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by

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Public Economics

Center for Economic Studies
Discussions Paper Series (DPS) 08.30
http://www.econ.kuleuven.be/ces/discussionpapers/default.htm

November 2008

DISCUSSION
PAPER

Identity and Educational Choice: A Behavioral Approach

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Abstract

It is puzzling that socioeconomic background greatly affects educational choice. Distinguished from the explanations based on expected utility theory, this paper attempts to explore the psychological mechanisms of generating educational identity[1] and schooling choice. It offers a self-signaling model where (1) it incorporates self-esteem concerns into the agent's payoff function, (2) the investment in schooling not only signals her cognitive ability but also brings the agent into cognitive dissonance and reduction when the perceptions of ability are time-dependent. Using this model, I show a more discriminating analysis of educational choice which combines multi-dimensional factors including socioeconomic background, cognitive and non-cognitive abilities. I identify the conditions under which the high ability agent fails to invest in education. The quality of school and the preschooling are key variables. The model suggests that public policy can help poor children by improving both the early and later education quality at school.

Keywords: identity, educational choice, poverty

JEL: D81, I30

[1]Educational identity refers to how one perceives one's own ability, the intrinsic cognition of own ability and the motivation to engage in education. It is generated by adding the analysis of emotional functionings such as self-esteem.

1 Introduction

Since the end of 1970s, overall wage inequality and educational differentials have expanded in most OECD countries. This trend is coupled with a rise in the pay-off to high education and skills because the demands for such labors exceed the inadequate supply (Atkinson, 2003). Despite substantial increasing premiums to high education, the college participation rates in the US increase more sharply in the high income groups than in the low ones. The big puzzle is why, if to improve education and skills becomes more financially rewarding, there are still so high and fairly constant drop-out rates at high school among the economically disadvantaged youth, especially when some of them are intellectually qualified?

One explanation to the puzzle refers to imperfect capital markets. This imperfection results in short term credit constraints that prevent the poor from affording an expensive education. Using data from the US and European countries, Carneiro and Heckman (2002), Blanden and Gregg (2004) and Vandenberghe (2007) conclude that only a very small proportion of the population in developed countries face such problems to finance college education. Another explanation claims that poor children lack a good cultural and social environment which is crucial to foster their cognitive, non-cognitive abilities and motivations required for success at school. For example, Fordham and Ogbu (1986) emphasize the role of culture and argue for the existence of an oppositional culture in the non-dominating social groups that prohibits behaviors traditionally seen as the prerogative for their mainstream counterparts.

These theories have shed some light on the relationship between persistent poverty and schooling choice. However, three aspects of recent empirical investigations about group poverty question their effectiveness and require a more concrete and discriminating theoretical analysis. Firstly, Burtless (1996) and Kozol (1991) point out that children living in a poor segregated community may only enter low quality schools with insufficient school expenditure on facilities and teachers. Low quality schools are poor in both teaching valuable skills and promoting a student's ideal image close to economically useful cultural norms. This negatively affects the outcomes of children's education. (Akerlof and Kranton (2002)) Secondly, there exist peer effects leading to imitative behaviors in disadvantaged neighborhoods. Elliott et al (2006) find that individual-level successes significantly own to good parenting practice, positive climate of school and peer groups. Thirdly, apart from cognitive ability, individual characteristics such as motivations affect the eductional level one can achieve. According to Carneiro and Heckman (2003), the percentage of drop-outs who are intellectually competent with GED certificate has exceeded 30% in the 17-year-old population. Heckman, Stixrud and Urzua (2006) highlight that non-cognitive abilities, measured as the degree of control individuals feel over their life and perceptions of self-worth, are important for their educational achievements.

This study proposes a behavioral model which allows for an inclusive discussion of the relationship between cognitive and non-cognitive abilities, socioeconomic status and educational choice. For the sake of simplicity, the cognitive ability is assumed to be independent of one's family. Non-cognitive abilities such as educational motivations are closely related to one's socioeconomic background. The latter often decides about the stock of preschool, the quality of school and the social environment one faces. (see Figure 1)

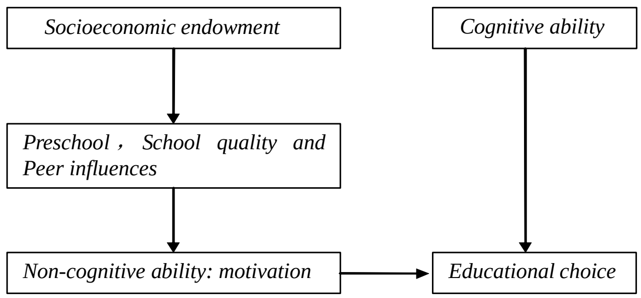

Figure 1. Educational choice, cognitive and non-cognitive abilities

Instead of assuming a highly rational agent who is competent for assessing future earning distributions, this paper assumes that emotions greatly influence judgement, and individuals' subjective beliefs about returns of schooling are often ambiguous and diverge from rational evaluations. Much of the new work in fields such as neuroscience and social psychology highlights the role played by emotions as informational and motivational inputs into the decision making process. Motivated by these insights, this paper on the one hand incoporates a self-esteem demand into the utility function. On the other hand, the model considers the supply side of the self-esteem. Investing in education is assumed to self-signal some good or bad news about the level of cognitive ability. As the perceptions of the cognitive ability are time-dependent, the agent experiences cognitive dissonance. In order to reduce the dissonance, the agent needs to ignore and reinterpret cognitions by modifying her memory at a cost. This demand-supply psychological process allows the agent to adapt her preference for schooling and finally form a stable educational identity. This approach of modelling is in line with Benabou and Tirole(2006), Oxoby(2004), Bodner and Prelee (2003).

Departing from Akerlof and Kranton (2000 & 2002) who introduce exogenous psychological costs related to one's social category, this paper attempts to endogenize the psychological costs by exploring the demand-supply psychological mechanism of self-esteem pursuit. The central conclusion of this paper is that the high ability agent could fail to invest in education, as cognitive ability is not the only reason to invest in education. One adapts her own preference of schooling according to the specific economic and social scenario she faces. The most influential factors are school quality, preschool education and social environment (progressive or conservative). The school quality is evaluated with respect to how much market-valued skills and knowledge the school imparts and the student's salience of self-esteem concerns of educational identity the school promotes. When the school quality is high enough, the agent invests in education. When the school quality is insufficient, there exist differing psychological self-fulfilling alternatives determined by her preschool education and social environment leading the agent to either investment or not. The threshold of preschool edcuation for the agent to invest is sensitive to the initial beliefs about the distribution of ability type. The paper is organized as follows: section 2 reviews the theory of identity and its empirical evidence on education which rejects the "rational expectation" assumption of expected utility theory; the model of the formation of educational identity is in section 3 where the quality of school, the preschool and initial beliefs about ability distribution matter in educational choice; section 4 endogenizes the memory and shows how social environment affects the resulting equilibria; it concludes in section 5.

2 Educational decision under uncertainty

Investment in education is risky. The uncertainty of the returns to schooling in the labor market mainly comes from three types of risk. Firstly, the individual experiences market risks. In a typical dynamic economy frequently exposed to technical and orgnizational changes, labor supply shocks, etc, the value of human capitals and skills often shifts over time. As a result, workers with the same educational levels may receive different wages. Secondly, the individual cannot be sure that she is able to successfully accomplish the education. Thirdly, given her cognitive ability, the individual also cannot predict what her relative position in the post-education earnings distribution will be. Bowles et al (2001a&b) claim that even in a "Walrasian equilibrium" workers with the same productivity may not obtain the same wage because of informational asymmetry that the employers do not know the worker's labor efforts. To reduce the cost of supervision, the employers pay differently to workers with different incentive-enhancing preferences which are independent of the cognitive ability. Using a collection of available empirical surveys, Bowles et al (2001b) estimate that, on average, only 18% of the returns to schooling are generated by cognitive abilities while a substantial portion, roughly 82%, remain unexplained.

Based on the expected utility theory, Levhari and Weiss (1974) adopt a standard model for schooling choice under uncertainty. The theory relies on a crucial assumption that subjective and objective beliefs of the wage distribution coincide. However, in reality, individuals often fail to obtain precise knowledge of wage distribution and instead create a subjective judgement with biases and errors.

An increasing amount of literature has cast doubt on the general validity of expected utility theory in disentangling the puzzle of schooling choice. Firstly, evidence especially from psychology and brain science (for a general review, see Loewenstein et al. (2001)) supports that emotional reactions to an uncertain situation often diverge from cognitive evaluation of the uncertainty and risk. Secondly, economists such as Dominitz and Manski (1996) have designed a computer-based survey of high school students and college undergraduates in US to elicit the various earning expectations and beliefs about earnings. Their main findings show a common opinion among the respondents that one's own future earnings are rather uncertain. A college degree is believed to have a positive effect on earnings, but the respondents are more uncertain about their earnings conditional on a bachelor's degree than that on a lower degree. There is also substantial within-group (e.g. the female high school group) variation in earning expectations and in beliefs about current earnings distributions. When asked about the distribution of earnings, most respondents tend to overestimate the degree of inequality in American society. Finally, Carneiro, Hansen and Heckman (2003), Cunha, Heckman, and Navarro (2005 & 2006) have identified the importance of psychological costs in the choice of schooling when accounting for uncertainty and sequential revelation of information.

If the subjective beliefs of wage distribution are different from the objective beliefs, the question arises what determines the formation of subjective beliefs about the returns to schooling. Psychological studies have long documented the profound effects of self-esteem on individuals' thinking process and action choice (Nathaniel Branden, 1969). Individuals experience self-esteem in the form of emotional feelings. This emotion is a product of self-evaluation of the beneficial or harmful effects of some aspects of reality on the individual. The feeling of liking and disliking serves as information directly influencing the decision process. For the individual uncertain about where she stands with respect to her own ability, each new choice can provide a bit of good or bad "news".

In response to the growing psychological literature on self-esteem, Akerlof and Kranton (2002) are the first to emphasize the psychological aspects of educational choice by introducing exogenous psychological gains and costs determined by their own social category. They propose a utility function that incorporates "identity" (self-image concerns) as a motivation for educational behavior. Identity, associated with a certain social category, defines how people in this category should behave. They also claim that each social category imposes an "identity" on its members, which creates the relevant psychological and social costs when the individuals violate the identity. The psychological and social costs are derived from (1) the difference between the agent's own characteristics and the ideal of the assigned category, (2) the difference between the agent's schooling choice and the schooling level in the ideal social category. Wichardt (2007) uses an evolutionary argument to justify the preference for identity-consistent behavior. Human beings are psychologically programmed to conform to certain social norms in their group, which could greatly facilitate the long-run cooperations with other group members.

Based on the previous utility settings, Akerlof and Kranton (2002) then construct a game-theoretic model where schools promote a single social category and the students choose between the ideal "academic identity" and an identity fitting their social backgrounds. When the students hold two contradictory ideas simultaneously, the phenomena is termed "cognitive dissonance". When experiencing this dissonance, individuals have a fundamental cognitive drive to reduce it by modifying the existing belief, or by rejecting one of the contradictory ideas at a psychological cost. A significant example is when the cognitive dissonance is so large that the psychological costs of keeping an ideal "academic identity" are greater than the benefits of future wages and of an ideal self-image. Students from low social class are often trapped in such a situation and then reject the main-stream schools and choose to be drop-outs. It is necessary to point out that the Akerlof-Kranton model is insightful in explaining why some individuals fail in educational achievements. However, this model also suggests that there is not much we can do to reduce the psychological costs which hamper the poor students to escape poverty. Further research is necessary to understand the nature of the psychological costs. This will allow us to identify instruments that the public policy can use to help the poor.

3 The Formation of an "Educational Identity"

3.1 The Assumptions

3.1.1 Endowment

There are three stages in the model depicting the formation process of an educational identity. At stage 0, the agent receives some signal σ indicating the agent's cognitive ability type as θ_i (either θ_H or θ_L). This signal σ contains information such as the result of an IQ test by professional valuations or judgements based on the daily learning activities etc[2]. Assume that the cognitive type (θ_H, θ_L) is exogenously distributed in the population and independent of socioeconomic background.

$$\theta_i = \begin{cases} \theta_H & \text{with probability } \rho \\ \theta_L & \text{with probability } 1-\rho \end{cases} \tag{1}$$

At stage 0, the agent has also formed initial subjective beliefs $(\rho, 1-\rho)$ over the cognitive type (θ_H, θ_L) as shown in (1). The subjective beliefs $(\rho, 1-\rho)$, however, are influenced by her own family, communities and social networks. The lower ρ is, the more pessimistic the agent becomes from the social communities. For example, when faced with a number of academic failures around her, the agent from a disadvantaged community has a lower prior belief ρ that she could be high ability type than her counterparts in an affluent community. Moreover, the agent is endowed with some preschool education $S_0 > 0$, such as intellectual recognitions from parents, or successful performances and experiences of acquiring knowledge, etc.

