Complementary Platforms*

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Abstract

We introduce an analytical framework close to the canonical model of platform competition investigated by Rochet and Tirole (2006) to study pricing decisions in two-sided markets when two or more platforms are needed simultaneously for the successful completion of a transaction. The model developed is a natural extension of the Cournot-Ellet theory of complementary monopoly featuring clear cut asymmetric single- and multihoming patterns across the market. The results indicate that the so-called anticommons problem generalizes to two-sided markets because individual platforms do not take into account the negative pricing externality they exert on the other platforms. As a result, mergers between such platforms may be welfare enhancing, but involve redistribution of surplus from one side of the market to the other. Moreover, the limit of an atomistic allocation of property rights however is not monopoly pricing, indicating that there also exist differences with the received theory of complementarity.

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JEL classification: D43 · D62 · K11 · L13 · L4 · L5

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1 Introduction

Cournot (1838, 1971) was the first to investigate a market structure in which two producers have a monopoly on goods that are complements in the production of a third composite good. The striking conclusion of Cournot's complementary monopoly theory is that welfare in this particular industry decreases with the number of individual producers, a result also known as the *anti*commons problem.[1]

Whereas the problem of the commons stems from inadequately defined property rights,[2] the problem of the anticommons is exactly opposite: the negative externality results from too many individual owners, who in their pricing decision do not take into account their impact on total demand, see Heller (1998) and Buchanan and Yoon (2000).

With this in mind, consider markets where effective communication is composite by nature in that it can only be produced by simultaneously conveying information to different agents. This would be the case if each agent in the market needs to be aware of a particular alternative's existence in order for that alternative to stand a chance of being chosen. To the extent that each agent uses his own channel to acquire this information, these channels are complements.

For example, if a tour operator wants to promote and sell a new destination as the ideal trip for the entire family to spend a holiday, he needs to send this information to all the decision makers in a household, implying the use of different complementary information channels. If the tour operator in some way communicated the exquisite features of this destination to both parents but forgot to inform the adolescent children, there is a distinct possibility that the proposed destination will not be the one withheld by the family as a whole, and hence that it will not be chosen as the next holiday destination.

The recent theory of two-sided markets (see e.g Armstrong, 2006; Parker and Van Alstyne, 2005; Rochet and Tirole, 2003, 2006), approaches information and communication channels as platforms connecting two distinct sides of a market. On one side of the market, there is a group of consumers who wants to get informed. They buy a magazine to find out about their field of interest. This is the magazine's readership and constitutes the "target group" to producers located on the other side of the market. This side of the market wants to "get information across," and does so by buying advertising space in the magazines. As we will deal with applications

[1] Cournot's findings with respect to the pricing of complementary goods by monopolists each providing a component are dual to his results on the quantity decisions taken by oligopolists in the presence of a Walrasian auctioneer, as shown by Sonnenschein (1968).

[2] See Gordon (1954), Scott (1955) and Hardin (1968) for the original contributions on the commons problem and Gibbons (1992) for an illuminating game-theoretic analysis.

other than newspapers and magazines later on, we will refer to the reader and advertiser sides of the market as "receivers" (consumers) and "senders" (producers) respectively.

A prominent research question in the two-sided markets literature addresses agents' single- and multihoming patterns. Much in the spirit of the chicken-and-egg problem that underlies the business model of platforms, one can raise two opposing arguments with respect to the localization of these patterns. The first states that one side of the market singlehomes because of preferences or tastes, and hence that the other side has to consider multihoming, thus explaining why competing platforms are sometimes used simultaneously (See Rochet and Tirole, 2003, Section 3).

Conversely, our paper deals with examples where the platforms are complements by necessity due to technical, biological, cultural or legal reasons,[3] forcing one side (senders) to multihome. As a consequence the receiver side rationally will singlehome. This kind of complementarity therefore constitutes a necessary and sufficient condition for explaining asymmetric single-and multihoming patterns across the market, i.e., why one side of the market singlehomes, together with complete multihoming on the other side. *Since this approach is novel yet quite prominently present in reality, we dedicate effort to illustrate and argument the complementarity of platforms.*

The paper's main contribution however aims to illustrate the *implications* of platform complementarity on platform pricing structures. A number of interesting research questions arise: is complementarity beneficial to the sender (multihoming) or receiver (singlehoming) side? What about mergers between complementary platforms? (Can Cournot's results be extended to two-sided markets?) Does extreme fragmentation of property rights induce monopoly outcomes in the presence of complementary platforms?

As such, the model developed in this paper lies at the crossroads of two important strands in the economic literature, borrowing elements and combining insights from (*i*) the two-sided markets literature (see e.g Armstrong, 2006; Parker and Van Alstyne, 2005; Rochet and Tirole, 2003, 2006), and (*ii*) the theory on complementary goods (see e.g. Cournot, 1838, 1971; Ellet, 1839, 1966; Economides and Salop, 1992; Gaudet and Salant, 1992; Feinberg and Kamien, 2001).

Many modern network industries feature both complementarity and two-sidedness, and as such three studies are related to our analysis. First, Carrillo and Tan

[3]Another source of complementarity could be political: if the decision-making process requires unanimity, all voters have to be convinced, for example all family members in choosing the holiday destination in the tour operator's example.

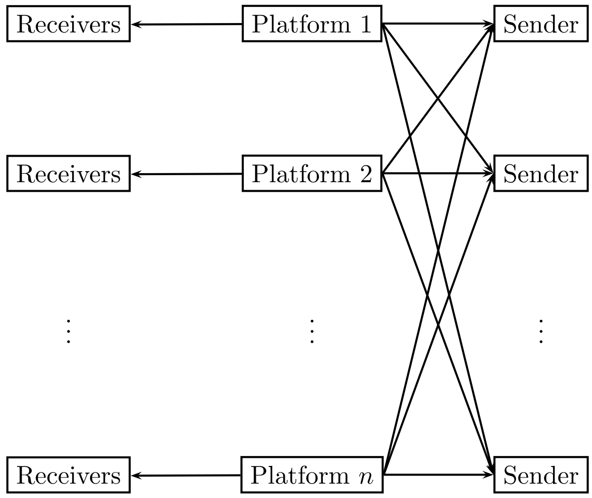
(2006) study consumers' single- or multihoming decisions in a setting where third parties offer goods and services that are complementary to the ones provided by two competing horizontally differentiated platforms. Whereas their focus lies on the impact of platform differentiation and the number of complementors on platform pricing structures, our paper—while simultaneously providing an explanation for asymmetric "homing" patterns—stresses the impact of *platform complementarity* on the pricing structure. We do so by comparing ensuing prices and profits under different platform ownership structures, taking our cue from the theory on complementary goods.

Second, and related to Carrillo and Tan (2006), Economides and Katsamakas (2006) tackle the same issue of the optimal two-sided pricing strategy but from the point of view of proprietary versus open source platforms. Our paper shares their framework of analysis in the presence of different industry structures. However, while these authors consider vertical integration between platforms and complements, our model emphasizes horizontal integration between complementary platforms.

Still another perspective is taken by Doganoglu and Wright (2006), studying the influence of consumer multihoming on compatibility decisions by firms. At the heart of their analysis lies the observation that although compatibility between firms increases consumers' network benefits, these can also be obtained when consumers choose to multihome should firms decide to remain incompatible. In our model, platform complementarity assures that singlehoming consumers (receivers) fully realize cross-market network benefits.

To shed light on the aforementioned issues, this paper is structured as follows: in Section 2 we further elaborate on the complementarity of platforms by examining a number of mini case studies that have been chosen because of policy relevance, as well as their illustrative nature with respect to the complementarity of platforms. Section 3 introduces the model we use to investigate pricing decisions by complementary platforms in two-sided markets and presents the basic results. Among other things, we show that the problem of the anticommons extends to two-sided markets, but not in a symmetric way. Reducing the number of independent platforms increases social welfare (with redistribution of wealth), but increasing the number of players does not destroy all sender surplus by convergence to the monopoly price. Section 4 generalizes the setting to include the analysis of pricing behavior by bundles composed of one- and two-sided components. This encompasses as special cases both Cournot's initial approach and the present model. Section 5 concludes.

Figure 1: Generalized industry configuration



2 On the Complementarity of Platforms

In this section we document three cases in which the prevailing business models of the industry are well captured by a model of complementary platforms. The discussion always proceeds along the same lines: first we provide some stylized facts that are relevant to the industry under consideration. Next we indicate why its two-sided nature is important and what explains for the complementarity of the providers. Given the latter, it follows that one side of the market (fully) multihomes, whereas for the other side it then becomes rational to singlehome.

