partial C(w,q)}{\partial w} = c'(w)q$. Furthermore, $\mathrm{d}c = c'(w)\mathrm{d}w = \frac{x}{q}\mathrm{d}w$. The change in the plaintiff's profits, given by (1), then simplifies to:

$$\mathrm{d}\pi = -q\mathrm{d}c + q\mathrm{d}p + (p - c)\mathrm{d}q. \tag{2}$$

[8] The assumption of constant marginal cost is not without consequences. If marginal cost would be increasing in output, the extent of pass-on can be expected to be smaller, which would also result in a lower output effect. The reverse is true if marginal cost is decreasing in output.

Equation (2) expresses the direct effect of the cartel in terms of the overall marginal cost increase, $\mathrm{d}c$, instead of the price overcharge, $\mathrm{d}w$. We will follow this practice in the rest of the paper. To reinterpret our results in terms of the price overcharge $\mathrm{d}w$, simply substitute $\mathrm{d}c = \frac{x}{q}\mathrm{d}w$.

4 Common cost increase

We begin with the situation in which the cartel affects all firms in the plaintiff's industry. More specifically, assume that all firms have the same (constant) marginal cost c prior to the cartel and are subject to a common marginal cost increase $\mathrm{d}c$ due to the cartel. Let the plaintiff's demand when all firms set the same price p be $q = H(p)$. This is the traditional Chamberlinian DD curve. Assume this is a constant fraction α of total industry demand when all firms set the same price, i.e. $H(p) = \alpha Q(p)$.[9] The industry-level price elasticity of demand is then given by $\varepsilon = -\frac{\partial Q(p)}{\partial p}\frac{p}{Q(p)} = -\frac{\partial H(p)}{\partial p}\frac{p}{H(p)}$. Assume that the cost, demand and competitive conditions generate a symmetric equilibrium, i.e. an equilibrium in which all firms sell their output at the same price p. Denote the equilibrium price as a function of the common marginal cost by $p = p^*(c)$. Assume this function is increasing in c, and define the industry-level pass-on rate by $\tau = \frac{\partial p^*(c)}{\partial c} > 0$. While this set-up imposes much symmetry, it allows for various sources of market power in the plaintiff's market: product differentiation, the number of competitors, and the competitive conduct (e.g. Bertrand versus Cournot). This will be illustrated with specific models below.

The plaintiff's equilibrium profits as a function of the common marginal cost are:

$$\pi(c) = (p^*(c) - c)H(p^*(c)).$$

The change in its profits due to the cartel, given in general by (2), is therefore:

$$\mathrm{d}\pi = \left(-q + q\frac{\partial p^*(c)}{\partial c} + (p - c)\frac{\partial H(p)}{\partial p}\frac{\partial p^*(c)}{\partial c}\right)\mathrm{d}c. \tag{3}$$

This confirms that the cartel has three effects: a direct cost effect (first term), and the pass-on and an output effects (second and third terms). But the additional structure on the plaintiff's downstream market now enables us to quantify the relative importance of these three effects in terms of familiar economic concepts. To see this, define

$$\lambda = \frac{p - c}{p}\varepsilon \tag{4}$$

[9]The assumption of a constant fraction generalizes the usual full symmetry assumption that firms obtain the same fraction of industry demand.

as the competition intensity parameter for the plaintiff's industry, as in Corts (1999). This is a number between zero and one, measuring the plaintiff's actual markup $\frac{p-c}{p}$ relative to the maximum markup it could achieve as a monopolist or as a member of a downstream cartel ($\frac{1}{\varepsilon}$). Substituting the definitions of ε, τ and λ in (3) and rearranging, we can write the change in the plaintiff's profit due to the cartel as:

$$\mathrm{d}\pi = -\left(1 - (1-\lambda)\tau\right) q\mathrm{d}c. \tag{5}$$

Equation (5) says that the cost effect of the cartel ($-q\mathrm{d}c$) forms a starting basis for computing cartel damages, but that a discount equal to $(1-\lambda)\tau$ should be applied. Since $\tau > 0$ and λ is between zero and one, this discount is positive or zero, but less than the pass-on rate. We therefore have:

Proposition 1 *Consider a symmetric industry with a common cost increase due to the cartel. The appropriate discount to the direct cost effect suffered by the plaintiff is positive or zero, and is given by:*

$$\textit{discount} = (1-\lambda)\tau \geq 0. \tag{6}$$

An adjusted passing-on defense is therefore justified, unless the plaintiff's industry is itself fully cartelized ($\lambda = 1$).[10]

The downward adjustment of the pass-on rate in computing the discount stems from the fact that pass-on may lead to a further output effect. In a perfectly competitive industry ($\lambda = 0$), the output effect is absent since lost sales do not matter at the margin. The discount to the cost effect is then simply the unadjusted pass-on rate. But as the plaintiff's industry becomes less competitive ($\lambda > 0$), the lost sales do matter, and the pass-on rate should be adjusted downwards. In the extreme case in which the plaintiff's industry is fully cartelized ($\lambda = 1$), the output effect actually fully offsets the pass-on effect and the discount to the cost effect becomes zero. The passing-on defense would thus not be justified in this extreme case, as has also been observed by Hellwig (2006) and Kosicki and Cahill (2006).

We now apply two standard oligopoly models to the plaintiff's industry to show how these results can be made operational.

4.1 Bertrand competition

With Bertrand competition in the plaintiff's industry, each firm chooses its price to maximize its own profits, taking as given the prices set by the other firms. Market power then stems

[10] While the discount to the direct cost effect is generally positive (unless $\lambda = 1$), it is not necessarily less than one. The discount may be greater than one if the pass-on rate $\tau > 1$ and if λ is sufficiently small. In this case, the plaintiff would actually *gain* from the common cost increase due to the cartel.

from the degree of product differentiation and the number of competing firms. Let the plaintiff's own demand when it sets a price p and its rivals all set the same price r be $D(p,r)$. If $p=r$, we obtain the Chamberlinian DD curve, $D(p,p)=H(p)$. The first-order condition defining a symmetric Bertrand-Nash equilibrium is:

$$(p-c)\frac{\partial D(p,p)}{\partial p}+D(p,p)=0. \tag{7}$$

Define the plaintiff's firm-level own-price elasticity of demand, evaluated at equal prices $p=r$, as $\eta=-\frac{\partial D(p,p)}{\partial p}\frac{p}{D(p,p)}$. Furthermore, define $\delta=-\frac{\partial D(p,p)}{\partial r}\Big/\frac{\partial D(p,p)}{\partial p}$, i.e. the ratio of the cross-price effect of a price increase by the rivals over the own-price effect. If there are no income effects, the firms' cross-price effects are symmetric, so that δ can be interpreted as the plaintiff's aggregate diversion ratio, i.e. the fraction of the sales lost by the plaintiff after a price increase that flows back to its rivals in the industry.[11] Differentiating $D(p,r)$ and $H(p)$ and evaluating at equal prices $p=r$, one can verify that the industry-level price elasticity ε is related to the product-level own-price elasticity η through $\varepsilon=\eta(1-\delta)$. The Bertrand-Nash equilibrium condition (7) can then be written in the following two ways:

$$\begin{aligned}\frac{p-c}{p}&=\frac{1}{\eta}\\&=\frac{1-\delta}{\varepsilon}.\end{aligned}$$

Substituting this in (4), we obtain $\lambda=\frac{\varepsilon}{\eta}=1-\delta$. We can then apply the discount formula (6) to obtain the following corollary to Proposition 1:

Corollary 1 *In a symmetric Bertrand industry with a common cost increase the appropriate discount to the direct cost effect is:*

$$\begin{aligned}\textit{discount}&=\left(1-\frac{\varepsilon}{\eta}\right)\tau\\&=\delta\tau.\end{aligned} \tag{8}$$

The first expression shows that the discount can be obtained by adjusting the pass-on rate using information on the firm-level and market-level price elasticities of demand. The second expression is even simpler and shows that the pass-on rate can be adjusted using information on the plaintiff's aggregate diversion ratio. For example, if $\tau=60\%$ and $\delta=50\%$, the defendant can claim a 30% discount from the cost effect due to the cartel.

