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Abstract

Many transport and other service problems come down to simple network choices: what mode and/or route to take, when some of the routes and modes are congested and their use can be priced or not priced by different operators. The operators can have different objectives and face different market environments: public or private monopoly, private duopoly, etc.. This standard problem has been studied in many variants, mostly using the assumption of perfect substitutability between alternatives, so that in the deterministic Wardrop equilibrium, all routes that are used have the same generalized cost. This paper examines in more detail the role of the substitutability assumption using varying degrees of unobserved individual heterogeneity. Users of a network consume transport services, which are differentiated in two ways. There are objective differences in quality (length of route, congestion level) perceived in the same way by all users but there are also individual idiosyncratic preferences or unobserved heterogeneity (e.g. in modal choice) for transport services. The resulting stochastic equilibrium is analysed on a simple parallel network for four types of ownership regimes: private ownership, coordinated public ownership, mixed public-private and public Stackelberg leadership. First we synthesize the literature and prove rigorously that when total demand is fixed and there is congestion, then by controlling one route a government can achieve the First Best allocation , irrespective of whether the second route is privately operated or unpriced. This result holds whatever the level of substitutability and whatever the levels of congestion on the two routes. Secondly, we rank ownership regimes when the government cannot control the pricing of any route. If there is no congestion, no pricing is obviously best and second best is to have only the inferior route privately priced. If there is congestion and the heterogeneity in the preferences is limited, it is still best to have the more congestible route privately tolled.

Keywords: network equilibrium, imperfect competition, imperfect substitutability, second best pricing , modal choice, transport policy

JEL : R48,R41,L11,L12

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1. INTRODUCTION

Users of transport services choose between routes or modes on the basis of objective characteristics and on the basis of subjective preferences. Objective characteristics are common to all users and consist of the quality of the transport link (length), the level of congestion and the access charge. The subjective preferences are individual specific. Both elements are important for the optimal pricing of the transport alternatives but most of the transport pricing literature has neglected the individual heterogeneity. The main ambition of this paper is to bridge this gap and analyse how the degree of heterogeneity in preferences affects the optimal pricing results.

How to organize the pricing of the different routes and modes such as to maximize welfare has been the subject of many contributions in the literature. Optimal pricing requires that the different routes on the network are all priced at their marginal social cost, as this guarantees an efficient allocation of users over the different mode and route alternatives. However often not every alternative can be priced and some links are priced by private monopolies pursuing different objectives. This raises interesting second best pricing issues: is it better for the transport regulator to allow pricing of some or all alternatives by private firms rather than to have no pricing at all? This is the problem of the pay lane. And what is the best pricing strategy when he can price part of the network but the rest is controlled by a private monopolist? For example, given the choice between congested private transport, such as roads or airlines, and uncongested public transport, such as rail, how should government price rail?

Most of the literature has dealt with this question implicitly using the assumption of perfect substitution between alternatives. Alternatives can differ in levels of congestion, comfort and operator type, so that there is imperfect competition, but whenever all users have the same preferences for these characteristics, the user equilibrium is deterministic and can rely on the Wardrop principle. Many papers concentrate on the case of a parallel network. de Palma and Leruth (1989) study a duopoly that first chooses capacities on their link and then set prices. Both Verhoef, Nijkamp, Rietveld (1996) and de Palma and Lindsey (2000) analyze imperfect competition in a deterministic setting between two competing parallel roads with congestion, assuming different ownership structures. Using social surplus to measure allocative efficiency, they find that a private duopoly can actually be more efficient than a mixed public-private one. De Borger, Proost and Van Dender (2005) examine toll competition between governments on a congested parallel network. Price competition in a duopoly where firms offer perfect substitutes but have congested access is also studied by Van Dender (2005). Scotchmer (1985) looks at price competition between congestible facilities in a symmetric setting when total demand is fixed. Other authors also study price competition between different operators on congested serial networks (De Borger, Dunkerley and Proost(2008), Mun and Ahn (2008)).

A second type of approach considers differences in preferences between users but the preferences are known and non stochastic. Small and Yan (2001) consider mixed ownership regimes on a congested parallel network, where consumer heterogeneity is accounted for by different values of time. They find that, when one route is untolled, accounting for consumer heterogeneity improves the performance of second best pricing on the other route, and that in some cases, no tolling at all may be preferable to a profit maximising toll on one route. Verhoef and Small (2004) extend this approach to a linked parallel and serial network. Liu et al (2009) consider a congested highway competing with an uncongested public transit system, In their model consumer heterogeneity is again introduced through a continuous value of time (VOT) distribution but total demand is fixed. They develop revenue neutral pricing schemes for the complete network, which result in positive tolls on the highway and negative fares for public transport.

When we drop the assumption of perfect substitutability between alternatives we need to rely on models that have idiosyncratic preferences, where the preference for a route or link is individual specific. This modelling of preferences has been the basis of the discrete choice approach in transport demand analysis. The approach to modelling imperfect competition with idiosyncratic preferences is surveyed in Anderson, de Palma and Thisse (1992). de Palma and Proost (2006) use this method to study competition between symmetric variants. Introducing imperfect substitutability between alternatives via idiosyncratic differences in individual preferences generates different results for two reasons. Firstly, offering more options generates welfare and, secondly, each variant has some intrinsic market power. Dunkerley, de Palma and Proost (2009) apply this type of model to study an asymmetric duopoly of airlines. In their study, private firms compete for customers and access to the firms can be congested but this cannot be controlled by the firms. Any policies aimed at reducing congestion, such as tolling or capacity expansion can only be imposed externally. Melo (2014) studies the entry and pricing decisions of different congestible service providers to the same destination. His paper concentrates on the private provision solution only but uses a more general demand formulation.

In this paper we focus on the impact of imperfect substitution on pricing and demand allocation on a congested parallel transport network using a discrete choice approach. Capacity investment decisions are not modelled, although the role of the closely related congestion parameter is analysed. Given a single government, which can potentially make policy interventions on different links, we examine the policy implications of publicly and privately operated and untolled alternatives. The results of our stochastic model are compared to the corresponding deterministic results; these differ from those of de Palma and Lindsey (2000) due to the assumption of fixed total demand.

On our duopoly transport network, we have two types of results. First when the government is able to control pricing on one route, then it is always possible to achieve the First Best demand allocation. This result holds whatever the degree of substitutability of the two alternatives and whatever the degree of congestability. This result is not new but we synthesize and generalize the literature and offer a rigorous proof.

The second result concerns the case where the government can control whether the operator of a route can implement a toll or not, but not the level of the toll. Here we offer a ranking of the welfare of alternatives where only one of the two routes can be given to a private monopolist. First, whenever there is no congestion, it is better to have no private pricing at all. If one route needs to be privatised, it is better to privatize the route with inferior quality. Adding congestion, changes the trade-off faced by the user when deciding which route to take. When all users value the routes in the same way (no stochastic preferences) , the superior quality route should be privatised as long as there is sufficient capacity on the lower quality route. In this case, while some users will be forced to take the inferior route, the remaining users of the best route will have improved travel times and the toll revenues will also have increased welfare. However, if the inferior route is also prone to congestion, the overall positive welfare contribution may no longer hold and it is better to leave both routes untolled. In the stochastic setting, a more complex relationship between congestibility on the two routes and the strength of consumer preferences determines whether it is welfare enhancing to privatise one of the routes. In general, the arguments from the deterministic case for privatising the best route, if it is the more congestible route, continue to hold, but with the caveat that consumer preferences for diversity should not be too large

We proceed in this paper as follows. We first discuss the model set up paying particular attention to the general setting and the duopoly results in the presence of idiosyncratic preferences. In section 3 we analyze in detail the case of two parallel routes and where the government can control the pricing on at least one route. In section 4 we dig deeper into the role of substitutability and congestion and rank the different ownership regimes. In this section we also offer second best results for the case where the government itself cannot price any of the roads. In section 5 we conclude.