As an example, consider Figure 1 depicting the generalized industry configuration where a range of complementary platforms serve two distinct sides of the same market, *senders* and *receivers* respectively.[4] In particular, note that while each platform serves its own segment of receivers (singlehoming), it simultaneously serves many senders who are forced to multihome in order to reach cross-market agents effectively. As a result, for a single transaction senders pay the sum of prices of all platforms present in the market, as opposed to the receivers who pay a single fee to the platform they exclusively patronize.

[4] In Figure 1 the arrows emanate from the price-setting entity.

2.1 Financial and Legal Advertising in a Multilinguistic Country

On December 20, 2005 the Belgian Antitrust Authority approved of a merger between the only two remaining financial newspapers in the country, however not without conducting lengthy further investigations and imposing restrictions.[5] The results of the present paper show why the merger was welfare enhancing and thus should not have been delayed. Moreover, the conditions imposed hardly made sense given the complementary nature of advertising in this particular market.

Following up on European Commission practice, see Recoletos/Unidesa and Gruner and Jahr/Financial Times/JV,[6] the Belgian Antitrust Authority partitioned the market for advertising in three distinct submarkets: (1) the market for thematic advertising, (2) the market for legal and financial advertising, and finally (3) the market for job advertisements, see Van Cayseele (2006). Especially the second market is important for the particular merger that was proposed since it involved the Dutch language financial newspaper "De Tijd" and the French language financial newspaper "L'Echo."

Reflecting both historical and cultural differences between the two major communities constituting Belgium, "De Tijd" and "L'Echo" respectively cater for readers in the Dutch-speaking part of the country, Flanders, situated in the North, and their French-speaking counterparts in Wallonia, situated in the South. In the market for legal and financial advertising, each paper connects investors from a specific linguistic regime with companies that want to convey "information," e.g., an announcement for the general assembly to be held in the near future.[7] As such, this particular market is two-sided and newspapers act as platforms connecting cross-market agents. *To protect investors' interests, companies situated in Belgium—irrespective of their regional origin—are required by law to publish their information in the different languages, Dutch and French.*

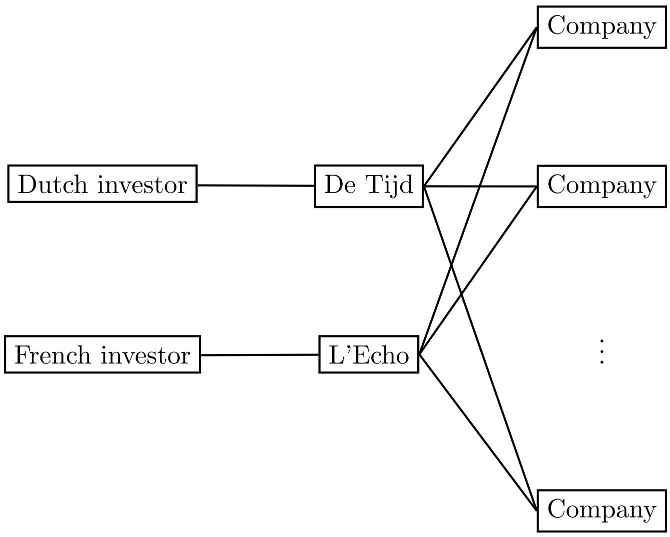
Thus, in order to reach Dutch-speaking investors, companies place their announcements in "De Tijd" and simultaneously buy advertisement space in "L'Echo" to interact with French-speaking investors. From the point of view of the companies both newspapers are necessary (and thus complementary) inputs into the provision

[5] See decision 2005–C/C–56 on cases CONC–C/C–03/050, N.V. Rossel & Cie/N.V. De Persgroep/N.V. Editeco, and MEDE–C/C–05/0068, N.V. Uitgeversbedrijf Tijd/N.V. Editeco.

[6] See respectively the European Commission decision of 1 February 1999 on case IV-M.1041, Recoletos/Unidesa, Pb. 17 March 1999, C 73/06, and the decision of 20 April 1999 on case IV-M.1455, Gruner and Jahr/Financial Times/JV, Pb. 31 August 1999, C 247/05.

[7] Related events are extra-ordinary meetings of the general assembly, with topics on the agenda such as stock splits, raising capital, ...

Figure 2: Belgian financial newspapers

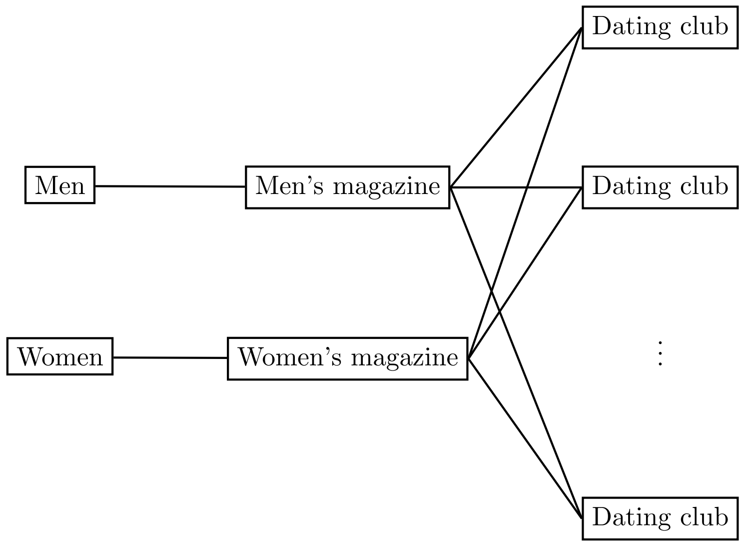


of corporate information, forcing companies to multihome. Investors on the other hand singlehome; they buy a single financial newspaper and through its complementary nature, consequently stay informed on all companies' activities. Figure 2 provides a schematic overview of this particular industry setup.

The results presented in Section 3 indicate that the merger between "De Tijd" and "L'Echo" actually increases welfare as measured by lower total prices and higher industry profits. Hence, the condition imposed by the Belgian Antitrust Authority so as to remedy the alleged negative consequences of the proposed merger did not make sense as it prohibited discounts for joint advertising in "De Tijd" and other newspapers belonging to the merged group, such as "L'Echo." This is particularly the case because financial and legal advertising was explicitly mentioned to be precluded from discounts for a combined advertisement. Moreover, we show that prices on the receiver side are likely to increase post-merger, but often what readers pay is subject to a price cap.

The complementary nature of newspaper and magazine advertising may well reach far beyond the example of legal and financial messages in a multilinguistic country. Besides the example of the tour operator who wants to sell a destination to a family with unanimity voting, we may have a look at advertising by platforms themselves. Considered separately, heterosexual dating clubs are platforms that connect the two distinct sides of the market they operate in, namely single men and single women. However, to advertise their activities to potential customers they rely on the services of other platforms embodied by magazines that specifically cater for

Figure 3: Gender-biased magazines and heterosexual dating clubs



the preferences of heterosexual men and women.

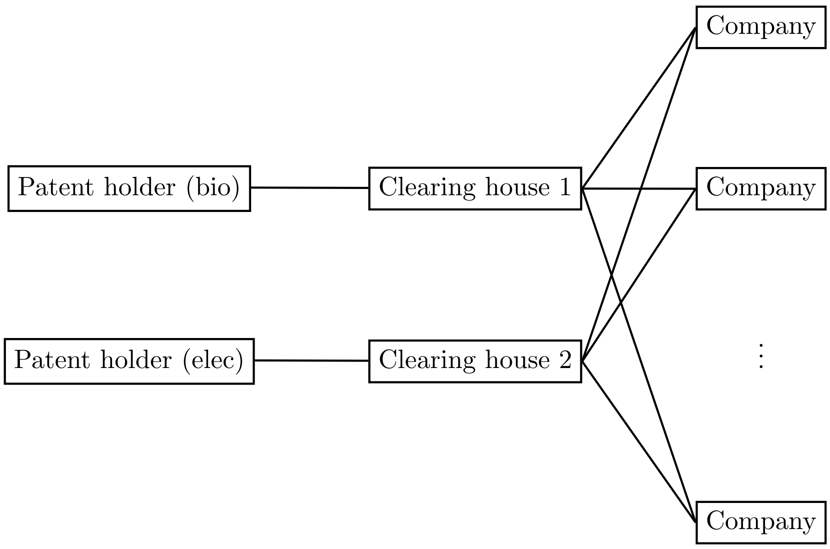
As such, these constitute two separate information channels through which dating clubs can reach agents on both sides of the market, see Figure 3. So, to the extent that heterosexual single men are inclined to only read gender-biased magazines (and similarly single women focus on one magazine), both sexes singlehome. These gender-biased magazines then are complements whose advertising services will need to be consumed as a bundle by dating clubs. As in the previous example, the demand for platform services by dating clubs will be governed by the sum of fees (total fee) charged by the platforms under consideration.

