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economies

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**DISCUSSION  
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# **Cost-Benefit analysis of transport investments in distorted economies**

**by**

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## **Abstract**

This paper deals with costs-benefit analysis of investment in transport infrastructure. Its contribution is twofold. Firstly, we develop a general equilibrium model to explore the impact of a small budgetary-neutral investment in transport infrastructure in a second-best setting, where other markets in the economy are distorted by taxes or external costs. The model incorporates different transport modes that are used both for intermediate inputs (freight) and for final consumption (passenger travel). An intuitive operational expression for the net economic benefit of an investment is derived that depends on the way the investment is financed. This expression generalizes recent findings in the literature. Secondly, we illustrate the results numerically using a small example. Our findings show that both the specific financing instrument used and the labour market consequences may have large implications for the net benefits of transport investments. Significant errors may be made in limiting cost-benefit analysis to transport markets only.

**Keywords:** cost-benefit analysis, transport investments, marginal cost of funds..

**JEL codes:** H23, H43, H54, R13, R42

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## **0. Introduction**

The traditional assessment of transport investments largely focuses on their direct costs and benefits on transport markets (Layard and Glaister (1994), Pearce and Nash (1984)). Of course, it is well known from welfare theory that this is only acceptable under rather heroic assumptions with respect to the social welfare function one assumes and regarding the functioning of other markets. Specifically, it requires that the government has perfect instruments available to redistribute income in combination with the total absence of distortions in all markets other than transport markets. None of these assumptions is realistic, so there is a need to adapt traditional cost-benefit rules to correct for distortions elsewhere in the economy and for distributive concerns.

The purpose of this paper is to embed cost-benefit analysis in a general equilibrium framework that takes into account distortions on all markets, as well as the distributional effects of both the infrastructure improvement and of the way it is financed. Specifically, we develop a general equilibrium model to explore the impact of a small budgetary-neutral investment in transport infrastructure in a second-best setting, where other markets in the economy are distorted by taxes or external costs. The model incorporates different transport modes that are used both for intermediate inputs (freight) and for final consumption (passenger travel). We first derive a very general but quite intuitive operational expression for the net economic benefit of an investment that takes into account the way the investment is financed. Next we illustrate the application of the methodology with a few numerical examples.

Although substantial progress has been made recently on incorporating effects within the transport sector in cost-benefit analysis (e.g., effects of a road investment elsewhere on a road network (Kidokoro (2004))), there is still a need to incorporate general equilibrium effects for at least three reasons<sup>1</sup>. First, it provides better and

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<sup>1</sup> Importantly, there are at least two common misunderstandings related to the role of general equilibrium corrections in cost-benefit analysis. One is that only large transport projects need a general equilibrium correction. This is not correct. Any project, even a very small one, will have effects on the rest of the economy; these effects can *proportionally* be as important as in the case of large projects. Hence, in percentage terms, the ‘mistake’ made by ignoring general equilibrium effects does not depend on project size. Another misunderstanding is that macro-economic models can do the general equilibrium corrections *ex post*. Again, this is not true. Macro models can be helpful to trace the ultimate effects of certain measures on economic activity and on the public budget. They are not suited for welfare analysis of projects, however, because they only track aggregate macro-indicators (like GDP, inflation) and neither individual nor social welfare.

more complete estimates of the implications of a transport infrastructure improvement by taking into account the feedback effects on other sectors. For example, an investment in inland waterways may stimulate freight-intensive economic activities. Second, it can assess the indirect efficiency<sup>2</sup> effects on other markets by computing the increase (or decrease) in the distortions on these markets. Distortions can take many different forms. They include the presence of indirect taxes or subsidies, imperfections due to oligopolistic market structures (e.g., car markets, airlines) or standard cases of market disequilibrium due to, e.g., classical unemployment on the labor market. In evaluating the effect of the transport investment on these distortions one can distinguish between the direct effect on other distorted markets (suppose, for example, that the investment reduces commuting costs which stimulate employment) and the indirect effects on the tax system: the investment indirectly affects tax revenues and causes increased public deficits or surpluses. These latter effects are often ‘hidden’ in a cost of funds parameter. Third, general equilibrium calculations of transport investment costs and benefits allows to estimate, in a rigorous way, the income distribution effects of a transport project; it takes into account not only the distributive consequences of the project itself but also the effects of raising the funds (e.g., via taxes) that finance the infrastructure investments.

There are two possible approaches to the integration of general equilibrium effects. One can build a large scale general equilibrium model that explicitly incorporates all markets in the economy and, by definition, integrates all effects of the investment on all these markets<sup>3</sup>. This ‘ideal’ approach often misses the necessary degree of detail for specific transport projects, however. We therefore choose another approach in this paper. We explicitly include only a limited number of important aggregate markets (transport markets, labor market, two types of commodity markets: freight-intensive and other<sup>4</sup>) into the model, and we integrate financing aspects via shadow cost parameters that are derived within the framework of the model itself. This approach yields more intuitive and transparent results and, as we will argue, offers more chances for a decentralized implementation in practice.

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<sup>2</sup> Efficiency on these other markets is increased when the effect of distortions between consumer prices (willingness to pay) and marginal costs is reduced. Note that efficiency disregards the distribution of losses and gains over individuals.

<sup>3</sup> Models have typically between 10 and 100 sectors, where the sectoral detail is determined by the available Input-Output matrix data.

<sup>4</sup> One implicitly invokes Hicks’ aggregation theorem here to limit the number of aggregate goods.

The theoretical approach developed in this paper mainly draws upon three strands of literature. First, since transport infrastructure can be seen as a congestible public good, the extensive literature on the cost of funding public goods via distortionary taxes is directly relevant. The question of the appropriate supply of public goods in an economy with distortionary labour taxes is a classic in the theoretical public finance literature since Pigou (more recently, see Atkinson and Stern (1974), King (1986) and Sandmo (1998)). When we ignore issues of income distribution, the message from this literature looks simple but, in the case of transport investment, it is not. The simple answer to the question of the optimal level of supply of public investment is that the extra efficiency cost of raising tax money via labor taxes increases the cost of public funds above one, calling for a smaller supply of the public good. However, in the case of transport investments this argument ignores the feedback effect of the investment on total tax revenues via the consumption of taxed commodities such as transport, freight-intensive consumer goods and labor (e.g., in the case of reduced commuting costs). Moreover, including equity concerns requires paying attention to the income of the beneficiaries of the investment and to the incomes of those ultimately paying for it. In parallel, a large literature has developed that estimates the marginal cost of public funds raised via different types of taxes, but without any feedback from the public expenditure to the tax base (Kleven and Kreiner (2006)).

A second strand of literature specifically deals with the role of distortions in the rest of the economy for transport pricing and investment projects. For example, Mayeres and Proost (1997) develop optimal tax and investment rules for congestion-type externalities. Parry and Bento (2001) incorporate the relation between transport and labor markets to evaluate a tax reform on the transport market and show that the feedback effects of congestion imply more favorable labor market outcomes of a congestion tax increase. Two other tax reform exercises include Mayeres and Proost (2001) and Calthrop, De Borger, Proost (2007). The first study considers the distributive effects of a transport tax reform, the latter studies a partial tax reform on the transport market, in the sense that the reform implies higher taxes on freight but not on passengers.

Third, we extend the recent literature on cost-benefit analysis of transport investments and compare our theoretical results with recent applications to concrete transport infrastructure projects. At the theoretical level, Kanemoto and Mera (1985)

and Jara Diaz (1986) offer first attempts to incorporate general equilibrium effects, but they largely focus on a first-best world, and they ignore congestion. More recently, Kidokoro (2004, 2006) generalizes earlier second-best analyses (see, e.g., Oppenheim (1995) and Williams et al. (2001)) by offering a consistent method for benefit estimation on transport networks. He shows that in a second-best world three methods are equivalent: calculate the benefits as (i) the changes in the sum of total consumer and producer surplus on all routes, (ii) the changes in the sum of consumer and producer surplus on the route where the investment takes place plus the change in deadweight losses<sup>5</sup> on all other routes, (iii) the benefits in the first-best case plus the changes in deadweight losses on all routes. However, he does not allow for distributive concerns, and does not explicitly take into account the relation with non-transport markets, nor the issue of financing the investment. Finally, Fosgerau and Pilegaard (2007) study cost-benefit rules in a general equilibrium framework that does incorporate the relation with the labor market and financing by distortionary taxes. They express benefits in terms of variables that can be obtained as the predicted output of traffic models, such as changes in commuting time, changes in taxes or resource costs, changes in the number of commuters, etc. It captures secondary effects and financing issues by applying an exogenous marginal cost of funds, which ignores the impact of the specific financing instrument on congestion. A disadvantage of the model is that it does not guarantee that the exogenous changes taken from the transport model are consistent with the behavior underlying the general equilibrium model, nor does it allow feedbacks from the general equilibrium model to the traffic model.

Our model generalizes both Kidokoro (2004, 2006) and Fosgerau and Pilegaard (2007); their results can be obtained as special cases of our model. We include distributive issues, we model the relation between the transport sector and the rest of the economy, and we model how the investment affects demands for transport and non-transport goods, congestion, etc. Moreover, we consider the marginal cost of funds of different financing instruments.

Our methodology is of direct policy relevance. We compare our findings with practical guides to cost-benefit analysis that are currently used by national public administrations and International Financing Institutions to inform policy decisions on

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<sup>5</sup> If the consumer price on these other routes differs from the marginal social cost due to existing taxes or tolls, the deadweight loss is the loss of consumer and producer surplus, net of tax or toll revenues.

investment decisions (e.g. World Bank, 1998; European Commission, 2002; European Investment Bank and European Commission, 2005).

The structure of the paper is as follows. In the next section, we present the structure of our model. Then, in Section 2, we derive the welfare effect of road and rail investments under different assumptions about how the investment is funded; specifically, we consider lump-sum<sup>6</sup>, labor and transport taxes. We compare the structure of the welfare effects obtained with previous results in the literature and with the current advice on practical cost-benefit analysis included in several well known manuals of international organizations. In Section 3 we briefly review some implementation issues that arise when applying the models in practice, using available data and empirical information. A small-scale numerical example is presented in Section 4. The application illustrates various theoretical results and corroborates the relevance of incorporating general equilibrium effects in analyzing the welfare effects of transport investments. Finally, Section 5 summarizes conclusions.