[11] To see this formally, we need some additional demand notation. Let $D_i(\mathbf{p})$ be firm i's own demand as a function of the vector of prices set by all firms $\mathbf{p}$. The aggregate diversion ratio of firm i is defined as $-\sum_{j\neq i}\frac{\partial D_j(\mathbf{p})}{\partial p_i}\Big/\frac{\partial D_i(\mathbf{p})}{\partial p_i}$. With symmetric price effects $\frac{\partial D_j(\mathbf{p})}{\partial p_i}=\frac{\partial D_i(\mathbf{p})}{\partial p_j}$, we can write this as $-\sum_{j\neq i}\frac{\partial D_i(\mathbf{p})}{\partial p_j}\Big/\frac{\partial D_i(\mathbf{p})}{\partial p_i}$ which is indeed equal to $\delta=-\frac{\partial D(p,p)}{\partial r}\Big/\frac{\partial D(p,p)}{\partial p}$

Example: logit demand To illustrate, consider the logit model, which has been popular in many areas of antitrust analysis; see for example Werden and Froeb (1994). There are N symmetrically differentiated products and one outside good, the no-purchase alternative. There are L potential consumers who either buy one of the differentiated products, or the "outside good" at an exogenous price p_0. The plaintiff's own demand as a function of its own price p and the identical rivals' prices r is

$$D(p,r) = \frac{\exp(-p)}{\exp(p_0) + \exp(-p) + (N-1)\exp(-r)} L,$$

and its portion of total industry demand as a function of a common industry price (the Chamberlinian DD curve) is:

$$H(p) = \frac{\exp(-p)}{\exp(p_0) + N\exp(-r)} L = s(p)L,$$

where $s(p)$ is the plaintiff's market share in the total number of potential consumers. One can easily verify that $\eta = s(p)(1 - s(p))p$, and $\varepsilon = s(p)(1 - Ns(p))p$. This implies that $\lambda = \frac{1-Ns(p)}{1-s(p)}$, so that the discount to the direct cost effect is

$$\text{discount} = \frac{(N-1)s(p)}{1-s(p)}\tau.$$

This discount can be computed using information on the pass-on rate, the number of firms and the plaintiff's market share in the total number of potential consumers.

4.2 Cournot competition with conjectural variations

Now suppose that the firms in the plaintiff's industry compete according to a homogeneous goods Cournot model with conjectural variations. Let $p = P(Q)$ denote the inverse industry demand function, where $Q = \sum_{i=1}^{N} q_i$ is total industry output, i.e. the sum of the quantities produced by the N firms. In the standard Cournot model each firm chooses its quantity to maximize its profits, taking as given the quantities of the other firms. In the conjectural variations extension each firm "conjectures" that a change in its own quantity induces the other firms to respond. Let the conjectural variations parameter θ be each firm's conjectured change in total output Q when a firm changes its own quantity by one unit. The first-order condition defining a symmetric conjectural variations equilibrium is

$$P(Q) - c + P'(Q)\theta\frac{Q}{N} = 0. \tag{9}$$

This condition nests several well-known special cases. If $\theta = 1$, each firm conjectures that total output increases by the same amount as its own quantity (i.e. it takes the quantities of the other firms as given), so that the standard Cournot condition obtains. If $\theta = N$, each firm conjectures that each rival will fully match a quantity increase, so that the condition of a perfect cartel obtains. If $\theta = 0$, each firm conjectures that the rivals contract their quantities in response to a change in its own quantity, in such a way that total output remains constant. In this case, the condition of perfect competition obtains. Outside such special cases, the conjectural variations model has little game-theoretic appeal, since it aims to capture dynamic responses within a static model. It has, however, often been used in empirical work to estimate the conduct or average collusiveness of firms without having to specify a full dynamic model. The critical debate on the interpretation and estimation of θ is ongoing, as illustrated by the critical discussions in Bresnahan (1989), Corts (1999) and Reiss and Wolak (2005). We nevertheless include it here to show how it fits nicely into our framework for computing discounts to the cost effect.

Using the industry-level price elasticity $\varepsilon = -\frac{1}{P'(Q)}\frac{P(Q)}{Q}$, the conjectural variations equilibrium condition (9) can be rewritten as

$$\frac{p-c}{p} = \frac{\theta}{N}\frac{1}{\varepsilon}.$$

Based on (4), we can compute $\lambda = \frac{\theta}{N}$, and apply this to the discount formula to obtain a second corollary to Proposition 1:

Corollary 2 *In a symmetric Cournot conjectural variation industry with a common cost increase the appropriate discount to the direct cost effect is:*

$$discount = \left(1 - \frac{\theta}{N}\right)\tau. \tag{10}$$

For example, in the standard Cournot model $\theta = 1$, so that only information on the number of firms is required to adjust the pass-on rate and obtain the discount.

5 Firm-specific cost increase

In a variety of settings it is not appropriate to assume that the cartel leads to a cost increase common to all firms in the plaintiff's industry. First, one or more of the cartel members may be vertically integrated and therefore also be active as a downstream competitor in the plaintiff's industry. Such a firm could then decide to favour its own downstream unit and only charge a high input price to the downstream competitors.[12] To the extent that such

[12] It is not obvious whether a vertically integrated firm would actually have an incentive to engage in such foreclosure; see e.g. Rey and Tirole (2006). So in real antitrust cases this should be separately investigated on a case by case basis.

behavior would occur, the plaintiff experiences a competitive disadvantage relative to some of its rivals, so that it is no longer appropriate to apply the above analysis of a common cost increase.

Second, the cartel members may not be able to perfectly control the supply of their input. Some of the plaintiff's downstream competitors may be able to purchase their inputs from suppliers outside the cartel, or from foreign suppliers if the cartel is national, etc. The plaintiff then also suffers a competitive disadvantage relative to its rivals, so that the analysis of a common cost increase is no longer valid.

This motivates an analysis of firm-specific cost increases due to the cartel. The general framework of section 3 still applies, i.e. there is a direct cost effect, a pass-on effect and an output effect. However, the relative magnitudes of these effects no longer follow the simple relations obtained for common cost increases in section 4. In particular, the output effect becomes potentially more important since the plaintiff loses to other firms in its industry as it passes on part of the cost increase. As we will show, it is even possible that the output effect dominates the pass-on effect.

To obtain concrete insights on firm-specific cost increases, we first consider a Bertrand model with differentiated products and subsequently a Cournot model with homogeneous products.

5.1 Bertrand competition

There are N price-setting firms, $i = 1 \cdots N$, selling differentiated products. Let I be the set of insiders, i.e. the firms who are affected by the cartel. One of the insiders is the plaintiff, denoted by firm 1. Each firm i sells a single product and sets a price p_i, operating at a constant marginal cost c_i. Firm i's profits in the but-for world (without the cartel) are

$$\pi_i = (p_i - c_i) D_i(\mathbf{p}),$$

where $q_i = D_i(\mathbf{p})$ is its demand, as a function of the $N \times 1$ price vector $\mathbf{p} = (p_1 \cdots p_N)$. Demand is downward sloping $\frac{\partial D_i(\mathbf{p})}{\partial p_i} < 0$, products are gross substitutes $\frac{\partial D_i(\mathbf{p})}{\partial p_k} > 0$ for $k \neq i$, there are no income effects so that the cross-price effects are symmetric $\frac{\partial D_i(\mathbf{p})}{\partial p_k} = \frac{\partial Dk(\mathbf{p})}{\partial p_i}$, and the Jacobian is negative-definite. Let the diversion ratio between the plaintiff firm 1 and any other firm $k \neq 1$ be $\delta_1^k = -\frac{\partial D_k(\mathbf{p})}{\partial p_1} \Big/ \frac{\partial D_1(\mathbf{p})}{\partial p_1}$. This is the fraction of firm 1's lost sales that diverts to firm k after a price increase by firm 1. Furthermore, let firm 1's aggregate diversion ratio be $\delta_1 = \sum_{k \neq 1} \delta_1^k < 1$, i.e. the fraction of firm 1's lost sales that flows to other firms within the industry.[13]

[13] This is the same as the aggregate diversion ratio δ defined earlier in the symmetric framework.

