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Signaling and indirect taxation

by

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Signaling and Indirect Taxation

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Abstract

Commodities communicate. Consumers choose a consumption bundle both for its intrinsic characteristics and for what this bundle communicates about their qualities (or 'identity') to spectators. We investigate optimal indirect taxation when consumption choices are motivated by two sorts of concerns: intrinsic consumption and costly signaling. Optimal indirect taxes are introduced into a monotonic signaling game with a finite typespace of consumers. We provide sufficient conditions for the uniqueness of the D1 sequential equilibrium in terms of strategies. In the case of pure costly signaling, signaling goods can in equilibrium be taxed without burden and the optimal quantity taxes on these goods are infinite. When commodities serve both intrinsic consumption and signaling, optimal taxes can be characterized by a generalization of the Ramsey rule, which also deals with the distortions resulting from signaling.

Keywords: Optimal Taxation, Indirect Taxation, Costly Signaling, Identity.
JEL Classification: C720, H210

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1 Introduction

Commodities communicate. In choosing a consumption bundle, consumers care about both the intrinsic qualities of the bundles and what their choice reveals about themselves to spectators. The intrinsic qualities of a commodity are the (mostly physical) characteristics of the commodity itself, which would remain unchanged if the commodity were consumed in total social isolation. What a commodity communicates can be understood as its social meaning, and the locus of this social meaning is the head of the spectators rather than the commodity itself. The social meaning of a commodity depends on the consumption choices of all consumers in society, thus introducing an interpersonal interdependency into the consumer problem. One can generally distinguish two irreducible dynamics governing social meaning: conformity and distinction. Conformity is driven by consumers confirming their membership of the groups or categories they claim to belong to, and can be thought of as cheap talk signaling. Distinction works by consumers distinguishing themselves from worse types and functions by a form of costly signaling. Clearly, these two mechanisms interact in various ways in reality, but are studied separately for the sake of exposition.

The dependence of the meaning of commodities on the choices of all other consumers reveals an interesting opportunity for indirect taxation. If taxation changes the choices of all consumers, it also changes the meaning of a commodity. Building on the words of Richard Layard (1980) "*In a poor society a man proves to his wife that he loves her by giving her a rose, but in a rich society he must give a dozen roses*", the reasoning of this article can be summarized as: if one taxes roses sufficiently, one rose will satisfy again to get the same job done in rich societies. If roses were consumed only for communication purposes and taxation does not break up the signaling equilibrium, then both the man and his wife remain equally happy after taxation and the tax revenues are pure gain. If roses are also consumed for intrinsic reasons and the signaling equilibrium is not broken, then the husband can still communicate the same message to his wife after taxation, and the policy maker should only care about intrinsic utility. In reality, consumers often employ their complete consumption pattern to communicate their identity. Some kinds of commodities, such as cars or clothing, are more apt for communication purposes than others. For such a commodities, consumption is motivated relatively more by signaling and less by intrinsic reasons, such that the intrinsic utility cost of taxing these commodities is relatively lower. The communicative use of consumption provides thus a new argument for differentiated indirect taxation.

This article characterizes optimal indirect taxation by means of a generalized many-person Ramsey rule in a setting where consumption patterns are used by consumers to signal their types. More precisely, consumer choices are modelled as equilibrium strategies in a monotonic signaling game with consumption patterns as multidimensional signals about a unidimensional dimension of heterogeneity: income. The model builds on two branches of the literature: the literature on costly signaling as initiated by Spence (1973) and surveyed in Riley (2001) and the literature on optimal indirect taxation, as discussed in e.g. Atkinson and Stiglitz (1980) and Myles (1995). The multidimensional signaling framework hinges on work by a.o. Cho and Sobel (1990). The analysis of taxation of pure costly signaling goods, i.e. commodities which are uniquely consumed for signaling, is related to and underpins the analysis of burden free taxes by Ng (1987) for the case of pure 'diamond goods' (cfr. infra). If goods are used for both signaling and intrinsic consumption, the optimal indirect taxation result formally resembles work on indirect taxation with externalities (e.g. Sandmo, 1975, Cremer, Gahvari and Ladoux, 1998) or merit goods (Blomquist and Micheletto, 2006), in the sense that the consumer choice differs from the social welfare optimum. Signaling induces consumers to consume too much of some commodities from a welfare perspective, such that a reduction of spending on such commodities due to taxes causes a relatively smaller welfare loss. More generally, this line of reasoning relates to taxes on consumption motivated by relative concerns. Frank (1985, 1999) calls for a progressive tax on overall consumption, because consumption is deemed to be largely driven by relative concerns and therefore self-defeating in terms of social welfare. This article explicitly models the mechanism driving the relative concerns, allows a characterization of the extent to which consumption is self-defeating in welfare terms and establishes a rule for differential indirect taxation of commodity groups which are used for communication. Ireland (1994) explores commodity taxes in a unidimensional signaling model and shows in a numerical example how a small tax on a signaling commodity can be a Pareto improvement.

The second section sets out the formal model and introduces a unique signaling equilibrium. The third section deals with the benchmark case of pure costly signaling. It is shown that these goods can be taxed without burden, such that the revenues from taxing pure costly signaling goods are a pure gain. In the fourth section, all visible consumption goods serve both intrinsic consumption and communication. Optimal commodity taxation is then characterized by a generalization of the conventional many-person Ramsey rule. The fifth section concludes.

2 Setting

Imagine a finite population of N consumers. Nature draws the type t of each consumer out of a finite set T with prior probabilities π_t. The type of a consumer is her private information and different consumer types differ ex ante only in one dimension: income $m_t \in \mathbb{R}_{++}$. Assume, without loss of generality, that $m_t < m_{t'} \iff t < t'$. Consumers spend this income on K different commodities, indexed k, summarized in a consumption vector $c \equiv (c_1, ..., c_K) \in \mathbb{R}_+^K$, a column vector. This consumption vector consists of one invisible good c_1 and a visible consumption bundle $\bar{c} \equiv (c_2, ..., c_K)$, also a column vector (if $K > 2$). The visible consumption pattern will function as a signal regarding the consumer's type to spectators (the other consumers), while the invisible good c_1 represents the opportunity cost of visible consumption. Producer prices are by assumption fixed and are normalized to 1. Consumer prices are denoted by a row vector $p \equiv (p_1, \bar{p}) = (p_1, ..., p_K) \in \bar{\mathbb{R}}_{++}^K$, such that the quantity tax imposed on good k amounts to $p_k - 1$, and forms a subsidy if $p_k < 1$. The budget constraint for a type t is then $pc \leq m_t$. A mixed strategy $\mu(t, p)$ of type t given prices p is a probability distribution over the visible budget set $\{\bar{c} | \bar{p}\bar{c} \leq m_t\}$.

Spectators do not know the true type of a consumer and form beliefs $\beta(\bar{c})$ about a consumer's income, conditional on the visible consumption vector $\bar{c}$, where β is a $|T|$ −dimensional vector, with the t-th entry $\beta_t(\bar{c})$ indicating the probability that a consumer with visible consumption pattern $\bar{c}$ is deemed a type t. Consumer preferences can, by assumption, be represented by a utility function[1]

$$u(c, \hat{m}(\bar{c})), \tag{1}$$

where $\hat{m}(\bar{c})$ is the average perceived income, as estimated by spectators given their beliefs

$$\hat{m}(\bar{c}) \equiv \sum_{t=1}^{T} \beta_t(\bar{c}) m_t.$$

Assume that this utility function satisfies the following regularity conditions.

[1] In most signaling games, a receiver observes and interprets the signal and chooses a utility maximizing action as a best reply to the signal $\bar{c}$ and beliefs $\beta(\bar{c})$. If these optimal responses are unique and strictly increasing in $\hat{m}(\bar{c})$ and if the sender's utility is strictly increasing in the receiver's best reply, then the utility function in formula 1 can be understood as a shorthand notation of this game, omitting the receiver's best reply. Alternatively, one can also interpret this specification as if $\hat{m}(.)$ is a good on itself, as consumers may also enjoy esteem for its own sake.

Condition 1 *Assume that u is*
i) twice continuously differentiable with $u_1' > 0$, $u_k' \geq 0$, $u_{K+1}' > 0$ and $u_{11}'' < 0$, and
ii) such that $u_1'(.)$ is bounded and $u_{K+1}'(.)$ is bounded away from zero.

Substitute the budget constraint into the utility function to obtain (with a slight abuse of notation)

$$u\left(p_1^{-1}\left(m_t - \overline{p}\overline{c}\right), \overline{c}, \hat{m}\right). \tag{2}$$

Next to these regularity conditions, the utility function is also assumed to be such that (2) satisfies a Spence-Mirrlees single crossing condition for each dimension of $\overline{c}$. This means that the slope of the indifference curve in the $(c_k, \hat{m})-$plane is decreasing with the type of a consumer, or that[2]

$$\left.\frac{d\hat{m}}{dc_k}\right|_{u=\text{constant}} = -\frac{\partial u\left(p_1^{-1}\left(m_t - \overline{p}\overline{c}\right), \overline{c}, \hat{m}\right)}{\partial c_k}\left(\frac{\partial u\left(p_1^{-1}\left(m_t - \overline{p}\overline{c}\right), \overline{c}, \hat{m}\right)}{\partial \hat{m}}\right)^{-1}$$

must be strictly decreasing with m_t for all $k = 2, ..., K$ and all $(\overline{c}, \hat{m})$.

