# Social Status in a Social Structure: Noisy Signaling in Networks

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**Abstract**

How do incentives to engage in costly signaling depend on social structure? This paper formalises and extends Thorstein Veblen's theory of how costly signaling by conspicuous consumption depends on social structure. A noisy signaling game is introduced in which spectators observe signals only imperfectly, and use Bayesian updating to interpret the observed signals. It is shown that this noisy signaling game has (under some weak regularity conditions) a unique plausible Perfect Bayesian Nash equilibrium. Then, a social information network is introduced as a second source of information about a player's type. Equilibrium signaling depends in the resulting game on the relative quality of the substitute sources of information, which depends again on the social network. For some highly stylised networks, the dependence of equilibrium costly signaling on network characteristics (network size, density and connectedness, the centrality of the consumer in the network) is studied, and a simple dominance result for more arbitrary networks is suggested.

## 1 Introduction

Distinction is one of the stronger motives of human and animal conduct. Through costly, conspicuously and often extravagantly wasting means, humans and animals try to reveal their abundance of purchasing power, talent or fitness to spectators in a convincing way. This conspicuous waste may be understood by direct motivations, as people generally enjoy a feeling of superiority (Frank, 1985b,1999), or instrumental reasons,

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as distinguishing oneself from competitors is crucial in any economic situation of cooperation with endogenous partner choice in which the joint surplus depends on the imperfectly visible qualities of the partners. Competition for the best or most partners, by revealing yourself as the most attractive party, drives sexual selection among all sexually reproducing species (Darwin, 1871, Miller, 2000), job market signaling (Spence, 1973), marriages (Cole, Mailath, Postlewaite, 1995), advertising in trade, facade building in banking and insurance, friendships... Most consumption goods in post-industrial societies, are valued both for their intrinsic (physical) characteristics and for what they communicate about the quality of their consumer to spectators.

This paper concerns the impact of the structure of the social network in which the consumers live and exchange information on the equilibrium level of costly signaling. The transmission mechanism from social network structure to the signaling equilibrium, is inspired by the '*The Theory of the Leisure Class*' (1899) of Thorstein Veblen. According to Veblen, not only income and the distribution of incomes determined the incentives to invest in wasteful, highly visible forms of consumption. The structure of a society had to be understood as a major factor as well in explaining differences in costly ostentatious display. Veblen (1899) notes that "*Conspicuous consumption claims a relatively larger portion of the income of the urban than of the rural population, and the claim is also more imperative. The result is that, in order to keep up a decent appearance, the former habitually live hand to mouth to a larger extent than the latter. So it comes, for instance, that the American farmer and his wife and daughters are notoriously less modish in their dress, as well as less urbane in their manners, than the city artisan's family with equal income.* [...] *And in the struggle to outdo one another the city population push their normal standard of conspicuous consumption to a higher point* [...]. *The requirement of conformity to this higher conventional standard becomes mandatory. The standard of decency is higher, class for class, and this requirement of decent appearance must be lived up to on pain of losing caste.*" The reason for this disparity in incentives for ostentatious consumption between rural and urban areas lies, according to Veblen, in some fundamental characteristics of social structure.[1] When the size of social groups increases, and when human mobility is enhanced, ostentatious display of wealth becomes the predominant way of establishing a good reputation. "*So long as the community or social group is small enough and compact enough to be effectually reached by*

[1] Veblen's argument is in fact about the impact of social structure on the incentives for conspicuous consumption compared to leisure, but they may be extended easily for the case of optimal signalling in general.

*common notoriety alone [...], so long [leisure] is about as effective as [conspicuous consumption].*" But with the development of trade, and the improvement of means of transportation and communication, the importance of ostentatious display is boosted. "*The means of communication and the mobility of the population now expose the individual to the observation of many persons who have no other means of judging of his reputability than the display of goods [...]. The modern organization of industry works in the same direction also by another line. The exigencies of the modern industrial system frequently place individuals and households in juxtaposition between whom there is little contact in any other sense than that of juxtaposition. One's neighbors, mechanically speaking, often are socially not one's neighbors, or even acquaintances; and still their transient good opinion has a high degree of utility. [...] In the modern community, there is also a more frequent attendance of large gatherings of people to whom one's everyday life is unknown; in such places as churches, theatres, ballroom, hotels, parks, shops and the like.*"

This paper develops Veblen's argument further, using a social information network as an additional source of information which serves as a substitute to costly signaling, and traces the impact of the network structure on optimal signaling and status induced consumption. In the second section, I develop a noisy signaling game by which consumers decide on their level of costly signaling when signals are only imperfectly observed and interpreted by Bayesian players. I introduce an equilibrium refinement which allows the selection of a unique Perfect Bayesian Nash equilibrium and then investigate the equilibrium properties and comparative statics. In the third section of the paper, a social network is inserted into this noisy signaling model as a second source of information. The impact of network characteristics (network size, density, connectedness) and position (centrality) in the network) on equilibrium signaling is studied for a class of highly stylized networks and a simple dominance result is suggested for more arbitrary networks. The fourth section concludes.

The impact of social structure on costly signaling has drawn little attention from researchers. The most related papers to this one are those of Corneo and Jeanne (1998,1999), who investigate social segmentation (the probability of meeting someone of the same type) on signaling and the 'counter signaling' paper of Feltovich, Harbaugh, and To (2002). Feltovich et alii analyze the possibility of an equilibrium in which, in the presence of a second, noisy signal of quality, the highest of three types pools with the lowest type to distinguish herself from the middle type. Frank (1985a) shortly considers the possibility of a second noisy source of information, and the use of an minimal variance unbiased estimator to pool these two sources of information about a consumer's type. More generally, this paper combines insights from the literatures on costly signaling (surveyed by Riley, 2001), relative concerns (Truyts, 2005), information pooling (Clemen, 1999) and, finally, social networks in general (Jackson, 2006) and small world networks in particular (Watts, 2004).

# 2 Bayesian Noisy Signaling

## 2.1 Setting

Imagine a population of $N$ consumers, denoted by the set $\mathcal{N} \equiv \{i = 1, ..., \ N\}$. These consumers are heterogeneous in income, $m$.[2] I restrict the typespace of a consumer $i$, denoted $\mathcal{M}_i$, to two types, a low and high income type, indexed by $t \in T \equiv \{L, H\}$, such that the income of consumer $i$, denoted $m(i)$, is in $\mathcal{M}_i \equiv \{m_L, m_H\}$ with $0 < m_L < m_H < +\infty$. Hence, the typespace is $\mathcal{M} \equiv (\mathcal{M}_i)^N = \{m_L, m_H\}^N$. Each consumer in $\mathcal{N}$ is of type $H$ with a prior probability $p \in ]0, 1[$ and of type $L$ with prior probability $1 - p$.

Each consumer $i$ may spend her income $m(i)$ on two perfectly divisible commodities, $I \in \mathbb{R}^+$ and $c \in \mathbb{R}^+$. Commodity $I$ (for 'status investment') only serves signaling and generates no other ('intrinsic') utility. Hence, in social isolation or in the absence of signaling motives, typically no $I$ is consumed and all income may be spent on commodity $c$, 'rest consumption', representing all other commodities which are consumed for intrinsic reasons.[3] Denoting consumer $i$'s consumption of the two commodities by $c_i$ and $I_i$ and assuming that units of consumption are such that prices may be normalized to 1.

The payoffs of each consumption bundle $(c_i, I_i)$ are represented by a preference ranking over commodity bundles which is identical over all consumers and may by assumption be represented by a utility function. To avoid risk complications intermingling with the signaling analysis, I assume that the utility function is risk neutral in the second argument, and take the utility function to be quasi-linear

$$U(c_i, \hat{m}_i(I_i)) = V(c_i) + \kappa \hat{m}_i(I_i). \tag{1}$$

[2] This may rather easily be generalized to productivity or fitness, to extend the model to labour market or handicap signaling types of sexual selection models.

[3] This assumption of no intrinsic utility from $I$ may of course easily be relaxed. In this case consumers typically consume some positive amount $I^o$ in social isolation, equating marginal intrinsic utility from $c$ and $I$. Signaling motives then provide an extra marginal utility term, driving equilibrium $I$ away from $I^o$. I omit this intrinsic utility term for clarity and simplicity, and hence consider the bias away from zero signaling caused by signaling incentives.

The term $\hat{m}_i(I_i)$ denotes the expected value of the spectators' estimate of consumer $i$'s income, which will be seen to depend on her level of costly signaling $I_i$ and to be based on Bayesian updating.[4] The parameter $0 < \kappa < \infty$ represents the constant marginal utility of an extra unit of $\hat{m}_i$. Furthermore, it is assumed that $V(c_i)$ such that $\frac{\partial V(0)}{\partial c_i} = +\infty$, $\frac{\partial V(.)}{\partial c_i} > 0$ and $\frac{\partial^2 V(.)}{\partial^2 c_i} < 0$. Since utility is everywhere strictly increasing in both arguments, utility maximisation implies that the budget constraint is always satisfied with equality: $c_i + I_i = m(i)$.

A distinguishing feature of the present consumer problem is that spectators, which are all other consumers in this game, observe the status investment $I_i$ of each other consumer $i$ only imperfectly. More specifically, they observe consumer $i$'s status investment distorted by some random error term $\varepsilon_i$: such that the observed status investment is[5]

$$y_i = I_i + \varepsilon_i. \tag{2}$$

The error distribution is assumed to be independent of the identity of the signaler and of her signaling level and of the draws of all other consumers, and distributed along some probability density function $\varphi(\varepsilon; \mu = 0, \sigma^2)$, in which $\mu$ is the mean and $\sigma^2$ the variance. The strict **monotone likelihood ratio property** with respect to the mean for some density distribution $\varphi(\varepsilon; \mu, \sigma^2)$ may be defined as the requirement that, for $\mu > \mu'$, the ratio $\frac{\varphi(\varepsilon;\mu,\sigma^2)}{\varphi(\varepsilon;\mu',\sigma^2)}$ is monotonically and strictly increasing over the whole support of $\varphi(.)$. I need the following restrictions on density function $\varphi(.)$:

**Condition 1** *Let $\varphi(\varepsilon; \mu, \sigma^2)$ be a continuous and $C^2$ probability density function which is i) symmetric and unimodal, ii) has the real line for support and iii) satisfies the strict monotone likelihood ratio property with respect to the mean.*

[4] This specification may be interpreted as shorthand notation for the following scheme: a spectator observes and interprets the noisy signal $y_i$, and chooses a utility maximising action as best reply to the noisy signal $y_i$, according to her beliefs about the type of the sender. It is assumed that the optimal responses of the receiver are described by a unique best reply function, which is strictly and monotonically increasing in $\hat{m}$ (see e.g. employers setting wages in Bertrand competition in Spence's job market game). The utility of the sender is assumed to increase strictly and monotonically in the reply of the receiver. The utility function in formula 1 takes the beliefs of the receiver, represented by the expected value $\hat{m}\,(.)$, directly as an argument, and is as such shorthand notation, omitting the receiver's best reply.

[5] This error term could just as well be made dependent on spectator identity: neither the distributions nor the results shown hereafter would change. For notational simplicity, the error is here taken to be the same for all spectators.

By part $ii$ of condition 1, out-of-equilibrium observed signals are not an issue, as all $y_i \in \mathbb{R}$ will have an equilibrium interpretation. Part $iii$ of condition 1 ensures that posterior probabilities will be strictly monotonic. Since $y_i = I_i + \varepsilon_i$, one may write the distribution of $y_i$ (for $i$ of type $t$) as $\varphi(y_i; I_t, \sigma^2)$. A prominent example of a density function satisfying condition 1 is the normal density function. When used under an indefinite integral over $y_i$, density function $\varphi(.)$ will be abbreviated as $\varphi_t \equiv \varphi(y_i; I_t, \sigma^2)$. The first order derivative of this density function towards its mean under an indefinite integral is denoted by $\varphi_t' \equiv \frac{\partial \varphi(y_i; I_t, \sigma^2)}{\partial I_t}$. Finally, it is assumed that all players have perfect knowledge of the game and all its parameters, and know that they all know this etcetera.

How should a consumer now choose an optimal strategy, and how should spectators interpret an observed level of signaling $y_i$? Let game $\Gamma$ denote the baseline noisy signaling game. The choice of a consumer $i$ can, by equation 2, be summarised by a mapping $I_i : \mathcal{M}_i \rightarrow \mathcal{I}_i : m(i) \rightarrow I\left(m(i)\right)$, called her signaling strategy, with $\mathcal{I}_i \equiv [0, m(i)]$ consumer $i$'s 'strategy space'. Hence, the strategy space of each consumer $i$ is an interval of the real line containing infinitely many strategies. Since in this section consumers only differ in income $m$ which comes only in 2 types, the signaling strategy will be the same for all consumers of the same type $t$. It will therefore be convenient in this section to write the strategy and strategy spaces only for both types as $I_t : \mathcal{M}_t \rightarrow \mathcal{I}_t \equiv [0, m_t] : m_t \rightarrow I_t$. A strategy profile is an $N$-tuple $\langle I_1, I_2, ..., I_N \rangle$, which is an element of the N-dimensional strategy space $\left(\mathcal{I}_i\right)^N$. The $(N-1)$-tuple $I_{-i} \in \left(\mathcal{I}_i\right)^{N-1}$ is to denote the strategy profile of the $N-1$ other players which some consumer $i$ faces, and likewise the ordered pair $I \equiv \langle I_L, I_H \rangle \in \mathcal{I}_L \times \mathcal{I}_H$ summarizes a strategy profile in terms of types, and $I_{-t}$ denotes the strategy of the type other than $t$.

Now, assume that spectators know (e.g. by deduction, as will be demonstrated) the actual status investments of both types of players. Then these spectators may after observing a distorted signal $y_i$ from consumer $i$, update the probability of $i$ being of the high type by Bayes' rule. If $P(t; y_i)$ denotes the posterior probability that $i$ is of type $t$ given observed signal $y_i$, one may write

$$P(H; y_i, I) = \frac{p\varphi\left(y_i; I_H, \sigma^2\right)}{(1-p)\varphi\left(y_i; I_L, \sigma^2\right) + p\varphi\left(y_i; I_H, \sigma^2\right)}, \tag{3}$$

while obviously $P(L; y_i, I) = 1 - P(H; y_i, I)$. Spectators will therefore estimate the income of consumer $i$ to be $m_H P(H; y_i, I) + m_L P(L; y_i, I)$ $= m_L + (m_H - m_L)P(H; y_i, I)$. Multiplying this expression with the probability density $\varphi\left(y_i; I_t, \sigma^2\right)$ that the signal of a consumer of type $t$ is observed as $y_i$, one may compute the expected value of the impression

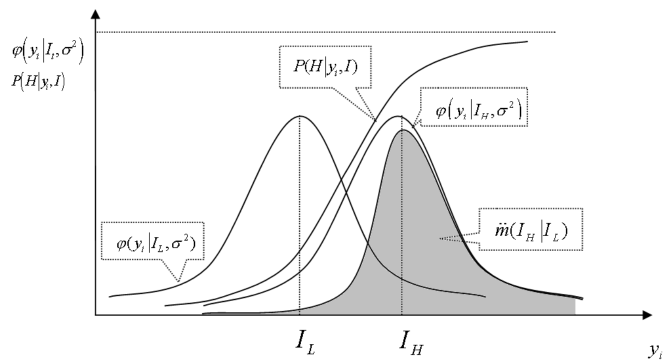


Figure 1: $\varphi(y_i \mid I_L, \sigma^2)$, $\varphi(y_i \mid I_H, \sigma^2)$, $P(H \mid y_i, I)$ and the surface $\ddot{m}(I_H \mid I_L)$ in gray.

on the spectators which a consumer of type $t$ makes with investments $I_t$ given $I_{-t}$ as

$$\hat{m}_t(I_t) = m_L + (m_H - m_L) \int P(H; y_i, I)\varphi_t dy_i. \tag{4}$$

Note that by the term $P(H; y_i, I)$, the interpretation of each observed signal $y_i$ depends on the the status investments of both types. As such, this interpretation or 'meaning' of each observed signal $y_i$ introduces an interpersonal interdependency into game $\Gamma$, making this consumer problem a problem of strategic interaction between the different types. Let the integral in 4 be denoted as

$$\ddot{m}_t(I_t; I_{-t}) \equiv \int P(H; y_i, I)\varphi_t dy_i.$$

The term $\ddot{m}_t(I_t; I_{-t})$ represents the expected probability of a type $t$ consumer, given her own investments $I_t$ and the other type's investments $I_{-t}$, being taken for a high type, i.e. the expected value of the posterior of being a high type, and will be a central concept in the remainder of this paper. Figure 1 illustrates the density functions $\varphi(y_i; I_H, \sigma^2)$ and $\varphi(y_i; I_L, \sigma^2)$, the posterior of being a high type $P(H; y_i, I)$, and the integral $\ddot{m}_H(I_H; I_L)$, which is represented by the gray area. Since $\ddot{m}_H(I_H; I_L)$ is the area under the product of $P(H; y_i, I)$ and $\varphi(y_i; I_H, \sigma^2)$, it can be seen to be the surface under the density function $\varphi(y_i; I_H, \sigma^2)$ minus a margin at the left side. Similarly, $\ddot{m}_L(I_L; I_H)$, the surface under the product $P(H; y_i, I)$ and $\varphi(y_i; I_L, \sigma^2)$, is a small bell shaped surface under the right tail of $\varphi(y_i; I_L, \sigma^2)$.