The system of first-order conditions defining a Bertrand-Nash equilibrium is

$$(p_i - c_i)\frac{\partial D_i(\mathbf{p})}{\partial p_i} + D_i(\mathbf{p}) = 0, \qquad\qquad i = 1 \cdots N \tag{11}$$

and assume that the second-order conditions are satisfied. Denote the equilibrium price vector, the solution to (11), as a function of the marginal cost vector $\mathbf{c} = (c_1 \cdots c_N)$, i.e. $\mathbf{p} = p^*(\mathbf{c})$. Let firm k's pass-on rate with respect to a cost increase of firm i be $\tau_i^k = \frac{\partial p_k^*(\mathbf{c})}{\partial c_i}$ and assume that these are all positive, i.e. $\tau_i^k > 0$ for all i, k. Furthermore, let firm k's insider-level pass-on rate be $\tau_I^k = \sum_{i \in I} \tau_i^k$, i.e. firm k's pass-on rate with respect to a cost increase of all insiders $i \in I$.

We are interested in the effect of the cartel on the plaintiff firm 1's profits. Firm 1's equilibrium profits as a function of the marginal cost vector $\mathbf{c}$ in the but-for world are

$$\pi_1(\mathbf{c}) = (p_1^*(\mathbf{c}) - c_1)D_1(p^*(\mathbf{c})).$$

Assume that the cartel raises all insiders' marginal costs by the same amount as the plaintiff and does not affect the outsiders' marginal costs, i.e. $\mathrm{d}c_i = \mathrm{d}c_1$ for $i \in I$ and $\mathrm{d}c_i = 0$ for $i \notin I$. The change in plaintiff firm 1's profits in response to the cartel is then equal to

$$\begin{aligned} \mathrm{d}\pi_1 &= \sum_{i=1}^{N} \frac{\partial \pi_1(\mathbf{c})}{\partial c_i}\mathrm{d}c_i \\ &= \sum_{i \in I} \frac{\partial \pi_1(\mathbf{c})}{\partial c_i}\mathrm{d}c_1 \\ &= \left(-q_1 + q_1 \sum_{i \in I} \frac{\partial p_1^*}{\partial c_i} + (p_1 - c_1)\sum_{i \in I}\left(\frac{\partial D_1}{\partial p_1}\frac{\partial p_1^*}{\partial c_i} + \cdots \frac{\partial D_1}{\partial p_N}\frac{\partial p_N^*}{\partial c_i}\right)\right)\mathrm{d}c_1. \end{aligned} \tag{12}$$

This reconfirms that the cartel has three effects. The first term is the cost effect and proportional to minus firm 1's sales, $-q_1$. The second term is the pass-on effect. It is positive and proportional to firm 1's sales, multiplied by the extent to which firm 1 passes on the insiders' marginal cost increases. The third term is the output effect. It is proportional to firm 1's profit margin, multiplied by the extent to which firm 1 loses sales through the equilibrium price responses of all firms (both the insiders and outsiders).

We now show that the positive pass-on effect dominates the output effect, so that an adjusted passing-on defense is valid under the general conditions of the Bertrand model. To see this, substitute firm 1's first-order condition (11) and the symmetric cross-effects $\frac{\partial D_1}{\partial p_k} = \frac{\partial D_k}{\partial p_1}$ in the profit change (12). Then apply the definitions of the diversion ratios δ_1^k and the pass-on rates τ_I^k to obtain:

$$\mathrm{d}\pi_1 = -\left(1 - \delta_1^2\tau_I^2 - \cdots - \delta_1^N\tau_I^N\right) q_1 \mathrm{d}c_1. \tag{13}$$

Equation (13) says that the cost effect of the cartel ($-q\mathrm{d}c$) should be discounted by the amount $\delta_1^2\tau_I^2 + \cdots + \delta_1^N\tau_I^N$. Since $\delta_1^k > 0$ for $k \neq 1$ and $\tau_I^k > 0$, this discount is generally positive. We therefore have:

Proposition 2 *Consider a Bertrand industry with a cost increase due to the cartel to the set of insiders I only, including the plaintiff firm 1. The appropriate discount to the direct cost effect suffered by plaintiff firm 1 is positive, and is given by*

$$\textit{discount} = \delta_1^2\tau_I^2 + \cdots + \delta_1^N\tau_I^N > 0. \tag{14}$$

An adjusted passing-on defense is therefore justified.[14]

The discount to the direct cost effect (14) reflects the combined pass-on and output effects. Intuitively, it is equal to a weighted-average of the insider-level pass-on rates over all firms except the plaintiff firm 1, where the weights are the diversion ratios with respect to firm 1. The adjusted passing-on defense should however be carefully interpreted. It is *not* the fact that plaintiff firm 1 is able to pass on the insiders' cost increase that justifies resorting to the passing-on defense. This term is actually only a second-order effect because it is fully compensated by an output effect.[15] In contrast, it is that fact that all *other* firms than the plaintiff also raise their prices in response to the insiders' cost increases that justifies the passing-on defense. These other firms' price responses are first-order effects and raise the plaintiff's profits through increased output.

The question of practical interest is of course how to measure the discount to the direct effect (14). A first approach is to obtain an econometric estimate of the insider-level pass-on rates for all firms that are active in the downstream market. The discount can then be computed from (14) using quantitative or qualitative information on the diversion ratios as weights. This approach may require a substantial amount of information in practice. As an alternative, one may make additional assumptions to obtain a more explicit expression of the discount formula. We discuss two examples next.

Identical firms without the cartel Suppose that all firms in the plaintiff's industry are identical in the but-for world, and are correspondingly in a symmetric Bertrand-Nash equilibrium. The cartel subsequently raises the insiders' marginal costs and leaves the other firms' unaffected. With identical firms in the but-for world, the insider-level pass-on rates

[14] The discount may be greater than one if some of the pass-on rates $\tau_I^k > 1$ and the corresponding diversion ratios δ_1^k are sufficiently close to 1.

[15] Formally, after substituting firm 1's first-order condition (11) the pass-on term $q_1 \sum_{i \in I} \frac{\partial p_1^*}{\partial c_i}$ cancels out with part of the output term in (12).

are all equal, i.e. $\tau_I^k = \tau_I$ for all k. Using the aggregate diversion ratio $\delta_1 = \sum_{k\neq 1} \delta_1^k$, the discount formula (14) simplifies to

$$\text{discount} = \delta_1 \tau^I. \tag{15}$$

This generalizes our earlier discount formula (8) for the symmetric Bertrand industry with a common cost increase. The crucial difference is that the insider-level pass-on rate τ^I now enters instead of the industry-level pass-on rate τ. Since the insider-level pass-on rate is typically lower than the industry-level pass-on rate, the discount to the cost effect is clearly also lower.

Symmetric substitution patterns and logit demand Now allow firms to be different but assume that the demand side is characterized by symmetric substitution patterns. This means that a loss in the market share of plaintiff firm 1 (or of any other firm) is associated with an increase in the market shares of the rivals in proportion to their market shares. This is a property of random utility discrete choice models of demand, when the independence of irrelevant alternatives (IIA) assumption is satisfied. The diversion ratio between firm 1 and firm k is then equal to $\delta_1^k = \frac{s_k}{1-s_1}$,where s_k is the market share of firm k in the total number of potential consumers L. The discount (14) can then be written as

$$\text{discount} = \frac{1}{1-s_1}\left(s_2\tau_I^2 + \cdots + s_N\tau_I^N\right). \tag{16}$$

Hence, the discount equals the weighted average of the rivals' pass-on rates where the market shares are the weights. Equivalently, one can interpret this discount as the effect of the insiders' cost increase on the price index for the whole industry except firm 1 (using fixed market shares as weights).