Condition 2 *Let $\frac{\frac{p_k}{p_1}u_1'\left(p_1^{-1}(m_t-\overline{p}\overline{c}),\overline{c},\hat{m}\right)-u_k'\left(p_1^{-1}(m_t-\overline{p}\overline{c}),\overline{c},\hat{m}\right)}{u_{K+1}'\left(p_1^{-1}(m_t-\overline{p}\overline{c}),\overline{c},\hat{m}\right)}$ be decreasing in m_t for all $k \in \{2, ..., K\}$ and all $(\overline{c}, \hat{m})$.*

Note that if u is strongly separable in all arguments, this condition is already guaranteed by condition 1, because u is strictly concave in c_1 and strictly increasing in $\hat{m}$. For more general specifications of u, condition 2 restricts the cross derivatives to preserve this higher marginal opportunity cost of signaling for lower income types.

Consumer behavior is characterized by the equilibrium strategies in a sequential equilibrium of the signaling game.

Definition 1 *A tuple $(\mu(t,p), \beta)$ is a Sequential Equilibrium (S.E.) if $\mu(t,p)$ maximizes utility for each type t given beliefs β and beliefs β are consistent given μ in the sense of Bayesian rationality, i.e.:*

1. *For each type t,*

$$\mu(t,p) \in \arg\max_{\mu}\left\{\int_{\overline{c}\in\{\overline{c}'|\overline{p}\overline{c}'\leq m_t\}} u\left(p_1^{-1}\left(m_t - \overline{p}\overline{c}\right), \overline{c}, \hat{m}(\overline{c})\right)\mu(\overline{c}|t,p)\,d\overline{c}\right\}$$

given β.

[2] Given this substitution of the budget constraint in u, we distinguish in notation between $u_k'\left(p_1^{-1}\left(m_t - \overline{p}\overline{c}\right), \overline{c}, \hat{m}\right)$, the partial derivative to the k-th argument, and $\frac{\partial u\left(p_1^{-1}(m_t-\overline{p}\overline{c}),\overline{c},\hat{m}\right)}{\partial c_k}$, the partial derivative to good c_k (with $1 < k \leq K$), as it is found in the first and k-th argument of u.

2. *Beliefs are updated according to Bayes' rule for all* $\bar{c} \in supp(\mu)$, *such that* [3]

$$\beta_t(\bar{c}) = \frac{\pi_t \mu(\bar{c}|t,p)}{\sum_{t'} \pi_{t'} \mu(\bar{c}|t',p)}.$$

How can we describe consumer behavior within this setting? A sequential equilibrium is called 'separating' if the supports of the equilibrium consumption patterns of different types do not intersect:

$$\forall t \neq t' : supp(\mu(t,p)) \cap supp(\mu(t',p)) = \varnothing,$$

such that equilibrium beliefs attribute equilibrium consumption patterns to a single type with probability 1. The sequential equilibrium concept imposes no restrictions on beliefs about out-of-equilibrium consumption patterns (except that they should 'support' the equilibrium) and this allows for an unappealing multitude of often counterintuitive equilibria. For this reason, a number of equilibrium refinements have been proposed which impose restrictions on beliefs about out-of-equilibrium consumption patterns. Although the key results of this paper are not limited to this particular selection criterion or outcome, we focus on the most commonly used D1 criterion[4] and the selected Riley outcome, after Riley (1979). This Riley outcome, also known as the "Pareto Dominant Separating Equilibrium" is generally deemed the most plausible solution for monotonic signaling games (see e.g. Fudenberg and Tirole, 1991).

[3] With $supp(\mu) = \bigcup_t supp(\mu(t,p))$.

[4] Divinity (D1) eliminates a consumer type t out of the support of the beliefs $\beta(\bar{c}^\diamondsuit)$ following an out-of-equilibrium consumption pattern $\bar{c}^\diamondsuit$, if a second type t' has a strict incentive to deviate to $\bar{c}^\diamondsuit$ whenever the first type t under investigation has a weak incentive to deviate. Formally, fix a S.E. and equilibrium utility levels u^t for each type t. Let

$$M^+(\bar{c}^\diamondsuit, t) = \left\{ \widetilde{m} | u\left(\frac{1}{p_1}\left(m_t - \bar{p}\bar{c}^\diamondsuit\right), \bar{c}^\diamondsuit, \widetilde{m}\right) > u^t \right\}$$

be the set of beliefs (only $\hat{m}$ matters) which make type t strictly better off with $\bar{c}^\diamondsuit$ then in the S.E. Similarly, let

$$M^o(\bar{c}^\diamondsuit, t) = \left\{ \widetilde{m} | u\left(\frac{1}{p_1}\left(m_t - \bar{p}\bar{c}^\diamondsuit\right), \bar{c}^\diamondsuit, \widetilde{m}\right) = u^t \right\}$$

be the set of beliefs which make type t indifferent between consumption pattern $\bar{c}^\diamondsuit$ and the S.E. payoffs. Then Divinity requires that all types t for whom there is a type t' for which

$$M^+(\bar{c}^\diamondsuit, t) \cup M^o(\bar{c}^\diamondsuit, t) \subseteq M^+(\bar{c}^\diamondsuit, t'),$$

are eliminated from the support of the out-of-equilibrium beliefs.

The Riley outcome is generated by a separating equilibrium in which all consumers choose consumption to maximize utility under the constraint that no other type can benefit from imitating their consumption patterns and while spectators concentrate all probability mass on their true type.

Definition 2 *The Riley outcome is generated by a S.E. in which all consumer types $t > 1$ choose their consumption strategies with support on visible consumption patterns $\bar{c}^t$ which are a solution to the constrained maximization problem*

$$\bar{c}^t \in \arg\max_{\bar{c}} \left\{ u\left(p_1^{-1}\left(m_t - \bar{p}\bar{c} \right), \bar{c}, m_t \right) \right\} \tag{3}$$

subject to $c \geq 0$ and

$$u\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c} \right), \bar{c}, m_t \right) \leq u^{t-1}, \tag{4}$$

with $u^{t-1} \equiv u\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}^{t-1} \right), \bar{c}^{t-1}, m_{t-1} \right)$. *Type $t = 1$ only chooses*

$$\bar{c}^1 \in \arg\max_{\bar{c}} \left\{ u\left(p_1^{-1}\left(m_1 - \bar{p}\bar{c} \right), \bar{c}, m_1 \right) \right\} \tag{5}$$

subject to $c \geq 0$.

Hence, the lowest income consumers, with no lower types to distinguish themselves from, maximize utility as if there were no signaling concerns. All higher types t choose a visible consumption pattern which maximizes utility under the constraint that a worse type $t-1$ cannot improve herself by mimicking. As shown in the proof of theorem 1, condition 2 ensures that no other type wishes to imitate type t if type $t-1$ does not. Since such an S.E. is separating, equilibrium beliefs satisfy $\beta_t\left(\bar{c}^t\right) = 1$ such that $\hat{m}\left(\bar{c}^t\right) = m_t$, which justifies setting $\hat{m} = m_t$ in equations (3) to (5).

Cho and Sobel (1990) demonstrate the existence of a D1 S.E. for the class of monotonic signaling games with a finite typespace and both a finite or infinite multidimensional signal space. They also show the uniqueness of such a S.E. in terms of outcome and prove that this outcome coincides with the Riley outcome. The following lemma summarizes their result.

Theorem 1 *If u satisfies conditions 1 and 2, then a D1 sequential equilibrium exists and the equilibrium outcome is the Riley outcome, as defined by equations 3 to 5.*

Proof. In appendix. ■

Motivated by this equilibrium selection literature, the remainder focusses solely on the D1 equilibrium or Riley outcome to characterize consumer behavior. The result of Cho and Sobel (1990) proves uniqueness in terms of outcome, but not necessarily in terms of strategies. We now provide sufficient conditions for the Riley problem in equations 3 to 5 to have a unique solution $\bar{c}^t$ for all types t.

Condition 3 *Let K and u satisfy either of the following conditions*
i) $K = 2$ and $u_2' = 0$ or $u_2' > 0$ and u is strictly concave in c, or
ii) $K = 3$, u is strictly increasing ($u_k' > 0$) and strictly concave in c and let for all $\bar{c}$

$$\frac{\partial^2 MRS\left(c_2, c_3\right)}{\partial c_3 \partial m_t} < 0, \tag{6}$$

with $MRS\left(c_2, c_3\right) \equiv -\frac{p_1 u_3'\left(p_1^{-1}(m_t - \bar{p}\bar{c}), \bar{c}, \hat{m}\right) - p_3 u_1'\left(p_1^{-1}(m_t - \bar{p}\bar{c}), \bar{c}, \hat{m}\right)}{p_1 u_2'\left(p_1^{-1}(m_t - \bar{p}\bar{c}), \bar{c}, \hat{m}\right) - p_2 u_1'\left(p_1^{-1}(m_t - \bar{p}\bar{c}), \bar{c}, \hat{m}\right)}.$

If $K > 2$, the Riley problem in equations 3 to 5 is nonconvex. The choice set of a consumer of type $t > 1$, i.e. the set of visible consumption bundles in the positive quadrant which satisfy both inequality 4 and the budget constraint (denoted L_t in figure 1), is typically nonconvex. The frontier formed by the visible consumption patterns for which inequality 4 is satisfied with equality (L_t^o in figure 1) can be both convex and concave. If inequality 4 is not binding, the strict concavity of u guarantees the uniqueness of a maximum. The inequality in (6) ensures that the indifference curves which take the budget constraint into account (i.e. the implied changes in c_1 of a change in bundle $\bar{c}$) are at all points more concave (or less convex) for higher income types. This is shown to be a sufficient condition for the utility function of a consumer type t to have a unique maximum on L_t^o, and by extension that the Riley problem in equations 3 to 5 has a unique solution $\bar{c}^t$ for all types t.