2. MODEL SETTING

The starting point for this model is the allocation of users over a simple parallel network with two competing routes from origin O to destination D as shown in Figure 1.

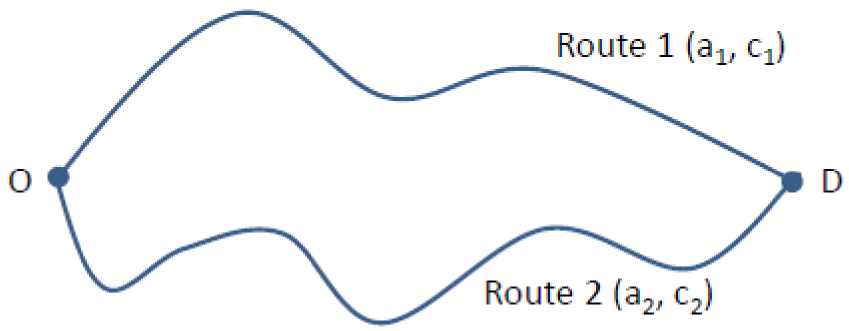

Figure 1 Generalised parallel network for 2 routes

Users travel from O to D and consume transport services to do this. These transport services are differentiated in that they offer a different level of quality, as perceived by all users, and users also have their own individual preferences for transport services. As discussed below, all users are constrained to select one route, although in principle this restriction can be overcome by including an outside option of not travelling. Each route linking O and D has marginal and fixed costs associated with its operation and may be controlled by different operators: private or public or be untolled.

For transport networks, the obvious interpretation is to consider alternative routes or modes for the same origin – destination but this need not be so. Each of the alternatives can be a different destination, the alternatives that differ not only in terms of the quality of transport but also in the quality of the destination. What we need, however, is that each individual chooses only one alternative and that each alternative has only one operator setting the price. If we have a transport operator setting an access price to the transport network and another firm setting access prices to the destination we have a serial network structure instead of a parallel structure.

2.1. Users of transport services

The focus of this paper is the role of substitutability in the allocation of users over the simple network when there are qualitative differences between the two routes and at least one route is congested. The underlying economic framework for the model we use is based on the structure proposed by de

Palma and Proost (2006)[2]. For our purposes, it is sufficient to concentrate on the utility a user derives from taking a particular route and his subsequent route choice.

Starting from a fixed, total population of N individuals, the indirect utility function conditional on the use of route *i* (*i*=1,2) by one individual is given by

$$V_i = \Gamma + a_i - \alpha_i n_i - p_i + \mu\varepsilon_i \tag{1}$$

where the first term (*Γ*) reflects his total income. The utility he obtains from transport alternative *i* is represented by a constant term a_i, which can be regarded as a quality parameter and is the same for all users. It is associated with the transport route itself or with the final destination, if this is different for each alternative. Congestion time losses are assumed to be a linear function of total consumption (n_i) of that transport alternative, so that the total transport time needed for one trip equals $\alpha_i n_i$, where α_i is again an objective parameter, representing the congestibility of the route[3]. By definition, the congestion parameter cannot be negative ($\alpha_i \geq 0$), and is zero only when a route is uncongested. A price p_i is also charged by the operator for use of route *i*. We assume that this will be equal to the marginal operating cost (c_i) associated with the route if there is no toll. Hence, the toll can always be interpreted as the mark-up over marginal operating cost and users of an untolled route are in fact subject to a minimum charge of c_i. This approach merely serves to simplify the following analysis and does not affect the results. Finally, the user's idiosyncratic preferences for transport services are denoted by the random term $\mu\varepsilon_i$, where μ is a strictly, positive scale parameter, which reflects the strength of consumers preferences for variety. The probability that an individual chooses to take route *i* is given by $\mathrm{P}_i = \mathrm{Prob}(V_i \geq V_j, \forall j)$, where each individual is constrained to travel unless a non-participation utility level is specified[4]. Then, assuming the ε_i are i.i.d. Gumbel distributed, this probability can be expressed as

$$\mathrm{P}_i = \frac{\exp\left(-\dfrac{p_i + \alpha_i n_i - a_i}{\mu}\right)}{\sum_{j=1,2} \exp\left(-\dfrac{p_j + \alpha_j n_j - a_j}{\mu}\right)} \in (0,1) \quad i = 1,2 \tag{2}$$

For our duopoly network, with fixed total demand, *N*, the demands for routes 1 and 2 can be simply expressed as $n_1 = N\mathrm{P}_1$ and $n_2 = N\left(1 - \mathrm{P}_1\right)$. When there is congestion on at least one of these routes, equation (2) becomes an implicit expression for the probability that a user selects a given route (and hence the demand for that route). Indeed, we mainly use the probability P_1 in our analysis and may refer to this as the demand for route 1. Equation (2) can then usefully be rewritten as

$$\mathrm{P}_1(\Delta p \equiv p_1 - p_2; \xi) = \left[1 + \exp\Psi\right]^{-1} \tag{3}$$

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[2] The reader is referred to this paper for a full description of the general equilibrium framework that is behind this model.

[3] This parameter can be regarded as inversely related to capacity.

[4] Including an outside option complicates the mathematics without really adding further insights. We therefore only present results for full participation.

where $\xi = \{\mu, a_1, a_2, \alpha_1, \alpha_2, c_1, c_2\}$ is the model parameter set, which is independent of the type of network operator, and $\Psi(\mathrm{P}_1, p_1, p_2; \xi) \equiv \left[p_1 - p_2 + N\alpha_1 \mathrm{P}_1 - N(1-\mathrm{P}_1)\alpha_2 + a_2 - a_1 \right] \mu^{-1}$.Therefore the demand exists and is always a function of the differences in price (including marginal cost), congestion and quality between the two routes. This is the utility difference between the two alternatives. In Appendix A it is shown that, for given prices, there exists a unique user equilibrium.

The indirect utility function (1) is only defined when there is a feasible solution to the consumer's optimisation problem. More specifically, we assume that there is at least one alternative *i* for which the generalised cost $p_i + \alpha_i N \mathrm{P}_i$ is smaller than the total income Γ.

The CES provides an alternative to the Logit formulation used here as the substitutability between alternatives can also be varied easily. But, as discussed in Anderson et al. (1992), the CES can be derived as a discrete choice model, with (adapting their original setting to add quality): $V_i = \Gamma - \alpha n_i - ln\left(p_i\right) + \mu \varepsilon_i$, which leads to a CES demand system, with endogenous quality. First, note that the CES suffers from the same IIA (Independence of Irrelevant Alternatives) restriction as the Logit (that is, the ratio of two demands is independent of the demand for other alternatives). Second, while the total demand is constant for the Logit, it is the total budget share that is constant for the CES, an equally debatable constraint. Third, using Roy's identity, we get for the CES, the individual conditional demand Y/p_i. This expression can be interpreted as the proportion of times (per month, for example), route *i* is used (up to a multiplicative factor). The inclusion of an outside alternative in the Logit, requires to add an alternative O is added, such that: $V_0 = \Gamma + a_0 + \mu e_0$, where a_0 denotes the quality of the outside alternative. The Logit approach seems to be superior (with respect to the CES), since it is more structural and flexible.

