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Asset prices and exchange rates: a time dependent approach

by

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DISCUSSION
PAPER

Asset Prices and Exchange Rates:
A time dependent approach

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Abstract

The paper studies the relationship between exchange rates and asset prices. It takes the approach of order flows to exchange rates. Specifically, it focuses on the effect of time-dependent risk aversion. The switch in the parameter causes the equilibrium of the system to alternate between two regimes: an optimistic and a pessimistic one. The paper is complete of a wide empirical section where the two equilibria are identified and specified for three of the main world markets. The regimes appear to be persistent and consistent with the existing literature on risk aversion. This also includes recent events of the financial crisis. The analysis uncovers a new development for exchange rate microstructure models. 3 of the 4 markets studied are consistent with both the order flow and the Markov switching models. The markets analyzed are the UK, Switzerland, Germany and Japan.

JEL categories: C2, F3, G1

Keywords: Exchange rates, Microstructure, Markov chains.

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1 Introduction

The relationship between the exchange rates and the stock prices has been a rich field of study for several decades. This is because we observe that the two markets have high (and similar) volatilities that are not justified by a purely fundamental analysis. Several papers have been written describing the statistical relationship between the two markets. However not always a solely statistical study may uncover the connection between the two (Hatemi-Ja and Roca, 2005; Hatemi-Ja and Irandoust, 2002; Ramasamy and Yeung, 2005; Granger et al., 1998). A theory drawing the underlying picture is at the core of most of the contributions. To this day there is no commonly accepted framework in the literature. The focus of the most recent research is the theory to choose in order to trace this relationship.

The papers on the subject may be divided in two classes. One is macro, observing the flows as a result of large economic fundamentals. The second one is micro, related to financial measures and how they affect prices and profit maximizing rules. Both make strong cases and are in fact probably working together in the determination of the exchange rate and asset price time series.

This paper will take a micro approach. Specifically it fits into the set of models describing the microstructure of exchange rates. The theoretical intuition comes from papers on capital flows. The application to exchange rate literature has been recent (Evans and Lyons, 2002). Since then it has quickly spread due to the simplicity of the concepts as well as to the strong empirical results obtained (Evans and Lyons, 2006).

More recently one of the papers on micro-structure has drawn a complete picture of the exchange rate through order flows and has tested the implications of the model on commonly available data. The great majority of the markets studied is consistent with the theory. The paper was published in the Review of Financial Studies by Hau and Rey (2006). Our paper is adopting the Hau-Rey framework.

Another stream of literature that has proven successful in increasing the forecasting power of previous models has been the time-dependent approach. There are several kinds of time-varying statistical models. The most flexible is typically GARCH. Then comes the literature on structural breaks. Finally, Markov Switching Models (MSM) are among the most regulated time-varying frameworks. In this paper we use these last set of models, due to the strong structure that it holds in the estimation (Otranto and Gallo, 2002; Alvarez-Plata and Schrooten, May 2003; Sims and Zha, 2004; Cheung and Erlandsson, December 2004; Psaradakis and Spagnolo, 2003; Bellone, 2005; Krolzig, 1996).

The main trait of this kind of models is that it alternates between a limited number of specifications. This means that the program is not allowed to create a better fitting model every time there is a break. Instead it is required to look for similarities throughout time. This is conceptually very similar to a representative agent's behavior, who looks at the past to interpret the present and forecast the future. In order to impose these preferences, a MSM is then the most desirable one.

The two streams of literature are merged in the model devised in the following paragraphs. Specifically, sections 2 and 3 show an order flow model that is subject to switching regimes. Section 4 carries out an empirical analysis to verify whether the theory is consistent with the data. We have also thought it interesting to describe how the results obtained by the model may be interpreted at the light of recent events and literature on the subject. Finally a short conclusion will summarize and highlight future research possibilities.

2 The order flow model

This paper is based on the theoretical framework described in Hau and Rey (2006). It focuses on the dynamics of the exchange rate in relation with the stock prices and capital flows. The value taken by the exchange rate is a function of the order flows originating from the capital market. This may be referred to as the micro-finance approach to exchange rates.

2.1 Intuition

There are two countries, Home and Foreign. Each country has two assets: stocks and bonds. This means that in the world there are four different assets. However only the stocks are traded internationally. Investors are not allowed to hedge their exchange rate risk when investing in the capital market. As a result only the stock market dynamics affects the evolution of the exchange rate.

The exchange rate takes on the value where the demand for foreign currency meets the supply. The foreign trade in capital assets necessarily affects the demand for foreign currency. Specifically, two major transactions cause the demand to rise. The first is given by new local investors wanting to buy foreign shares, the second is due to local companies giving out dividends that are going to be returned to the foreign investors. Both of these actions cause an increase in the demand for foreign currency balanced by a sale of local currency. In a symmetric way foreign investors buying local company shares and dividends given to local investors by foreign companies will contribute to the Home currency demand.