One may simplify the problem further by exploiting the structure of $\ddot{m}_t$ in two ways. Firstly, one may easily see that game $\Gamma$ is constant sum in the expected social impression term $\hat{m}$ by writing down the total amount of $\ddot{m}_t$ to be divided among the $N$ consumers (abusing notation in the first term) as

$$\begin{aligned}\sum_{i=1}^{N} \ddot{m}_i(I_i; I_{-i}) &= N\left[(1-p)\ddot{m}_L(I_L; I_H) + p\ddot{m}_H(I_H; I_L)\right] \\ &= N\begin{bmatrix} (1-p)\int \frac{p\varphi_H}{(1-p)\varphi_L + p\varphi_H}\varphi_L dy_i \\ +p\int \frac{p\varphi_H}{(1-p)\varphi_L + p\varphi_H}\varphi_H dy_i \end{bmatrix} \\ &= Np\int \varphi_H dy_i = Np.\end{aligned}$$

Hence, Bayesian consistency logically implies that $(1-p)\ddot{m}_L(I_L; I_H) + p\ddot{m}_H(I_H; I_L) = p$, which means that

$$\ddot{m}_H(I_H; I_L) = 1 - \frac{(1-p)}{p}\ddot{m}_L(I_L; I_H) \tag{5}$$

or

$$\ddot{m}_L(I_L; I_H) = \frac{p}{(1-p)}\left(1 - \ddot{m}_H(I_H; I_L)\right). \tag{6}$$

Secondly, one may exploit the standard and well known insensitivity of continuous density functions for certain affine transformations, such that $\varphi\left(y_i + A; I_t + A, \sigma^2\right) = \varphi\left(y_i; I_t, \sigma^2\right)$, for $\forall A \in \mathbb{R}$, and $\varphi\left(\alpha y_i; \alpha I_t, (\alpha\sigma)^2\right) = \varphi\left(y_i; I_t, \sigma^2\right)$, for $\alpha \in \mathbb{R}^+$. Since $\ddot{m}$ is computed as a indefinite integral, the addition of $A$ or multiplication with $\alpha$ are $y_i$ of course neutral within the integral. Setting $A = -I_L$ and $\alpha = \frac{1}{\sigma}$, one may finally rewrite the term $\ddot{m}$ as only a function of the **normalized difference in status investments**

$$\Delta \equiv \frac{I_H - I_L}{\sigma},$$

which further only depends on the prior $p$ and density function $\varphi(.)$. One may therefore finally write down the rescaled expected spectators' estimates of the high types as

$$\ddot{m}(\Delta) \equiv \ddot{m}_H(I_H; I_L) = \int \frac{p\left(\varphi\left(y_i; \Delta, 1\right)\right)^2}{(1-p)\varphi\left(y_i; 0, 1\right) + p\varphi\left(y_i; \Delta, 1\right)} dy_i, \tag{7}$$

and note that

$$
\begin{aligned}
\ddot{m}_L(I_L;I_H) &= \int \frac{p\varphi\left(y_i;\Delta,1\right)\varphi\left(y_i;0,1\right)}{(1-p)\,\varphi\left(y_i;0,1\right)+p\varphi\left(y_i;\Delta,1\right)}dy_i \\
&= \frac{p}{(1-p)}\left(1-\int \frac{p\left(\varphi\left(y_i;\Delta,1\right)\right)^2}{(1-p)\,\varphi\left(y_i;0,1\right)+p\varphi\left(y_i;\Delta,1\right)}dy_i\right) \\
&= \frac{p}{(1-p)}\left(1-\ddot{m}(\Delta)\right).
\end{aligned}
$$

The two last simplifications suggest some properties of game $\Gamma$ which are interesting in their own right, and play an important role in the welfare analysis of the equilibria in the remainder of the paper. They are summarized in the following proposition.

**Proposition 2** *In the noisy signaling game $\Gamma$ it is true for any strategy profile $\langle I_L, I_H\rangle$ that:*

1. *$\Gamma$ is constant sum in $\hat{m}$, as $\ddot{m}_H(I_H;I_L) = 1 - \frac{(1-p)}{p}\ddot{m}_L(I_L;I_H)$.*

2. *The integral $\hat{m}$ depends only on $\Delta \equiv \frac{I_H - I_L}{\sigma}$, and not on the actual level of $I$. Subtracting $I_L$ (assuming that $I_L < I_H$) from the consumption bundle of all $i \in \mathcal{N}$ does not affect utility. The amount $I_L.N$ is purely wasted.*

3. *Therefore, average utilitarian social welfare $W = \frac{1}{N}\sum_{i=1}^{N} U(c_i, \hat{m}_i(I_i))$ is maximized if all players play their 'intrinsic' optimum:$\forall i \in \mathcal{N}$ : $\langle c_i, I_i\rangle = \langle m(i), 0\rangle$.*

Part i) of proposition 3 (and equations 5 and 6) mean that gains in expected impression of one type are necessarily compensated exactly by the losses in expected impression for the other type, when appropriately accounting for differences in relative numbers. If the high types manage to increase their expected impression $\ddot{m}_H(.)$ by separating themselves from the low types (increasing $\Delta$), and hence by being confused less with low types, then the expected impression of the low types necessarily decreases by just that much (compensated for relative numbers), because they are confused less with high types. Since, by condition 1 *ii,* $\varepsilon$ has the real line for support, the high income types can never fully separate themselves from the low types. High income types can however, by buying some of the signaling good $I$, purchase some degree of separation from the lower types and push up their expected impression on spectators. Low types, in their turn, can buy some quantity of the signaling commodity $I$ to undo some of the separation achieved by the high types. In doing so, they increase the confusion and thereby their expected impression on spectators. The low types maximally achievable expected impression is achieved when $I_L = I_H$, the pooling outcome, when it is true that $\ddot{m}_L = \ddot{m}_H = p$. High income types try to move away from this outcome, low income types try to move closer to it. The essence of noisy signaling game $\Gamma$ can be seen as an arms race between two types, both wasting considerable amount of means to pull as much as possible of the fixed quantity $\hat{m}$ to their side. Hence, in terms of figure 1, for a given $\sigma$, the distribution of expected impression $\ddot{m}$ is determined by the distance $I_H - I_L$ only. The distance from the origin to $I_L$ is purely wasted: if both $I_H$ and $I_L$ were shifted to the left by a distance $I_L$, $\ddot{m}_L$ and $\ddot{m}_H$ and utility would remain unchanged. So the quantity $I_L$ which all consumers invested in status investments, is pure waste, whereas the amount $I_H - I_L$ which high income consumers purchase in excess of $I_L$ merely shifts expected impression from low types to high types. Hence, utilitarian welfare is maximised when all consumers spend their total income on rest consumption.

What happens to $\ddot{m}(\Delta)$ by marginally increasing $\Delta$? By applying the chain rule in differentiating formula 7 to $\Delta$, one obtains

$$\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} = \int \frac{p\,(1-p)\,\varphi_0 \varphi'_\Delta}{\left((1-p)\,\varphi_0 + p\varphi_\Delta\right)^2} \varphi_\Delta dy_i + \int \frac{p\varphi_\Delta}{(1-p)\,\varphi_0 + p\varphi_\Delta} \varphi'_\Delta dy_i.$$

The first integral represents the 'interpretation effect': the changes in the interpretation of each observed signal $y_i$ as a consequence of a marginal increase in $\Delta$, keeping the expected occurence of each $y_i$ constant. The second integral contains what one might call the 'occurrence effect': the effect of shifting the probability distribution of observing some signal $y_i$ marginally to the right, keeping the interpretation of each $y_i$ fixed. The first integral will typically be negative, whereas the second term is always positive, as it implies a rightward shift of probability mass over a strictly increasing function. Writing these two effects together again, one may obtain $\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} = -p \int \left[ \frac{(1-p)\varphi_0}{(1-p)\varphi_0 + p\varphi_\Delta} \right]^2 \varphi'_\Delta dy_i$ which is strictly positive for all distributions $\varphi(.)$ that satisfy condition 1. This is summarized in the following lemma for $\Delta \geq 0$, which will be motivated soon.

**Lemma 3** *For all continuous probability densities $\varphi(\varepsilon; \mu, \sigma^2)$ satisfying the requirements in condition 1, it is true that $\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} > 0$ for all $\Delta \in \mathbb{R}^+$.*

**Proof.** In appendix. ■

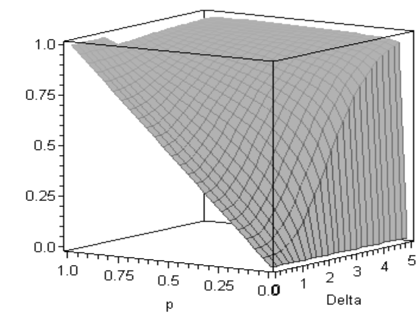


Figure 2: $\dddot{m}(\Delta)$ for $\Delta \in [0,5]$, $p \in ]0,1[$, and $\varphi(.)$ the normal density function

The shape of the integral $\dddot{m}(\Delta)$ as a function of $\Delta \in [0,5]$ and $p \in ]0,1[$, and with $\varphi(.)$ the normal density function, is depicted in figure 2. Note that the pooling value is of course linear in $p$ : i.e. $\dddot{m}(0) = p$. As $p$ gets lower, the pooling outcome deteriorates, and thereby the stakes for both types to establish of undo separation: $\dddot{m}(\Delta)$ becomes steeper as $p$ gets lower. Note finally that for bell shaped density functions such as the normal density function, $\dddot{m}(\Delta)$ takes a sigmoid form, and is hence partly convex and partly concave, which slightly complicates equilibrium analysis in the next section.

## 2.2 Equilibrium analysis

The problems of the high and low income consumers may now be stated more conveniently, using equations 2, 4, 6 and 7 as

$$Max_{I_H} \qquad V(m_H - I_H) + \kappa \left( (m_H - m_L)\, \dddot{m} \left( \frac{I_H - I_L}{\sigma} \right) + m_L \right) \tag{8}$$

$$Max_{I_L} \qquad V(m_L - I_L) + \kappa \left( \frac{(m_H - m_L)\, p}{(1-p)} \left( 1 - \dddot{m} \left( \frac{I_H - I_L}{\sigma} \right) \right) + m_L \right) \tag{9}$$

$$s.t. \quad I_L, I_H \geq 0.$$

The equilibrium concept used in the analysis of noisy signaling game $\Gamma$ is the Perfect Bayesian Nash equilibrium in pure strategies, which requires that information is processed according to Bayes' rule and that the equilibrium strategy profile is a fixed point of the best reply mapping, just like the common Nash equilibrium.

**Definition 4 (Perfect Bayesian Nash Equilibrium)** *The Perfect Bayesian*

*Nash Equilibrium (BNE) of the noisy signaling game* $\Gamma$ *is a strategy profile* $(I_L^*, I_H^*)$ *and a specification of posterior beliefs* $P(t; y_i)$ *such that:*

1. *For all types* $t$, $I_t^*$ *is the expected utility maximizing strategy given that the other consumers play their equilibrium strategies* $I_{-t}^*$*. Or formally:*

$$I_t^* \in \arg\max V(m_t - I_t) + \kappa\left[(m_H - m_L)\,\ddot{m}_t(\Delta) + m_L\right]$$

2. *All consumers, as spectators, use Bayes rule to update information* $P(m_t; y_i)$*, as shown in formula 3.*

*A Pure Strategy Perfect Bayesian Nash Equilibrium (PBNE) is a Perfect Bayesian Nash Equilibrium in pure strategies.*

In general, the noisy signaling game will have multiple PBNE, of which many are uninteresting, e.g. the case in which no investments are made by neither of the types and in which signals are not interpreted (such that $P(H; y_i, I) = p$ for all $y_i$ and $I$) by spectators is a PBNE. The common equilibrium refinements in signaling games (Intuitive Criterion, Divinity, Universal Divinity, Credible Deviations...) solve little since they rely on out-of-equilibrium signals, which do not perceivably exist in noisy signaling game $\Gamma$ under condition 1. More general Nash equilibrium selection tools, such as perfection, seem to miss cutting power as well in the present case. I propose to select the PNBE which is robust to learning by spectators, in the sense that equilibrium signaling strategies are the best reply to the equilibrium strategy of the other type if the noisy signals are correctly interpreted by spectators, and call this PBNE an True PBNE.

**Definition 5 (True PNBE)** *Let a True Perfect Bayesian Nash Equilibrium (T-PBNE) of noisy signaling game* $\Gamma$ *be a PBNE in which no type would have an incentive to deviate if her deviation were correctly observed by spectators.*

Hence, a True Perfect Bayesian Nash Equilibrium is a fixed point of the best reply correspondences in signaling strategies if that all noisy signals are correctly interpreted. One may think of at least two justifications for the selection of this PBNE. Firstly, spectators may be expected to learn to interpret signals correctly, such that if a consumer type deviates from her original signaling strategy, this strategy will be recognised by the spectators on a sufficiently short term. Each consumer type may therefore be expected to choose a best reply signaling strategy against the strategy of the other type, assuming that spectators interpret each distorted signal correctly. Secondly, spectators facing a noisy signal $y_i$, which they cannot trace this signal back to the original undistorted signal, might replay the noisy signaling game $\Gamma$ in their imagination. They may then infer that some types would want to deviate in a PBNE which is not a T-PBNE, if they would only be recognised as such. Spectators may find it therefore plausible to imagine the best replies of each of the types, to predict an T-PBNE in their head and use its equilibrium strategies to interpret observed noisy signals. These two interpretations greatly benefit from the uniqueness and global asymptotic stability of the T-PBNE in game $\Gamma$, which will be shown below.

Given the correct interpretation of noisy signals, the Kuhn-Tucker condition to the problem of the high income consumer is

$$I_H \left[ -\frac{\partial V(.)}{\partial c} + \frac{\kappa \left( m_H - m_L \right)}{\sigma} \frac{\partial \ddot{m}}{\partial \Delta}(\Delta) \right] = 0$$

such that either $I_H = 0$ and $-\frac{\partial V(.)}{\partial c} + \frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial \ddot{m}}{\partial \Delta}(\Delta) < 0$ or $I_H > 0$ and $-\frac{\partial V(.)}{\partial c} + \frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial \ddot{m}}{\partial \Delta}(\Delta) = 0$. In the case of an interior optimum $I_H^*$, the above first order condition may be written as an equality of marginal utility of consumption (opportunity costs) and the marginal benefits of status consumption,

$$\frac{\partial V(.)}{\partial c} = \frac{\kappa \left( m_H - m_L \right)}{\sigma} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}. \tag{10}$$

Similarly optimal status investments mean for the low income types that $0 = I_L \left[ -\frac{\partial V(.)}{\partial c} + \frac{\kappa(m_H - m_L)p}{\sigma(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \right]$, such that the first order condition for an interior solution ($I_L > 0$) to the problem of the low income consumers may be stated as:

$$\frac{\partial V(.)}{\partial c} = \frac{\kappa \left( m_H - m_L \right) p}{\sigma(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}. \tag{11}$$

Hence, marginally increasing $I_t$ implies for type $t$ consumers a decrease in rest consumption by one unit (by equation 2) and that the expected impression $\ddot{m}(\Delta)$ is affected by an increase of $\frac{1}{\sigma}$ in $\Delta$ for the high types, and a decrease of $\frac{1}{\sigma}$ in $\Delta$ for low types. The first order conditions in 10 and 11 may be interpreted as a tangency condition between $\ddot{m}_t(\Delta)$ and the indifference curve in the $(I_t, \ddot{m}_t)$-plane. Tangency then requires $\frac{\frac{\partial V(.)}{\partial c}}{\kappa(m_H - m_L)} = \frac{\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}}{\sigma}$ for the high types, in which the LHS depicts the marginal rate of substitution between rest consumption (the opportunity cost of $I_t$) and $\ddot{m}_t$, and the RHS the increases in $\ddot{m}_t$ by marginally increasing $I_t$ (and equivalent for the low types). By equality

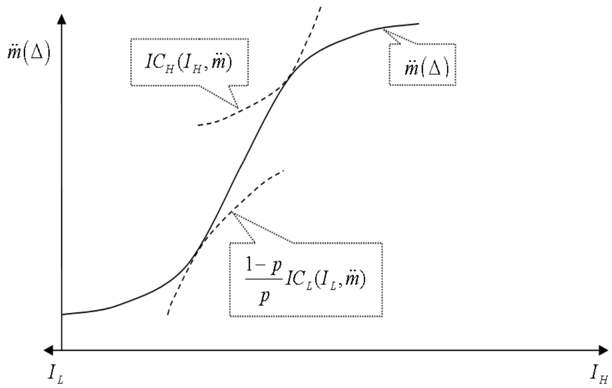


Figure 3: Tangency of indifference curves to $\ddot{m}(\Delta)$.