Independent of the type of operators of the routes and their price setting behaviour, there are three facets of our network that will be important in the analysis of demand allocation. First, there are cost and quality parameters from which transport users derive utility and the values of these parameters[5] may vary across the network. Second, alternatives may differ in their congestibility. Third, consumers may have idiosyncratic preferences for transport services. We make a number of assumptions in order to study these aspects of the model in more detail. Specifically, we denote route 1 as being intrinsically better than route 2, when the difference between the non- stochastic quality component of utility from transport services on the two routes is greater than the difference in costs[6] ($\Delta a - \Delta c \equiv \left(a_1 - a_2\right) - \left(c_1 - c_2\right) > 0$). Next, we assume that at least one of the routes is congested, so that $\alpha_1 + \alpha_2 > 0$. Finally, allowing consumers to have idiosyncratic preferences for transport services (μ>0), means that the degree of substitutability between routes may vary with consequently imperfect substitution between routes. This is the basis of the model presented above. When there is perfect substitutability, we rely on the standard deterministic approach, subject to the constraint of fixed total demand.

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[5] The parameters are symmetric as we use the same parameter set to describe each route but these are asymmetric in the values they can take.

[6] It is assumed throughout that the marginal and fixed costs are specified for each route and are therefore the same whatever the status of the operator (private or public) of a given route.

In addition, the results for a network without congestion clearly represent a limiting case for our set-up and we include and discuss these results throughout the following sections. A summary table of the results for the cases with congestion can be found in Appendix B.

Having established the route characteristics and consumer behaviour, we next turn our attention to the operators of the network.

2.2. Transport service providers

Transport services on the simple network described above can be provided by different types of operators, with different objectives. They can be tolled by the government (PUB), a private operator (PRIV) or be effectively untolled (FREE)[7]. From a road privatisation and tolling policy perspective, it is therefore instructive to analyse the effect of the different operators on pricing, transport volumes and welfare, when there is congestion and imperfect substitution on the network. The possible combinations to be considered on the network are presented in

Table 1.

Description	**Operator route 1**	**Operator route 2**	**Section**
First Best	PUB	PUB	3.1
Priced at marginal operator cost	FREE (no operator)	FREE (no operator)	3.2
Monopoly	PRIV	FREE (no operator)	3.3
Duopoly	PRIV	PRIV	3.4
Second Best	FREE (no operator)	PUB	3.5
Mixed Nash	PUB	PRIV	3.5

Table 1 Duopoly regimes

In all of the above regimes, prices are set simultaneously. Although, in theory, we could consider two Stackelberg scenarios as both the government and the private operator could act as leader, it turns out that, due to the assumption of fixed total demand, there are no first mover incentives. In the next section, we consider each of these combinations and determine how they can affect the setting of transport policy.

3. RESULTS FOR SIMPLE PARALLEL NETWORK

If a government's policy objective is to achieve the First Best demand allocation, its options can be summarised in a simple proposition.

[7] In fact, as discussed in Section 2.1, this corresponds to an exogenous toll equal to marginal cost c_i.

Proposition 1

When the total demand is fixed, the First Best can be decentralized by a toll on only one route , there exists a unique, stochastic user equilibrium for two congested routes. The First Best can be achieved when government either

i) Controls pricing of both routes;

ii) Controls the price of one route while the other route is unpriced;

iii) Controls the price of one route while the second route is priced by a private monopoly.

We develop this proposition in the following sections. It is discussed further in Section 3.5. and the full proof is given in Appendix A.

Note that constant total demand is crucial for this proposition. This is indeed a strong assumption necessary for some parts of PROP 1. Existence and uniqueness of an equilibrium are likely to hold with elastic demand. However, when demand is elastic, two things may go wrong: too many drivers in total; and too many drivers on one route (wrong split, as in the case of inelastic demand). As a consequence, it is likely that two instruments are needed in general (one toll on each route) to decentralize the social optimum. Computations in the stochastic case (Logit) are much more involved, and it is not obvious that an analytical solution could be derived.

3.1. First Best (PUB-PUB)

In terms of government policy, achieving the First Best demand allocation is often the desired goal. In the First Best, all quantities (here transport volumes) can be chosen freely and the only constraints are the production possibilities. The same result can, however, be achieved when the government cannot directly control quantities but can impose a toll to optimise the welfare of all users over both routes. In this case, using (1), the (per capita) welfare for a given demand and price is defined as the sum of consumer surplus plus the producer surplus, both per capita.

$$W = \mu \log\left\{ \sum_{j=1,2} \exp\left(-\frac{p_j + \alpha_j N \mathrm{P}_j - a_j}{\mu} \right) \right\} + \sum_{j=1,2} \left[(p_j - c_j)\mathrm{P}_j - \frac{F_j}{N} \right] \tag{4}$$

where the F_j are the fixed costs for the construction of each route. From (2), the demand for route 1 is a function of the prices on the two routes. Again noting that $\mathrm{P}_2 = 1 - \mathrm{P}_1$, (4) can be differentiated to obtain the optimal price on route 1, taking the price on route 2 as given, so that

$$\frac{\partial W}{\partial p_1} = \left[(p_1 - c_1) - (p_2 - c_2) - \left(N\mathrm{P}_1\alpha_1 - N(1-\mathrm{P}_1)\alpha_2 \right) \right] \frac{\partial \mathrm{P}_1}{\partial p_1} = 0 \tag{5}$$

or

$$\Delta p_{FB} = \Delta c + N\mathrm{P}_1\alpha_1 - N\mathrm{P}_2\alpha_2 \tag{6}$$

with the interpretation that, in the First Best, the difference in the toll (mark-up over marginal cost) should be equal to the difference in external congestion costs[8]. The duopoly demand, given by (3), is a function of the difference in price only and, hence, (5) specifies the First Best demand uniquely. Clearly one solution to this equation is that a toll equal to resource cost plus external cost is imposed on each route, such that $p_1 = c_1 + N\mathrm{P}_1\alpha_1$ on route 1 and we assume that this is the case in our First Best scenario, in order to make comparisons with the pricing regimes imposed by other operators. The fact that only the difference in price is important for the First Best allocation implies that it should be possible for a government to achieve the First Best when it can only control one route. We return to this later.

By definition the First Best demand probability for route 1 is given by

$\mathrm{P}_1^{FB} = \left[1 + \exp\Psi_{FB}\right]^{-1}$, where (7)

the demand for the two routes clearly depends on the difference in the model parameters: marginal cost $\left(\Delta c = c_1 - c_2\right)$, the quality of the route $\left(\Delta a = a_1 - a_2\right)$ and the level of congestion $\left(NP_1\alpha_1 - N\mathrm{P}_2\alpha_2\right)$, which is a function of capacity. Since the congestion function explicitly depends on the demand level, the expression for P_1 given by (7) is an implicit function, which can be shown to have a unique solution[9]. This is illustrated in Figure **2** below where we denote the right hand side of (7) by $Z(n_1;\xi_i)$, for some given set of parameter values $\xi_i = \left\{\mu, a_1, a_2, \alpha_1, \alpha_2, c_1, c_2\right\}$. Further assuming that route 1 is always intrinsically better than route 2 in the sense that $\Delta a - \Delta c > 0$, we can investigate how the different model parameters affect the level of demand for route 1; in particular the role of congestion. We consider three different cases with parameter sets ξ_1, ξ_2 and ξ_3.

