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revealed preference analysis
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**DISCUSSION
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Degrees of cooperation in household consumption models:

A revealed preference analysis

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Abstract

We develop a revealed preference approach to analyze non-unitary consumption models with intrahousehold allocations deviating from the cooperative (or Pareto efficient) solution. At a theoretical level, we establish revealed preference conditions of household consumption models with varying degrees of cooperation. Using these conditions, we show independence (or non-nestedness) of the different (cooperative-noncooperative) models. At a practical level, we show that our characterization implies testable conditions for a whole spectrum of cooperative-noncooperative models that can be verified by means of mixed integer programming (MIP) methods. This MIP formulation is particularly attractive in view of empirical analysis. An application to data drawn from the Russia Longitudinal Monitoring Survey (RLMS) demonstrates the empirical relevance of consumption models that account for limited intrahousehold cooperation.

JEL Classification: D11, D12, D13, C14.

Keywords: household consumption, intrahousehold cooperation, revealed preferences, Generalized Axiom of Revealed Preference (GARP), mixed integer programming (MIP).

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1 Introduction

We present a nonparametric revealed preference characterization of non-unitary household consumption models that are identified by varying degrees of cooperation. This characterization allows us to develop a practical method for analyzing a whole spectrum of noncooperative-cooperative consumption models. In addition, it enables us to derive some interesting theoretical results, such as independence (or non-nestedness) of consumption models with different degrees of cooperation. We use our method to analyze household consumption data taken from the Russia Longitudinal Monitoring Survey (RLMS). To the best of our knowledge, this is the first empirical application of consumption models that account for household behavior that is not fully cooperative. This introductory section motivates our main research questions, and relates them to the existing literature.

Non-unitary household consumption and cooperation. There is a growing consensus that multi-person household consumption behavior should no longer be treated as if the household were a single decision maker that optimizes a household utility function subject to the household budget constraint. Indeed, this so-called unitary model of household consumption imposes empirically testable restrictions on the household demand function (e.g. Slutsky symmetry) that are frequently rejected when confronted with consumption or labor supply data of multi-person households. See, for example, Fortin and Lacroix (1997), Browning and Chiappori (1998) and Cherchye and Vermeulen (2008).

Because of these empirical problems of the unitary model, an emerging literature explicitly acknowledges that households are composed of distinct individuals who are endowed with their own preferences, and that household consumption decisions are determined by an underlying intrahousehold decision mechanism. We refer to this approach as the non-unitary approach to household consumption. Typically, non-unitary consumption models allow for privately consumed goods as well as publicly consumed goods within the household. In addition, following Apps and Rees (1988) and Chiappori (1988, 1992), the usual assumption is that household allocations are Pareto efficient; in the household consumption literature, Pareto efficiency corresponds to the so-called cooperative within-household solution of the intrahousehold allocation problem.¹ However, the Pareto efficiency assumption has been questioned for the publicly consumed goods. Most notably, it has been argued that the informational requirement and the resulting cost of implementing cooperation may often be unrealistic. See, for example, Browning, Chiappori and Lechene (2007) and Lechene and Preston (2005, 2008).

In this paper we develop a framework for distinguishing between different non-

¹Following Chiappori (1988, 1992), the consumption literature often refers to the cooperative model as the ‘collective’ model of household behavior.

unitary consumption models in terms of the degree of cooperation. At this point, it is worth noting that we see at least two reasons why it is important to know the magnitude of intrahousehold cooperation. First, from a welfarist perspective, it gives an idea of the welfare improvement that is possible within a certain household. If it is possible to link the level of cooperation to household characteristics, it may be possible to use this knowledge for welfare enhancement measures that correct the efficiency loss originating from household behavior that is not fully cooperative. Second, the issue has also important implications for the structure of optimal taxation and policies that target to alter the intrahousehold income distribution. See, for example, Blundell, Chiappori and Meghir (2005) for a discussion on such targeting issues in a non-unitary setting. In this respect, different (cooperative-noncooperative) consumption models may lead to other intrahousehold allocations. In fact, the literature has revealed a need for non-unitary household consumption models situated between a fully cooperative case and a fully noncooperative situation, in order to obtain a realistic modeling of observed behavior. See, for example, d'Aspremont and Dos Santos Ferreira (2009) for discussion.

In what follows, we will provide a characterization of the whole cooperative-noncooperative spectrum. At the one extreme, the fully cooperative solution corresponds to the Pareto efficient within-household allocation mentioned before. At the other extreme, the fully noncooperative solution corresponds to a Nash equilibrium allocation within the household. Finally, we also characterize the semicooperative case, which is situated on a continuum between the cooperative case and the non-cooperative case. We will argue that our characterization of this semicooperative case has a natural interpretation in terms of the degree of cooperation within the household.

The cooperative-noncooperative spectrum: literature review. By now, the modeling of the fully cooperative case is quite complete. Browning and Chiappori (1998) provide a local differential characterization of the cooperative model. A general finding is that if the household acts cooperatively, then the unitary condition of Slutsky symmetry no longer holds. By contrast, cooperative behavior imposes that there exists a household pseudo-Slutsky matrix that can be decomposed as the sum of a symmetric negative semi-definite matrix and a matrix of rank 1 (in the case of two household members). As shown by Chiappori and Ekeland (2006), this condition, together with homogeneity and adding up, is also locally sufficient for the existence of individual utility functions and Pareto weights that reproduce the observed behavior. Cherchye, De Rock and Vermeulen (2007, 2009a,b) complement these local differential results by presenting a global revealed preference characterization of the same cooperative model. In the tradition of Afriat (1967) and Varian

(1982),² they derived necessary and sufficient conditions for household consumption data to be consistent with the model. For the publicly consumed quantities, these conditions require the existence of suitable Lindahl prices such that each individual in the household satisfies the Generalized Axiom of Revealed Preference (GARP; see Section 2) when using these individual Lindahl prices to evaluate the public goods.

At the other extreme of the spectrum, the fully noncooperative model assumes that each individual within the household maximizes her/his own utility given the consumption of the other household members. In this case, the household consumption decision is determined by the Nash equilibrium solution with voluntary contributions for the publicly consumed goods. See, among others, Lundberg and Pollak (1993), Browning, Chiappori and Lechene (2007) and Lechene and Preston (2005, 2008). As for the (local) differential characterization of this model, data consistency with the noncooperative model requires the existence of a pseudo-Slutsky matrix that can be decomposed as a symmetric negative semidefinite matrix and a matrix with rank less than the number of public goods plus 1 (again in the case of two household members). Three remarks are important in view of our following exposition. First, at present it is not known whether these noncooperative conditions are also (locally) sufficient. Second, these noncooperative conditions are nested with the (differential) cooperative conditions mentioned above: data consistency with the cooperative conditions always implies data consistency with the noncooperative conditions, but not vice versa. Finally, to the best of our knowledge a complementary global revealed preference characterization of the noncooperative household consumption model is nonexistent in the literature.³

Between the two (cooperative and noncooperative) extremes, we can conceive a continuum of semicooperative cases. These cases differ in the degree to which a certain household member behaves cooperatively towards the other household members. We are aware of only one study that investigates these intermediate cases. Specifically, d'Aspremont and Dos Santos Ferreira (2009) consider a semicooperative model where the willingness to pay for public goods is between the Lindahl price vector associated with the cooperative equilibrium and the market price vector. Focusing on the local differential characterization of this semicooperative behavior, they also derive corresponding rank conditions on the pseudo-Slutsky matrix. We will discuss the characterization of d'Aspremont and Dos Santos Ferreira in more depth when we relate it to our characterization of semicooperative behavior. At this point, it is important to indicate that the above three remarks for the noncooperative model extend to the semicooperative model of d'Aspremont and Dos Santos Ferreira.

²See also Samuelson (1938), Houthakker (1950) and Diewert (1973) for seminal contributions on the revealed preference approach to analyzing consumption behavior.

³However, see Sprumont (2000) for a revealed preference characterization of the noncooperative Nash solution in a choice-theoretic framework à la Richter (1966).

This study. We will develop an alternative framework for modeling household consumption behavior characterized by varying degrees of cooperation; this framework will contain the fully cooperative model and the fully noncooperative model as limiting cases. We will explicitly discuss the relationship between our framework and the one of d’Aspremont and Dos Santos Ferreira (2009). In contrast to most research in the literature, we focus on the revealed preference characterization of (cooperative, noncooperative and semicooperative) consumption behavior.

The revealed preference approach has a number of attractive features. First of all, our characterization is global, which contrasts with the local characterization obtained by the standard differential approach. Specifically, we get global conditions that enable checking consistency of a given data set with a particular consumption model; in the spirit of Varian (1982), we refer to this as ‘testing’ data consistency with the model under study. Second, we are able to verify these conditions while keeping their inherent nonparametric nature, i.e. the associated tests do not require an a priori (typically non-verifiable) parametric specification of the intrahousehold decision process (e.g. individual preferences). By contrast, the differential approach (until present) usually maintains additional assumptions concerning the functional form for the demand function (and thus individual preferences) when verifying the abovementioned rank conditions of the pseudo-Slutsky matrix (e.g. Browning and Chiappori (1998) start from a quadratic almost ideal demand system in their empirical analysis). More specifically, our nonparametric tests apply mixed integer programming (MIP) methods, which combine linear constraints with binary integer variables. This MIP formulation is particularly attractive from a practical point of view: for a given data set, it allows for testing data consistency with a specific consumption model by applying standard MIP solution techniques.

Two further features imply notable differences with the differential results described above. First, the testable revealed preference conditions are not only necessary but also sufficient for data consistency with specific (cooperative, noncooperative and semicooperative) consumption models. Second, we will show that the conditions for the semicooperative model (or, in a limiting case, the conditions of the fully noncooperative model) are not nested with the cooperative conditions: data consistency with the (global) semicooperative conditions is neither necessary nor sufficient for data consistency with the (global) cooperative conditions. This makes it interesting to compare the empirical validity of different models. In fact, we can meaningfully verify data consistency with a given model (and compare different models) even if there are only a few observations and without restriction on the number of privately consumed goods (see Section 4.4 and Section 5).

We demonstrate the practical usefulness of our approach through an empirical application to data taken from the RLMS. As indicated above, as far as we know, this is

the first application of noncooperative and semicooperative household consumption models to a real-life data set. Interestingly, this application demonstrates the empirical relevance of our theoretical insights on independence (or non-nestedness) of, on the one hand, the revealed preference conditions for the fully cooperative model and, on the other hand, the revealed preference conditions for other (not fully cooperative) models. As such, it motivates considering noncooperative and semicooperative models in addition to the (more common) cooperative model in empirical analysis of household consumption behavior.

The rest of this study is organized as follows. To set the stage, Section 2 recaptures the revealed preference characterization of individually rational behavior. Section 3 introduces a general household game concept, which applies to the consumption decisions of multi-person households. This concept will provide the starting point for our discussion in Section 4, which gives a revealed preference characterization of the cooperative-noncooperative spectrum introduced above. This section also discusses independence (or non-nestedness) between the conditions of non-unitary consumption models characterized by different degrees of cooperation. Section 5 introduces the MIP approach for empirical verification of the different conditions, and presents our empirical application. Section 6 summarizes and formulates a number of concluding remarks.