5, i.e. because $\ddot{m}(\Delta) = \ddot{m}_H(I_H; I_L) = 1 - \frac{(1-p)}{p}\ddot{m}_L(I_L; I_H)$, the indifference curve of the high types is tangential to $\ddot{m}(\Delta)$ from above, and $\frac{(1-p)}{p}$ times the indifference curve of the low income types is tangential to $\ddot{m}(\Delta)$ from below. In figure 3, the tangency between the sigmoid $\ddot{m}(\Delta)$ and the indifference curves, of the high types $(IC_H(I_H, \ddot{m}))$ and the low types ($IC_L(I_L, \ddot{m})$ times $\frac{(1-p)}{p}$) are depicted. Note that these indifference curves should be read differently: points above the indifference curve are better points for the high types, and worse for the low types (higher $\ddot{m}(\Delta)$), and a rightward move for the low types should be read as a decrease in $I_L$.

This tangency condition nicely illustrates the second order condition one needs to impose to guarantee equilibrium existence. The indifference curves (better than sets) are convex by assumption, but since the $\ddot{m}$-surface may be both convex and concave, the indifference curves may be tangential to $\ddot{m}(\Delta)$ over an interval of the player's strategy space or -worse- on two disconnected subsets of her strategy set. In order to ensure that the best reply set is convex, it is sufficient to require that the curvature of the indifference curve is always stronger than the curvature of the $\ddot{m}$-surface. If one requires this condition to be satisfied strictly, then the set of best replies is always a singleton, and the best-reply correspondence is in fact a function. This is obviously equivalent to imposing strict concavity on the consumer problem.

**Condition 6 (Second order conditions)** *Let $V(.)$, $\kappa$, $\sigma^2$, $(m_H - m_L)$, $p$ and $\varphi(.)$ be such that is holds for $\forall I \in \mathcal{I}$ that*

$$\frac{\partial^2 V(m_H - I_H)}{\partial^2 c} + \frac{\kappa\,(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} < 0$$

*and*
$$\frac{\partial^2 V(m_L - I_L)}{\partial^2 c} - \frac{\kappa\,(m_H - m_L)\,p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} < 0$$

This second order condition may be interpreted as a restriction on the curvature of $V(.)$ given the parameters $\kappa$, $\sigma^2, (m_H - m_L)$ and $p$ and the functional form of $\varphi(.)$ or rather as a restriction on the parameters, e.g. on the relative importance of expected impression $\kappa$, given $V(.)$, $\sigma^2, (m_H - m_L)$ , $p$ and $\varphi(.)$.

When condition 6 is satisfied, the first order conditions in equations 10 and 11.define two best reply functions. These are continuous and differentiable by the implicit function theorem if condition 6 is satisfied (and as both $V(.)$ and $\varphi(.)$ are $C^2$ by assumption): differentiating the first order condition of the high type towards $I_L$ and isolating $\frac{\partial I_H}{\partial I_L}$ one gets
$$\frac{\partial I_H^*}{\partial I_L} = \frac{\frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}}{\frac{\partial^2 V(.)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}}$$
which may be seen to be continuous over the whole strategy set $[0, m_H]$ as long as condition 6 is satisfied (such that the denominator remains strictly negative). One may recover the best reply function $I_H^{BR}(I_L)$ by computing the optimal $I_H$ for $I_L = 0$ from 10 and using it as an initial condition, and setting $I_H^{BR}(I_L) = 0$ whenever the obtained solution is negative. Proceeding equivalently, one may also obtain a continuous and differentiable best reply function for the low types $I_L^{BR} : [0, m_H] \rightarrow [0, m_L]$.The existence of a Nash equilibrium in pure strategies is now easily seen. The Cartesian product $[0, m_L] \times [0, m_H]$ of the strategy spaces of the two types is a rectangle with $[0, m_L]$ as a basis and $[0, m_H]$ as a left side. $I_L^{BR}$ is a continuous and surjective best-reply mapping (hence a continuous curve from top to bottom) which indicates the best reply of the low income types to each possible strategy of the high income consumers in $[0, m_H]$. Equivalently, $I_H^{BR}$ is a function indicating the best reply of the high types to each possible strategy of the low types, thus continuously connecting the two sides of the rectangle. It is obvious that these two best reply curves must cross at least once, establishing the existence of a T-PBNE. A more formal statement of the existence and qualification of a T-PBNE in game $\Gamma$ is presented in the following proposition.

**Proposition 7** *Let $\{I^*, P(H; I^*)\}$, with $I^* = \langle I_L^*, I_H^* \rangle$ denote the True Pure Strategy Perfect Bayesian Nash Equilibrium (T-PBNE) of the noisy signaling game $\Gamma$.*

1. *Then $\{I^*, P(H; I^*)\}$ exists if the conditions 1 and 6 hold.*

2. *Then $I_L^* \leq I_H^*$ or $\Delta^* \equiv I_H^* - I_L^* \geq 0$ and $I_H^* - I_L^* = 0 \Longrightarrow I_H^* = I_L^* = 0$ and/or $p > 1/2$*

3. *Then $\{I^*, P(H; I^*)\}$ is the unique T-PBNE of $\Gamma$ if the conditions 1 and 6 hold.*

4. *And then $\{I^*, P(H; I^*)\}$ is globally asymptotically stable if the conditions 1 and 6 hold.*

**Proof.** In appendix. ■

The properties 3 and 4 of theorem 1 are in general very nice, since they imply that the fixed point $I^*$ is the unique global attractor of the best reply function. This suggests that the T-PBNE will be reached in a tâtonnement process, as long as the signals are correctly interpreted trough e.g. a learning process. Secondly, it is important to note that contrary to the undistorted signaling model of Spence (1973), the low income types as well as the high income types have strictly positive marginal benefits from status signaling. This means that, if the optimum of the low types is not a corner solution, then the consumption choice of all types is distorted by signaling motives. Since low types may always be mistaken for a high type, and more so if they invest in costly signaling, there is distortion at the bottom.

What can one say about the influence of the exogenous parameters on the equilibrium status investments $I^*$ (under conditions 1 and 6)? Since one may in general not obtain an explicit solution for both the best reply functions $I_L^{BR}(I_H)$ and $I_H^{BR}(I_L)$ and $I^*$, one has to rely on the implicit function theorem to learn something about the direction of changes in equilibrium investments as a result of shifting exogenous parameters. The easiest case is the parameter $\kappa$, measuring the marginal utility of $\hat{m}$. The effect of increasing $\kappa$ is rather straightforward: it uniformly increases the marginal utility of status investments, but leaves the opportunity costs of status investments unchanged. Alternatively, the indifference curves of the two types become flatter. As a result, the best reply functions of both types is shifted upwardly and hence $I^*$ is increasing in $\kappa$, and $I_t^*$ increases strictly if it is in the interior of $\mathcal{I}_t$.

The comparative statics of $\sigma$, $m_L$ and $m_H$, are mostly less straightforward, as most of these parameters affect the equilibrium investments $I^*$ non-monotonically. A first parameter of interest may be the standard deviation $\sigma$. Firstly, when observations get more distorted, it takes more efforts to establish or undo some level of separation, while the opportunity costs remain constant. Secondly, however, when $\sigma$ is marginally increased, this entails a change in $\Delta$, and hence also a change in the marginal effects of $\Delta$ on $\ddot{m}$. The sign of this effect depends on the second order derivative $\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}$, and has its opposite sign. If $\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}$ is negative, then reducing $\Delta$ raises the marginal effect of $\Delta$, pushing up $I^*$ again. The overall effect of marginally increasing $\sigma$ has the opposite sign of $\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\Delta$, where $\frac{\partial \ddot{m}}{\partial \Delta}(.)$ is always positive and $\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}$ is partly positive and partly negative, such that $\frac{\partial I^*}{\partial \sigma}$ is mostly negative.

As far as the income endowments $m_H$ and $m_L$ are concerned, one may distinguish two separate effects: an 'income effect' and a 'type gap effect'. The first is common in most economic modelling: if income rises (and since $V(c)$ is concave), the opportunity cost of some level $I_t$ decreases. Therefore the income effect will push up $I^*$. The second effect is also straightforward: changes in income endowments change the income gap $(m_H - m_L)$ between the two types, and hence what is at stake, due to the specification of $\hat{m}$ as an estimator of income levels. Changes in this gap are comparable to changes in $\kappa$, and as such, these 'stakes' are of course increasing in $m_H$ and decreasing in $m_L$. Finally, there are of course again indirect effects through changes in the marginal benefits of status signaling, due to the behavioural reactions of own and other type, of which the sign depends on the second order derivative $\frac{\partial^2 \ddot{m}}{\partial^2 \Delta}(.)$. Bringing all this together, it is shown that $\frac{\partial I_H^*}{\partial m_H} > 0$, as both direct effects are clearly positive. For $\frac{\partial I_L^*}{\partial m_H}$, only the 'type gap' and the indirect effect play, and as such $\frac{\partial I_L^*}{\partial m_H}$ takes the sign of $\frac{(m_H - m_L)}{\sigma}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}$, which implies that $\frac{\partial I_L^*}{\partial m_H}$ is positive most of the time. For the low income endowment $m_L$, the 'type gap effect' makes that there is less at stake for the high income individual, but that the low income types behavioural reaction may still affect the marginal status benefits otherwise. As such, $\frac{\partial I_H^*}{\partial m_L}$ takes the opposite sign of $\frac{(m_H - m_L)}{\sigma}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}$ and is mostly negative. The least clearcut are the own income effects of the low income types. Here, the income effect is opposed to the 'type gap effect', while the changes in marginal effect of $\Delta$ can go either way. As such, the sign of $\frac{\partial I_L^*}{\partial m_L}$ is difficult to predict, and takes the sign of $\frac{\partial V^2(m_L - I_L^*)}{\partial^2 c}\left(\frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) + \frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c}\frac{\kappa p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}$. These results are summarised in the following theorem.

**Proposition 8 (Comparative Statics)** *The equilibrium signaling levels $I^*$ in the T-PBNE of noisy signaling game $\Gamma$ depends in such a way on the exogenous parameters $\kappa$, $\sigma$, $m_H$ and $m_L$ that, if conditions 1 and 6 hold:*

1. *$I^*$ is increasing in the marginal utility of social status $\kappa$.*

2. *$I^*$ varies non-monotonically with the degree of noisiness $\sigma$ such that $\frac{\partial I_H^*}{\partial \sigma}\left(\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\Delta\right) < 0$ and $\frac{\partial I_L^*}{\partial \sigma}\left(\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\Delta\right) < 0$.*

3. *$I^*$ varies with $m_H$ such that $\frac{\partial I_H^*}{\partial m_H} > 0$ and $\frac{\partial I_L^*}{\partial m_H}\left(\frac{(m_H - m_L)}{\sigma}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right) > 0$*

4. *$I^*$ varies with $m_L$ such that $\frac{\partial I_H^*}{\partial m_L}\left(\frac{(m_H - m_L)}{\sigma}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right) < 0$ and $\left[\begin{array}{c}\frac{\partial V^2(m_L - I_L^*)}{\partial^2 c}\left(\frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) \\ + \frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c}\frac{\kappa p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\end{array}\right]\frac{\partial I_L^*}{\partial m_L} > 0.$*

**Proof.** In appendix ■

# 3 Optimal status investments in a social information network

Fortunately, wasteful costly signaling is not the only way to learn about other consumers. People can share information by talking. Usually, they talk to people they are related to in some way (neighbors, friends and family, colleagues), along the lines of what might be stylized as a social network. This paper introduces social networks as an additional source of information about the type of other consumers as an information supplement to signaling. By social acquaintance, people learn about the true quality of other consumers, and then tend to share this knowledge with related consumers in their social network (one might also call this gossip). But as information spreads further on the network, information quality deteriorates. Whereas the quality of information from conspicuous consumption remains of a constant quality, the quality of the gossip information one receives depends on the structure of the social network. And as the quality of gossip as a supplementary source of information depends on the social structure, so does the equilibrium investment in signaling.

## 3.1 Optimal signaling in networks: the case of information substitutes

Let $\Gamma_G$ denote a noisy signaling game with information substitutes, played on a social network denoted by $G$. Game $\Gamma_G$ is identical to game $\Gamma$, except that a second source of information needs to be integrated. Let $\tilde{y}_{i,j} \in \mathbb{R}$ denote an unbiased estimator of consumer $i$'s income $m(i)$, as obtained by some other consumer $j$ from her social relations: $\tilde{y}_{i,j} = m(i) + \tilde{\varepsilon}_{i,j}$, where $\tilde{\varepsilon}_{i,j}$ is an error term distributed along some density function $\tilde{\varphi}\left(\tilde{\varepsilon}_{i,j}; 0, \tilde{\sigma}^2_{i,G}\right)$, where $E(\tilde{\varepsilon}_{i,j}) = 0$ and the variance depends on the structure of the social network and is denoted by $\tilde{\sigma}^2_{i,G}$ for now. Hence, spectators in game $\Gamma_G$ observe two different signals: the status signaling $y_i = I_i + \varepsilon_i$ with $\varepsilon_i \sim \varphi(\varepsilon_i; 0, \sigma)$ and the gossip information $\tilde{y}_{i,j} = m(i) + \tilde{\varepsilon}_{i,j}$ with $\tilde{\varepsilon}_{i,j} \sim \tilde{\varphi}(\varepsilon_{i,j}; 0, \tilde{\sigma}^2_{i,G})$ and all know the prior probabilities $p$ of being type $H$ and $(1-p)$ for type $L$. Based on each of these separate signals, spectators might derive from these signals two separate posterior probabilities of $i$ being a high type, i.e. $P(H; y_i)$ as before and

$$\tilde{P}(H; \tilde{y}_{i,j}) = \frac{p\tilde{\varphi}(\tilde{y}_{i,j}; m_H, \tilde{\sigma}^2_{i,G})}{p\tilde{\varphi}(\tilde{y}_{i,j}; m_H, \tilde{\sigma}^2_{i,G}) + (1-p)p\tilde{\varphi}(\tilde{y}_{i,j}; m_L, \tilde{\sigma}^2_{i,G})}$$

Similar to $\ddot{m}\left(\Delta\right)$ for the status signaling information, one may define

$$\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})) \equiv \frac{p\left(\tilde{\varphi}(\tilde{y}_{i,j}; m_H, \tilde{\sigma}^2_{i,G})\right)^2}{p\tilde{\varphi}(\tilde{y}_{i,j}; m_H, \tilde{\sigma}^2_{i,G}) + (1-p)p\tilde{\varphi}(\tilde{y}_{i,j}; m_L, \tilde{\sigma}^2_{i,G})}$$

(with $\Delta m(\tilde{\sigma}^2_{i,G}) \equiv \frac{m_H - m_L}{\tilde{\sigma}_{ij,G}}$) as the expected impression for the high income types for the gossip information part. This quantity is only a function of exogenous parameters $p, m_H, m_L, \tilde{\sigma}^2_{i,G}$ and density function $\tilde{\varphi}$. Quite equivalent to earlier derivations for $\ddot{m}$, one may write the expected impression of the low income types as $\tilde{m}_L(\Delta m(\tilde{\sigma}^2_{i,G})) = \frac{p}{(1-p)}\left(1 - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)$.

How can spectators $j \in \mathcal{N}$ handle these two different sources of information and generically different estimates of consumer $i$'s income? Should one choose the best of the two unbiased estimators? A more efficient way is to combine them into one estimator, thus using all available information. I assume that spectators use linear opinion pooling with minimal variance weight to aggregate information. Let $\breve{P}(H; y_i, \tilde{y}_{i,j})$ denote the composed posterior probability that consumer $i$ is a high income type after the observation of distorted signals $y_i$ and $\tilde{y}_{i,j}$. The 'linear opinion pooling rule' is then

$$\breve{P}(H; y_i, \tilde{y}_{i,j}) = \zeta P(H; y_i) + (1-\zeta)\tilde{P}(H; \tilde{y}_{i,j}) \tag{12}$$

with $\zeta \in [0, 1]$. As a convex combination of unbiased estimators with $\zeta \in [0, 1]$, $\breve{P}(H; y_i, \tilde{y}_{i,j})$ is unbiased itself, and $\zeta$ may then be chosen to minimize the variance of $\breve{P}(.)$.[6] Minimizing the variance of formula 12

[6] Alternative uses of $\zeta$ are reviewed by Genest and McConway (1990), but these alternative motivations do not apply to the simple context of game $\Gamma_G$

brings one to the commonly known inverse variance weights:[7]

$$\zeta^* = \frac{Var(\tilde{P}(H; \tilde{y}_{i,j})}{Var(P(H; y_i) + Var(\tilde{P}(H; \tilde{y}_{i,j})}. \tag{13}$$

By applying $\zeta^*$ in formula 12, each posterior estimate of $i$'s type is given relatively more weight if this estimate is relatively more precise compared to the other posterior estimation. The variance of the resulting composed estimator can be shown to be weakly smaller than the variances of the constituent parts (Bates & Granger, 1969). The use of the composed posterior estimate in equations 12 and 13 is motivated by the literature on the problem of 'opinion pooling', 'information pooling' or 'forecast combination' which has attracted quite some attention in the management and forecasting literature.[8]

If all spectators use linear opinion pooling, then the normalized expected composed impression of a high income consumer, denoted $\ddddot{m}(\Delta)$, may be written:

---

[7] $Min_\zeta\ Var(\breve{P}(H \mid y_i, \tilde{y}_i)) = Var(\zeta P(H \mid y_i) + (1-\zeta)\tilde{P}(H \mid \tilde{y}_i)) = \zeta^2 Var(P(H \mid y_i) \ +(1-\zeta)^2 Var(\tilde{P}(H \mid \tilde{y}_i))$. This generates the first order condition

$2\zeta(Var(P(H \mid y_i) \ + Var(\tilde{P}(H \mid \tilde{y}_i)) \qquad -2Var(\tilde{P}(H \mid \tilde{y}_i) = 0$, which may be rewritten to $\zeta^*$.