## **1. The structure of the model**

In this section we describe the structure of the model. We discuss consecutively how we model consumer behavior, producer behavior, congestion, behavior of the government, and the structure of the welfare function. Note that we model a steady state economy. There are no transfers between periods, so no budget deficit is allowed. Moreover, infrastructure is treated as fully variable. Finally, capacity costs are interpreted as rental costs for one period.

### *The consumer*

We have  $N$  different individuals in our economy. These individuals are allowed to differ in their consumption pattern and in their productivity. We will further in the paper interpret differences in consumption pattern as differences in location and in consumption of freight-intensive goods. This implies that the model does capture the realistic case in which some individuals are directly affected by the transport investment; whereas others are not not.

Let preferences for individual  $i$  be given by the utility function:

$$U_i = u_i(C_i, D_i, l_i, T_i^1, T_i^2, T_i^3, T_i^4) \quad (1)$$

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<sup>6</sup> A lump sum tax is a tax that does not distort relative prices and has therefore no direct deadweight losses. It affects the economy via income effects only. A head tax is a good example of a lump sum tax.

where variables are defined as follows:

$C_i$  : clean consumption good; numeraire

$D_i$  : dirty good

$l_i$  : leisure

$T_i^1$  : commuting by road

$T_i^2$  : commuting via a non-congested mode (rail)

$T_i^3$  : non-commuting transport by road

$T_i^4$  : non-commuting transport via a non-congested mode (rail)

The individual faces the following budget and time constraints:

$$q_D D_i + C_i + q_{T^1} T_i^1 + q_{T^2} T_i^2 + q_{T^3} T_i^3 + q_{T^4} T_i^4 = w e_i L_i + G \quad (2)$$

$$L_i + l_i + (T_i^1 + T_i^3) \phi^1 + (T_i^2 + T_i^4) \phi^2 = \bar{L} \quad (3)$$

where  $q_D$  and  $q_{T^j}$  ( $j=1,2,3,4$ ) are consumer prices,  $w$  is the net of tax wage,  $e_i$  is an individual specific productivity index,  $\bar{L}$  is total available time,  $L_i$  is labour supply,  $G$  is a lump-sum transfer. Finally, the  $\phi^1, \phi^2$  are congestion functions for road and rail; they indicate the time needed to travel one kilometer. For rail, this can be taken as a constant as there is by definition no congestion. See below for more details.

Combining the time and budget restrictions, maximization of utility leads to the indirect utility function; this can be written as:

$$V_i \left[ w e_i, q_D, P_{T^1}, P_{T^2}, P_{T^3}, P_{T^4}, G + w e_i \bar{L} \right]$$

where

$$P_{T^j} = q_{T^j} + w e_i \phi^1, \quad j = 1, 3$$

$$P_{T^j} = q_{T^j} + w e_i \phi^2, \quad j = 2, 4$$

are generalized prices per kilometer for the various transport services. They consist of the monetary price plus the time cost. The latter is the time needed to travel one kilometer times the value of time; in the above setup of the model, the value of time is just the effective wage per time unit. Note that extending the model to allow more flexible specifications of the value of time is conceptually straightforward, but it complicates some of the derivations and comes at a loss of transparency. For example, if the utility specification (1) also explicitly depends on the time spent in the different



transport modes and periods, the value of time for an individual will also capture the disutility of the time losses in transport. However, the advantage of the approach taken in this section is that indirect utility is defined in terms of generalized prices that depend on exogenous time values. In section 2.6, we briefly consider implementation of the model based on time values taken from the empirical literature.

### Producer behavior

Production is described by the following expressions:

$$\sum_{i=1}^N (\sum_{j=1}^4 T_i^j + C_i) + F^1 + F^2 + X + I^1 + I^2 \leq \sum_{i=1}^N e_i L_i$$

$$D = S(F^1, F^2, X, \phi^1, \phi^2)$$

The first expression assumes linear production possibilities for some general multi-purpose good; the second equation specifies the production function of the dirty good. The left-hand side of the first expression allocates the multi-purpose good to private transport ( $T$ ), other consumption ( $C$ ), freight transport ( $F$ ) and intermediate inputs ( $X$ ) needed for the production of the dirty consumption good, and to cover the rental costs of transport capacity ( $I$ ). The weak inequality implies that production possibilities are limited by the total effective labour inputs supplied. Note that two types of investment in transport capacity are distinguished:  $I^1$  is road investment and  $I^2$  is rail investment. The second equation captures the idea that the dirty good is produced under constant returns to scale, using two types of freight transport ( $F^1, F^2$  for road and rail, respectively), a clean intermediate input ( $X$ ) and transport time ( $\phi$ ). We assume the unit requirement of freight of type 1 and 2 per unit of the dirty good to be  $F_{ND}^1, F_{ND}^2$ , hence  $F^1 = DF_{ND}^1, F^2 = DF_{ND}^2$ . Note that cost minimizing behavior by producers implies that the unit requirements depend on prices and congestion levels. It also implies that the unit cost function, which we denote  $c_D(\cdot)$ , depends on the taxes or tolls on freight transport by road and rail ( $\tau_{F^1}, \tau_{F^2}$ , respectively) and on unit trip time by both modes. So  $c_D(\tau_{F^1}, \tau_{F^2}, \phi^1, \phi^2)$

### Congestion

The congestion function for roads and rail relates the trip time per unit to traffic levels and capacity of infrastructure. It is given by, respectively:

$$\begin{aligned}\phi^1 &= f^1(T^1 + T^3 + F^1, I^1) \\ \phi^2 &= f^2(I^2)\end{aligned}\quad (4)$$

where  $f^1(\cdot), f^2(\cdot)$  are the congestion functions. Transport variables reflect total demand across individuals:

$$T^j = \sum_{i=1}^N T_i^j$$

As argued above,  $I^1$  is road investment and  $I^2$  is rail investment. Note that we assume travel times for rail to be independent of traffic levels for simplicity, but it does depend on investment in capacity. Rail therefore operates at constant speed for given infrastructure, independent of demand; in this sense it is un-congested. Further note that we assume the contribution of commuting, non-commuting and freight transport to congestion to be equal.

### The government

The government levies taxes on labour ( $\tau_L$ ), passenger and freight transport (freight taxes  $\tau_{F^1}, \tau_{F^2}$ ; passenger taxes are denoted  $\tau_{T^j}$ , for  $j=1, \dots, 4$ ) and the dirty good ( $\tau_D$ ). It pays for two types of infrastructure investment, viz. in road and rail capacity ( $I^1, I^2$ , respectively) and for the lump-sum transfer  $G$ . Its budget restriction reads:

$$\sum_{j=1}^4 \tau_{T^j} \left( \sum_{i=1}^N T_i^j \right) + \tau_L \sum_{i=1}^N e_i L_i + \left[ \tau_D + \tau_{F^1} F_{ND}^1 + \tau_{F^2} F_{ND}^2 \right] \sum_{i=1}^N D_i = NG + I^1 + I^2 \quad (5)$$

### Welfare

Welfare is captured by a standard social welfare function defined on individual utilities:

$$W = W(V_1, V_2, \dots, V_N) \quad (6)$$

where indirect utilities have been defined before. We normalize all producer prices as well as the pre-tax wage per efficient unit of labor at one. Prices and wages are therefore defined as:

$$\begin{aligned}q_D &= (1 + \tau_D) c_D(\tau_{F^1}, \tau_{F^2}, \phi^1, \phi^2) \\ q_{T^j} &= (1 + \tau_{T^j}) \quad j=1, 2, 3, 4 \\ w &= (1 - \tau_L)\end{aligned}\quad (7)$$

where  $c_D(\cdot)$  is the unit cost function for production of the dirty good. Indirect utility functions can therefore be rewritten as:

$$V_i \left[ (1 - \tau_L) e_i, (1 + \tau_D) c_D(\tau_{F^1}, \tau_{F^2}, \phi^1, \phi^2), P_{T^1}, P_{T^2}, P_{T^3}, P_{T^4}, G + (1 - \tau_L) e_i \bar{L} \right] \quad (8)$$

where

$$\begin{aligned} P_{T^j} &= (1 + \tau_{T^j}) + (1 - \tau_L) e_i \phi^1, & j = 1, 3 \\ P_{T^j} &= (1 + \tau_{T^j}) + (1 - \tau_L) e_i \phi^2, & j = 2, 4 \end{aligned} \quad (9)$$

Finally, we define the marginal social utility of giving one extra unit of income to individual  $i$  as

$$\lambda_i = \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial G}$$

Because the welfare function and utility functions are only defined up to a constant positive linear transformation, we will later normalize the  $\lambda$ 's such that their average equals one. The  $\lambda$  parameters allow us to integrate the distributional preferences of the decision maker in a systematic way. If one does not care about income distribution, all the  $\lambda$ 's equal one and the distribution of benefits and costs is no longer important. The more one cares about the income distribution the larger will be the difference between the marginal social utilities of income. The distribution of the  $\lambda$ 's can either be based on ethical or normative principles, or it can be based on observed political choices.

## **2. The welfare effect of transport investment in general equilibrium**

In this section, we consider the welfare effects of transport investments. More precisely, we look at the welfare effect of a small permanent increase  $dI^i$  in the level of transport capacity  $i$ . We look at both road and rail investments, and we consider lump-sum transfers, labor taxes, and taxes on transport users (i.e., road charges) as possible financing instruments (see sections 2.1-2.4). We summarize our findings in section 2.5. Finally, in section 2.6 we provide a brief comparison of our results with the guidelines found in several manuals of international organizations on cost-benefit analysis of transport investments.