[8] In the more general case, the solution is $\sum_j \left(p_j - c_j - N\mathrm{P}_j\alpha_j\right)\frac{\partial \mathrm{P}_j}{\partial p_i} = 0$. Again, setting the mark-up on each route equal to the external congestion cost on that route is a solution, it is not unique since we can always write $\frac{\partial \mathrm{P}_N}{\partial p_k} = -\sum_{i=1}^{N-1}\frac{\partial \mathrm{P}_i}{\partial p_k}$. The optimal demand is, however, uniquely specified.

[9] The left hand side of (7) is non-negative, strictly increasing and vanishes at zero. The right hand side is non-negative and strictly decreasing and tends to zero as n tends to infinity. Both functions are continuous. Hence there is exactly one fixed point for every given set of parameters.

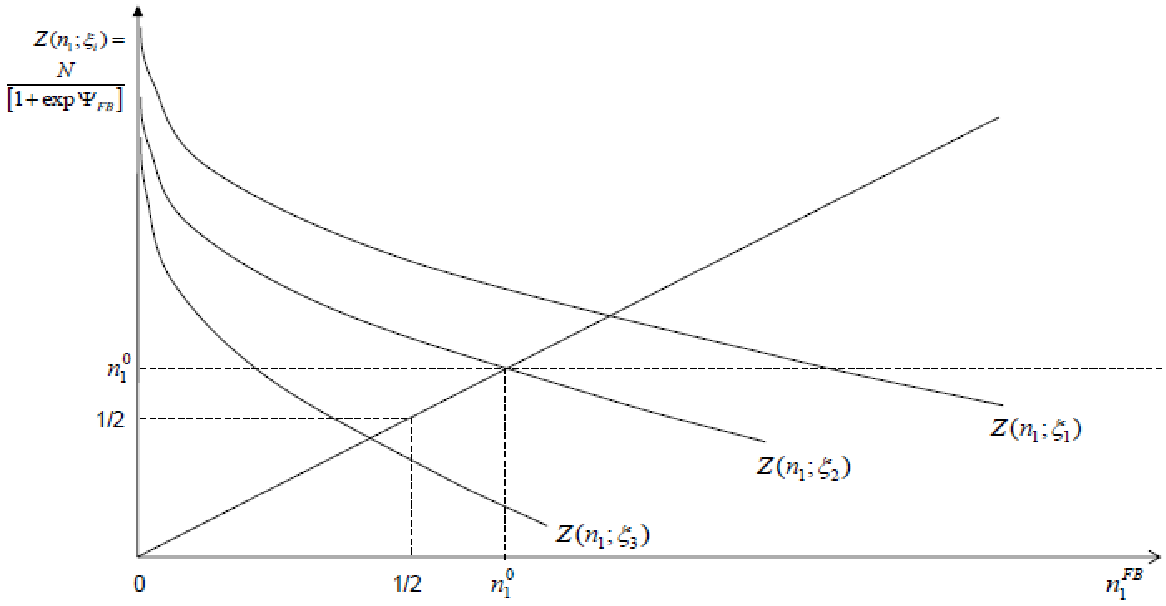

Figure 2 Unique First Best demand for route 1 under different levels of congestion

$Z(n_1;\xi_1)$ corresponds to the case where route 1 is less congested than route 2. Under our assumption that route 1 is the intrinsically better route, it follows from (7) that route 2 must be more congestible than route 1 ($\alpha_2 > \alpha_1$). Here we find that the use of route 1 is much larger than that of route 2. This accords with our intuition: route 1 is the more generally preferred, higher quality, less congested route. $Z(n_1;\xi_2)$ represents the case where both routes are equally congested, so that $\alpha_1 \mathrm{P}_1^{FB} = \alpha_2 \mathrm{P}_2^{FB}$ (or not congested at all). It can then be directly seen from (7), that the demand in this case is given by $\mathrm{P}_1^{FB} = \left[1+\exp\left((\Delta c - \Delta a)/\mu\right)\right]^{-1}$, which is the same as the uncongested demand (P_1^0) and is clearly always greater than one half when $\Delta a - \Delta c > 0$. Hence, the optimal flow on route 1 is larger than on route 2. However, equal congestion also implies that $P_1^{FB} = \alpha_2 / (\alpha_1 + \alpha_2)$ and thus for the case $Z(n_1;\xi_2)$ to occur, we further require that route 2 is still more congestible than route 1[10]. Finally, $Z(n_1;\xi_3)$ represents the case where route 1 is more congested than route 2. In this case, the demand for route 1 could become smaller than route 2. This would occur when route 1 is the more congestible route ($\alpha_1 > \alpha_2$) and this congestability outweighs the quality advantage of route 1. Otherwise, when the quality advantage of route 1 is sufficiently large or route 2 is slightly more congestible than route 1, then the demand for route 1 may still be greater than N/2[11].

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[10] The exact condition on the model parameters is in fact $\mu \ln\left[\alpha_2/\alpha_1\right] = \Delta a - \Delta c$.

[11] The condition for $P_1^{FB} \geq 1/2$ is that $\Delta a - \Delta c \geq N(\alpha_1 - \alpha_2)$

Turning now to the role of consumer preferences for diversity, the First Best demand under perfect substitution ($\mu = 0$), can be written as $\tilde{n}_1^{FB} = \dfrac{2N\alpha_2 + \Delta a - \Delta c}{2(\alpha_1 + \alpha_2)}$ and, reformulating equation (7), noting that $n_1^{FB} = N\mathrm{P}_1^{FB}$, we obtain

$$n_1^{FB} = N\left[1 + \exp\left(\frac{2(\alpha_1 + \alpha_2)\left[n_1^{FB} - \tilde{n}_1^{FB}\right]}{\mu}\right)\right]^{-1} \quad (8)$$

For a given set of model parameters, the stochastic demand will always be less than the deterministic demand, when route 1 is preferred, since some users have a stronger intrinsic preference for route 2. A corresponding result holds when route 2 is preferred. It is also clear from (8) that the stochastic and deterministic First Best only coincide when demand is equal on both routes[12], although the optimal pricing rules are the same (see Table 2 and Appendix B).

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Finally, the First Best welfare is given by

$$\begin{aligned} W_{FB} &= a_1 - c_1 - \mu \log[\mathrm{P}_1] + N\mathrm{P}_2[\alpha_2\mathrm{P}_2 - \alpha_1\mathrm{P}_1] - \alpha_1 N\mathrm{P}_1 - \frac{F_1 + F_2}{N} \\ &= a_1 - c_1 - \mu \log[\mathrm{P}_1] - (\alpha_1 + \alpha_2) N\mathrm{P}_1[2 - \mathrm{P}_1] - N\alpha_2 - \frac{F_1 + F_2}{N} \end{aligned}$$

It can be shown that, (at least if $\Delta a - \Delta c \geq N(\alpha_1 - \alpha_2)$ so that $P_1^{FB} \geq 1/2$), then First Best Welfare increases when there is greater consumer preference for diversity. As μ increases, this preference outweighs the intrinsic benefits or congestion effects of a particular route and more travellers are able to opt for their preferred route.

This section has clearly illustrated that the relative magnitudes of the intrinsic model parameters, including congestability, and the degree of consumer heterogeneity affect the distribution of demand over the network as well as welfare. We return to this theme in section 4, when we look in more detail at the roles of these two parameters in a comparative statics exercise.