To avoid econometric estimation of the pass-on rates, one may further specify the demand model and compute the pass-on rates as a function of observables or a limited set of parameters. To illustrate this, consider the earlier discussed logit model of demand, but now allowing firms to differ in quality or costs. The logit model satisfies the IIA assumption and correspondingly entails symmetric substitution patterns. In the Appendix we derive explicit formula for the pass-on rates τ_i^k as a function of observable market shares s_i, i.e. shares of each firm i in the total potential sales. Substituting these in (16) and rearranging, one can write the formula for the discount to the cost effect as:

$$\text{discount} = \frac{\sum_{i\in I} T_i - s_1(1-s_1)}{(1-s_1)^2 + s_1}, \tag{17}$$

where

$$T_i = \frac{s_i(1-s_i)^2}{(1-s_i)^2+s_i} \Big/ \left(s_0 + \sum_{k=1}^N \frac{s_k(1-s_k)^2}{(1-s_k)^2+s_k}\right),$$

and s_0 is the market share of the outside good.[16] This shows that the discount to the direct cost effect can be computed using only information on the market shares of all firms, including the outside good, and on the identity of the insiders. While the market share of the outside good is typically not known, it can be calibrated using information on the market-level price elasticity of demand. For example, if the market-level elasticity is approximately zero, then $s_0 = 0$.

One can immediately verify that the logit discount is indeed always positive. In addition, the discount is always less than 1. Furthermore, it approaches 1 if there is a common cost increase and perfectly inelastic industry-level demand (i.e. the number of insiders is equal to N and the market share of the outside good $s_0 \to 0$).

To gain additional intuition on the logit discount formula, Table 1 computes the discounts for alternative values of the number of firms in the plaintiff's market, the number of outsiders (not affected by the cartel), and the plaintiff's market share. The table assumes that the outside good has a market share of 10%, that the plaintiff firm 1 has a market share of either 10% or 50%, and that the other firms share the rest of the market equally. The table confirms that the discount is less than 100% but always positive, even if all firms except the plaintiff are outsiders (bold numbers on diagonal). We can make three additional observations. First, a comparison across the rows shows that the discount decreases with the number of unaffected outsiders. Second, a comparison across the columns shows that the discount increases with the degree competition in the plaintiff's market (holding the number of outsiders constant). Third, a comparison between the left and right panel shows that the discount is often larger if plaintiff has a small market share.

5.2 Cournot competition

There are $N > 1$ quantity-setting firms, selling a homogeneous product. Let I again be the set of insiders, i.e. the firms affected by the cartel, and let N_I be the number of these insiders. One of the insiders is the plaintiff, again denoted by firm 1. Each firm $i = 1 \cdots N$ produces a quantity q_i at a constant marginal cost $c_i > 0$. Total industry output is $Q = \sum_{k=1}^{N} q_k$ and the average of all firms' marginal costs is $\bar{c} = \sum_{k=1}^{N} c_k \Big/ N$. Let the inverse industry demand function be $p = P(Q)$, with $P'(Q) < 0$. The price elasticity of industry demand is $\varepsilon = -\frac{1}{P'(Q)} \frac{P(Q)}{Q}$. A measure of the curvature of industry demand is the elasticity of the slope of the inverse demand curve, i.e. $\rho = -P''(Q) \frac{Q}{P'(Q)}$; see e.g. Vives (1999). If $\rho < 0$, demand

[16] The market share of the outside good enters the discount formula differently from the other shares, because the price of this product remains constant after the cost change unlike the prices of the other products.

is concave; if $\rho = 0$, demand is linear; and if $\rho > 0$, demand is convex. A well-known example of convex demand is the constant elasticity demand case, for which $\rho = \frac{1+\varepsilon}{\varepsilon}$.[17] Each firm i chooses to produce its quantity q_i to maximize profits

$$\pi_i = (P(Q) - c_i)q_i,$$

taking as given the quantities chosen by the rival firms. The system of necessary first-order conditions defining a Cournot-Nash equilibrium is

$$P(Q) - c_i + P'(Q)q_i = 0, \qquad i = 1 \cdots N. \tag{18}$$

Assume that $P'(Q) + P''(Q)q_i \leq 0$ for all i. Given constant marginal costs c_i, this assumption ensures the existence of a unique Cournot-Nash equilibrium.[18] The assumption is equivalent to $1 - \rho s_i \geq 0$ for all i, where $s_i = q_i/Q$ is firm i's market share. It also implies that $N - \rho \geq 0$.

To perform comparative statics of the cost increase due to the cartel, first define the equilibrium quantities and price as a function of the marginal costs. Adding up the first-order conditions gives

$$(P(Q) - \overline{c})\, N + P'(Q)Q = 0. \tag{19}$$

Under the above assumptions, the left-hand-side of (19) is decreasing in Q and $\overline{c}$ and implicitly defines the equilibrium industry output function, $Q = Q^*(\overline{c})$, decreasing in $\overline{c}$. Furthermore, using the inverse industry demand function we can also define the equilibrium price function $p = p^*(\overline{c}) = P(Q^*(\overline{c}))$, increasing in $\overline{c}$. Implicitly differentiating (19), using the definition of ρ, and rearranging, we obtain

$$\tau = \frac{\partial p^*(\overline{c})}{\partial \overline{c}} = \frac{N}{N + 1 - \rho}. \tag{20}$$

This can be interpreted as the industry-level pass-on rate since $\tau = \frac{\partial p^*(\overline{c})}{\partial \overline{c}} = \sum_{i=}^{N} \frac{\partial p^*(\overline{c})}{\partial \overline{c}} \frac{\partial \overline{c}}{\partial c_i}$. Note that $\tau > 0$ since $N + 1 - \rho > N - \rho \geq 0$. Note also that pass-on is incomplete, i.e. $\tau < 1$, if and only if $\rho < 1$. Finally, substituting $Q = Q^*(\overline{c})$ in the first-order condition (18) gives

$$P(Q^*(\overline{c})) - c_i + P'(Q^*(\overline{c}))q_i = 0. \tag{21}$$

Equation (21) implicitly defines firm i's equilibrium output function $q_i = q_i^*(\overline{c}, c_i)$. Under the above assumptions (21) is decreasing in q_i, increasing in $\overline{c}$ and decreasing in c_i, so that $q_i^*(\overline{c}, c_i)$ is increasing in $\overline{c}$ and decreasing in c_i.

[17] An often-used related measure for the demand curvature is the elasticity of the elasticity. This is defined by $E = \varepsilon'(Q) \frac{1}{P'(Q)} \frac{P(Q)}{\varepsilon(Q)}$. It can be verified that $\rho = \frac{\varepsilon + 1 - E}{\varepsilon}$. Our measure gives simpler expressions below.

[18] Vives (1999) provides a more detailed discussion of these and weaker conditions for the existence, uniqueness and stability of Cournot equilibrium.

As before, we are interested in the effect of the cartel on the plaintiff firm 1's profits. Firm 1's equilibrium profits in the but-for world can be written as a function of the average of all firms' marginal costs $\bar{c}$ and its own marginal cost c_1:

$$\pi_1(\bar{c}, c_1) = (p^*(\bar{c}) - c_1)\, q_1^*(\bar{c}, c_1).$$

Assume again that the cartel affects all insiders' marginal costs by the same amount as the plaintiff and does not affect the other firms, $\mathrm{d}c_i = \mathrm{d}c_1$ for $i \in I$ and $\mathrm{d}c_i = 0$ for $i \notin I$. The change in plaintiff firm 1's profits in response to the cartel then equals

$$\begin{aligned}
\mathrm{d}\pi_1 &= \sum_{i=1}^{N} \frac{\partial \pi_1(\bar{c}, c_1)}{\partial c_i} \mathrm{d}c_i \\
&= \sum_{i \in I} \frac{\partial \pi_1(\bar{c}, c_1)}{\partial c_i} \mathrm{d}c_1 \\
&= \left(-q_1 + q_1 \sum_{i \in I} \left(\frac{\partial p^*}{\partial \bar{c}} \frac{1}{N} \right) + (p - c_1) \sum_{i \in I} \left(\frac{\partial q_1^*}{\partial \bar{c}} \frac{1}{N} + \frac{\partial q_1^*}{\partial c_i} \right) \right) \mathrm{d}c_1. \qquad (22)
\end{aligned}$$

The first term is the direct cost effect of the cartel and is clearly negative. The second term is the insider-level pass-on effect and it is positive since $\tau > 0$. Using (20), it can be written as $q_1 \frac{N_I}{N} \tau$, i.e. it is proportional to the industry-wide pass-on rate times the fraction of insider firms $\frac{N_I}{N}$. The third term is the output effect and is typically negative.