## 2.1 Welfare effect of investment in roads financed by reduction in transfer $G$

Consider the welfare change of a marginal and budgetary neutral investment in road capacity, where the investment is financed by reducing the transfer  $G$ . As the transfer is the same for everyone, this amounts to financing by a uniform lump sum tax. The welfare effect can be written as:

$$\left(\frac{dW}{dI^1}\right)^G = \frac{dW}{dI^1}\Big|_G + \frac{dW}{dG} \frac{dG}{dI^1} \quad (10)$$

The superscript on the left-hand side indicates that the investment is financed by the transfer  $G$ . The welfare effect consists of the direct welfare effect of the investment, independent of financing (holding the transfer  $G$  constant), plus the induced effects due to financing (i.e., via the changes in the transfer). Differentiating the welfare function, we have, after simple algebra:

$$\frac{dW}{dI^1}\Big|_G = -\sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dI^1}\Big|_G \quad (11a)$$

$$\frac{dW}{dG} = \sum_{i=1}^N \lambda_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dG} \quad (11b)$$

Note that in (11a) the investment impact on congestion is measured at constant  $G$ . Importantly, the term  $A_i$  appearing in (11a)-(11b) is defined as:

$$A_i = -\frac{\frac{\partial V_i}{\partial \phi^1}}{\frac{\partial V_i}{\partial G}} = \left[ (1 + \tau_D) D_i \frac{\partial c_D}{\partial \phi^1} + (1 - \tau_L) e_i (T_i^1 + T_i^3) \right] \quad (12)$$

It captures the welfare cost for individual  $i$  of an increase in the time needed to make a trip, i.e., an increase in  $\phi^1$ . More congestion raises the price of the dirty good via effects on freight input costs, and it affects passenger transport; this latter welfare cost is captured by the value of time (equal to the net wage) times transport demands of the congestible transport type (road). Substituting (11a)-(11b) into (10) gives, after normalizing average marginal social utility at one (so that  $\sum_{i=1}^N \lambda_i = N$ ):

$$\left(\frac{dW}{dI^1}\right)^G = \left[ -\sum_{i=1}^N \lambda_i (A_i) \right] \left[ \frac{d\phi^1}{dI^1}\Big|_G \right] + \left[ N - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dG} \right] \left( \frac{dG}{dI^1} \right) \quad (13)$$

On the right hand side of (13), the first term is the direct benefit of the investment. It represents the decrease in transport time costs and production costs, weighted by the marginal social utilities of income  $\lambda_i$ . The second term represents the full effect of the lump sum tax (reduction of transfer  $G$ ) that is needed to pay for the investment.

We assume the investment is budgetary neutral. To find out what increase in the transfer is needed to finance a small increase in road investment (i.e.,  $\frac{dG}{dI^1}$ ), we rewrite the government's budget constraint (5) as:

$$R(\tau_L, \tau_D, \tau_{T^1}, \tau_{T^2}, \tau_{T^3}, \tau_{T^4}, \tau_{F^1}, \tau_{F^2}, G, \phi^1, \phi^2) = NG + I^1 + I^2 \quad (14)$$

The left-hand side captures tax revenues:

$$R(\tau_L, \tau_D, \tau_{T^1}, \tau_{T^2}, \tau_{T^3}, \tau_{T^4}, \tau_{F^1}, \tau_{F^2}, G, \phi^1, \phi^2) = \sum_{j=1}^4 \tau_{T^j} \left( \sum_{i=1}^N T_i^j \right) + \tau_L \sum_{i=1}^N e_i L_i + \left[ \tau_D + \tau_{F^1} F_{ND}^1 + \tau_{F^2} F_{ND}^2 \right] \sum_{i=1}^N D_i \quad (15)$$

The implicit function theorem applied to (14) then yields, after simple algebra:

$$\frac{dG}{dI^1} = - \frac{1 - \left. \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G}{N - \frac{\partial R}{\partial G} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dG}} \quad (16)$$

Next define the marginal cost of funds of the financing instrument  $G$ . The marginal cost of funds of a given tax instrument is the welfare cost of using this instrument to raise one unit of extra government revenue. At this level no account is taken of the effects of spending the extra unit of revenue on public goods as the latter effect is measured separately. The definition we use takes into account that the extra lump sum tax affects (be it marginally) the spending on transport related goods by consumers and firms and, therefore, congestion; this in turn affects consumption and government revenue. Note that these latter effects are probably trivial for the tax instrument  $G$ , but not for other financing instruments such as transport taxes that we consider later. Specifically, define the marginal cost of funds as:

$$MCF_G = \frac{\frac{dW}{dG}}{\frac{dRev}{dG}} = \frac{N + \sum_{i=1}^N \frac{\partial W}{\partial V_i} \frac{\partial V_i}{\partial \phi^1} \frac{d\phi^1}{dG}}{N - \frac{\partial R}{\partial G} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dG}}$$

where  $Rev = NG + I^1 + I^2 - R(\tau_L, \tau_D, \tau_{T^1}, \tau_{T^2}, \tau_{T^3}, \tau_{T^4}, \tau_{F^1}, \tau_{F^2}, G, \phi^1, \phi^2)$ . Using the definition of  $A_i$ , see (12), we have:

$$MCF_G = \frac{N - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dG}}{N - \frac{\partial R}{\partial G} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dG}} \quad (17)$$

Note that this marginal cost of funds can not be directly taken from the literature for two reasons: unlike estimates in the literature (Kleven and Kreiner (2006)), it applies to the tax instrument  $G$ , and it corrects for congestion effects.

Next use (16) and (17) in the expression for the welfare effects of investment (expression (13)) to find:

$$\left( \frac{dW}{dI^1} \right)^G = \left[ -\sum_{i=1}^N \lambda_i (A_i) \right] \left[ \frac{d\phi^1}{dI^1} \Big|_G \right] - MCF_G \left[ 1 - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_G \right] \quad (18)$$

The first term captures the welfare-weighted benefit of the road investment for households (remember that the  $A$  terms contain freight transport benefits as well as passenger transport benefits). The second term deals with the financing aspects: it consists of the cost of the investment (here equal to 1) and the induced tax revenue changes, both measured at the marginal cost of funds. The tax revenue changes induced by the investment may be important if transport is a highly taxed good or if it facilitates commuting and increases labour supply.

The revenue effects can be further specified as follows: differentiate (15) to find:

$$\begin{aligned} \frac{\partial R}{\partial \phi^1} = & \sum_{j=1}^4 \tau_{T^j} \left( \sum_{i=1}^N \frac{\partial T_i^j}{\partial \phi^1} \right) + \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} \\ & + \left[ \tau_D + \tau_{F^1} F_{ND}^1 + \tau_{F^2} F_{ND}^2 \right] \sum_{i=1}^N \frac{\partial D_i}{\partial \phi^1} + \sum_{i=1}^N D_i \left[ \tau_{F^1} \frac{\partial F_{ND}^1}{\partial \phi^1} + \tau_{F^2} \frac{\partial F_{ND}^2}{\partial \phi^1} \right] \end{aligned}$$

Substituting in (18) then gives:

$$\begin{aligned} \left( \frac{dW}{dI^1} \right)^G = & \left[ -\sum_{i=1}^N \lambda_i (A_i) \right] \left[ \frac{d\phi^1}{dI^1} \Big|_G \right] - MCF_G \\ & + MCF_G \left\{ \sum_{j=1}^4 \tau_{T^j} \frac{\partial T_i^j}{\partial \phi^1} + \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} + \left[ \tau_D + \tau_{F^1} F_{ND}^1 + \tau_{F^2} F_{ND}^2 \right] \frac{\partial D}{\partial \phi^1} + D \left[ \tau_{F^1} \frac{\partial F_{ND}^1}{\partial \phi^1} + \tau_{F^2} \frac{\partial F_{ND}^2}{\partial \phi^1} \right] \right\} \frac{d\phi^1}{dI^1} \Big|_G \end{aligned} \quad (19)$$

where we have written aggregate demand derivatives wherever possible.

The final step is to note that, differentiating (4), the congestion reduction term satisfies:

$$\left. \frac{d\phi^1}{dI^1} \right|_G = \frac{\partial f^1}{\partial I^1} + \left[ \frac{\partial f^1}{\partial T^1} \frac{\partial T^1}{\partial \phi^1} + \frac{\partial f^1}{\partial T^3} \frac{\partial T^3}{\partial \phi^1} + \frac{\partial f^1}{\partial F^1} \frac{\partial F^1}{\partial \phi^1} \right] \left. \frac{d\phi^1}{dI^1} \right|_G \quad (20)$$

The first component on the right hand side is the effect of the investment on travel time at given traffic levels: the investment reduces congestion for a given traffic flow. The other terms capture the effect of the investment on congestion via induced changes in the demand for passenger and freight road transport. Substituting (20) into (19), for the term associated with the  $A_i$ , leads to:

$$\begin{aligned} \left( \frac{dW}{dI^1} \right)^G &= \left[ -\sum_{i=1}^N \lambda_i (A_i) \right] \left[ \left. \frac{\partial f^1}{\partial I^1} \right|_G \right] - MCF_G \\ &+ \sum_{j=1,3} (MCF_G \tau_{T^j} - MECC_{T^j}) \left. \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G + \sum_{j=2,4} (MCF_G \tau_{T^j}) \left. \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G \\ &+ (MCF_G \tau_{F^1} - MECC_{F^1}) \left. \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G + (MCF_G \tau_{F^2}) \left. \frac{\partial F^2}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G + (MCF_G \tau_D) \left. \frac{\partial D}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G \\ &+ MCF_G \tau_L \sum_{i=1}^N e_i \left. \frac{\partial L_i}{\partial \phi^1} \frac{d\phi^1}{dI^1} \right|_G \end{aligned} \quad (21)$$

where  $MECC_k$  stands for the equity-weighted marginal external congestion cost of a given transport type  $k$  ( $k = T^1, \dots, T^4, F^1, F^2$ ). For example (similar for the other types):

$$MECC_{T^1} = \sum_{i=1}^N \lambda_i A_i \frac{\partial f^1}{\partial T^1}$$

Expression (21) finally shows that the welfare effect of a small road investment financed by a reduction in the transfer consists of six well-defined terms:

- The direct benefit of the investment, measured at constant traffic levels
- The direct cost of the investment measured at the marginal cost of public funds
- The benefits or costs of induced rail and road passenger traffic
- The benefits or costs of induced rail and road freight traffic

- The benefits of changes in demand for the dirty good due to lower unit cost
- The benefits of the extra labor tax revenues generated by the transport investment and the way it is financed.

To interpret (21), first note that it is well known that in a welfare analysis of a given price or capacity change on one market, the effects on the other markets need only to be considered to the extent they are distorted. The distortions take here the form of a difference between a tax rate and the marginal external cost (that may be zero). All tax revenues are measured at the marginal cost of funds for a lump sum tax, as the lump sum tax is here the marginal tax used to balance the budget. Alternatively one can normalize all benefits and marginal external costs by the marginal cost of public funds. Second, note that benefits and the marginal external costs are equity-weighted but the revenue terms are not. The reason is that all tax revenue effects are shared equally by all individuals via a head tax, while the congestion costs or benefits are not. A third thing to note is that, with strict proportionality between commuting and labour supply, one can add the change in labour supply terms to the changes on the commuter transport market. The distortion on the commuter (or labour supply) market contains in this case the difference between the sum of the labour tax and the transport tax and the marginal external congestion cost.