Facing this demand there is a supply mechanism. Foreign currency is supplied by institutions that give out liquidity with a certain elasticity (i.e. k). So the liquidity is given out as the pressure on the price of the foreign currency (exchange rate) gets heavier. This means that the elasticity has an important role in the determination of how much liquidity is supplied to the market, and therefore in setting the exchange rate.

Given the importance of the elasticity of liquidity supply we will describe it further. We will consider a world where the exchange rate follows a Ornstein-Uhlenbeck process that reverts to a stable equilibrium $\overline{S}$ at a constant speed α_e. The investors in foreign currency maximize a standard mean variance objective. Solving the model, the value of k is a function of the degree of risk aversion and of the statistical characteristics governing the O-U process (namely the speed of convergence to equilibrium and the exchange rate variance).

This means that a change in elasticity is caused by either a change in risk aversion or a change in the exogenous statistical properties of the exchange rate. More intuitively, if risk aversion decreases the investors are going to be happy with a lower profit for the same variance of the exchange rate. If the equilibrium value of the exchange rate does not change, the excess supply of foreign currency will decrease through a lower elasticity. In the next section we will make the assumption, following a branch of behavioral finance literature, that the degree of risk aversion will change over time. We will derive what should happen if it changes and compare the theoretical implications with the data.

Back to the description of the model, the dynamics of the stock prices is studied. These prices are a function of the local dividends. The steady states of the dividends and their expected values are also important in the determination of the value of the stocks. The dividends also follow a O-U process.

The model solves for a unique solution under market incompleteness (with respect to foreign exchange risk). This solution becomes equivalent to the one in a complete market in the limit where the currency elasticity of supply is infinite. Therefore the final solution is also a function of the value of k (for a formal description of the model please refer to appendix A).

2.2 Implications

The theory described has several important implications that allow its testing in an intuitive manner. The first result is that (for certain parameters) exchange rate returns have almost as much volatility as equity returns. The next result is the most relevant. It may be shown that foreign stock returns and exchange rate returns are negatively correlated[1]. In other words a positive foreign stock return will cause a depreciation of the foreign currency.

The intuition behind this is that equity trade is the source of order flows: a positive foreign dividend innovation will create a sequence of consequences. It increases the dividend of foreign stock. An increase in the dividend has two effects. The first one is that the price of the stock rises (equations 9 and 10 in the appendix), causing the returns to increase as well. The second one

[1] For the sake of synthesis, I will spare the formal proofs of these implications, that may be found in Hau and Rey, 2006.

is that the demand for foreign currency decreases. This is mainly due to the repatriation of the dividends (of the home investors). Foreign currency is sold versus the home currency, causing the foreign currency to depreciate[2].

Following this implication the exchange rate is an automatic partial hedge against foreign stock risk. With a similar intuition, the relative stock returns[3] have an exactly inverse correlation with the returns of the exchange rates. Empirically a simple negative correlation will be considered sufficient due to the presence of exogenous shocks as well as of country asymmetries.

Finally, the authors interpret the change of their empirical data over time (and in cross-sections) as a function of equity market development. So the negative correlation between the two markets increases as the equity market is more developed. While the article shows other implications, the ones described above are enough for our discussion.

3 What if parameters change over time?

The contribution of this paper is to assume that the economy changes over time within the framework offered by Hau and Rey's model. Specifically we will formulate the assumption that one of the exogenous parameters evolves according to a Markov chain. Of the "constants" considered in the model there is one in particular that is commonly considered to be non-varying in a shorter period. This is risk aversion.

On the one hand, to consider this as a constant in the long run would mean to assume that the overall perception of risk stays similar throughout time. On the other hand, it would be possible to make the opposite hypothesis that this is always changing according to no scheme at all. This would imply the forceful assumption that the agents' understanding and interpretation of the world keeps adjusting in manners always new and not based on past

[2]There is also another reason why the foreign currency is sold. It may be called the risk re-balancing channel after the paper by Hau and Rey. It means that the home investor will reduce its holding of foreign stock as a consequence of the larger exposure from the increased value of its shares.

[3]The relative stock returns are defined as simply the difference between the local and the foreign stock return at every time period.

schemes.

Therefore it is natural to assume for the time being that the parameter for risk aversion is indeed time-dependent, but within a certain scheme. This means that agents recognize the current situation as similar (under the risk approach point of view) to other past occurrences. As a consequence they will behave accordingly, in a similar fashion to how they did in the past.

In the remainder of this paper we will also assume that the other exogenous statistic properties of the exchange rate remain stable (in mean) through time. The motivation for this assumption is that the order flow model already has rules on the evolution of the exchange rate. To modify them and to assume that they change exogenously would be to forcefully add explanatory power without adding a sound economic reasoning[4]. The focus of this paper is to integrate an assumption that is popular in the financial literature with a promising new scheme for the foreign exchange market.