3.2. No pricing on any route (FREE-FREE)

Untolled routes are a situation often encountered in practice. To simplify the analysis, we have assumed in this case that both routes are priced at marginal operating cost $p_i = c_i$, so no operator makes a profit. From (3), it can be seen directly that the demand for route 1 is given by the implicit function

$$P_1^{FF} = \left[1 + \exp\left(\frac{\Delta c + N\mathrm{P}_1\alpha_1 - N\mathrm{P}_2\alpha_2 - \Delta a}{\mu}\right)\right]^{-1} \quad (9)$$

Congestion is the only externality that requires correction here. So, when there is equal congestion on both routes ($\alpha_1\mathrm{P}_1 = \alpha_2\mathrm{P}_2$ in (9)), the FREE-FREE demand allocation is the same as the First Best (case $F(n_1;\xi_2)$ from the previous section). If we again assume that route 1 is intrinsically better than

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[12] Indeed, the demand is only equal on both routes (P=1/2) when the parameters are such that $\Delta a - \Delta c = N(\alpha_1 - \alpha_2)$.

route 2, it can then be shown, that, when congestion is lower (respectively higher) on route 1, the FREE-FREE demand for this route is too high (low) compared with the First Best. Here we notice an important difference with the perfect substitution case where there is no consumer heterogeneity.($\mu = 0$). In that case the demand for route 1, $\tilde{n}_1^{FF}$, is given by $\left(N\alpha_2 + \Delta a - \Delta c\right)/\left(\alpha_1 + \alpha_2\right)$ and the FREE-FREE demand for route 1 is always too high, whatever the congestion levels.

3.3. Duopoly with private operator and untolled route (PRIV-FREE)

A second situation that a government may encounter, is a network with one privately operated and tolled route and one untolled route. For this scenario, the monopolist operates route [1] and sets the access charge, p_1, while route [2] is untolled (price equals marginal cost). Using (3), the demand function can be written as $P_1^{PF} = \left[1 + \exp \Psi_{PRIV-FREE}\right]^{-1}$, where, in this case,

$$\Psi_{PRIV-FREE} = \left(\frac{p_1 - c_2 + N\mathrm{P}_1(\alpha_1 + \alpha_2) - N\alpha_2 - \Delta a}{\mu} \right) \tag{10}$$

The private operator of route 1 maximises his profits, according to

$$\pi_1 = \left(p_1 - c_1\right) N\mathrm{P}_1 - F_1, \tag{11}$$

and sets his price so that
$$\frac{d\pi_1}{dp_1} = N\mathrm{P}_1 + \left(p_1 - c_1\right) N \frac{\partial \mathrm{P}_1}{\partial p_1} = 0 \tag{12}$$

Then differentiating (10) directly, we obtain

$$\frac{\partial \mathrm{P}_1}{\partial p_1} = -\frac{1}{\left[\dfrac{\mu}{\mathrm{P}_1\left(1 - \mathrm{P}_1\right)} + N\left(\alpha_1 + \alpha_2\right) \right]} < 0.$$

Substituting this expression in (12) and re-arranging, results in the equilibrium price

$$p_1 - c_1 = \frac{\mu}{\left(1 - \mathrm{P}_1\right)} + \left(\alpha_1 + \alpha_2\right) N\mathrm{P}_1 \tag{13}$$

This result confirms our intuition. Firstly, the profit margin is increasing in μ : a preference for diversity protects the private monopolist. Secondly, the margin is increasing in the "total" level of congestion ($\alpha_1 + \alpha_2$): more congestion on its own network decreases the incentives for the monopolist to lower its price as the extra demand it can attract will be discouraged by increasing congestion losses. The more congestion there is on the unpriced route 2, the less the monopolist has to fear from leakage effects when he increases his price.

In addition

$$\Psi_{PRIV-FREE} = \left(\frac{\Delta c - \Delta a + 2N\mathrm{P}_1(\alpha_1 + \alpha_2) - N\alpha_2}{\mu} + \frac{1}{1 - \mathrm{P}_1} \right)$$

and it can be seen from (7) and the above that the PRIV-FREE demand for route 1 is always smaller than optimal, independent of the level of congestion[13] and route characteristics. Moreover, since route 2 is priced at marginal operator cost (c_2), $\Delta p_{PRIV-FREE}$ is always higher than in the first best ($\Delta p_{FB} = \Delta c + \alpha_1 N \mathrm{P}_1 - \alpha_2 N \mathrm{P}_2$). The private operator is always able to charge a price on route 1 that is higher than the First Best because of the consumer heterogeneity and hence the price difference between the two routes is always too high. The only way to raise demand on route 1 to the optimal level is for the government to impose a toll on route 2.

3.4. Private operators on both routes (PRIV-PRIV)

If both routes are now privately run, the operator on route 1 behaves in the same manner as in the previous section, where route 2 was untolled, although the demand function he faces is different as it now depends on the price charged by the private operator of route 2. The Nash price equilibrium is now given by

$$p_1 - c_1 = \frac{\mu}{1-\mathrm{P}_1} + (\alpha_1 + \alpha_2) N P_1 \qquad (14)$$

The private operator sets the toll on his route in the same way, whether the competing route is or is not tolled. It is apparent from the symmetry of that $\partial \mathrm{P}_1 / \partial p_1 = \partial \mathrm{P}_2 / \partial p_2$ and, since $P_1 + P_2 = 1$, $\partial P_1 / \partial p_2 = -\partial P_2 / \partial p_2$, so that the demand for route 1 is an increasing function of the price on route 2. Thus, when the operator of route 2 increases his price from zero, P_1 increases and it is then clear from (14) that the price on route 1 also increases. The private operator will not reduce his price, once the operator of route 2 imposes a non-zero toll, as he will gain customers at the existing price. It is in fact in his interest to further raise his own toll as, although this will have an impact on how many customers switch routes, it will still increase his profits.

The demand probability for route 1 can be written in the usual form $\mathrm{P}_1^{PP} = \left[1 + \exp \Psi_{PP}\right]^{-1}$ where

$$\Psi_{PP} = \left[\Delta c - \Delta a + N(\alpha_1 + \alpha_2)(2\mathrm{P}_1 - \mathrm{P}_2) - N\alpha_2 + \frac{\mu}{(1-\mathrm{P}_1)} - \frac{\mu}{\mathrm{P}_1} \right] \mu^{-1} \qquad (15)$$

To gain further insights into which operator sets the highest toll or has the largest market share, we can draw on the analysis of Dunkerley, de Palma and Proost (2009). In that paper it was shown that, for an asymmetric duopoly without congestion, the firm with the higher facility rank parameter, $B \equiv \Delta a - \Delta c$, would have a larger share of the profits and a greater market share. Indeed, the larger the B, the greater the difference in profits and market share between the two firms.

The price difference between the routes in this case is

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[13] Indeed, if $\mathrm{P}_1^{PF} \geq \mathrm{P}_1^{FB}$, this implies $\Psi_{PF} \leq \Psi_{FB}$, with the resulting condition $2N(\alpha_1 + \alpha_2)\left(\mathrm{P}_1^{PF} - \mathrm{P}_1^{FB}\right) + \mu\left(1 - \mathrm{P}_1^{PF}\right)^{-1} + N\alpha_2 < 0$, which cannot hold.

$$\Delta p^{PRIV-PRIV} = \Delta c + \frac{\mu}{\left(1-\mathrm{P}_1\right)} - \frac{\mu}{\mathrm{P}_1} + N\left(\alpha_1 + \alpha_2\right)\left(\mathrm{P}_1 - \mathrm{P}_2\right) \tag{16}$$

According to this specification, the price difference can be higher or lower than the First Best price difference, depending on the relative importance of the congestion and consumer heterogeneity model parameters. In terms of achieving the First Best by using other means than tolling, such as investing in capacity on a route (and thereby reducing its congestion parameter), we can consider whether there exists a set of model parameters, for which the PRIV-PRIV and First Best equilibria are equivalent. This is indeed possible in both the deterministic and non-deterministic cases by adjusting the quality or capacity (as measured by α) of one of the two routes. This is in contrast to the PRIV-FREE set-up where tolling is the only available tool.