2 The rational individual benchmark

In this section, we provide a brief introduction to the theory of revealed preferences. Specifically, we consider the optimization problem of a rational single individual. This will ease our following discussion of non-unitary consumption models, which assume rational individuals.

Consider an individual with a utility function U . Throughout, we will assume that utility functions U are continuous, concave, non-satiated and non-decreasing in their arguments. Let $T = \{1, \dots, |T|\}$ be a set of observations. Given a (strictly positive) price vector p_t and income Y_t ($t \in T$), we assume that the rational individual chooses the consumption bundle q in her/his budget set that maximizes her/his utility. In particular, the rational individual solves the following optimization problem (**OP-I**):⁴

$$q_t \in \arg \max_q U(q) \text{ s.t. } \langle p_t, q \rangle \leq Y_t$$

A data set $S = \{p_t, q_t\}_{t \in T}$ consists of a collection of strictly positive price vectors p_t and a collection of positive demand vectors q_t . We use the following concept of

⁴Observe that, under our maintained assumptions for the utility function U , we have that in equilibrium the budget restriction should hold with equality. The same applies to problems OP-H.A and OP-H.B below.

individual-rationalizability.

Definition 1 (individual-rationalizability) Consider a data set $S = \{p_t, q_t\}_{t \in T}$. The set S is individual-rationalizable if there exist a utility function U such that for all $t \in T$, the bundle q_t solves OP-I given the price vector p_t and income $Y_t = \langle p_t, q_t \rangle$.

Varian (1982) established that the set S is individual-rationalizable if and only if it satisfies the Generalized Axiom of Revealed Preference (GARP).

Definition 2 (garp) Consider a data set $S = \{p_t, q_t\}_{t \in T}$. The set S satisfies GARP if there exists a binary relation R such that the following holds. If $\langle p_t, q_t \rangle \geq \langle p_t, q_v \rangle$ then $q_t R q_v$. Next, $q_t R q_v$ if $q_t R q_s$, $q_s R q_l$, ..., $q_z R q_v$ for some sequence s, l, \dots, z . Finally, if $q_t R q_v$ then $\langle p_v, q_v \rangle \leq \langle p_v, q_t \rangle$.

In words, R captures the revealed preference relation in the data set S . We have $q_t R q_v$ if q_t is directly revealed preferred to q_v (i.e. $\langle p_t, q_t \rangle \geq \langle p_t, q_v \rangle$) or indirectly revealed preferred to q_v (i.e. there exists a sequence s, l, \dots, z such that $q_t R q_s$, $q_s R q_l$, ..., $q_z R q_v$). Finally, if $q_t R q_v$, then we must have $\langle p_v, q_v \rangle \leq \langle p_v, q_t \rangle$, i.e. q_v cannot be more expensive than any revealed preferred q_t .

The following theorem is probably the single most important result in revealed preference theory (see Varian, 1982, based on Afriat, 1967).

Theorem 1 Consider a data set $S = \{p_t, q_t\}_{t \in T}$. The following conditions are equivalent:

1. There exists a utility function U that individual-rationalizes S .
2. S satisfies GARP.
3. For all $t \in T$, there exist a positive number U_t and a strict positive number λ_t such that, for all $t, v \in T$,

$$U_t - U_v \leq \lambda_v \langle p_v, q_t - q_v \rangle.$$

This result has two important implications. First, data consistency with GARP is necessary and sufficient for individual-rationalizability of the data; see condition 2. Next, condition 3 provides an equivalent characterization in terms of the so-called Afriat inequalities, which allow an explicit construction of the utility levels associated with each observation t (i.e. utility level U_t for observed q_t). In our following discussion of consumption models, we will mainly concentrate on the GARP characterization of rational individual behavior. As we will show, this focus on GARP enables us to formulate testable implications of consumption models in mixed integer programming (MIP) terms (see Section 5). However, in principle our GARP-based characterization of consumption models can equivalently be expressed in terms of Afriat inequalities (by building on Theorem 1; see also the proof of Theorem 4).

3 The household game

To keep our exposition simple, we focus on 2-person (A and B) households in what follows. However, extensions to households with more than 2 members are fairly straightforward. Individuals have to decide over the consumption of a bundle of $|J|$ private goods ($J = \{1, \dots, |J|\}$) and a bundle of $|K|$ public goods ($K = \{1, \dots, |K|\}$). Given private and public consumption in the household, the utility of the individuals A and B is given by the functions $U^A(q^A, Q^A + Q^B)$ and $U^B(q^B, Q^A + Q^B)$, with q^A and q^B the private consumption bundles of A and B , and Q^A and Q^B the contributions to the public goods from A and B .⁵ The fact that we explicitly distinguish between A and B 's contributions to the public consumption may seem a bit unconventional. However, this distinction will be essential for modeling behavior that deviates from fully cooperative (or Pareto efficient) household behavior (e.g. Lechene and Preston, 2005, 2008, and d'Aspremont and Dos Santos Ferreira (2009) make similar distinctions).

The household consumption levels depend on the intrahousehold decision making process. As discussed in the Introduction, we will consider three types of non-unitary household models, which will have different equilibrium characterizations: the cooperative case, which assumes a Pareto efficient intrahousehold solution; the noncooperative case, which assumes a noncooperative Nash equilibrium solution; and the semi-cooperative case, which is situated on a continuum between the cooperative and the non-cooperative solutions. To formalize this idea, we will discuss the three models as particular cases of what we call the *household game*. Essentially, this household game describes each consumption decision as resulting from a two-step process. In a certain sense, this two-step representation generalizes the two-step representation of the cooperative consumption model; see, for example, Chiappori (1988, 1992). Just like for the cooperative model, it is important to remark that our two-step representation of the household game should not necessarily correspond to the actual decision making process within the household. We only assume that observed household behavior can be represented *as if* it follows from a two-step procedure.

In the first step of the household game, the total household income Y is divided between A and B , which defines the individual incomes Y^A and Y^B (with $Y^A + Y^B = Y$). In this study, we abstract from explicitly modeling this first step. In general, however, this intrahousehold income distribution can be seen as a function of exogenous variables such as prices, household expenditures and other variables that affect household decisions but not the preferences or the household budget (i.e.

⁵Throughout, we will abstract from externalities associated with privately consumed quantities. Importantly, however, our setting can actually account for such externalities. Specifically, if an individual is the exclusive consumer of a particular private good, then we can account for externalities for this good by formally treating it as a public good.

so-called extra-environmental parameters in the terminology of McElroy (1990) or distribution factors in the terminology of Browning, Bourguignon, Chiappori, and Lechene (1994)). In the second step, each individual (A and B) decides on the optimal level of the own private consumption and the own contribution to the level of public goods, by maximizing her/his own utility subject to a personalized budget constraint defined by the individual income. In doing so, the individual faces the price vectors p and P for her/his choice of private consumption and public contribution. In addition, in the general version of the household game, each individual receives a donation from the other individual per unit of public good that she/he purchases. We denote these donations for each good $k \in K$ by τ_k^A and τ_k^B ; τ^A and τ^B represent the vectors of donations. We see at least two interpretations for these intrahousehold donations related to public goods. First, one can see these donations as voluntary contributions: as B benefits from the purchase of Q_k^A , it may be the case that she/he is willing to contribute to the purchase of this bundle. Next, one can also interpret them as representing an implicit tax that B has to pay for the benefit of receiving Q_k^A . Both interpretations express that intrahousehold donations (i.e. a given specification of τ_k^A and τ_k^B) refer to the degree of (voluntary or obligatory) cooperation within the household. This donation concept will play a crucial role in our further exposition. Specifically, it will allow us to characterize a whole spectrum of cooperative-noncooperative household consumption models.

The empirical analysis of the household game starts from a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. For every observation $t \in T$, the vectors Q_t and q_t ($= q_t^A + q_t^B$) represent the household bundles of public and private goods demanded at t ; and we write $q_{t,j}$, and $Q_{t,k}$ for the demanded quantity of private good j or public good k at t ($j \in J, k \in K$). Thus, using p_t for the (strictly positive) price vector of the private commodities, P_t for the (strictly positive) price vector of the public commodities and Y_t for household income, the household faces the following budget constraint:

$$\langle p_t, q \rangle + \langle P_t, Q \rangle \leq Y_t,$$

The first step of the household game then defines the individual incomes Y_t^A and Y_t^B ($Y_t^A + Y_t^B = Y_t$). Throughout, we will assume that the empirical analyst only observes Y_t and not Y_t^A and Y_t^B . In the second step, individual A pays $\langle p_t, q^A \rangle$ and $\langle P_t, Q^A \rangle$ for her/his purchase of private and public goods corresponding to any choice of q^A and Q^A . In addition, she/he receives the amount $\tau_t^B Q^A$ from B while she/he pays $\tau_t^A Q^B$ to B for any Q^B representing B 's contribution to the public goods. Thus, A 's optimization problem in this second step is given by **(OP-H.A)**

$$\{q_t^A, Q_t^A\} \in \arg \max_{q^A, Q^A} U(q^A, Q^A + Q_t^B) \text{ s.t. } \langle p_t, q^A \rangle + \langle P_t - \tau_t^B, Q^A \rangle + \langle \tau_t^A, Q_t^B \rangle \leq Y_t^A.$$

Similarly, B solves **(OP-H.B)**

$$\{q_t^B, Q_t^B\} \in \arg \max_{q^B, Q^B} U(q^B, Q_t^A + Q^B) \quad s.t. \quad \langle p_t, q^B \rangle + \langle P_t - \tau_t^A, Q^B \rangle + \langle \tau_t^B, Q_t^A \rangle \leq Y_t^B.$$

In what follows, we define an equilibrium of the household game as an allocation $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ that simultaneously solves OP-H.A and OP-H.B for a particular choice of Y_t^A, Y_t^B, τ_t^A and τ_t^B . In the next section, we will see that the three equilibrium concepts in this paper depend entirely on the choice of τ_t^A and τ_t^B .