[2] θ_i is the cognitive type the agent perceives upon signal σ at stage 0; at stage 1, based on a new signal, the agent also gets another perception which might be different from θ_i. The agent's educational decision is informationally and motivationally oriented. Therefore, the true cognitive ability of the agent is not discussed in this paper.

3.1.2 Self-perceived ability from action a

At stage 1, the agent makes her investment decision of education $a \in \{0, 1\}$ at school. The investment cost of schooling c_i depends only on her cognitive ability[3] shown in (2). The high cognitive type agent spends less time and efforts than the low type to accomplish the same tasks, performances and degrees. With a constant return r to the educational investment a, the stock of schooling thus adds to S_1. The return r is independent of the ability type, but only affected by the school teaching quality, with respect to the market-valued skills and knowledge it imparts to the agent.[4] Conditional on θ_i, the returns to schooling the agent expects to receive in the future labor market is $\theta_i S_1$. These returns to schooling are determined only by cognitve factors.

$$c_i = \begin{cases} c_H & \text{for the high ability} \\ c_L & \text{for the low ability} \end{cases}, \quad c_H < c_L \tag{2}$$

$$S_1 = S_0 + ar \tag{3}$$

Distinguished from its conventional dual-role[5], putting efforts and investing in schooling now becomes a process of exploring and reevaluating one's own ability. Action a is purely self-signaling, from which the agent adopts the perspective of an outside observer to infer the level of her own ability. Figure 2 shows the extensive form of a two-stage self-signaling game. To be specific, at stage 1, without knowing her true ability, the agent only holds prior beliefs about her type (ρ, $1 - \rho$) with respect to (θ_H, θ_L). At stage 2, the agent gains new evidence from action $a = 0$ or 1 and infers this ability. For θ_H, the probabilities of investing (I) and not investing (NI) in schooling are x_H and $1 - x_H$; for θ_L, the corresponding probabilities are x_L and $1 - x_L$. After action a, the agent changes her beliefs from $(\rho, 1 - \rho)$ to $(\widehat{\rho}(a), 1 - \widehat{\rho}(a))$. Assume the agent is sophisticated enough to calculate Bayesian probability, using Bayes' rule, the relationship between the posterior and the prior beliefs are described below in (4) and (5). As a result, the perceived ability changes to $v(a) \in [\theta_L, \theta_H]$ in (6) and (7).

[3] Assume a perfect credit market so that only ability matters and there is no credit constraint for the low-income kids.

[4] In the relatively short-run, we reasonably assume that the market environment doesn't change. At school, the uncertainties of the returns to schooling do not come from market risks.

[5] On the one hand, schooling is basically regarded as a process for skill and human capital accumulations which has a direct impact on labor productivity. On the other hand, schooling demonstrates one's ability to future employers in the labor market. Spence(1973) describes how agents choose different levels of education to signal a known ability in the labor market.

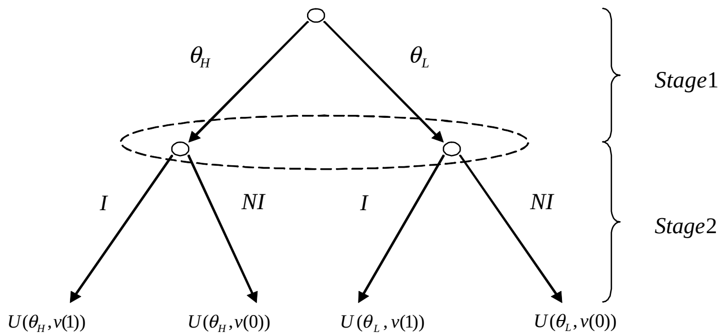

Figure 2. Two-stage self-signalling game

$$\widehat{\rho}(1) = \frac{\rho x_H}{\rho x_H + (1-\rho)x_L} \tag{4}$$

$$\widehat{\rho}(0) = \frac{\rho(1-x_H)}{\rho(1-x_H) + (1-\rho)(1-x_L)} \tag{5}$$

$$v(1) = \widehat{\rho}(1)\theta_H + (1-\widehat{\rho}(1))\theta_L \tag{6}$$

$$v(0) = \widehat{\rho}(0)\theta_H + (1-\widehat{\rho}(0))\theta_L \tag{7}$$

3.1.3 Memory and Self-esteem Utility

Apart from the long-term monetary returns to schooling $\theta_i S_1$, the agent also takes into account the self-esteem reasons for engaging in schooling at stage 2. From a psychological point of view, the agent has a dignified and motivated need to invest in schooling. This need becomes a source of pleasure and intrinsically affects her sense of emotional well-being. The higher the cognitive ability the agent perceives, the better self-image and the higher self-esteem she enjoys. However, the cognitions of her ability are ambivalent and time-dependent. At stage 0, the agent regards her ability as θ_i from the past signal σ, while at stage 2, she reinterprets her ability as $v(a)$ in the light of the new evidence "action

a". The two cognitions may conflict each other and the agent experiences a psychological conflicting state termed as "cognitive dissonance".

To reduce the amount of dissonance, there is some information loss from stage 0 (before action a) and some added information from action a. As a result, the educational identity (an intrinsic cognition of the ability) combines both θ_i and $v(a)$, which is written as $(\lambda\theta_i + (1-\lambda)v(a))$. Variable λ objectively measures the memory lapse the agent experiences about θ_i at stage 2. To be specific, at stage 2, the agent is aware of her initial cognition θ_i with probability λ, she no longer recalls it and replaces it to the new cognition $v(a)$ with probability $1-\lambda$. Assume that memory can be imperfect and $0 \leq \lambda \leq 1$. Furthermore, this λ also meaningfully describes the degree of difference between being a "progressive" type ($\lambda = 0$, only looking at the updated evidence action a about her cognitive ability) and a "conservative" type ($\lambda = 1$, only recalling the past experience about her cognitive ability). The larger λ is, the more conservative the agent is, the more the agent regards her initial cognition θ_i more important than the new one $v(a)$, and vice versa. Extreme cases are when $\lambda = 0$ or 1, the agent totally eliminates the dissonance and conflict, and realizes absolute consistency.

Assume the agent at stage 0 has optimally assigned weights λ and $1 - \lambda$ to θ_i and $v(a)$. The value of λ is chosen as a trade-off between the agent's social pressure influences and her pursuit of individual interests. The endogeneity of this variable λ is discussed in the later section. At stage 1, given memory variable λ, the total utility combining the monetary returns $\theta_i S_1$ and educational identity-specific emotion $q(\lambda\theta_i + (1-\lambda)v(a))S_1$ is indicated in (8). Note that savouring parameter $q > 0$ measures the salience of the agent's self-esteem concerns of her educational identity at stage 2, which is significantly related to how strongly the school shapes its students to conform to its educational cultures and norms. For a brief overview of the agent's actions and utilities, refer to table 1.

$$\theta_i S_1 + q(\lambda\theta_i + (1-\lambda)v(a))S_1 \tag{8}$$

Stage 0	Stage 1	Stage 2
(1) Signal σ indicates the cognitive ability $\theta_i \in \{\theta_H, \theta_L\}$ (2) Initial subjective beliefs $(\rho, 1-\rho)$ over (θ_H, θ_L), influenced by socioeconomic background. Preschool education S_0 (3) Optimally choose recall rate λ : with probability λ the agent is aware of her initial cognition θ_i, with probability $1-\lambda$ she adopts a new cognition $v(a)$.	School decision: $a = 0$ or 1, to maximize total utility in stage 2	Total Utility includes the monetary returns and the self-esteem concerns: $[\theta_i + q(\lambda\theta_i + (1-\lambda)v(a))](S_0 + ar) - ac_i$ (1) The new motivation $v(a)$ is perceived using Bayes' Rule. (2) r is the imparted skills and human capital after investment at school. (3) The investment cost $c_i \in \{c_H, c_L\}$ (4) q is the parameter measuring self-esteem or identity concerns.

Table 1: Settings and actions in the three stages

3.2 Investment decision

3.2.1 Maximization and self-signaling equilibria

At stage 1, the agent optimally selects action a, taking into account not only the long-term cognitive component of the returns to schooling, but also her anticipated self-esteem experienced from action a at the stage 2. Therefore, the solution to a maximizes the difference between total utility from schooling and the incurred cost:

$$\underset{a \in \{0,1\}}{Max}\, U(\theta_i, a, \lambda, S_0) = [\theta_i + q\,(\lambda\theta_i + (1-\lambda)v(a))]\,(S_0 + ar) - ac_i \qquad (9)$$

The agent chooses to invest $a = 1$, when the total payoffs from investing are larger than not investing. The decision criterion (the incentive to invest) writes

$$\begin{aligned} W_i &= U(\theta_i, 1, \lambda, S_0) - U(\theta_i, 0, \lambda, S_0) \qquad (10) \\ &= \theta_i r + q\lambda\theta_i r + q(1-\lambda)\,[v(1)(S_0 + r) - v(0)S_0] - c_i \geq 0 \end{aligned}$$

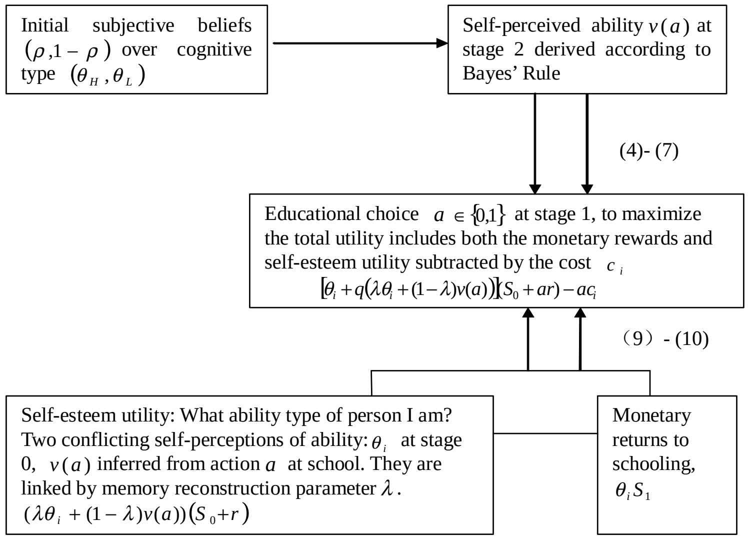

Figure 3. Perfect Bayesian Equilibrium in Self-signalling Game

In order to obtain a Perfect Bayesian Equilibrium in this self-signaling game, assume that the agent is rational in the two senses: she maximizes her utility in (9) and there is a set of strategies and beliefs $(a, \widehat{\rho}(a))$ (see equation (4)-(7))such that, at any stage of the game, strategies are optimally given the beliefs, and the beliefs are obtained from equilibrium strategies and observed actions using Bayes' rule. (see Figure 3)

Proposition 1 *Given a set of parameters* $(\theta_H, \theta_L, c_H, c_L)$*, internal traits* (q, ρ, λ)*, external constraints* (S_0, r)*, there exists a uniquely dominating equilibrium under the following specific conditions:*

Equilibrium A: "no investment equilibrium", $x_H = x_L = 0$, with $v(0) = \overline{v}$[6] and $v(1) = \theta_H$[7]*,when*

$$\theta_H r + q\lambda\theta_H r + q(1-\lambda)\left[\theta_H(S_0 + r) - \overline{v}S_0\right] \leq c_H \tag{C1}$$

Equilibrium B: "separating equilibrium", $x_H = 1, x_L = 0$*, with* $v(1) = \theta_H$ and $v(0) = \theta_L$*,when*

$$\theta_H r + q\lambda\theta_H r + q(1-\lambda)\left[\theta_H(S_0 + r) - \theta_L S_0\right] \geq c_H \tag{C2}$$

$$\theta_L r + q\lambda\theta_L r + q(1-\lambda)\left[\theta_H(S_0 + r) - \theta_L S_0\right] \leq c_L \tag{C3}$$

[6] Define $\overline{v} = \rho\theta_H + (1-\rho)\theta_L$.