We now show that in contrast to the Bertrand model, the negative output effect may be so strong that it actually dominates the positive pass-on effect. This implies that the discount to the cost effect may actually be negative, i.e. the plaintiff may actually incur damages that are *larger* than the direct cost effect. A passing-on defense may therefore no longer necessarily be justified.

To see this, we follow the same approach as in the Bertrand case and rewrite firm 1's profit change (22), after substituting firm 1's first-order condition from (18), substituting the pass-on expression (20), and performing the required implicit differentiations $\frac{\partial q_1^*(\bar{c}, c_i)}{\partial \bar{c}}$ and $\frac{\partial q_1^*(\bar{c}, c_i)}{\partial c_i}$ on (21). This gives:

$$\mathrm{d}\pi_1 = -\left(1 - \left(\frac{2 - \rho s_1}{N + 1 - \rho} N_I - 1\right)\right) q_1 \mathrm{d}c_1. \qquad (23)$$

Equation (23) implies that the cost effect of the cartel $(-q_1 \mathrm{d}c_1)$ should be discounted by the amount $\frac{2-\rho s_1}{N+1-\rho} N_I - 1$. It is easy to see that this discount is not necessarily positive, i.e. the pass-on effect may be fully dominated by the output effect. For example, under linear demand ($\rho = 0$) the discount is negative if and only if the number of $N_I < \frac{N+1}{2}$. As another example, under constant and unitary elasticity demand ($\varepsilon = 1$ and $\rho = 1 + \frac{1}{\varepsilon} = 2$) the discount is negative if and only if $N_I < \frac{N-1}{2(1-s_1)}$. Intuitively, in the Cournot model negative discounts to the direct effect may arise and invalidate the passing-on defense because the outsiders respond aggressively by expanding their output when the plaintiff and the other insiders reduce output after the cost increase. These aggressive output responses may make the output effect fully dominate the pass-on effect. We therefore have:

Proposition 3 *Consider a Cournot industry with a cost increase due to the cartel to the set of insiders I only, including the plaintiff firm 1. The appropriate discount to the direct cost effect suffered by plaintiff firm 1 may be positive or negative, and is given by*

$$\textit{discount} = \frac{2-\rho s_1}{N+1-\rho} N_I - 1 \gtrless 0 \tag{24}$$

An adjusted passing-on defense is therefore not necessarily justified.[19]

The discount formula (24) is written in terms of the demand curvature condition ρ. Using (20), it is however also possible to rewrite it in terms of the pass-on rate τ. This gives:

$$\text{discount} = (1-s_1)\frac{N_I}{N}\tau - (1-Ns_1)(1-\frac{N_I}{N}\tau) - Ns_1(1-\frac{N_I}{N}).$$

This formula may be preferred if ρ is difficult to observe and instead an estimate of τ is available. This is also how we expressed the discount formulas for the Bertrand model or for the symmetric models with a common cost increase.[20]

Because of the Cournot assumption $1-\rho s_i \geq 0$ for all i, we have $\frac{2-\rho s_1}{N+1-\rho} > 0$. This immediately implies that the discount (24) increases as the number of insiders N_I increases. It is not obvious, however, how many insiders are required for the discount to become positive and a passing-on defense to be justified. This depends on the plaintiff's market share s_1, on the number of firms N and especially on the curvature of demand ρ, of which both the sign and the magnitude are unknown and difficult to measure empirically (since it captures a second-order property of the demand curve). We can nevertheless show:

Proposition 4 *(a) In a Cournot industry with a common cost increase ($N_I = N$), the discount is positive and hence an adjusted passing-on defense is justified unless $\rho < -(N-1)$ and $s_1 < \frac{1}{N} - \frac{N-1}{N}\frac{1}{(-\rho)}$.*

(b) In a Cournot industry with a cost increase to the plaintiff only ($N_I = 1$), the discount is negative and hence an adjusted passing-on defense is not justified unless $\rho > (N-1)$ and $s_1 < \frac{1}{N} - \frac{N-1}{N}\frac{N-\rho}{\rho}$.

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[19] Furthermore, the discount may be greater than one, if demand is sufficiently convex.

[20] Note that if the cost increase applies to all firms $N_I = N$, and the plaintiff has a symmetric market share $s_1 = \frac{1}{N}$, the second and third terms vanish so that the discount reduces to our earlier symmetric Cournot formula (10) (with $\theta = 1$).

Proof. See the Appendix. ■

Proposition 6 provides easy to interpret necessary and sufficient conditions under which the passing-on defense is justified after a common cost increase ($N_I = N$) and not justified after a cost increase to the plaintiff only ($N_I = 1$). While a cost increase to the plaintiff only is clearly not representative for most cartels, it serves as a benchmark to stress that a passing-on defense against a Cournot plaintiff is only valid if a sufficiently large number of firms is affected by the cartel.

Proposition 6 can be simplified to the following possible sufficient conditions:

Corollary 3 *For the Cournot industry suppose that one of the following conditions applies: (i) $-(N-1) \leq \rho \leq N-1$ or equivalently $\frac{1}{2} < \tau < \frac{N}{2}$; or (ii) $s_1 \geq \frac{1}{N}$. An adjusted passing-on defense is then always justified after a common cost increase ($N_I = N$) and never justified after a cost increase to the plaintiff only ($N_I = 1$).*

Proof. The demand curvature condition $-(N-1) \leq \rho \leq N-1$ follows immediately from Proposition 6, and is equivalent to the condition on the pass-on rate $\frac{1}{2} \leq \tau \leq \frac{N}{2}$ by (20). The market share condition $s_1 \geq \frac{1}{N}$ also follows immediately, since both market share thresholds in Proposition 6 are below $\frac{1}{N}$. ■

The demand curvature condition on ρ (or the equivalent pass-on rate condition) is satisfied for a wide range of demand functions, including linear and exponential demand, but not necessarily under constant elasticity demand. The market share condition generalizes our earlier result of Corollary 2 that the passing-on defense is justified in a symmetric Cournot model with a common cost increase:[21] this continues to be true in an asymmetric Cournot model as long as the plaintiff has a higher than average market share. In sum, under a wide variety of circumstances the passing-on defense is justified when all firms are affected by the cost increase, and not justified when only the plaintiff is affected. This shows the key importance of assessing how many firms have been affected by the cost increase before resorting to the passing-on defense in a Cournot industry.

Specific functional forms of demand A more concrete picture of the discount formula (24) and our subsequent results emerges from specific functional forms of demand. Consider Genesove and Mullin's (1998) demand specification $Q = \beta(\alpha - p)^\gamma$, according to which the demand curvature is $\rho = \frac{\gamma-1}{\gamma}$. This specification nests various special demand functions with an increasingly convex curvature: linear demand ($\gamma = 1$, so that $\rho = 0$), quadratic demand

[21] This is the special case for which $N_I = N$ and $s_i = \frac{1}{N}$ for all i.

($\gamma = 2$, so that $\rho = \frac{1}{2}$), exponential demand ($\alpha, \gamma \to \infty$, $\frac{\alpha}{\gamma}$ constant, so that $\rho = 1$), and log-linear or constant elasticity demand ($\alpha = 0$, $\gamma < 0$, so that $\rho = 1 + \frac{1}{\varepsilon}$. We then have incomplete pass-on ($\tau < 1$) for linear and quadratic demand; complete pass-on ($\tau = 1$) for exponential demand; and more than complete pass-on ($\tau > 1$) for log-linear demand.

Table 2 computes the discount to the direct cost effect for these four demand specifications, for alternative values of the number of firms N, the number of outsiders not affected by the cartel $N - N_I$, and the plaintiff's market share s_1 (either 10% or 50%).[22] Table 2 confirms our findings summarized in Propositions 5 and 6 and Corollary 7. In contrast to the Bertrand model, the discount to the cost effect is not generally positive. It is, however, positive for a common cost increase (no outsiders, on first row of each panel). It decreases as the number of outsiders increases and it is almost always negative for a cost increase to the plaintiff only (all firms but the plaintiff are outsiders, bold numbers on diagonal).[23] Table 2 also illustrates how the discount varies in a complex way with the number of competing firms N, the plaintiff's market share s_1 and especially how this interacts with the demand curvature ρ. When the plaintiff has a small market share of 10%, the most conservative discounts obtain in the linear demand case, and they increase as demand becomes more convex. The reverse is true however when the plaintiff has a large market share of 50% and the number of firms is sufficiently large (6 or 10).[24] This discussion shows the importance of a robustness analysis in applying the passing-on defense when the demand curvature ρ is not observed. Alternatively, one may indirectly retrieve ρ from an empirical estimate of the pass-on rate τ and applying the pass-on formula (20).