Before considering these equilibrium concepts in more detail, we characterize the equilibrium for the general version of the household game. To this end, we consider the marginal utility levels $U_{Q_t}^A$ and $U_{Q_t}^B$ of A and B at the equilibrium allocation.⁶ Further, let us introduce the notation $\tilde{P}_t^A = U_{Q_t}^A / \lambda_t^A$ and $\tilde{P}_t^B = U_{Q_t}^B / \lambda_t^B$, with λ_t^A and λ_t^B the Lagrange multipliers (associated with the budget constraint) at the equilibrium for A and B evaluated in observation t . The vectors \tilde{P}_t^A and \tilde{P}_t^B represent the marginal willingness to pay (MWTP) for the bundle of public goods associated with A and B . In particular, $\tilde{P}_{t,k}^A$ represents the amount of income that A is willing to give up (at equilibrium) in order to receive one additional unit of the public good k . For each public good k (which is consumed by a strictly positive amount), the first order equilibrium conditions for OP-H.A and OP-H.B (for $Q_{t,k} > 0$) imply

$$\max\{\tilde{P}_{t,k}^A + \tau_{t,k}^B; \tilde{P}_{t,k}^B + \tau_{t,k}^A\} = P_{t,k}. \quad (1)$$

This equality requirement is easily interpreted as an equilibrium condition. To see this, let us consider the two possible inequality situations. First, if $\tilde{P}_{t,k}^A + \tau_{t,k}^B > P_{t,k}$ then the MWTP of A for one additional unit of k (i.e. $\tilde{P}_{t,k}^A$) is larger than the price A has to pay for it (i.e. $P_{t,k} - \tau_{t,k}^B$). Hence, A will increase her/his contribution to good k . A directly similar interpretation applies to the situation $\tilde{P}_{t,k}^B + \tau_{t,k}^A > P_{t,k}$. And, thus, $\max\{\tilde{P}_{t,k}^A + \tau_{t,k}^B; \tilde{P}_{t,k}^B + \tau_{t,k}^A\} > P_{t,k}$ implies a disequilibrium. Next, if $\max\{\tilde{P}_{t,k}^A + \tau_{t,k}^B; \tilde{P}_{t,k}^B + \tau_{t,k}^A\} < P_{t,k}$ then either A or B (whoever contributes positively to good k) will want to decrease her/his contribution to k . Again, this implies a disequilibrium situation.

As a concluding remark, we point out two assumptions that we will maintain in the next two sections. First, we will assume that the empirical analyst only observes the aggregate private demands q_t , and not the individual bundles q_t^A and q_t^B . However, it is easy to extend our analysis to include information on q_t^A and q_t^B , i.e. the private consumption of the individuals A and B is partly observed. See our empirical application in Section 5 for a specific example. Next, we will assume that all components of the aggregate demands Q_t are strict positive. Again, we could

⁶If U^A and U^B are not differentiable, we can take the subdifferentials that characterize the optimal allocation.

easily relax this assumption by introducing some additional notation, but this would only complicate the discussion while not really adding any new insights. In fact, our empirical application in Section 5 will consider data sets with some components of Q_t equal to zero; our basic theoretical insights developed below apply with equal strength to this setting.

4 The cooperative-noncooperative spectrum

The household game discussed in the previous section allows us to provide a revealed preference characterization of a whole spectrum of cooperative-noncooperative models of household consumption. As indicated above, different degrees of intrahousehold cooperation then correspond to different specifications of the intrahousehold donations (i.e. τ_k^A and τ_k^B). To formalize this idea, we first discuss the two extreme cases mentioned in the Introduction, i.e. the fully cooperative model and the fully noncooperative model. Subsequently, we present the semicooperative model, which is situated on a continuum between these two limiting models.

4.1 The cooperative solution

The cooperative model assumes that the household consumption decision coincides with a Pareto optimal allocation. An allocation $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ is Pareto optimal if for all allocations $\{q'^A, q'^B, Q'^A + Q'^B\}$ that satisfy the same household budget constraint, $U^A(q'^A, Q'^A + Q'^B) > U^A(q_t^A, Q_t^A + Q_t^B)$ implies $U^B(q'^B, Q'^A + Q'^B) < U^B(q_t^B, Q_t^A + Q_t^B)$. For our setting with concave utility functions, a Pareto optimal allocation $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ is usually characterized as maximizing a weighted sum of individual utilities U^A and U^B subject to the given budget constraint; in this characterization, the weights of U^A and U^B are commonly referred to as Pareto weights. The revealed preference characterization of this cooperative model has been discussed by Cherchye, De Rock and Vermeulen (2007, 2009b). In what follows, we briefly recapture this characterization by integrating it with the household game framework set out in the previous section. This will set the stage for our next discussion of the noncooperative and semicooperative models.

As discussed in the previous section, the household game defines an equilibrium bundle $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ for a given income distribution Y_t^A, Y_t^B and vectors τ_t^A, τ_t^B . Now assume that (in equilibrium) A considers to buy an additional unit of good k . In order to pay for this extra consumption, A receives from B a contribution of $\tau_{t,k}^B$. In a cooperative equilibrium, a natural assumption is that B pays according to her/his valuation of this extra consumption. In other words, B agrees to pay exactly her/his MWTP for this additional consumption; and this value is given by

$\tilde{P}_{t,k}^B$. Thus, for the cooperative solution we have

$$\tau_t^B = \tilde{P}_t^B,$$

so that the equilibrium condition (1) becomes

$$\begin{aligned} \max\{\tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B, \tilde{P}_{t,k}^B + \tilde{P}_{t,k}^A\} &= P_{t,k}, \text{ or} \\ \tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B &= P_{t,k}. \end{aligned} \quad (2)$$

In words, for every public good k and at each observation t , the sum of the MWTP of individuals A and B ($\tilde{P}_{t,k}^A$ and $\tilde{P}_{t,k}^B$) must equal the price $P_{t,k}$. As such, $\tilde{P}_{t,k}^A$ and $\tilde{P}_{t,k}^B$ can be interpreted as Lindahl prices and, thus, in this case the household game equilibrium corresponds to an equilibrium with Lindahl prices. This conforms to the well-known one-to-one correspondence between Lindahl price equilibria (with varying incomes Y_t^A and Y_t^B) and the set of Pareto optimal allocations (with varying Pareto weights for the individuals).

Given all this, we can next introduce the revealed preference characterization of this cooperative consumption model. We first define the concept of cooperative-rationalizability.

Definition 3 (cooperative-rationalizability) Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. The set S is cooperative-rationalizable if there exist utility functions U^A and U^B , individual private consumption bundles $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ that sum to q_t and public consumption bundles $Q_t^A, Q_t^B \in \mathbb{R}_+^{|K|}$ that sum to Q_t such that $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ simultaneously solves OP-H.A and OP-H.B under the condition $\tau_t^A = \tilde{P}_t^A$ and $\tau_t^B = \tilde{P}_t^B$.

The next result gives the revealed preference conditions corresponding with such cooperative-rationalizability.

Theorem 2 Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. The following conditions are equivalent:

1. There exists a pair of utility functions U^A, U^B that cooperative-rationalizes S .
2. For all $t \in T$, there exist price vectors $\tilde{P}_t^A, \tilde{P}_t^B \in \mathbb{R}_+^{|K|}$ and quantity vectors $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ such that

$$q_t^A + q_t^B = q_t, \quad (C.1)$$

$$\tilde{P}_t^A + \tilde{P}_t^B = P_t, \text{ and} \quad (C.2)$$

$$\{p_t, \tilde{P}_t^A, q_t^A, Q_t\}_{t \in T} \text{ and } \{p_t, \tilde{P}_t^B, q_t^B, Q_t\}_{t \in T} \text{ satisfy GARP.} \quad (C.3)$$

Condition C.3 implies that cooperative-rationalizability implies a GARP condition (i.e. individual-rationalizability) at the level of individuals A and B . The specificity of the cooperative model is that these GARP conditions use Lindahl prices (\tilde{P}_t^A and \tilde{P}_t^B) for evaluating the publicly consumed quantities; see condition C.2. It will be interesting to compare this condition with the conditions that apply to the fully noncooperative model and the semicooperative model.

Before doing so, we briefly recapture the so-called sharing rule concept that is intrinsic to the cooperative model of consumption behavior. Essentially, the sharing rule defines the individual income shares Y_t^A and Y_t^B corresponding to cooperative-rationalizable household consumption behavior; see, for example, Chiappori (1988, 1992) for extensive discussion. In this respect, we recall that a data set S only contains information on Y_t and not on Y_t^A and Y_t^B . However, in principle it is possible to empirically identify Y_t^A and Y_t^B (i.e. the sharing rule) if the set S is cooperative-rationalizable. Note that, from (2) the intrahousehold income distribution is given as

$$\langle p_t, q_t^A \rangle + \langle \tilde{P}_t^A, Q_t \rangle = Y_t^A \text{ and } \langle p_t, q_t^B \rangle + \langle \tilde{P}_t^B, Q_t \rangle = Y_t^B. \quad (3)$$

Thus, for a given data set S , if we can identify q_t^A , q_t^B , \tilde{P}_t^A and \tilde{P}_t^B (given the empirical conditions C.1-C.3), then we can identify the income shares Y_t^A and Y_t^B that underlie the observed cooperative consumption behavior. We refer to Cherchye, De Rock and Vermeulen (2009b) for a detailed discussion of this identifiability result that starts from the revealed preference characterization in Theorem 2.⁷ In what follows, we will see that this identifiability result does not hold in general for consumption models that are not fully cooperative.

4.2 The noncooperative solution

Fully noncooperative household behavior means that individual A is not willing to contribute to the purchase of Q^B and vice versa, which implies $\tau_t^A = \tau_t^B = 0$. In this instance, the programs OP-H.A and OP-H.B correspond to the usual definition of a Nash equilibrium; see, for example, Lechene and Preston (2005, 2008). Given $\tau_t^A = \tau_t^B = 0$, the equilibrium condition (1) for the household game reduces to

$$\max\{\tilde{P}_{t,k}^A, \tilde{P}_{t,k}^B\} = P_{t,k}. \quad (4)$$

The interpretation of this condition is directly analogous to the one of (1) discussed above.

Let us then consider the revealed preference conditions of this noncooperative model for a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. Similar to before, we define the concept

⁷Chiappori and Ekeland (2009) provide related identifiability results that start from a differential characterization of the cooperative model.

of noncooperative-rationalizability.

Definition 4 (noncooperative-rationalizability) Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. The set S is noncooperative-rationalizable if there exist utility functions U^A and U^B , individual private consumption bundles $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ that sum to q_t and public consumption bundles $Q_t^A, Q_t^B \in \mathbb{R}_+^{|K|}$ that sum to Q_t such that $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ simultaneously solve OP-H.A and OP-H.B under the condition $\tau_t^A = \tau_t^B = 0$.

We obtain the following result.

Theorem 3 Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. The following conditions are equivalent:

1. There exists a pair of utility functions U^A, U^B that noncooperative-rationalizes S .
2. For all $t \in T$ and $k \in K$, there exist price vectors $\tilde{P}_t^A, \tilde{P}_t^B \in \mathbb{R}_+^{|K|}$ and quantity vectors $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ such that

$$q_t^A + q_t^B = q_t, \quad (\text{NC.1})$$

$$\max \left\{ \tilde{P}_{t,k}^A, \tilde{P}_{t,k}^B \right\} = P_{t,k}, \text{ and} \quad (\text{NC.2})$$

$$\{p_t, \tilde{P}_t^A, q_t^A, Q_t\}_{t \in T} \text{ and } \{p_t, \tilde{P}_t^B, q_t^B, Q_t\}_{t \in T} \text{ satisfy GARP.} \quad (\text{NC.3})$$

Moreover, it follows that

$$\tilde{P}_{t,k}^A < P_{t,k} \text{ if and only if } Q_{t,k}^A = 0 \text{ and } Q_{t,k}^B = Q_{t,k}, \text{ and} \quad (\text{NC.4})$$

$$\tilde{P}_{t,k}^B < P_{t,k} \text{ if and only if } Q_{t,k}^B = 0 \text{ and } Q_{t,k}^A = Q_{t,k}. \quad (\text{NC.5})$$

The interpretation of NC.1-NC.3 is similar to the one of C.1-C.3 in Theorem 2. The main difference is restriction NC.2 in Theorem 3, which replaces restriction C.2 in Theorem 2. The restrictions NC.4 and NC.5 follow from the fact that, if $\tilde{P}_{t,k}^A < P_{t,k}$ ($\tilde{P}_{t,k}^B < P_{t,k}$), then A (B) will sell back any positive amount of the public good k . This implies $Q_{t,k}^A = 0$ ($Q_{t,k}^B = 0$) and, thus, $Q_{t,k}^B = Q_{t,k}$ ($Q_{t,k}^A = Q_{t,k}$). Note that we can have $\tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B > P_{t,k}$, which contrasts with (2) that applies to the cooperative case. In fact, this difference between $\tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B$ and $P_{t,k}$ indicates an efficiency loss in the consumption of public goods caused by Pareto inefficient (or not fully cooperative) behavior.