3.1 Changing equilibrium

It is reasonable to assume that the agents behave according to two separate rules. In other words we are going to assume that there are two different (and alternating) approaches to risk. Obviously one will be more optimistic, while the other will be more pessimistic. This type of agent behavior may be statistically described by a Markov Switching model. Both values taken by the parameter for risk aversion ρ will be within the limits of the theory described above[5]. By this we assume that in both cases there will exist one and one only equilibrium of the system, described formally in Appendix A by equations (10) through (12).

This is defined as the unique equilibrium of the relationship between the exchange rate and the stock prices of the two markets. While the theory concentrates on the definition of the steady states of the three variables (the

[4]However this assumption is thoroughly checked against the data in Appendix B.

[5]The risk aversion should not be too high since otherwise the behavior of the investors would not converge to any equilibrium. Similarly, the risk aversion is also related inversely with the parameter for price elasticity of forex supply, k. This should also be high enough to allow the system to compensate properly for the changes in stock prices and reach an equilibrium.

exchange rate and the two countries' stock prices) we will focus our analysis on the coefficient (also affected by the change in risk aversion) that relates the first difference of the exchange rate to the relative stock returns.

Since risk aversion varies according to regimes also the equilibrium of the system (which is function of it) will shift regime when the risk aversion parameter changes. This is as from Hidden Markov Chain (HMC) mechanism. The observable variable in this case is going to be the steady state of the system, when the real variable following it is hidden from observation. We assume that the system's steady state changes as a result of the HMC, but the actual timing of this shift might be slightly slower than in the hidden variable. In other words, from the moment the risk aversion parameter changes, the system will start moving to the new equilibrium. The length of this "switching" period will be dependent on the flexibility of the financial system and on the other parameters involved.

To test whether this theoretical framework is consistent with the empirical data the relationship between the exchange rate and the stock prices will be analyzed through a Markov Switching Model with 2 regimes. We will run a regression with the exchange rate returns as a dependent variable and the stock return differentials as the independent one. We expect to find a negative and significant coefficient of the stock returns in both the regimes. If it is so (and we get the same results over the whole sample period), then it is possible to say that the theory by Hau and Rey is consistent with the studied market. Then we will proceed to check whether the series could be the result of two distinct alternating regimes. Our hypothesis is that in the two specifications they will be different and persistent enough to uphold (be consistent with) the assumption of switching risk aversion.

4 Empirics

Since the theoretical model uses the foreign exchange approach to order flows, the empirical analysis is going to be carried out focussing on the exchange rate series. The object in study will be the exchange rate equation as explained by the difference of the stock returns of the two countries. Formally:

$$\Delta S_t = a + b(\Delta R_t^{f*} - \Delta R_t^h)/\overline{P} \quad (1)$$

where ΔS_t is the first difference of the exchange rate series (in logs) and the regressor is given by the returns on stocks as defined in the model above. According to the above theory the coefficient b should always be significant and negative.

We study the markets of the United Kingdom, Switzerland, pre-Euro Germany and Japan. All exchange rates are against the United States dollar. The sample period is from October 1983 through October 2008. Within this period homogenous data is available for all the countries. For Germany the observations will end on December 1998 to avoid any data mispecification from the introduction of the Euro. The frequency is monthly. The first returns of the stock indexes are used for each country. For the United States the index used is the Standard and Poor's. For United Kingdom it is the FTSE. For Germany we used the DAX and for Switzerland the index is the MSCI. For Japan we used the NIKKEI. All data is taken from Datastream.

At first there will be a non time-varying analysis over the whole sample. This will be needed to verify that the theory used is indeed consistent with the data in the long run. Then it will be interesting to start the time-dependent study. The markets that might be rejecting the theory will also be studied with a time-varying approach. However the interpretation of the results will be different.

Table 1 shows the results of the regression above. It is possible to see that for three out of the four countries the theoretical framework introduced works perfectly. This is because the table shows the b coefficients to be most always negative and significant (the probability values are shown in parenthesis). The theory predicts indeed a negative correlation between the two variables. The only market that clearly rejects the theory is the Japanese one. The coefficient is not only positive but also insignificant. This means that the negative correlation predicted by the theory is definitely missing[6].

[6]This conclusion is based on the data as described. However it matches perfectly with the data shown in Hau, Rey 2006. The correlations between the markets over these samples are -0.26 for the UK, -0.23 for Switzerland, -0.23 for Germany and 0.07 for Japan. This is in line with the coefficient results described above.

	a	b	R^2
UK	-0.051	-0.086	0.057
	(0.48)	(0.00)	
Switzerland	-0.252	-0.086	0.015
	(0.20)	(0.03)	
Germany	-0.002	-0.141	0.051
	(0.32)	(0.00)	
Japan	-0.127	0.049	0.007
	(0.10)	(0.12)	

Table 1: Coefficients of the regressions on the whole sample

Since the theory is consistent with the data it is interesting to check whether the time varying characteristic adds to the understanding of the data. In order to test our theory we are going to take a two step approach. First we are going to check whether it makes any sense to apply a time-dependent specification on these markets. This is done through a Markov Switching Model (2) that is regressed over equation 1. Only if this analysis shows consistent results then it is possible to proceed. Specifically the regimes will have to be persistent enough to allow for periods of system equilibrium. If this was not to happen it would be hard to see the economic interest of this approach.