Proposition 1 *The Riley problem, as defined by equations 3 to 5 has a unique solution $\bar{c}^t$ for all types t if u satisfies conditions 1, 2 and 3.*

Proof. In appendix. ■

Given that consumer behavior is uniquely characterized by equations 3 to 5, we can now engage in finding an optimal system of indirect taxes. Given a S.E., the indirect utility function of a type t consumer is written $v\left(t, p\right)$. Let $W\left(\mathbf{v}\left(p\right)\right)$ represent a general Bergson-Samuelson social welfare function which represents the preference ordering of the

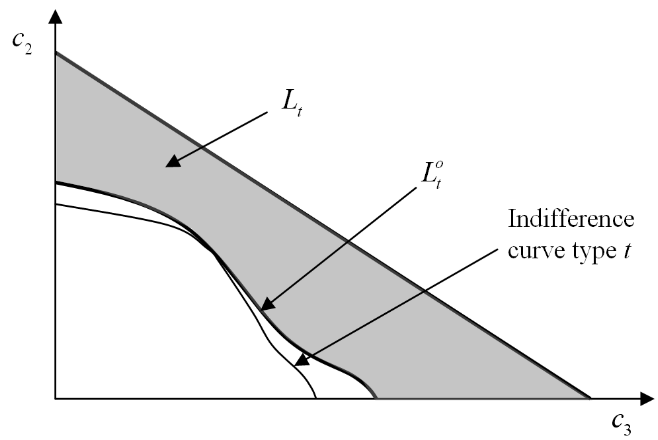

Figure 1: Graphical illustration of condition ii) in condition 3.

social planner, with $\mathbf{v}(p)$ a vector with the (anticipated) indirect utility functions of all consumers. Given the unique D1 equilibrium strategies $\bar{c}^t$, the tax income of a tax schedule characterized by p is

$$R(p) \equiv N \sum_t \pi_t \left((p_1 - 1) \frac{m_t - \bar{p}\bar{c}^t}{p_1} + (\bar{p} - \mathbf{1}) \bar{c}^t \right),$$

with $\mathbf{1}$ a $K-1$ dimensional column vector of ones. The optimal tax problem, given a S.E,. can then be written as

$$\max_p W(\mathbf{v}(p)) \text{ s.t. } R(p) \geq \bar{R}, \tag{7}$$

with $\bar{R}$ an exogenously fixed level of required tax revenue.[5]

The main information issue in this model concerns consumers signaling their quality to other consumers. The government is assumed to know the preference orderings of consumers and the prior distribution of the different consumer types, but does not need to know the actual type and consumption patterns of consumers. The assumption of known preference orderings serves to allow the government to maximize social welfare and can, in principle, be known from a representative sample. The assumption that individual consumption is not observed by the government excludes nonlinear indirect tax schemes, which would be vulnerable to arbitrage. By assumption, the policy maker cannot choose nonlinear direct tax schemes, which are beyond the scope of this paper (cfr. infra).

[5] The possibility of nonconcavities in $W(\mathbf{v}(p))$ or nonconvexities in its domain and the justification of the first order approach is a known problem, but not discussed further here (see e.g.. Mirrlees (1986)).

3 Pure costly signaling

Consider the baseline case of pure costly signaling, in which visible consumption $\bar{c}$ serves only one purpose: distinguishing oneself from worse types. This pure costly signaling case is formalized by condition 4.

Condition 4 *Let u be such that for $2 \leq k \leq K$: $u'_k = 0$, $u''_{1k} = 0$ and $u''_{kK+1} = 0$.*

Since the vector $\bar{c}$ does not affect utility directly, the utility function can be rewritten as a function of two arguments:

$$u^{CS}\left(p_1^{-1}\left(m_t - \bar{p}\bar{c}\right), \hat{m}\left(\bar{c}\right)\right).$$

Consumers derive utility from consuming the invisible good c_1 and from making an impression $\hat{m}\left(.\right)$ on others, such that the visible goods $\bar{c}$ affect utility only through $\hat{m}\left(\bar{c}\right)$. Under condition 4, all $K-1$ visible goods are equivalent ways of conspicuously wasting purchasing power. Without loss of generality, we simplify the analysis for this case by setting $K = 2$ to avoid a multiplicity of signaling equilibria. The results obtained for $K = 2$ in this section easily extend to $K > 2$.

The Riley outcome for the pure costly signaling case solves (omitting $c \geq 0$):

$$\max_{\bar{c}} u^{CS}\left(p_1^{-1}\left(m_t - \bar{p}\bar{c}\right), m_t\right)$$

$$u^{CS}\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), m_t\right) \leq u^{CS}\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}_{t-1}\right), m_{t-1}\right). \tag{8}$$

Note that visible consumption only appears multiplied by its own price in this problem. To ensure separation, consumer type t needs to spend exactly enough on pure costly signaling, $\bar{p}\bar{c}^t$, to make a lower type indifferent between wasting $\bar{p}\bar{c}^t$ from her income m_{t-1}, in order to be perceived as a t type, and wasting $\bar{p}\bar{c}^{t-1}$ to appear of a type $t-1$. Hence, denoting by $e(t,p) \equiv \bar{p}\bar{c}^t$ the total expenditure on visible consumption goods of a consumer type t, the Riley outcome is characterized by the unique amount of expenditures $e(t,p)$ which solves for each type $t > 1$

$$\max_{e} u^{CS}\left(p_1^{-1}\left(m_t - e\right), m_t\right) \tag{9}$$

$$u^{CS}\left(p_1^{-1}\left(m_{t-1} - e\right), m_t\right) \leq u^{CS}\left(p_1^{-1}\left[m_{t-1} - e\left(t-1, p\right)\right], m_{t-1}\right). \tag{10}$$

Type 1 does not waste any income on visible consumption $e(1,p) = 0$. Note that these equilibrium expenditures on pure costly signals are independent of the price of visible goods $\bar{p}$, such that one can abuse notation to write $e(t, p_1)$. Consumers only need to waste enough in terms of opportunity costs to keep worse types from mimicking, and there is no reason why the optimal amount of wasted c_1 should depend on $\bar{p}$. These equilibrium features are summarized in the following lemma.

Lemma 1 *If u satisfies conditions 1, 2 and 4, then:*

1. *All D1 sequential equilibria are characterized by $\bar{p}\bar{c}_t = e(t, p_1)$, with $e(t, p_1)$ solving problem 9 and 10 for each type t.*
2. *Equilibrium expenditures on visible consumption are independent of $\bar{p}$.*

The equilibrium demand for visible consumption as a function of the own post tax price can then be written

$$\bar{c}^t = \frac{e(t, p_1)}{\bar{p}},$$

such that the own price elasticity of the demand for visible goods is then necessarily -1 everywhere:

$$\frac{\partial \bar{c}^t}{\partial \bar{p}} \frac{\bar{p}}{\bar{c}_t} = -\frac{e(t, p_1)}{\bar{p}^2} \frac{\bar{p}}{\bar{c}^t} = -1.$$

Thus, in the pure costly signaling case the equilibrium demand functions for pure costly signals of all type $t > 1$ consumers are necessarily rectangular hyperbola as a function of the own price. Since $t = 1$ consumers do not buy any costly signal, the aggregate demand function of $\bar{c}$ is a rectangular hyperbola as well. The insensitivity of equilibrium expenditures on pure costly signaling for changes in the own price implies an interesting feature: visible goods can, in equilibrium, be taxed without burden.

Lemma 2 *In equilibrium, the utility of consumers is unaffected by taxes on the pure costly signal:*

$$\frac{\partial v^{CS}(t, p)}{\partial \bar{p}} = 0,$$

with $v^{CS}(t, p) \equiv u^{CS}\left(p_1^{-1}\left[m_t - e(t, p_1)\right], m_t\right)$ and $u^{CS}(.)$ satisfying conditions 1, 2 and 4.

The reason is that both arguments of the utility function are unaffected by taxes on $\bar{c}$, as long as the signaling equilibrium is maintained. First, expenditures on visible goods are independent of $\bar{p}$, such that the consumption of invisible goods is unaffected by a tax on visible goods. Second, as long as the fully separating equilibrium is maintained, the second argument is not affected by changes in $\bar{p}$ either. It is easily seen that increasing $\bar{p}$ does not break up conditions 1, 2 and 4, needed to maintain the separating equilibrium. An optimal system of taxes maximally exploits this potential for burden free taxation and the income wasted on costly signaling is recuperated as much as possible.