[8] An important alternative to the linear pooling rule consists of a fully Bayesian approach, which requires the formulation and computation of the full joint conditional and posterior distributions. For our model $\Gamma_G$ this can be written

$$\breve{P}(H \mid y_i, \tilde{y}_{i,j}) = \frac{p\tilde{\varphi}(\tilde{y}_{i,j} \mid m_H, \tilde{\sigma}^2_{i,G})\varphi(y_i \mid I_H, \sigma)}{p\tilde{\varphi}(\tilde{y}_{i,j} \mid m_H, \tilde{\sigma}^2_{i,G})\varphi(y_i \mid I_H, \sigma) + (1-p)\tilde{\varphi}(\tilde{y}_{i,j} \mid m_H, \tilde{\sigma}^2_{i,G})\varphi(y_i \mid I_L, \sigma)}$$

such that the expected impression given the two information sources, denoted $\dddot{m}$, of the high types may in this case be written

$$\ddddot{m} = \int\int \frac{p\tilde{\varphi}(\tilde{y}_{i,j} \mid m_H, \tilde{\sigma}^2_{i,G})\varphi(y_i \mid I_H, \sigma)}{\begin{array}{c} p\tilde{\varphi}(\tilde{y}_{i,j} \mid m_H, \tilde{\sigma}^2_{i,G})\varphi(y_i \mid I_H, \sigma) \\ +(1-p)\tilde{\varphi}(\tilde{y}_{i,j} \mid m_L, \tilde{\sigma}^2_{i,G})\varphi(y_i \mid I_L, \sigma) \end{array}} \varphi(y_i \vert\ I_H, \sigma)\tilde{\varphi}(\tilde{y}_{i,j} \vert\ m_H, \tilde{\sigma}) dy_i d\tilde{y}_{i,j}.$$

This approach seems theoretically preferable, but is hard to handle in practice. This Bayesian approach, however, motivates for an interesting class of distribution functions $\varphi(.)$ and $\tilde{\varphi}(.)$ the use of linear pooling formulas 12 and 13. Winkler (1981) and Bordley (1982) show that this Bayesian formulation reduces to the linear opinion pooling formula with minimum variance weights for normally distributed errors as far as the mean is concerned, which is the relevant statistic in this paper. This result was extended further by Lindley (1983) and Genest and Schervisch (1985) to a broader family of symmetric exponential density functions.

$$
\begin{aligned}
\dddot{m}(\Delta) \equiv & \zeta^* \int P(H; y_i)\varphi(y_i; \Delta, \sigma) dy_i + \\
& (1-\zeta^*) \int \tilde{P}(H; \tilde{y}_{i,j})\tilde{\varphi}(\tilde{y}_{i,j}; m_H, \tilde{\sigma}^2_{i,G}) d\tilde{y}_{i,j} \\
= & \zeta^* \ddot{m}(\Delta) + (1-\zeta^*)\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})).
\end{aligned}
$$

It may easily be verified that the normalized expected composed impression of a low type is again $\dddot{m}_L(\Delta) = \frac{p}{(1-p)}\left(1 - \dddot{m}(\Delta)\right)$.

Keeping the variance of gossip information fixed at $\tilde{\sigma}^2_{i,G}$, the problem of the high and low income consumers in game $\Gamma_G$ may respectively be put as

$$
Max_{I_H} \quad V(m_H - I_H) + \kappa \left[m_L + (m_H - m_L)\dddot{m}(\Delta)\right] \tag{14}
$$

$$
Max_{I_L} \quad V(m_L - I_L) + \kappa \left[m_L + (m_H - m_L)\frac{p}{(1-p)}\left(1 - \dddot{m}(\Delta)\right)\right] \tag{15}
$$

with $I_H, I_L \geq 0$

The first order conditions to the consumer problems in equations 14 and 15 can (restricting attention to interior solutions only) respectively be written as:

$$
\frac{\partial V(m_H - I_H)}{\partial c} = \frac{\kappa(m_H - m_L)}{\sigma}\left[\zeta^* \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial \zeta^*}{\partial \Delta}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)\right] \tag{16}
$$

and equivalently for the low types:

$$
\frac{\partial V(m_L - I_L)}{\partial c} = \frac{\kappa(m_H - m_L)}{\sigma}\frac{p}{(1-p)}\left[\zeta^* \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial \zeta^*}{\partial \Delta}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)\right] \tag{17}
$$

The right hand side of the first order conditions now contains two marginal benefit effects of increased status investments: the first term between square brackets represents as before an increase in expected impression. The second term represents a new effect: the change in the relative importance of costly signaling. When $\Delta$ increases, this generates more separation in noisy signaling. And this not only raises the expected impression of the high types, but also the reliability of conspicuous consumption as a signal, and hence its importance to spectators as a source of information. The left hand side still represents the marginal opportunity costs of status investments.

How should one understand this impact of changes in $\Delta$ on the minimal variance weight $\zeta^* = \frac{Var(\tilde{P}(H;\tilde{y}_{i,j})}{Var(P(H;y_i)+Var(\tilde{P}(H;\tilde{y}_{i,j})}$? The variance of the posterior distribution $P(H;y_i)$ (the computation for $\tilde{P}(H;\tilde{y}_{i,j})$ is of course completely similar) at some point $y_i$ may be written:

$$\begin{aligned} Var(P(H;y_i);y_i) &= (1-P(H;y_i))^2 P(H;y_i) + (1-P(H;y_i))(P(H;y_i))^2 \\ &= (1-P(H;y_i))P(H;y_i), \end{aligned}$$

such that the variance over the whole range of $y_i$, i.e. over $\mathbb{R}$, may be written

$$\begin{aligned} Var(P(H;y_i) &= \int (1-P(H;y_i))P(H;y_i)\left[p\varphi(y_i;\Delta,1)+(1-p)\varphi(y_i;0,1)\right]dy_i \\ &= \int \frac{p\varphi(y_i;\Delta,1)(1-p)\varphi(y_i;0,1)}{p\varphi(y_i;\Delta,1)+(1-p)\varphi(y_i;0,1)}dy_i. \end{aligned}$$

As such,

$$\begin{aligned} \frac{\partial Var(P(H;y_i)}{\partial\Delta} &= p\int\left[\frac{(1-p)\varphi(y_i;0,1)}{p\varphi(y_i;\Delta,1)+(1-p)\varphi(y_i;0,1)}\right]^2\frac{\partial\varphi(y_i;\Delta,1)}{\partial\Delta}dy_i \\ &= -\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} < 0, \end{aligned}$$

and similarly $\frac{\partial Var(\tilde{P}(H;\tilde{y}_{i,j})}{\partial\Delta m(\tilde{\sigma}^2_{i,G})} < 0$ if one assumes that $\varphi(.)$ and $\tilde{\varphi}(.)$ satisfies condition 1. When the normalized difference in status investments gets larger, the assessment of one's income through status investments becomes more distinguishable, and hence the variance of the status investments posterior decreases. As such, more weight is shifted to the more precise estimator of quality, or $\frac{\partial\zeta^*}{\partial\Delta} > 0$.

The double effect of increased status investments in equations 16 and 17 is somewhat more complicated. The requirement of strict concavity now becomes for high and low income consumers respectively:

**Condition 9 (Second order conditions)** *Let* $V(.), \kappa, \sigma^2, \tilde{\sigma}^2_{i,G}, (m_H - m_L)$, $p, \varphi(.)$ *and* $\tilde{\varphi}(.)$ *be such that is holds for* $\forall I \in \mathcal{I}$ *that*

$$\frac{\partial V^2(m_H-I_H)}{\partial^2 c} + \frac{\kappa(m_H-m_L)}{\sigma^2}\begin{bmatrix} \zeta^*\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta} + 2\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} \\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)\end{bmatrix} < 0$$

$$\frac{\partial^2 V(m_L-I_L)}{\partial^2 c} - \frac{\kappa(m_H-m_L)}{\sigma^2}\frac{p}{(1-p)}\begin{bmatrix} \zeta^*\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta} + 2\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} \\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)\end{bmatrix} < 0$$

Condition 9 requires that the parameters $\kappa$, $\sigma^2, \tilde{\sigma}^2_{i,G}, (m_H - m_L)$, $p$ and the functional forms of $V(.)$, $\varphi(.)$ and $\tilde{\varphi}(.)$ are such that the consumer problem is strictly concave, which is satisfied if the indifference curve in the $I_t$-$\dddot{m}$-plane of both consumer types is more strongly curved than the $\dddot{m}(\Delta)$ function over the whole strategy space $\mathcal{I}$. In the term between square brackets, the second term is always negative. But in the first and third term, the sign of both $\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}$ and $\frac{\partial^2 \zeta^*}{\partial^2 \Delta}$ can in general not be determined without further specifying $\varphi(.)$ and $\tilde{\varphi}(.)$. One possible interpretation of condition 9 is again that of a restriction on the marginal utility of $\hat{m}$, $\kappa$, which should not be too great relative to the marginal utility from rest consumption. When condition 9 applies and condition 1 holds for density functions $\varphi(.)$ and $\tilde{\varphi}(.)$, then the existence of a unique T-PBNE of game $\Gamma_G$ may easily be shown along the same lines as proposition 7.

**Proposition 10** *If condition 9 is satisfied and condition 1 holds for $\varphi(.)$ and $\tilde{\varphi}(.)$, then:*

1. *Game $\Gamma_G$ has a unique and globally asymptotically stable T-PBNE, denoted by $\left\{\breve{P}(H; y_i, \tilde{y}_{i,j}), I^*_G\right\}$ with $I^*_G = \left\langle I^*_{L,G}, I^*_{H,G}\right\rangle$ the equilibrium status investments.*
2. *For $I^*_G$, it is true that $\frac{\partial I^*_G}{\partial \tilde{\sigma}^2_{i,G}} \geq 0$, and $\frac{\partial I^*_{i,G}}{\partial \tilde{\sigma}^2_{i,G}} > 0$ for $I^*_{i,G}$ in the interior of $\mathcal{I}_i$.*

**Proof.** In appendix. ■

Part 2 of proposition 10 says that equilibrium status investments of both types of consumers are monotonically increasing in $\tilde{\sigma}^2_{i,G}$, and hence monotonically decreasing in the quality of gossip information. The final step in this paper is to endogenize $\tilde{\sigma}^2_{i,G}$ in function of the structure of the social network $G$.

## 3.2 Optimal signaling and social networks

Consumers obtain information about other consumers by observing their status investments $I$ and by hearing gossip through their social network. Such a network may be understood as a collection of bilateral relationships or 'links'. Two consumers having a link may be friends, neighbors, colleagues or be acquainted in some other way: anything goes, as long as they share information about others. A social network $G \in \mathcal{G}$ (with $\mathcal{G} \subseteq \mathbb{N} \times \mathbb{N}^2$ the set of all possible graphs) is defined by a set of vertices $\mathcal{N} = \{i = 0, ..., \ N\ \}$, representing consumers, and a set of unordered pairs $E = \{(i,j) : i,j \in \mathcal{N}\}$, which represent relations ('edges' or 'links') between the $N$ consumers. For convenience, where it is obvious $G$ denotes the network $G(\mathcal{N}, E)$, $(i,j) \in G$ means $(i,j) \in E$, and $i \in G$ means similarly $i \in \mathcal{N}$. A social network $G$ may also be represented as either a graph, with $N$ nodes or 'vertices' representing the consumers and a set of lines between these nodes representing the links. A 'path' between two different nodes $i$ and $j$ is a series of consecutive links $(ik)(ks)(sv)...(tj) \in E$, with $i, k, s, v, t, j \in \mathcal{N}$, which connects node $i$ to node $j$, and which passes no vertex twice. If such a path exists, then the two nodes are called 'connected'. The number of links in a path is called the 'length' of the path. The number of links of the shortest path or 'geodesic' between node $i$ and some node $j$ is called the 'geodesic distance' or simply 'distance' of $i$ to $j$ in network $G$, denoted $d(i,j;G)$. If no path connects two nodes $i$ and $j$, then they are called disconnected, and their geodesic distance is set to $+\infty$. If all $i, j \in \mathcal{N}$ are connected, then we call the network $G(\mathcal{N}, E)$ 'connected'. A network is disconnected if and only if it consists of more than one 'component'. A component of a network $G$ is a maximal connected subgraph, i.e. $C$ is a component of the network $G$ if $(i,j) \in C \Rightarrow (i,j) \in G$, if $(i,j), (kl) \in C$ implies that there is a path from $i$ to $k$ in $C$ and if $(i,j) \in C$ and $(j,k) \in G \Rightarrow (j,k) \in C$. Hence, a component $C$ of a network $G$ is a subset of vertices and links, which is such that all the nodes in $C$ are connected to each other, but not connected to any node outside the subgraph $C$. The component to which node $i$ belongs is simply denoted by $C(i)$. The diameter of a network or a component $G$ is the maximum geodesic distance between two nodes that are addressed by links of the component, and is denoted $\bar{d}(G) = \max_{i,j} d(i,j;G)$. Finally, the 'degree' of a node is the number of links that connect a node to direct neighbors. The degree of a node $i$, is hence the number edges adjacent to vertex $i$, or the number of direct neighbors that are connected to $i$ by a single link, and it is denoted $\deg(i;G)$. If all vertices have the same degree $k$, then the graph is called $k-$regular. The set of all nodes that are directly connected to some node $i$, is called the neighborhood of $i$, and denoted $\tilde{N}(i;G) = \{k; (ik) \in E\}$, such that $\deg(i;G) = \left|\tilde{N}(i;G)\right|$.[9] One may also define a neighborhood setwise, for some subgraph $S \subset G$, letting $\tilde{N}(S;G) = \{k; (ik) \in G, i \in G, k \in G \backslash S\}$, hence the neighborhood of some subgraph $S$ consists of the vertices which have a link to a vertex within $S$, and are not an element of $S$ themselves. The second neighborhood of some vertex $i$ in a graph $G$ is the neighborhood of the neighborhood of $i$, denoted by $\tilde{N}^2(i;G) = \tilde{N}(\tilde{N}(i;G);G)$, and

[9] $|S|$ of a set $S$ denotes the number of elements in that set, as is common notation in network theory.

equivalently the $j-$th neighborhood of the vertex $i$, denoted $\tilde{N}^j(i;G)$ consists of all vertices which are at geodesic distance $j$ of vertex $i$. Using the $j-$th neighborhood concept, one may then define the geodesic (distance) distribution in $G$ from a vertex $i$ as $\Phi_j(i;G) = \left|\tilde{N}^l(i;G)\right|$. This geodesic distribution characterizes the distribution of geodesic distances from some initial vertex $i$, and hence also the dissemination of information from some vertex $i$ throughout the social network. Finally, the most crucial characteristic for the development of noisy status signaling game $\Gamma_G$ is the average geodesic of player $i$, defined $\tilde{d}(i;G) = \sum_{j=1}^{\infty} \frac{j\Phi_j(i;G)}{N-1}$. Define the class of distance-homogeneous networks, $\mathcal{G}^H = \left\{G(\mathcal{N}, E); \tilde{d}(i;G) = \tilde{d}(j;G),\ \forall i,j \in \mathcal{N}\right\}$, as the set of all networks in which the average geodesic is equal for all players.

As information travels along this social network, its quality declines with the number of links it passes. The longer gossip information takes to reach someone, the more unreliable it becomes. This decay of information quality is modelled in a most simple fashion: every time information passes from one consumer $i$ to another $j$, it is distorted by some random error term $\varepsilon_{i,j}$, drawn from an independent and identical distribution with $E(\varepsilon_{i,j}) = 0$ and $Var(\varepsilon_{i,j}) = \tilde{\sigma}^2$. An important simplification is also that consumers only use one single source of information, and that they prefer information which has come to them through a minimal number of nodes, and hence with a minimal burden of error. In this way, the information about the quality of consumer $j$, which reached consumer $i$ through a path $(j,k)(k,l)(l,m)(m,i)$ is characterized as $\tilde{y}_{i(j)} = m(i) + \varepsilon_{j,k} + \varepsilon_{k,l} + \varepsilon_{l,m} + \varepsilon_{m,i} = m_i + \tilde{\varepsilon}_{i,j}$ (with $i,j,k,l,m \in \mathcal{N}$). The variance of this unbiased estimator can now be written as $Var(\tilde{y}_{i(j)}) = Var(m(i) + \varepsilon_{j,k} + \varepsilon_{k,l} + \varepsilon_{l,m} + \varepsilon_{m,i}) = 4\tilde{\sigma}^2$. More generally, the variance of any estimation of quality obtained from gossip may then be written as a function of the geodesic distance between the receiver $j$ and the subject $i$ of the information, or $Var(\tilde{y}_{i(j)}) = d(i,j;G)\tilde{\sigma}^2$. Another heroic simplification made in this paper is that every consumer $i$ cares just as much about the impression she makes on all others. This now implies for the consumer problem in the case of a connected network $G$ that the optimal weights using the two sources of information are a function of average geodesic distance

$$\tilde{\sigma}^2_{i,G} = \tilde{d}(i;G)\tilde{\sigma}^2. \tag{18}$$

such that (finally!) the consumer problem of the high and low consumer types as a function of social structure becomes:

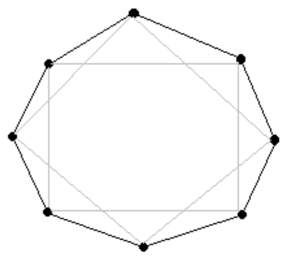

Figure 4: Simple and extended cyclical network for $N = 8$.

$$Max_{I_H} U = V(m_H - I_H) + \kappa \left[ m_L + (m_H - m_L) \left[ \zeta^* \ddot{m}(\Delta) + (1-\zeta^*)\tilde{m}\left( \frac{m_H - m_L}{\tilde{d}(i \mid G)\tilde{\sigma}^2} \right) \right] \right]$$

$$Max_{I_L} U = V(m_L - I_L) + \kappa \left[ m_L + \frac{(m_H - m_L)p}{(1-p)} \left( 1 - \left[ \begin{array}{c} \zeta^* \ddot{m}(\Delta) + \\ (1-\zeta^*)\tilde{m}\left( \frac{m_H - m_L}{\tilde{d}(i|G)\tilde{\sigma}^2} \right) \end{array} \right] \right) \right]$$

One may now inspect the incentives for costly signaling in some highly stylized and simple social networks, given the assumptions of signaling and gossip being information substitutes and of information decay along path length.