## 2.2. Welfare effect of a rail investment financed by a reduction in the transfer $G$

Consider now, largely by analogy, the welfare change due to investing an extra euro in rail capacity, financing the investment by reducing the transfer  $G$ . We consider an investment that raises average rail speed for simplicity. In Appendix 1 we show that the welfare effect is given by:



$$\begin{aligned}
\left(\frac{dW}{dI^2}\right)^G &= -\left[\sum_{i=1}^N \lambda_i B_i\right] \left[\frac{\partial f^2}{\partial I^2}\right] - MCF_G \\
&+ \sum_{j=1,3} (MCF_G \tau_{T^j} - MECC_{T^j}) \left[\frac{\partial T^j}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^2}\right]_G + \sum_{j=2,4} (MCF_G \tau_{T^j}) \frac{\partial T^j}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} \\
&+ (MCF_G \tau_{F^1} - MECC_{F^1}) \left[\frac{\partial F^1}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^2}\right]_G + (MCF_G \tau_{F^2}) \frac{\partial F^2}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + (MCF_G \tau_D) \frac{\partial D}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} \\
&+ MCF_G \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^2} \frac{\partial f^2}{\partial I^2}
\end{aligned} \tag{22}$$

where

$$B_i = -\frac{\frac{\partial V_i}{\partial \phi^2}}{\frac{\partial V_i}{\partial G}} = \left[ (1 + \tau_D) D_i \frac{\partial c_D}{\partial \phi^2} + (1 - \tau_L) e_i (T_i^2 + T_i^4) \right]$$

The term  $B_i$  is the welfare cost of time losses in rail transport. Note that rail investment  $I^2$  directly affects time used for a rail trip  $\phi^2$ . It also indirectly affects time in road transport  $\phi^1$ : the investment reduces generalized prices of rail, which (through cross-price effects) influence demand for road transport, hence affecting congestion  $\phi^1$ . In the case of road investment (see (21)), there was no such indirect effect because congestion levels in rail are kept constant. Comparing (22) with (21), the main difference is that rail investment affects road congestion, whereas (by our assumptions) road investment has no impact on rail speed.

The first term on the right hand side of (22) is the direct weighted welfare benefit of lower rail travel times for consumers; the second term is the investment cost evaluated at the cost of funds. The other terms reflect, as before, the distortions on the various markets. Rail investment affects the demand for passenger and freight road transport, where the existing distortion is given by the deviation of tax (corrected for the cost of funds) and marginal external congestion cost. The investment further affects tax revenues on the markets for rail freight, the dirty good, and the labor market.

### 2.3. Welfare effect of investment in roads financed by raising the tax on labor

Consider the welfare change due to investing an extra euro in road capacity, financing the investment by raising the labor tax. Using the methodology of the previous sections, we find that the welfare effect can be written as (see Appendix 2):

$$\begin{aligned}
\left(\frac{dW}{dI^1}\right)^{\tau_L} &= \left[-\sum_{i=1}^N \lambda_i (A_i)\right] \left[\frac{\partial f^1}{\partial I^1}\right] - MCF_{\tau_L} \\
&+ \sum_{j=1,3} (MCF_{\tau_L} \tau_{T^j} - MECC_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} + \sum_{j=2,4} (MCF_{\tau_L} \tau_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \\
&+ (MCF_{\tau_L} \tau_{F^1} - MECC_{F^1}) \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} + (MCF_{\tau_L} \tau_{F^2}) \frac{\partial F^2}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} + (MCF_{\tau_L} \tau_D) \frac{\partial D}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \\
&+ MCF_{\tau_L} \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L}
\end{aligned} \tag{23}$$

This is identical to (21), except for the use of a different marginal cost of funds. In expression (23), the marginal cost of raising a euro through the labor tax is given by (again, see Appendix 2):

$$MCF_{\tau_L} = \frac{\frac{dW}{d\tau_L}}{\frac{dRev}{d\tau_L}} = \frac{-\sum_{i=1}^N \lambda_i e_i L_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_L}}{-\frac{\partial R}{\partial \tau_L} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{d\tau_L}}$$

#### 2.4. Welfare effect of investment in roads financed by a tax on road use

Assume the investment in roads needs to be fully financed through an increase in the tax on road use. As we have 2 taxes on road use (one on commuting transport and one on non-commuting transport), we can either increase both taxes in a uniform way, or focus on only one of them. In what follows, we select the tax  $\tau_{T^1}$  on  $T^1$  to close the budget; this allows a direct comparison with the expressions obtained for financing via the lump sum subsidy or the labour tax.

Going through the same procedure as before, one finally obtains the welfare effect of a road investment financed by an increase in the road tax:

$$\begin{aligned}
\left(\frac{dW}{dI^1}\right)^{\tau_{T^1}} &= \left[-\sum_{i=1}^N \lambda_i(A_i)\right] \left[\frac{\partial f^1}{\partial I^1}\right] - MCF_{\tau_{T^1}} \\
&+ \sum_{j=1,3} (MCF_{\tau_L} \tau_{T^j} - MECC_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_{T^1}} + \sum_{j=2,4} (MCF_{\tau_L} \tau_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_{T^1}} \\
&+ (MCF_{\tau_L} \tau_{F^1} - MECC_{F^1}) \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_{T^1}} + (MCF_{\tau_L} \tau_{F^2}) \frac{\partial F^2}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_{T^1}} + (MCF_{\tau_L} \tau_D) \frac{\partial D}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_{T^1}} \\
&+ MCF_{\tau_L} \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_{T^1}}
\end{aligned} \tag{24}$$

where the marginal cost of raising funds via the transport tax is defined as:

$$MCF_{\tau_{T^1}} = \frac{\frac{dW}{d\tau_{T^1}}}{\frac{dRev}{d\tau_{T^1}}} = \frac{-\sum_{i=1}^N \lambda_i T_i^1 - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_{T^1}}}{-\frac{\partial R}{\partial \tau_{T^1}} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{d\tau_{T^1}}}$$

## 2.5. Welfare effect of investment in a general equilibrium model: summing up

For a given investment, whatever the source of funds, the expressions for the welfare assessment (21)-(24) all have the same structure. We always have three types of terms:

- the direct benefit of the investment keeping all traffic flows and all taxes constant; this term is independent of the source of funding
- the direct cost of financing the investment, measured at the marginal cost of funds of the tax instrument used;
- the induced distortions of the investment on all the markets, measured at constant taxes. The distortion equals the difference between the tax, corrected for the marginal cost of funds, minus the marginal external costs.

Note that only the first element is independent of the source of funding. Moreover, the financing instrument used only enters the expressions via a different marginal cost of funds, but leaves all other terms unaffected.

One would expect that, if the externality is initially insufficiently taxed, an externality-correcting tax would generate additional benefits compared to using a labor or lump-sum tax as financing instrument. Considering expression (24), at first sight there seems to be no such extra benefit from using transport taxes as financing instrument. However, the additional benefits are present, but they are hidden in the

definition of the marginal cost of funds. The term  $-\sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_{T^1}}$  in the marginal cost of funds of a transport tax will give extra credit to the externality reduction effect of a transport tax: the effect of this tax on congestion is more direct and likely to be numerically much more important when using a transport tax, compared to labor and lump-sum taxes.

One can relate the expressions for the net welfare gain (or loss) with earlier findings in the literature. First, consider the literature on the optimal supply of public goods. If one uses only one distorting tax to finance the investment (say the tax on labor), dividing through by the marginal cost of funds generates an expression that is comparable to Bovenberg and Van der Ploeg (1994) and Mayeres and Proost (1997): the benefits of the public good supply (the reduction of external costs) are all scaled down by the marginal cost of government revenue.

Second, with proper re-interpretation, our expressions also immediately produce simple results on tax reform. Consider the net welfare effect of raising one euro of tax revenue by increasing an arbitrary tax A, where the budget is kept constant by decreasing tax revenue by one Euro through a reduction in another tax B. Specifically; let us consider an increase of the transport tax  $\tau_{T^1}$  financed by a reduction in the labor tax. In our model, this can be obtained by (i) subtracting the welfare expression for a given investment financed by the labor tax from the corresponding expression for transport tax financing, and (ii) noting that the operation implies that there is no net change in investment. We find:

$$dW = MCF_{\tau_{T^1}} - MCF_{\tau_L} = \frac{-\sum_{i=1}^N \lambda_i T_i^1 - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_{T^1}}}{\frac{\partial R}{\partial \tau_{T^1}} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{d\tau_{T^1}}} - \frac{-\sum_{i=1}^N \lambda_i e_i L_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_L}}{\frac{\partial R}{\partial \tau_L} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{d\tau_L}}$$

This is just the difference in marginal cost of funds for the two taxes. It is also precisely the tax reform formula obtained by Mayeres and Proost (2001), although our expressions are more general because they also include taxes on intermediate goods.

Third, we can compare with the recent results on cost-benefit of transport projects, most notably obtained by Kidokoro (2004, 2006). He ignores freight transport as well as the markets for labor and the dirty consumption good. Moreover,

he considers distortions on other routes within a transport network rather than distortions on markets for other transport modes, but this is just a semantic distinction between his and our model. Not surprisingly, our results reduce to those he obtained under some simplifying assumptions. For example, if distributive concerns are irrelevant and the marginal cost of funds equals one, if pricing on competing transport markets is at marginal external cost, then our model reproduces Kidokoro's first-best case (see p. 283-284). The benefits are then given by the direct consumer surplus on the market where the investment took place plus the extra tax revenues on this market. If the competing transport mode is not optimally priced, our formulas reproduce (keeping all other assumptions) Kidokoro's second expression (see p. 289) for the second-best case: the benefit is the consumer surplus on the market where investment took place plus that tax revenues on this market, plus the change in deadweight loss on the competing market due to the deviation of tax from marginal external cost. Of course, we generalize his findings by considering the cost of funds, the distortions on several other markets, and distributive issues.