[7] Equation (4) and (6) is invalid in the "no investment" equilibrium. Now reasonably suppose that given the agent invests in schooling, $v(1) = \theta_H$ in equilibrium A.

Equilibrium C: "semi-separating equilibrium", $x_H = 1, 0 < x_L < 1$, with $v(0) = \theta_L, \overline{v} < v(1) < \theta_H$,*when*

$$\theta_L r + q\lambda\theta_L r + q(1-\lambda)\left[v(1)(S_0 + r) - \theta_L S_0\right] = c_L \tag{C4}$$

Equilibrium D: "full investment equilibrium", $x_H = x_L = 1$, with $v(0) = \theta_L$[8]*, $v(1) = \overline{v}$,when*

$$\theta_L r + q\lambda\theta_L r + q(1-\lambda)\left[\overline{v}(S_0 + r) - \theta_L S_0\right] \geq c_L \tag{C5}$$

Proposition 1 tells us that the agent only falls into one of the equilibria A, B, C or D. It depends on which of the conditions (C1)-(C5) are satisfied considering her external constraints (S_0, r) and internal traits (q, ρ, λ)[9]. In figure 4, A, B, C and D respectively represent the space of the four equilibria. The boundaries separating the four equilibria are λ_A, λ_B and λ_D (from the right to the left). They are derived from conditions (C1), (C3) and (C5). A particular equilibrium is a result of the agent's S_0, r and λ.

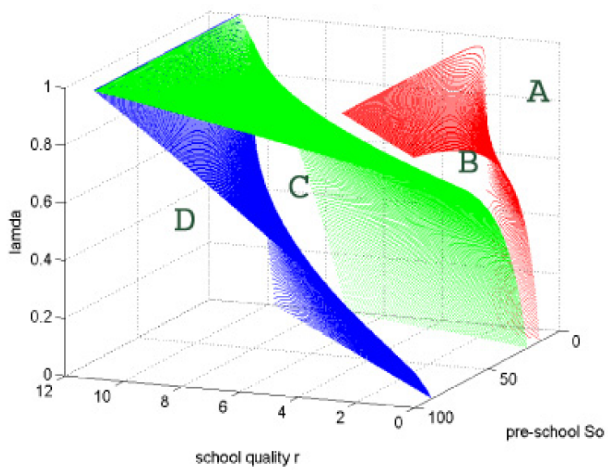

Figure 4. Four equilibria, S_0, r and λ

3.2.2 The relationship between λ, r and S_0 and the equilibria

Proposition 2 *The probability of investing increases when:*

(1) the preschool education S_0 increases
(2) the teaching quality r increases

[8]Equation (5) and (7) are invalid in the "full investment" equilibrium. Now reasonably suppose that given the agent does not invest in schooling, $v(0) = \theta_L$ in equilibrium D.

[9]There are situations which satisfy both condition (C1) and (C2). For this overlapping part, the agent is better-off in equilibrium A of no investment than equilibrium B. Therefore, the agent chooses not to invest. For details, refer to the appendix.

(3) the more progressive (the lower λ) the agent is

In Figure 4, we can see that (1) for the high ability agent, as S_0 (or r) increases or λ decreases, the agent is more likely to be in space B, C or D than in space A, (2) for the low ability agent, as S_0 (or r) increases or λ decreases, the agent is more likely to be in space C or D than in space A or B.

Corollary 3 *Given the parameters $(\theta_H, \theta_L, c_H, c_L)$, internal traits (q, ρ, λ), there exist three scenarios according to the school quality:*

(1) At good quality schools where $r \geq \frac{c_L}{(1+q)\theta_L}$, only the "full investment" equilibrium D exists (Figure 5).

(2) At medium quality schools where $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}$, there exist three potential equilibria B, C and D (Figure 6).

(3) At poor quality schools where $r < \frac{c_H}{(1+q)\theta_H}$, there exist four potential equilibria A, B, C and D[10] (Figure 7).

In Figure 5, as the quality of the school is good, both the high and the low type can acquire enough market-valued skills and knowledge. Therefore, it is beneficial to invest in schooling.

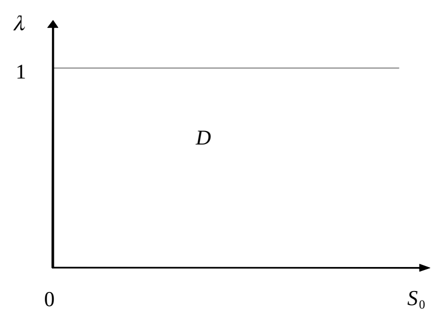

Figure 5. Full investment when $r \geq \frac{c_L}{(1+q)\theta_L}$

In Figure 6, where the school quality is medium, the agent is in equilibria B, C or D. The high type always invests as the returns to education at the medium quality school is profitable to her. For the low type, the equilibrium she is in depends on (S_0, λ). Now assign a value to λ, as shown in the figure, when $0 < S_0 \leq S_0^{\lambda_B}$, the low type agent is in equilibrium B where she does not invest; when $S_0^{\lambda_B} < S_0 < S_0^{\lambda_D}$, she is in equilibrium C where she invests randomly at probability p[11];when $S_0 \geq S_0^{\lambda_D}$, she is in equilibrium D where she invests. Figure 6 also shows that when r increases, the equilibrium boundaries λ_B and λ_D shift to the left, therefore equilibrium B shrinks while the domain of the equilibria C and D expands.

[10] The boundaries separating equilibria A, B, C and D are, respectively, λ_A, λ_B and λ_D. They are shown in the following two dimensional figure 6 and 7.

[11] Probability $p = \frac{\rho}{1-\rho}(\frac{q(\theta_H - \theta_L)(\lambda-1)(S_0+r)}{-c_L+(1+q)r\theta_L} - 1)$ which is derived from condition (C4)

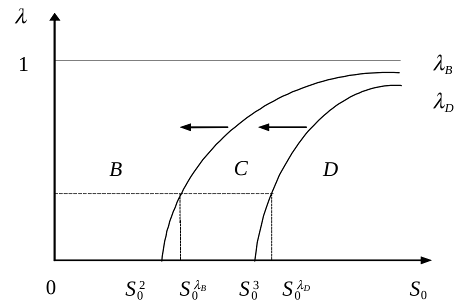

Figure 6. Three equilibria as r increases when $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}$

In Figure 7, where the school quality is poor, the agent is in equilibria A, B, C or D. The equilibrium she is in depends on (S_0, λ). Assign a value to λ,as shown in the figure, for the high type, only when $S_0 \geq S_0^{\lambda_A}$ she invests, otherwise she does not invest. For the low type, when $0 < S_0 \leq S_0^{\lambda_B}$, she is in equilibria A or B where she does not invest; when $S_0^{\lambda_B} < S_0 < S_0^{\lambda_D}$, the low type is in equilibrium C where she invests randomly at probability p;when $S_0 \geq S_0^{\lambda_D}$, she is in equilibrium D where she invests. Moreover, as r increases, the equilibrium boundaries λ_A, λ_B and λ_D shift to the left, therefore the domain of equilibrium A shrinks while the overall domain of equilibria B, C and D expands.

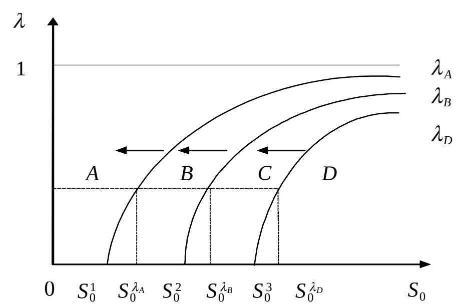

Figure 7. Four equilibria as r increases when $r < \frac{c_H}{(1+q)\theta_H}$

3.3 Impacts of salience parameter q and initial belief ρ

3.3.1 Salience parameter q on the thresholds of r

According to Corollary 3, the thresholds of the school quality are:

$$r_1 = \frac{c_L}{(1+q)\theta_L}, r_2 = \frac{c_H}{(1+q)\theta_H}$$

$\frac{\partial r_1}{\partial q} < 0$ and $\frac{\partial r_2}{\partial q} < 0$ show that the higher the salience of self-esteem concerns of educational identity, the lower the thresholds r_1 and r_2 are. Other conditions equal, both the high and low type agents are more likely to invest in schooling. From the school's point of view, the standards of school quality ("good", "medium" or "poor") not only depends on how much market-valued skills and knowledge the school can impart to its students, but also on how the school develops a climate that emphasizes the self-esteem concerns of academic achievement among its students.

3.3.2 Salience parameter q on the thresholds of S_0

When the school quality is relatively low, the choice of investment is sensitive to the initial condition S_0. For the high type agent at a poor quality school, she invests in schooling when $S_0 > S_0^{\lambda_A}$ (Figure 7). For the low type, at a medium or poor quality school, she invests when $S_0 \geq S_0^{\lambda_D}$ (Figure 6 or 7). The thresholds satisfy,

$$\frac{dS_0^{\lambda_A}}{dq} < 0, \frac{dS_0^{\lambda_D}}{dq} < 0$$

Given the level of preschool education S_0, the higher q the lower the thresholds are, the more likely both type agents invest in schooling. For the high type, the perceived ability of investing in equilibrium B $v(1) = \theta_H$ is higher than that of not investing in equlibrium A $v(0) = \overline{v}$. The higher emotional concerns about educational identity, the more weight she puts on the difference between θ_H and $\overline{v}$, the more motivated the agent invests in schooling. Similarly, for the low type, the perceived ability of investing in equilibrium D $v(1) = \overline{v}$ is higher than that of not investing in equlibrium C $v(0) = \theta_L$. The higher emotional concerns about the perceived ability, the more weight she puts on the difference between $\overline{v}$ and θ_L, the more motivated the low type agent invests in schooling.

3.3.3 Initial belief ρ on the thresholds of S_0

As for the impact of the initial belief ρ on the threshold of S_0, it differs in the high and the low type agent. The thresholds satisfy:

$$\frac{dS_0^{\lambda_A}}{d\rho} > 0, \frac{dS_0^{\lambda_D}}{d\rho} < 0$$

For the high type, the threshold $S_0^{\lambda_A}$ increases with initial belief ρ. It implies that at a poor quality school the more optimistic the high type agent initially is, the lower the probability that she invests. This is because when ρ is greater, the difference between the perceived ability from equilibrium B "investing" and equilibrium A "not investing", $\theta_H - \overline{v}$, becomes smalller, the less motivated the agent wants to invest. Strikingly, the initial belief ρ has a contrasting influence on the low type agent. As the initial belief ρ increases, the threshold $S_0^{\lambda_D}$ decreases. At a medium or poor quality school, the more optimistic the low type agent initially is, the more likely the agent is to invest in schooling. The same reasoning holds for the low type agent: the difference between the perceived ability from equilibrium D of investing and equilibrium C of not investing, $\overline{v}-\theta_L$, increases when ρ becomes larger.

4 Cognitive investment: the endogeneity of λ and the influence of social environment

4.1 The decision function

Remembering is an active reconstructive process rather than an exact record of the actual experiences. Assume that at stage 0 the agent optimally chooses her memory λ to reduce the cognitive dissonance by balancing between the following two aspects. On the one hand, referring to (10) and proposition 2(3), the agent has a demand for certain memory level λ to motivate investment in schooling. The lower λ is, the more likely she invests. On the other hand, when confronting the contradictory cognitions, the agent makes efforts to ignore, distort and reinterpret some evidence. To modify memory exhausts real resources, time, psychic stress from repression, etc. The cost of modifying memory is related to the social environment and peer influences the agent has access to. For the sake of simplicity, assume a linear memory supply function $M(\lambda)$, where $0 \leq \lambda \leq 1$. Hence, the agent chooses the optimal level of memory λ according to (11). The first derivative is shown in (12):

$$\underset{0\leq\lambda\leq1}{Max}\, U(\theta_i, a, \lambda, S_0) = (\theta_i + q\lambda\theta_i + q(1-\lambda)v(a))\,(S_0 + ar) - ac_i \; - M(\lambda) \quad (11)$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_i - v(a))(S_0 + ar) - M^{'}(\lambda) \quad (12)$$

Specify the cost function in two symmetric extremes. In the conservative social environment, set $M(\lambda) = m(1-\lambda)$, $m > 0$. Figure 8 shows that being a pure conservative ($\lambda = 1$) is costless; the smaller λ, the more costs the agent pays, m is the marginal cost of reducing a unit of λ. The more conservative the environment is, the higher marginal cost m is. On the contrary, in the progressive social environment, set $M(\lambda) = m\lambda$. Figure 9 describes that being a pure progressive $\lambda = 0$ requires no cost; when the agent chooses larger λ, it incurs more cost, m is the marginal cost of adding a unit of λ. The more progressive the environment is, the higher marginal cost m is.