Compared to legal and financial advertising in a multilinguistic country, the segmentation of the groups in this case is not cultural (by language) but biological (by sex), and, the cause of complementarity is not the law, but taste (heterosexual preferences). The effect however is exactly the same: the advertisement services offered by the gender-biased magazines are tied together in the same way the legal and financial advertising opportunities offered by the financial newspapers are linked: as complements.

2.2 Clearing Houses, Patent Pools, and Technology Licensing

Clearing houses match technology suppliers with potential users. They can have a very general approach providing a marketplace for a variety of technologies. Or they

Figure 4: Patent pools and clearing houses



can be specialized, well aware of the potential of a certain technology, and actively searching for potential licensees, providing an array of supporting services.[8]

In the present context, we focus on the case of specialized clearing houses trying to match the patentholders they represent with a variety of licensees. From their specialized knowledge of the technology underlying the patent they will actively seek for applications. As a result of these activities they collect payments from both sides of the market, and hence can be modeled as platforms. Figure 4 provides a configuration of the economic relationship between patentholder and licensee meeting over a clearing house.

Innovations nowadays build on a variety of patented inventions. The result is that patents are complements and the patentholders, when negotiating royalties, again generate a negative pricing externality upon each other. Shapiro (2001) showed in a (one-sided) Cournot (complementary monopoly) model that patent pools, combining the ownership of the patents involved in the innovations, increase welfare. Subsequent research contributions by Lerner and Tirole (2004) have relaxed the

[8]An example of the former could be the internet marketplace yet2.com, an example of the latter pharmalicensing.com. Interestingly, yet2.com's revenue structure is detailed on its website, see `http://www.yet2.com/app/about/usingsite`: reflecting its two-sided nature revenue is generated from two activities, (1) searching for technology, a basic search tool which in an Adobe Reader-style is free but can be upgraded at a cost, and (2) selling technology, where costs depend on the type of membership, e.g., from individuals, selling one technology at a time, to unlimited annual listing memberships. They also charge a commission (with a minimum of $10.000) on every technology transfer arrangement facilitated by its services.

complementary feature of patents to allow patents to become substitutes as the price of technology (license fees to be paid) for the innovations increases to the level where "dropping" a patent from the bundle comes into consideration.

The model presented in this paper re-assumes perfect complementarity, but in a two-sided context.[9] The analysis shows that clearing houses, when allowed to act as a pool, increase their profits and surplus to the end-users. This result thus shows explicitly that pools facilitate the dissemination of inventions, yet at a price to the patentholders. The net effect of patent pools on the incentive to innovate therefore is ambiguous: on the one hand the patent pool increases the number of end-users and hence the amount of royalties paid, but on the other hand the patentees pay a higher fee to the platform.

The patent pool problem shows that besides cultural or biological segmentation, also technological specialization can be a source of segmentation. Technological clearing houses specialize in certain technologies (biomedical, electronic, ...) and a patent-holding innovator with a specific technology will offer his technology to the market over that platform. Or there is specialization on the supply side of technology, but on the demand side users need many complementary inventions managed by several clearing houses, which therefore are complementary platforms.

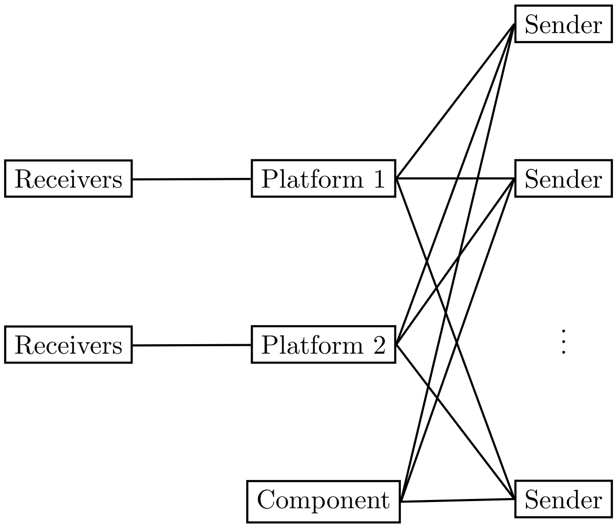
2.3 Urban Location and Conglomeration

Another area of research where complementarity was stressed is the economics of urban location, shopping malls and supermarkets, see Stahl (1987) and Klemperer (1992). In this literature it is well recognized that the presence of one retail shop may attract another, illustrating the complementarity between e.g. a grocery store, a fast-food outlet and a pharmacy. It is also well-known that the higher individual shop prices, the less attractive the overall shopping area becomes. Even supermarkets in a multi-stop shopping context face such pricing externality, see Manachotphong and Smith (2007).

At the same time the externality across market sides has been noted as well, and especially shopping malls are seen to be platforms that connect retailers with consumers. As it is often the case in two-sided markets, one side of the market

[9] Undoubtedly, many examples exist in the context of combining several patented technologies into one innovation, getting a new product on the market. A particular one which fits the present model well involves ultra high-speed cameras and ultra slow-motion image reproduction, see for example i-movix' *SprintCam* which combines internally developed server technology with external Photron cameras. Over 60% of the contracts the provider of the "recording technology" is involved in, is in combination with the ultra-slow reproduction technology provider. Both activities however involve different technologies, covered by different patents. But the market for sports television needs both, regardless whether it covers soccer, cycling, tennis, golf, ...

Figure 5: Bundle with complementary one- and two-sided components



is a loss leader and needs to be subsidized by the other side of the market, which generates profits. For example, in shopping malls consumers get parking space for free.

Nothing however prevents other providers of complementary services trying to become part of the new "agglomeration." The free parking space provided by the shopping mall might convince car drivers to travel somewhat longer, which in turn might attract a gas station to provide fuel. Typically, a gas station (or bus service) is a one-sided component that links to the bundle, and this raises the question how the price structure of the two-sided platform is affected by the inclusion of a one-sided component. Figure 5 shows the industry structure when this happens.

Shopping malls certainly are not an isolated example of the presence of a one-sided component in the bundle. Bundles can be made up by several one- and two-sided components. As an example of the latter, consider large-scale amusement parks and modern theme parks such as Disneyland. These resorts try to lure tourists by offering them entertainment to be found nowhere else, ranging from adrenaline soaked roller coaster rides to thematic shows, movies and performances. Connecting tourists with performers and production houses (or even suppliers/manufacturers of attractions) amusement parks can be considered as platforms in the market for family-oriented entertainment.

Next, consider the typical on site McDonald's at this very same Disneyland resort. It connects hungry tourists with a specific desire to eat at McDonald's with producers of fastfood menu ingredients (such as hamburgers, buns, ketchup, fries and soft drinks), all vying to secure contracts so as to become exclusive suppliers to that fastfood chain. Together, Disneyland Resort and McDonald's are complements to these visitors' theme park experience as a whole. The fact that the two platforms mentioned here—Euro Disney and McDonald's—operate global purchasing centers to deal with suppliers—which, once selected, will supply to the entire chain of outlets of all players—serves to strengthen the complementary nature of this particular industry.

Finally, consider the airline or railroad companies bringing tourists to Disneyland. These are typically one-sided operators, but are as complementary as any of the other two components. The current example can even be extended to include hotel chains such as the Sheraton which can be added to the bundle should tourists plan a prolonged stay at the amusement park.

As in the previous cases (see Subsections 2.1 and 2.2), some factors explain for the segmentation of one side of the market while others for complementarity at the other. Here it is again specialization in production together with complementarity in consumption that entails the industry configuration shown in Figure 1. While complementarity beyond any doubt is less than perfect in the present case (Disneyland visitors do not need to buy a hamburger on site or stay in a hotel while they can drive their own car to get there), the implications for the pricing structure merit close attention from the perspective of zoning laws. Often, these will confine economic activity to the area of the shopping mall, limiting the number of complements that are bundled at the same location.

3 The Model

Assume a market with platforms $i = 1, \ldots, n$ exclusively providing their services to two distinct sides of the market, referred to as senders and receivers. Given the discussion in Section 2 and without loss of generality, assume that the receiver side of the market singlehomes, whereas the sender side multihomes. As such, senders rely on the services of all n platforms; receivers on the other hand only need a single platform to successfully complete their transactions. We will refer to the characteristic of receiver singlehoming and sender multihoming as the *complementarity assumption* (CA).

Following the work by Rochet and Tirole (2003, 2006) and Bolt and Tieman (2006), receivers are heterogeneous in the gross benefit $b_i^R \in \left[0, \bar{b}_i^R\right]$ they receive from a transaction mediated by a specific platform i, with benefits distributed according to a probability density function g_i^R and corresponding cumulative distribution function

G_i^R, and where $0 < \bar{b}_i^R \leq \infty$. Similarly, senders are heterogeneous in benefits $b^S \in [0, \bar{b}^S]$ with probability density function g_S and cumulative distribution function G_S, and $0 < \bar{b}^S \leq \infty$.