Proposition 2 *In the D1 equilibrium of the pure costly signaling game, in which the utility function satisfies conditions 1, 2 and 4, optimal indirect taxes satisfy*

$$p_1 = \frac{\sum_{t \in T} \pi_t (m_t - e(t, p_1))}{\sum_{t \in T} \pi_t (m_t - \bar{c}^t) - \frac{\bar{R}}{N}} \tag{11}$$
$$p_2 \to +\infty.$$

Proof. In appendix. ■

The first equality in 11 can be rewritten

$$\frac{p_1 - 1}{p_1} = \frac{\bar{R} - \frac{p_2 - 1}{p_2} N \sum_{t \in T} \pi_t e(t, p_1)}{N \sum_{t \in T} \pi_t (m_t - e(t, p_1))}, \tag{12}$$

in which $\frac{p_2-1}{p_2} N \sum_{t \in T} \pi_t e(t, p_1)$ are the tax revenues from taxing the visible consumption good, such that the tax rate on invisible goods should equal the ratio of the difference between required tax revenue and tax revenue raised on visible goods to total expenditure on invisible goods. Hence, one should subsidize invisible consumption $p_1 < 1$ if the burden-free tax revenues from taxing visible consumption exceed the required revenues. If the required revenues exceed the burden-free revenue from taxing visible consumption, then these revenues should be raised by a tax on invisible consumption.

The result is that each consumer type, by buying the infinitely taxed visible consumption good, conspicuously contributes just enough to the required tax revenue to distinguish herself from any lower income types. The policy maker collects almost all the means wasted for social distinction. This result underpins the burden free tax result of Ng (1987) for the case of pure diamond goods. Ng defines pure 'diamond goods' as goods which are uniquely valued for their value and therefore enter the utility function together with their price. For x a 'diamond good' and y a regular good, p_x and p_y their respective prices and m income, the consumer problem is then

$$\max U(p_x x, y)$$
$$\text{s.t. } p_x x + p_y y \leq m.$$

This section shows that pure costly signaling goods are necessarily pure diamond goods. Clearly, the infinite tax on pure costly signals crucially hinges, as a limit case, on the perfect divisibility and observability of the visible good. Very high taxes on the visible good only leave the individual unaffected if the very small consumed quantities of the visible good of different types can be distinguished, such that the different consumer types can be identified by spectators. The basic intuition of this section, however, remains valid under much broader assumptions.

4 The mixed signaling case

In reality, not too many - if any - consumption goods only serve costly signaling. Mostly, consumers use every day consumption goods, often with luxurious characteristics, to distinguish themselves from worse consumer types. These conspicuous premium characteristics are excessively costly for the extra utility they generate, compared to some other less conspicuous good which would generate more marginal utility in social isolation. Formally, preferences are assumed such that u satisfies conditions 1, 2 and 3,ii). Hence, u is such that all visible goods now generate intrinsic utility and the marginal utility they deliver is decreasing in the number of already consumed units.

The problem of the lowest income type 1 is:

$$\max_{c} \mathcal{L} = u\left(c, m_1\right) - \lambda_1 \left(pc - m_1\right),$$

in which λ_1 is the Lagrange multiplier associated with the budget constraint of type 1. The optimum c^1 is characterized by the usual equality of the marginal rate of substitution to relative prices, taking $\hat{m} = m_1$ as given:

$$\forall k, l = 1, ..., K : \frac{u'_k\left(c^1, m_1\right)}{u'_l\left(c^1, m_1\right)} = \frac{p_k}{p_l}. \tag{13}$$

The optimal choice is the consumption bundle chosen in the absence of signaling concerns, i.e. the 'intrinsic optimum'. The unique D1 equilibrium consumption pattern c^t of each type $t > 1$ characterizes the Riley outcome and is the unique solution, given c^{t-1}, to:

$$\begin{aligned} \max_{c}\ \mathcal{L} &= u\left(c, m_t\right) - \lambda_t \left(pc - m_t\right) \\ &-\psi_t \left(u\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right) - u^{t-1}\right), \end{aligned}$$

where λ_t and ψ_t are the Lagrange multipliers associated with respectively the budget and Riley constraint of a type t consumer.[6] The first order

[6]The optimal solution of the constrained maximization problem in equations 3 to 5 is characterized by Lagrange optimization, as is common practice in the optimal taxation literature despite known problems (see e.g. Mirrlees, 1986). The proof of proposition 1 contains a partially constructive proof of the optimum, from which the optimality of the solution to the Lagrange maximization problem can be verified.

conditions for a type $t > 1$ consumer are

$$u_1'\left(c^t, m_t\right) - \lambda_t p_1 = 0 \tag{14}$$

$$\forall k > 1 : u_k'\left(c^t, m_t\right) - \lambda_t p_k - \psi_t \begin{bmatrix} -\frac{p_k}{p_1} u_1'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right) + \\ u_k'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right) \end{bmatrix} = 0.$$

Optimal consumer choices are summarized in the following lemma.

Lemma 3 *If u satisfies conditions 1, 2 and 3,ii), the unique D1 equilibrium consumption pattern is characterized by*

$$\forall k, l = 1, ..., K : \frac{u_k'\left(c^1, m_1\right)}{u_l'\left(c^1, m_1\right)} = \frac{p_k}{p_l} \tag{15}$$

$$\forall t > 1 : \frac{u_2'\left(c^t, m_t\right) + B_{t,2}}{u_3'\left(c^t, m_t\right) + B_{t,3}} = \frac{p_2}{p_3}$$

$$\text{and } \forall k, t > 1 : \frac{u_k'\left(c^t, m_t\right) + B_{t,k}}{u_1'\left(c^t, m_t\right)} = \frac{p_k}{p_1} \tag{16}$$

$$\text{with } B_{t,k} \equiv \psi_t \begin{bmatrix} \frac{p_k}{p_1} u_1'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}^t, m_t\right) - \\ u_k'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}^t, m_t\right) \end{bmatrix} \tag{17}$$

$$\lambda_t \geq 0,\ \psi_t \geq 0,\ c \geq 0.$$

How to interpret this equilibrium consumption pattern? Clearly, if incomes are so distant that the Riley constraint is not binding, such that $\psi_t = 0$, type t chooses her intrinsic optimum. If $\psi_t \neq 0$, then it must be that $u_k'\left(c^t, m_t\right) < \frac{p_k}{p_1} u_1'\left(c^t, m_t\right)$ and

$$\frac{p_k}{p_1} u_1'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}^t, m_t\right) > u_k'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}^t, m_t\right), \tag{18}$$

such that $\psi_t > 0$ and $B_{t,k} > 0$.[7] The term between square brackets is the marginal utility cost of an increased consumption of good k to

[7] To see this, verify in the proof of theorem 1 that if type t consumers choose $\bar{c}^t$ in the S.E., then spectators attribute any consumption pattern $\bar{c} > \bar{c}^t$ to a type $t' < t$ consumer with zero probability. Suppose then that $\psi_t \neq 0$ and $u_k'\left(c^t, m_t\right) > \frac{p_k}{p_1} u_1'\left(c^t, m_t\right)$. Then type t consumers can strictly improve themselves by increasing their consumption of good k at the expense of the invisible good (while keeping the rest of the visible consumption pattern constant). Note that this is always possible, because if $\bar{c}^t$ exhausts the whole income of the type t consumer, it must be that $\psi_t = 0$. But then $\bar{c}^t$ cannot be utility maximizing, a contradiction. If $u_k'\left(c^t, m_t\right) < \frac{p_k}{p_1} u_1'\left(c^t, m_t\right)$, then (18) is true by condition 1.

a type $t-1$ mimicker of the t type. More precisely, it is the utility gain of a marginal shift in consumption from good k to good 1 for a consumer of type $t-1$, if she were imitating a type t consumer. The Lagrange multiplier ψ_t measures the marginal utility which a marginal unit of equilibrium utility for a type $t-1$ consumer generates maximally for the t type consumer. Hence, ψ_t quantifies the marginal utility of relaxing the Riley constraint to consumer t by marginally increasing u^{t-1}. Rewrite (16) to see that $B_{t,k}$ measures the gain in intrinsic utility which a marginal shift of consumption from visible good c_k to invisible good c_1 would generate for a type t consumer:

$$\frac{p_k}{p_1} u_1' \left(c^t, m_t\right) - u_k' \left(c^t, m_t\right) = B_{t,k}.$$

If utility were strongly separable in invisible consumption, such that $\forall k \in \{2,3\} : u_{1,k}''(.) = 0$, then

$$u_k' \left(c^t, m_t\right) = u_k' \left(p_1^{-1} \left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}^t, m_t\right)$$

and the first order conditions of all $K-1$ visible goods can be written

$$\forall t > 1 : \frac{u_2' \left(c^t, m_t\right)}{u_3' \left(c^t, m_t\right)} = \frac{p_2}{p_3} \tag{19}$$

$$\forall t, k > 1 : \frac{\left(1 - \psi_t\right) u_k' \left(c^t, m_t\right)}{u_1' \left(c^t, m_t\right) - \psi_t u_1' \left(p_1^{-1} \left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}, m_t\right)} = \frac{p_k}{p_1}, \tag{20}$$

such that

$$\frac{u_k' \left(c^t, m_t\right)}{u_1' \left(c^t, m_t\right)} < \frac{p_k}{p_1}.$$

This means that if the utility function were separable in invisible consumption, all types $t > 1$ would mimic the intrinsically optimal visible consumption pattern of a (generically fictitious) higher income type. Equation 19 states that the marginal rate of substitution of all visible commodities should equal their relative prices, as in the intrinsic optimum. Equation 20 however, states that the marginal rate of substitution of a visible and the invisible good is smaller than their relative prices if the Riley constraint is binding. The type $t > 1$ consumers inflate their visible consumption pattern to a visible consumption pattern which consumers with a higher income would choose in the absence of signaling motives, to discourage imitation by lower types.