Two further remarks are in order. First, if we had imposed the additional assumption that for all $t \in T$ and $k \in K$ the contributions $Q_{t,k}^A$ and $Q_{t,k}^B$ are everywhere

strictly positive, then we would have derived a simpler characterization of noncooperative behavior. Specifically, it can be verified that condition NC.3 in Theorem 3 would have reduced to requiring bundles q_t^A and q_t^B that sum to q_t such that the sets $\{p_t, P_t, q_t^A, Q_t\}_{t \in T}$ and $\{p_t, P_t, q_t^B, Q_t\}_{t \in T}$ both satisfy GARP. However, the assumption that all $Q_{t,k}^A$ and $Q_{t,k}^B$ are positive is problematic. Specifically, Browning, Chiappori and Lechene (2007) have shown that generically (i.e. in all but a particular set of cases) the number of public goods to which both individuals contribute is less than or equal to one. This suggests only assuming that $Q_{t,k}^A$ and $Q_{t,k}^B$ are non-negative, which effectively obtains the characterization in Theorem 3.⁸

The final remark pertains to our earlier discussion of (3) for the cooperative model. We have indicated that, in principle, under cooperative-rationalizability the (unobserved) within-household income distribution (i.e. the sharing rule) can be identified from the observed set S . This identifiability result does not generally hold under noncooperative-rationalizability. Specifically, it directly follows from the budget constraints in OP-H.A and OP-H.B that, under noncooperative-rationalizability, the income shares of the two individuals are given by:

$$\langle p_t, q_t^A \rangle + \langle P_t, Q_t^A \rangle = Y_t^A \text{ and } \langle p_t, q_t^B \rangle + \langle P_t, Q_t^B \rangle = Y_t^B. \quad (5)$$

Given this, conditions NC.4 and NC.5 imply that Y_t^A and Y_t^B are uniquely identified only if for all k and t we have $\tilde{P}_{t,k}^A < P_{t,k}$ (so that $Q_{t,k}^A = 0$ and $Q_{t,k}^B = Q_{t,k}$) or $\tilde{P}_{t,k}^B < P_{t,k}$ (so that $Q_{t,k}^B = 0$ and $Q_{t,k}^A = Q_{t,k}$). This last situation conforms to the so-called separate spheres Nash equilibrium concept; see Lundberg and Pollak (1993) and Browning, Chiappori and Lechene (2007). On the other hand, as soon as there is one public good k to which both individuals contribute for some t (i.e. $\tilde{P}_{t,k}^A = \tilde{P}_{t,k}^B = P_k$), it is impossible to exactly recover the income shares Y_t^A and Y_t^B that underlie the observed noncooperative behavior. Specifically, in this case $Q_{t,k}^A$ and $Q_{t,k}^B$ can take any value (under the sole condition $Q_{t,k}^A + Q_{t,k}^B = Q_{t,k}$) and, thus, the expenditures on good k can not be assigned to the individual household members. Interestingly, this result complies with the so-called local income pooling result, which also applies to situations where both individuals contribute to the same public good in a noncooperative setting; see Kemp (1984), Bergstrom, Blume and Varian (1986) and Browning, Chiappori and Lechene (2007). However, even though we cannot identify Y_t^A and Y_t^B in such a situation, it is still possible to recover upper and lower bounds on values for Y_t^A and Y_t^B that are consistent with a noncooperative-rationalization of the given data set. These bounds then account for the total (non-assignable) expenditures on the jointly contributed public goods.

⁸However, see Lechene and Preston (2005) for some example settings where the assumption of strictly positive $Q_{t,k}^A$ and $Q_{t,k}^B$ is always satisfied.

4.3 The semicooperative solution

In contrast to the cooperative and noncooperative cases discussed before, there is no obvious way to model semicooperative household consumption behavior. In this section, we forward a model that extends the interpretation of the previous (limiting) models to situations characterized by intermediate levels of intrahousehold cooperation. We believe this model captures most characteristics of both models in a realistic and intuitive way. To enhance the intuition of the model, we will compare it with -to the best of our knowledge- the only alternative semicooperative model that has been suggested in the literature, i.e. the model of d'Aspremont and Dos Santos Ferreira (2009).

Following our reasoning in the previous sections, we characterize the semicooperative model in terms of the parameters τ_t^A and τ_t^B . We recall that the cooperative case corresponds to $\tau_t^A = \tilde{P}_t^A$ and $\tau_t^B = \tilde{P}_t^B$, while the noncooperative case corresponds to $\tau_t^A = \tau_t^B = 0$. This naturally suggests to characterize the semicooperative case by τ_t^A, τ_t^B such that

$$\begin{aligned} 0 \leq \tau_t^A \leq \tilde{P}_t^A & \quad \text{and} \quad 0 \leq \tau_t^B \leq \tilde{P}_t^B, \text{ or} \\ \tau_{t,k}^A = \theta_{t,k}^A \tilde{P}_{t,k}^A & \quad \text{and} \quad \tau_{t,k}^B = \theta_{t,k}^B \tilde{P}_{t,k}^B \text{ with} \\ 0 \leq \theta_{t,k}^A \leq 1 & \quad \text{and} \quad 0 \leq \theta_{t,k}^B \leq 1. \end{aligned}$$

Let us interpret this semicooperative model in terms of the household game described above. Assume that (in equilibrium) individual A wants to increase her/his contribution to public good k by one unit, and individual B 's MWTP for this increase is $\tilde{P}_{t,k}^B$. However, individual B is not fully cooperative and, thus, she/he is unwilling to pay this entire amount to A when purchasing the additional public good. On the other hand, she/he is not fully noncooperative either. Therefore, individual B will contribute $\tau_{t,k}^B$ situated between zero and $\tilde{P}_{t,k}^B$ and, thus, there exists a constant $\theta_{t,k}^B$ such that B is willing to contribute $\theta_{t,k}^B \tilde{P}_{t,k}^B$ to the purchase of the good. This corresponds to the interpretation of the intrahousehold donations (τ_k^A and τ_k^B) in terms of voluntary intrahousehold cooperation; in this case, $\theta_{t,k}^B$ represents a subsidy from B to A . When interpreting the same donations in terms of obligatory cooperation, we can think of $\theta_{t,k}^B$ as a tax rate which individual B faces on his MWTP for the fact that A purchases the public good. Finally, we obviously have that $\theta_{t,k}^A = \theta_{t,k}^B = 1$ for all t and k corresponds to fully cooperative behavior, and $\theta_{t,k}^A = \theta_{t,k}^B = 0$ for all t and k complies with fully noncooperative behavior.

For given $\theta_{t,k}^A$ and $\theta_{t,k}^B$, the equilibrium condition (1) for the household game is given as

$$\max\{\tilde{P}_{t,k}^A + \theta_{t,k}^B \tilde{P}_{t,k}^B; \tilde{P}_{t,k}^B + \theta_{t,k}^A \tilde{P}_{t,k}^A\} = P_{t,k}. \quad (6)$$

The intuition of this condition is directly similar to the one of (1), which was explained before. We remark that we can have $\tilde{P}_{t,k}^B + \tilde{P}_{t,k}^A > P_{t,k}$ for some k if $\theta_{t,k}^A \neq 0$ or $\theta_{t,k}^B \neq 0$. Like for the fully noncooperative case, such an inequality indicates an efficiency loss due to limited cooperation.

Let us then formulate the corresponding revealed preference conditions. We first define the concept of semicooperative-rationalizability.

Definition 5 (semicooperative-rationalizability) Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. The set S is semicooperative-rationalizable if there exist $\theta_{t,k}^A$ and $\theta_{t,k}^B \in [0, 1]$, utility functions U^A and U^B , individual private consumption bundles $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ that sum to q_t and public consumption bundles $Q_t^A, Q_t^B \in \mathbb{R}_+^{|K|}$ that sum to Q_t such that $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ simultaneously solve OP-H.A and OP-H.B under the condition $\tau_{t,k}^A = \theta_{t,k}^A \tilde{P}_{t,k}^A$ and $\tau_{t,k}^B = \theta_{t,k}^B \tilde{P}_{t,k}^B$.

The following theorem characterizes the collection of data sets that are semicooperative-rationalizable.

Theorem 4 Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. The following conditions are equivalent:

1. There exists a pair of utility functions U^A, U^B that semicooperative-rationalizes S .
2. For all $t \in T$ and $k \in K$, there exist numbers $\theta_{t,k}^A, \theta_{t,k}^B \in [0, 1]$, price vectors $\tilde{P}_t^A, \tilde{P}_t^B \in \mathbb{R}_+^{|K|}$ and vectors $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ such that

$$q_t^A + q_t^B = q_t, \quad (\text{SC.1})$$

$$\max \left\{ \tilde{P}_{t,k}^A + \theta_{t,k}^B \tilde{P}_{t,k}^B, \tilde{P}_{t,k}^B + \theta_{t,k}^A \tilde{P}_{t,k}^A \right\} = P_{t,k}, \text{ and} \quad (\text{SC.2})$$

$$\{p_t, \tilde{P}_t^A, q_t^A, Q_t\}_{t \in T} \text{ and } \{p_t, \tilde{P}_t^B, q_t^B, Q_t\}_{t \in T} \text{ satisfy GARP.} \quad (\text{SC.3})$$

Moreover, it follows that

$$\tilde{P}_{k,t}^A + \theta_{t,k}^B \tilde{P}_{k,t}^B < P_{k,t} \text{ if and only if } Q_{k,t}^A = 0 \text{ and } Q_{k,t}^B = Q_{k,t}, \text{ and} \quad (\text{SC.4})$$

$$\theta_{t,k}^A \tilde{P}_{k,t}^A + \tilde{P}_{k,t}^B < P_{k,t} \text{ if and only if } Q_{k,t}^B = 0 \text{ and } Q_{k,t}^A = Q_{k,t}. \quad (\text{SC.5})$$

The interpretation is readily similar to the one of Theorem 3. Like in the noncooperative case, we have that the individual income shares (Y_t^A and Y_t^B) underlying observed semicooperative behavior are not identifiable in general. Mutatis mutandis, our discussion of (5) carries over to this semicooperative case.