3.5. Discussion of Proposition 1

Figure 3 demonstrates how the First Best allocation can be achieved by tolling (or subsidizing) the untolled route 2, when route 1 is free (case FREE-FREE) or privately operated (case PRIV-FREE). The results are shown for the case when route 1 is more congested than route 2. When the situation is reversed, the free route can be tolled (as indicated by the parentheses).

In Figure 3, the horizontal line “FIRST BEST” indicates the optimal demand level on route 1. Government can only control the price of route 2 (p_2). There are two possible cases: either demand on route 1 is too high (upper curve, which is only the case for FREE-FREE) or too low (lower curve). When demand on route 1 is too high, government can only correct the demand level away from point A to point B by subsidising route 2. The second situation arises when the demand for route 1 is too low. This will always be the case when route 1 is controlled by a private operator but can also occur when both routes are untolled. Then the government needs to set a high toll to discourage too high use of route 2.

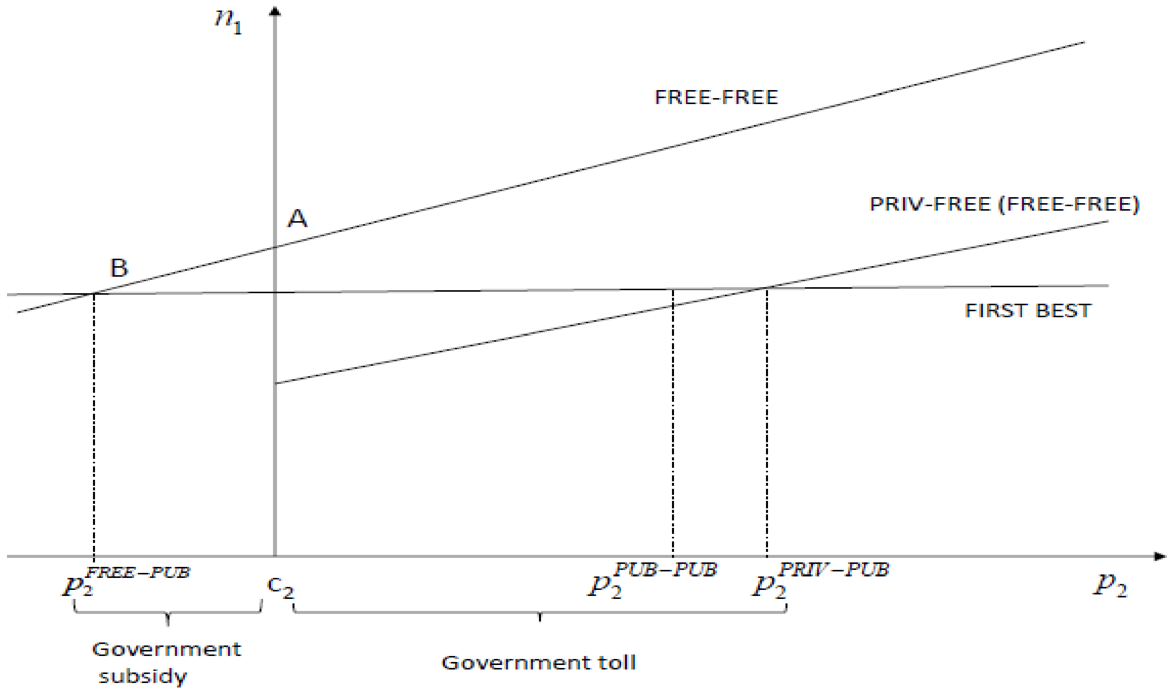

Figure 3 Achieving the First Best demand allocation through government toll or subsidy

In Figure 4 the effect of government actions on the difference in price between the two routes are summarised. The price difference ($\Delta p = p_1 - p_2$) is shown on the vertical axis and the price on route 2, on which the government can intervene, on the horizontal axis. Again, a horizontal line indicates the First Best solution. The price difference is always too high when one route is privately operated, regardless of the level of congestion. In the FREE-FREE case, when route 1 is the more highly congested, subsidising route 2 achieves Δp^{FB}, whereas a toll is required when congestion is higher on this route (dotted line in figure).

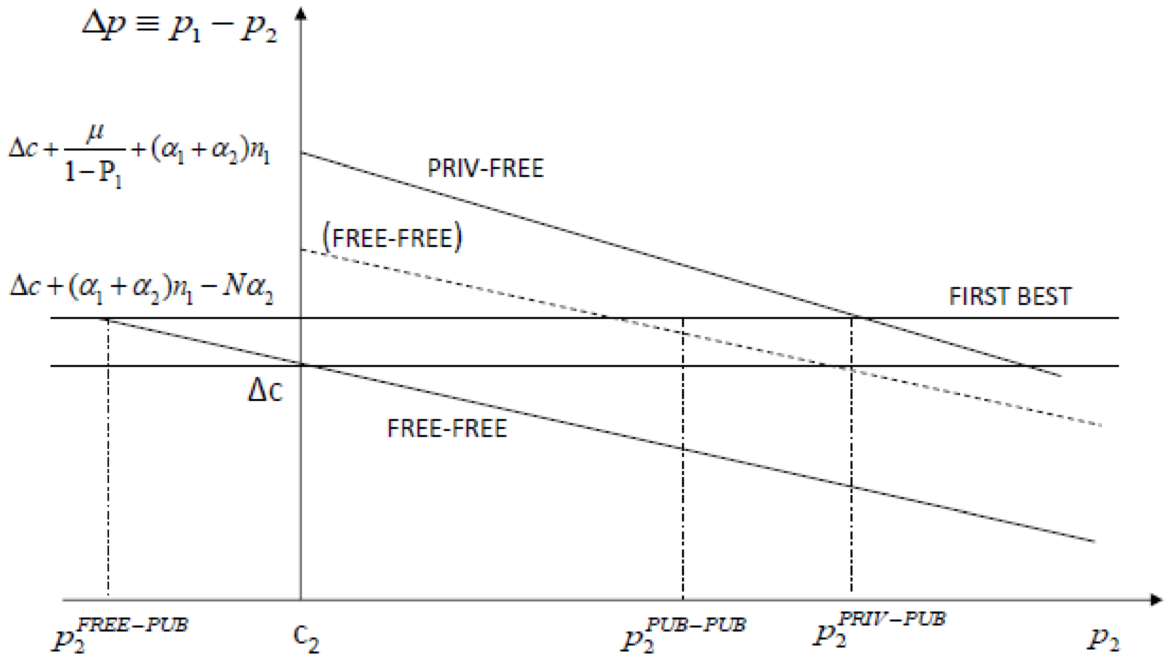

Figure 4 The effect of government tolls (subsidies) on the price difference between routes

4. THE ROLE OF SUBSTITUTABILITY AND CONGESTION

4.1 Synthesizing the pricing rules

Perhaps the most direct way in which the role of imperfect substitution and congestion can be understood is to consider their impact on the pricing rules implemented in the different scenarios. These rules are summarised in Table 2 for the case when route 1 is the "better" option in the sense that it is preferred to route 2 in objective characteristics ($\Delta a - \Delta c > 0$) and when both routes are either congested ($\alpha_1 > 0,\ \alpha_2 > 0$) or uncongested ($\alpha_1 = \alpha_2 = 0$)[14]. The shaded cells indicate where the First Best has been achieved.