If it is shown that the time dependent analysis gives interesting results, step 2 will test whether the structural breaks may be attributed to a change in risk aversion. However (as discussed above) the risk aversion parameter is not observable from these series. Therefore we will have to devise a way to check if at least the data is consistent with the assumption on risk aversion. This will be done by observing the value of a function of k in the periods when the two regimes have reached an equilibrium. This analysis will also show what periods are characterized by an optimistic and by a pessimistic regime[7].

The results of the first step are shown in Figures 1 through 4. The figures

[7]The robustness checks for this assumptions are carried out and listed in Appendix B.

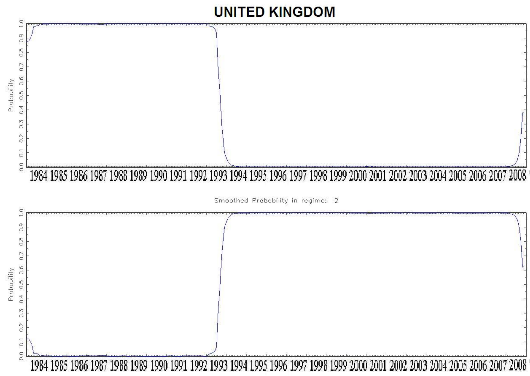

Figure 1: Smoothed probabilities of the two regimes for the British market.

represent the probability that at every point in time the market is in regime 1 or 2. Obviously the sum of the two is always equal to 1, as the market has to be in one of the states at all times. When this probability is 1 for the first regime and 0 for the other it is sure that the market is in the first regime. On the other hand when the probabilities are closer to a 50-50 scenario, it is not possible to tell in what regime the market is in. This could be seen as a period of switching between the two states.

It is interesting to notice how in the markets where our general framework is confirmed, there is a clear distinction between the periods in which the two regimes are acting. A peculiarity emerging from the pictures is also that in all the Hau-Rey type markets there is one single switch between the regimes. In all cases this switch takes a few time periods to be complete. The length of this "switching" time is due to the characteristics of the economies (i.e. their flexibility in adjusting to a new scenario). It is possible to show two separate time ranges in which the two regimes alternate. This is consistent with the time-varying assumption. Indeed it is reasonable to believe that if the risk aversion changes in time it is not going to keep switching back and forth. It is going to slowly bring the system towards another equilibrium. This is exactly the picture shown by the empirical analysis.

As shown earlier the Japanese market is not consistent with the general framework used. This means that even in the case that the risk aversion

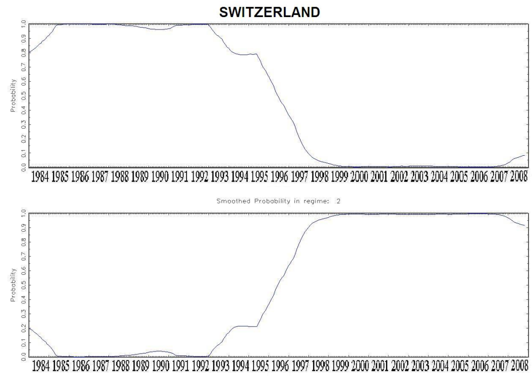

Figure 2: Smoothed probabilities of the two regimes for the Swiss market.

were to change, the system would not necessarily reach a stable equilibrium where the indipendent variable is relevant. That is because the exchange rate seems not to be dependent on the asset prices as heavily as it is in the other markets (see Table 1 for the overall R^2 values). Then it is natural that also the time-varying study gives different results than for the other countries.

Table 2 shows the coefficient b in the two regimes of the MSM given that the first regime to chronologically appear in the data is referred to as Regime A and the second as Regime B. It is possible to notice how in every market the two states have very different coefficients. This shows how this analysis adds value in the estimation of the variable[8].

However the fact that the markets switch regime does not necessarily prove that the source of the switch is the risk aversion. This is the reason why the second step is needed. We shall look at another statistic shown in Table 2. The variance ratio is formally defined as:

$$Var\ Ratio = \sqrt{\frac{Var(\Delta S_t)}{Var(\Delta R_t^{f*}/\overline{P})}}$$

It is a function of the variable k described in the previous section. k is the elasticity of the exchange rate supply, but in this model is also a function of the risk aversion and of the speed of return to equilibrium. From the table

[8]For a discussion of the last few observations of every series please refer to next section.