If visible consumption is not separable from invisible consumption, then the term

$$u_k' \left(p_1^{-1} \left(m_{t-1} - \bar{p}\bar{c}^t\right), \bar{c}^t, m_t\right)$$

in equation 17 causes a deviation in visible consumption away from the visible intrinsic optimum of higher income types, towards visible goods with a higher complementarity with the invisible good c_1 and hence relatively smaller

$$u'_k\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}^t\right),\bar{c}^t,m_t\right).$$

A visible good with a higher complementarity with invisible consumption implies less marginal utility if consumed with less invisible goods. As such, imitating higher types' consumption of this visible good is more costly for lower types and this feature is exploited in the optimal visible consumption pattern of all but the lowest income type.

The bias away from the intrinsic optimum caused by signaling suggests interesting opportunities for optimal indirect taxation. As long as conditions 1, 2 and 3,ii) are satisfied, all types are in equilibrium recognized as their true self, $\hat{m}\left(\bar{c}^t\right)=m_t$. However, signaling generates an inequality in the marginal intrinsic utility which a marginal cent spent on each of the K commodities produces. Thus, marginally reducing the expenditures on invisible consumption generally induces more utility loss than an equivalent reduction of expenditures on a visible good, and more so for visible goods which are complementary to c_1.

The policy maker's problem is to choose a tax policy p which satisfies the tax revenue constraint and which is such that the D1 signaling equilibrium maximizes social welfare. The problem of the policy maker is

$$\max_p \mathcal{L}=W\left(\mathbf{v}\left(p\right)\right)-\xi\left(\bar{R}-N\sum_{t\in T}\pi_t\sum_{k=1}^{K}\left(p_k-1\right)c_k^t\right),$$

with ξ the Lagrange multiplier associated with the government tax revenue constraint.

The first order condition for any tax p_j is

$$\begin{aligned} &N\sum_{t\in T}\pi_t\frac{\partial W}{\partial u(c^t,m_t)}\sum_{k=1}^{K}\frac{\partial u(c^t,m_t)}{\partial c_k^t}\frac{\partial c_k^t}{\partial p_j} \\ +&N\xi\sum_{t\in T}\pi_t\left(\sum_{k=1}^{K}\left(p_k-1\right)\frac{\partial c_k^t}{\partial p_j}+c_j^t\right)=0 \qquad \forall j=1,...,K. \end{aligned} \tag{21}$$

Using equations 14 and 17, these first order conditions in (21) can be written:

$$\sum_{t\in T}\pi_t\frac{\partial W}{\partial u(c^t,m_t)}\left(\sum_{k=1}^{K}\lambda_t p_k\frac{\partial c_k^t}{\partial p_j}-\sum_{k=2}^{K}B_{t,k}\frac{\partial c_k^t}{\partial p_j}\right) \tag{22}$$
$$+\xi\sum_{t\in T}\pi_t\left(\sum_{k=1}^{K}(p_k-1)\frac{\partial c_k^t}{\partial p_j}+c_j^t\right)=0 \qquad \forall j=1,...,K.$$

Define the marginal social benefit of an extra unit of income for a type t consumer as $\omega_t \equiv \frac{\partial W}{\partial u(c^t,m_t)}\lambda_t$, let $\widetilde{c_j} \equiv \sum_{t\in T}\pi_t c_j^t$ represent average equilibrium consumption of good j and take the derivative of the budget constraint for some type t with respect to p_j, such that

$$\sum_{k=1}^{K}p_k\frac{\partial c_k^t}{\partial p_j}=-c_j^t.$$

Using this, the first order conditions can be written

$$-\sum_{t\in T}\pi_t\omega_t c_j^t-\sum_{t\in T}\pi_t\omega_t\sum_{k=2}^{K}\tilde{B}_{k,t}\frac{\partial c_k^t}{\partial p_j} \tag{23}$$
$$+\xi\left(\sum_{t\in T}\pi_t\sum_{k=1}^{K}(p_k-1)\frac{\partial c_k^t}{\partial p_j}+\widetilde{c_j}\right)=0 \qquad \forall j=1,...,K,$$

with $\tilde{B}_{k,t} \equiv \frac{B_{k,t}}{\lambda}$. The expression in equation 23 can already be understood as an uncompensated version of the many-person Ramsey rule, with an extra term due to signaling motives. Note the resemblance of this optimal indirect tax prescription to that of Sandmo (1975) for optimal indirect taxes with externalities. Following Diamond and Mirrlees (1971) and Diamond (1975), one may obtain a variant of the usual compensated many-person Ramsey rule by employing the Slutsky decomposition on the $\frac{\partial c_k^t}{\partial p_j}$ between brackets, i.e. use

$$\frac{\partial c_k^t}{\partial p_j}=\sigma_{k,j}^t-c_j^t\frac{\partial c_k^t}{\partial m_t}.$$

Here $\sigma_{k,j}^t$ denotes element k,j of the Slutsky matrix of a type t consumer in equilibrium, reflecting the first order effect of a change in the compensated demand of commodity k because of a marginal increase in the price of commodity j (and keeping $\hat{m}=m_t$). One only considers the distortion along the compensated demand curve because the income effects $c_j^t\frac{\partial c_k^t}{\partial m_t}$ are generated by any form of taxation (also lump sum), such that this distortion is inevitable for any tax scheme. Substituting the Slutsky equation into equation 23, one obtains

$$\xi \sum_{k=1}^{K} (p_k - 1) \sum_{t \in T} \pi_t \sigma_{k,j}^t = -\xi \widetilde{c_j} + \xi \sum_{t \in T} \pi_t \sum_{k=1}^{K} (p_k - 1) \, c_j^t \frac{\partial c_k^t}{\partial m_t}$$
$$+ \sum_{t \in T} \pi_t \omega_t c_j^t + \sum_{t \in T} \pi_t \omega_t \sum_{k=2}^{K} \tilde{B}_{k,t} \frac{\partial c_k^t}{\partial p_j} \quad \forall j = 1, ..., K$$

or

$$\frac{\sum_{k=1}^{K} (p_k - 1) \sum_{t \in T} \pi_t \sigma_{k,j}^t}{\widetilde{c_j}} = -\left[1 - \sum_{t \in T} \pi_t \frac{\omega_t}{\xi} \frac{c_j^t}{\widetilde{c_j}} - \sum_{t \in T} \pi_t \sum_{k=1}^{K} (p_k - 1) \frac{c_j^t}{\widetilde{c_j}} \frac{\partial c_k^t}{\partial m_t}\right]$$
$$+ \sum_{t \in T} \frac{\pi_t}{\widetilde{c_j} \xi} \omega_t \sum_{k=2}^{K} \tilde{B}_{k,t} \frac{\partial c_k^t}{\partial p_j} \quad \forall j = 1, ..., K$$

Finally, defining

$$b_t = \frac{\omega_t}{\xi} + \sum_{k=1}^{K} (p_k - 1) \frac{\partial c_k^t}{\partial m_t}$$

as the net social marginal valuation of income for a type t consumer (i.e. net in the sense that it also counts the extra taxes the government receives when giving a type t consumer an extra unit of income), the main result of this section is stated in the following proposition.

Proposition 3 *If u satisfies conditions 1, 2 and 3,ii) and all consumers choose their unique D1 equilibrium consumption pattern, then an optimal system of indirect taxes should satisfy*

$$\forall j = 1, ..., K : \frac{\sum_{k=1}^{K} (p_k - 1) \sum_{t \in T} \pi_t \sigma_{k,j}^t}{\widetilde{c_j}} = -\left[1 - \sum_{t \in T} \pi_t \left(b_t \frac{c_j^t}{\widetilde{c_j}}\right)\right] + \Omega_j, \tag{24}$$

with

$$\Omega_j \equiv \frac{1}{\widetilde{c_j} \xi} \sum_{t \in T} \pi_t \omega_t \sum_{k=2}^{K} \tilde{B}_{k,t} \frac{\partial c_k^t}{\partial p_j}. \tag{25}$$

How should one understand the optimal indirect tax rule in equations 24 and 25? If $\forall t : \psi_t = 0$, such that all types would choose their intrinsic optimum and therefore $\forall j : \Omega_j = 0$, the tax rule in equation 24 reduces to the usual Ramsey rule in a many-person setting. The left hand side represents the proportional reduction in consumption of the j-th commodity along the compensated demand function due to taxes. In a single person setting without signaling, efficiency requires that this proportional reduction is the same for all commodities, as shown by Frank Ramsey's (1929) seminal paper. In a many-person setting, distributional concerns cause the policy maker to deviate from this equality of proportional reductions, as induced by the term between square brackets in equation 24. The term $\sum_{t\in T}\pi_t\left(b_t\frac{c_j^t}{\tilde{c}_j}\right)$ can be understood as the covariance between the net marginal social valuation of income and the relative consumption of a commodity j. If this covariance is positive, indicating that the commodity is consumed relatively more by consumers with a high b_t, then the many-person Ramsey rule prescribes that taxes should induce a relatively smaller proportional reduction in the consumption of commodity j. If the covariance is negative, such that the good is relatively more consumed by consumers with a lower net social marginal valuation of income, then optimal commodity taxes should generate a relatively larger proportional reduction of (compensated) demand for the good.