3.1 Complementary Platforms

We additionally state the following conditions:

(C1) "Effective" interaction between both sides of the market occurs between a singleton sender and an n-tuple of receivers;

(C2) Receiver segments served by each platform are of equal size;

(C3) The receiver side singlehomes.

The complementarity assumption (CA) and condition C1 merely serve to characterize this particular industry setup and constitute the "Maintained Assumption." C1 is relaxed by focussing on variable production ratio's, see Subsection 3.3 below. Conditions C2 and C3 are additional assumptions made out of convenience: C2 substantially facilitates calculations, and C3 is a possible explanation for each platform's local monopoly over the receivers it serves (*perfect segmentation*). C3 states that a receiver makes a single discrete choice, which is perfectly rational in the present setting as argued above.

The complementarity assumption (CA) induces full multihoming on the sender side of the market: senders require the provision of services by all n platforms, entailing a total fee $A \equiv \sum_i p_i^S$, where $p_i^S \geq 0$ denotes the sender fee charged by platform i. As such, sender quasi-demand for the bundle of platform services becomes a function of the total fee charged and is defined as

$$D^S : \mathbb{R}_+ \to [0,1] : A \mapsto D^S(A) = D^S\left(p_1^S + p_2^S + \cdots + p_n^S\right), \tag{1}$$

with $D^S(A) = \text{Prob}\left\{b^S \geq A\right\} = 1 - G_S(A)$. Obviously, demand is zero for all $A \geq \bar{b}^S$. It follows from the definition of g_S that sender quasi-demand is a decreasing function of the bundle price A:

$$\frac{\mathrm{d}}{\mathrm{d}A}\left[D^S(A)\right] = \frac{\mathrm{d}}{\mathrm{d}A}\left[1 - G_S(A)\right] = -g_S(A) < 0.$$

Zero conjectural variations (Bowley, 1924), i.e.

$$\frac{\partial A}{\partial p_i^S} = \frac{\partial}{\partial p_i^S}\left(\sum_{k=1}^{n} p_k^S\right) = \begin{cases} 0 & \forall\, k \neq i \\ 1 & \text{if } k = i, \end{cases} \tag{2}$$

also imply that sender quasi-demand is decreasing in the individual platform sender prices:

$$\frac{\partial}{\partial p_i^S}\left[D^S(A)\right] = -\frac{\partial}{\partial p_i^S}\left[G_S(A)\right] = -\frac{\partial G_S(A)}{\partial A}\frac{\partial A}{\partial p_i^S} = -g_S(A) < 0.$$

To guarantee the existence and uniqueness of an optimum we additionally impose log-concavity on sender quasi-demand, or $\partial^2\left[\ln D^S(A)\right]/\partial\left(p_i^S\right)^2 < 0$.

As can be seen from (1), sender quasi-demand D^S is the same for all platforms due to the complementarity assumption. Also, from the point of view of the senders, an implicit assumption is that the successful completion of a transaction requires the platforms to be combined in the bundle in a $1:1$ ratio, i.e., each platform is only needed once in the interaction with the receivers on the other side of the market.

At first sight the multihoming characteristic of the sender side of the market seems to have important consequences for the quasi-demand structure on the receiver side as demand for platform i's services becomes a function of all the platforms' prices charged to receivers (see Rochet and Tirole, 2003). In this case, let $\mathbf{p}^R \in \mathbb{R}_+^n$ be the vector of receiver prices. Then the fraction of receivers choosing platform i when all senders are affiliated with platform i is given by

$$d_i^R : \mathbb{R}_+^n \to [0,1] : \mathbf{p}^R \mapsto d_i^R\left(\mathbf{p}^R\right),$$

where $d_i^R\left(\mathbf{p}^R\right) = \text{Prob}\left\{b_i^R - p_i^R > \max\left(0, b_k^R - p_k^R\right)\forall k \neq i\right\}$.[10] As receivers only need acces to a single platform to complete a transaction with any of the senders on the other side of the market, it is plausible to assume that they will singlehome.

The complementarity assumption however induces perfect segmentation on the receiver side of the market to the extent that each platform exclusively serves its own segment. In fact, complementarity and perfect segmentation are two sides of the same coin, as illustrated extensively in Section 2. As a consequence, receiver quasi-demand is a function of the own price only and is defined as

$$D_i^R : \mathbb{R}_+ \to [0,1] : p_i^R \mapsto D_i^R\left(p_i^R\right), \tag{3}$$

where $D_i^R\left(p_i^R\right) = \text{Prob}\left\{b_i^R \geq p_i^R\right\} = 1 - G_R\left(p_i^R\right)$. Similar to sender quasi-demand, we require D_i^R to be decreasing, $\partial D_i^R\left(p_i^R\right)/\partial p_i^R < 0$, and log-concave in prices, $\partial^2\left[\ln D_i^R\left(p_i^R\right)\right]/\partial\left(p_i^R\right)^2 < 0$.

[10] Note that this is equivalent to a discrete choice model where a receiver chooses the platform that maximizes utility, see e.g. Anderson and Gabszewicz (2005).

Common to the literature on standard two-sided markets, the utility a receiver derives from the services delivered by platform i is increasing in the number of cross-market participants, i.e. the senders. It is equal to the net benefit from a transaction, $b_i^R - p_i^R$, times the number of senders, which is the same as sender quasi-demand $D^S(A)$, or

$$\left(b_i^R - p_i^R\right) D^S(A).$$

While still increasing in the number of receivers $D_i^R\left(p_i^R\right)$, the complementary feature of the market entails an expression for the utility that senders derive from the services delivered by the bundle of platforms that is quite different from the standard case: it is equal to the net benefit from a transaction, $b^S - A$, times the number of receivers, summed across the n platforms, or

$$\left(b^S - A\right) \sum_{i=1}^{n} D_i^R\left(p_i^R\right).$$

Given (1) and (3), and assuming independence between sender and receiver benefits, platform i's expected transaction demand D is the product of receiver and sender quasi-demand:[11]

$$D : \mathbb{R}_+^2 \to [0,1] : \left(p_i^R, A\right) \mapsto D\left(p_i^R, A\right) = D_i^R\left(p_i^R\right) \cdot D^S(A).$$

Assuming for simplicity that platforms incur a constant marginal cost $c = 0$ per transaction, each platform i's optimization problem then becomes

$$\max_{p_i^R, p_i^S} \pi_i = \left(p_i^R + p_i^S\right) D_i^R\left(p_i^R\right) D^S(A). \tag{4}$$

Additionally, assume that it is costless to produce the composite good, i.e., the bundle of platform services. Imposing log-concavity on receiver and sender quasi-demand ensures that the first-order conditions for program (4) are both necessary and sufficient for a maximum: as log-concavity is preserved under multiplication and positive scaling (see Boyd and Vandenberghe, 2004, pp. 105–106), π_i becomes log-concave and its unique maximum is found by differentiating with respect to receiver and sender prices. For ease of notation, let subscript I henceforth denote actions taken by independent platforms, and let $\phi \equiv \bar{b}^R \cdot \bar{b}^S$. It now becomes possible to prove the following proposition:

[11] Exogenously fixing the number of potential transactions in the market at N, platform i's total expected transaction demand is $N \cdot D_i^R\left(p_i^R\right) D^S(A)$. For simplicity we normalize N to 1.

Proposition 1 (Independent Complementary Platforms)**.** *Independent symmetric complementary platforms charge*

$$p_I^R = \frac{(n+1)\bar{b}^R - \bar{b}^S}{2n+1} \tag{5}$$

$$p_I^S = \frac{2\bar{b}^S - \bar{b}^R}{2n+1} \tag{6}$$

and make profits equal to

$$\pi_I = \frac{\left(n\bar{b}^R + \bar{b}^S\right)^3}{\phi(2n+1)^3}. \tag{7}$$

The price level, the bundle price and industry profits respectively equal

$$p_I \equiv p_I^R + p_I^S = \frac{n\bar{b}^R + \bar{b}^S}{2n+1} \tag{8}$$

$$A_I = np_I^S = \frac{n\left(2\bar{b}^S - \bar{b}^R\right)}{2n+1} \tag{9}$$

$$\Pi_I = n\pi_I = \frac{n\left(n\bar{b}^R + \bar{b}^S\right)^3}{\phi(2n+1)^3}. \tag{10}$$

Proof. The first-order conditions for profit maximization are

$$\frac{\partial \pi_i}{\partial p_i^R} = D^S(A)\left\{D_i^R\left(p_i^R\right) + \left(p_i^R + p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]'\right\} = 0 \tag{11}$$

$$\frac{\partial \pi_i}{\partial p_i^S} = D_i^R\left(p_i^R\right)\left\{D^S(A) + \left(p_i^R + p_i^S\right)\left[D^S(A)\right]'\frac{\partial A}{\partial p_i^S}\right\} = 0. \tag{12}$$

Zero conjectural variations (2) imply that we obtain a system of $2n$ equations in $2n$ unknowns which implicitly define the optimal sender and receiver prices:

$$p_i^R + p_i^S = -\frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'} \tag{13}$$

$$p_i^R + p_i^S = -\frac{D^S(A)}{[D^S(A)]'}. \tag{14}$$

Combining (13) and (14) we obtain

$$-\frac{D_i^R(p_i^R)}{[D_i^R(p_i^R)]'} = -\frac{D^S(A)}{[D^S(A)]'}, \tag{15}$$

which replicates the result of Rochet and Tirole (2003): when platforms set prices

p_i^R and p_i^S to maximize volume for a given total price $p_i = p_i^R + p_i^S$, the volume impact of a small variation in prices has to be the same on both sides, keeping in mind that here the volume impact on the sender side is triggered by a change in the total sender fee $A = \sum_i p_i^S$.