#### 3.2.1 Veblen's town and villages: Cyclical networks

Consider first a cyclical network $C^N \in \mathcal{G}^H$, which is such that all of the $N \geq 3$ consumers have exactly two links and hence two 'neighbors', such that each cyclic network is isomorphic to a network with as a set of edges $E^c = \{(1,2),(2,3),...,(N-1,N),(N,1)\}$. This network may be represented by a simple circle, as illustrated by the black network in figure 4 for $N = 8$. Each node of a cyclical network is hence necessarily of degree 2. If game $\Gamma_{C^N}$ is the noisy signaling game $\Gamma_G$ played on a cyclical social network $C^N$, then corollary 11 shows that $\tilde{\sigma}^2_{i,C^N} = \tilde{d}(i \mid G)\tilde{\sigma}^2$ is increasing in $N$ for all $i \in \mathcal{N}$. Therefore, in game $\Gamma_{C^N}$ it holds by proposition 10 that $\frac{\partial I^*_{C^N}}{\partial N} > 0$, and that utilitarian welfare is higher in a network with smaller $N$.

**Corollary 11** *If game $\Gamma_{C^N}$ is the noisy signaling game $\Gamma_G$ played on a cyclical social network $C^N$ and $I^*_{C^N}$ represents equilibrium status investments in the unique T-PBNE of $\Gamma_G$, then $\frac{\partial I^*_{C^N}}{\partial N} \geq 0$ (with $\frac{\partial I^*_{C^N}}{\partial N} > 0$*

*for $I^*_{t,C^N}$ in the interior of $\mathcal{I}_t$). Average utilitarian social welfare $W = \frac{1}{N}\sum_{i=1}^{N} U(m(i) - I^*_{i,C^N}, \hat{m}_i(I^*_{i,C^N}))$ is decreasing in $N$.*

**Proof.** In appendix. ■

Instead of varying the number of consumers, one might also vary the number of social relations which consumers maintain directly in the network, i.e. one may change the degree of the vertices in the networks. To explore this idea further, one can assume that consumers now also can establish links to the direct neighbors of their direct neighbors. This network is now no longer a cyclical network *strictu sensu*, but a unidimensional lattice of degree $k$. I abuse the notation $C^{N|k} \in \mathcal{G}^H$ to denote an average geodesic homogeneous '$k$-degree cyclical network' of $N$ nodes, such that the degree of each node is increased to $k$ in the fashion described above. Clearly, this generalization of the cyclical network requires that $k$ is even and also that $N \geq k+1$. In figure 4, $C^{8|4}$ is illustrated by both black and grey lines. In general, it is shown that average geodesic distance decreases as may be expected in the degree of all nodes, such that the incentives to engage in costly signaling investments are decreasing in the number of social relations consumers maintain in social network $C^{N|k}$. When the social network is denser and consumers maintain more social relations with their peers, information obtained through the social network will be more reliable, such that equilibrium investments in status signaling are lower. With respect to the information substitutes effect of social information networks, social welfare is increasing with the number of relations consumers maintain.

**Corollary 12** *Let game $\Gamma_{C^{N|k}}$ be the noisy signaling game $\Gamma_G$ played on a $k-$degree cyclical social network, $C^{N|k} \in \mathcal{G}^H$ and let $I^*_{C^{N|k}}$ be equilibrium status investments in the unique T-PBNE of $\Gamma_{C^{N|k}}$, then $I^*_{C^{N|k}} \geq I^*_{C^{N|k'}}$ for $k < k'$ (with $I^*_{C^{N|k}} > I^*_{C^{N|k'}}$ for $k < k'$ for $I^*_{t,C^{N|k'}}$ in the interior of $\mathcal{I}_t$). Average utilitarian social welfare $W = \frac{1}{N}\sum_{i=1}^{N} U(m(i) - I^*_{C^{N|k}}, \hat{m}_i(I^*_{C^{N|k}}))$ is increasing in $k$.*

***Proof.*** *In appendix.* ■

Both the networks $C^N$ and $C^{N|k}$ are in fact unidimensional examples of a more general class of $d$-lattices in $\mathcal{G}^H$, with $d$ the number of dimensions. For two dimensions, such a network may be represented as a grid on the surface of a torus. For three dimensions, it becomes a cubic lattice. This sort of networks is shortly developed in the appendix.

## 3.3 Segregation: Social status in disconnected networks

A very limited way of loosening the stringent assumption of homogeneity in average geodesic distance may be found by giving up the connectedness of $G$. If a network consists of more than one component, consumers have no gossip information about the $N-|C(i)|$ consumers outside their own component, and can only rely on costly signaling to judge these 'out-group' consumers. The composed estimator of the impression one may expect to make on the other $N-1$ consumers in $\mathcal{N}$ can in a disconnected network be written

$$\begin{aligned}\ddddot{m}(\Delta) &= \left(1-\frac{|C(i)|}{N-1}\right)\ddot{m}(\Delta)+\frac{|C(i)|}{N-1}\left(\zeta^{*}\ddot{m}(\Delta)+(1-\zeta^{*})\tilde{m}(\Delta m(\tilde{\sigma}^{2}_{i,G}))\right)\\ &= \left(1-\frac{|C(i)|}{N-1}\left(1-\zeta^{*}\right)\right)\ddot{m}(\Delta)+\frac{|C(i)|}{N-1}(1-\zeta^{*})\tilde{m}(\Delta m(\tilde{\sigma}^{2}_{i,G})).\end{aligned}$$

Let $\Gamma_{DC}$ be the noisy signaling game $\Gamma_G$ played on a disconnected network $DC$, consisting of $n$ components $C_n$ $(n=1,...,n)$ and with each component $C_n \in \mathcal{G}^H$ and $\tilde{d}(i;C(i))$ constant. In a sense, the game $\Gamma_{DC}$ can be considered a convex combination of games $\Gamma$ and $\Gamma_G$. It is assumed that consumers are randomly spread over the different components, such that the priors $p$ and $(1-p)$ are homogeneous over the whole network, and that all consumers care exactly as much about the impression they make on all $i \in \mathcal{N}$. Hence, one cares just as much about the judgement of close neighbors and unconnected strangers, although this assumption could easily be relaxed by adding some extra weights to the opinions of different consumers. Finally, it is also necessary that all consumers know the network $G$ next to the fundamentals of the game (distributions, utility functions...) to deal with the extra dimension of heterogeneity. Given this knowledge, all consumers have all the information needed to predict the equilibrium and interpret each observed signaling level $y_i$ accordingly, and the game can be played as before, with the T-PBNE consisting of a tuple of equilibrium status investments and signal interpretations for each component separately. Under assumptions of strict concavity similar to those above, and stated in condition 17 in the appendix, one may show the results summarized in the following corollary.

**Corollary 13** *Let game $\Gamma_{DC}$ be the noisy signaling game $\Gamma_G$ played on a disconnected network $DC$, consisting of $n'$ components $C_n$ $(n=1,...,n')$ and with each component $C_n \in \mathcal{G}^H$. If consumers are randomly spread over the different components, and all care about the impression they make on all $i \in \mathcal{N}$, then (if the conditions 17 and 1 for $\varphi(.)$ and $\tilde{\varphi}(.)$*

*hold) one may find for each $C_n$ a tupple $I^*_{C_n}$ characterizing equilibrium status investments in the unique T-PBNE of $\Gamma_{DC}$ for the consumers in component $C_n$. For this $I^*_{C_n}$ it is true that, keeping $\tilde{d}(i;C(i))$ constant, the equilibrium status investments levels are decreasing in the relative size of the own component $\frac{\partial I^*_{C_n}}{\partial \frac{|C(i)|}{N-1}} \leq 0$ (and $\frac{\partial I^*_{C_n}}{\partial \frac{|C(i)|}{N-1}} < 0$ for $I^*_{i,C_n}$ in the interior of $\mathcal{I}_i$). Average utilitarian social welfare $W = \frac{1}{|Cn|}\sum_{j=1}^{|Cn|} U(m(j) - I^*_{I^*_{C_n}}, \hat{m}_j(I^*_{I^*_{C_n}}))$ is on average higher in larger components of a disconnected social network..*

**Proof.** In appendix ■

This result may be interpreted as members of small, socially isolated groups, who care about the impression they make on the broad public, having *ceteris paribus* a higher incentive to engage in wasteful conspicuous consumption than identical consumers in the majority group. One example of such a small network components may be socially isolated ethnic minorities, with no close social connections to the majority but caring about making a good impression nevertheless. More generally, any newly arrived migrants, and likewise Veblen's travelling salesmen, face *ceteris paribus* higher incentives to spend money wastefully but ostentatiously then locals, with firm social roots in the community.

The possibility of disconnected networks also suggests upper and lower bounds to equilibria of the game $\Gamma_G$ (as a function of the parameters of course). The lower bound for the average distance of any connected social network is that of the complete network, which may be denoted by $C^{N|N-1}$, and for which $\tilde{d}(C^{N|N-1}) = 1$. The complete network is the most dense network, and implies minimal status investments with respect to $G$, as the reliability of information is now fully determined by the relative sizes of $\sigma^2$ and $\tilde{\sigma}^2$. At the other extreme lies the empty network, which may be written as $C^{N|0}$, in which all geodesic distances equal infinity. This is of course the upper bound for average geodesic distance and diameter in $\mathcal{G}$, and one may write that $\Gamma_{C^{N;0}} = \Gamma$. Therefore equilibrium status investments of game $\Gamma$, $I^*$, is the upper bound for all $I^*_G$ with respect to $G$. This is summarized in the following corollary.

**Corollary 14** *For all T-PBNE and equilibrium status investments $I^*_G$ of game $\Gamma_G$, it is true that*

$$I^*_{C^{N|N-1}} \leq I^*_G \leq I^*_{C^{N|0}} = I^*.$$

*If $W(G)$ represents average utilitarian welfare given a social network $G$, i.e. $W(G) = \frac{1}{N}\sum_{i=1}^{N} U(m(i) - I^*_G, \hat{m}_i(I^*_G))$, then it is true for all $G \in \mathcal{G}$*

*that*

$$W(C^{N|N-1}) \geq W(G) \geq W(C^{N|0}).$$

### 3.4 Hierarchies and Centrality: Nested Star Networks

It is also possible to deviate from the average geodesic homogeneity assumption within a component or connected network. In this section, I investigate in a highly stylized manner how differences in centrality in the network result in differences in average geodesic among consumers in the same network (or component) and hence in differences to engage in status signaling. A star network is a simple network in which $N-1$ nodes all have one single link ('arms') to the same node $i$, called 'the centre'. If consumer 1 is the centre of a star network, then the set of edges is $E^S = \{(1,2), (1,3), ..., (1,N)\}$. One may generalize this simple star network into a nested star network $S_{k,l}$ of $l$ levels and $k$ downward arms from each nested centre. A nested star $S_{k,l}$ has one single centre (at level 1) from which $k$ arms depart to $k$ vertices at level 2. Each of these $k$ vertices is in it's turn a centre and has $k$ 'downward' arms to vertices on level 3, which are $k^2$ in total. All these vertices again have k downward arms, and so on, until one reaches the $k^{l-1}$ vertices at level $l$. Alternatively, each branch of the star is a tree, which splits $l-1$ times into $k$ branches. Therefore, the degree of the centre of the star is $k$, that of the vertices at intermediate levels 2 until $l-1$ is $k+1$, and that of the bottom level $l$ is 1. Observe that for $k=1$, the nested star $S_{k,l}$ is simply a line network. The total number of vertices on a $S_{k,l}$ network is $N = \sum_{s=0}^{l-1} k^s = \frac{1-k^l}{1-k}$. Figure 5 illustrates a simple nested star network $S_{5,4}$.

Let $\Gamma_{S_{k,l}}$ represent the game $\Gamma_G$ played on a nested star $S_{k,l}$. Then the vector of equilibrium status investments in the T-PBNE of $\Gamma_{S_{k,l}}$, denoted $I^*_{S_{k,l}}$, consists of $l$ ordered pairs $I^*_{j,S_{k,l}}$ representing the T-PBNE signaling strategies for each type a each level $j$ of the nested star. As before, all fundamentals of the game are known to all spectators, i.e. utility functions, prior distributions and network structure, such that spectators may calculate the equilibrium status investment values $I^*_{j,S_{k,l}}$ for any consumer at some level j and correctly interpret any pair of observed signals $y_i$ and $\tilde{y}_i$. Since the average geodesic distance is increasing in the levels of the nested star, it follows by proposition 10 that equilibrium status investments are lower for consumers which have a more central position in the nested star social information network. Therefore, conditional on the type, expected utility is higher for consumers with a more

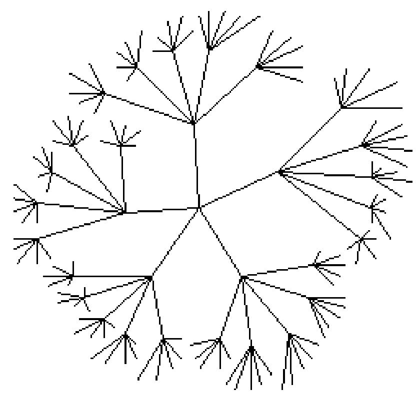

Figure 5: A nested star $S_{5,4}$

central position in the nested star social information network. These results may be summarized in the following corollary.

**Corollary 15** *Let $\Gamma_{S_{k,l}}$ represent the game $\Gamma_G$ played on a nested star $S_{k,l}$. Under the conditions 9 and 1 on $\varphi(.)$ and $\tilde{\varphi}(.)$, $\Gamma_{S_{k,l}}$ has a unique T-PBNE, characterized by a $2l-$tuple $I^*_{S_{k,l}}$, which specifies the optimal status investment strategy for all types at all levels of the nested star, and the according posterior beliefs. By proposition 10, it is true for any level $j \in \{1, ..., l-1\}$ that $I^*_{j,S_{k,l}} \leq I^*_{j+1,S_{k,l}}$. Therefore, expected utility is decreasing in the consumer's level in the nested star $S_{k,l}$ :*

$$U(m(i) - I^*_{j,S_{k,l}}, \hat{m}_i(I^*_{j,S_{k,l}})) \geq U(m(i) - I^*_{j+1,S_{k,l}}, \hat{m}_i(I^*_{j+1,S_{k,l}})).$$

***Proof.*** *In appendix.* ■

This result may be interpreted as a complementary mechanism adding to the countersignaling principle of Feltovich, Harbaugh and To (2002), who show that in a three types signaling model with a second noisy source of information, the highest types may in some cases distinguish themselves from the middle types by not signaling (pooling with the low types) if they are confident enough that the second signal will distinguish them from the low types (which the middle types can insufficiently expect to happen). Feltovich, Harbaugh and To (2002) state that this mechanism may explain why the very rich and powerful often abstain from conspicuous wasteful behavior or pressing their power, and I fully agree with their view. The mechanism described in present paper may be considered an an additional explanation to some of their results. The highest types often also occupy very central positions in the social information network, e.g. because of media attention or the central role they play in their organizations. Therefore most other consumers have relatively reliable information about their type, and as such, they do not need to engage in conspicuous consumption behavior. Old aristocracy can afford to be low profile, because they are well rooted in high society life, whereas '*nouveau riches*' need to show their wealth most ostentatiously to let the power of their money work in social life. Also, if one would assume a correlation between income level and centrality, the income effect and the 'centrality effect' work in opposite directions on the equilibrium level of signaling, but signaling incentives are more pressing for low income types at low levels of centrality.

## 3.5 Random Networks and Dominance

Finally, one may also wonder whether one can say anything about less stylized, more random networks. Can one make any predictions about signaling levels in a fully arbitrary network, without the strong structure of the strongly stylized networks above? As shown above, spectators who are in complete knowledge of the fundamentals of the game $\Gamma_G$, may correctly predict equilibrium behavior of consumer $i$ for any type, conditional on her average geodesic $\tilde{d}(i; S_{k,l})$, and will hence correctly interpret the observed signals $y_i$ and $\tilde{y}_i$ in equilibrium. Given this equilibrium interpretation by spectators, the optimal strategy of consumer $i$ is the (under the conditions in equations 9) unique T-PBNE strategy $I^*_{i,G}$. By proposition 10, we know that this equilibrium status signaling strategy is increasing in the average geodesic distance of consumer $i$ to the other $N-1$ consumers. This motivates some dominance results.