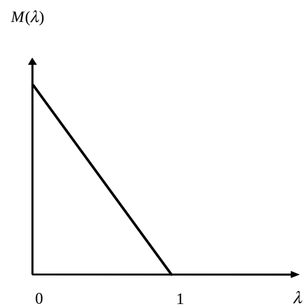

Figure 8. $M(\lambda) = m(1-\lambda)$

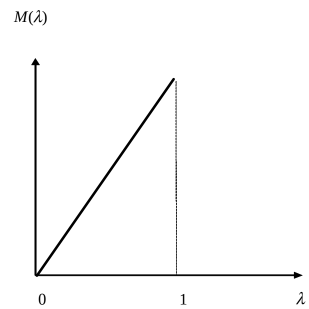

Figure 9. $M(\lambda) = m\lambda$

4.2 The results

4.2.1 The high type

In the case of conservative environment, the high type optimally chooses $\lambda = 1$ because it is costless to maintain a good self-image θ_H as a conservative. Investment decision only depends on the quality of school r. At a good or medium quality school (Figure 10 or 11), the high type agent always invests. At a poor quality school, to invest not only fails to improve the self-image $\lambda\theta_H + (1-\lambda)v(1)$ as good as θ_H, but also incurs extra psychological cost of lowering her memory λ.Therefore, the high type does not invest and remains to be a conservative (see Figure 12).

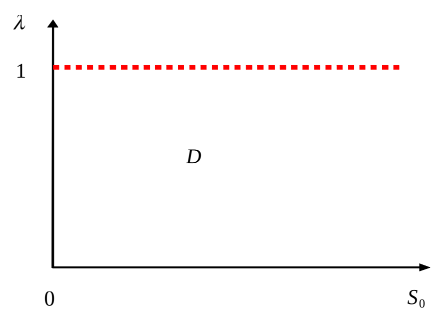

Figure 10. Good school, high type, conservative environment

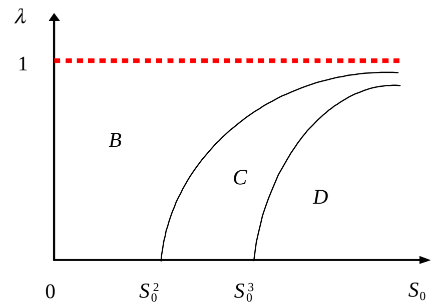

Figure 11. Medium quality school, high type, conservative environment

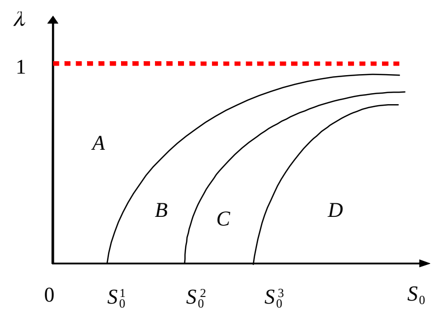

Figure 12. Poor quality school, high type, conservative environment

In the progressive environment, being conservative $\lambda = 1$ is more costly than in the conservative environment. The high type increases λ only when the preschool education S_0 exceeds some critical threshold.

At a good quality school (Figure 13), the agent will always be in equilibrium D where she invests, $\lambda = 0$ and the self-image is $\overline{v}$. When $S_0 > S^{GP}$, the agent invests and finds it better-off to obtain a good self-image θ_H by increasing λ from 0 to 1.

$$\lambda = \begin{cases} 0 & \text{if } S_0 \leq S^{GP} \\ 1 & \text{if } S_0 > S^{GP} \end{cases}, S^{GP} = \frac{m}{q(\theta_H - \overline{v})} - r > 0$$

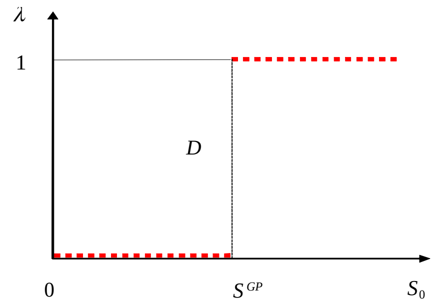

Figure 13. Good quality school, high type, progressive environment

At a medium quality school (Figure 14), the agent invests. In equilibrium B, $\lambda = 0$ because it allows the agent to maintain a good self-image θ_H at no psychological cost. In equilibrium C, λ remains 0 and self-image $\overline{v}$ when $S_0 \leq S^{MP}$. When $S_0 > S^{MP}$, the agent finds it affordable to obtain a good self-image θ_H by increasing λ from 0 to λ_B. It switches from equilibrium C to equilibrium B.

$$\lambda = \begin{cases} 0 & \text{if } S_0 \leq S^{MP} \\ \lambda_B & \text{if } S_0 > S^{MP} \end{cases}, S^{MP} = \frac{m}{q(\theta_H - \theta_L)} - r > S_0^2$$

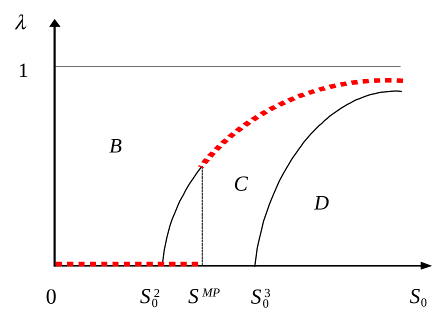

Figure 14. Medium quality school, high type, progressive environment

At a poor quality school, λ depends not only on the preschool education S_0, but also on the marginal cost of increasing λ:

When $m < c_H - (1+q)\theta_H r$ (Figure 15), as S_0 increases, λ changes from 0 to 1. Because the cost of increasing λ is lower than the sum of opportunity cost $-(1+q)\theta_H r$ and investment cost c_H, the high type agent maintains a good self-image θ_H by being conservative instead of investing. The agent is in equilibrium A of no investment.

$$\lambda = \begin{cases} 0 & \text{if } S_0 \leq S^{PP1} \\ 1 & \text{if } S_0 > S^{PP1} \end{cases}, S^{PP1} = \frac{m}{q(\theta_H - \overline{v})} \leq S_0^1$$

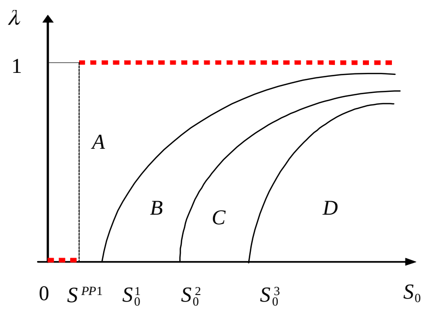

Figure 15. Poor quality school, high type, progressive environment $-(1+q)\theta_H r + c_H > m$

When $m \geq c_H - (1+q)\theta_H r$ (Figure 16), the cost of being conservative is higher than the sum of the opportunity cost $-(1+q)\theta_H r$ and the investment cost c_H. The high type agent chooses to be progressive $\lambda = 0$ regardless of her self-image. Only when S_0 is higher than S^{PP2}, the high type agent is in equilibrium B, where she chooses $\lambda = \lambda_B$ and obtains a good self-image θ_H. In this highly progressive environment, whatever λ is, the agent invests in schooling once $S_0 > S_0^1$.

$$\lambda = \begin{cases} 0 & \text{if } S_0 \leq S^{PP2} \\ \lambda_B & \text{if } S_0 > S^{PP2} \end{cases}, S^{PP2} = \frac{m}{q(\theta_H - \theta_L)} - r > S_0^2$$

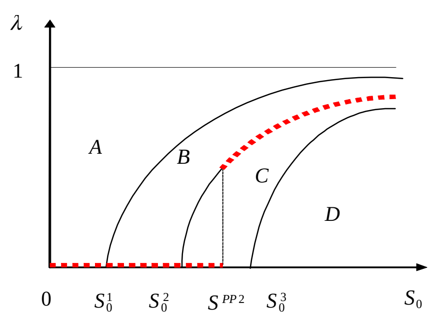

Figure 16. Poor quality school, high type, progressive environment $c_H - (1+q)\theta_H \leq m$

4.2.2 The low type

In the conservative environment, the choice of memory λ is more sophisticated for the low type agent than for the high type agent. On the one hand, a lower λ brings a better self-image, on the other hand, it adds additional psychological costs to go against the conservative social environment. At a good or medium quality school, there exists a certain critical threshold above which the low type agent can afford to be progressive.

At a good quality school (Figure 17), the low type agent is in equilibrium D and she invests.

$$\lambda = \begin{cases} 1 & \text{if } S_0 \leq S^{GC} \\ 0 & \text{if } S_0 > S^{GC} \end{cases}, \quad S^{GC} = \frac{m}{q(\overline{v} - \theta_L)} - r > 0$$

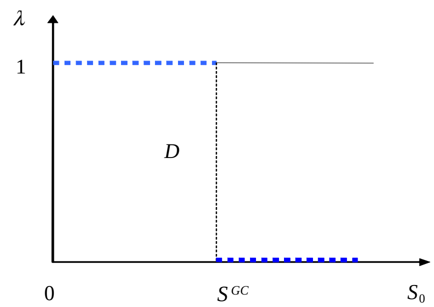

Figure 17. Good school, low type, conservative environment

At a medium quality school (Figure 18),

$$\lambda = \begin{cases} 1 & \text{if } S_0 \leq S^{MC} \\ 0 & \text{if } S_0 > S^{MC} \end{cases}, S^{MC} = \frac{m + c_L - \theta_L r - \overline{v} r}{q(\overline{v} - \theta_L)} > S_0^3$$

When the preschool education $S_0 \leq S^{MC}$, the low type agent is conservative and in equilibrium B of no investment. When $S_0 > S^{MC}$, the low type agent is progressive and in equilibrium D where she invests.

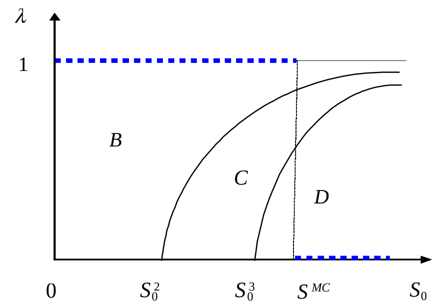

Figure 18. Medium quality school, low type, conservative environment

At a poor quality school, the conservative environment worsens the situation for the low type agent. She does not invest at all because the economic and emotional payoffs do not compensate the investment cost c_L. In equilibrium A, "not investing" brings better news $\overline{v}$ than the initial self-image θ_L. In equilibrium B, C or D, "not investing" brings θ_L as self-image. Therefore, the agent adapts λ to stay in this no-investment equilibrium A rather than equilibria B, C or D. The optimal chosen value of λ does not change monotonically as S_0 increases. When $S_0 \leq S^{PC}$, the benefits from a higher self-image, $\lambda\theta_L + (1-\lambda)v(0)$, cannot compensate the psychological costs of lowering λ. When $S_0 > S^{PC}$, the agent gets better-off by being more progressive but keeping the value of λ on the edge of equilibrium A, λ_A. When the marginal psychological costs $m \leq \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$, the threshold $S^{PC} < S_0^1$ (see Figure 19).

$$\lambda = \begin{cases} 1 & \text{if } S_0 \leq S^{PC} \\ 0 & \text{if } S^{PC} < S_0 \leq S_0^1 \\ \lambda_A & S_0 > S_0^1 \end{cases}, S^{PC} = \frac{m}{q(\overline{v} - \theta_L)} \leq S_0^1$$