Summing equations (13) and (14) over i we obtain

$$\sum_i \left(p_i^R + p_i^S\right) = -\sum_i \frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'}$$
$$\sum_i \left(p_i^R + p_i^S\right) = -\frac{nD^S(A)}{\left[D^S(A)\right]'}.$$

Then, under perfect segmentation and assuming equal supports of the distribution of receiver benefits, meaning $\underline{b}_i^R = \underline{b}^R$ and $\bar{b}_i^R = \bar{b}^R \,\forall\, i$, we obtain in a symmetric equilibrium where $p_i^S = p_I^S$ and $p_i^R = p_I^R \,\forall\, i$ (such that receiver quasi-demand $D_i^R\left(p_I^R\right) \equiv D^R\left(p_I^R\right)$ is symmetric), the system of best-response functions above can be simplified and written in matrix notation as

$$\begin{bmatrix} 2 & 1 \\ 1 & n+1 \end{bmatrix} \begin{bmatrix} p_I^R \\ p_I^S \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \end{bmatrix}.$$

Notice we have used a uniform distribution of receiver and sender benefits to obtain a closed form for equations (13) and (14).[12] Applying Cramer's rule, $p_I^k = \frac{|A_k|}{|A|}$ for $k = R, S$, yields the desired results. ■

For $n = 2$, competition is characterized by a duopoly. The industry configuration for two complementary platforms under Bertrand price competition is shown in Figure 6.

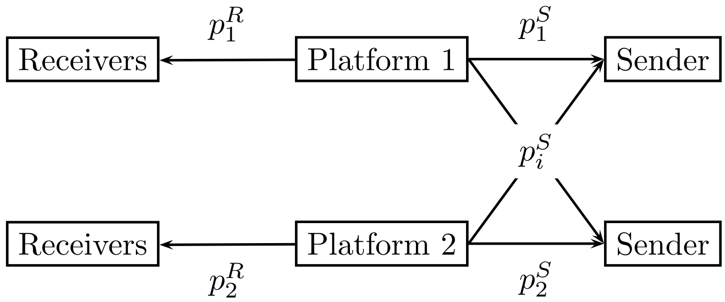
Corollary 1 (Duopoly)**.** *If $n = 2$, platforms charge*

$$p_I^R = \frac{3\bar{b}^R - \bar{b}^S}{5} \tag{16}$$

$$p_I^S = \frac{2\bar{b}^S - \bar{b}^R}{5} \tag{17}$$

[12] Log-concave quasi-demand functions are easily obtained from a uniform distribution of benefits with, respectively, probability density and cumulative distribution function $g_k(x) = 1/\bar{b}^k$ and $G_k(x) = x/\bar{b}^k$ for $k = R, S$. Quasi-demand then follows from the definition $D^k(x) := \text{Prob}\{b^k \geq x\} = 1 - G_k(x) = (\bar{b}^k - x)/\bar{b}^k$ and is decreasing and log-concave in its argument: $\mathrm{d}D^k(x)/\mathrm{d}x = -1/\bar{b}^k < 0$ and $\mathrm{d}^2[\ln D^k(x)]/\mathrm{d}x^2 = -1/(\bar{b}^k - x)^2 < 0$. As such, the quasi-demand function in models of two-sided markets is the equivalent of the so-called *reliability* or *survival* function $\bar{G}(\cdot) = 1 - G(\cdot)$ commonly used in reliability theory, see e.g. Bagnoli and Bergstrom (2005, Section 3). Additionally, the use of uniform distributions avoids corner solutions arising from skewed pricing distributions, see e.g. Bolt and Tieman (2004, 2006).

Figure 6: Industry configuration for $n = 2$ (Bertrand pricing)



and profits equal

$$\pi_I = \frac{\left(2\bar{b}^R + \bar{b}^S\right)^3}{125\phi}. \tag{18}$$

A first important result—from a welfare point of view—is that, unlike the Cournot-Ellet complementary monopoly theory (see Corollary 4), the bundle price as set by independent complementary platforms does not approach the senders' "choke" level in the limit as the number of components (platforms) approaches infinity:

Corollary 2 (Bundle Limit Price in Two-Sided Markets)**.** *As the number of platforms grows to infinity, the bundle price does not attain the upper bound imposed by the sender choke level $\bar{b}^S$.*

Proof. By taking the limit of the bundle price as $n \to \infty$, we have that $\lim_{n\to\infty} A_I = \lim_{n\to\infty} np_I^S = \lim_{n\to\infty} \frac{n(2\bar{b}^S - \bar{b}^R)}{2n+1} = \frac{2\bar{b}^S - \bar{b}^R}{2}$, where the last equality follows from de l'Hôpital's rule. Because $\bar{b}^S - \frac{1}{2}\bar{b}^R \leq \bar{b}^S$, this limit value is smaller than the sender choke level. ■

The presence of the receiver side thus acts as a counterweight that limits the upward pressure on the bundle price exerted by an increasing number of components (platforms). The reason that the choke level is never reached in two-sided markets is that it is not beneficial for platforms to do so: pushing the price on one side to its choke level would effectively kill off all quasi-demand on that side and hence profits given the "multiplicative" nature of revenues (and profit function).

3.2 Complementary Platforms: Joint Ownership

Suppose now that the platforms are owned by a single entity which sets the price of the *bundle*, $A \equiv \sum_i p_i^S$, on the sender side and the receiver prices, p_i^R, on the other side so as to maximize the following additively separable profit function:

$$\Pi \equiv \sum_{i=1}^{n} \pi_i = \sum_{i=1}^{n} \left(p_i^R + p_i^S\right) D_i^R\left(p_i^R\right) D^S(A).$$

With subscript J referring to the actions taken by the joint entity, we can now state the following:

Proposition 2 (Complementary Platforms: Joint Ownership)**.** *Under joint ownership symmetric complementary platforms charge*

$$p_J^R = \frac{2n\bar{b}^R - \bar{b}^S}{3n} \tag{19}$$

$$p_J^S = \frac{2\bar{b}^S - n\bar{b}^R}{3n} \tag{20}$$

and make profits equal to

$$\pi_J = \frac{\left(n\bar{b}^R + \bar{b}^S\right)^3}{27n^2\phi}. \tag{21}$$

The price level, the bundle price and industry profits respectively equal

$$p_J \equiv p_J^R + p_J^S = \frac{n\bar{b}^R + \bar{b}^S}{3n} \tag{22}$$

$$A_J = np_J^S = \frac{n\left(2\bar{b}^S - n\bar{b}^R\right)}{3n} \tag{23}$$

$$\Pi_J = n\pi_J = \frac{\left(n\bar{b}^R + \bar{b}^S\right)^3}{27n\phi}. \tag{24}$$

Proof. The first-order conditions with respect to receiver prices are identical to the ones obtained under independent platforms, embodied by equation (13):

$$\frac{\partial \Pi}{\partial p_i^R} = D^S(A)\left\{D_i^R\left(p_i^R\right) + \left(p_i^R + p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]'\right\} = 0.$$

This yields the familiar expression

$$p_i^R + p_i^S = -\frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'}.$$

Summing over i we obtain

$$\sum_i \left(p_i^R + p_i^S\right) = -\sum_i \frac{D_i^R\left(p_i^R\right)}{\left[D_i^R\left(p_i^R\right)\right]'}. \tag{25}$$

Taking the first derivative with respect to sender prices we obtain

$$\begin{aligned}\frac{\partial \Pi}{\partial p_i^S} =& D_i^R\left(p_i^R\right)\left\{D^S(A) + \left(p_i^R + p_i^S\right)\left[D^S(A)\right]'\right\} \\ &+ \left[D^S(A)\right]' \sum_{k \neq i} \left(p_k^R + p_k^S\right) D_k^R\left(p_k^R\right) = 0.\end{aligned} \tag{26}$$