## 2.6. Comparison with existing guidelines

Finally, we can compare the results with the more practical guides to cost-benefit analysis. In three recent guides (European Commission (EC, 2002), European Investment Bank and European Commission (2006) and the World Bank (1998)) reference is made to the need to account, in addition to the direct benefits and costs of a project, to the marginal cost of public funds, distortions on secondary markets and equity impacts. However, in the specific numerical examples given for transport projects, effects are limited to direct impacts only. As an example, we refer to Table 3.17 in EC (2002). There it is implicitly assumed that (i) two competing roads are perfect substitutes; (ii) no distortions exist on other markets<sup>7</sup>; (iii) the marginal cost of funds is one, i.e. non-distortionary lump-sum taxes are available; and (iv) by implication, lump-sum taxes are used to ensure an equal marginal social utility of income (i.e., equity impacts are not relevant). A similar set of implicit assumptions are made within the rail investment case studies developed in European Investment Bank and European Commission (2006). The theory developed in this section

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<sup>7</sup> Admittedly, the example does measure costs in terms of shadow prices. By applying a conversion factor of around 0.5 to the share of unskilled labour costs within total investment costs, there is an indirect attempt to capture labour market imperfections. Our approach, by contrast, is a direct measure of this impact.

indicates that this approach ignores several potentially important effects, due to distortions on other markets and the role of the marginal cost of funds. We illustrate these issues with a small numerical example in the next section.

### **3. Towards operational cost-benefit rules**

In this section, we discuss some implementation problems that arise when trying to operationalize the theoretical results for realistic projects, using available data. Next we briefly discuss the problem of using extraneous estimates on time values in the model.

In practice, an individual road or rail investment is unlikely to affect all individuals. This has several implications for making the cost-benefit rules operational. It is useful, therefore, to distinguish between individuals that are directly affected by the project and those that are not. The individuals directly affected are those that benefit directly from lower passenger transport prices and from lower prices for freight-intensive consumer goods. The set of all individuals  $N$  is therefore divided into the set  $N_u$  of other individuals not directly affected by the project and the set  $N_d$  of those directly affected. Related to this, if the investment project is “small”, say affecting only one region or one city in a large country, we expect that the effect of a change in general taxes on local congestion (the terms  $\frac{d\phi^1}{dG}, \frac{d\phi^1}{d\tau_L}$ ) will become very small. The reason is that the change in general taxes necessary to finance a local project becomes very small and, therefore, the behavioral effect on local users will be very small too.

When a small project is financed by a nationwide tax instrument, the consequence of this argument is clear. For example, suppose a small local investment project is financed by a change in the lump-sum transfer for all individuals  $N$ . In that case, the congestion terms drop out of the definition of the marginal cost of public funds; it can be approximated by:

$$MCF^*_G = \frac{N}{N - \frac{\partial R}{\partial G}}$$

Similarly, in the case of financing via a reduction in the general labor tax on all individuals  $N$ , the cost of funds reads:

$$MCF^*_{\tau_L} = \frac{-\sum_{i=1}^N \lambda_i e_i L_i}{\frac{\partial R}{\partial \tau_L}} \quad (25)$$

Note the advantage of this latter expression: it can be approximated by estimates directly taken from the literature. Indeed, these estimates typically refer to national labour tax changes and they obviously ignore congestion effects (Kleven and Kreiner (2006)).

Consider as, an example, the practical implementation of expression (23), assuming that a small project is financed by a reduction in the labour tax. Taking previous remarks into account, the cost-benefit rule can be rewritten as:

$$\begin{aligned} \left( \frac{dW}{dI^1} \right)^{\tau_L} &= \left[ -\sum_{i=1}^N \lambda_i (A_i) \right] \left[ \frac{\partial f^1}{\partial I^1} \right] \\ &+ \sum_{j=1,3} (MCF^*_{\tau_L} \tau_{T^j} - MECC_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Bigg|_{\tau_L} + \sum_{j=2,4} (MCF^*_{\tau_L} \tau_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Bigg|_{\tau_L} \\ &+ (MCF^*_{\tau_L} \tau_{F^1} - MECC_{F^1}) \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Bigg|_{\tau_L} + (MCF^*_{\tau_L} \tau_{F^2}) \frac{\partial F^2}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Bigg|_{\tau_L} + (MCF^*_{\tau_L} \tau_D) \frac{\partial D}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Bigg|_{\tau_L} \\ &+ MCF^*_{\tau_L} \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Bigg|_{\tau_L} \\ &- MCF^*_{\tau_L} \end{aligned}$$

All terms except the final one represent the welfare and demand effects on the small subset of individuals  $N_d$  that are directly affected by the project. The marginal cost of funds represents the investment cost financed by a general labour tax increase on the whole economy. This marginal cost of funds is approximated by (25); as suggested above, an estimate can be taken from available literature.

A similar procedure can be followed for the case of financing by a lump-sum tax on all individuals  $N$ . However, when a project is entirely financed by an increase in transport taxes on the users of the transport capacity that is being extended by the investment, no such simplification can be applied. The reason is that in that case the financing of the investment does have non-negligible implications for congestion. The marginal cost of funds in this case takes into account these effects, see section 2.4 above; it cannot be approximated by estimates available in the literature, but has to be

endogenously determined within the model. We return to this issue in the numerical application below.

Finally, consider the issue of time values to be used when implementing the model. Standard practice in applied models is to take advantage of time values that are available in the empirical literature, and to introduce these exogenously into the model. This raises two issues. First, at the theoretical level, our model did not explicitly include the disutility of travel and labor time and, therefore, it implied time values equal to the net wage. As argued before, adapting the theory to allow for more flexibility in time values is conceptually simple, but complicates the derivations and goes at a loss of transparency. Moreover, doing so results in time values that are endogenously determined by the model. Second, the estimates found in the empirical literature are typically estimates of the value of an exogenous reduction in travel times; although related, this is a different concept than the value of time underlying the theoretical model which, at the consumer optimum, is independent of the activity in which time is spent. A compromise is necessary, therefore. This consists of (i) approximating the value of time by empirically estimated values, corrected for the disutility of time wasted in traffic and (ii) assuming that time values are not affected by the investment policy under consideration so that it can be treated as constant.

#### **4. Numerical example**

In order to apply the theory developed above, a simple numerical example of a marginal investment project has been developed. The “base case” (without project) is defined as an existing motorway with 2 lanes in each direction. The “project” is defined as an investment to widen the motorway from 2 lanes to 3 in each direction along a 20-kilometre section. In order to keep the example tractable, we make a number of restrictions to the general case developed above. We assume (i) a single transport good (i.e. passenger transport) and a single numeraire non-transport good; (ii) an explicitly separable, additive specification of the utility function<sup>8</sup>, see below; (iii) an explicit linear specification for congestion, also discussed in detail below; (iv) a single representative consumer. Specifically, we assume there are  $n$  identical individuals who are all directly affected by the project. Moreover, (v) we assume that

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<sup>8</sup> The additive separable utility function simplifies the analytical results because there are no income effects on transport demand and labor supply.



the project is small relative to the whole economy. As argued in Section 3, this implies that the marginal cost of funds of financing the investment through the labour tax is not influenced by the investment project: the national labour tax changes induced by the project have a negligible impact on local congestion. Hence we can approximate the marginal cost of funds by exogenous estimates taken from the literature. By contrast, the marginal cost of funds when financing the investment by congestion tolls, cannot be assumed to be independent of the project: the changes in local congestion that result from financing the investment cannot be ignored. Consequently, in this case the marginal cost of funds is computed endogenously within the model.

Finally, it turns out to be easier for the interpretation of the numerical results not to normalize wages and prices at one in the reference equilibrium. We denote wages and the resource cost of transport by  $w$  and  $r$ , respectively.

#### 4.1. Setup of the problem

Specifically, let there be  $n$  identical individuals affected directly by the project. Let the typical consumer maximize the following utility function subject to a budget and time constraint:

$$\underset{C,T,l}{Max} \quad u(C,T,l) = C + (a_1T - \frac{1}{2}a_2T^2) + (b_1l - \frac{1}{2}b_2l^2)$$

subject to

$$C + r(1 + \tau)T = (1 - t)wL + G$$

$$l + L + \gamma T = \bar{L} = 1$$

where  $C, T, l$  are, respectively, a numeraire non-transport good, passenger transport and leisure. We have normalized available time at one. Moreover, transport faces congestion; the time needed per unit of  $T$ , denoted  $\gamma$ , depends on the traffic level and on capacity. We assume a linear specification as function of the volume-capacity ratio (where  $c$  represents the time at maximal speed):

$$\gamma = f(T, I) = c + \frac{mT * n}{I}$$

Here  $I$  is interpreted as the transport capacity available.

Combining budget and time constraint yields:

$$C + PT + (1 - t)wl = (1 - t)w + G$$

where  $P$  is the generalized price of transport:

$$P = r(1 + \tau) + w(1 - t)\gamma$$

Demand functions are given by:

$$T = \frac{a_1 - P}{a_2}, \quad l = \frac{b_1 - w(1 - t)}{b_2}$$

It is worth noting that these specifications imply (using  $L = \bar{L} - \gamma T - l$ ):

$$\frac{\partial T}{\partial \gamma} = -\frac{(1 - t)w}{a_2} < 0$$

$$\frac{\partial l}{\partial \gamma} = 0$$

$$\frac{\partial L}{\partial \gamma} = -(\gamma \frac{\partial T}{\partial \gamma} + T)$$

The impact of travel time on transport demand is negative, so investment does increase demand for transport. Due to our additive utility formulation, leisure is not affected by a change in the transport time per unit; only the real wage plays a role. This means that a decrease in the transport time  $\gamma$  will affect the number of trips, and if this reduces overall transport time, the remaining time savings are used for labour supply. The sign of the effect of a decrease in unit transport time on labour supply is therefore ambiguous; it will be positive if the elasticity of transport with respect to travel time is greater than one. Finally, note that substitution of the demand function in the utility function gives indirect utility. This can be written as:

$$v(P, t, w, G) = G + w(1 - t) + \frac{1}{2a_2}[a_1 - P]^2 + \frac{1}{2b_2}[b_1 - w(1 - t)]^2$$

For this particular setup, we derive the welfare effect of a marginal capacity increase financed by a reduction in the lump sum subsidy  $G$  (or by an increase in the labour tax  $\tau_L$ ). This implies that all individuals in the economy contribute to the financing of the project. Specifically, we show in Appendix 3 that the equivalent of expression (21) of the theoretical section for our specific example is given by:

$$\frac{dv}{dI} = mn(1 - t) \left[ \frac{T}{I} \right]^2 - MCF + MCF * tw \left[ \frac{\gamma(1 - t)w}{a_2} - T \right] \frac{d\gamma}{dI} - [MCF * \tau r - MEC] \left( \frac{(1 - t)w}{a_2} \right) \frac{d\gamma}{dI} \quad (26)$$

where

$$\left[ \frac{\gamma(1-t)w}{a_2} - T \right] = \frac{\partial L}{\partial \gamma}$$

$$- \left[ \frac{(1-t)w}{a_2} \right] = \frac{\partial T}{\partial \gamma}$$

The first term on the right hand side of (26) is the direct effect at constant transport demand. The second term is the relevant marginal cost of funds of financing by G (or  $\tau_L$ ), levied on the whole economy. The third term captures the effect of transport investment on labor tax revenues paid by those directly affected by the investment project, and the final term captures the product of the change in the number of trips of those directly affected and the difference between the transport tax revenue (weighted by the *MCF*) and the marginal external cost.