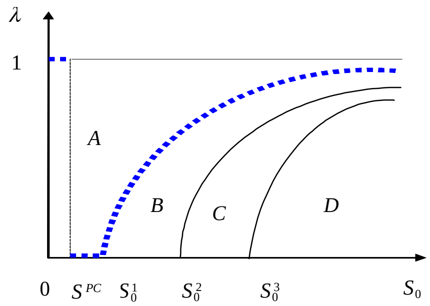

Figure 19. Poor quality school, low type, conservative environment $m \leq \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$

When the mental costs $m > \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$, the threshold $S^{PC} \geq S_0^1$ (see Figure 20).

$$\lambda = \begin{cases} 1 & \text{if } S_0 \leq S^{PC} \\ \lambda_A & \text{if } S_0 > S^{PC} \end{cases}, S^{PC} > S_0^1$$

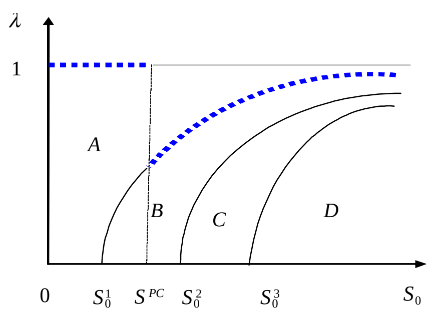

Figure 20. Poor quality school, low type, conservative environment $m > \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$

In the progressive environment, the low type agent always finds it optimal to keep $\lambda = 0$ as a new self-image $v(a)$ is at least as good as the old one θ_L. It requires no extra psychological costs. The investment choice for the low type agent depends on the school quality. At a good quality school (Figure 21), the agent is in equilibrium D. At a medium or poor quality school (in Figure 22), if $0 < S_0 \leq S_0^2$, the agent does not invest (in equilibrium A or B); if $S_0^2 < S_0 < S_0^3$, she is in equilibrium C and invests randomly at probability $\frac{c_L - r\theta_L + q\theta_L S_0}{q(r+S_0)}$; if $S_0 \geq S_0^3$, she is in equilibrium D and invests.

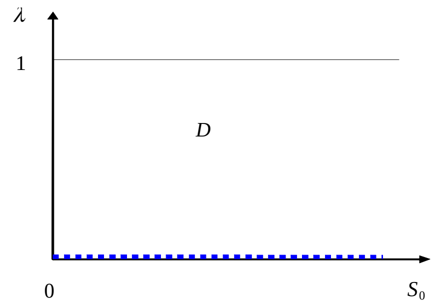

Figure 21. Good quality school, low type, progressive environment

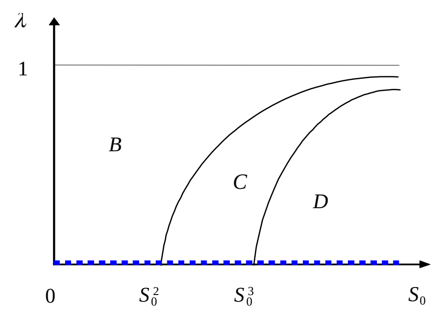

Figure 22. Medium quality school, low type, progressive environment

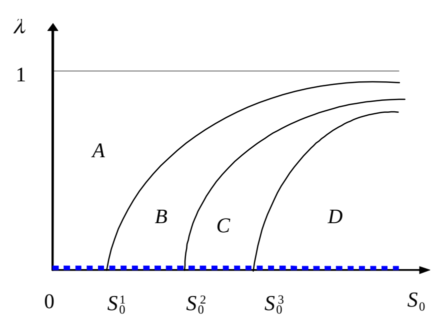

Figure 23. Poor quality school, low type, progressive environment

To sum up, the agent forms various self-fulfilling preferences for schooling according to (S_0, r) by shaping her memory λ at a cost under different social environment. For the high type agent, investing sometimes brings bad news about self-image and therefore she is anti-progress. At a good or medium quality school, she always invests whatever λ is. At a bad quality school, when $S_0 > S_0^1$, the agent may invest. The decision depends on the social pressures and environment. When the psychological cost of being conservative is relatively low, the high type agent is conservative and does not invest. In a highly progressive environment where the psychological costs of being conservative is very high, the high type agent becomes progressive. This psychological self-fulfilling alternative leads her to investment. For the low type agent, investing always brings good news about self-image and therefore she is pro-progress. At a good quality school, the agent always invests whatever λ is. At a medium or poor quality school with progressive environment, the agent becomes progressive and invests once $S_0 \geq S_0^3$. In the conservative environment, the agent becomes progressive and invests only when $S_0 \geq S^{MC} > S_0^3$. However, at a poor school with conservative environment, the low type agent cannot afford to invest and then remains conservative.

5 Conclusion

Nowadays, schools are important social institutions for individual development. This paper provides a framework to comprehend the relationship between the socioeconomic background, cognitive and non-cognitive abilities, and one's educational choice. Several variables, in particular the school quality, the preschool education and the social environment, are key to affect the youth's schooling choice. The conditions under which one fails to invest in education are carefully identified.

This paper also provides a rich framework to reflect on the deeply rooted problems of low achievement for economically disadvantaged youth. It rejects the "centrality of culture" view that an oppositional culture—prior values and patterns of behavior in the non-dominating social groups— determines their failure. Instead, it agrees with a new approach developed by Appadurai(2004) and Sen and Nussbaum (1993) and Nussbaum (2000), which links the poverty trap and the capacity to aspire. This capacity to aspire refers to meaningful and reciprocal experiences relating motivations to commodities and to what people actually do. However, given that the required economic and social rescources are unequally distributed at both early age at home and later at school, the capacity to aspire and to pursue personal development through education will not be equal in the society. Compared to the privileged who have sufficient stock of experiences and good opportunities to promote development, the poor have a much smaller stock of experience and less frequent opportunities so that they lack the motivation to invest in high education regardless of the rising economic incentives in the labor market.

The theoretical framework developed in this paper also calls for more empirical support concerning its assumptions and results. Both the theoretical and empirical progress will enhance the understanding of the "poverty trap and low education" black box and help to evaluate what direct interventions to raise academic attainment of the disadvantaged young will be effective.

Reference

Akerlof, George and Rachel Kranton (2000), “Economics and Identity”, Quarterly Journal of Economics, 115, 3, p.715-753

Akerlof, George and Rachel Kranton (2002), “Identity and Schooling: Some Lessons for the Economics of Education”, Journal of Economic Literature, Vol. XL, p.1167-1201

Appadurai (2004), "The capacity to aspire: Culture and the Terms of Recognition", in V. Rao and M. Walton (Eds). "Culture and Public Action", IBRD-World Bank, Washington DC.

Atkinson, Tony (2003), "Income Inequality in OECD Countries: Data and Explanations CESifo Economic Studies", Vol. 49, p.479-513

Austen-Smith, David and Roland Fryer (2005), “An Economic Analysis of ‘Acting White”, Quarterly Journal of Economics, May, p.551-583

Banerjee Abhijit V., Roland Benabou and Dilip Mookherjee (2006), “Understanding Poverty”, Oxford University Express

Benabou, Roland and Jean Tirole (2006), “Identity, Dignity and Taboos: Beliefs as Assets”, Discussion Paper , No. 2583

Bernheim, B. Douglas (2005), "Memory and Anticipation", The Economic Journal, 2005, 115 (April), p.271-304

Bertrand, Marianne, Sendhil Mullainathan and Eldar Shafir, (2004), "A Behaviroal-economics View of Poverty", American Economic Review, 94(2), p.419-423

Blanden, Jo and Paul Gregg (2004), "Family Income and Educational Attainment: A Review of Approaches and Evidence for Britain", CEE Discussion Papers 0041, Centre for the Economics of Education, LSE.

Bodner, Ronit and Drazen Prelee (2003), "Self-signaling and Diagnostic Utility in Everyday Decision Making," in I. Brocas and J. Carrillo eds. The Psychology of Economic Decision. Vol. 1: Rationality and Well-being, Oxford University Press

Bowles, Samuel, Herbert Gintis and Melissa Osborne(2001a), “Incentive-enhancing preferences: Personality Behaviour and Earning", American Economic Review, 91, p.155–158

Bowles, Samuel, Herbert Gintis and Melissa Osborne(2001b), "The determinants of Earnings: A Behavioral Approach", Journal of Economic Literature, 2001, vol. 39, issue 4, p.1137-1176

Branden, Nathaniel (1969), "The psychology of self-esteem", published by arrangement with Nash Publishing Corporation

Burtless, Gary, (1996), "Does Money Matter?" Washington, DC: Brookings Institution

Carneiro, Pedro and James Heckman (2002, October), "The Evidence on Credit Constraints in Post-Secondary Schooling", Economic Journal 112 (482), p.705—734

Carneiro, Pedro, Karsten Hansen, and James Heckman (2003), "Estimating distributions of treatment effects with an application to the returns to schooling and measurement of the effects of uncertainty on college choice", International Economic Review 44(2), p.361-422

Cunha, Flavio, James Heckman, and Salvador Navarro (2005), "Counterfactual analysis of inequality and social mobility", In S. L. Morgan, D. B. Grusky, and G. S. Fields (Eds.), Mobility and Inequality: Frontiers of Research from Sociology and Economics, Chapter 4. Palo Alto: Stanford University Press, forthcoming.

Cunha, Flavio, James Heckman, and Salvador Navarro (2005), "The evolution of uncertainty in the US economy", Presented at the 9th World Congress of the Econometric Society, London. Previously "Separating Heterogeneity from Uncertainty in an Aiyagari-Laitner Economy", presented at the Goldwater Conference on Labor Markets, Arizona State University, March 2004.

Cunha, Flavio, James Heckman, and Salvador Navarro (2005), "A framework for the analysis of inequality", Journal of Macroeconomics, forthcoming.

Cunha, Flavio, James Heckman, and Salvador Navarro (2005), "Separating uncertainty from heterogeneity in life cycle earnings", the 2004 Hicks Lecture, Oxford Economic Papers 57(2), p.191—261.

Dalton, Patricio and Sayantan Ghosal (2008), "Behavioural Decisions and Welfare", working paper

Damasio, Antonio R. (1996), "Descartes' Error, Emotion, Reason and the Human Brain", Papermac

Dominitz, Jeff and Charles Manski, (1996), "Eliciting Student Expectations of the Return to Schooling", The Journal of Human Resources, p.1-26

Elliott, Delbert S., Scott Menard, Bruce Rankin, Amanda Elliott, William Julius Wilson and David Huizinga (2006), "Good Kids from Bad Neighborhoods: Successful Development in Social Context", Cambridge University Press

Fordham, Signithia and John Ogbu (1986), "Black Students' School Successes: Coping with the Burden of 'Acting White'", The Urban Review, XVIII

Heckman, James and Alan B. Krueger edited (2003), "Inequality in America What Role for Human Capital Policies", The MIT Press, London

Heckman, James and Yona Rubinstein (2001), "The importance of Noncognitive Skills: Lessons from the GED Testing Program", The American Economic Review, vol. 91, no.2, p.145-149

Heckman, James, Jora Stixrud and Sergio Urzua (2006), "The Effects of Cognitive and Noncognitive Abilities on Labour Market Outcomes and Social Behaviour", Working paper 12006, National Bureau of Economic Research.

Heifetz, Aviad and Enrico Minelli (2006), "Aspiration Traps", Discussion Paper n.0610

Kozol, Jonathan (1991), "Savage Inequalities", New York: Crown

Levhari, David and Yoram Weiss (1974), "The Effect of Risk on the Investment in Human Capital", American Economic Review 64(6), p. 950—963.