Let $G+(k,l)$ denote the network $G(\mathcal{N}, E \cup \{(k,l)\})$, where it is assumed that $(k,l) \notin E$. Hence, $G+(k,l)$ represents the old network with the number of vertices kept constant, but with one previously inexistent link added between vertices $k$ and $l$. Secondly, let $G+k$ denote the network $G(\mathcal{N} \cup \{(k)\}, E)$ with k$\notin \mathcal{N}$, or hence the network $G$ with one unconnected node added. Firstly, one may obviously state that by adding one link to the social network, the average geodesic distance of all players weakly decreases $\tilde{d}(i; G+(k,l)) \leq \tilde{d}(i; G), \forall i \in \mathcal{N}$, and this inequality is strict for all players for whom the absolute value of the difference in geodesic distance to the newly linked vertices is two or more, i.e. for all vertices $i$ for which $|d(i,k;G) - d(i,l;G)| \geq 2$, the above inequalities will be strict, $\tilde{d}(i; G+(k,l)) < \tilde{d}(i; G), \forall i : |d(i,k;G) - d(i,l;G)| \geq 2$, such that by proposition 10, the equilibrium investments of all these players will be weakly lower in the extended network $I^*_{G+(k,l)} \leq I^*_G$, and strictly lower $I^*_{i,G+(k,l)} < I^*_{i,G}$ if $I^*_{i,G}$ is in the interior of $\mathcal{I}_i$. Otherwise, if $|d(i,k;G) - d(i,l;G)| < 2$, then $I^*_{G+(k,l)} = I^*_G$. This also implies that average utilitarian welfare is higher in the extended network:

$W(G) \leq W(G+(k,l))$. For adding an unconnected extra consumer to the network, the predictions are even more clear-cut. As shown in corollary 13, adding one more unconnected consumer to the network increases the marginal utility of status signaling weakly, and strictly for connected consumers (on vertices with a strictly positive degree). Therefore $I^*_{G+k} \geq I^*_G$, and $I^*_{i,G+k} > I^*_{i,G}$ for $i$ connected and $I^*_{i,G}$ nonzero. This implies for average utilitarian welfare that $W(G) \geq W(G+k)$. The following corollary summarizes these results.

**Corollary 16** *Let $G+(k,l)$ denote $G(\mathcal{N}, E \cup \{(k,l)\})$ with $(k,l) \notin E$ and $G+k$ denote $G(\mathcal{N} \cup \{(k)\}, E)$ with $k \notin \mathcal{N}$. Then:*

1. *$I^*_{G+(k,l)} \leq I^*_G$, and $I^*_{G+(k,l)} < I^*_G$ for $\forall i : |d(i,k;G) - d(i,l;G)| \geq 2$ if $I^*_{i,G+(k,l)} > 0$, and*

$$W(G) \leq W(G+(k,l))$$

2. *2. $I^*_{G+k} \geq I^*_G$ and $I^*_{i,G+k} > I^*_{i,G}$ for all $i$ connected and $I^*_{i,G}$ nonzero and*

$$W(G) \geq W(G+k).$$

Hence, an extra friendship weakly increases social welfare in society. And one marginal stranger in the community, about whose judgement one cares, chases equilibrium wasteful signaling incentives weakly upwards, weakly decreasing social welfare.

# 4 Conclusion

The noisy status signaling game $\Gamma$ shows the constant sum nature of status signaling in a continuous fashion. High income consumers spend a part of their means to distinguish themselves in the eyes of Bayesian spectators from the low income types, but because of imperfect observation of the status investments (and the noise distribution having the real line for support), separation can never be perfect. For the same reason, the low income types always find marginal benefits in status investments themselves, thus undoing some of the separation achieved by high types. Unless the low types' consumer optimum is a corner solution, the consumption of the low types will in equilibrium be distorted as well. From a social (utilitarian) point of view, any amount of status signaling is wasteful, as the gains from status signaling are constant sum among all of the consumers.

In the second part of the paper, a social information network is introduced, providing a second source of information about player quality.

This 'gossip information' functions as an information substitute to status investments. Because the reliability of information depends on the structure of the social network, and spectators make in equilibrium more use of more reliable information, optimal status signaling investments depend on the structure of the social information network in game $\Gamma_G$. It is shown for some simple, highly stylized networks, that the incentives to engage in costly signaling increase with network size, decrease with network density and decrease with the centrality of a consumer's position in the network. One may also find some simple dominance results for networks of an arbitrary form, establishing that equilibrium status investments weakly decrease by adding one more link to the networks, and weakly increase by adding one more consumer.

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# A Appendix

## A.1 Proof of lemma 3

One may write

$$\begin{aligned}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} &= -(1-p)\int \frac{p\varphi'_\Delta \varphi_0\left[(1-p)\varphi_0 + p\varphi_\Delta\right] - p\varphi_\Delta\varphi_0 p\varphi'_\Delta}{\left[(1-p)\varphi_0 + p\varphi_\Delta\right]^2} dy_i \\ &= -p\int \left[\frac{(1-p)\varphi_0}{(1-p)\varphi_0 + p\varphi_\Delta}\right]^2 \varphi'_\Delta dy_i.\end{aligned}$$

a) The squared term between brackets at the RHS is the posterior probability of any $i$ being a low type. This probability is strictly decreasing in $y_i$ given condition 1, iii, as $\frac{(1-p)\varphi_0}{(1-p)\varphi_0+p\varphi_\Delta} = \frac{1}{1+\frac{p}{(1-p)}\frac{\varphi_\Delta}{\varphi_0}}$. b) By part i) of condition 1, the mean of $\varphi(.)$ is the mode. Part iii) of condition 1 implies that $\varphi(.)$ increases monotonically on the interval $]-\infty,\mu[$, since if $\varphi(.)$ would be downward sloping on this interval, $\mu'$ could be chosen such that $\frac{\partial\varphi(y;\mu,\sigma)}{\partial y} < 0$ and $\frac{\partial\varphi(y;\mu',\sigma)}{\partial y} > 0$ for $\mu' < \mu$, such that $\varphi(.)$ violates iii). Therefore $\varphi'_\Delta$ is negative for all $y_i < \frac{\Delta+d\Delta}{2}$, and positive for $y_i > \frac{\Delta+d\Delta}{2}$ to the same extent, as $\varphi(.)$ is symmetric. By a), the negative values outweight the positive values right of $\frac{\Delta+d\Delta}{2}$, such that the integral is negative, and $\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} > 0$.

## A.2 Proof of proposition 7.

**Proof.** *Proof of 1*: This is rather standard and straightforward. The strategy space $I$ is a compact, convex and non-empty subspace of the Euclidean space One may for every tuple $I = \langle I_L, I_H \rangle$ define the best reply function $f(I) : \mathcal{I} \twoheadrightarrow \mathcal{I} : I \twoheadrightarrow \left\langle I_L^{BR}(I_H), I_H^{BR}(I_L) \right\rangle$, which is defined over the whole space $I$ and continuous by the continuity of it's components. The individual best reply functions $I_L^{BR}(I_H)$ are continuous be the continuity of the utility function and of it's components, and is globally concave under conditions 1 and 6 over it's whole domain, such that the implicit function theorem guarantees the continuity of the best reply functions. Then, by Brouwer's fixed point theorem, $f(I)$ has at least one fixed point $I^*$ which is such that $f(I^*) = I^*$, and hence under this condition a T-PBNE exists for $\Gamma$.

*Proof of 2*: Suppose that $I^{\bowtie} \equiv \langle I_L^{\bowtie}, I_H^{\bowtie} \rangle$ were part of a T-PBNE $\{I^{\bowtie}, P(H; I^{\bowtie})\}$ such that $I_L^{\bowtie} > I_H^{\bowtie}$. Then spectators, knowing the equilibrium by assumption, take advantage of the fact that $|\Delta^{\bowtie}| > 0$ to partially distinguish between the low and high income consumers, such that $\ddot{m}_L(I_L^{\bowtie}; I_H^{\bowtie}) < \ddot{m}_L(I_H^{\bowtie}; I_H^{\bowtie}) = p$, while $V(m_L - I_L^{\bowtie}) < V(m_L - I_H^{\bowtie})$. Therefore $I_L^{\bowtie}$ cannot be the low income consumer's best reply to $I_H^{\bowtie}$, and $\{I^{\bowtie}, P(H; I^{\bowtie})\}$ not a T-PBNE.

The second statement follows from the Kuhn-Tucker first order conditions to the problems of the low and the high income consumers. When $I_H^* = I_L^*$, this is either because the first derivatives of the utility function towards own status investments are negative over the whole strategy space, such that $I_H^* = I_L^* = 0$, or, in the case of an interior solution, this implies that $\frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial \ddot{m}(0)}{\partial \Delta} = \frac{\partial V(m_H - I_H^*)}{\partial c} < \frac{\partial V(m_L - I_H^*)}{\partial c} = \frac{\kappa(m_H - m_L)p}{\sigma(1-p)} \frac{\partial \ddot{m}(0)}{\partial \Delta}$ which implies $\frac{p}{1-p} = \frac{\frac{\partial V(m_H - I_H^*)}{\partial c}}{\frac{\partial V(m_L - I_H^*)}{\partial c}} > 1$ as $V''(.) < 0$, from which follows that $p < 1/2$.

*Proof of 3 and 4:* Fudenberg and Tirole (1991,p.24) show how a sufficient condition for asymptotic stability of a fixed point (that all eigenvalues of $\partial f(I^*)$ have real parts whose absolute value is $< 1$) is satisfied if $\frac{\partial^2 U_L}{\partial I_H \partial I_L} \frac{\partial^2 U_H}{\partial I_H \partial I_L} < \frac{\partial^2 U_H}{\partial^2 I_H} \frac{\partial^2 U_L}{\partial^2 I_L}$ holds in a region $G(I^*, \varepsilon)$ around the equilibrium. This means in our case that

$$\left(-\frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) \left(\frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) <$$
$$\left(-\frac{\partial^2 V(L.)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right) \left(-\frac{\partial^2 V(H)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right),$$
which is equivalent to
$$-\frac{p}{(1-p)} \left(\frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right)^2 <$$
$$\left(-\frac{\partial^2 V(L.)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right) \left(-\frac{\partial^2 V(H)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right)$$

in which the first term is clearly weakly negative and both factors at the right hand side are strictly positive under the conditions 1 and 6 , and hence this equality holds for all $I \in \mathcal{I}$, or $G(I^*, \varepsilon) = \mathcal{I}$, which proves global asymptotic stability. Uniqueness follows from global asymptotic stability. ■

## A.3 Proof of proposition 8.

**Proof.** The applied method is the same for all comparative statics established below. The equilibrium signaling investments are determined by the system of first order conditions in equations 10 and 11. After taking the derivative of both first order conditions towards the exogenous parameter of interest, one may write the thus obtained equations as a system in matrix form, i.e. as $A * dI = b$, in which $dI \equiv \begin{pmatrix} \frac{\partial I_L^*}{\partial x} \\ \frac{\partial I_H^*}{\partial x} \end{pmatrix}$ (with x representing the exogenous parameter of interest). By the implicit function theorem, this system has a unique solution as long as is holds that the determinant of the coefficients matrix $|A| \neq 0$ over $I$ (see e.g. Currier, 2000), which is of course always satisfied if condition 6 holds:

$$A = \begin{pmatrix} -\frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & \frac{\partial^2 V(.)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \\ \frac{\partial^2 V(.)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \end{pmatrix}$$

$|A| = -\frac{p}{(1-p)}\left(\frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right)^2$
$- \left(\frac{\partial^2 V(.)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right)\left(\frac{\partial^2 V(.)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) < 0$ where the inequality holds as the two factors in the second term are negative everywhere by condition 6. Knowing this, one may solve the system by Cramer's rule, setting

$$\frac{\partial I_L^*}{\partial x} = \frac{\begin{vmatrix} b_1 & A_{1,2} \\ b_2 & A_{2,2} \end{vmatrix}}{|A|} \quad \text{and} \quad \frac{\partial I_H^*}{\partial x} = \frac{\begin{vmatrix} A_{1,1} & b_1 \\ A_{2,1} & b_2 \end{vmatrix}}{|A|}.$$

*1. Parameter $\kappa$:* The derivative of the first order conditions towards $\kappa$is for the high income types $\frac{\partial^2 V(.)}{\partial^2 c}\frac{\partial I_H}{\partial \kappa} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\left(\frac{\partial I_H}{\partial \kappa} - \frac{\partial I_L}{\partial \kappa}\right) + \frac{\kappa(m_H - m_L)}{\sigma}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} = 0$ and for the low income consumers: $\frac{\partial^2 V(.)}{\partial^2 c}\frac{\partial I_L}{\partial \kappa} + \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\left(\frac{\partial I_H}{\partial \kappa} - \frac{\partial I_L}{\partial \kappa}\right)$
$+ \frac{\kappa(m_H - m_L)p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} = 0$.As such $b = \begin{pmatrix} -\frac{\kappa(m_H - m_L)}{\sigma}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ -\frac{\kappa(m_H - m_L)}{\sigma}\frac{p}{1-p}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \end{pmatrix}$

Hence, we may write
$\begin{vmatrix} b_1 & A_{1,2} \\ b_2 & A_{2,2} \end{vmatrix} = \begin{vmatrix} -\frac{\kappa(m_H - m_L)}{\sigma}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} & \frac{\partial^2 V(m_H - I_H.)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \\ -\frac{\kappa(m_H - m_L)}{\sigma}\frac{p}{1-p}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} & \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \end{vmatrix}$
$= \frac{\kappa(m_H - m_L)}{\sigma}\frac{p}{1-p}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\frac{\partial^2 V(m_H - I_H.)}{\partial^2 c} < 0$. Hence $\frac{\partial I_L^*}{\partial \kappa} > 0$.
In the same fashion we may derive

$\begin{vmatrix} A_{1,1} \, b_1 \\ A_{2,1} \, b_2 \end{vmatrix} = \begin{vmatrix} -\frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & -\frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ \frac{\partial^2 V(m_L - I_L)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & -\frac{\kappa(m_H - m_L)}{\sigma} \frac{p}{1-p} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \end{vmatrix}$
$= \frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \frac{\partial^2 V(m_L - I_L)}{\partial^2 c} < 0$, such that $\frac{\partial I_H^*}{\partial \kappa} > 0$.Hence $\frac{\partial I_H^*}{\partial \kappa} \geq 0$ and $\frac{\partial I_L^*}{\partial \kappa} \geq 0$, where the weak inequalities also deal with the possibility that the restrictions $I_L \geq 0$ and $I_H \geq 0$ are binding, such that $I_L^* = 0$ or $I_L^* = 0$ and $I_H^* = 0$.

*2. Parameter $\sigma$:* The derivatives of the first order conditions for the high income types:

$$\frac{\partial^2 V(.)}{\partial^2 c} \frac{\partial I_H}{\partial \sigma} + \frac{\kappa \left(m_H - m_L\right)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \left( \frac{\partial I_H}{\partial \sigma} - \frac{\partial I_L}{\partial \sigma} \right) - \frac{\kappa \left(m_H - m_L\right)}{\sigma^2} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ - \frac{\kappa \left(m_H - m_L\right)}{\sigma} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} = 0,$$

and for the low types:

$$\frac{\partial^2 V(.)}{\partial^2 c} \frac{\partial I_L}{\partial \sigma} + \frac{\kappa \left(m_H - m_L\right) p}{\sigma^2 (1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \left( \frac{\partial I_H}{\partial \sigma} - \frac{\partial I_L}{\partial \sigma} \right) - \frac{\kappa \left(m_H - m_L\right) p}{\sigma^2 (1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ - \frac{\kappa \left(m_H - m_L\right) p}{\sigma (1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} = 0.$$

Also, $b = \begin{pmatrix} \frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} \\ \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa(m_H - m_L)p}{\sigma(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} \end{pmatrix}$, such that
$\begin{vmatrix} b_1 \, A_{1,2} \\ b_2 \, A_{2,2} \end{vmatrix} = \begin{vmatrix} \frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} & \frac{\partial^2 V(.)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \\ \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa(m_H - m_L)p}{\sigma(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} & \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \end{vmatrix}$
$= -\frac{\kappa(m_H - m_L)}{\sigma^2} \frac{p}{1-p} \frac{\partial^2 V(.)}{\partial^2 c} \left( \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \Delta \right)$ such that $\frac{\partial I_L^*}{\partial \sigma} \left( \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \Delta \right) < 0$.

Equivalently
$\begin{vmatrix} A_{1,1} \, b_1 \\ A_{2,1} \, b_2 \end{vmatrix} = \begin{vmatrix} -\frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & \frac{\kappa(m_H - m_L)}{\sigma^2} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa(m_H - m_L)}{\sigma} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} \\ \frac{\partial^2 V(m_L - I_L)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa(m_H - m_L)p}{\sigma(1-p)} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{I_H - I_L}{\sigma^2} \end{vmatrix}$
$= -\frac{\partial^2 V(m_L - I_L)}{\partial^2 c} \frac{\kappa(m_H - m_L)}{\sigma^2} \left( \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \Delta \right)$ and hence $\frac{\partial I_H^*}{\partial \sigma} \left( \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \Delta \right) < 0$.