Summing over i and grouping common elements yields

$$D^S(A)\sum_i D_i^R\left(p_i^R\right) + n\left[D^S(A)\right]'\sum_i \left(p_i^R + p_i^S\right) D_i^R\left(p_i^R\right) = 0,$$

or

$$\sum_i \left(p_i^R + p_i^S\right) D_i^R\left(p_i^R\right) = -\frac{D^S(A)}{n\left[D^S(A)\right]'}\sum_i D_i^R\left(p_i^R\right). \tag{27}$$

In a symmetric equilibrium equations (25) and (27) can again be simplified and written as

$$\begin{bmatrix} 2 & 1 \\ n & 2n \end{bmatrix}\begin{bmatrix} p_J^R \\ p_J^S \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \end{bmatrix}.$$

Applying Cramer's rule, $p_J^k = \frac{|A_k|}{|A|}$ for $k = R, S$, yields the desired results. ∎

As can be seen from equation (26), the major difference with the results under independent platforms is the presence of the term $\left[D^S(A)\right]' \sum_{k \neq i}\left(p_k^R + p_k^S\right) D_k^R\left(p_k^R\right)$ in the first-order condition with respect to the individual sender prices. It is exactly this which allows us to state the following proposition, extending the anticommons problem to two-sided markets:

Proposition 3 (The Anticommons Problem in Two-Sided Markets)**.** *Compared with independent complementary platforms, under joint ownership platforms set receiver and sender prices such that*

(i) the price level is lower:

$$p_J < p_I, \tag{28}$$

(ii) the price structure is beneficial to senders and detrimental to receivers:

$$p_J^S < p_I^S \tag{29}$$

$$A_J < A_I \tag{30}$$

$$p_J^R > p_I^R, \tag{31}$$

(iii) platform and industry profits are higher:

$$\pi_J > \pi_I \tag{32}$$

$$\Pi_J > \Pi_I. \tag{33}$$

Proof. By comparison of the results in Propositions 1 and 2. ∎

As in the classic anticommons result, independent platforms charge too high a sender price, exerting a negative pricing externality on the other platforms. As a result, sender quasi-demand for the bundle of platform services decreases. Being a two-sided market, this increase in sender pricers is offset by a decrease in receiver prices. This decrease however fails to compensate for the losses incurred on the sender side, causing individual and industry profits to decrease.

Similar to a multi-product monopoly (see Tirole, 1988, pp. 69–72), complementary platforms under joint ownership internalize negative pricing externalities, charging lower sender prices so as to decrease the bundle price [see equation (30)], thereby increasing sender quasi-demand. The two-sidedness of the market is mirrored however by higher receiver prices, as indicated by the price structure [see equations (29) and (31)]. Contrary to the previous situation, the gains on the sender side now outweigh the losses on the receiver side.[13] This result resembles the *topsy-turvy* principle of platform pricing in standard two-sided markets (Rochet and Tirole, 2006): an exogenous factor that leads to higher prices (and higher margins) on one side of the market (the receiver side), induces platforms to lower prices on the other side of the market (the sender side) since increasing volume on that side now becomes more profitable. The exogenous factor that increases the receiver prices here is the change in industry structure as platforms evolve from independent entities to subsidiaries under a single entity.

Summarizing, a lower price level p_J combined with higher platform profits π_J entails that welfare in this particular industry decreases with the number of independent platforms and that Cournot's complementary monopoly theory extends

[13] Note that for $n = 1$, prices and profits are the same under both ownership structures.

to two-sided markets. Consequently, mergers between such agents are not to be discouraged from an antitrust point of view, a result already hinted at in Subsection 2.1.[14]

3.3 Variable Production Ratio

In this subsection we drop the implicit assumption that platforms are only needed once in the "production" of the composite good. As such, let a_i denote the number of times platform i is needed in composing the bundle of platform services required by a sender to successfully interact with receivers. The production ratio between any two components (platforms) i and j thus equals $a_i : a_j$, and the bundle price

$$A \equiv a_1 p_1^S + a_2 p_2^S + \cdots + a_n p_n^S = \sum_{i=1}^{n} a_i p_i^S,$$

with p_i^S the sender price as charged by platform i. As before, let

$$D^S(A) = D^S\left(\sum_{i=1}^{n} a_i p_i^S\right)$$

be sender quasi-demand for the bundle, and, following Cournot (1838, 1971), denote by

$$D_i^S(A) \equiv a_i D^S(A)$$

sender quasi-demand for each of the individual components. We can then write platform profits as

$$\begin{aligned} \pi_i &= \left(p_i^R + p_i^S\right) D_i^R\left(p_i^R\right) D_i^S(A) \\ &= \left(p_i^R + p_i^S\right) D_i^R\left(p_i^R\right) a_i D^S(A), \end{aligned}$$

which independent platforms tend to maximize when setting receiver and sender prices, allowing us to state the following proposition:

Proposition 4. *With $1/\alpha = \sum_i a_i / \sum_i a_i^2$ and $\beta = \sum_i a_i$, independent complemen-*

[14] To be precise, a potential measure of welfare can be defined as the unweighted (utilitarian) sum of (i) receivers' surplus, (ii) senders' surplus, and (iii) industry profits, or

$$W_k = CS_k^R + CS_k^S + \Pi_k,$$

for $k = I, J$. The extent to which welfare increases under joint ownership is then the difference $\Delta W = W_J - W_I$. As both senders' surplus and industry profits increase ($\Delta CS^S = CS_J^S - CS_I^S > 0, \Delta\Pi = \Pi_J - \Pi_I > 0$), this will ultimately depend on the redistribution of surplus from receivers ($\Delta CS^R = CS_J^R - CS_I^R < 0$) to the latter, or $\Delta W > 0$ if $\Delta CS^S + \Delta\Pi > -\Delta CS^R$.

tary platforms, needed in a production ratio $a_i : a_j$ for the successful completion of a transaction between senders and receivers, charge

$$p_I^R = \frac{(\alpha+\beta)\bar{b}^R - \bar{b}^S}{\alpha+2\beta} \tag{34}$$

$$p_I^S = \frac{2\bar{b}^S - \alpha\bar{b}^R}{\alpha+2\beta}, \tag{35}$$

and make profits equal to

$$\pi_I = \frac{a_i\alpha\left(\beta\bar{b}^R + \bar{b}^S\right)^3}{(\alpha+2\beta)^3\phi}, \tag{36}$$

with profits distributed according to $a_i\alpha$.

Proof. The FOCs with respect to receiver prices are

$$\frac{\partial\pi_i}{\partial p_i^R} = a_i D^S(A)\left\{D_i^R\left(p_i^R\right) + \left(p_i^R + p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]'\right\} = 0.$$

Summing over i yields

$$D^S(A)\sum_{i=1}^n a_i D_i^R\left(p_i^R\right) + D^S(A)\sum_{i=1}^n a_i\left(p_i^R + p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]' = 0.$$

Invoking symmetry, we once more obtain

$$p^R + p^S = -\frac{D^R\left(p^R\right)}{\left[D^R\left(p^R\right)\right]'}.$$

Noting that

$$\frac{\partial A}{\partial p_i^S} = a_i,$$

the FOCs with respect to sender prices are

$$\frac{\partial\pi_i}{\partial p_i^S} = a_i D_i^R\left(p_i^R\right)\left\{D^S(A) + a_i\left(p_i^R + p_i^S\right)\left[D^S(A)\right]'\right\} = 0.$$

Summing over i gives

$$D^S(A)\sum_{i=1}^n a_i D_i^R\left(p_i^R\right) + \left[D^S(A)\right]'\sum_{i=1}^n a_i^2\left(p_i^R + p_i^S\right)D_i^R\left(p_i^R\right) = 0.$$

Invoking symmetry, the latter simplifies to

$$p^R + p^S = -\frac{D^S(A)}{[D^S(A)]'} \left(\frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n a_i^2} \right).$$

Let $1/\alpha = \sum_i a_i / \sum_i a_i^2$ and $\beta = \sum_i a_i$. Then, noting that $A = \sum_i a_i p^S = \beta p^S$, we obtain the following system of equations:

$$\begin{bmatrix} 2 & 1 \\ \alpha & \alpha + \beta \end{bmatrix} \begin{bmatrix} p_I^R \\ p_I^S \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \end{bmatrix}.$$

Obtaining a solution for p_I^k $(k = R, S)$ then is a straightforward application of Cramer's Rule. ■

Similarly, under joint ownership a single entity sets receiver and sender prices so as to maximize joint profits

$$\Pi = \sum_{i=1}^n \left(p_i^R + p_i^S\right) D_i^R \left(p_i^R\right) a_i D^S(A).$$

Proposition 5. *With $1/\gamma = \sum_i a_i / (\sum_i a_i)^2$ and $\beta = \sum_i a_i$, under joint ownership complementary platforms needed in a production ratio $a_i : a_j$ for the successful completion of a transaction between senders and receivers, charge*

$$p_J^R = \frac{(\beta + \gamma)\bar{b}^R - \bar{b}^S}{2\beta + \gamma} \tag{37}$$

$$p_J^S = \frac{2\bar{b}^S - \gamma \bar{b}^R}{2\beta + \gamma}, \tag{38}$$

and make profits equal to

$$\pi_J = \frac{a_i \gamma \left(\beta \bar{b}^R + \bar{b}^S\right)^3}{(2\beta + \gamma)^3 \phi}, \tag{39}$$

with profits distributed according to $a_i\gamma$.