Loewenstein, George, Elke U. Weber, Christopher K. Hsee and Ned Welch (2001), "Risk as feelings", Psychological Bulletin, Vol. 127, No. 2, p.267-286

Nussbaum, Martha C. (2007), "The upheavals of thought, the intelligence of emotions", Cambriage University Press

Nussbaum, Martha C. (2000), "Women and Human Development", Cambridge University Press

Nussbaum, Martha, and Sen Amartya (1993), "The Quality of Life", Oxford: Clarendon Press

Oxoby, Robert J. (2003), "Attitudes and Allocations: Status, Cognitive Dissonance, and the Manipulation of Attitudes", Journal of Economic Behavior and Organization, Vol.52, p.365-385

Oxoby, Robert J. (2004), "Cognitive Dissonance, Status and Growth of the Underclass", The Economic Journal, 114 October, p.727-749

Rryer, Roland G. (2006), "An Empirical Analysis of 'Acting White'", Working paper, October, Harvard University and NBER

Steel Claude M. (1997), "How Stereotype Shape Intellectual Identity and Performance", American Psychologist, Vol. 52, No.6, P.613-629

Vandenberghe Vincint (2007), "Family Income and Tertiary Education Attendance across the EU. An empirical assessment using sibling data", Case DP (2007) No 123, Case-Sticerd, LSE, London

Wichardt, Phillipp C. (2007), "Why and How Identity Should Influence Utility", University of Bonn, Working Paper

Wilson, William J. (1990), "The Truly Disadvantaged", University of Chicago Press, Chicago and London

Appendix

Proposition 1: Four Equilibria

According to the investment criteria (10), the difference of the incentive to invest between the two types:

$$W_H - W_L = (\theta_H - \theta_L)(1 + q\lambda)r + c_L - c_H > 0$$

Because of this monotonicity property, it is impossible that the low type invest while the high type doesn't. Equation $x_L(1-x_H) = 0$ always holds. This finally leads to several potential equilibria:

Equilibrium A: $x_H = x_L = 0$, then $v(0) = \overline{v}$ and $v(1) = \theta_H$, it satisfies,

$$W_H \leq 0 \text{ (C1)}, W_L \leq 0$$[12]

Equilibrium B: $x_H = 1, x_L = 0$, then $v(1) = \theta_H$ and $v(0) = \theta_L$, it satisfies,

$$W_H \geq 0 \text{ (C2)}, W_L \leq 0 \text{ (C3)}$$

Equilibrium C: $x_H = 1, 1 > x_L > 0$, then $v(0) = \theta_L, \overline{v} < v(1) < \theta_H$, it satisfies,

[12] As $W_H > W_L$, if $W_H \leq 0$, $W_L \leq 0$ also holds; if $W_L \geq 0$, $W_H \geq 0$ also holds.

$$W_H \geq 0,\ W_L = 0 \text{ (C4)}$$

Equilibrium D: $x_H = x_L = 1$, then $v(0) = \theta_L, v(1) = \overline{v}$, it satisfies,

$$W_H \geq 0,\ W_L \geq 0 \text{ (C5)}$$

Equilibrium E: $1 > x_H > x_L = 0$, then $\theta_L < v(0) < \overline{v}$ and $v(1) = \theta_H$, it satisfies,

$$W_H = 0 \text{ (C6)},\ W_L < 0$$

From (4),(5),(6) and (7), in all the equilibria, $\widehat{\rho}(1) > \widehat{\rho}(0)$ and $v(1) > v(0)$.

Once condition (C6) holds, (C1) holds too. Therefore, Equilibria A and E are always coexist. Both types are better off in the no-investment Equilibrium A than E according to her utility function (10), therefore using Pareto criterion we can select Equilibrium A and rule out E. Moreover, we can always find an domain that Equilibrium A and B coexist because c_H satisfies both (C1) and (C2). Both types are better off in the no-investment Equilibrium A than B if we compare the utilities according to(10). Therefore, we can rule out Equilibrium B whenever it coexists with A.

Derived from **proposition 1** (C1)[13], (C3) and (C5), curve λ_A, λ_B and λ_D, if exist, are the bounds of equilibria A, B, C and D (see Figue 4):

$$\lambda_A = 1 - \frac{c_H - (1+q)\theta_H r}{q(\theta_H - \overline{v})S_0} \tag{D1}$$

$$\lambda_B = 1 - \frac{c_L - (1+q)\theta_L r}{q(\theta_H - \theta_L)(S_0 + r)} \tag{D2}$$

$$\lambda_D = 1 - \frac{c_L - (1+q)\theta_L r}{q(\overline{v} - \theta_L)(S_0 + r)} \tag{D3}$$

$$\text{which satisfy } \ 0 \leq \lambda_D < \lambda_B < \lambda_A \leq 1$$

Proposition 2:

Whatever the high or low type, given $\theta_H > v(1) > v(0) > \theta_L$, $S_0 > 0$, $r > 0$,

$$\frac{\partial W_i}{\partial S_0} = q(1-\lambda)(v(1) - v(0)) \geq 0$$

$$\frac{\partial W_i}{\partial r} = \theta_i(1 + \lambda q) + q(1-\lambda)v(1) > 0$$

It is obvious that the higher S_0 (or r) is, the higher the incentive (W_i) to invest for both high and low ability agent.

[13] When Equilibrium B and A coexist, the latter equilibrium A dominates the former equilibrium B. Therefore, the upper bound is not determined by (C2), but (C1).

$$\frac{\partial W}{\partial \lambda}=-q\left[(v(1)-\theta_i)(S_0+r)-(v(0)-\theta_i)S_0\right]$$

For the low type agent, $\frac{\partial W}{\partial \lambda}<0$ always holds, therefore the lower λ is, the higher the incentive to invest.

For the high type agent, in equilibria B, C and D, she always invest. While in equilibrium A with $v(1)=\theta_H$ and $v(0)=\overline{v}$, therefore, $\frac{\partial W}{\partial \lambda}<0$ also holds.

Corollary 3: Relationships between λ, r and S_0 and the equilibria

From (D1), (D2) and (D3), given the value of λ, S_0 can be formulated as:

$$S_0^{\lambda_A}=\frac{c_H-(1+q)\theta_H r}{q(\theta_H-\overline{v})(1-\lambda)},\ S_0^{\lambda_B}=\frac{c_L-(1+q)\theta_L r}{q(1-\lambda)(\theta_H-\theta_L)}-r,\ S_0^{\lambda_D}=\frac{c_L-(1+q)\theta_L r}{q(1-\lambda)\left(\overline{v}-\theta_L\right)}-r$$

when $\lambda=0$, S_0 equals:

$$S_0^1=\frac{c_H-(1+q)\theta_H r}{q(\theta_H-\overline{v})},\ S_0^2=\frac{c_L-q\theta_H r-\theta_L r}{q(\theta_H-\theta_L)},\ S_0^3=\frac{c_L-(\theta_L+q\overline{v})r}{q\left(\overline{v}-\theta_L\right)}$$

From (D1), (D2), (D3), when $S_0=0, \lambda$ equals:

$$\lambda_A^0=\lim_{S_0=0}(1-\frac{c_H-(1+q)\theta_H r}{q(\theta_H-\overline{v})S_0}),\ \lambda_B^0=1-\frac{c_L-(1+q)\theta_L r}{q(\theta_H-\theta_L)r},\ \lambda_D^0=1-\frac{c_L-(1+q)\theta_L r}{q(\overline{v}-\theta_L)r}$$

$$\text{Additionally, define: the slope of curve }\lambda_A=\frac{d\lambda_A}{dS_0}=\frac{c_H-(1+q)\theta_H r}{qS_0^2\left(\theta_H-\overline{v}\right)}$$

$$\text{the change of the slope}=\frac{d^2\lambda_A}{dS_0^2}=-\frac{2\left(c_H-(1+q)\theta_H r\right)}{qS_0^3\left(\theta_H-\overline{v}\right)}$$

$$\text{the slope of curve }\lambda_B=\ \frac{d\lambda_B}{dS_0}=\frac{c_L-(1+q)\theta_L r}{q\left(r+S_0\right)^2\left(\theta_H-\theta_L\right)}$$

$$\text{the change of the slope}=\frac{d^2\lambda_B}{dS_0^2}=-\frac{2\left(c_L-(1+q)\theta_L r\right)}{q\left(r+S_0\right)^3\left(\theta_H-\theta_L\right)}$$

$$\text{the slope of curve }\lambda_D=\frac{d\lambda_D}{dS_0}=\frac{c_L-(1+q)\theta_L r}{q\left(r+S_0\right)^2\left(\overline{v}-\theta_L\right)}$$

$$\text{the change of the slope}:\frac{d^2\lambda_D}{dS_0^2}=\frac{-2\left(c_L-(1+q)r\theta_L\right)}{q\left(r+S_0\right)^3\left(\overline{v}-\theta_L\right)}$$

Now under each condition of r, discuss the relationship between λ and S_0.

(1) When $r\geq\dfrac{c_L}{(1+q)\theta_L}$: (see Figue 7) $\lambda_A>1,\lambda_B\geq 1,\lambda_D\geq 1$

There is no equilibrium A, B or C. Equilibrium D exists.

(2) When $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}$: (see Figue 8) $\quad \lambda_A \geq 1, \lambda_B < 1, \lambda_D < 1$

There is no equilibrium A. Equilibria B, C and D exist.
To be specific about the feature of curve λ_B and λ_D :

$$\lambda_B < 1; S_0^2 > 0, \lambda_B^0 < 0, \frac{d\lambda_B}{dS_0} > 0, \frac{d^2\lambda_B}{dS_0^2} < 0; \text{when } S_0 \to \infty, \lambda_B \to 1$$

$$\lambda_D < 1; S_0^3 > 0, \lambda_D^0 < 0, \frac{d\lambda_D}{dS_0} > 0, \frac{d^2\lambda_D}{dS_0^2} < 0; \text{when } S_0 \to \infty, \lambda_D \to 1$$

$\frac{d\lambda_B}{dr} > 0, \frac{d\lambda_D}{dr} > 0 \implies$ As r rises, λ_B and λ_D shift to the left in Figure 8.

$$\text{From (C4), we get } v(1) = \frac{-c_L + r\theta_L + q\theta_L S_0 (\lambda - 1) + qr\lambda\theta_L}{q(\lambda - 1)(r + S_0)}$$

$$\text{From (4) and (6), } x_L = \frac{\rho(\theta_H - v(1))}{(1-\rho)(v(1) - \theta_L)} = \frac{\rho}{1-\rho}\left(\frac{q(\theta_H - \theta_L)(\lambda - 1)(S_0 + r)}{-c_L + (1+q)r\theta_L} - 1\right)$$

(3) When $r < \frac{c_H}{(1+q)\theta_H}$: (see Figue 9) $\quad \lambda_A < 1, \lambda_B < 1, \lambda_D < 1$

There exist equilibria A, B, C and D.
To be specific about the feature of curve λ_A, λ_B and λ_D :

$$\lambda_A < 1; S_0^1 > 0, \lambda_A^0 < 0, \frac{d\lambda_A}{dS_0} > 0, \frac{d^2\lambda_A}{dS_0^2} < 0; \text{when } S_0 \to \infty, \lambda_A \to 1$$

$$\lambda_B < 1; S_0^2 > 0, \lambda_B^0 < 0, \frac{d\lambda_A}{dS_0} > 0, \frac{d^2\lambda_A}{dS_0^2} < 0; \text{when } S_0 \to \infty, \lambda_B \to 1$$

$$\lambda_D < 1; S_0^3 > 0, \lambda_D^0 < 0, \frac{d\lambda_D}{dS_0} > 0, \frac{d^2\lambda_A}{dS_0^2} < 0; when S_0 \to \infty, \lambda_D \to 1$$

$$\frac{d\lambda_A}{dr} > 0, \frac{d\lambda_B}{dr} > 0, \frac{d\lambda_D}{dr} > 0$$
$\implies$ As r rises, λ_A, λ_B and λ_D shift to the left in Figure 9.

The endogeneity of λ

The main approach and steps to solve λ from (11) and (12) are: (a) Categorize agent according to her (ability, preschool, school, environment). (b) Find the potential equilibria according to her categorization. (c) In each potential equilibrium, find λ that maximizes the utility in (11) considering the condition (12). (d) Compare between those λs from different equilibria, the utility of which dominates the others is the solution λ to (11).

I. In the "conservative scenario":

1. For the high type**, (11)** and (12) are specified as:

In equilibrium A:

$$U^A(\theta_H, 0, \lambda, S_0) = (\theta_H + q\lambda\theta_H + q(1-\lambda)\overline{v})\, S_0 - m(1-\lambda) \tag{HA1}$$

$$\frac{\partial U}{\partial \lambda} = q\,(\theta_H - \overline{v})\, S_0 + m > 0 \tag{HA2}$$

In equilibrium B:

$$U^B(\theta_H, 1, \lambda, S_0) = \theta_H(1+q)\,(S_0 + r) - c_H - m(1-\lambda) \tag{HB1}$$

$$\frac{\partial U}{\partial \lambda} = m > 0 \tag{HB2}$$

In equilibrium C:

$$U^C(\theta_H, 1, \lambda, S_0) = (\theta_H + q\lambda\theta_H + q(1-\lambda)v(1))\,(S_0 + r) - c_H - m(1-\lambda) \tag{HC1}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_H - v(1))\,(S_0 + r) + m > 0 \tag{HC2}$$

In equilibrium D:

$$U^D(\theta_H, 1, \lambda, S_0) = (\theta_H + q\lambda\theta_H + q(1-\lambda)\overline{v})\,(S_0 + r) - c_H - m(1-\lambda) \tag{HD1}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_H - \overline{v})\,(S_0 + r) + m > 0 \tag{HD2}$$

(1) When $r \geq \frac{c_L}{(1+q)\theta_L}$, $S_0 > 0$, there only exists equilibrium D. (Figure 10) (HD2) shows the agent optimally chooses $\lambda = 1$.