6 The cartel's total harm

Our analysis has so far exclusively looked at the cartel's effects on the plaintiff's profits. In a recent paper, Basso and Ross (2007) take a different focus and analyze the total harm of the cartel. This is the harm to all affected parties, i.e. all firms (including the plaintiff) and the

[22] The elasticity is irrelevant for the results in all specifications, except under log-linear demand. In that case, we set it equal to 2 so that $\rho = 1 + \frac{1}{\varepsilon} = \frac{3}{2}$.

[23] There is only one case with a positive discount (23%) after a cost increase to the plaintiff, i.e. under the log-linear demand with $N = 2$, $N_I = 1$ and plaintiff's market share $s_1 = 0.1$. In this case, the two possible sufficient conditions of Corollary 7 are violated since (i) $\rho = \frac{3}{2} > 1$, and $s_1 = 0.1 > \frac{1}{2}$.

[24] Furthermore, for linear and quadratic demand the discount increases as the number of competing firms N increases, as in the Bertrand case. This is also true for exponential demand if there is at least one outsider. However, for exponential demand without an outsider the discount is independent of the number of competitors. Furthermore, for log-linear demand, the discount may actually decrease as competition increases.

final consumers purchasing from the firms. Interestingly, Basso and Ross also relate their findings to the traditional cost effect, arguing that the total harm is larger for two reasons. First, the cartel generates a deadweight loss associated with the output reduction. Second, the cartel generates an additional harm because the firms purchasing from the cartel are not the final consumers. Our framework essentially treats the first reason (deadweight loss) as a second-order effect, since it considered "small" cartel overcharges dc.[25] The second reason, and Basso and Ross' main point, is however a first-order effect, and we now show how it relates to our own framework, and in particular to the output effect.

We return to the case of a common cost increase of section 4, but now consider the cartel's effect on all firms (including the plaintiff) and on the consumers. As before, total industry demand when all firms set the same price p is $Q = Q(p)$. Furthermore, let aggregate consumer surplus be $v(p)$. The industry equilibrium price as a function of the common marginal cost c is again $p = p^*(c)$. Total surplus as a function of marginal cost c is the sum of aggregate industry profits $\Pi(c)$ and aggregate consumer surplus $v(p^*(c))$:

$$\begin{aligned} S(c) &= \Pi(c) + v(p^*(c)) \\ &= (p^*(c) - c)\, Q\left(p^*(c)\right) + v(p^*(c)), \end{aligned}$$

The change in total surplus due to the cartel's cost increase, or the cartel's total harm, is therefore:

$$\mathrm{d}S = \left(-Q + Q\frac{\partial p^*(c)}{\partial c} + (p-c)\frac{\partial Q(p)}{\partial p}\frac{\partial p^*(c)}{\partial c} \right)\mathrm{d}c + \frac{\partial v(p)}{\partial p}\frac{\partial p^*(c)}{\partial c}\mathrm{d}c.$$

First, the cartel's effect on aggregate profits consists of an aggregate cost, pass-on and output effect. This parallels our previous analysis of the plaintiff's profit; see (3) in section 4. Second, the effect on consumers is the pass-on effect, as transferred by the firms purchasing from the cartel. Applying the aggregate version of Roy's identity, $Q = -\frac{\partial v(p)}{\partial p}$, and using our earlier defined price elasticity of demand ε, pass-on rate τ and competition intensity parameter λ, the cartel's total harm can be written as:

$$\begin{aligned} \mathrm{d}S &= -\left(1 - (1-\lambda)\tau\right) Q\mathrm{d}c + \tau Q\mathrm{d}c \\ &= -\left(1 + \lambda\tau\right) Q\mathrm{d}c. \end{aligned} \tag{25}$$

[25] This does not mean, however, that the deadweight loss is not important in practice. On the basis of public information on approximately 300 cartels, Connor (2004) finds a median cartel overcharge of 25% across all types of cartels over all time periods. On the basis of evidence from the trade press, Levenstein and Suslow (2006, p80) report price overcharges ranging from 10 to 100% for international cartels. Hellwig (2006) and Leslie (2006) discuss the economics and legal implications of accounting for the cartel's deadweight loss. Incorporating it in our framework would entail integrating over the small overcharges dc. This does not give general closed form solutions, but simulation analysis based on specific functional forms would provide an alternative solution (as in merger analysis).

The final consumers are hurt by the extent of pass-on τ, but this is merely a transfer from the cartel's purchasers so it cancels out in the second line of (25). The total harm is therefore the direct cost effect, plus a percentage *premium* $\lambda\tau$. In fact, this premium reflects the output effect that was also used in the downward adjustment of the discount when applying a passing-on defense to the purchaser plaintiff. We therefore have:

Proposition 5 *Consider a symmetric industry with a common cost increase due to the cartel. The total harm from the cartel consists of the aggregate direct cost effect, plus a percentage premium of $\lambda\tau$. This premium reflects the output effect.*

It is straightforward to generalize Proposition 5 to allow for one or multiple layers of indirect purchaser industries.[26]

The result that the total harm from the cartel is larger than the aggregate cost effect is consistent with Basso and Ross (2007). We show how this premium (or multiplier as they call it) reflects the output effect and can be nicely written in terms of the pass-on rate τ and our competition intensity parameter λ.[27] Interestingly, an adjusted passing-on defense against the purchaser plaintiffs may therefore actually turn against the defendant, since the evidence required to adjust for the output effect in the passing-on defense may also be used to demonstrate by how much the cartel's total harm exceeds the direct cost effect.

7 Concluding discussion

We have developed a general economic framework to assess cartel damages to a purchaser plaintiff, starting from the anticompetitive price overcharge (or direct cost effect) as the commonly used basis. We have identified the circumstances under which an adjusted passing-on defense against the purchaser plaintiff is justified. This defense takes into account that

[26] Consider an upstream industry consisting of the cartel's direct purchasers and a downstream industry consisting of the indirect purchasers. Aggregate profits in the downstream industry are $\Pi_D(w) = (p^*(w) - w)Q(p^*(w))$, where $p^*(w)$ is the equilibrium price as a function of the wholesale price w. Aggregate profits in the upstream industry are $\Pi_U(c) = (w^*(c) - c)Q(p^*(w^*(c)))$, where $w^*(c)$ is the equilibrium wholesale price as a function of marginal cost c. Add up aggregate upstream and downstream profits and aggregate consumer surplus, and differentiate with respect to c. This gives the same total harm formula (25), where λ should be interpreted as the overall markup of direct and indirect purchasers $(p - c)$, multiplied by the elasticity of final industry demand, and τ should be reinterpreted as the combined pass-on rate of the direct and indirect purchasers.

[27] Our discussion of Basso and Ross is clearly stylized and is only intended to relate it to our own framework. We refer to Basso and Ross (2007) for more specific results, and computations of the premium under specific models of oligopoly and demand.

any pass-on of the price overcharge by the plaintiff may subsequently also lead to a reduction in its output. While incorporating this output effect inevitably complicates the analysis, the informational requirements increase only moderately relative to a simple passing-on defense that ignores the output effect. In particular, we derive explicit formulas for the discount to the cost effect under a wide variety of circumstances. These include various models of imperfect competition, and allow for either common cost increases to all of the plaintiff's competitors or cost increases to only a subset of them. We note, however, that a proper account of the output effect in the passing-on defense may actually turn against the defendant, when one considers the cartel's total harm (i.e. including the effect on final consumers).