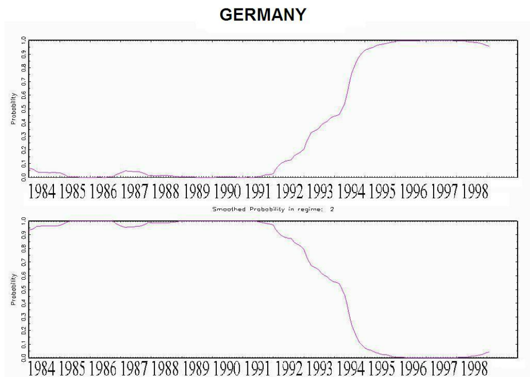

Figure 3: Smoothed probabilities of the two regimes for the German market.

we can see that the Variance Ratio changes considerably between the two regimes, and is always lower in Regime B. This means that the variable k becomes higher, which is consistent with a lower risk aversion parameter. In other words the analysis shows that Regime A may be defined a "pessimistic" regime, while Regime B is an "optimistic" regime. As it will be illustrated in the next paragraph, this is coherent with the literature on risk aversion.

The same statistic is also shown for the Japanese market. Of course in Japan the two periods to measure the ratio were arbitrarily chosen, only to compare with the other markets. The Variance Ratio falls over time also here. This is consistent with the time varying assumption that risk aversion changes normally in economies with capitalistic features. However the effect on the equilibrium of the system may only be seen when the variables considered are important in the determination of the economy steady state.

For completeness of the study it is important to also calculate the correlation coefficient in the different regimes. This is also shown in Table 2. The correlation increases in Germany and the UK, while it decreases in Switzerland. The model shows how the theoretical correlation assuming no asymmetries or exogenous shocks would be equal to 1. So the difference in these coefficients over time is attributed to equity market integration Hau and Rey (2006) or to other exogenous factors. For more information on robustness checks studying breaks in variance and in mean, please refer to Appendix B.

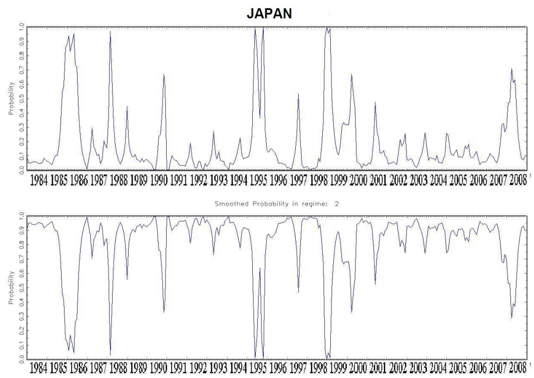

Figure 4: Smoothed probabilities of the two regimes for the Japanese market.

The Model vs The Real World

The theory described in the first part of the paper was tested empirically on the data. The clear result that was obtained in most of the markets is that the data is consistent with the order flow approach and that a time-dependent study upholds the varying risk aversion hypothesis. However this is not the only new information that came from the data.

First of all it may be noticed how in the three countries with a negative correlation, the regimes switch in similar periods. This means that the model has found a generalized lowering of the risk aversion during the nineties. This has began to change starting in the end of 2007. So the big question becomes: how does this relate to our literature on the subject?

Risk has always been an interesting topic for research, so the literature on the field is very wide. However it was not until the 80s and 90s that a varying risk aversion has been studied more in detail Hansen and Singleton (1983); Brunnermeier and Nagel (2004); Brandt and Wang (2002). Specifically, it is now usually accepted that risk aversion may change, although there is not yet agreement on the exact schedule on which it does so. What is more interesting is the fact that the risk aversion literature points to a strong decrease of the perception of risk that started in the nineties (Brandt and Wang, 2002; Brunnermeier and Nagel, 2004).

Furthermore financial papers and journals have dedicated a lot of atten-

		Regime A	Regime B
UK	t	Oct 1983 - Nov 1992	Dec 1994 - Nov 2007
	corr	-0.20	-0.40
	coef	-0.21	-0.28
	p-value	0.04	0.00
	var ratio	0.70	0.48
Switzerland	t	Oct 1983 - Oct 1992	Apr 1999 - Nov 2007
	corr	-0.25	-0.19
	coef	-0.14	-0.09
	p-value	0.00	0.01
	var ratio	0.73	0.63
Germany	t	Oct 1983 - Apr 1991	Dec 1995 - Dec 1998
	corr	-0.08	-0.53
	coef	-0.08	-0.33
	p-value	0.17	0.00
	var ratio	0.54	0.45
Japan	t	\\	\\
	corr	\\	\\
	coef	-0.08	0.13
	p-value	0.25	0.00
	var ratio	0.62	0.44

Table 2: Coefficients of the MSM

tion to varying risk premia in the last years (among the others, see Canto (2005)). It was widely recognized that they have been decreasing sharply starting in the 90s. Risk premia may be related to risk aversion since their fall implies that agents require a lower price for taking the same amount of risk, which could be taken as measure of a decreasing risk aversion.