Signaling provides a second reason to deviate from Ramsey's equality of proportional reductions in compensated demand and this deviation is captured by the term Ω_j. First, consider the last part in equation 25, $\sum_{k=2}^{K}\tilde{B}_{k,t}\frac{\partial c_k^t}{\partial p_j}$. Remember that the $B_{k,t}$'s represent marginal intrinsic utility gain which a shift in consumption from good c_k to c_1 would imply for the type t consumers in equilibrium. The $\tilde{B}_{k,t}$ express this utility loss due to signaling in monetary terms. If an increase in p_j shifts the consumption pattern of a type t consumer more towards goods with a high $B_{k,t}$, then taxing good j implies a greater increase in $\sum_{k=2}^{K}\tilde{B}_{k,t}\frac{\partial c_k^t}{\partial p_j}$, the inefficiencies in monetary terms due to signaling in type t's consumption pattern. If taxing good j shifts consumption away from goods with a high $\tilde{B}_{k,t}$, this term can be negative for some types, indicating an efficiency gain from taxing good j for type t consumers. This monetary value of the change signaling inefficiencies due to a change in p_j is then multiplied with ω_t, the marginal social benefit of an additional unit of income for a type t consumer, to measure the change in social welfare due to a change in the signaling inefficiencies in the consumption pattern of a consumer of type t. This measure is then aggregated for all consumers (i.e. summed over all types weighted by their proportion in the population π_t). The factor $\frac{1}{\xi}$ translates this aggregate measure of the social value of changes in signaling efficiencies into monetary terms. Finally, the factor $\frac{1}{\tilde{c}_j}$ relates this effect inversely to the average consumption of good j. Remember that $\widetilde{c}_j$ constitutes the additional per capita tax revenue of a marginal increase in p_j. As such, Ω_j measures the change due to a marginal increase of the tax on good j in the average social value (in monetary terms) of signaling efficiencies per unit of tax revenue gained from taxing good j. If a marginal increase of p_j reduces the average social inefficiency from signaling, then the optimal tax scheme should allow for a greater proportional reduction of commodity j along the compensated demand curve (remember that the LHS in negative!). Similarly, if raising p_j would aggravate the average social inefficiencies due to signaling, such that Ω_j is high, then this calls for a smaller proportional reduction of commodity j along the compensated demand curve than the traditional many-person Ramsey rule without signaling would prescribe.

5 Conclusions

When consumers use their visible consumption bundle for intrinsic reasons as well as for costly signaling, their consumption choices are biased away from what they would choose in the absence of signaling motives. They buy relatively too many visible commodities and spend too little on invisible commodities, such that the intrinsic utility of a marginal cent invested in invisible consumption is strictly higher than that of a marginal cent invested in visible consumption. The welfare loss induced by signaling has been recognized since ages and has motivated Roman and medieval policy makers to forbid certain forms of conspicuous consumption by sumptuary laws. It is easily understood within the model above that imposing restrictions on the consumption of one or a few visible consumption goods cannot solve the problem and will merely lead to a shift of costly overconsumption towards other visible commodities. If the elimination of costly signaling is the goal of the policy maker, then altering the incentives to engage in signaling by targeting the information structure (e.g. revealing more information about the true qualities of people) or the rewards to relative superiority seems the surest way to proceed. Rather, this article has explored and advocated how costly signaling can also be an opportunity for policy makers. The endogeneity of the social meaning of visible consumption with respect to taxes implies that the equilibrium meaning of the pre-tax and post-tax equilibrium consumption bundles is identical. As a consequence, no utility which stems from the communicative function of visible consumption is affected by taxes, and the policy maker needs to focus only on the intrinsic utility of consumption. In this sense, the welfarist approach exposed above and a non-welfarist approach which takes only intrinsic utility from consumption as an argument in the social welfare function, result in identical optimal tax rules. In both cases, optimal indirect taxes exploit the differences in marginal intrinsic utility which marginal investments in different commodities generate. In the extreme case of pure costly signaling, this even results in burden-free tax revenue. Signaling thus provides an additional motive for differentiated indirect taxation. To the extent that the consumption of luxury goods is motivated more by communicative purposes, the above analysis presents an efficiency argument, next to equity, for taxing luxury goods relatively more. However, the same analysis also suggests that communication through consumption is unlikely to be restricted to the wealthiest consumers, and that efficiency gains can also made for lower income types. Which commodity groups are consumed most for signaling, and to what extent, is essentially an empirical question, and beyond the scope of this text.

Doesn't taxing one signaling commodity imply that it will be merely substituted by another and won't tax authorities be caught in a race they can never win? Within the model exposed in this text, such a signaling substitution is not an issue. Substitution effects are in this model entirely driven by the structure of intrinsic utility, which determines both the intrinsic utility of the commodity and the costs to imitators. These substitution effects are taken into account in the generalized Ramsey rule in equation 24. There is no other reason within the framework exposed here why taxes should shift choices towards untaxed commodities. Twenty euros spent on roses mean *ceteris paribus* the same before and after a tax on roses and in terms of communication it is of no importance whether one buys twenty or two roses for that amount. In general, requiring that one or more visible commodities are exempted from taxation should not pose too much of a problem within the current framework either. This merely implies a restriction on the policy makers problem, while consumer behavior is still described just as much by equations 15 to 17. There is no reason from a communicative point of view why consumers would prefer to shift to the untaxed visible goods.

If the tax authority cannot tax all specimens of a visible commodity and if spectators cannot distinguish taxed from untaxed specimens, then taxes may impair the informational value of a commodity. This can occur in the case of visible durables and other visible commodities of which consumers keep a stock, for commodities produced at home by some consumers or because of fraud. If spectators cannot distinguish between taxed and untaxed specimens, then the meaning of these commodities should be some convex combination (by Bayesian updating) of the pre-tax and perfectly taxed post tax meaning. In such a case, consumers will want to substitute this commodity with other, more taxable, commodities.

Finally, three remarks about the limitations of the analysis exposed in this article seem in place. First, the signaling game in this text has considered consumption patterns as multidimensional signals about a single dimension of heterogeneity. Clearly, real world consumption communicates about many different qualities of its consumers. Although we believe that the key intuitions of this text remain valid, the introduction of multiple dimensions of heterogeneity into the signaling model will certainly complicate a.o. the analysis of the substitution effects due to taxation. Second, the partial equilibrium analysis of the optimal taxation problem has neglected the effects of taxation on the production side of the market. In a general equilibrium setting, one expects the tax rule suggested above to generate efficiency gains by shifting production towards commodities which generate more intrinsic utility, and away from commodities which are mostly consumed for communication purposes. Finally, our analysis has not considered the possibility of nonlinear direct taxation in the above setting. Atkinson and Stiglitz (1976) show that if the policy maker can choose nonlinear direct tax schemes and if the consumers' utility function is separable between labor and all commodities, no indirect taxed need be employed. The present analysis suggests an additional argument in favor of the use of indirect taxes, next to arguments based on e.g. Pigovian taxes, merit goods or the prevention of tax fraud. If tax revenue can be raised without burden, then an optimal tax scheme should first exploit this potential before relying on direct taxes. This line of reasoning is reserved for future research.

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A Appendix: Proofs

A.1 Proof of theorem 1

A large part of this proof is in essence an adaptation of the proofs of lemma 4.1 and propositions 4.1-4.5 in Cho and Sobel (1990) to the present setting.

1. **The problem in equations 3 to 5 is well defined** for this framework.
   The set of $\bar{c}$ satisfying inequality 4 is compact and nonempty since $m_t > m_{t-1}$ for all $t > 1$ (hence, any type $t > 1$ can always separate from lower types), and u is continuous. If for any visible consumption pattern $\bar{c}$, $\bar{p}\bar{c} > m_t$, then obviously $\beta(t|\bar{c}) = 0$.