In what follows, we will assume constant $\theta_{t,k}^A$ and $\theta_{t,k}^B$, i.e. $\theta_{t,k}^A = \theta^A$ and $\theta_{t,k}^B = \theta^B$ for all t and k . This will substantially simplify our exposition. The fact that the

parameters θ^A and θ^B are independent of t means that the subsidy (i.e. voluntary donations) or tax rate (i.e. obligatory donations) does not change over observations. It is possible to relax this assumption, but this comes at the cost of a considerable increase of the computational complexity of the testable MIP conditions. Next, constant θ^A and θ^B also imposes that the donations τ_t^A and τ_t^B are proportional to the MWTP vectors \tilde{P}_t^A and \tilde{P}_t^B . In other words, if individual B 's MWTP for some good k_1 is twice her/his MWTP for some other good k_2 , then her/his contribution per unit of k_1 will also be twice as large as her/his contribution per unit of k_2 . Although it is also possible to relax this assumption (again at the cost of additional computational burden), we will stick to it as we believe it is quite intuitive and plausible in the current context.

As a final note, it is useful to compare our model with the one of d'Aspremont and Dos Santos Ferreira (2009). Consider a data set S and let $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ be a cooperative (Pareto efficient) equilibrium for the household game, with π_t^A and π_t^B the associated Lindahl prices (i.e. π_t^A and π_t^B give the MWTP vectors for A and B at this cooperative equilibrium; see our above discussion of the cooperative model). Then, the model of d'Aspremont and Dos Santos Ferreira is characterized by the following first order condition:⁹

$$\max\{\tilde{P}_{t,k}^A + \theta^B \pi_{t,k}^B; \tilde{P}_{t,k}^B + \theta^A \pi_{t,k}^A\} = P_{t,k}. \quad (7)$$

This equilibrium condition is closely similar to the condition (6) that applies to our model. However, there is one crucial difference. Specifically, in the model of d'Aspremont and Dos Santos Ferreira the intrahousehold donations per unit of any public good k (captured by θ^A and θ^B in (7)) is proportional to the MWTP for this public good in the Pareto efficient equilibrium ($\pi_{t,k}^A$ and $\pi_{t,k}^B$ in (7)). By contrast, in our model these donations are proportional to the MWTP for the same goods in the semicooperative equilibrium ($\tilde{P}_{t,k}^A$ and $\tilde{P}_{t,k}^B$ in (6)). Thus, depending on the value of θ^A and θ^B , the two models may lead to different outcomes. In addition, our above discussion makes clear that the two semicooperative models have a rather different interpretation, even though they have a similar structure.

A main motivation to focus on our version of the semicooperative model, and not on the one of d'Aspremont and Dos Santos Ferreira, is that our model only uses information on the MWTP for quantities that are effectively observed (i.e. in the data set S), while the alternative model of d'Aspremont and Dos Santos Ferreira requires information on the MWTP for quantities in some unobserved cooperative equilibrium (associated with the same data set S). The fact that we only use observ-

⁹d'Aspremont and Dos Santos Ferreira (2009) originally expressed their equilibrium condition in a different form. However, the formulation in (7) is easily obtained by slightly rearranging this original formulation.

able quantity information is interesting from a practical point of view. Specifically, as we will explain in Section 5, it allows us to reformulate the revealed preference condition in Theorem 4 in MIP terms. As far as we can see, it is not possible to obtain a similar MIP formulation for the revealed preference characterization of the model of d’Aspremont and Dos Santos Ferreira, precisely because this model requires unobservable quantity information.

4.4 Independence

We can show that the revealed preference conditions for noncooperative behavior are independent of the revealed preference conditions for cooperative behavior: a data set that satisfies the cooperative conditions does not necessarily satisfy the noncooperative conditions, and vice versa. Specifically, the two examples in Appendix 2 show that there is neither any inclusion nor any exclusion relation between the collection of data sets that satisfy the conditions in Theorem 2 and the collection of data sets that satisfy the conditions in Theorem 3. For simplicity, these examples focus on the (limiting) fully cooperative and fully noncooperative cases. However, in principle we can construct similar (but substantially more complex) examples that pertain to the (intermediate) semicooperative model characterized in Theorem 4. Thus, we can conclude that models characterized by different degrees of cooperation are generally independent of each other.

This independence/non-nestedness conclusion is important for at least two reasons. Firstly, this result stands in sharp contrast with the findings in the (local) differential approach to modeling non-unitary consumption behavior. As discussed in the Introduction, the rationalizability conditions for the noncooperative and semi-cooperative models derived in that approach are generally nested with the rationalizability conditions for the cooperative model: if a given data set passes the (local) condition for cooperative rationalizability, then it should also pass the test for noncooperative rationalizability, but not vice versa. Secondly, our empirical application in Section 5 will show that this independence is not a theoretical curiosity but also has empirical relevance. Specifically, this application does effectively include data that are cooperative-rationalizable but not noncooperative-rationalizable, and (different) data that are noncooperative-rationalizable but not cooperative-rationalizable.

Apart from independence, the examples in Appendix 2 demonstrate two further features of our revealed preference conditions that are important in view of empirical applications. First, they show that we can meaningfully test data consistency with specific household consumption models (and compare the empirical validity of different models) even if only a few observations are available. Second, because all consumption is public in both examples, such empirical analysis in principle does not require privately consumed goods. In fact, this last feature implies an additional

difference with the existing differential characterizations of noncooperative and semi-cooperative models: these differential characterizations typically require (much) more privately consumed goods than publicly consumed goods in order to obtain empirically testable restrictions; see Lechene and Preston (2005, 2008) and d’Aspremont and Dos Santos Ferreira (2009).

5 Empirical application

We apply our method to data drawn from the Russia Longitudinal Monitoring Survey (RLMS). Cherchye, De Rock and Vermeulen (2009b,c) studied the same data set. These authors focused on consistency of these data with the cooperative model of household consumption. We extend these earlier studies by providing complementary results pertaining to noncooperative and semicooperative consumption models. In doing so, we also generalize the MIP methodology introduced by these authors (for the cooperative case) to apply to models of noncooperative and semicooperative household behavior.

Our following analysis will concentrate on consistency testing, and will particularly illustrate the empirical relevance of the independence result articulated above (see Section 4.4). If household behavior is found consistent with a particular (cooperative-noncooperative) model, then subsequent analysis can focus on recovering/identifying the specificities of the decision model that underlies the (rationalizable) observed consumption behavior. For brevity, we do not consider recovery issues in this application. However, we will return to recovery (based on our MIP methodology) in the concluding section.

5.1 Verification

To be able to verify the GARP conditions in Theorems 2-4, we reformulate these conditions in mixed integer programming (MIP) terms. We focus on formulating the MIP program for the semicooperative model, with endogenous variables $\theta^A, \theta^B \in [0, 1]$. It follows from our above discussion that the program for the fully cooperative and fully noncooperative models correspond to $\theta^A = \theta^B = 1$ and $\theta^A = \theta^B = 0$, respectively.

To obtain the MIP formulation, we define the binary variables $x_{t,v}^M \in \{0, 1\}$, with $x_{t,v}^M = 1$ interpreted as $(q_t^M, Q_t) R^M (q_v^M, Q_v)$ (where $(q_t^M, Q_t) R^M (q_v^M, Q_v)$, $M = A, B$, has a straightforwardly similar meaning as $q_t R q_v$ in Section 2). Then, a data set S satisfies the necessary and sufficient condition for semicooperative-rationalizability in Theorem 4 if and only if the following MIP problem is feasible:

For all $t, v \in T$ and all $k \in K$, there exist strictly positive vectors $\tilde{P}_t^A, \tilde{P}_t^B$, binary

variables $z_{t,k}, x_{t,v}^M \in \{0,1\}$, and parameters $\theta^A, \theta^B \in [0,1]$ such that (for $s, t, v \in T$, $k \in K$, $M = A, B$):¹⁰

$$\tilde{P}_t^A + \theta^B \tilde{P}_t^B \leq P_t, \quad (8)$$

$$\theta^A \tilde{P}_t^A + \tilde{P}_t^B \leq P_t, \quad (9)$$

$$P_{t,k} - \tilde{P}_{t,k}^A - \theta^B \tilde{P}_{t,k}^B \leq z_{t,k} C_t, \quad (10)$$

$$P_{t,k} - \theta^A \tilde{P}_{t,k}^A - \tilde{P}_{t,k}^B \leq (1 - z_{t,k}) C_t, \quad (11)$$

$$q_t^A + q_t^B = q_t, \quad (12)$$

$$\langle p_t, q_t^M - q_v^M \rangle + \langle \tilde{P}_t^M, Q_t - Q_v \rangle < x_{t,v}^M C_t, \quad (13)$$

$$x_{t,s}^M + x_{s,v}^M \leq 1 + x_{t,v}^M, \quad (14)$$

$$(1 - x_{t,v}^M) C_v \geq \langle p_v, q_v^M - q_t^M \rangle + \langle \tilde{P}_v^M, Q_v - Q_t \rangle, \quad (15)$$

with $C_t > P_{t,k}$ and $C_t > Y_t$ for all t and k .

The interpretation is as follows. Constraint (12) imposes that the private consumption bundles q_t^A and q_t^B sum to the observed aggregate quantities q_t , as required by condition SC.1. Further, constraints (8)-(11) comply with condition SC.2 in Theorem 4. Specifically, (8) and (9) impose the given upper bound constriction for \tilde{P}_t^A and \tilde{P}_t^B . Next, (10) imposes $P_{t,k} \leq \tilde{P}_{t,k}^A + \theta^B \tilde{P}_{t,k}^B$ if $z_{t,k} = 0$, while (11) imposes $P_{t,k} \leq \theta^A \tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B$ if $z_{t,k} = 1$. Because $z_{t,k} \in \{0,1\}$, this implies $\max\{\tilde{P}_{t,k}^A + \theta^B \tilde{P}_{t,k}^B, \tilde{P}_{t,k}^B + \theta^A \tilde{P}_{t,k}^A\} = P_{t,k}$ and thus condition SC.2 is satisfied. Finally, constraints (13)-(15) correspond to the GARP conditions for each individual M ($= A$ or B) (condition SC.3 in Theorem 4). Specifically, (13) states that $\langle p_t, q_t^M - q_v^M \rangle + \langle \tilde{P}_t^M, Q_t - Q_v \rangle \geq 0$ implies $x_{t,v}^M = 1$ (or $(q_t^M, Q_t) R^M (q_v^M, Q_v)$). Next, constraint (14) imposes transitivity of the individual revealed preference relations R^M : if $x_{t,s}^M = 1$ (i.e. $(q_t^M, Q_t) R^M (q_s^M, Q_s)$) and $x_{s,v}^M = 1$ (i.e. $(q_s^M, Q_s) R^M (q_v^M, Q_v)$) then $x_{t,v}^M = 1$ (i.e. $(q_t^M, Q_t) R^M (q_v^M, Q_v)$). And (15) requires $\langle p_v, q_v^M - q_t^M \rangle + \langle \tilde{P}_v^M, Q_v - Q_t \rangle \leq 0$ if $x_{t,v}^M = 1$ (i.e. $(q_t^M, Q_t) R^M (q_v^M, Q_v)$).