An important element of the analysis is that it also shows the effects of the current financial crisis. The break originating at the end of 2007 in UK and Switzerland is easy to interpret. After the sub-prime crisis broke out on last year's summer period, people have corrected their risk aversion once again. The system is now switching to the regime that is characterized by a higher risk aversion.

At the light of this research and financial press, it is possible to see the pattern that has been highlighted in this paper. Although the data does not prove the model in itself, it is very coherent with it.

5 Conclusion

This paper described an order flow approach to the exchange rate market. This is studied in relationship with the stock prices. The model adopted is of recent development and may be the beginning of a new approach to exchange rate study. The innovation has been to make this approach time-varying.

The analysis that was carried out shows how this modification has potential in explaining the time series. This is due to the clear results provided by the empirical estimation. The regimes found are persistent and in agreement with the theory. Indeed the data shows the changes in the two regimes in a consistent manner with the financial literature and press. That is a crucial result for the theory's consistency.

This shows why the approach taken in this paper is encouraging. However future space for research is left in several aspects. First of all it would be possible to develop a further specification of the exogenous parameters. Secondly a more explicit process of exchange rate (other than an O-U process) would probably improve the fit of the model. Also, this would render the approach of order flows flexible to be incorporated in exchange rate literature of most schools of thought. Thirdly, characterizing the out of equilibrium dynamics would be interesting to see whether the length of the switching time between regimes is a function of economic variables.

Overall this theoretical framework seems to be promising and ready for more constraining assumptions. The simplicity of the intuition makes it a powerful tool for the explanation of the exchange rates and the stock prices. The accuracy of the test results testifies to the potential of this theory. The little number of variables used should encourage a wider analysis to include also exogenous observable changes.

Appendix A - Hau and Rey framework and time dependent add-on

We will take a closer look at the theoretical framework. In a 2 country world (Home and Foreign) there exist 4 assets. These are Home and Foreign riskless bonds and Home and Foreign risky stocks. For the sake of simplicity the representative local investor cannot invest in foreign bonds. The stocks provide a continuous (stochastic) dividend flow D_t^h and D_t^f. The bonds give a constant return in the local currency.

The goal of the two representative local investors is to maximize their excess returns (over the riskless rate), given their risk aversion. They have the possibility to diversify their equity portfolios so that they include both countries equities, $K_t = (K_t^h, K_t^f)$ and $K_t^* = (K_t^{h*}, K_t^{f*})$. Given the excess payoffs in local currency over the local riskless bond, $dR_t = (dR_t^h, dR_t^f)$ and $dR_t^* = (dR_t^{h*}, dR_t^{f*})$ profit flows are given by:

$$d\Pi = K_t dR_t \tag{2}$$
$$d\Pi = K_t dR_t$$

So the representative investors for the two countries solve the mean-variance problems:

$$\max_{\{K_t^h, K_t^f\}} E_t \int_{s=t}^{\infty} e^{-r(s-t)} [d\Pi_s - \frac{1}{2}\rho d\Pi_s^2] \tag{3}$$
$$\max_{\{K_t^{f*}, K_t^{h*}\}} E_t \int_{s=t}^{\infty} e^{-r(s-t)} [d\Pi_s^* - \frac{1}{2}\rho d\Pi_s^{*2}]$$

where E is the rational expectations operator and ρ is a parameter for risk aversion. When markets clear, both the stocks are normalized to one. The foreign exchange order flow is endogenous. It can be calculated as a clearing condition of the exchange market. Since there is no trade in bonds, the stock flows are the only ones that make the order flow for foreign currency in the forex market. Intuitively, the equity related capital flow is given by:

$$dQ_t = S_t K_t^{h*} D_t^h dt - K_t^f D_t^f dt + dK_t^f P_t^f - S_t dK_t^{h*} P_t^h \tag{4}$$

where dQ_t is the total capital flow from equity trade (out of the home country in foreign currency terms) and S_t is the exchange rate. Specifically, the outflows of dividends are shown first and any increase in foreign equity holding is shown in the last two elements of the equation. This form may be approximated linearly by:

$$dQ_t^D = (S_t - \overline{S})\overline{K}\overline{D}dt + (\overline{S}K_t^{h*} - K_t^f)\overline{D}dt + (\overline{S}D_t^h - D_t^f)\overline{K}dt + (dK_t^f - \overline{S}dK_t^{h*})\overline{P} \tag{5}$$

with barred variables being unconditional means of stochastic variables.

In order for the forex market to clear, the excess supply of foreign exchange must equal the demand. We look now at the supply side. It is characterized by liquidity-supplying banks which can ease foreign exchange disequilibria. The elasticity of the supply curve k plays an active role in clearing the foreign exchange market. Excess supply is given by $Q_t^S = -k(S - \overline{S})$, where $\overline{S}$ is the steady state exchange rate.