#### 4.2.Data

The following typical investment costs are assumed (all cost figures relate to flat terrain in Europe, excluding VAT (see Conseil général des ponts et chaussées, 2006)):

- 7 m EUR/km for 2 lanes (i.e. the base case);
- 10 m EUR/km for 3 lanes.

This implies near constant-returns-to-scale in lane expansion. Given an asset life of 30 years and a discount rate of 5%, the annuitized value of investment without the project (i.e. for 2-lanes in each direction only) is equal to 9.1 m EUR per year.

The capacity of the motorway is assumed to be 2,000 vehicles per lane per hour per direction. Morning peak-period transport demand in the base case is assumed equal to capacity for 3 hours i.e.12,000 vehicles. All benefits are assumed to occur during the peak period, with speed at capacity equal to 30 kph, compared with a free-flow speed of 130 kph.

Finally, assumptions made for all other data are collected in Table 1.

Type	Parameters	Value	Comment/Source
Individuals directly affected	$N$	12,000	In text
Demand	$C$	1	Normalised
	$T$	1	Single morning commute
	$\bar{L}$ – time endowment	12	See text
	$L$ – labour supply	5	See text
Taxes	$\tau$ – tax rate on transport	0.2	
	$t$ – tax on labour	0.3	See text
Budget constraint	$r$ – gross vehicle operating resource cost/km	0.3	De Borger and Proost (2001)
	$w$ – gross wage rate in EUR/hour	20	Implies net of tax wage (or value of time) equal to 15 EUR/hr (De Borger and Proost, 2001)

**Table 1: parameter assumptions**

#### 4.3. Calibration

The model has five unknowns. The transport unknowns –  $a_1, a_2$  – have been calibrated to ensure a generalized price elasticity of demand of -0.6, equivalent to a money elasticity of demand equal to -0.3 (Parry and Bento, 2001). The leisure unknowns –  $b_1, b_2$  – have been calibrated to ensure an elasticity of labour supply with respect to net-of-tax wage of 0.3 (Parry and Bento, 2001). In addition, the parameters are selected to ensure reasonable shares of money expenditures on transport (8% - with the remainder on the numeraire good) and time shares (53% on labour supply, 42% on leisure; 6% transport). See, for example, Mayeres and Proost (1997) for detailed analysis of expenditure shares.

For the congestion function (i.e. unknown  $m$ ), applying the assumptions made on base case speed and flow, the congestion function can be solved for  $m$ . In order to test the reasonableness of this specification, it is insightful to compute the base case marginal elasticity of travel time with respect to capacity. This equals -0.6, i.e., a ten

percent increase in investment reduces average travel time by 6%. This seems reasonable – adding a third lane broadly equals a 30% in annuitised capacity cost and would result in an approximately 20% increase in average peak-period speed (at new traffic levels), i.e., from 30 kph to 36kph.

#### 4.4. Results

For a discrete investment, it is possible to compute the difference in indirect utility from the project (net of the base case). However, in order to illustrate the continuous case developed above, we report results under a notional 1 euro marginal investment.

Table 2 shows the central results of the model. We assume that the project is financed via a labour tax on the whole economy. Hence, following the justification in section 3 above, we assume an exogenous marginal cost of public funds equal to 1.5. We find that a one euro investment in capacity leads to a net social welfare gain of 2.58 euro. This gain can be divided into the four components of equation (26). Line 1 in Table 2 shows that the direct benefit from time savings to those directly affected by the project equals 3.45, while the cost, adjusted for the marginal cost of public funds obviously equals 1.50 (see Line 2). In addition, the transport investment boosts taxed labour market supply of those directly affected by the investment, this is due to our functional forms where transport time savings result in higher labour supply only. This provides an additional benefit of 1.16 euros (see Line 3). Finally, Line 4 shows that the investment also boosts under-taxed transport demand for those directly affected, and this reduces welfare by 0.54 euro. Taking all impacts into account, the total gain in welfare equals 2.58 euros.

**Table 2: Central Result**

1	Direct benefit	3.45
2	Direct cost	-1.50
3	Labour market	1.16
4	Transport market	-0.54
WELFARE	all impacts: 1-4	2.58

In Table 3 we show how the results vary according to other assumptions on the type of taxes that are used to finance the investment. This is done via changes in the marginal cost of funds  $MCF$ . In three cases, following the justification in Section 3, different exogenous values for  $MCF$  have been assumed, reflecting the fact that taxes are imposed on the whole economy and, hence, have negligible effects on local congestion: (i) in the first column, results are shown for financing via a lump sum tax. Given the specification of utility and the absence of income effects, our example then implies that the  $MCF=1$ ; (ii) in the last two columns, results are given for two different exogenous values for the labour tax: we use as before 1.5 in the central case, and also consider a higher cost of funds equal to 2.0. Finally, we also present results for the case where the investment is financed by raising the transport tax ( $\tau$ ) on users of the infrastructure, As argued before, in this case the  $MCF$  has to be determined endogenously by the model; for the base case calibration, we find a  $MCF$  equal to 1.12. This is not inconsistent with other estimates in the literature (e.g. Mayeres and Proost, 1997). The results under this last financing instrument are shown in the second column. All calculations assume an initial transport tax  $\tau = 0.2$  and an elasticity of labour supply with respect to travel time (denoted  $e$ ) equal to  $-0.07$ . We find that the larger the  $MCF$ , the smaller the overall welfare gain from the project:  $MCF=2$  causes the total welfare gain to drop to 2.53.

**Table 3: Influence of  $MCF$**

<b>MCF (<math>\tau=0.2</math>; <math>e = -0.07</math>)</b>		<b>LUMP</b>	<b>TOLL</b>	<b>LAB</b>	<b>LAB HI</b>
		<b>SUM</b>			
		<b>MCF=1</b>	<b>MCF=1.12</b>	<b>MCF=1.5</b>	<b>MCF = 2</b>
1	Direct benefit	3.45	3.45	3.45	3.45
2	Direct cost	-1.00	-1.12	-1.50	-2.00
3	Labour market	0.77	0.87	1.16	1.55
4	Transport market	-0.60	-0.58	-0.54	-0.48
<b>WELFARE</b> all impacts: 1-4		2.63	2.62	2.58	2.53
simple: 1+2		2.45	2.33	1.95	1.45
transport only: 1,2+4		1.86	1.75	1.42	0.98

Table 3 further provides an interesting comparison point for different types of transport models. In many practical cases, project evaluation is done on the basis of a standard transport model which does not consider  $MCF$  or labour market impacts. The

impact of ignoring the different elements of equation (26) is shown in the last two rows of Table 3. Assume the central result is correct. A simple cost-benefit analysis may (implicitly) assume in the case of labour taxes a  $MCF=1$  – rather than 1.5 - and add direct impacts only. Table 3 shows this approach (marked 'simple') would measure the net benefit at 2.45 – underestimating the real impact by around 5%. A more sophisticated model, taking into account congestion impacts, may also measure the impact on the distortion on the transport market – i.e. line 4. In this case, shown as 'transport only', the net social impact would be measured as 1.86, as the investment generates additional congestion. However, this underestimates the true benefit of the investment, equal to 2.58, by nearly 30%. Of course, this result depends crucially on the labour market response of a transport investment. In fact, one might suspect that this response is on the high side in our model. First, separable utility implies that leisure does not increase in response to changes in transport time. Second, the labour market response is weighted by the  $MCF$ . At the other extreme, if the labour supply response were zero, then the “transport only” approach is the right approximation.

Table 4 shows the impact of different assumptions with respect to existing tolls on the project whose capacity is being extended. The central case, as usual, is shown in the middle column. Recall that, in this case, passenger transport is underpriced: passengers pay taxes, measured in public revenue terms, that are significantly under marginal external costs ( $MEC$ ). In the model, setting a toll equal to around eighty percent of resource costs ( $\tau =0.8$ ) approximately equals the  $MEC$ . At this level of toll, the net benefits of the investments are much higher: 2.99 rather than 2.58. This is consistent with results of the literature on investment with imperfect tolling (e.g. Small, 1992).

**Table 4: Influence of toll**

<b>tau (MCF = 1.5; e = -0.07)</b>		tau = 0.1	tau = 0.2	tau = 0.8
1	Direct benefit	3.45	3.45	3.45
2	Direct cost	-1.50	-1.50	-1.50
3	Labour market	1.18	1.16	1.04
4	Transport market	-0.62	-0.54	0.00
WELFARE	all impacts: 1-4	2.52	2.58	2.99
	simple: 1+2	1.95	1.95	1.95
	transport only: 1,2+4	1.34	1.42	1.96

Finally, the impact of the investment on ameliorating pre-existing labour market distortions depends to a large degree on the elasticity of labour supply with respect to travel time (denoted  $e$ ). In the central case,  $e$  was approximately -0.07. Table 5 shows the sensitivity of the overall results to this assumption. The more sensitive labour supply is to travel time, the greater the additional benefit of the investment. Indeed, in this example, as the first column shows, if labour supply is wholly inelastic, the net benefit of the project is modest, given the cost of public funds and the magnitude of the pre-existing distortion on the transport market.

**Table 5: Influence of labour response**

**e (MCF = 1.5; tau = 0.2)**

	e = 0	e = -0.07	E = -0.1
1 Direct benefit	3.45	3.45	3.45
2 Direct cost	-1.50	-1.50	-1.50
3 Labour market	0.00	1.16	1.96
4 Transport market	-1.12	-0.54	-0.13
WELFARE all impacts: 1-4	0.83	2.58	3.78
simple: 1+2	1.95	1.95	1.95
transport only: 1,2+4	0.83	1.42	1.82

#### 4.5. Lessons from the numerical example

The numerical model provides a simple application of the general theory developed above. The different components of the welfare impact of a marginal investment in a motorway network are illustrated under a variety of plausible parameter values.