Clearly, all constraints are linear for the cooperative case (with $\theta^A = \theta^B = 1$) and the noncooperative case (with $\theta^A = \theta^B = 0$). Linearity implies that the above program can be solved by standard MIP methods for a given data set S . As for the semi-cooperative case, the constraints are obviously linear if we know the values of θ^A and θ^B . If we do not know these values (which is usually the case), then we suggest to conduct a grid search that checks the above problem (through MIP methods) for

¹⁰The strict inequality $\langle p_t, q_t^M - q_v^M \rangle + \langle \tilde{P}_t^M, Q_t - Q_v \rangle < x_{t,v}^M C_t$ is difficult to use in IP analysis. Therefore, in practice we can replace it with $\langle p_t, q_t^M - q_v^M \rangle + \langle \tilde{P}_t^M, Q_t - Q_v \rangle + \epsilon \leq x_{t,v}^M C_t$ for ϵ (> 0) arbitrarily small.

a whole range of possible values for θ^A and θ^B . In fact, this can provide a definite answer on whether the data satisfy the rationalizability condition in Theorem 4 with arbitrarily large probability. To see this, we first note that the parameter space $\theta^A \times \theta^B$ is $[0, 1] \times [0, 1]$, which is of size 1. Let us assume that the subspace with θ^A and θ^B obtaining a semicooperative-rationalizability of the given data set has at least size ε . Then, we get that any random draw out of $[0, 1] \times [0, 1]$ has at least probability ε to lead to a rationalization. In other words, with probability less than $(1 - \varepsilon)$ no rationalization is found. So, if we take n random draws from $[0, 1] \times [0, 1]$, then with probability at least $1 - (1 - \varepsilon)^n$, we must find a rationalization. By taking n large enough, we can make this probability as close to 1 as desirable for any given ε . In our following application, we will use an equally spaced grid search with step 0.1 for $\theta^A, \theta^B \in [0, 1]$, which implies $n = 121$.

5.2 Data

We refer to Cherchye, De Rock and Vermeulen (2009b,c) for a detailed discussion of the RLMS data that we use. These authors also provide more specific information on the assignability procedure that we present below. For compactness, we restrict ourselves to a brief summary here.

Our sample consists of 148 adult couples, with both (female and male) household members employed. We consider each of the 148 households separately, which avoids (often debatable) preference homogeneity assumptions across male or female members of different households. This illustrates the use of our method for a panel data set. However, it is worth emphasizing that revealed preference methods such as ours are equally applicable to (repeated) cross-section data sets. In this respect, we refer to Blundell, Browning and Crawford (2003, 2008) for some recent methodological advances.

Our data set covers the period from 1994 to 2003. We have consumption data for each year except for the years 1997 and 1999, so that we end up with 8 ($= |T|$) observations (prices and quantities) per household. We consider bundles consisting of 21 ($= |J| + |K|$) nondurable goods: (1) food outside the home, (2) clothing, (3) car fuel, (4) wood fuel, (5) gas fuel, (6) luxury goods, (7) services, (8) housing rent, (9) bread, (10) potatoes, (11) vegetables, (12) fruit, (13) meat, (14) dairy products, (15) fat, (16) sugar, (17) eggs, (18) fish, (19) other food items, (20) alcohol and (21) tobacco. We assume that wood fuel, gas fuel and housing rent are public ($|K| = 3$), while the other goods are private ($|J| = 18$).

Our application will show the possibility of including specific information on q_t^A and q_t^B , i.e. we can assign private consumption to individuals A and B . Formally, this means that assignable quantities q_t^{aM} ($M = A, B$) act as lower bounds for the

quantities q_t^M , i.e.

$$q_t^M \geq q_t^{aM}.$$

Essentially, the procedure starts from a base scenario for the distribution of the privately consumed quantities across the two household members. Because assignable quantity information is not directly available from the RLMS data set, this base scenario uses the observed consumption of male and female singles (or one-person households).¹¹ In subsequent steps, we consider less and less assignability, i.e. we account for (ever larger) deviations from the base scenario distribution. Formally, using q_t^{bM} for the private quantities of member M that correspond to the hypothesized base scenario, we define

$$q_t^{aM} = \kappa q_t^{bM},$$

with $0 \leq \kappa \leq 1$. The parameter κ captures the extent to which we allow for deviations from the base scenario distribution. For example, $\kappa = 1$ implies $q_t^{aM} = q_t^{bM}$, while $\kappa < 1$ implies $q_t^{aM} < q_t^{bM}$. Generally, lower κ values imply less stringent restrictions for the private quantities. Varying the value of κ will allow us to compare different cooperative-noncooperative models under varying degrees of assignability.

5.3 Results

To structure our discussion, we first provide empirical results for the (limiting) cooperative (with $\theta^A = \theta^B = 1$) and noncooperative (with $\theta^A = \theta^B = 0$) models for the full sample of households. Subsequently, we report on the (intermediate) semicooperative model (with $\theta^A, \theta^B \in [0, 1]$) for specific households.

Table 1 presents pass rates for the cooperative model and the noncooperative model under different degrees of assignability (captured by κ). The table reveals that pass rates increase if κ decreases. This is not surprising given that lower κ values comply with less assignable information for the privately consumed quantities. For one household, we need $\kappa = 0.60$ for a rationalization in terms of the cooperative model as well as the noncooperative model. If we look at the aggregate pass rates in Table 1, we do not find much difference between the cooperative model and the noncooperative model. To some extent, this provides empirical support for both types of non-unitary models (conditional on the base scenario that is assumed).

Still, even though the two models provide a rather good overall fit of observed household behavior, there are some notable differences for specific households. For example, for $\kappa = 0.90$ the noncooperative model rationalizes the behavior of two more households than the cooperative model, while for $\kappa = 0.80$ we observe a bet-

¹¹For example, it is observed that the average budget share of alcohol for male singles is (about) 5 times the corresponding budget share for female singles. Given this, in the base scenario the male consumes 5/6 of all alcohol bought by the household and the female consumes 1/6.

ter fit of the cooperative model. Table 2 provides more detailed results pertaining to individual households. Specifically, it reports on (i) the number of households that are noncooperative-rationalizable but not cooperative-rationalizable and (ii) the number of households that are cooperative-rationalizable but not noncooperative-rationalizable. The table suggest that the adequate behavioral model varies with the household under consideration: for some households the cooperative model provides a better fit of observed behavior than the noncooperative model, while the opposite holds for other households. Generally, this motivates the empirical relevance of considering noncooperative models of household behavior in addition to the (more common) cooperative model.

As a further base of comparison, we have also calculated power results for the different model specifications. Specifically, for each household and each κ we compute a power measure that quantifies the probability of detecting random behavior. Random behavior is then modeled using a bootstrap method: for each observation, with given prices and income, we define quantities by randomly drawing budget shares (for the 21 goods) from the set of 1184 (= 148 x 8) observed household choices.¹² Thus, our power assessment gives information on the expected distribution of violations under random choice, while incorporating information on the households' actual choices.

Table 1 reports on the distribution of the power measure defined over the 148 households under study. These results are based on Monte Carlo-type simulations that include 1000 iterations. We find that the power varies a lot across households and models: while it is reasonably high for some households (see in particular the maximum and 3rd quartile values for higher κ), it is also very low for other households (see the minimum and 1st quartile values). Generally, these results suggest that assignable quantity information can be particularly helpful to enhance the power of tests for non-unitary models (with or without cooperation). Next, we recall that our analysis uses only 8 observations per household. Obviously, power can only improve when more observations become available.

In the context of the present study, it seems particularly interesting to compare the power of the cooperative and noncooperative models.¹³ For the data under consideration, we observe that the power distribution for the noncooperative model is situated somewhat below the one for the cooperative model for each value of κ . However, the difference is very small; we can safely conclude that the power distributions are generally close to each other. In our opinion, this provides additional

¹²See Bronars (1987) and Andreoni and Harbaugh (2006) for general discussions on alternative procedures to evaluate power in the context of revealed preference tests such as ours.

¹³To compute the power results in Tables 1, we have used the same distribution of randomly drawn budget shares to evaluate the cooperative and noncooperative models. Obviously, this is needed to meaningfully compare the power of the two types of models. A similar qualification applies to the power results in Table 3.

motivation for considering non-unitary models with limited cooperation in addition to the fully cooperative model.

[Table 1 about here]

[Table 2 about here]

As a final exercise, we consider the semicooperative model. Specifically, Table 3 reports on two households selected on the basis of the results in Table 2: for $\kappa = 1.00$, household 1 can be rationalized in terms of the noncooperative model but not in terms of the cooperative model, and household 2 can be rationalized in terms of the cooperative model but not in terms of the noncooperative model. Table 3 gives test results (1 = pass; 0 = fail) and power estimates for the semicooperative-rationalizability conditions corresponding to 121 combinations of $\theta^A, \theta^B \in [0, 1]$, when using $\kappa = 1.00$.¹⁴

The results suggest that our methodology can be useful to define bounds on the values of θ^A and θ^B that are consistent with semicooperative-rationalizable behavior for specific households. Given that θ^A and θ^B indicate the degree of cooperation of each individual household member, these bounds tell us about the extent to which observed household consumption behavior is characterized by (limited) intrahousehold cooperation. Interestingly, the results in Table 3 show that different household members may well be characterized by other degrees of cooperation in the semicooperative equilibrium (i.e. θ^A and θ^B have different bounds). In our opinion, an interesting following step can relate these findings on (varying) intrahousehold cooperation to specific characteristics of the household and/or household members. Such an exercise falls beyond the scope of the current study (also because of limited data availability). In this respect, our discussion in the concluding section will point out that combining experimental data with our methodology constitutes an interesting avenue for addressing this type of questions.

[Table 3 about here]

6 Concluding discussion

We have presented a revealed preference toolkit for analyzing non-unitary household consumption behavior identified by varying degrees of cooperation. We started

¹⁴Results for other κ values and other households are available upon request.

from global characterizations of non-unitary rationalizable behavior, which complement the existing local differential characterizations. Our toolkit allows for empirical analysis of such behavior while avoiding (typically nonverifiable) parametric structure for the household decision process. Such analysis can make use of MIP techniques, and is thus easy-to-implement. Our application to RLMS data suggests the empirical relevance of considering household consumption models that account for limited cooperation in addition to the (more common) model that assumes fully cooperative behavior.

To focus our discussion, we have concentrated on the characterization of consumption models with different degrees of cooperation, and testing consistency of observed behavior with alternative model specifications. If observed behavior is consistent with a particular model (i.e. can be rationalized), then a natural next question pertains to recovering/identifying the decision model that underlies the (rationalizable) observed consumption behavior. Such recovery can start from the MIP methodology presented in this paper. In this respect, see Cherchye, De Rock and Vermeulen (2009b), who consider these questions for the cooperative model; their analysis is directly extended to the noncooperative and semicooperative models discussed here. Their basic argument is that nonparametric revealed preference recovery on the basis of an MIP characterization of rational behavior boils down to defining feasible sets characterized by the MIP constraints.