Risk averse currency traders solve a problem similar to the asset traders (shown in equation 2), with $d\Pi_t = Q_t^S dS_t$ and ρ as the currency traders' risk aversion[9]. The optimal supply of liquidity may be found to be $Q_t^S = E_t(dS_t)/\rho\sigma_s^2 dt$. If the exchange rate follows a Ornstein-Uhlenbeck process with a constant equilibrium $\overline{S}$ and return speed of α_s, then it is possible to set $E_t(dS_t) = \alpha_s(\overline{S} - S_t)dt$. Therefore we can write:

$$Q_t^S = \alpha_s(\overline{S} - S_t)/\rho\sigma_s^2 = -k(S_t - \overline{S}) \tag{6}$$

In order for the market to clear $Q_t^S = Q_t^D$, therefore:

$$-kdS_t = (S_t - \overline{S})\overline{K}\overline{D}dt + (\overline{S}K_t^{h*} - K_t^f)\overline{D}dt + (\overline{S}D_t^h - D_t^f)\overline{K}dt + (dK_t^f - \overline{S}dK_t^{h*})\overline{P} \tag{7}$$

We can normalize $\overline{S}$ to 1. Furthermore, having established the foreign exchange market clearing condition it is possible to show[10] that the Home excess payoffs are given by $dR_t^h = dP_t^h - rP_t^h dt + D_t^h dt$. The Foreign excess payoffs are approximated around $\overline{S} = 1$ and the steady state prices $\overline{P}$ so that $dR_t^f \approx -dS_t\overline{P} + dP_t^f - dS_t dP_t^f - r[P_t^f - \overline{P}(S_t - 1)]dt + [D_t^f - \overline{D}(S_t - 1)]dt$. Excess returns are then formalized as $dR_t^h/\overline{P}$ and $dR_t^f/\overline{P}$.

[9] We will assume the two risk aversion parameters to be equal

[10] See Hau and Rey for the details of the derivations.

The dividends from Home and Foreign follow independent Ornstein-Uhlenbeck processes that have equal variance and speed of mean reversion as follows:

$$dD_t^h = \alpha_D(\overline{D} - D_t^h)dt + \sigma_D dw_t^h \quad (8)$$
$$dD_t^f = \alpha_D(\overline{D} - D_t^f)dt + \sigma_D dw_t^f \quad (9)$$

with dw_t^h and dw_t^f being independent and $\alpha > 0$. The expected present values of the future flow of dividends may be shown as variables F_t^h and F_t^f to be formally defined as follows:

$$F_t^h = E_t \int_{s=t}^{\infty} D_s^h e^{-r(s-t)} ds = f_0 + f_D D_t^h$$
$$F_t^f = E_t \int_{s=t}^{\infty} D_s^f e^{-r(s-t)} ds = f_0 + f_D D_t^f$$

with $f_D = 1/(\alpha_D + r)$ and $f_0 = (r^{-1} - f_D)\overline{D}$

The exchange rate dynamics is defined according to the assumption of incomplete risk-sharing[11]. Given the symmetry between countries, the exchange rate has to be a function of the relative changes in dividend innovations, so $dw_s = dw_s^h - dw_s^f$. Every other variable within the exchange rate determination is going to be endogenous. The dynamics may be formalized as follows:

$$S_t = 1 + e_\Theta \Theta_t + e_\Lambda \Lambda_t,$$

with

$$\Theta_t = D^h - D_t^f = \int_{-\infty}^{t} exp[-\alpha_D(t-s)]\sigma_D dw_s$$
$$\Lambda_t = \int_{-\infty}^{t} exp[z(t-s)] dw_s$$

with $z < 0$ being a first characteristic root associated with the supply induced mean reversion. The constraint from above becomes:

[11] In this general framework, two benchmarks are examined for financial specification: financial autarky (with no trade in stocks) and complete risk sharing (including the exchange rate risk). However the economic implications are analyzed from the incomplete risk sharing assumption. This is also the most interesting setting for our contribution.

$$dS_t = k_1\Theta_t dt + k_2(E_t - 1)dt + k_3 dw_t$$

where k_1, k_2 and k_3 are undetermined coefficients.

We have now described the key assumptions of the model in the incomplete risk sharing framework. It is possible to prove that for a sufficiently low risk aversion (ρ) and for a sufficiently high price elasticity of foreign exchange supply (k) there exists a unique stable (linear) equilibrium:

$$P_t^h = p_0 + p_F F_t^h + p_\Theta \Theta_t + p_\Lambda \Lambda_t \tag{10}$$

$$P_t^f = p_0 + p_F F_t^f - p_\Theta \Theta_t - p_\Lambda \Lambda_t \tag{11}$$

$$S_t = 1 + e_\Theta \Theta_t + e_\Lambda \Lambda_t \tag{12}$$

This is the equilibrium described in the main text. All the implications described spring off of these equations. Those implications are tested in the beginning part of our empirical section. this is also the equilibrium that changes in case of variations of the parameters involved. For example a change in the risk aversion parameter as described in section 3 would affect this equilibrium in the following way. For two different realizations of ρ being ρ_1 and ρ_2, there will be two alternating equilibria of the system. When the realized risk aversion will be equal to ρ_1 we will have:

$$P_{1,t}^h = p_{1,0} + p_F F_t^h + p_\Theta \Theta_t + p_{1,\Lambda} \Lambda_{1,t} \tag{13}$$

$$P_{1,t}^f = p_{1,0} + p_F F_t^f - p_\Theta \Theta_t - p_{1,\Lambda} \Lambda_{1,t} \tag{14}$$

$$S_{1,t} = 1 + e_\Theta \Theta_t + e_{1,\Lambda} \Lambda_{1,t} \tag{15}$$

On the other hand, when the risk aversion of the agents switches to ρ_2, then the equilibrium of the system moves to:

$$P_{2,t}^h = p_{2,0} + p_F F_t^h + p_\Theta \Theta_t + p_{2,\Lambda} \Lambda_{2,t} \tag{16}$$

$$P_{2,t}^f = p_{2,0} + p_F F_t^f - p_\Theta \Theta_t - p_{2,\Lambda} \Lambda_{2,t} \tag{17}$$

$$S_{2,t} = 1 + e_\Theta \Theta_t + e_{2,\Lambda} \Lambda_{2,t} \tag{18}$$

There is one remark to make. The set of equations above would represent the equilibrium after the switch[12]. It would not show how long the system takes to achieve the new equilibrium.

Finally, the regression tested in the main text of this paper is a simplification of the three equations shown above. When the new equilibrium is achieved, also the coefficient b will be different as a consequence of all the variables that are functions of k. That is why it is possible to study the simpler form of the system equilibrium. This simplification allows us to focus on the direct relationship between the exchange rate and the stock prices.

[12]In order to see how the indexed variables above are related to ρ it is possible to refer to the appendix of Hau, Rey, 2006 Hau and Rey (2006). All the connections are clearly drawn and the proofs are shown.

Appendix B - Robustness checks

Within this paper we tested that the relationship between exchange rates returns and stock returns changes over time. We have assumed that the origin of the change of regime was risk aversion. Specifically, we assumed that a break in the level of risk aversion would cause a switch of regime of the system. Indeed the switches have been found. The different markets showed consistent results through time and space. However only indicative support has been shown for the fact that indeed it is risk aversion, and not other variables, that causes the switches.

This appendix looks at the behavior of exchange rate variance and mean over time. We can start from the model equilibrium elasticity of:

$$k = \frac{\alpha_s}{\rho\sigma_s^2}$$

As described above, k is the elasticity of supply, and in equilibrium (and for the assumptions listed in appendix A) it is a function of the investors risk aversion and of some statistical characteristics of the exchange rate, namely the variance and the speed of return to equilibrium.

While it would be possible that all other variables than risk aversion stay constant, an overwhelming amount of literature has showed that they also vary through time. The question is simply whether they are indeed also responsible for the switches pointed out in this paper, or not.

In order to achieve this goal, we studied the variance of the logs exchange rate for any structural breaks (Bai and Perron, 1998a,b). Table 3 shows the results.

As predicted, the variance is anything but constant. However, despite the several structural breaks, it seems that in none of the markets the breaks

Market	Breaks in Exchange Rate variance
UK	April 87, May 94, April 00, December 2003
Switzerland	April 87, June 94, January 98, November 2003
Germany	January 86, June 90, March 94, June 96

Table 3: Structural breaks in Variance of Exchange Rates

Market	α_s	
	Period 1	Period 2
UK	0.04	0.01
Switzerland	0.11	0.02
Germany	0.01	0.05

Table 4: Values of Mean-Reversion parameters of the Exchange Rates

could explain the sudden changes in regimes shown in the paper. This is due to the fact that several structural breaks occur in each of the time series. However in our Markov Switches analysis only one major switch was seen found. This means that, most likely, whatever the source of the change in the variable of elasticity is, there is something more relevant in the data than the breaks in the variance.

Another important parameter to study is the speed of return to equilibrium (α_s) as defined by the Ornstein-Uhlenbeck process. Equation 6 shows how it is part of the equilibrium value of k, together with risk aversion and the exchange rate standard deviation. Table 4 shows the evolution of this parameter over time, in the 3 markets where the system switches between two persistent regimes. The values are from the same time periods as the ones found in Table 2. A higher α_s would justify the higher k value found in the previous empirical test in the second period. However the data is not consistent in this direction. On the contrary, in two of the three markets the value of the speed of mean reversion seems to decrease in the second period.

The evidence shown in this appendix is supporting the assumption that the major switch in system equilibrium is caused by a changing risk aversion. The main empirical analysis and these robustness checks are indeed in line with a declining risk aversion through the nineties and a new, opposite switch since 2007.

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