(2) When $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}$, there could exist equilibria B, C and D when $S_0 > 0$. (Figure 11)

1) If $0 < S_0 \leq S_0^2$, the agent is in equilibrium B. (HB2) shows optimally $\lambda = 1$.

2) If $S_0^2 < S_0 < S_0^3$, the agent is in equilibrium B or C.

In equilibrium B, (HB2) shows optimally $\lambda = 1$; In equilibrium C, (HC2) shows optimally $\lambda = \lambda_B - \varepsilon$, $\varepsilon \to +0$. The agent optimally chooses $\lambda = 1$ as

$$U^B(\theta_H, 1, 1, S_0) > U^C(\theta_H, 1, \lambda_B - \varepsilon, S_0),$$

3) If $S_0^2 < S_0 \geq S_0^3$, the agent is in equilibrium B, C or D.

In equilibrium B shows optimally $\lambda = 1$; In equilibrium C, (HC2) shows optimally $\lambda = \lambda_B - \varepsilon$; In equilibrium D, (HD2) shows optimally $\lambda = \lambda_D$.The agent optimally chooses $\lambda = 1$ as

$$\begin{aligned} U^B(\theta_H, 1, 1, S_0) &> U^C(\theta_H, 1, \lambda_B - \varepsilon, S_0) \\ U^B(\theta_H, 1, 1, S_0) &> U^D(\theta_H, 1, \lambda_D, S_0) \end{aligned}$$

(3) When $r < \frac{c_H}{(1+q)\theta_H}$, there could exist equilibria A, B, C and D when $S_0 > 0$. (Figure 12)

1) If $0 < S_0 \leq S_0^1$,the agent is in equilibrium A. (HA2) shows optimally $\lambda = 1$.

2) If $S_0^1 < S_0 \leq S_0^2$,the agent could be in equilibrium A or B.

In equilibrium A, (HA2) shows optimally $\lambda = 1$; In equilibrium B, (HB2) shows optimally $\lambda = \lambda_A - \varepsilon$. The agent then chooses $\lambda = 1$ as

$$U^A(\theta_H, 0, 1, S_0) > U^B(\theta_H, 1, \lambda_A - \varepsilon, S_0)$$

3) If $S_0^2 < S_0 < S_0^3$,the agent could be in equilibrium A, B or C.

In equilibrium A, (HA2) shows optimally $\lambda = 1$; In equilibrium B, (HB2) shows optimally $\lambda = \lambda_A - \varepsilon$; In equilibrium C, (HC2) shows optimally $\lambda = \lambda_B - \varepsilon$. The agent then chooses $\lambda = 1$ as

$$\begin{aligned} U^A(\theta_H, 0, 1, S_0) &> U^B(\theta_H, 1, \lambda_A - \varepsilon, S_0) \\ U^A(\theta_H, 0, 1, S_0) &> U^C(\theta_H, 1, \lambda_B - \varepsilon, S_0) \end{aligned}$$

4) If $S_0 \geq S_0^3$, the agent could be in equilibrium A, B, C or D.

In equilibrium A, (HA2) shows optimally $\lambda = 1$; In equilibrium B, (HB2) shows optimally $\lambda = \lambda_A - \varepsilon$; In equilibrium C, (HC2) shows optimally $\lambda = \lambda_B - \varepsilon$; In equlibrium D, (HD2) shows optimally $\lambda = \lambda_D$. The agent then chooses $\lambda = 1$ as

$$\begin{aligned} U^A(\theta_H, 0, 1, S_0) &> U^B(\theta_H, 1, \lambda_A - \varepsilon, S_0) \\ U^A(\theta_H, 0, 1, S_0) &> U^C(\theta_H, 1, \lambda_B - \varepsilon, S_0) \\ U^A(\theta_H, 0, 1, S_0) &> U^D(\theta_H, 1, \lambda_D, S_0) \end{aligned}$$

2. For the low type: (11) and (12) are specified as:

In equilibrium A:

$$U^A(\theta_L, 0, \lambda, S_0) = (\theta_L + q\lambda\theta_L + q(1-\lambda)\overline{v})\, S_0 - m(1-\lambda) \tag{LA1}$$

$$\frac{\partial U}{\partial \lambda} = q\,(\theta_L - \overline{v})\, S_0 + m \tag{LA2}$$

In equilibrium B:

$$U^B(\theta_L, 0, \lambda, S_0) = (1+q)\theta_L S_0 - m(1-\lambda) \tag{LB1}$$

$$\frac{\partial U}{\partial \lambda} = m > 0 \tag{LB2}$$

In equilibrium C:

$$U^C(\theta_L, 0, \lambda, S_0) = (1+q)\theta_L S_0 - m(1-\lambda) \tag{LC1}$$

$$\frac{\partial U}{\partial \lambda} = m > 0 \tag{LC2}$$

In equilibrium D:

$$U^D(\theta_L, 1, \lambda, S_0) = (\theta_L + q\lambda\theta_L + q(1-\lambda)\overline{v})(S_0 + r) - c_L - m(1-\lambda) \tag{LD1}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_L - \overline{v})(S_0 + r) + m \tag{LD2}$$

(1) When $r \geq \frac{c_L}{(1+q)\theta_L}$, $S_0 > 0$, there only exists equilibrium D. (Figure 13)

$$\begin{aligned} \text{Given } q(\theta_L - \overline{v})(S_0 + r) + m &\geq 0, \text{ or } S_0 \leq S^{GC}, \text{ optiamlly } \lambda = 1; \\ \text{Given } q(\theta_L - \overline{v})(S_0 + r) + m &< 0, \text{ or } S_0 > S^{GC}, \text{ optiamlly } \lambda = 0. \\ \text{Define } S^{GC} &= \frac{m}{q(\overline{v} - \theta_L)} - r \end{aligned}$$

(2) When $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}$, there exists equilibria B, C and D. (Figure 14)

1) if $0 < S_0 \leq S_0^2$: there exists equilibrium B.
Condition (LB2) shows optimally $\lambda = 1$.

2) if $S_0^2 < S_0 < S_0^3$: there could exist equilibrium B or C.
In equilibrium B, optimally $\lambda = 1$;In equilibrium C, condition (LC2) shows optimally $\lambda = \lambda_B - \varepsilon$. The low type chooses $\lambda = 1$ as,

$$U^B(\theta_L, 0, 1, S_0) > U^C(\theta_L, 0, \lambda_B - \varepsilon, S_0)$$

3) if $S_0 \geq S_0^3$: there exist equilibrium B, C or D.

In equilibrium B and C, $\quad U^B(\theta_L, 0, 1, S_0) > U^C(\theta_L, 0, \lambda_B - \varepsilon, S_0)$

In equilibrium D, condition FOC (LD2) shows two possibilities:
The first one, optimally $\lambda = 0$,when

$$q(\theta_L - \overline{v})(S_0 + r) + m < 0,$$

Compare with equilibrium B, then

$$\text{If } m < q(\overline{v} - \theta_L)(S_0 + r) - (c_L - (1+q)\theta_L r)\,,$$

$$U^B(\theta_L, 0, 1, S_0) < U^D(\theta_L, 1, 0, S_0), \text{optimally } \lambda = 0$$

$$\text{If } q(\overline{v} - \theta_L)(S_0 + r) - (c_L - (1+q)\theta_L r) \leq m < q(\overline{v} - \theta_L)(S_0 + r),$$

$$U^B(\theta_L, 0, 1, S_0) \geq U^D(\theta_L, 1, 0, S_0), \text{optimally } \lambda = 1$$

The second one, optimally $\lambda = \lambda_D$,when

$$q(\theta_L - \overline{v})\,(S_0 + r) + m \geq 0,$$

Compare with equilibrium B, then

$$U^B(\theta_L, 0, 1, S_0) > U^D(\theta_L, 1, \lambda_D, S_0), \text{optimally } \lambda = 1$$

To conclude, when $S_0 \leq S^{MC}, \lambda = 1$; when $S_0 > S^{MC}$, $\lambda = 0$, where

$$S^{MC} = \frac{m + c_L - \theta_L r - q\overline{v}r}{q\,(\overline{v} - \theta_L)} > S_0^3.$$

(3) When $r < \frac{c_H}{(1+q)\theta_H}$,there could exist equilibrium A, B, C or D.
Case 1 assume $m \leq \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$, see Figure 15.
1) if $0 < S_0 \leq S_0^1$
In equilibrium A, conditions (LA2) shows,

$$\text{When } q(\theta_L - \overline{v})S_0 + m \geq 0, \lambda = 1; \quad \text{when } q(\theta_L - \overline{v})S_0 + m < 0, \lambda = 0.$$

2) if $S_0^1 < S_0 \leq S_0^2$
In equilibrium A, $\lambda = \lambda_A$ as conditions (LA2) satisfies

$$q(\theta_L - \overline{v})S_0 + m < 0$$

In equilibrium B, conditions (LB2) shows that, $\lambda = \lambda_A - \varepsilon$
Compare equilibrium A and B, optimally $\lambda = \lambda_A$, as

$$U^A(\theta_L, 0, \lambda_A, S_0) > U^B(\theta_L, 0, \lambda_A - \varepsilon, S_0)$$

3) if $S_0^2 < S_0 < S_0^3$
Similarly, in equilibrium A and B, optimally $\lambda = \lambda_A$.
In equilibrium C, (LC2) show $\lambda = \lambda_B - \varepsilon$
Compare equilibria A, B and C, optiamlly $\lambda = \lambda_A$ as,

$$U^A(\theta_L, 0, \lambda_A, S_0) > U^C(\theta_L, 0, \lambda_B - \varepsilon, S_0)$$

4) if $S_0 \geq S_0^3$,the low type could be in equilibrium A, B, C or D.
Similarly, in equilibria A, B and C, $\lambda = \lambda_A$
In equilibrium D, optimally $\lambda = 0$, as condition (LD2) satisfies,

$$q(\theta_L - \overline{v})\,(S_0 + r) + m < 0, \lambda = 0$$

Compare A, B, C and D, optimally $\lambda = \lambda_A$ as,

$$\text{When } q(\theta_L - \overline{v})\,(S_0 + r) + m < 0,\ U^A(\theta_L, 0, \lambda_A, S_0) > U^D(\theta_L, 1, 0, S_0)$$

<u>*Case 2*</u> assume $m > \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$, see Figure 16.
1) if $0 < S_0 \leq S_0^1$
In equilibrium A, optimally $\lambda = 1$, as conditions (LA2) satisfies,

$$q(\theta_L - \overline{v})S_0 + m \geq 0$$

2) if $S_0^1 < S_0 \leq S_0^2$
In equilibrium A, $\lambda = \lambda_A$ when conditions (LA2) satisfies

$$q(\theta_L - \overline{v})S_0 + m < 0$$

$\lambda = 1$ when condition (LA2) satisfies

$$q(\theta_L - \overline{v})S_0 + m \geq 0$$

In equilibrium B, conditions (LB2) shows that, $\lambda = \lambda_A - \varepsilon$
Compare equilibria A and B,
When $q(\theta_L - \overline{v})S_0 + m < 0$, optimally $\lambda = \lambda_A$, as

$$U^A(\theta_L, 0, \lambda_A, S_0) > U^B(\theta_L, 0, \lambda_A - \varepsilon, S_0)$$