2. **If type t' chooses $\bar{c}'$ in the S.E., then no type $t'' < t'$ chooses a bundle $\bar{c}'' > \bar{c}'$ with positive probability.** Let $t'' < t'$ and assume that $\bar{c}'' > \bar{c}'$ are both feasible for both types and denote

   $$z(\bar{c}, t, \hat{m}) \equiv u\left(p_1^{-1}(m_t - \bar{p}\bar{c}), \bar{c}, \hat{m}\right).$$

   Consider a S.E. in which type t' chooses $\bar{c}'$ to get $\hat{m}'$ with a positive probability. Let $\hat{m}''$ be the value for $\hat{m}$ making types t' indifferent between $\bar{c}'$ and $\bar{c}''$. The single crossing condition implies that if type t' is indifferent between bundles $(\bar{c}', \hat{m}')$ and $(\bar{c}'', \hat{m}'')$, then type t'' strictly prefers $(\bar{c}', \hat{m}')$ to $(\bar{c}'', \hat{m}'')$. Since the indifference curves of the t'' type are steeper for each dimension of visible consumption k in the $(c_k, \hat{m})$ plane, t'' needs to be compensated with more $\hat{m}$ for the increase from $\bar{c}'$ to $\bar{c}''$ than t'. Suppose that

   $$M^+(\bar{c}'', t') \cup M^o(\bar{c}'', t') \subseteq M^+(\bar{c}'', t'') \cup M^o(\bar{c}'', t''),$$

   then this implies that type t'' would strictly prefer bundle $(\bar{c}', \hat{m}')$ to her equilibrium bundle, a contradiction. Hence, if the S.E. survives the D1 criterion, it must be that $\beta(t''|\bar{c}'') = 0$ and no t'' chooses a signal $\bar{c}''$.

3. What if $\bar{c}''$ and $\bar{c}'$ cannot be vector ordered? Then construct a $\bar{c}^{\diamondsuit}$ such that type t' is indifferent between the bundles $(\bar{c}', \hat{m}')$ and $(\bar{c}', \hat{m}')$, and (choose $\bar{c}''$ and $\bar{c}'$ such that) $\bar{c}'' > \bar{c}^{\diamondsuit}$. Then proceed as above to find that for any $\bar{c}''$ above the iso-utility surface of a type t' consumer, for any $t'' < t'$ it is true that $\beta(t''|\bar{c}'') = 0$.

4. **There can be no pooling above** 0. Assume otherwise that different types pool at $\vec{c}'$, to get $\hat{m}(\vec{c}')$ and that $t' > t''$ are the highest types in the pool. Then $m_t > \hat{m}(\vec{c}')$ and for any $\vec{c}'' > \vec{c}'$ by point 2, $\beta(t''|\vec{c}') = 0$. Choose $\vec{c}''$ such that type t'' would be indifferent between the bundles $(\vec{c}', \hat{m}(\vec{c}'))$ and $(\vec{c}'', m_t)$. Then by condition 2, type t' would strictly prefer $(\vec{c}'', m_t)$ to $(\vec{c}', \hat{m}(\vec{c}'))$, but then t' cannot choose $\vec{c}'$ in equilibrium.

5. **There is no pooling at** 0. Let $\bar{m}(t) \equiv \frac{1}{\sum_{t'=t}^{T} \pi_{t'}} \sum_{t'=t}^{T} \pi_{t'} m_{t'}$ be average income of the types weakly smaller than t. Type t separates from the lower types iff for at least one $k \in \{2, ..., K\}$, it is true that

$$\left[-\frac{p_k}{p_1} u_1' \left(\frac{m_t}{p_1}, 0, \bar{m}(t) \right) + u_k' \left(\frac{m_t}{p_1}, 0, \bar{m}(t) \right) \right] dc_k + \left[u \left(\frac{m_t}{p_1}, 0, m_t \right) - u \left(\frac{m_t}{p_1}, 0, \bar{m}(t) \right) \right] > 0.$$

Hence, marginal utility of the invisible consumption should not be so high that it inhibits even the smallest investment in a visible commodity to distinguish oneself from lower types and gain by a discrete jump in $\hat{m}$. Condition 1, ii) requires that $u_1'(.)$ is bounded and $u_{K+1}'(.)$ is bounded away, such that a $dc_k > 0$ can always be found for which the above inequality applies.

6. **All D1 S.E. generate the Riley outcome.** If u^t are the utility levels obtained in the Riley outcome, then in any S.E. that is D1, type t gets utility u^t. Imagine a S.E. in which type t gets a different payoff u_o. As type t separates in all S.E. under the above conditions, and is thus recognized as a type t, it must be that $u_o \leq u^t$, since the latter is a maximum. At the other hand, it must be that $u_o \geq u^t$. Imagine otherwise, i.e. that $u_o < u^t$. Then a visible consumption pattern $\bar{c}$ exists such that types $t' < t$ can not benefit from imitation and $u\left(\frac{m_t - \bar{p}\bar{c}}{p_1}, \bar{c}, m_t\right) > u_o$. Type t' does not wish to imitate since then $\hat{m}(\bar{c}) < m_t$, such that $\beta(\bar{c}|t') = 0$, but this contradicts individual rationality.

7. **Existence**. An equilibrium exists in which consumers play strategies with support on solutions to the Riley problem $\vec{c}^t$, equilibrium strategies are interpreted $\beta_t(\vec{c}^t) = 1$ (or $\hat{m}(\vec{c}^t) = m_t$). For each out-of-equilibrium consumption pattern $\vec{c}^\diamond$, define for each type $\widetilde{m}_t(\vec{c}^\diamond)$ by

$$z\left(\vec{c}^\diamond, t, \widetilde{m}_t\left(\vec{c}^\diamond\right)\right) = u^t$$

and

$$\widetilde{t}\left(\bar{c}^{\lozenge}\right) \equiv \min_{T}\left\{t|\widetilde{m}_t\left(\bar{c}^{\lozenge}\right) = \min_{t'\in T}\widetilde{m}_{t'}\left(\bar{c}^{\lozenge}\right)\right\}.$$

Then set the beliefs at

$$\begin{cases} \beta_t\left(\bar{c}^{\lozenge}\right) = 1 \Leftrightarrow t = \widetilde{t}\left(\bar{c}^{\lozenge}\right) \\ \quad = 0 \text{ otherwise.} \end{cases}$$

Hence, D1 dictates to concentrate all mass of the out-of-equilibrium beliefs on the consumer most willing to consume out-of-equilibrium pattern $\bar{c}^{\lozenge}$, i.e. who has the lowest $\widetilde{m}\left(\bar{c}^{\lozenge}\right)$. Note that if $\bar{c}^{\lozenge} > \bar{c}^{t-1}$ and $\bar{c}^{\lozenge} < \bar{c}^{t}$, then single crossing implies that $\beta_{t-1}\left(\bar{c}^{\lozenge}\right) = 1$.
These beliefs are D1 by construction. Strategies are optimal by construction. No equilibrium consumption pattern of another type is an improvement: no visible consumption pattern of a higher type can be an improvement by construction. No visible consumption pattern of a lower type can be an improvement: if the Riley constraint is binding, then $z\left(\bar{c}^{t-1}, t-1, m_{t-1}\right) = z\left(\bar{c}^{t}, t-1, m_t\right)$ and the single crossing condition means that

$$z\left(\bar{c}^{t-1}, t, m_{t-1}\right) < z\left(\bar{c}^{t}, t, m_t\right).$$

If the constraint is not binding, then

$$z\left(\bar{c}^{t}, t, m_t\right) \geq z\left(\bar{c}^{t-1}, t, m_t\right) > z\left(\bar{c}^{t-1}, t, m_{t-1}\right),$$

where the first inequality obtains because type t is maximizing utility without constraints, and second by monotonicity of utility in $\hat{m}$. No other out-of-equilibrium pattern can be strictly better than an equilibrium consumption pattern: consumers get the same $\hat{m}$ for a bundle that is equivalent or strictly higher (and more costly) than (a bundle equivalent to) the equilibrium bundle.

A.2 Proof of proposition 1

1. **Uniqueness for condition i).** If condition $i)$ applies, then $\bar{c} > \bar{c}^{t-1}$ and $u\left(\frac{1}{p_1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right) \leq u^{t-1}$ defines a convex set on which the continuous and strictly concave function u is maximized.

2. **Uniqueness for condition ii).** If $K = 3$, then define

$$L_t^o \equiv \left\{\bar{c}|u\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right) = u^{t-1} \text{ and } c \geq 0\right\}$$

and

$$L_t \equiv \left\{\bar{c}|\exists \bar{c}' \in L_t^o : \bar{c} \geq \bar{c}' \text{ and } pc \leq m_t \text{ and } c \geq 0\right\}.$$

We must show that $u\left(p_1^{-1}\left(m_t-\bar{p}\bar{c}\right),\bar{c},m_t\right)$ has a unique maximum on L_t. Note that this maximum then lies either in $L_t \backslash L_t^o$ or on L_t^o. If it lies in $L_t \backslash L_t^o$, it is the global maximum for consumer t, and is unique because of the strict concavity of u. In this case, the Riley constraint is not binding for type t, and type t chooses her intrinsic optimum characterized by $\frac{u'_k\left(p_1^{-1}(m_t-\bar{p}\bar{c}),\bar{c},m_t\right)}{u'_{k'}\left(p_1^{-1}(m_t-\bar{p}\bar{c}),\bar{c},m_t\right)}=\frac{p_k}{p_{k'}}$.
If the Riley constraint in inequality 4 is binding for a type t consumer, then her maximum lies on L_t^o, and we show that $u\left(p_1^{-1}\left(m_t-\bar{p}\bar{c}\right),\bar{c},m_t\right)$ has a unique maximum on L_t^o under the conditions 1, 2 and 3,ii).