Our suggested discount formulas depend on relatively easy-to-observe variables for the purchaser plaintiff's industry. They naturally lead to two empirical approaches. The first is a reduced-form approach and relates most closely to a traditional pass-on analysis. It entails estimating the appropriate pass-on rate (firm-specific of industry wide) and then adjusting that pass-on rate based on our derived discount formulas for alternative industries.[28] The second approach is the structural approach. This requires substituting the pass-on rate out of the discount formulas (in those cases where we have not already done so), and then amounts to estimating all relevant supply and demand parameters entering the discount formula.[29]

Our analysis is particularly timely in light of the recent recommendations of the U.S. Antitrust Modernization Commission and the efforts by the European Commission to stimulate private cartel damages claims. Both authorities recommend making private damages claims in line with the actually lost profits. A question that then arises is whether it would not be better to directly focus on estimating lost profits, rather than follow an indirect approach which starts from the anticompetitive price overcharge and subsequently computes discounts. In principle, both approaches should be equivalent. From a practical point of view, however, focusing on the measurement of the anticompetitive price overcharge may entail an important advantage. There is extensive experience with estimating the anticompetitive price overcharge based on the econometric estimation of the "but-for" price; as reviewed

[28] The empirical literature on estimating pass-on rates is very large, and relates to various areas, including the literature on exchange rate pass-through literature, on tax incidence, on price transmission in agricultural economics, and on market power and competition. See Stennek and Verboven (2005) for an overview. A paper of particular interest in our context is by Ashenfelter, Ashmore, Baker and McKerman (1998), showing how to empirically estimate the firm-specific pass-through rate (in the context of evaluating efficiency gains from mergers).

[29] For example, Proposition 4 writes the Bertrand discount (14) in terms of pass-on rates but our subsequent logit example in section 5.1 eliminates the pass-on rate and writes the discount formulas in terms of demand parameters and market shares. Conversely, Proposition 5 writes the Cournot discount formula (24) in terms of a demand curvature parameter and market variables, but we subsequently rewrite it in terms of the pass-on rate and market variables.

in e.g. van Dijk and Verboven (2007) or Davis and Garces (2007). Applying a passing-on defense to this overcharge then shifts the burden of proof to the cartel, who should collect the information on the pass-on effect and the proper adjustment for the output effect. It is likely that the defendant would use this defense with care, since the same evidence may actually also be used to show that the cartel's total harm is higher than the traditional cost effect.

8 References

Anderson, S., A. de Palma and J.-F. Thisse, 1992, Discrete Choice Theory of Product Differentiation, MIT Press, 423 pp.

Ashenfelter, O., D. Ashmore, J.B. Baker and S. Kernan, 1998, Identifying the Firm-Specific Pass-Through Rate, Federal Trade Commission, Bureau of Economics Working Paper No. 217.

Antitrust Modernization Commission, 2007, Civil and Criminal Remedies, Chapter 3 in Report and Recommendations)

(http://www.amc.gov/report_recommendation/amc_final_report.pdf)

L.J. Basso and T.W. Ross, 2007, Measuring the True Harm from Price-Fixing to Both Direct and Indirect Purchasers", Working Paper 25 April 2007, Sauder School of Business, University of British Columbia.

Bresnahan, T., 1989, Empirical Methods for Industries with Market Power, Chapter 17 in Handbook of Industrial Organization, Vol. II, R. Schmalensee and R. Willig, eds., Amsterdam, Elsevier Publisher Sciences.

Connor, J.M., 2004, Price-fixing Overcharges: Legal and Economic Evidence, Working Paper, Department of Agricultural Economics, Purdue University.

Corts, K., 1999, Conduct Parameters and the Measurement of Market Power, Journal of Econometrics, 88, p. 227-250.

Davis, P. and E. Garces, 2007, Damage estimation, Chapter 7 in: Davis, P. and E. Garces: Quantitative techniques for competition analysis, European Commission, forthcoming.

European Commission, 2005, Green Paper: Damages Actions for Breach of the EC Antitrust Rules, DG-Competition.

Genesove, D. and W.P. Mullin, 1998, Testing Static Oligopoly Models: Conduct and Cost in the Sugar Industry: 1890-1914, RAND Journal of Economics, 29, 355-377.

Harris, R.G. and L.A. Sullivan, 1979, Passing on the Monopoly Overcharge: A Comprehensive Policy Analysis, University of Pennsylvania Law Review, 128, p. 269-360.

Hellwig, M., 2006, Private Damage Claims and the Passing-On Defense in Horizontal Price-Fixing Cases: An Economist's Perspective, working paper, Max Planck Institute, 29pp.

Hussain, S., D. Garrett and V.M. Howell, 2001, Economics of Class Action Certification in Indirect Purchaser Antitrust Cases, Journal of the Antitrust and Unfair Competition Law Section of the State Bar of California.

Kosicki, G. and M.B. Cahill, 2006, Economics of Cost Pass-Through and Damages in Indirect Purchaser Antitrust Cases, Antitrust Bulletin, 51(3), p. 599-630.

Landes, W. M. and R.A. Posner, 1979, Should Indirect Purchasers Have Standing to Sue Under Antitrust Laws? An Economic Analysis of the Rule of Illinois Brick", University of Chicago Law Review, 602.

Leslie, C.R. , 2006, Antitrust damages and deadweight loss, The Antitrust Bulletin, 51 (3), 521-567.

Levenstein, M.C. and V.Y. Suslow, 2006, What Determines Cartel Success?, Journal of Economic Literature,. 44 (1), 43-95.

Norberg, S,.2005, Some Elements to Enhance Damages Actions for Breach of the Competition Rules in Articles 81 and 82 EC, mimeo presented at 32nd Annual International Antitrust Law & Policy Conference, Fordham, New York, 31pp (http://europa.eu.int/comm/competition/speeches/text/sp2005_019_en.pdf)

Reiss, P. and F. Wolak, Structural Econometric Modeling: Rationale and Examples from Industrial Organization, forthcoming Chapter in Handbook of Econometrics, Vol. V.

Rey, P. and J. Tirole, 2006, A Primer on Foreclosure, forthcoming Chapter in Handbook of Industrial Organization, Vol. III, M. Armstrong and R. Porter.

Schinkel, M. P., J. Tuinstra and J. Rüggeberg, 2005, Illinois Walls: How Barring Indirect Purchaser Suits Facilitates Collusion", CeNDEF Working Paper

Series 05/10, 2005.

Stennek, J. and F. Verboven, 2001, Merger Control and Enterprise Competitiveness: Empirical Analysis and Policy Recommendations, European Economy, 5, 130-194.

van Dijk, T. and F. Verboven, 2007, Quantification of Damages, forthcoming chapter in W.D. Collins (Ed.): Issues in Competition Law and Policy, ABA Publications in Antitrust.

Vives, X., 1999, Oligopoly Pricing: Old Ideas and New Tools, MIT Press, 425 pp.

Waelbroeck, D., D. Slater, and G. Even-Shoshan, 2004, Study on the conditions for claims for damages in case of infringement of EC competition rules: Comparative Report, Ashurst Study for Directorate General Competition of the EU Commission.

Werden, G. and L. Froeb, 1994, The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy," Journal of Law, Economics and Organization, 10, 407-426.