When $q(\theta_L - \overline{v})S_0 + m \geq 0$, optimally $\lambda = 1$, as

$$U^A(\theta_L, 0, 1, S_0) > U^B(\theta_L, 0, \lambda_A - \varepsilon, S_0)$$

3) if $S_0^2 < S_0 < S_0^3$
In equilibria A and B, as $q(\theta_L - \overline{v})S_0 + m < 0$, optimally $\lambda = \lambda_A$.
In equilibrium C, (LC2) show $\lambda = \lambda_B - \varepsilon$
Compare equilibria A, B and C, optiamlly $\lambda = \lambda_A$ as,

$$U^A(\theta_L, 0, \lambda_A, S_0) > U^C(\theta_L, 0, \lambda_B - \varepsilon, S_0)$$

4) if $S_0 \geq S_0^3$,the low type could be in equilibrium A, B, C or D.
Similarly, in equilibria A, B and C, $\lambda = \lambda_A$
In equilibrium D, optimally $\lambda = 0$, as condition (LD2) satisfies,

$$q(\theta_L - \overline{v})\,(S_0 + r) + m < 0$$

Compare A, B, C and D, optimally $\lambda = \lambda_A$ as

$$U^A(\theta_L, 0, \lambda_A, S_0) > U^D(\theta_L, 1, 0, S_0)^{14}$$

To conclude:
When $m \leq \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$ holds, $S^{PC} \leq S_0^1$
$if\ S_0 \leq S^{PC}, \lambda = 1; if\ S^{PC} < S_0 \leq S_0^1, \lambda = 0; if\ S_0 > S_0^1, \lambda = \lambda_A.$
When $m > \frac{\rho}{1-\rho}(c_H - (1+q)\theta_H r)$ holds, $S^{PC} > S_0^1$
$if\ S_0 \leq S^{PC}, \lambda = 1; if\ S_0 > S^{PC}, \lambda = \lambda_A + \varepsilon$,where

$$S^{PC} = \frac{m}{q(\overline{v} - \theta_L)}$$

II. In the "progressive scenario":
1. For the high type: (11) and (12) are specified as:
In equilibrium A:

$$U^A(\theta_H, 0, \lambda, S_0) = (\theta_H + q\lambda\theta_H + q(1-\lambda)\overline{v})\, S_0 - m\lambda \tag{HA3}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_H - \overline{v})S_0 - m \tag{HA4}$$

In equilibrium B:

$$U^B(\theta_H, 1, \lambda, S_0) = (1+q)\theta_H\,(S_0 + r) - c_H - m\lambda \tag{HB3}$$

$$\frac{\partial U}{\partial \lambda} = -m < 0 \tag{HB4}$$

In equilibrium C:

$$U^C(\theta_H, 1, \lambda, S_0) = (\theta_H + q\lambda\theta_H + q(1-\lambda)v(1))\,(S_0 + r) - c_H - m\lambda \tag{HC3}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_H - v(1))\,(S_0 + r) - m \tag{HC4}$$

In equilibrium D:

$$U^D(\theta_H, 1, \lambda, S_0) = (\theta_H + q\lambda\theta_H + q(1-\lambda)\overline{v})\,(S_0 + r) - c_H - m\lambda \tag{HD3}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_H - \overline{v})\,(S_0 + r) - m \tag{HD4}$$

(1) When $r \geq \frac{c_L}{(1+q)\theta_L}$, $S_0 > 0$,there exists equilibrium D. (Figure 17)

[14]When proving, use $S_0 \geq S_0^3 = \frac{c_L - (\theta_L + q\overline{v})r}{q(\overline{v} - \theta_L)}$

(HD4) shows when $S_0 > S^{GP}, \lambda = 1;$ when $S_0 \leq S^{GP}, \lambda = 0,$

$$\text{where } S^{GP} = \frac{m}{q(\theta_H - \overline{v})} - r > 0$$

(2) When $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}, S_0 > 0$,there exists equilibrium B, C or D. (Figure 18)

1) When $0 < S_0 \leq S_0^2$, there exists equilibrium B.

(HB4) shows optimally $\lambda = 0$.

2) When $S_0^2 < S_0 < S_0^3$, there could exist equilibrium B or C.

In equilibrium B, (HB4) shows optimally $\lambda = \lambda_B$.

In equilibrium C, (HC4) shows

When $q(\theta_H - v(1))\,(S_0 + r) - m \geq 0,\ \lambda = \lambda_B - \varepsilon,\ \varepsilon \longrightarrow +0$,as

$$U^B(\theta_H, 1, \lambda_B, S_0) > U^C(\theta_H, 1, \lambda_B - \varepsilon, S_0)$$

When $q(\theta_H - v(1))\,(S_0 + r) - m < 0,\ \lambda = 0$

$$U^B(\theta_H, 1, \lambda_B, S_0) - U^C(\theta_H, 1, 0, S_0) = q(\theta_H - v(1))\,(S_0 + r) - m\lambda_B$$

If $q(\theta_H - \theta_L)\,(S_0 + r) - m \leq 0$,

$$U^B(\theta_H, 1, \lambda_B, S_0) \leq U^C(\theta_H, 1, 0, S_0)$$

If $q(\theta_H - \theta_L)\,(S_0 + r) - m > 0$,

$$U^B(\theta_H, 1, \lambda_B, S_0) > U^C(\theta_H, 1, 0, S_0)$$

3) When $S_0 \geq S_0^3$, there exist equilibrium B, C or D.

Likewise, compare equilibria B ,C and D, it shows

optimally $\lambda = \lambda_B$,when

$$q(\theta_H - \theta_L)(S_0 + r) - m > 0$$

optimally $\lambda = 0$,when

$$q(\theta_H - \theta_L)(S_0 + r) - m \leq 0$$

Therefore, from 1), 2) and 3) we obtain, when $S_0 \leq S^{MP}, \lambda = 0$;when $S_0 > S^{MP}, \lambda = \lambda_B$, where

$$S^{MP} = \frac{m}{q(\theta_H - \theta_L)} - r > S_0^2$$

(3) When $r < \frac{c_H}{(1+q)\theta_H}$,there exists equilibrium A, B, C or D.

Case1: assume $m < c_H - (1 + q)\theta_H r$ (Figure 19)

1) When $0 < S_0 \leq S_0^1$

In equilibrium A, condition (HA4) shows that,

when $q(\theta_H - \overline{v})S_0 - m \leq 0$, or equivalently $S_0 \leq S^{PP1} = \frac{m}{q(\theta_H - \overline{v})}$,optimally $\lambda = 0$

when $q(\theta_H - \overline{v})S_0 - m > 0$, or equivalently $S_0 > S^{PP1} = \frac{m}{q(\theta_H - \overline{v})}$,optimally $\lambda = 1$;where

$$0 < S^{PP1} < S_0^1$$ [15]

2) When $S_0 > S_0^1$

In equilibrium A, optimally $\lambda = 1$, as

$$q(\theta_H - \overline{v})S_0 - m > 0$$ [16]

$U^A(\theta_H, 0, 1, S_0) - U(\theta_H, 1, \lambda, S_0)$[17]

$= (1 - \lambda)\left[q(\theta_H - v(1))(S_0 + r) - m\right] + \left[-(\theta_H + q\theta_H)r + c_H\right] > 0$ (when $m < c_H - (1+q)\theta_H r$)

Therefore what other equilibria are, equilibrium A with $\lambda = 1$ is the optimal choice.

<u>*Case 2*</u>*:* assume $m \geq c_H - (1+q)\theta_H r$ (Figure 20)

1) When $0 < S_0 \leq S_0^2$, obviously the agent chooses $\lambda = 0$

2) When $S_0^2 < S_0 < S_0^3$, there exists equilibrium A, B or C.

Equilibrium A is not the candidate as it's dominated by equilibrium B or C.

In equilibrium B, optimally $\lambda = \lambda_B$

In equilibrium C, $\lambda = 0$ if

$$q(\theta_H - v(1))(S_0 + r) - m \leq 0$$

otherwise, $\lambda = \lambda_B - \varepsilon$

Compare equilibria B and C:

$$U^B(\theta_H, 1, \lambda_B, S_0) - U^C(\theta_H, 1, 0, S_0) = \lambda_B\left[q(\theta_H - \theta_L)(S_0 + r) - m\right]$$

When $q(\theta_H - \theta_L)(S_0 + r) - m > 0$, $\lambda = \lambda_B$ as

$$U^B(\theta_H, 1, \lambda_B, S_0) > U^C(\theta_H, 1, 0, S_0)$$

When $q(\theta_H - \theta_L)(S_0 + r) - m \leq 0$, $\lambda = 0$ as

$$U^B(\theta_H, 1, \lambda_B, S_0) \leq U^C(\theta_H, 1, 0, S_0)$$

Moreover, in equilibrium C, $\lambda = \lambda_B - \varepsilon$ if

$$q(\theta_H - v(1))(S_0 + r) - m > 0$$

Compare equilibria B and C, optimally $\lambda = \lambda_B$ as

[15] Derived from $m < c_H - (1+q)\theta_H r$

[16] Derived from $S_0 > S_0^1 > S^{PP1}$

[17] The utility of equilibrium B, C or D that the high type agent chooses to invest.

$$U^B(\theta_H, 1, \lambda_B, S_0) > U^C(\theta_H, 1, \lambda_B - \varepsilon, S_0)$$

4) When $S_0 \geq S_0^3$,there exists equilibrium A, B, C or D.

Apply the same procedure above to compare the equilibria A, B, C and D, we get:

When $q(\theta_H - \theta_L)(S_0 + r) - m \leq 0$, optimally $\lambda = 0$

When $q(\theta_H - \theta_L)(S_0 + r) - m > 0$, optimally $\lambda = \lambda_B$

Therefore, in case 2:

If $0 < S_0 \leq S^{PP2}$, $\lambda = 0$;If $S_0 > S^{PP2}$, $\lambda = \lambda_B$, where

$$S^{PP2} = \frac{m}{q(\theta_H - \theta_L)} - r > S_0^2$$

2. For the low type: (11) and (12) are specified as:

In equilibrium A:

$$U^A(\theta_L, 0, \lambda, S_0) = (\theta_L + q\lambda\theta_L + q(1-\lambda)\overline{v})\, S_0 - m\lambda \tag{LA3}$$

$$\frac{\partial U}{\partial \lambda} = q\,(\theta_L - \overline{v})\, S_0 - m < 0 \tag{LA4}$$

In equilibrium B:

$$U^B(\theta_L, 0, \lambda, S_0) = (1+q)\theta_L S_0 - m\lambda \tag{LB3}$$

$$\frac{\partial U}{\partial \lambda} = -m < 0 \tag{LB4}$$

In equilibrium C:

$$U^C(\theta_L, 0, \lambda, S_0) = (1+q)\theta_L S_0 - m\lambda \tag{LC3}$$

$$\frac{\partial U}{\partial \lambda} = -m < 0 \tag{LC4}$$

In equilibrium D:

$$U^D(\theta_L, 1, \lambda, S_0) = (\theta_L + q\lambda\theta_L + q(1-\lambda)\overline{v})\,(S_0 + r) - c_L - m\lambda \tag{LD3}$$

$$\frac{\partial U}{\partial \lambda} = q(\theta_L - \overline{v})\,(S_0 + r) - m < 0 \tag{LD4}$$

(1) When $r \geq \frac{c_L}{(1+q)\theta_L}$, $S_0 > 0$,there exists equilibrium D. (Figure 21)

(LD4) shows optimally $\lambda = 0$.

(2) When $\frac{c_H}{(1+q)\theta_H} \leq r < \frac{c_L}{(1+q)\theta_L}$,there could exist equilibrium B, C or D. (Figure 22)

1) if $0 < S_0 \leq S_0^2$, the agent is in equilibrium B. (LB4) shows $\lambda = 0$

2) if $S_0^2 < S_0 < S_0^3$, the agent is in equlibrium B or C.

In equilibrium B, (LB4) shows $\lambda = \lambda_B$;In equilibrium C, (LC4) shows $\lambda = 0$.

Compare equilibria A and B, and optiamlly $\lambda = 0$ as,

$$U^C(\theta_L, 0, 0, S_0) > U^B(\theta_L, 0, \lambda_B, S_0)$$

3) if $S_0 \geq S_0^3$,the agent is in equlibrium B, C or D.

Similarly, we find optiamlly $\lambda = 0$.

(3) When $r < \frac{c_H}{(1+q)\theta_H}$,there exists equilibrium A, B, C or D. (Figure 23)

Compare the candidate equilibria by using the similar procedure and it shows when $S_0 > 0$, optimally $\lambda = 0$.