Proof. By the same token, the first-order conditions with respect to sender prices remain unchanged under joint ownership. However, with respect to sender prices we have that

$$\frac{\partial \Pi}{\partial p_i^S} = a_i D_i^R \left(p_i^R\right) \left\{ D^S(A) + \left(p_i^R + p_i^S\right) \left[D^S(A)\right]' a_i \right\} + \sum_{k \neq i}^{n} \left(p_k^R + p_k^S\right) D_k^R \left(p_k^R\right) a_k a_i \left[D^S(A)\right]' = 0.$$

Summing over i and invoking symmetry we then find

$$p^R + p^S = -\frac{D^S(A)}{[D^S(A)]'} \left[\frac{\sum_{i=1}^n a_i}{\left(\sum_{i=1}^n a_i\right)^2} \right].$$

Let $1/\gamma = \sum_i a_i / (\sum_i a_i)^2$ and $\beta = \sum_i a_i$. In this case we obtain the system of equations

$$\begin{bmatrix} 2 & 1 \\ \gamma & \beta + \gamma \end{bmatrix} \begin{bmatrix} p_J^R \\ p_J^S \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \end{bmatrix},$$

and a solution for p_J^k $(k = R, S)$ is found using Cramer's Rule. ■

It is clear that should $a_i = 1 \, \forall i$ (entailing $\alpha = 1$ and $\beta, \gamma = n$), this variable production ratio model reduces to the 1 : 1 production ratio ("perfect complements") case discussed in the previous subsections.

4 On the Two-Sidedness of Complements

As mentioned in Subsection 2.3, in reality bundles exist that simultaneously combine two-sided (platforms) with one-sided components (traditional firms). We refer to these as *asymmetric* bundles, with the term "symmetry" pointing to the unique presence of components of a specific type, either one- or two-sided. This evidently calls for a reinterpretation of the number of platforms n. Therefore, redefine n as the total number of components present in the bundle, and respectively denote by n_1 and n_2 the number of one- and two-sided components, yielding

$$n \equiv n_1 + n_2.$$

This definition allows for a variety of bundle types, with extreme cases being symmetric compositions of either one-sided components ($n_1 = n, n_2 = 0$), or two-sided components ($n_1 = 0, n_2 = n$). Any combination in between is an "asymmetric" bundle ($n_1, n_2 < n$ and $n_1 + n_2 = n$). As the symmetric two-sided case has been treated by Propositions 1, 2 and 3, we focus attention on the remaining symmetric and asymmetric cases.

Maintaining the implicit assumption that components are used in a 1 : 1 ratio for a successful completion of a transaction, the bundle price A in this more general a case is the sum of prices of one- and two-sided components:

$$A \equiv \sum_{h=1}^{n_1} p_h^C + \sum_{i=1}^{n_2} p_i^S.$$

With demand for the bundle still a function of the bundle price, two cases remain to be analyzed: (1) the pricing of symmetric one-sided bundles, and (2) the pricing of asymmetric bundles. With respect to the latter, we investigate the effect of the number of one- and two-sided components present in the bundle (i.e. the fraction $\frac{n_1}{n_2}$) on the limiting price of the bundle itself.

4.1 Symmetric One-Sided Bundles: Complementary Monopoly

With $n_1 = 0, n_2 = n$ and following Economides and Salop (1992), D^S denotes demand for the bundle composed of n_1 *one*-sided complementary goods, and produced by firms 1 to n_1, each having a monopoly on the production of their respective component. For ease of exposition, assume that $n_1 = 2$. The defining feature of the complementary monopoly setting is that both monopolists face the same demand, i.e. the demand for the bundle as a whole, which is a function of the bundle price $A \equiv \sum_h p_h^C$, where $p_h^C \geq 0$ is the price charged for complement $h = 1, 2$.

Assuming for simplicity that each good is produced at constant marginal cost $c_1 = c_2 = 0$, each firm will independently set price so as to maximize

$$\pi_h^C = p_h^C D^S(A),$$

for $h = 1, 2$. First-order conditions (FOCs) are

$$\frac{\partial \pi_1^C}{\partial p_1^S} = D^S(A) + p_1^C \left[D^S(A)\right]' = 0$$
$$\frac{\partial \pi_2^C}{\partial p_2^S} = D^S(A) + p_2^C \left[D^S(A)\right]' = 0.$$

Summing across both FOCs yields

$$2D^S(A) + \left(p_1^C + p_2^C\right) \left[D^S(A)\right]' = 0,$$

and hence the price for the entire bundle is given by

$$A \equiv p_1^C + p_2^C = -\frac{2D^S(A)}{[D^S(A)]'}. \tag{40}$$

For a bundle consisting of n_1 components, equation (40) naturally extends to

$$A \equiv \sum_{h=1}^{n_1} p_h^C = -\frac{n_1 D^S(A)}{[D^S(A)]'}.$$

Now, suppose that both complements are produced by a single entity which sets the price of the bundle A so as to maximize joint profits

$$\Pi = \pi_1^C + \pi_2^C = \left(p_1^C + p_2^C\right) D^S(A) = AD^S(A).$$

Following the lead taken in Section 3, the FOCs with respect to p_h^C are[15]

$$\frac{\partial \Pi}{\partial p_1^C} = D^S(A) + \left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0$$
$$\frac{\partial \Pi}{\partial p_2^C} = D^S(A) + \left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0.$$

Summing across gives

$$2D^S(A) + 2\left(p_1^C + p_2^C\right)\left[D^S(A)\right]' = 0,$$

which yields the bundle price under joint ownership:

$$A^* \equiv p_1^C + p_2^C = -\frac{D^S(A^*)}{[D^S(A^*)]'}. \tag{41}$$

Note that this result holds regardless the number of one-sided components. Therefore we can state the following:

Corollary 3 (Complementary Monopoly). *For a bundle exclusively consisting of one-sided components ($n_1 = n, n_2 = 0$), the two-sided complementary monopoly result stated in Propositions 1, 2 and 3 leads to the complementary monopoly result (Cournot, 1838, 1971) for one-sided markets.*

Proof. Comparing (40) and (41), we find that $A > A^*$. It follows that the price for the bundle under complementary monopoly is twice (n_1 times) as large as under

[15] Alternatively, taking the FOC with respect to the bundle price A immediately yields the same result.

integrated complementary monopoly, thus replicating Cournot's anticommons result for a *one*-sided market. ■

What explains for this remarkable result? As stated by Dari-Mattiacci and Parisi (2006), any producer increasing the price of his component exerts a negative externality on the producers of the remaining complementary goods because demand for the composite good D^S decreases. This seller reaps the full benefit of his price increase in additional revenue but bears only part of the associated cost, which is the corresponding reduction in the quantity demanded. An integrated complementary monopolist, bearing the full cost of such price increases, internalizes the negative externality and sets a lower, profit-maximizing bundle price. As such, this is the horizontal equivalent of vertical integration to avoid the problem of double marginalization (Spengler, 1950).

Corollary 4. *(i) Independent symmetric one-sided components charge*

$$p_I^C = \frac{\bar{b}^S}{n_1 + 1}, \tag{42}$$

while symmetric one-sided components under joint ownership charge

$$p_J^C = \frac{\bar{b}^S}{2n_1}. \tag{43}$$

The respective bundle prices are

$$A_I = \frac{n_1 \bar{b}^S}{n_1 + 1}, \quad \text{and} \quad A_J = \frac{n_1 \bar{b}^S}{2n_1}. \tag{44}$$

(ii) In the limit the bundle price as set by independent components approaches the choke level, while under joint ownership it attains half the choke level.