We see at least two interesting applications of recovery. First, recovery can focus on the individuals' MWTP for the publicly consumed goods. As indicated above, lack of intrahousehold cooperation implies that the sum of these individual MWTP deviates from the observed prices for the publicly consumed goods. The MIP method can be used for quantifying this discrepancy between MWTP and observed prices (as a measure for the efficiency loss caused by limited cooperation) in empirical applications. Next, one can try to recover the income distribution that is associated with rationalizable behavior while accounting for limited cooperation. As a matter of fact, the literature on cooperative household consumption behavior has paid considerable attention to analyzing the intrahousehold distribution underlying observed cooperative-rationalizable behavior. See, for example, Browning, Bourguignon, Chiappori and Lechene (1994), Blundell, Chiappori and Meghir (2005), Browning, Chiappori and Lewbel (2006) and Lewbel and Pendakur (2008), who focus on various welfare-related questions associated with sharing rule recovery. The methodology presented in this paper allows for analyzing similar questions for noncooperative and semicooperative models.¹⁵

¹⁵However, we recall our discussion (at the end of Section 4.2) on identifiability problems for noncooperative and semicooperative household consumption behavior when both individuals contribute the same public goods. In this case, it is only possible to recover upper and lower bounds on the individual income shares that account for the total (non-assignable) expenditures on these

Finally, the current study has concentrated on analyzing household consumption behavior. However, the same methodology can also be used to analyze multi-person group behavior. Indeed, a lot of situations involve groups of individuals spending a joint budget; e.g. decisions of committees, clubs, villages and other local organizations, or firms with multiple decision makers. Chiappori and Ekeland (2006, 2009) suggest the cooperative (Pareto efficient) model as a natural benchmark for assessing the collective rationality of such group decisions. Our methodology allows for assessing group decisions that do not meet this benchmark. In this respect, an interesting avenue for follow-up research consists of analyzing group consumption behavior on the basis of data gathered by means of a laboratory experiment. In fact, it has been argued that the nonparametric revealed preference methodology is particularly useful in combination with such experimental data. See, for example, Sippel (1997), Harbaugh, Krause and Berry (2001) and Andreoni and Miller (2002) for earlier applications that experimentally analyze individually rational behavior. For example, experiments can use our methodology to focus on specific conditions (e.g. individual and group characteristics or other exogenous circumstances that can be manipulated) that ‘trigger’ (different degrees of) cooperative/noncooperative behavior in multi-person consumption decisions.

Appendix 1: proof of Theorem 4

We will only prove Theorem 4. The proofs of Theorems 2 and 3 are directly similar to this one and, therefore, we leave them to the reader.

1 \Rightarrow 2. Pick any $t \in T$ and consider the OP-H.A and OP-H.B. Let $U_{q_t^M}^M$ and $U_{Q_t}^M$ ($M = A, B$) be the subgradients for the function U^M at bundle (q_t^M, Q_t) , and λ_t^A and λ_t^B the Lagrange multipliers for the budget constraints in OP-H.A and OP-H.B. The first order conditions for OP-H.A and OP-H.B are:

$$\begin{aligned} U_{q_t^A}^A &\leq \lambda_t^A p_t, \\ U_{q_t^B}^B &\leq \lambda_t^B p_t, \\ U_{Q_t}^A &\leq \lambda_t^A (P_t - \tau_t^B), \\ U_{Q_t}^B &\leq \lambda_t^B (P_t - \tau_t^A). \end{aligned}$$

The inequalities are replaced by equalities in case the quantities of the goods under consideration are strictly positive. Next, concavity of the utility functions U^A and U^B jointly contributed public goods.

U^B implies for all $t, v \in T$:

$$\begin{aligned} U^A(q_t^A, Q_t) - U^A(q_v^A, Q_v) &\leq \left\langle U_{q_v^A}^A, q_t^A - q_v^A \right\rangle + \left\langle U_{Q_v}^A, Q_t - Q_v \right\rangle, \\ U^B(q_t^B, Q_t) - U^B(q_v^B, Q_v) &\leq \left\langle U_{q_v^B}^B, q_t^B - q_v^B \right\rangle + \left\langle U_{Q_v}^B, Q_t - Q_v \right\rangle. \end{aligned}$$

For all $t \in T$, define $U_{Q_t}^A/\lambda_t^A = \tilde{P}_t^A$ and $U_{Q_t}^B/\lambda_t^B = \tilde{P}_t^B$, $U^A(q_t^A, Q_t) = U_t^A$ and $U^B(q_t^B, Q_t) = U_t^B$. This gives:

$$\begin{aligned} U_t^A - U_v^A &\leq \lambda_v^A \left(\left\langle p_v, q_t^A - q_v^A \right\rangle + \left\langle \tilde{P}_v^A, Q_t - Q_v \right\rangle \right), \\ U_t^B - U_v^B &\leq \lambda_v^B \left(\left\langle p_v, q_t^B - q_v^B \right\rangle + \left\langle \tilde{P}_v^B, Q_t - Q_v \right\rangle \right). \end{aligned}$$

Using Theorem 1, we know that these two conditions are equivalent to the conditions that $\{p_t, \tilde{P}_t^A, q_t^A, Q_t\}_{t \in T}$ and $\{p_t, \tilde{P}_t^B, q_t^B, Q_t\}_{t \in T}$ satisfy GARP. This obtains SC.3.

Also, observe that, for the semi-cooperative equilibrium, $\tau_{t,k}^B = \theta_{t,k}^B \tilde{P}_{t,k}^B$ and $\tau_{t,k}^A = \theta_{t,k}^A \tilde{P}_{t,k}^A$ for all $k \in K$ and $t \in T$. If $\tilde{P}_{t,k}^A + \theta_{t,k}^B \tilde{P}_{t,k}^B < P_{t,k}$, we know that $Q_{t,k}^A = 0$ and, thus, $Q_{t,k}^B = Q_{t,k} > 0$. Then, the first order condition for $k \in K$ in OP-H.B must be binding, so that $\theta_{t,k}^A \tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B = P_{t,k}$. This obtains the first part of SC.1. Reversing the roles of A and B shows the other part of SC.1. Similarly, one can verify SC.4 and SC.5.

2 \Rightarrow 1. From the GARP conditions and Theorem 1 we know that there exist positive numbers U_t^A , U_t^B and strict positive numbers λ_t^A and λ_t^B such that:

$$\begin{aligned} U_t^A - U_v^A &\leq \lambda_v^A \left(\left\langle p_v, q_t^A - q_v^A \right\rangle + \left\langle \tilde{P}_v^A, Q_t - Q_v \right\rangle \right), \\ U_t^B - U_v^B &\leq \lambda_v^B \left(\left\langle p_v, q_t^B - q_v^B \right\rangle + \left\langle \tilde{P}_v^B, Q_t - Q_v \right\rangle \right). \end{aligned}$$

Define the functions U^A and U^B such that:

$$\begin{aligned} U^A(q^A, Q) &= \min_{v \in T} \left\{ U_v^A + \lambda_v^A \left(\left\langle p_v, q^A - q_v^A \right\rangle + \left\langle \tilde{P}_v^A, Q - Q_v \right\rangle \right) \right\}, \\ U^B(q^B, Q) &= \min_{v \in T} \left\{ U_v^B + \lambda_v^B \left(\left\langle p_v, q^B - q_v^B \right\rangle + \left\langle \tilde{P}_v^B, Q - Q_v \right\rangle \right) \right\}. \end{aligned}$$

Notice that U^A and U^B are continuous, concave, strictly monotone and that for all $t \in T$, $U^A(q_t^A, Q_t) = U_t^A$ and $U^B(q_t^B, Q_t) = U_t^B$. See, for example, Varian (1982).

We need to show that the functions U^A and U^B provide a semicooperative rationalization of the data set. For brevity, we only provide the argument for U^A , but a straightforwardly analogous reasoning applies to U^B . For all $t \in T$, define Q_t^A and Q_t^B so that if $\tilde{P}_{t,k}^A + \theta_{t,k}^B \tilde{P}_{t,k}^B < P_{t,k}$ then $Q_{t,k}^A = 0$ and $Q_{t,k}^B = Q_{t,k}$, and if $\theta_{t,k}^A \tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B < P_{t,k}$ then $Q_{t,k}^B = 0$ and $Q_{t,k}^A = Q_{t,k}$ (see SC.4 and SC.5). (If $\tilde{P}_{t,k}^A + \theta_{t,k}^B \tilde{P}_{t,k}^B = P_{t,k}$ and $\theta_{t,k}^A \tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B = P_{t,k}$ then we can randomly allocate $Q_{t,k}$ between $Q_{t,k}^A$ and $Q_{t,k}^B$.)

Next, consider $t \in T$ and a bundle (q^A, Q^A) with $Q = Q^A + Q_t^B$ such that

$$\begin{aligned}
& \langle p_t, q^A \rangle + \sum_k \left[(P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) Q_k^A + \theta_{t,k}^A \tilde{P}_{t,k}^A Q_{t,k}^B \right] \\
& \leq \langle p_t, q_t^A \rangle + \sum_k \left[(P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) Q_{t,k}^A + \theta_{t,k}^A \tilde{P}_{t,k}^A Q_{t,k}^B \right], \text{ or} \\
& \langle p_t, q^A \rangle + \sum_k (P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) Q_k^A \\
& \leq \langle p_t, q_t^A \rangle + \sum_k (P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) Q_{t,k}^A
\end{aligned} \tag{16}$$

Then, we have to prove $U^A(q^A, Q) \leq U^A(q_t^A, Q_t)$. To obtain this result, we first note that, by construction, $\langle \tilde{P}_t^A, Q_t^A \rangle = \sum_k (P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) Q_{t,k}^A$. Thus, because $\tilde{P}_{t,k}^A + \theta_{t,k}^B \tilde{P}_{t,k}^B \leq P_{t,k}$ (which implies $\langle \tilde{P}_t^A, Q^A \rangle \leq \sum_k (P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) Q_k^A$), we get $\langle \tilde{P}_t^A, Q^A - Q_t^A \rangle \leq \sum_k (P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) (Q_k^A - Q_{t,k}^A)$. Using this, we then obtain

$$\begin{aligned}
& U^A(q^A, Q) \\
& = \min_{v \in T} \left\{ U_v^A + \lambda_v^A \left(\langle p_v, q^A - q_v^A \rangle + \langle \tilde{P}_v^A, Q - Q_v \rangle \right) \right\} \\
& \leq U_t^A + \lambda_t^A \left(\langle p_t, q^A - q_t^A \rangle + \langle \tilde{P}_t^A, Q - Q_t \rangle \right) \\
& = U_t^A + \lambda_t^A \left(\langle p_t, q^A - q_t^A \rangle + \langle \tilde{P}_t^A, Q^A - Q_t^A \rangle \right) \\
& \leq U_t^A + \lambda_t^A \left(\langle p_t, q^A - q_t^A \rangle + \sum_k (P_{t,k} - \theta_{t,k}^B \tilde{P}_{t,k}^B) (Q_k^A - Q_{t,k}^A) \right) \\
& \leq U_t^A
\end{aligned}$$

This provides the wanted result, i.e. $\{q_t^A, Q_t^A\}$ solves OP-H.A.

Appendix 2: independence - examples

Throughout, we will use ε to represent a strictly positive but sufficiently small number.