In general terms, the simple model suggests that labour market impacts may play an important role in determining the net benefits of transport investment. In our simple model, the investment serves to boost labour supply. In reality this will depend on the precise project. This seems plausible for motorways feeding employment centres. In theory, investment may equally serve to reduce labour supply, as the



implicit boost in private income may lead consumers to substitute in favour of additional leisure.

## **5. Conclusions**

In this paper we embedded cost-benefit analysis of a transport investment in a general equilibrium framework that takes into account distortions on all markets, as well as the distributional effects of both the infrastructure improvement and of the way it is financed. We first developed a simple theoretical general equilibrium model to explore the impact of a small budgetary-neutral investment in transport infrastructure in a second-best setting, where other markets in the economy are distorted by taxes or external costs. The model incorporates different transport modes that are used both for intermediate inputs (freight) and for final consumption (passenger travel). We derived and interpreted a general but quite intuitive operational expression for the net economic benefit of an investment that takes into account the way the investment is financed. Next we illustrate the application of the methodology with a few numerical examples.

Conclusions are easily summarized. First, both the theoretical model and the numerical application suggest that only capturing direct costs and benefits of an investment, and hence ignoring the distortions on labor and transport markets, may lead to very misleading cost-benefit outcomes. Second, the numerical example illustrated the relevance of the indirect effects. In the example, the investment serves to raise labour supply, affecting substantially the net benefits of the project. Although this seems plausible for motorways feeding employment centres, it remains an empirical matter which needs to be established in the context of any particular project.

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## Appendices:

### 1. The welfare effect of a rail investment financed by reducing the transfer G

The welfare effect of a time-reducing rail investment is given by:

$$\left(\frac{dW}{dI^2}\right)^G = \frac{dW}{dI^2}\Big|_G + \frac{dW}{dG} \frac{dG}{dI^2}$$

The first term on the right hand side can be worked out to yield:

$$\frac{dW}{dI^2}\Big|_G = -\sum_{i=1}^N \lambda_i B_i \frac{\partial f^2}{\partial I^2} - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dI^2}\Big|_G$$

where

$$B_i = -\frac{\frac{\partial V_i}{\partial \phi^2}}{\frac{\partial V_i}{\partial G}} = \left[ (1 + \tau_D) D_i \frac{\partial c_D}{\partial \phi^2} + (1 - \tau_L) e_i (T_i^2 + T_i^4) \right]$$

is the welfare cost of time losses in rail transport. The term  $A_i$  (the welfare cost of time losses in road passenger transport) was defined before. The term  $\frac{d\phi^1}{dI^2}\Big|_G$  will be considered in more detail below.

Also as before, we have:

$$\frac{dW}{dG} = \sum_{i=1}^N \lambda_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dG}$$

Substituting then gives:

$$\left(\frac{dW}{dI^2}\right)^G = -\left[\sum_{i=1}^N \lambda_i B_i\right] \left[\frac{\partial f^2}{\partial I^2}\right] - \left[\sum_{i=1}^N \lambda_i A_i\right] \left[\frac{d\phi^1}{dI^2}\Big|_G\right] + \left[N - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dG}\right] \left(\frac{dG}{dI^2}\right)$$

From the government budget constraint we derive the increase in the lump-sum tax needed to finance a small increase in rail investment:

$$\frac{dG}{dI^2} = -\frac{1 - \frac{\partial R}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dI^2}\Big|_G}{N - \frac{\partial R}{\partial G} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dG}}$$

Substitution and using the cost of funds defined above (see (17)), we have:

$$\left(\frac{dW}{dI^2}\right)^G = -\left[\sum_{i=1}^N \lambda_i B_i\right] \left[\frac{\partial f^2}{\partial I^2}\right] - \left[\sum_{i=1}^N \lambda_i A_i\right] \left[\frac{d\phi^1}{dI^2}\right]_G - MCF_G \left[1 - \frac{\partial R}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dI^2}\right]_G$$

This expression has a nice interpretation. The welfare effect consists of three terms. The first one is the direct benefit of faster rail service for consumers. The second term captures the extra benefit for consumers due to reductions in road congestion induced by rail investment: faster rail service reduces the generalized price of rail relative to road and reduces road demand; this reduces congestion. The third term measures the welfare implications of financing the investment: apart from the direct cost, the investment induces revenue changes. All budgetary implications are measured at the cost of funds.

Further interpretation can be obtained as follows. First, rewrite the last expression as:

$$\left(\frac{dW}{dI^2}\right)^G = -\left[\sum_{i=1}^N \lambda_i B_i\right] \left[\frac{\partial f^2}{\partial I^2}\right] - MCF_G + MCF_G \frac{\partial R}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + \left\{ \left[ MCF_G \frac{\partial R}{\partial \phi^1} - \left(\sum_{i=1}^N \lambda_i A_i\right) \right] \left[\frac{d\phi^1}{dI^2}\right]_G \right\}$$

Now observe that the effect of rail investment on road congestion is given by (see (4)):

$$\frac{d\phi^1}{dI^2}\Big|_G = \left[ \frac{\partial f^1}{\partial T^1} \frac{\partial T^1}{\partial \phi^2} + \frac{\partial f^1}{\partial T^3} \frac{\partial T^3}{\partial \phi^2} + \frac{\partial f^1}{\partial F^1} \frac{\partial F^1}{\partial \phi^2} \right] \frac{\partial f^2}{\partial I^2} + \left[ \frac{\partial f^1}{\partial T^1} \frac{\partial T^1}{\partial \phi^1} + \frac{\partial f^1}{\partial T^3} \frac{\partial T^3}{\partial \phi^1} + \frac{\partial f^1}{\partial F^1} \frac{\partial F^1}{\partial \phi^1} \right] \frac{d\phi^1}{dI^2}\Big|_G$$

The rail investment reduces travel time by rail; this affects commuting and non-commuting passenger road transport as well as freight road transport; hence, congestion on the road rises (see the first term). However, more congestion reduces road transport demand, generating feedbacks, see the second term. Substituting this final expression for the term associated with the  $A_i$ , we find:

$$\begin{aligned} \left(\frac{dW}{dI^2}\right)^G &= -\left[\sum_{i=1}^N \lambda_i B_i\right] \left[\frac{\partial f^2}{\partial I^2}\right] - MCF_G + MCF_G \frac{\partial R}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + \left\{ \left[ MCF_G \frac{\partial R}{\partial \phi^1} \right] \left[\frac{d\phi^1}{dI^2}\right]_G \right\} \\ &\quad - \sum_{i=1}^N \lambda_i A_i \left[ \frac{\partial f^1}{\partial T^1} \frac{\partial T^1}{\partial \phi^2} + \frac{\partial f^1}{\partial T^3} \frac{\partial T^3}{\partial \phi^2} + \frac{\partial f^1}{\partial F^1} \frac{\partial F^1}{\partial \phi^2} \right] \frac{\partial f^2}{\partial I^2} \\ &\quad - \sum_{i=1}^N \lambda_i A_i \left[ \frac{\partial f^1}{\partial T^1} \frac{\partial T^1}{\partial \phi^1} + \frac{\partial f^1}{\partial T^3} \frac{\partial T^3}{\partial \phi^1} + \frac{\partial f^1}{\partial F^1} \frac{\partial F^1}{\partial \phi^1} \right] \frac{d\phi^1}{dI^2}\Big|_G \end{aligned}$$

Finally, note that the effects of time changes in road and rail transport on revenues are given by, respectively:

$$\begin{aligned} \frac{\partial R}{\partial \phi^1} &= \sum_{j=1}^4 \tau_{T^j} \left( \sum_{i=1}^N \frac{\partial T_i^j}{\partial \phi^1} \right) + \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} \\ &+ \left[ \tau_D + \tau_{F^1} F_{ND}^1 + \tau_{F^2} F_{ND}^2 \right] \sum_{i=1}^N \frac{\partial D_i}{\partial \phi^1} + \sum_{i=1}^N D_i \left[ \tau_{F^1} \frac{\partial F_{ND}^1}{\partial \phi^1} + \tau_{F^2} \frac{\partial F_{ND}^2}{\partial \phi^1} \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial R}{\partial \phi^2} &= \sum_{j=1}^4 \tau_{T^j} \left( \sum_{i=1}^N \frac{\partial T_i^j}{\partial \phi^2} \right) + \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^2} \\ &+ \left[ \tau_D + \tau_{F^1} F_{ND}^1 + \tau_{F^2} F_{ND}^2 \right] \sum_{i=1}^N \frac{\partial D_i}{\partial \phi^2} + \sum_{i=1}^N D_i \left[ \tau_{F^1} \frac{\partial F_{ND}^1}{\partial \phi^2} + \tau_{F^2} \frac{\partial F_{ND}^2}{\partial \phi^2} \right] \end{aligned}$$

Substitute these last two equations and use the definitions of marginal external congestion costs given before:

$$MECC_{T^1} = \sum_{i=1}^N \lambda_i A_i \frac{\partial f^1}{\partial T^1}, \quad MECC_{T^3} = \sum_{i=1}^N \lambda_i A_i \frac{\partial f^1}{\partial T^3}, \quad MECC_{F^1} = \sum_{i=1}^N \lambda_i A_i \frac{\partial f^1}{\partial F^1}$$

We then easily show:

$$\begin{aligned} \left( \frac{dW}{dI^2} \right)^G &= - \left[ \sum_{i=1}^N \lambda_i B_i \right] \left[ \frac{\partial f^2}{\partial I^2} \right] - MCF_G \\ &+ \sum_{j=1,3} (MCF_G \tau_{T^j} - MECC_{T^j}) \left[ \frac{\partial T^j}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^2} \right]_G + \sum_{j=2,4} (MCF_G \tau_{T^j}) \frac{\partial T^j}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} \\ &+ (MCF_G \tau_{F^1} - MECC_{F^1}) \left[ \frac{\partial F^1}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^2} \right]_G + (MCF_G \tau_{F^2}) \frac{\partial F^2}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} + (MCF_G \tau_D) \frac{\partial D}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} \\ &+ MCF_G \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^2} \frac{\partial f^2}{\partial I^2} \end{aligned}$$

## **2. The welfare effect of investment in roads financed by raising the tax on labor**

Consider the welfare change due to investing an extra euro in road capacity, financing the investment by raising the labor tax. This welfare effect can be written as:

$$\left( \frac{dW}{dI^1} \right)^{\tau_L} = \frac{dW}{dI^1} \Big|_{\tau_L} + \frac{dW}{d\tau_L} \frac{d\tau_L}{dI^1}$$