9 Appendix.

9.1 The logit model

The potential number of consumers is L. Each consumer may buy one of the N differentiated products or the outside good. The demand for product $i = 1 \cdots N$ is $D_i(\mathbf{p}) = s_i(\mathbf{p})L$, where $s_i = s_i(\mathbf{p})$ are the market shares given by

$$s_i(\mathbf{p}) = \frac{\exp(v_i - \alpha p_i)}{1 + \sum_{k=1}^{N} \exp(v_k - \alpha p_k)}.$$

The market share of the outside good 0 is simply $s_0(\mathbf{p}) = 1 - \sum_{i=1}^{N} s_i(\mathbf{p})$, so that the demand for the outside good is $D_0(\mathbf{p}) = s_0(\mathbf{p})L$. The market share derivatives are

$$\begin{aligned} \frac{\partial s_i(\mathbf{p})}{\partial p_i} &= \alpha s_i(1 - s_i) \\ \frac{\partial s_k(\mathbf{p})}{\partial p_i} &= \alpha s_i s_k \qquad \text{for } k \neq i. \end{aligned}$$

Using the diversion ratio between firm 1 and firm k,

$$\delta_1^k = -\frac{\partial D_k(\mathbf{p})}{\partial p_1} \Big/ \frac{\partial D_1(\mathbf{p})}{\partial p_1} = \frac{s_k}{(1 - s_1)},$$

we can apply (14) to write the discount to the cost effect as:

$$\text{discount} = \frac{s_2}{(1-s_1)}\tau_I^2 + \cdots + \frac{s_i}{(1-s_1)}\tau_I^N. \tag{26}$$

If the pass-on rates are known or estimated, this can be used to calculate the discount. Alternatively, the pass-on rates can be computed by performing comparative statics on the system of first-order conditions defining the Bertrand-Nash equilibrium:

$$p_i - c_i - \frac{1}{\alpha}\frac{1}{1-s_i} = 0 \quad \text{for all } i.$$

To perform the comparative statics of a cost increase by firm i on prices, totally differentiate this system with respect to p_k, $k = 1 \cdots N$, and c_i. The tedious calculations are somewhat similar to Anderson, de Palma and Thisse (1992, p. 266-267), except that the comparative statics are in cost rather than in quality and that an outside good is included. This results in the following pass-on rates

$$\begin{aligned}
\tau_i^i &= \frac{\partial p_i^*(\mathbf{c})}{\partial c_i} = \frac{s_i}{(1-s_i)^2+s_i}T_i + \frac{(1-s_i)^2}{(1-s_i)^2+s_i} \\
\tau_i^k &= \frac{\partial p_k^*(\mathbf{c})}{\partial c_i} = \frac{s_k}{(1-s_k)^2+s_k}T_i \qquad \text{for } k \neq i.
\end{aligned}$$

where

$$T_i = \frac{s_i(1-s_i)^2}{(1-s_i)^2+s_i}\bigg/\left(s_0 + \sum_{k=1}^{N}\frac{s_k(1-s_k)^2}{(1-s_k)^2+s_k}\right).$$

Note that $T_i = \sum_{k=1}^{N} s_k \frac{\partial p_k^*(\mathbf{c})}{\partial c_i}$, i.e. T_i can be interpreted as the effect of a cost increase of firm i on the industry price index, using market shares as weights. Inserting the pass-on effects in τ_I^k, $k = 2 \cdots N$ and then in (26), and rearranging gives the following expression for the discount

$$\text{discount} = \frac{\left(\sum_{i\in I} T_i\right) - s_1(1-s_1)}{(1-s_1)^2+s_1}.$$

This can be computed based on information on the market shares of all products including the outside good.

9.2 Proof of proposition 4

The assumptions of the Cournot model involve the following inequalities: (i) $\rho s_1 \leq 1$, (ii)$\rho \leq N$, (iii) $N > 1$ and (iv) $0 < s_1 < 1$. To show the proposition, we have to derive the sign of the discount (24) or equivalently the sign of $2N_I - N - 1 - \rho(s_1 N_I - 1)$ under (a) $N_I = N$ and (b) $N_I = 1$.

To show (a), we have to show that $N-1-\rho(s_1N-1) > 0$. Consider all possible cases for ρ. First, if $\rho > 1$, then $N-1-\rho s_1 N+\rho \geq N-1-N+\rho = \rho-1 > 0$ by (i). Second, if $0 \leq \rho \leq 1$ and $s_1N-1 \geq 0$, then $N-1-\rho(s_1N-1) \geq N-1-(s_1N-1) = N(1-s_1) > 0$ by (iv). Third, if $0 \leq \rho \leq 1$ and $s_1N-1 < 0$, then $N-1-\rho(s_1N-1) \geq N-1 > 0$ by (iii). Fourth, if $\rho < 0$ and $s_1N-1 \geq 0$, then $N-1-\rho(s_1N-1) \geq N-1 > 0$ by (iii). Fifth, if $-(N-1) \leq \rho < 0$ and $s_1N-1 < 0$, then $N-1-\rho(s_1N-1) \geq N-1+(N-1)(s_1N-1) = (N-1)s_1N > 0$ by (iii). Finally, if $\rho < -(N-1)$ and $s_1N-1 < 0$, then $N-1-\rho(s_1N-1) > 0$ is equivalent with the market share condition $s_1 > \frac{1}{N} - \frac{N-1}{N}\frac{1}{(-\rho)}$. This shows that the discount is always positive, unless possibly in the final case, namely. if $\rho < -(N-1)$ and $s_1 < \frac{1}{N} - \frac{N-1}{N}\frac{1}{(-\rho)}$.

To show (b), we have to show that $1-N+\rho(1-s_1) < 0$. Consider again all possible cases for ρ. First, if $\rho \leq 0$, then $1-N+\rho(1-s_1) \leq 1-N < 0$ by (iii). Second, if $0 < \rho \leq 1$, then $1-N+\rho(1-s_1) < 1-N+\rho \leq 1-N+1 \leq 0$ by (iii). Third, if $\rho > 1$ and $s_1N-1 \geq 0$, then $1-N+\rho(1-s_1) < 1-N+N(1-s_1) = 1-s_1N \leq 0$ by (ii). Fourth, if $N-1 > \rho > 1$ and $s_1N-1 < 0$, then $1-N+\rho(1-s_1) < 1-N+(N-1)(1-s_1) = -(N-1)s_1 < 0$ by (iii). Finally, if $\rho > (N-1)$ and $s_1N-1 < 0$, then $1-N+\rho(1-s_1) < 0$ is equivalent to the market share condition $s_1 > \frac{1}{N} - \frac{N-1}{N}\frac{N-\rho}{\rho}$. This shows that the discount is always negative unless possibly in the final case, namely if $\rho > (N-1)$ and $s_1 < \frac{1}{N} - \frac{N-1}{N}\frac{N-\rho}{\rho}$.

Table 1: Discount to the cost effect of the cartel: Bertrand competition

	Plaintiff's market share: 10%			Plaintiff's market share: 50%		
Outsiders	Number of firms in the plaintiff's market					
	2	6	10	2	6	10
	logit demand					
0	52%	87%	88%	71%	79%	79%
1	**33%**	70%	78%	**15%**	63%	71%
5	–	**2%**	40%	–	**2%**	36%
9	–	–	**1%**	–	–	**1%**

Notes: The numbers are based on the discount formula (17). The market shares are as follows: outside good = 10%; plaintiff = 10% or 50%; other firms: identical share of remaining part. The numbers in bold refer to common cost increases (all firms are insiders).

Table 2: Discount to the cost effect of the cartel: Cournot competition

	Plaintiff's market share: $s_1 = 10\%$			Plaintiff's market share: $s_1 = 50\%$		
Outsiders	Number of firms in the downstream market (N)					
($N - N_I$)	2	6	10	2	6	10
linear demand ($\rho = 0$, so $\tau = \frac{N}{N+1}$)						
0	33%	71%	82%	33%	71%	82%
1	**-33%**	43%	64%	**-33%**	43%	64%
5		**-71%**	-9%		**-71%**	-9%
9			**-82%**			**-82%**
quadratic demand ($\rho = \frac{1}{2}$, so $\tau = \frac{N}{N+1/2}$)						
0	56%	80%	86%	40%	62%	67%
1	**-22%**	50%	67%	**-30%**	35%	50%
5		**-70%**	-7%		**-73%**	-17%
9			**-81%**			**-83%**
exponential demand ($\rho = 1$, so $\tau = 1$)						
0	90%	90%	90%	50%	50%	50%
1	**-5%**	58%	71%	**-25%**	25%	35%
5		**-68%**	-5%		**-75**	-70%
9			**-81%**			**-85%**
log-linear demand with $\varepsilon = 2$ ($\rho = \frac{3}{2}$, so $\tau = \frac{N}{N-1/2}$)						
0	147%	102%	95%	67%	36%	32%
1	**23%**	68%	75%	**-17%**	14%	18%
5		**-66%**	-3%		**-77%**	-31%
9			**-81%**			**-87%**

Notes: The numbers are based on the discount formula (24) after substituting the relevant demand parameters ρ. The market shares are as follows: plaintiff = 10% or 50%; other firms: identical share of remaining part. The numbers in bold refer to common cost increases (all firms are insiders).