*3. Parameter $m_H$:* The first order derivative towards $m_H$, for the high types:

$$-\frac{\partial V^2(m_H - I_H^*)}{\partial^2 c} + \frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c} \frac{\partial I_H^*}{\partial m_H} + \frac{\kappa}{\sigma} \frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\kappa \left(m_H - m_L\right)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{\partial I_H^*}{\partial m_H} \\ - \frac{\kappa \left(m_H - m_L\right)}{\sigma^2} \frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \frac{\partial I_L^*}{\partial m_H} = 0$$

and for low income consumers:

$$\frac{\partial^2 V(m_L-I_L^*)}{\partial^2 c}\frac{\partial I_L^*}{\partial m_H}+\frac{\kappa p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}+\frac{\kappa\,(m_H-m_L)\,p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\frac{\partial I_H^*}{\partial m_H}$$
$$-\frac{\kappa\,(m_H-m_L)\,p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\frac{\partial I_L^*}{\partial m_H}=0$$

$b=\begin{pmatrix}\frac{\partial V^2(m_H-I_H^*)}{\partial^2 c}-\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ -\frac{\kappa p}{\sigma(1-p)}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\end{pmatrix}$, and hence

$\begin{vmatrix}A_{1,1}\,b_1\\ A_{2,1}\,b_2\end{vmatrix}=\begin{vmatrix}-\frac{\kappa(m_H-m_L)}{\sigma^2}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta} & \frac{\partial V^2(m_H-I_H^*)}{\partial^2 c}-\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ \frac{\partial^2 V(m_L-I_L^*)}{\partial^2 c}-\frac{\kappa(m_H-m_L)p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta} & -\frac{\kappa p}{\sigma(1-p)}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\end{vmatrix}$

$=-\frac{\partial V^2(m_H-I_H^*)}{\partial^2 c}\left(\frac{\partial^2 V(m_L-I_L^*)}{\partial^2 c}-\frac{\kappa(m_H-m_L)p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\right)+\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\frac{\partial^2 V(m_L-I_L^*)}{\partial^2 c}<0$

and as the term between brackets is negative under condition 6, it follows that $\frac{\partial I_H^*}{\partial m_H}>0$.

As for the low income consumers:

$\begin{vmatrix}b_1\,A_{1,2}\\ b_2\,A_{2,2}\end{vmatrix}=\begin{vmatrix}\frac{\partial V^2(m_H-I_H^*)}{\partial^2 c}-\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} & \frac{\partial^2 V(m_H-I_H^*)}{\partial^2 c}+\frac{\kappa(m_H-m_L)}{\sigma^2}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\\ -\frac{\kappa p}{\sigma(1-p)}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} & \frac{\kappa(m_H-m_L)p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\end{vmatrix}$

$=\frac{\partial V^2(m_H-I_H^*)}{\partial^2 c}\frac{\kappa p}{\sigma(1-p)}\left(\frac{(m_H-m_L)}{\sigma}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\right)$ and hence $\frac{\partial I_L^*}{\partial m_H}\left(\frac{(m_H-m_L)}{\sigma}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\right)>0$

*4. Parameter $m_L$*:The derivatives of the first order conditions are for the high income consumers:

$$\frac{\partial^2 V(m_H-I_H^*)}{\partial^2 c}\frac{\partial I_H^*}{\partial m_L}-\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\kappa\,(m_H-m_L)}{\sigma^2}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\frac{\partial I_H^*}{\partial m_L}-\frac{\kappa\,(m_H-m_L)}{\sigma^2}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\frac{\partial I_L^*}{\partial m_L}=0$$

and for the low income consumers

$$-\frac{\partial V^2(m_L-I_L^*)}{\partial^2 c}+\frac{\partial^2 V(m_L-I_L^*)}{\partial^2 c}\frac{\partial I_L^*}{\partial m_L}-\frac{\kappa p}{\sigma(1-p)}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\kappa\,(m_H-m_L)\,p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\frac{\partial I_H^*}{\partial m_L}$$
$$-\frac{\kappa\,(m_H-m_L)\,p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\frac{\partial I_L^*}{\partial m_L}=0$$

Hence $b=\begin{pmatrix}\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ \frac{\partial V^2(m_L-I_L^*)}{\partial^2 c}+\frac{\kappa p}{\sigma(1-p)}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\end{pmatrix}$ and therefore

$\begin{vmatrix}b_1\,A_{1,2}\\ b_2\,A_{2,2}\end{vmatrix}=\begin{vmatrix}\frac{\kappa}{\sigma}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} & \frac{\partial^2 V(m_H-I_H^*)}{\partial^2 c}+\frac{\kappa(m_H-m_L)}{\sigma^2}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\\ \frac{\partial V^2(m_L-I_L^*)}{\partial^2 c}+\frac{\kappa p}{\sigma(1-p)}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} & \frac{\kappa(m_H-m_L)p}{\sigma^2(1-p)}\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\end{vmatrix}$

$$= -\frac{\partial V^2(m_L - I_L^*)}{\partial^2 c}\left(\frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) - \frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c}\frac{\kappa p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}$$

in which the first term is negative and the second term is positive, such that the overall effect depends on the relative magnitude of these two terms, and hence

$$\left[-\frac{\partial V^2(m_L - I_L^*)}{\partial^2 c}\left(\frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta}\right) - \frac{\partial^2 V(m_H - I_H^*)}{\partial^2 c}\frac{\kappa p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right]\frac{\partial I_L^*}{\partial m_L} > 0$$

For the high income consumers

$$\begin{vmatrix} A_{1,1} & b_1 \\ A_{2,1} & b_2 \end{vmatrix} = \begin{vmatrix} -\frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & \frac{\kappa}{\sigma}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ \frac{\partial^2 V(m_L - I_L^*)}{\partial^2 c} - \frac{\kappa(m_H - m_L)p}{\sigma^2(1-p)}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} & \frac{\partial V^2(m_L - I_L^*)}{\partial^2 c} + \frac{\kappa p}{\sigma(1-p)}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \end{vmatrix}$$

$$= -\frac{\partial V^2(m_L - I_L^*)}{\partial^2 c}\left(\frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{\kappa}{\sigma}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right) \text{ such that } \frac{\partial I_H^*}{\partial m_L}\left(\frac{\kappa(m_H - m_L)}{\sigma^2}\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{\kappa}{\sigma}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\right) < 0$$

■

## A.4 Proof of proposition 10.

**Proof.** *Part 1:* For existence, uniqueness and global asymptotic stability: these proofs are essentially the same as for game $\Gamma$. If condition 9 applies, then the implicit function theorem guarantees the existence of continuous best reply functions $I_L^{BR}(I_H)$ and $I_H^{BR}(I_L)$. Therefore there exists a continuous best reply mapping $f(I) : I \twoheadrightarrow I : I \twoheadrightarrow \langle I_L^{BR}(I_H), I_H^{BR}(I_L)\rangle$ of $I$ onto itself. And since $I$ is convex, Brouwers' fixed point theorem guarantees the existence of a fixed point, $I_G^*$. This fixed point, together with $\breve{P}(H; y_i, \tilde{y}_{i,j})$ with $I_G^*$ in $\Delta$,defines the T-PBNE of game $\Gamma_G$. Uniqueness and global asymptotic stability follow again from the condition $\frac{\partial^2 U_L}{\partial I_H \partial I_L}\frac{\partial^2 U_H}{\partial I_H \partial I_L} < \frac{\partial^2 U_H}{\partial^2 I_H}\frac{\partial^2 U_L}{\partial^2 I_L}$, of which the right hand side is strictly positive by condition 9, and the left hand side is

$$-\frac{p}{(1-p)}\left(\frac{\kappa(m_H - m_L)}{\sigma^2}\right)^2\left[\begin{array}{c}\frac{\partial \zeta^*}{\partial \Delta}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \zeta^*\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} \\ +\frac{\partial^2 \zeta^*}{\partial^2 \Delta}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,G}^2)\right) + \frac{\partial \zeta^*}{\partial \Delta}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta}\end{array}\right]^2 < 0 \tag{19}$$

for all $\langle I_H, I_L\rangle \in \mathcal{I}$. Hence, global asymptotic stability holds and uniqueness follows

*Part 2:* As before, start from the first order conditions in equations 16 and 17, and take their derivative towards $\tilde{\sigma}_{i,G}^2$.

The derivatives to $\tilde{\sigma}_{i,G}^2$ are for the high and low type consumers respectively

$$\frac{\partial^2 V(m_H - I_H)}{\partial^2 c}\frac{\partial I_{H,G}^*}{\partial \tilde{\sigma}_{i,G}^2} + \frac{\kappa(m_H - m_L)}{\sigma^2}\left[\begin{array}{c}\zeta^*\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + 2\frac{\partial \zeta^*}{\partial \Delta}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ +\frac{\partial^2 \zeta^*}{\partial^2 \Delta}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,G}^2)\right)\end{array}\right]\left(\frac{\partial I_{H,G}^*}{\partial \tilde{\sigma}_{i,G}^2} - \frac{\partial I_{L,G}^*}{\partial \tilde{\sigma}_{i,G}^2}\right)$$

$$+\frac{\kappa(m_H - m_L)}{\sigma}\left[\frac{\partial \zeta^*}{\partial \tilde{\sigma}_{i,G}^2}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} + \frac{\partial^2 \zeta^*}{\partial \Delta \partial \tilde{\sigma}_{i,G}^2}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,G}^2)\right) - \frac{\partial \zeta^*}{\partial \Delta}\frac{\partial \tilde{m}(\Delta m(\tilde{\sigma}_{i,G}^2)}{\partial \tilde{\sigma}_{i,G}^2}\right] = 0$$

$$\frac{\partial^2 V(m_L - I_L)}{\partial^2 c}\frac{\partial I_{L,G}^*}{\partial \tilde{\sigma}_{i,G}^2} + \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{p}{(1-p)}\left[\begin{array}{c}\zeta^*\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + 2\frac{\partial \zeta^*}{\partial \Delta}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ +\frac{\partial^2 \zeta^*}{\partial^2 \Delta}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,G}^2)\right)\end{array}\right]\left(\frac{\partial I_{H,G}^*}{\partial \tilde{\sigma}_{i,G}^2} - \frac{\partial I_{L,G}^*}{\partial \tilde{\sigma}_{i,G}^2}\right)$$

$+\frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\left[\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)-\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}}\right]=0$

These derivatives may again be written as a system in matrix form $A * dI = b$, with

$A_{1,1}=-\frac{\kappa(m_H-m_L)}{\sigma^2}\left(\zeta^*\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+2\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial^2\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)\right)$,

$A_{1,2}=\frac{\partial^2V(m_H-I_H)}{\partial^2c}+\frac{\kappa(m_H-m_L)}{\sigma^2}\left(\zeta^*\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+2\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial^2\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)\right)$,

$A_{2,1}=\frac{\partial^2V(m_L-I_L)}{\partial^2c}-\frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\left(\zeta^*\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+2\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial^2\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)\right)$,

$A_{2,2}=\frac{\kappa(m_H-m_L)}{\sigma^2}\frac{p}{(1-p)}\left[\zeta^*\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+2\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial^2\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)\right]$

and

$$b=\begin{pmatrix}-\frac{\kappa(m_H-m_L)}{\sigma}\left[\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)-\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}}\right]\\ -\frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\left[\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)-\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}}\right]\end{pmatrix}$$

As before, this system has a unique solution, as it holds everywhere that $|A|\neq 0$ by condition 9.

As before, by Cramer's rule, then, $\frac{\partial I^*_{L,G}}{\partial\tilde{\sigma}^2_{i,G}}=\frac{\begin{vmatrix}b_1 & A_{1,2}\\ b_2 & A_{2,2}\end{vmatrix}}{|A|}$ and $\frac{\partial I^*_{H,G}}{\partial\tilde{\sigma}^2_{i,G}}=\frac{\begin{vmatrix}A_{1,1} & b_1\\ A_{2,1} & b_2\end{vmatrix}}{|A|}$

in which

$$\begin{vmatrix}b_1 & A_{1,2}\\ b_2 & A_{2,2}\end{vmatrix}=\frac{\partial^2V(m_H-I_H)}{\partial^2c}\frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\begin{bmatrix}\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}-\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}}\\ +\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)\end{bmatrix}$$

such that $\frac{\partial I^*_{L,G}}{\partial\tilde{\sigma}^2_{i,G}}$ has the same sign as the factor between square brackets. Similarly,

$$\begin{vmatrix}A_{1,1} & b_1\\ A_{2,1} & b_2\end{vmatrix}=\frac{\partial^2V(m_L-I_L)}{\partial^2c}\frac{\kappa(m_H-m_L)}{\sigma}\begin{bmatrix}\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)\\ -\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}}\end{bmatrix}$$

Hence, the sign of both $\frac{\partial I^*_{H,G}}{\partial\tilde{\sigma}^2_{i,G}}$ and $\frac{\partial I^*_{L,G}}{\partial\tilde{\sigma}^2_{i,G}}$ equals the sign of the expression

$\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right)-\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}}$

In the first term, $\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}$ is positive, and for the factor $\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}$, one first needs to establish the effect of a marginal increase of $\sigma$ or $\tilde{\sigma}^2_{i,G}$ on the posterior variances. For the case of the status investments posterior (the case for $\tilde{\sigma}^2_{i,G}$ is similar), it is true that

$\frac{\partial Var(P(H;y_i)}{\partial\sigma}=-p\frac{I_H-I_L}{\sigma^2}\int\left(\frac{(1-p)\varphi(y_i;0,1)}{p\varphi(y_i;\Delta,1)+(1-p)\varphi(y_i;0,1)}\right)^2\frac{\partial\varphi(y_i;\Delta,1)}{\partial\Delta}dy_i=\Delta\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}>0$ (for $\Delta>0$). Similarly, one finds that $\frac{\partial Var(\tilde{P}(H;\tilde{y}_{i,j})}{\partial\tilde{\sigma}^2_{i,G}}>0$.

Because $\zeta^*=\frac{Var(\tilde{P}(H;\tilde{y}_{i,j})}{Var(P(H;y_i)+Var(\tilde{P}(H;\tilde{y}_{i,j})}$ and hence $\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}=\frac{\frac{\partial Var(\tilde{P}(H;\tilde{y}_{i,j})}{\partial\tilde{\sigma}^2_{i,G}}Var(P(H;y_i)}{\left(Var(P(H;y_i)+Var(\tilde{P}(H;\tilde{y}_{i,j})\right)^2}>$

0.