Example 1: cooperative-rationalizable but not noncooperative-rationalizable

We first construct a data set that is cooperative-rationalizable but not noncooperative-rationalizable. The data set contains 3 observations ($T = \{t, v, w\}$) and 3 public goods ($K = \{1, 2, 3\}$). More specifically, the set S contains the following informa-

tion:

$$Q_t = \begin{pmatrix} 1 \\ \varepsilon \\ \varepsilon \end{pmatrix}, \quad Q_v = \begin{pmatrix} \varepsilon \\ 1 \\ \varepsilon \end{pmatrix}, \quad Q_w = \begin{pmatrix} \varepsilon \\ \varepsilon \\ 1 \end{pmatrix} \text{ and}$$

$$P_t = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix}, \quad P_v = \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix}, \quad P_w = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}.$$

To show cooperative-rationalizability, we consider the following specification:

$$\tilde{P}_t^A = \begin{pmatrix} 7 - \varepsilon^2 \\ 4 - \varepsilon \\ 4 - \varepsilon \end{pmatrix}, \quad \tilde{P}_v^A = \begin{pmatrix} 4 - \varepsilon \\ 3.5 \\ \varepsilon \end{pmatrix}, \quad \tilde{P}_w^A = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon^2 \end{pmatrix} \text{ and}$$

$$\tilde{P}_t^B = \begin{pmatrix} \varepsilon^2 \\ \varepsilon \\ \varepsilon \end{pmatrix}, \quad \tilde{P}_v^B = \begin{pmatrix} \varepsilon \\ 3.5 \\ 4 - \varepsilon \end{pmatrix}, \quad \tilde{P}_w^B = \begin{pmatrix} 4 - \varepsilon \\ 4 - \varepsilon \\ 7 - \varepsilon^2 \end{pmatrix}.$$

This specification clearly meets the condition $\tilde{P}_s^A + \tilde{P}_s^B = P_t$ ($s \in T$). By computing for both members all inner vector-products, $\tilde{P}_s^M Q_u$ ($s, u \in T, M = A, B$), it is straightforward to verify that $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ and $\{\tilde{P}_t^B, Q_t\}_{t \in T}$ both satisfy GARP. As such the data set meets the necessary and sufficient conditions for cooperative-rationalizability in Theorem 2.

We still need to prove that the data set S is not noncooperative-rationalizable. Recall that we must have $\max \{\tilde{P}_{s,k}^A, \tilde{P}_{s,k}^B\} = P_{s,k}$ for all $s \in T$ and $k \in K$. Thus, we have to specify $\tilde{P}_{t,1}^A = 7$ or $\tilde{P}_{t,1}^B = 7$. Without loss of generality, we assume $\tilde{P}_{t,1}^A = 7$. Then, given that ε is small enough, it directly follows that $Q_t R^A Q_v$ and $Q_t R^A Q_w$. Similarly, for observation v , there must be an individual M ($= A$ or B) so that $\tilde{P}_{v,2}^M = 7$. Because the set $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ has to satisfy GARP (and $Q_t R^A Q_v$), we have to choose $M = B$ and thus $Q_v R^B Q_t$ and $Q_v R^B Q_w$. Finally, we must specify M ($= A$ or B) so that $\tilde{P}_{w,3}^M = 7$. Any choice of M makes that GARP is violated either by the set $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ (because $Q_t R^A Q_w$) or by the set $\{\tilde{P}_t^B, Q_t\}_{t \in T}$ (because $Q_v R^B Q_w$). We conclude that the given data set does not meet the necessary and sufficient conditions for noncooperative-rationalizability in Theorem 3.

Example 2: noncooperative-rationalizable but not cooperative-rationalizable

We next construct a data set that is noncooperative-rationalizable but not cooperative-rationalizable. Specifically, we consider the following data set S with 4 observations

($T = \{t, v, w, z\}$) and 4 public goods ($K = \{1, 2, 3, 4\}$):

$$Q_t = \begin{pmatrix} 100 \\ 11 \\ \varepsilon \\ 20 \end{pmatrix}, Q_v = \begin{pmatrix} 20 \\ \varepsilon \\ 11 \\ 100 \end{pmatrix}, Q_w = \begin{pmatrix} 5 \\ 10 \\ 10 \\ 5 \end{pmatrix}, Q_z = \begin{pmatrix} 10 \\ 4 \\ 4 \\ 10 \end{pmatrix} \text{ and}$$

$$P_t = \begin{pmatrix} 1 \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}, P_v = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ 1 \end{pmatrix}, P_w = \begin{pmatrix} \varepsilon \\ 1 \\ 1 \\ \varepsilon \end{pmatrix}, P_z = \begin{pmatrix} 1 \\ \varepsilon \\ 1 \\ 1 \end{pmatrix}.$$

We first demonstrate that this data set is noncooperative-rationalizable. To see this, we consider the following specification:

$$\tilde{P}_t^A = \begin{pmatrix} 1 \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}, \tilde{P}_v^A = \begin{pmatrix} \varepsilon^3 \\ \varepsilon \\ \varepsilon^3 \\ \varepsilon^3 \end{pmatrix}, \tilde{P}_w^A = \begin{pmatrix} \varepsilon \\ 1 \\ \varepsilon \\ \varepsilon \end{pmatrix}, \tilde{P}_z^A = \begin{pmatrix} 1 \\ \varepsilon \\ 1 \\ \varepsilon \end{pmatrix} \text{ and}$$

$$\tilde{P}_t^B = \begin{pmatrix} \varepsilon^3 \\ \varepsilon^3 \\ \varepsilon \\ \varepsilon^3 \end{pmatrix}, \tilde{P}_v^B = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ 1 \end{pmatrix}, \tilde{P}_w^B = \begin{pmatrix} \varepsilon \\ \varepsilon \\ 1 \\ \varepsilon \end{pmatrix}, \tilde{P}_z^B = \begin{pmatrix} \varepsilon \\ \varepsilon \\ 1 \\ 1 \end{pmatrix}.$$

This specification clearly meets the condition $\max \{ \tilde{P}_{s,k}^A, \tilde{P}_{s,k}^B \} = P_{s,k}$ ($s \in T$ and $k \in K$). Again, it is straightforward to verify that the sets $\{ \tilde{P}_t^A, Q_t \}_{t \in T}$ and $\{ \tilde{P}_t^B, Q_t \}_{t \in T}$ both satisfy GARP. Therefore, we conclude that the given data set meets the necessary and sufficient conditions for noncooperative-rationalizability in Theorem 3.

Next, it can be verified that the given data set does not pass the condition for consistency with the cooperative model that is given in Proposition 2 of Cherchye, De Rock and Vermeulen (2007); the reasoning is similar to the one in their Example 1. For brevity, we do not include the argument here, but it can be obtained upon request. We thus conclude that the given data set violates the necessary and sufficient condition in Theorem 2.

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Table 1: Pass rates and power; cooperative and noncooperative models

Cooperative model ($\theta^A = \theta^B = 1$)						
Value of κ	Pass (on total of on 148 households)	Power (probability of detecting random behavior)				
		minimum	1st quartile	median	3rd quartile	maximum
1.00	137	0.000	0.063	0.087	0.123	0.230
0.90	143	0.000	0.029	0.040	0.056	0.110
0.80	147	0.000	0.011	0.018	0.025	0.056
0.70	147	0.000	0.003	0.007	0.011	0.039
0.60	148	0.000	0.000	0.002	0.005	0.018
Noncooperative model ($\theta^A = \theta^B = 0$)						
Value of κ	Pass (on total of on 148 households)	Power (probability of detecting random behavior)				
		minimum	1st quartile	median	3rd quartile	maximum
1.00	137	0.000	0.062	0.087	0.115	0.215
0.90	145	0.000	0.022	0.034	0.045	0.093
0.80	146	0.000	0.009	0.015	0.022	0.048
0.70	147	0.000	0.003	0.006	0.010	0.036
0.60	148	0.000	0.000	0.002	0.004	0.011

Table 2: Independence; cooperative and noncooperative models

Rationalizability: not cooperative but noncooperative		
Value of κ	Number: not cooperative ($\theta^A = \theta^B = 1$)	... but noncoop ($\theta^A = \theta^B = 0$)
1.00	11	1
0.90	5	3
0.80	1	0
0.70	1	0
0.60	0	0
Rationalizability: not noncooperative but cooperative		
Value of κ	Number: not noncoop ($\theta^A = \theta^B = 0$)...	... but coop ($\theta^A = \theta^B = 1$)
1.00	11	2
0.90	3	1
0.80	2	1
0.70	1	0
0.60	0	0

Table 3: Pass rates and power; semicooperative models

Household 1	θ^A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
θ^B	0	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.160	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173
0.1	Pass (0/1)	1	1	1	1	1	1	1	1	0	0	0	0
	Power	0.158	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173	
0.2	Pass (0/1)	1	1	1	1	1	1	1	1	0	0	0	0
	Power	0.157	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173	
0.3	Pass (0/1)	1	1	1	1	1	1	1	1	0	0	0	0
	Power	0.157	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173	
0.4	Pass (0/1)	1	1	1	1	1	1	1	1	0	0	0	0
	Power	0.157	0.157	0.155	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173	
0.5	Pass (0/1)	1	1	1	1	1	1	1	1	0	0	0	0
	Power	0.154	0.154	0.154	0.158	0.161	0.168	0.168	0.169	0.169	0.170	0.173	
0.6	Pass (0/1)	1	1	1	1	1	1	0	0	0	0	0	0
	Power	0.160	0.160	0.160	0.159	0.162	0.169	0.169	0.169	0.169	0.170	0.173	
0.7	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0	0
	Power	0.170	0.170	0.170	0.170	0.171	0.170	0.169	0.169	0.169	0.170	0.173	
0.8	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0	0
	Power	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.169	0.170	0.173
0.9	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0	0
	Power	0.170	0.170	0.170	0.170	0.170	0.170	0.170	0.170	0.170	0.169	0.170	0.173
1	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0	0
	Power	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173

Household 2	θ^A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
θ^B	0	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1
		Power	0.108	0.102	0.102	0.102	0.100	0.104	0.106	0.103	0.105	0.107	0.108
0.1	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1	1
	Power	0.108	0.103	0.102	0.102	0.099	0.103	0.105	0.103	0.105	0.107	0.108	
0.2	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1	1
	Power	0.109	0.103	0.101	0.101	0.099	0.103	0.105	0.103	0.105	0.107	0.108	
0.3	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1	1
	Power	0.107	0.104	0.101	0.101	0.099	0.102	0.104	0.103	0.105	0.107	0.108	
0.4	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.108	0.107	0.105	0.103	0.101	0.102	0.104	0.103	0.105	0.107	0.108	
0.5	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.107	0.107	0.105	0.103	0.101	0.103	0.105	0.104	0.105	0.107	0.108	
0.6	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.103	0.103	0.104	0.103	0.101	0.103	0.104	0.104	0.105	0.107	0.108	
0.7	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.102	0.102	0.102	0.102	0.102	0.102	0.104	0.104	0.105	0.107	0.108	
0.8	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.105	0.106	0.108	0.108
0.9	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.105	0.106	0.108	0.108
1	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1	1
	Power	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108