[14] This differs slightly from our original set-up in Section 2, in which at least one route is assumed to be congested, in order to make the role of congestion clear in the most general case. It is in fact possible to allow the congestion parameter of one of the routes to be set to zero.

Route 1	Route 2	Deterministic preferences		Stochastic preferences	
		No congestion	congestion	No congestion	congestion
FREE	FREE	No tolls	No tolls	No tolls	No tolls
PUB (First Best)	PUB	No tolls	$Toll_1=MEC_1$ $Toll_2=MEC_2$	No tolls	$Toll_1=MEC_1$ $Toll_2=MEC_2$
FREE	PUB	No tolls	$Toll_2=MEC_2-MEC_1$	No tolls	$Toll_2=MEC_2-MEC_1$
PRIV	FREE	$Toll_1=mc_2-mc_1-\varepsilon$	$Toll_1=MEC_1+\alpha_2 n_1$	$Toll_1=\mu N/n_2$	$Toll_1=\mu N/n_2+MEC_1+\alpha_2 n_1$
PRIV	PRIV	$\Delta toll= mc_2-mc_1-\varepsilon$	$Toll_1=MEC_1+\alpha_2 n_1$ $Toll_2=MEC_2+\alpha_1 n_2$	$Toll_1=\mu N/n_2$ $Toll_2=\mu N/n_1$	$Toll_1=\mu N/n_2+MEC_1+\alpha_2 n_1$ $Toll_2=\mu N/n_1+MEC_2+\alpha_1 n_2$
PRIV	PUB	$Toll_1=mc_2-mc_1-\varepsilon$ $Toll_2=0$	$Toll_1=MEC_1+\alpha_2 n_1$ $Toll_2=MEC_2+\alpha_2 n_1$	$Toll_1=\mu N/n_2$ $Toll_2=\mu N/n_2$	$Toll_1=\mu N/n_2+MEC_1+\alpha_2 n_1$ $Toll_2=\mu N/n_2+MEC_2+\alpha_2 n_1$

Table 2 Pricing results for two parallel routes [16]

We take each of the six scenarios in turn. In the top half of the table, the routes are either untolled or government run. When neither route is priced (FREE-FREE) and there is no congestion, there is no need for public intervention and the no toll case is First Best. This holds for both deterministic and stochastic preferences. Whenever there is congestion and in the absence of tolls, the First Best is not achieved. Next, when both routes are priced by the government (PUB-PUB), the tolls are set at marginal external congestion cost (MEC_1, MEC_2) and the First Best is achieved. Finally, when only one route is controlled by the public sector, the other route is untolled and total demand is fixed, the First Best can be obtained by charging the difference in marginal external cost between the two links. Table 2 shows clearly that, when there are only government operators or untolled routes (the top half of the table), the pricing rules are the same, whatever the degree of consumer heterogeneity. This does not, however, mean that the stochastic and deterministic demands are identical since, as noted in Section 3.1, in the stochastic case consumer preferences for variety lead to fewer users selecting the “better” route.

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[16] Marginal external costs on route *i* are denoted $MEC_i \equiv \alpha_i n_i$, where $n_i = N\mathrm{P}_i$, for stochastic demand and $\alpha_i \tilde{n}_i$ for deterministic, mc_i represents marginal operating costs on route *i* (c_i in our model terminology).

Turning now to the scenarios in the bottom half of Table 2, where at least one route is privately operated, we see the effect of market power on the pricing rules. Firstly, the presence of congestion allows the private operator to set a toll that is greater than the marginal external cost on his route (columns 4 and 6). Secondly, consumer heterogeneity also results in higher tolls that exceed marginal external costs (columns 5 and 6), as consumer preferences for a given route mean that the operator can raise prices without losing many customers and hence increase profits.

When only one route is privately tolled (PRIV-FREE), preferences are deterministic, and there is no congestion (column 3), the private monopoly can capture the whole market by charging a toll such that the travel cost on his route is just lower than the travel cost on the untolled route: he reduces the toll by an arbitrarily small factor, ε. This is First Best because the total social cost is minimized by having all users on route 1 which is objectively the best.

Adding congestion on both routes (column 4), the private operator will first of all charge the MEC on his route, as this minimises total transport costs on the route. He then adds a mark-up equal to the additional congestion costs that consumers would face if they switched to the untolled route. It is only when there is private pricing involved that the pricing rules for stochastic preferences will be different from those for deterministic preferences. Imperfect substitutability is a source of market power for every operator. When there is one private operator (PRIV-FREE), stochastic preferences and no congestion, he charges the monopoly mark-up $\mu N/n_2$ (column 5). If congestion is then added, the resulting toll combines the deterministic congestion pricing rule with the purely stochastic mark-up (column 6).

For the case where both routes are privately operated (PRIV-PRIV), the First Best can still be achieved when there is no congestion and preferences are deterministic. In that case it is only the difference in tolls that matters and the operator of route 1 can adjust his toll so as to capture the entire market. The demand allocation becomes sub-optimal however, whenever there is congestion or preferences are stochastic. Now both operators try to make the most of their intrinsic market power and/or the relative congestion on the two routes to maximise profits and set their tolls accordingly. Finally, in the last row of Table 2, we see that the First Best can also be achieved when one of the two routes can be publicly operated (PRIV-PUB), as discussed in Section 3.5. Since total demand is fixed and given the profit maximising behaviour of the private operator, the government can always set a toll so that the difference in tolls is equal to the difference in marginal external costs between the two routes.

4.2 Comparative statics

The pricing rules presented in Table 2 result in different levels of demand on the two routes, according to the type of operator, level of congestion and degree of imperfect substitution. The effect of increasing the degree of imperfect substitution on demand, for fixed levels of congestibility on the two routes, is illustrated in Figure 5. This figure allows us to give a relative ranking of different regimes compared to the First Best. As usual, we take route 1 to be the intrinsically better route[17] . The deterministic demands are shown on the axis where $\mu = 0$. In this case, the demand for route 1 is

[17] For illustrative purposes we further impose that $\Delta a - \Delta c \geq N(\alpha_1 - \alpha_2)$ so that $\Psi_{FB} < 0$ and the First Best demand is always greater than one half. For $\Psi_{FB} > 0$, the curves asymptote from below.

always too high when both routes are untolled (FREE-FREE) and too low when one route is privately operated (PRIV-FREE). In Section 3.2, we saw that the stochastic FREE-FREE demand for route 1 coincided with the First Best only when there was equal congestion on both routes. Since the deterministic FREE-FREE demand is too high on the intrinsically better route 1, as μ increases, more users will prefer to take route 2 until, at some point both routes are equally congested. As the strength of consumer preferences continues to increase, this effect outweighs congestion or quality aspects of route choice. A similar story arises for the private duopoly (see Section 3.4). In this case, when route 2 is more congested, the market power exerted by the operator of the "better" route 1, due to congestion, leads to a suboptimal demand, as he sets a very high toll to maximise profits. This advantage gradually diminishes as consumer heterogeneity becomes stronger. It should be noted here that, whenever one route is privately operated, then, once μ becomes large enough, the generalised price of using this route will become too large: $p_i + \alpha_i N \mathrm{P}_i > \Gamma + a_i$. Demand will then drop to zero.