Proof. (i) Under symmetry, equations (42) and (43) follow directly from (40) and (41); (ii) As the number of one-sided components approaches infinity, we have respectively $\lim_{n_1\to\infty} A_I = \lim_{n_1\to\infty} \frac{n_1\bar{b}^S}{n_1+1} = \bar{b}^S$ and $\lim_{n_1\to\infty} A_J = \lim_{n_1\to\infty} \frac{n_1\bar{b}^S}{2n_1} = \frac{1}{2}\bar{b}^S$. ■

4.2 Asymmetric Bundles

This subsection details the analysis of price-setting behavior in markets where platforms team up with one-sided firms—referred to as *components*—to create a bundle which senders need to consume as a whole to successfully interact with cross-market agents (see Subsection 2.3). From a methodological point of view, we apply the blueprint developed in Section 3 to derive prices and profits, and emphasize the role played by the number of one- and two-sided components in this particular industry setup.[16]

Sender demand D^S is now governed by the total fee $A \equiv \sum_{h=1}^{n_1} p_h^C + \sum_{i=1}^{n_2} p_i^S$, where p_h^C is the sender fee charged by one-sided components, and p_i^S the platforms' sender fees. Just as in the standard one-sided complementary monopoly setting (see Subsection 4.1), transaction volume for the components is equal to the demand for the entire bundle, $D^S(A)$. With c_h^C and c_i respectively denoting the components' and the platforms' marginal costs per transaction, the profit function for the components can then be written as

$$\pi_h^C = \left(p_h^C - c_h^C\right) D^S(A),$$

and for the platforms

$$\pi_i = \left(p_i^R + p_i^S - c_i\right) D_i^R \left(p_i^R\right) D^S(A).$$

Next, assume that $c_i = c \geq 0$ (i.e., platforms are symmetric), and that $c_h^C = \theta c$ with $\theta \in [0, 1]$. For example, with $\theta = 1$, components incur the same marginal cost per transaction as do the platforms. For simplicity we again assume that $c = 0$.

Proposition 6 (Asymmetric Bundles)**.** *In asymmetric bundles composed of n_1 one-sided and n_2 two-sided complementary goods, symmetric platforms charge*

$$p_I^R = \frac{(n_1 + n_2 + 1)\bar{b}^R - \bar{b}^S}{n_1 + 2n_2 + 1} \tag{45}$$

$$p_I^S = \frac{2\bar{b}^S - (n_1 + 1)\bar{b}^R}{n_1 + 2n_2 + 1} \tag{46}$$

and make profits

$$\pi_I = \frac{\left(n_2\bar{b}^R + \bar{b}^S\right)^3}{\phi(n_1 + 2n_2 + 1)^3}, \tag{47}$$

[16] We do not focus on prices under different ownership structures here as it is easy to see that the analysis of (i) mergers between two-sided components only where a single entity sets platform prices on both sides of the market is a simple extension of the results found in Subsections 3.1 and 3.2, (ii) mergers between one-sided components only hinges on the classic Cournot (one-sided) complementary monopoly result, see Subsection 4.1, and (iii) mergers between one- and two-sided components where a single entity sets all prices, in particular the bundle price, combines elements from both (i) and (ii).

while symmetric components charge

$$p_I^C = \frac{n_2\bar{b}^R + \bar{b}^S}{n_1 + 2n_2 + 1} \tag{48}$$

and make profits

$$\pi^C = \frac{\left(n_2\bar{b}^R + \bar{b}^S\right)^2}{\bar{b}^S(n_1 + 2n_2 + 1)^2}. \tag{49}$$

The bundle price equals

$$A_I = \frac{(n_1 + 2n_2)\bar{b}^S - n_2\bar{b}^R}{n_1 + 2n2 + 1}. \tag{50}$$

Proof. As platforms individually set prices to maximize profits, we again obtain the system of $2n_2$ FOCs (11) and (12). Summing over i, we have

$$\begin{aligned}
D^S(A)\sum_{i=1}^{n_2} D_i^R\left(p_i^R\right) + D^S(A)\sum_{i=1}^{n_2}\left(p_i^R + p_i^S\right)\left[D_i^R\left(p_i^R\right)\right]' &= 0 \\
D^S(A)\sum_{i=1}^{n_2} D_i^R\left(p_i^R\right) + \left[D^S(A)\right]'\sum_{i=1}^{n_2}\left(p_i^R + p_i^S\right)D_i^R\left(p_i^R\right) &= 0.
\end{aligned}$$

Invoking symmetry, the latter respectively simplify to

$$\begin{aligned}
D^S(A)n_2D^R\left(p^R\right) + D^S(A)n_2\left(p^R + p^S\right)\left[D^R\left(p^R\right)\right]' &= 0 \\
D^S(A)n_2D^R\left(p^R\right) + \left[D^S(A)\right]' n_2\left(p^R + p^S\right)D^R\left(p^R\right) &= 0,
\end{aligned}$$

thus yielding

$$p^R + p^S = -\frac{D^R\left(p^R\right)}{\left[D^R\left(p^R\right)\right]'} \tag{51}$$

$$p^R + p^S = -\frac{D^S(A)}{\left[D^S(A)\right]'}. \tag{52}$$

Taking the first derivative with respect to component prices we find the following n_1 FOCs:

$$\frac{\partial \pi_h^C}{\partial p_h^C} = D^S(A) + p_h^C\left[D^S(A)\right]' = 0.$$

Summing over h we have

$$n_1 D^S(A) + \sum_{h=1}^{n_1} p_h^C \left[D^S(A)\right]' = 0,$$

which, when invoking symmetry ($p_h^C = p^C \,\forall\, h$), simplifies to

$$n_1 D^S(A) + n_1 p^C \left[D^S(A)\right]' = 0,$$

or

$$p^C = -\frac{D^S(A)}{\left[D^S(A)\right]'}. \tag{53}$$

Noting that the bundle price under symmetry amounts to

$$A = n_1 p^C + n_2 p^S,$$

and combining (51), (52) and (53), we obtain the following system of three equations in three unknowns:

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & n_2+1 & n_1 \\ 0 & n_2 & n_1+1 \end{bmatrix} \begin{bmatrix} p^R \\ p^S \\ p^C \end{bmatrix} = \begin{bmatrix} \bar{b}^R \\ \bar{b}^S \\ \bar{b}^S \end{bmatrix}.$$

The application of Cramer's Rule, $p_I^k = \frac{|A_k|}{|A|}$ for $k \in \{R, S, C\}$, gives the desired results. ■

A closer look reveals that this general result encompasses both symmetric cases: for $n_1 = 0$ and $n_2 = n$, the asymmetric model replicates results (5) and (6) for receiver and sender prices set by independent complementary platforms. For $n_1 = n$ and $n_2 = 0$, the asymmetric model yields (42), the price for one-sided components.

A final result following from equation (50) is that the two-sided characteristic of the market tends to disappear as the number of one-sided components grows large. Despite the presence of platforms, the market behaves as if it were one-sided when the number of one-sided components approaches infinity. On the other hand, if the number of platforms approaches infinity, the one-sided components become relatively unimportant and the market tends to a two-sided market with complementary platforms.

Corollary 5 (Asymmetric Bundle Limit Prices)**.** *(i) If the number of one-sided components approaches infinity the bundle price approaches the senders' choke level, replicating the Cournot-Ellet complementary monopoly result of Corollary 4; (ii)* *if the number of two-sided components grows large, the bundle price does not approach the senders' choke level and replicates the complementary platform result from Corollary 2.*

Proof. (i) In this case we have $\lim_{n_1\to\infty} A_I = \lim_{n_1\to\infty} \frac{(n_1+2n_2)\bar{b}^S - n_2\bar{b}^R}{n_1+2n2+1} = \frac{n_1\bar{b}^S}{n_1} = \bar{b}^S$; (ii) Here we have that $\lim_{n_2\to\infty} A_I = \lim_{n_2\to\infty} \frac{(n_1+2n_2)\bar{b}^S - n_2\bar{b}^R}{n_1+2n2+1} = \frac{2n_2\bar{b}^S - n_2\bar{b}^R}{2n_2} = \bar{b}^S - \frac{1}{2}\bar{b}^R$. ■

5 Conclusion

We introduced a model that allows for the investigation of pricing decisions by complementary platforms, extending Cournot's anticommons problem to two-sided markets. At the same time, this setting offers a natural explanation for asymmetric single- and multihoming patterns across the market.

We show that welfare in markets characterized by the presence of independent complementary platforms is lower than with complementary platforms under joint ownership, as the total fee charged is higher and platform and industry profits are lower. Similar to the anticommons problem in traditional one-sided markets, this result arises because independent platforms fail to internalize the negative pricing externality they exert on others. Under joint ownership, platforms charge lower sender prices (and therefore a lower bundle price) and correspondingly make higher profits. The two-sidedness of the market however also induces the charging of higher receiver prices, thus creating both "winners" and "losers" at the same time. However, the gains realized by the senders and the increase in profits made by the platforms allow for the compensation of the losses incurred on the receiver side. Conversely, the adverse effects of the dilution of property rights on equilibrium pricing are mitigated vis-à-vis the case where only one side of the market is present.

Finally, we also show that both the complementary platform theorem and the Cournot-Ellet complementary monopoly result arise as special cases of a general setting where bundles consist of both one- and two-sided components. Given the many problems in economics that arise from cultural or biological complementarity, also often enforced legally, as well as the presence of network externalities, we are confident that the present results will contribute to a better economic understanding of some laws governing the interaction between individuals over platforms.

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