As for $\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}$, we know that $\frac{\partial\zeta^*}{\partial\Delta} = \frac{-Var(\tilde{P}(H;\tilde{y}_{i,j})\frac{\partial Var(P(H;y_i)}{\partial\Delta}}{\left(Var(P(H;y_i)+Var(\tilde{P}(H;\tilde{y}_{i,j})\right)}$ in which $\frac{\partial Var(P(H;y_i)}{\partial\Delta} = -\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} < 0$. Then

$\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}} == \frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\left(-\frac{\partial Var(P(H;y_i)}{\partial\Delta}\right) = \frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}}{\partial\Delta} > 0$. Finally, the sign of $\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}} = -\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\Delta m(\tilde{\sigma}^2_{i,G})}\frac{(m_H-m_L)}{\tilde{\sigma}^2_{i,G}} < 0$. Hence, in

$\left[\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} - \frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}} + \frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)\right]$ the first two terms are positive and the third takes the sign of $\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)$, which can be both positive and negative. However, using that

$\frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}} = \frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\left(-\frac{\partial Var(P(H;y_i)}{\partial\Delta}\right) = \frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}}{\partial\Delta}$, one may write

$\frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta} - \frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}} + \frac{\partial^2\zeta^*}{\partial\Delta\partial\tilde{\sigma}^2_{i,G}}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}))\right)$

$= \frac{\partial\zeta^*}{\partial\tilde{\sigma}^2_{i,G}}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\left(1 + \ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})\right) - \frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G})}{\partial\tilde{\sigma}^2_{i,G}} > 0$ (as $\tilde{m}(\Delta m(\tilde{\sigma}^2_{i,G}) \in ]0,1[$ ). Hence, $\frac{\partial I^*_{H,G}}{\partial\tilde{\sigma}^2_{i,G}} \geq 0$ and $\frac{\partial I^*_{L,G}}{\partial\tilde{\sigma}^2_{i,G}} \geq 0$, where the weak inequality takes the possibility of corner solutions into account. ■

## A.5 Proof corollary 11

**Proof.** Define $\nu = \max\left\{n; n \in \mathbb{N}, n \leq \frac{N-1}{2}\right\}$ and note that for $j \leq \nu, \Phi_j(i;G) = 2\ \forall i \in N$. The average geodesic distance in a cyclical network (which is independent of $i$) is $\tilde{d}(i;C^N) = \tilde{d}(C^N) = \frac{1}{N-1}\left(\sum_j^{\nu} 2j + (\nu+1)\left((N-1)-2\nu\right)\right)$ $= \frac{(\nu+1)(N-1-\nu)}{N-1}$. The effect of a marginal increase of $N$ (and adapting the links in $E^c$) on the average geodesic distance $\tilde{d}(C^N)$ is $\Delta_N\tilde{d}(C^N) = \tilde{d}(C^{N+1}) - \tilde{d}(C^N) = \frac{(\nu'+1)(N-\nu')}{N} - \frac{(\nu+1)(N-1-\nu)}{N-1}$, with $\nu' = \max\left\{n; n \in \mathbb{N}, n \leq \frac{N}{2}\right\}$. For $N$ even, it is true that $\nu' = \nu$, such that $\Delta_N\tilde{d}(C^N) = \frac{(\nu+1)\nu}{N(N-1)} > 0$, whereas for $N$ odd, $\nu' = \nu+1$, and hence $\Delta_N\tilde{d}(C^N) = \frac{(\nu+2)(N-\nu+1)}{N} - \frac{(\nu+1)(N-1-\nu)}{N-1} = \frac{N^2+N+\nu^2+\nu-2}{N(N-1)} > 0$. ■

## A.6 Proof of corollary 12

**Proof.** Let $\nu = \max\left\{n; n \in \mathbb{N}, n \leq \frac{N-1}{k}\right\}$ such that the diameter $\bar{d}(C^{N;k}) = \nu + I(N-1 > k\nu)$, with $I(.)$ an indicator function. The average distance in network $C^{N;k}$ may be written as $\tilde{d}(C^{N;k}) = \frac{1}{N-1}\left(\sum_i^{\nu} ki + ((N-1)-\nu k)(\nu+1)\right)$ $= \frac{(\nu+1)(2(N-1)-k\nu)}{2(N-1)}$. When increasing the degree $k$ by two units (to maintain homogeneity in average geodesic), one finds that

$$\begin{aligned}\Delta_k \tilde{d}(C^{N;k}) &= \tilde{d}(C^{N;k+2}) - \tilde{d}(C^{N;k}) \\ &= -\frac{1}{N-1}\sum_{j=1}^{\nu'} 2j + (\nu k - \nu'(k+2))(\nu - \nu' - 1) + (N - 1 - \nu k)(\nu - \nu')\end{aligned}$$

if $\nu k > \nu'(k+2)$ and just $-\frac{1}{N-1}\sum_{j=1}^{\nu'} 2j + (N - 1 - \nu'(k+2))(\nu - \nu')$ otherwise, such that

$$\Delta_k \tilde{d}(C^{N;k}) = -\frac{1}{N-1}\begin{bmatrix}(\nu'+1)\nu' + (N-1-\nu'(k+2))(\nu-\nu') \\ +Min\{0, \nu'(k+2) - \nu k\}\end{bmatrix} < 0,$$

since if one would order all $N-1$ vertices by distance from any randomly chosen vertex $i$ from low to high, then the distance to $i$ in $C^{N;k}$ is a stepwise increasing function and the difference between the stepwise increasing distance functions, summed and divided by $N-1$, reduces to the function elaborated above. Hence, $\frac{\Delta_k \tilde{d}(C^{N;k})}{\Delta k = 2} < 0$, such that $\tilde{\sigma}_{i,C^{N;k}} = \tilde{d}(i; C^{N;k})\tilde{\sigma}^2$ decreases in $k$, and hence $I^*_{C^{N;k}} \geq I^*_{C^{N;k'}}$ for $k < k'$ by proposition 10 (with $I^*_{t,C^{N;k}} > I^*_{t,C^{N;k'}}$ for an interior equilibrium). ■

## A.7 Generalisation to d-lattices

Let $L_d^{N;k}$ denote a $d$-dimensional lattice, in which each of the $N$ consumers is linked to the $j = \frac{k}{2d}$ last and $j$ next neighbors along each dimension $d$, and is hence of degree $k$, with $k \in 2d.\mathbb{N}$ to preserve homogeneity in degree. In order to be homogeneous in average geodesic, we need the lattice to be cyclical along each dimension. For two dimensions, this implies that $L_2^{N;k}$ is in fact a two-dimensional grid on the surface of a torus. For simplicity, I assume that the $N$ consumers are spread equally over all dimensions, i.e. that from each node $i$ one may move along each dimension $d$ and pass $N_d - 1$ nodes before returning to node $i$. Hence, along a single dimension, $L_d^{N;k}$ is a $C^{N_d;k}$ network and $N = \prod N_d$ and $N_j = N_l$ for $j, l = 1, .., d$. For now, let us assume that $N_d \in \frac{k}{d}\mathbb{N}$. For the two-dimensional lattice of degree k, define $\nu$ as $\nu = \max\left\{n; n \in \mathbb{N}, n \leq \frac{N_d - 1}{k/d} = \frac{N_d - 1}{2j}\right\}$. One can, imagining a two-dimensional grid, see that $\Phi_s\left(L_2^{N;k}\right) = k + 2^d j^d (s-1)$ for $s = 1, .., \nu$ and $\Phi_s\left(L_2^{N;k}\right) = s\left(2^d j^d (2\nu - s - 1)\right)$ for $s = \nu + 1, ..., 2\nu - 1$. Therefore, the average geodesic of $L_2^{N;k}$ (a square grid on the surface of a torus) is $\tilde{d}(L_2^{N;k}) = \frac{1}{N-1}\sum_{s=1}^{\infty} s\Phi_s\left(L_2^{N;k}\right) = \frac{1}{N-1}\left[\sum_{s=1}^{\nu} s\left(k + 2^d j^d (s-1)\right) + \sum_{s=1}^{\nu-1}(\nu + s)\left(2^d j^d (\nu - s - 1)\right)\right]$.
For the three-dimensional lattice (a cubic lattice in which the first elements

of every row and column are neighbors of the last elements), one may see that $\Phi_s\left(L_3^{N;k}\right) = k + \sum_{f=1}^{s} f2^d j^d = k + 2^d j^d \frac{(s-1)s}{2}$ for $s = 1, .., \nu$ and $\Phi_s\left(L_3^{N;k}\right) = 2^d j^d \frac{(2\nu-s-2)(2\nu-s-1)}{2}$ such that

$$\tilde{d}(L_3^{N;k}) = \frac{1}{N-1}\left[\sum_{s=1}^{\nu} s\left(k + 2^d j^d \frac{(s-1)s}{2}\right) + \sum_{s=1}^{\nu-1}(\nu+s)2^d j^d \frac{(\nu-s-2)(\nu-s-1)}{2}\right].$$

One may see that for two- and three-dimensional lattices of degree k, it holds as well that $\tilde{d}(L_d^{N;k})$ is increasing in $N_d$ and decreasing in $k$, such that $I^*_{L_d^{N;k}}$ may be expected to increase in $N$ and to decrease in $k$.

## A.8 Proof of corollary 13.

**Proof.** The problem of a high and low income consumer $i$ in some component $C(i)$ is respectively: ■

$$Max_{I_H} U = V(m_H - I_H) + \kappa\left[(m_H - m_L)\begin{bmatrix}\left(1 - \frac{|C(i)|}{N-1}(1-\zeta^*)\right)\ddot{m}(\Delta) \\ + \frac{|C(i)|}{N-1}(1-\zeta^*)\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\end{bmatrix} + m_L\right]$$

$$Max_{I_L} U = V(m_L - I_L) + \kappa\left[(m_H - m_L)\frac{p}{(1-p)}\left(1 - \begin{bmatrix}\left(1 - \frac{|C(i)|}{N-1}(1-\zeta^*)\right)\ddot{m}(\Delta) \\ + \frac{|C(i)|}{N-1}(1-\zeta^*)\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\end{bmatrix}\right) + m_L\right]$$

The first order condition for the high and low income type is respectively:

$$-\frac{\partial V(m_H - I_H)}{\partial c} + \frac{\kappa(m_H - m_L)}{\sigma}\begin{bmatrix}\left(1 - \frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ + \frac{\partial \zeta^*}{\partial \Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{bmatrix} = 0 \tag{20}$$

$$-\frac{\partial V(m_L - I_L)}{\partial c} + \frac{\kappa(m_H - m_L)}{\sigma}\frac{p}{(1-p)}\begin{bmatrix}\left(1 - \frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ + \frac{\partial \zeta^*}{\partial \Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{bmatrix} = 0 \tag{21}$$

This problem needs an adaptation of the second order conditions in condition 9:

**Condition 17** *Let $V(.), \kappa, \sigma^2, \tilde{\sigma}^2_{i,G}, (m_H - m_L), p, \varphi(.)$ and $\tilde{\varphi}(.)$ be such that is holds for $\forall I \in I$ that*

$$\frac{\partial^2 V(m_H - I_H)}{\partial^2 c} + \frac{\kappa(m_H - m_L)}{\sigma^2}\begin{bmatrix}\left(1 - \frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2 \Delta} + \frac{|C(i)|}{N-1}\frac{\partial \zeta^*}{\partial \Delta}\frac{\partial \ddot{m}(\Delta)}{\partial \Delta} \\ + \frac{\partial^2 \zeta^*}{\partial^2 \Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta) - \tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{bmatrix} < 0$$

$$\frac{\partial^2 V(m_L - I_L)}{\partial^2 c} - \frac{\kappa(m_H - m_L)}{\sigma^2}\frac{p}{(1-p)}\left[\begin{array}{c}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2 \ddot{m}(\Delta)}{\partial^2\Delta}+\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right] < 0$$

*Under condition 17, the proof of existence and uniqueness is completely similar to that of proposition 10. For the monotonicity in* $\frac{|C(i)|}{N-1}$*, first take again the partial derivative first order conditions in equations 20 and 21 towards* $\frac{|C(i)|}{N-1}$

**Proof.** $\frac{\partial^2 V(m_H-I_H)}{\partial^2 c}\frac{\partial I_H^*}{\partial\frac{|C(i)|}{N-1}} + \left[\begin{array}{c}\frac{\kappa(m_H-m_L)}{\sigma^2}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$
$\left(\frac{\partial I_H^*}{\partial\frac{|C(i)|}{N-1}}-\frac{\partial I_L^*}{\partial\frac{|C(i)|}{N-1}}\right)+\frac{\kappa(m_H-m_L)}{\sigma}\left[\begin{array}{c}-(1-\zeta^*)\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial\zeta^*}{\partial\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$

and the same for the low income type

$\frac{\partial^2 V(m_H-I_H)}{\partial^2 c}\frac{\partial I_L^*}{\partial\frac{|C(i)|}{N-1}} + \left[\begin{array}{c}\frac{\kappa(m_H-m_L)}{\sigma^2}\frac{p}{(1-p)}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}\\ +\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}+\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$
$\left(\frac{\partial I_H^*}{\partial\frac{|C(i)|}{N-1}}-\frac{\partial I_L^*}{\partial\frac{|C(i)|}{N-1}}\right)+\frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\left[\begin{array}{c}-(1-\zeta^*)\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial\zeta^*}{\partial\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$

Again, this may be written as a system, with

$A_{1,1} = -\frac{\kappa(m_H-m_L)}{\sigma^2}\left[\begin{array}{c}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$

$A_{2,1} = \frac{\partial^2 V(m_L-I_L)}{\partial^2 c}-\frac{\kappa(m_H-m_L)}{\sigma^2}\frac{p}{(1-p)}\left[\begin{array}{c}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$

$A_{1,2} = \frac{\partial^2 V(m_H-I_H)}{\partial^2 c}+\frac{\kappa(m_H-m_L)}{\sigma^2}\left[\begin{array}{c}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$

$A_{2,2} = \frac{\kappa(m_H-m_L)}{\sigma^2}\frac{p}{(1-p)}\left[\begin{array}{c}\left(1-\frac{|C(i)|}{N-1}(1-\zeta^*)\right)\frac{\partial^2\ddot{m}(\Delta)}{\partial^2\Delta}+\frac{|C(i)|}{N-1}\frac{\partial\zeta^*}{\partial\Delta}\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\\ +\frac{\partial^2\zeta^*}{\partial^2\Delta}\frac{|C(i)|}{N-1}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\end{array}\right]$

and $b = \begin{pmatrix}\frac{\kappa(m_H-m_L)}{\sigma}\left[(1-\zeta^*)\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}-\frac{\partial\zeta^*}{\partial\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\right]\\ \frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\left[(1-\zeta^*)\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}-\frac{\partial\zeta^*}{\partial\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\right]\end{pmatrix}$

as before, $|A|<0$ under the second order conditions in condition 17, and

$\begin{vmatrix}b_1 & A_{1,2}\\ b_2 & A_{2,2}\end{vmatrix} = -\frac{\kappa(m_H-m_L)}{\sigma}\frac{p}{(1-p)}\frac{\partial^2 V(m_H-I_H)}{\partial^2 c}\left[(1-\zeta^*)\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}-\frac{\partial\zeta^*}{\partial\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right)\right]$

such that $\frac{\partial I_L^*}{\partial\frac{|C(i)|}{N-1}}$ has the opposite sign of $(1-\zeta^*)\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}-\frac{\partial\zeta^*}{\partial\Delta}\left(\ddot{m}(\Delta)-\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right) = \zeta^*\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}\left(1-\ddot{m}(\Delta)+\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\right) > 0$, where the last equality holds because $\frac{\partial\zeta^*}{\partial\Delta}=\zeta^*\frac{\partial\ddot{m}(\Delta)}{\partial\Delta}$, and the inequality because $\ddot{m}(\Delta),\tilde{m}(\Delta m(\tilde{\sigma}_{i,C(i)}))\in ]0,1[$. Hence $\frac{\partial I_L^*}{\partial\frac{|C(i)|}{N-1}}<0$.

The proof for $\frac{\partial I_L^*}{\partial\frac{|C(i)|}{N-1}}<0$ if fully equivalent to the one above. ■

### A.9 Proof corollary 15

**Proof.** How to compute the average geodesic for a nested star $S_{k,l}$. As before, one proceeds by studying the geodesic distance distribution $\Phi_j(i; S_{k,l})$ for all consumers $i \in N$. Note that all consumers on the same level $n = 1, ..., l$ are in fact identical with respect to the geodesic distance distribution, such that it is sufficient to characterize this distribution for each level $\Phi_j(n; S_{k,l})$, $n = 1, ..., l$. For the centre node, this is easy: the geodesic distribution is $\Phi_j(1; S_{k,l}) = k^j$, and hence the average geodesic distance equals $\tilde{d}(1; S_{k,l}) = \frac{1}{N-1} \sum_{s=1}^{l-1} sk^s$. For lower levels, things get a bit more complex. At level 2, the average geodesic distribution is

$$\tilde{d}(2; S_{k,l}) = \frac{1}{N-1} \sum_{j=1}^{\infty} j\Phi_j(2; S_{k,l}) = \frac{1}{N-1} \left[ \sum_{s=1}^{l-2} sk^s + 1 + (k-1) \sum_{s=0}^{l-2} (s+2) k^s \right],$$

in which the first term represents the downward vertices in the own tree, the second term is the distance to the centre node, and the third term collects the geodesic distances to the nodes on the other trees of the star. At level 3, the geodesic distribution is similar, but with one extra term:

$$\tilde{d}(3; S_{k,l}) = \frac{1}{N-1} \sum_{j=1}^{\infty} j\Phi_j(3; S_{k,l}) = \frac{1}{N-1} \left[ \begin{array}{c} \sum_{s=1}^{l-3} sk^s + \sum_{s=1}^{2} s + (k-1) \sum_{s=0}^{l-3} (s+2) k^s \\ + (k-1) \sum_{s=0}^{l-2} (s+3) k^s \end{array} \right],$$

in which the extra term represents the $k-1$ trees originating from the node at level two which is between the studied node (at level three) and the centre of the star. Also note that the nodes on the other trees from the centre are now one unit further away. For level 4, one proceeds equivalently, adding an extra term for $k-1$ more trees more on the path to the centre, resulting in

$$\tilde{d}(4; S_{k,l}) = \frac{1}{N-1} \left[ \begin{array}{c} \sum_{s=1}^{l-4} sk^s + \sum_{s=1}^{3} s + (k-1) \sum_{s=0}^{l-4} (s+2) k^s + (k-1) \sum_{s=0}^{l-3} (s+3) k^s \\ + (k-1) \sum_{s=0}^{l-2} (s+4) k^s \end{array} \right].$$

Generalizing this, we find that the average geodesic distance of a node $i$ at level $j$ to the other $N-1$ nodes in $S_{k,l}$ can be written as:

$$\tilde{d}(j; S_{k,l}) = \frac{1}{N-1} \left[ \sum_{s=1}^{l-j} sk^s + \sum_{s=1}^{j-1} s + (k-1) \sum_{n=1}^{j-1} \left( \sum_{s=0}^{l-1-n} (s+1+n) k^s \right) \right]$$

in which the first term again concerns the downward tree leaving from $i$, the second term concerns the nodes on the path to the centre node and the third term collects all the trees departing from one of the nodes on this path, the centre node included. It may now easily be seen that the average geodesic distance increases when one moves away from the centre of the star.

More specifically, a simple computation shows that

$$\begin{aligned}\Delta_j \tilde{d}(j; S_{k,l}) &= \tilde{d}(j+1; S_{k,l}) - \tilde{d}(j; S_{k,l}) \\ &= j + (k-1) \sum_{s=0}^{l-2-j} (s+1+j)\, k^s + l k^{l-1-j} + j k^{l-j} > 0\end{aligned}$$

■