3. **Uniqueness on L_t^o.** The indifference curve L_t^o is characterized by

$$MRS^{t-1}\left(\bar{c},L_t^o\right)=\left.\frac{\partial c_2}{\partial c_3}\right|_{L_t^o}^{t-1}$$
$$=-\frac{p_1u'_3\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}\right),\bar{c},m_t\right)-p_3u'_1\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}\right),\bar{c},m_t\right)}{p_1u'_2\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}\right),\bar{c},m_t\right)-p_2u'_1\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}\right),\bar{c},m_t\right)}.$$

Define in the same fashion

$$MRS^{t}\left(\bar{c},L_t^o\right)=-\frac{p_1u'_3\left(p_1^{-1}\left(m_{t}-\bar{p}\bar{c}\right),\bar{c},m_t\right)-p_3u'_1\left(p_1^{-1}\left(m_{t}-\bar{p}\bar{c}\right),\bar{c},m_t\right)}{p_1u'_2\left(p_1^{-1}\left(m_{t}-\bar{p}\bar{c}\right),\bar{c},m_t\right)-p_2u'_1\left(p_1^{-1}\left(m_{t}-\bar{p}\bar{c}\right),\bar{c},m_t\right)}.$$

Note that at any maximum on L_t^o, $MRS^t\left(\bar{c},L_t^o\right)<0$ and

$$p_1u'_3\left(p_1^{-1}\left(m_t-\bar{p}\bar{c}\right),\bar{c},m_t\right)<p_3u'_1\left(p_1^{-1}\left(m_t-\bar{p}\bar{c}\right),\bar{c},m_t\right),$$

i.e. both numerator and denominator of $MRS^t\left(\bar{c},L_t^o\right)$ are negative. Assume otherwise, such that e.g.

$$p_1u'_3\left(p_1^{-1}\left(m_t-\bar{p}\bar{c}\right),\bar{c},m_t\right)>p_3u'_1\left(p_1^{-1}\left(m_t-\bar{p}\bar{c}\right),\bar{c},m_t\right),$$

then consumer type t can improve herself by shifting consumption from c_1 to c_3, thus moving strictly above L_t^o, and this point cannot be a maximum.
If at a

$$\bar{c}\in L_t^o: MRS^{t-1}\left(\bar{c},L_t^o\right)<MRS^{t}\left(\bar{c},L_t^o\right)<0,$$

then type t can improve herself by moving on L_t^o towards more c_3, since this allows a greater reduction in c_2 than necessary to compensate the increase in c_3 and

$$p_1u'_2\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}\right),\bar{c},m_t\right)<p_2u'_1\left(p_1^{-1}\left(m_{t-1}-\bar{p}\bar{c}\right),\bar{c},m_t\right).$$

If, on the other hand, at a

$$\bar{c} \in L_t^o : MRS^t\left(\bar{c}, L_t^o\right) < MRS^{t-1}\left(\bar{c}, L_t^o\right) < 0,$$

then type t can improve herself by moving on L_t^o towards less c_3, as this reduction in c_3 is then compensated by a smaller increase in c_2 than needed to maintain the same utility level. A sufficient condition for the uniqueness of a maximum on L_t^o requires that a unique c_3^* exists such that for $\bar{c} \in L_t^o$ and $c_3 < c_3^*$ means that $MRS^{t-1}\left(\bar{c}, L_t^o\right) < MRS^t\left(\bar{c}, L_t^o\right) < 0$ and for $\bar{c} \in L_t^o$ and $c_3 > c_3^*$ rather $MRS^t\left(\bar{c}, L_t^o\right) < MRS^{t-1}\left(\bar{c}, L_t^o\right) < 0$. Since u is C^2 by condition 1, i), at this maximum it must be that $MRS^t\left(\bar{c}, L_t^o\right) = MRS^{t-1}\left(\bar{c}, L_t^o\right)$.

4. **If u is strongly separable in all dimensions of** c, then

$$\begin{aligned} & MRS^t\left(\bar{c}, L_t^o\right) - MRS^{t-1}\left(\bar{c}, L_t^o\right) \\ = & \frac{\left(u_1'\left(m_t\right) - u_1'\left(m_t\right)\right)}{\left(p_1 u_2' - p_2 u_1'\left(m_t\right)\right)\left(p_1 u_2' - p_2 u_1'\left(m_{t-1}\right)\right)}\left(p_2 u_3' - p_3 u_2'\right), \end{aligned}$$

with

$$\begin{aligned} u_1'\left(m_t\right) &= u_1'\left(p_1^{-1}\left(m_t - \bar{p}\bar{c}\right), \bar{c}, m_t\right), \\ u_1'\left(m_{t-1}\right) &= u_1'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right), \\ u_2' &= u_2'\left(p_1^{-1}\left(m_t - \bar{p}\bar{c}\right), \bar{c}, m_t\right) = u_2'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right) \text{ and} \\ u_3' &= u_3'\left(p_1^{-1}\left(m_t - \bar{p}\bar{c}\right), \bar{c}, m_t\right) = u_3'\left(p_1^{-1}\left(m_{t-1} - \bar{p}\bar{c}\right), \bar{c}, m_t\right). \end{aligned}$$

It is easily seen that this is zero only at $p_2 u_3' = p_3 u_2'$, i.e. where $\frac{p_2}{p_3} = \frac{u_2'}{u_3'}$, which also characterizes the intrinsic optimum of a (possibly fictitious) higher income type.

5. **If u is as in condition ii)** of condition 3, this ensures that $MRS^t\left(\bar{c}, L_t^o\right) - MRS^{t-1}\left(\bar{c}, L_t^o\right)$ is strictly decreasing if we move along L_t^o towards a higher c_3, guaranteeing a unique point $\bar{c}$ where $MRS^t\left(\bar{c}, L_t^o\right) = MRS^{t-1}\left(\bar{c}, L_t^o\right)$, as required for a unique maximum on L_t^o. Alternatively, it ensures that the indifference curves of type t are more concave (less convex) functions of c_3 than those of type $t-1$ everywhere, thus ensuring a unique tangency point where $MRS^t\left(\bar{c}, L_t^o\right) = MRS^{t-1}\left(\bar{c}, L_t^o\right)$, as illustrated in figure 1.

A.3 Proof of proposition 2

The optimal tuple of indirect taxes solves the problem:

$$\max_p \mathcal{L} = W\left(v^{CS}(p)\right) - \lambda\left[N \sum_{t \in T} \pi_t \left(\begin{array}{c} \frac{p_1 - 1}{p_1}\left(m_t - e(t, p_1)\right) + \\ \frac{p_2 - 1}{p_2} e(t, p_1) \end{array}\right) - \bar{R}\right],$$

in which λ represents the Lagrange multiplier. The first order condition for p_1 is

$$N\sum_{t\in T}\pi_t\frac{\partial W\left(.\right)}{\partial v^{CS}}\frac{\partial v^{CS}\left(.\right)}{\partial p_1}- \qquad (26)$$

$$\lambda N\sum_{t\in T}\pi_t\left[\frac{c_1^t}{p_1}+\left(\frac{p_2-1}{p_2}-\frac{p_1-1}{p_1}\right)\frac{\partial e\left(t,p_1\right)}{\partial p_1}\right]=0$$

and the first order condition for p_2 is

$$-\lambda N\sum_{t\in T}\pi_t\frac{e\left(t,p_1\right)}{\left(p_2\right)^2}=0. \qquad (27)$$

The government's revenue constraint remains

$$N\sum_{t\in T}\pi_t\left(\frac{p_1-1}{p_1}\left(m_t-e(t,p_1)\right)+\frac{p_2-1}{p_2}e(t,p_1)\right)=\bar{R}. \qquad (28)$$

From equation 26, one sees that $\lambda\neq 0$ as long as p_1 affects social welfare, i.e. as long as $\sum\limits_{t\in T}N\pi_t\frac{\partial W(.)}{\partial V^{CS}}\frac{\partial V^{CS}(.)}{\partial p_1}\neq 0$, which is generically the case. Because in equilibrium $e\left(t,p_1\right)>0$ for all but the lowest type, equation 27 can only be satisfied if

$$p_2\rightarrow+\infty,$$

which implies that $\bar{c}^t\rightarrow 0$. Since taxing the visible good does not, in equilibrium, affect utility, this potential is maximally exploited and the income wasted on costly signaling is recuperated as much as possible. The overall revenues from taxing visible consumption can be written

$$N\frac{p_2-1}{p_2}\sum_{t\in T}\pi_t e(t,p_1)=N\sum_{t\in T}\pi_t\left(e(t,p_1)-\bar{c}^t\right), \qquad (29)$$

in which the last term on the right hand side, $\bar{c}^t$, represents the part of expenditures which the producers of visible goods still collect. Substituting equation 29 in equation 28 and solving for p_1, one obtains

$$\frac{p_1-1}{p_1}=\frac{\frac{\bar{R}}{N}-\frac{p_2-1}{p_2}\sum\limits_{t\in T}\pi_t e(t,p_1)}{\sum\limits_{t\in T}\pi_t\left(m_t-e(t,p_1)\right)}$$

$$p_1=\frac{\sum\limits_{t\in T}\pi_t\left(m_t-e(t,p_1)\right)}{\sum\limits_{t\in T}\pi_t\left(m_t-c_t\right)-\frac{\bar{R}}{N}}.$$