It consists of the direct welfare effect of the investment, independent of financing plus the induced effects due to financing. Differentiating the welfare function, using Roy's identity and the time constraint of the consumer implies that we can write:

$$\frac{dW}{dI^1} \Big|_{\tau_L} = - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{dI^1} \Big|_{\tau_L}$$

$$\frac{dW}{d\tau_L} = - \sum_{i=1}^N \lambda_i e_i L_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_L}$$

Note that the welfare effect of a labor tax change consists of the weighted effect on employment plus the indirect effects via congestion. Using these results and rearranging gives:

$$\left( \frac{dW}{dI^1} \right)^{\tau_L} = \left[ - \sum_{i=1}^N \lambda_i (A_i) \right] \left[ \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \right] + \left[ - \sum_{i=1}^N \lambda_i e_i L_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_L} \right] \left( \frac{d\tau_L}{dI^1} \right)$$

From the government budget constraint we derive the increase in the labor tax needed to finance a small increase in road investment. We find:

$$\frac{d\tau_L}{dI^1} = - \frac{1 - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L}}{\frac{\partial R}{\partial \tau_L} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{d\tau_L}}$$

Define the marginal cost of raising a euro through the labor tax as:

$$MCF_{\tau_L} = \frac{\frac{dW}{d\tau_L}}{\frac{d \text{Rev}}{d\tau_L}} = \frac{- \sum_{i=1}^N \lambda_i e_i L_i - \sum_{i=1}^N \lambda_i A_i \frac{d\phi^1}{d\tau_L}}{\frac{\partial R}{\partial \tau_L} - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{d\tau_L}}$$

Using the last two expressions we can rewrite the welfare effects of investment:

$$\left( \frac{dW}{dI^1} \right)^{\tau_L} = \left[ - \sum_{i=1}^N \lambda_i (A_i) \right] \left[ \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \right] - MCF_{\tau_L} \left[ 1 - \frac{\partial R}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \right]$$

Interestingly, this only differs from the expression for the welfare effects of road investment financed by a reduction in the transfer by the fact that the marginal cost of funds differs (compare with expression (18)). Going through the same procedures as before (i.e., working out the revenue effects and the congestion term) we finally get:



$$\begin{aligned}
\left(\frac{dW}{dI^1}\right)^{\tau_L} &= \left[-\sum_{i=1}^N \lambda_i(A_i)\right] \left[\frac{\partial f^1}{\partial I^1}\right] - MCF_{\tau_L} \\
&+ \sum_{j=1,3} (MCF_{\tau_L} \tau_{T^j} - MECC_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} + \sum_{j=2,4} (MCF_{\tau_L} \tau_{T^j}) \frac{\partial T^j}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \\
&+ (MCF_{\tau_L} \tau_{F^1} - MECC_{F^1}) \frac{\partial F^1}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} + (MCF_{\tau_L} \tau_{F^2}) \frac{\partial F^2}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} + (MCF_{\tau_L} \tau_D) \frac{\partial D}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L} \\
&+ MCF_{\tau_L} \tau_L \sum_{i=1}^N e_i \frac{\partial L_i}{\partial \phi^1} \frac{d\phi^1}{dI^1} \Big|_{\tau_L}
\end{aligned}$$

Again, this is identical to what we had before, except for the use of a different marginal cost of funds.

### **3. Structure of the numerical example**

There are  $n$  identical individuals. Let the typical consumer maximize utility subject to a budget and time constraint:

$$\underset{C,T,l}{Max} \quad u(C,T,l) = C + (a_1 T - \frac{1}{2} a_2 T^2) + (b_1 l - \frac{1}{2} b_2 l^2) \quad (A1)$$

subject to

$$\begin{aligned}
C + r(1 + \tau)T &= (1 - t)wL + G \\
l + L + \gamma T &= \bar{L} = 1
\end{aligned} \quad (A2)$$

where  $C, T, l$  are, respectively, a numeraire non-transport good, passenger transport and leisure. We have normalized available time at one. Moreover, transport faces congestion; the time needed per unit of  $T$ , denoted  $\gamma$ , depends on the traffic level and on capacity. We assume, in line with the bottleneck model, a linear specification as function of the volume-capacity ratio:

$$\gamma = f(T, I) = c + \frac{mT * n}{I} \quad (A3)$$

Here  $I$  is interpreted as the annuitized transport investment.

Combining budget and time constraint yields:

$$C + PT + (1 - t)wl = (1 - t)w + G \quad (A4)$$

where  $P$  is the generalized price of transport:

$$P = r(1 + \tau) + w(1 - t)\gamma \quad (A5)$$

Maximizing (A1) subject to (A4) gives the demand functions for transport and leisure:

$$T = \frac{a_1 - P}{a_2}, \quad l = \frac{b_1 - w(1-t)}{b_2} \quad (\text{A6})$$

Substitution of (A4) and (A6) in (A1) gives indirect utility. It can be written as:

$$v(P, t, w, G) = G + w(1-t) + \frac{1}{2a_2} [a_1 - P]^2 + \frac{1}{2b_2} [b_1 - w(1-t)]^2 \quad (\text{A7})$$

Now consider the welfare effect of a budgetary neutral increase in capacity investment  $I$ , where the investment increase is financed by adjusting the tax on labor:

$$\frac{dv}{dI} = \left. \frac{dv}{dI} \right|_t + \frac{dv}{dt} \frac{dt}{dI} \quad (\text{A8})$$

The first term on the right hand side can be written, using (A3), (A5) and (A7):

$$\left. \frac{dv}{dI} \right|_t = -T(1-t)w \frac{d\gamma}{dI} \quad (\text{A9})$$

Similarly, the effect of a labor tax increase on welfare is, see (A7), (A5) and (A6):

$$\frac{dv}{dt} = -w + \left( \frac{a_1 - P}{a_2} \right) \left( -\frac{dP}{dt} \right) + w \left[ \frac{b_1 - w(1-t)}{b_2} \right] = -w - T \left[ \gamma + wt \frac{d\gamma}{dt} \right] + wl \quad (\text{A10})$$

Finally, the labor tax change needed to finance the extra investment is derived from the government budget constraint:

$$twL + \tau rT = G + I \quad (\text{A11})$$

Reformulating and using the time constraint of the consumer in (A2) yields the implicit function

$$R(t, I) = G + I - wt(1-l - \gamma T) - \tau rT = 0 \quad (\text{A12})$$

It follows:

$$\frac{dt}{dI} = - \frac{\frac{dR}{dI}}{\frac{dR}{dt}} = - \frac{1 + wt \left[ \gamma \frac{dT}{dI} + T \frac{d\gamma}{dI} \right] - \tau r \frac{dT}{dI}}{-w + wt \frac{dl}{dt} + wl + \gamma wT + twT \frac{d\gamma}{dt} + (tw\gamma - \tau r) \frac{dT}{dt}} \quad (\text{A13})$$

Define the marginal cost of funds of raising extra tax revenues by raising the labor tax as the welfare cost per extra euro raised:

$$MCF = \frac{\frac{dv}{dt}}{\frac{dR}{dt}} \quad (A14)$$

Note an analogous expression can be defined for the marginal cost of alternate tax instruments, including the congestion toll. Combining (A13)-(A14) gives:

$$\frac{dv}{dt} \frac{dt}{dI} = \frac{\frac{dv}{dt}}{\frac{dR}{dt}} \frac{dR}{dt} \left[ -\frac{\frac{dI}{dR}}{\frac{dI}{dt}} \right] = -MCF \frac{dR}{dI} \quad (A15)$$

Next, substitute (A9) and (A15) into the definition of the total welfare effect (A8), and use (A13). We find:

$$\frac{dv}{dI} = -T(1-t)w \frac{d\gamma}{dI} - MCF \left[ 1 + tw \left[ \gamma \frac{dT}{dI} + T \frac{d\gamma}{dI} \right] - \tau r \frac{dT}{dI} \right] \quad (A16)$$

Note from (A5)-(A6) that

$$\frac{dT}{dI} = -\frac{1}{a_2} \frac{dP}{dI} = -\frac{1}{a_2} (1-t)w \frac{d\gamma}{dI}$$

and substitute into (A16):

$$\frac{dv}{dI} = -T(1-t)w \frac{d\gamma}{dI} - MCF + MCF * tw \left[ \frac{\gamma(1-t)w}{a_2} - T \right] \frac{d\gamma}{dI} - MCF * \tau r \left( \frac{(1-t)w}{a_2} \right) \frac{d\gamma}{dI} \quad (A17)$$

This is the perfect analogy to expression (19) in the theory section, given that we only have transport and labor markets and no distribution. The term MCF is here interpreted for a labour tax – but note it can equally refer to an alternate financing instrument.

Finally, to get the equivalent of (21) in the theoretical section, we use the definition of the time cost per travel time given in (A3) to get:

$$\frac{d\gamma}{dI} = \frac{\partial f(T, I)}{\partial I} + \frac{\partial f(T, I)}{\partial T} \frac{\partial T}{\partial \gamma} \frac{d\gamma}{dI} = -\frac{mTn}{I^2} - \frac{mn}{I} \left( \frac{(1-t)w}{a_2} \right) \frac{d\gamma}{dI} \quad (A18)$$

Substitute in the first term on the right hand side of (A17), use the definition of the marginal external cost:

$$MEC = T(1-t)w \frac{mn}{I} \quad (A19)$$

and work out. This leads to the final expression for the welfare effect:

$$\frac{dv}{dI} = mn(1-t) \left[ \frac{T}{I} \right]^2 - MCF + MCF * tw \left[ \frac{\gamma(1-t)w}{a_2} - T \right] \frac{d\gamma}{dI} - [MCF * \tau r - MEC] \left( \frac{(1-t)w}{a_2} \right) \frac{d\gamma}{dI} \quad (A20)$$

The first term of (A20) is the direct effect at constant transport demand (see (A18)), the second term is the marginal cost of funds, the third term captures the effect of investment on labor tax revenues, and the final term captures the impact on transport tax revenues corrected for marginal external costs. Expression (A20) is the perfect analogy to formula (21). To see this, it suffices to note that, using the time constraint,

$$\frac{\partial L}{\partial \gamma} = \left[ \frac{\gamma(1-t)w}{a_2} - T \right]$$

Moreover, we have 
$$\frac{\partial T}{\partial \gamma} = - \left[ \frac{(1-t)w}{a_2} \right]$$