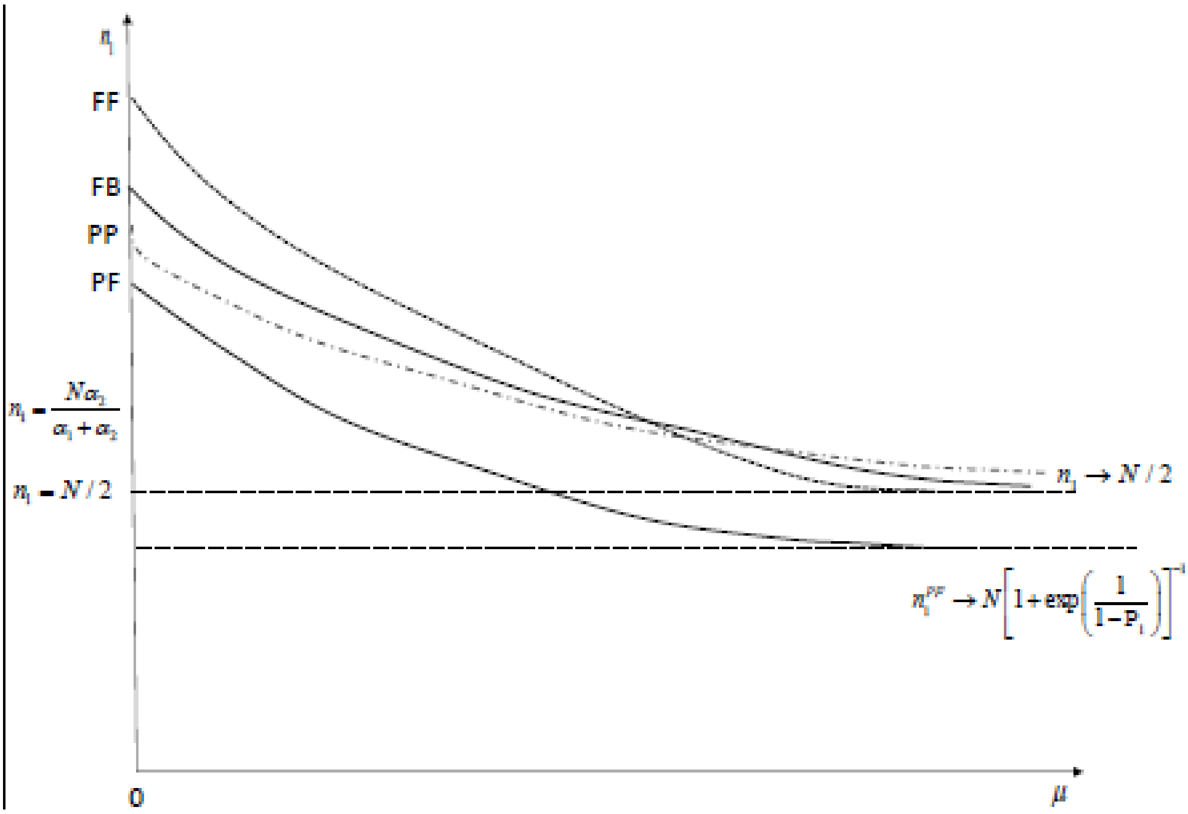

Figure 5 Effect on demand of increasing consumer heterogeneity (μ) when route 1 is intrinsically better than route 2 and there is congestion

There are two ways in which we can consider the impact of increasing congestion on the network: either the total congestibility of the network increases, but the relative level of congestibility on the two routes remains unchanged; or the congestion parameter associated with each route increases independently. In the former case, we can write total congestion as $\Upsilon = \gamma(\alpha_1 + \alpha_2)$ and examine the effect of increasing the scale parameter γ[18]. Then, for all degrees of consumer heterogeneity, we find

[18] We can then regard the analysis of the previous section as effectively corresponding to the case $\gamma = 1$.

that, for the case $\Delta a - \Delta c > 0$, the demand for route 1 decreases under all operator regimes, as the role of congestion becomes more important than the intrinsic advantage that route 1 enjoys[19].

If, instead, we consider the effect of increasing total congestibility by increasing the congestion parameter for one route, while keeping the other fixed, this gives rise to a number of additional interesting results. Again, these are subject to the participation constraint, as the tolls on the two routes cannot become infinitely large. Firstly, for the deterministic case, when the level of congestion on route 1 is too low, all consumers will prefer to take this route, whatever the level of congestion on route 2. This is in contrast to the stochastic case, where consumer preferences for diversity mean that some consumers will always take route 2. The stochastic results are illustrated in Figure 6. Increasing the congestion parameter of route 1 (α_1)[20], for any fixed level of congestability on route 2 (α_2), leads to a decrease in the demand for route 1, for all operator regimes and all levels of consumer heterogeneity.

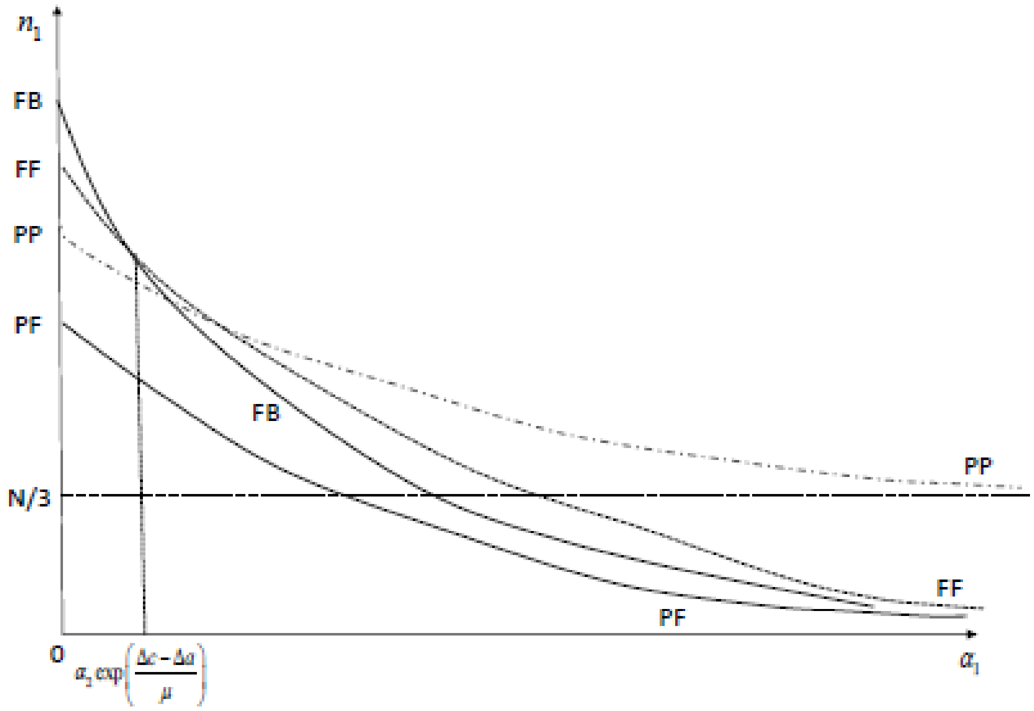

Figure 6 Effect on demand of increasing the congestibility of route 1 (α_1) when route 1 intrinsically better than route 2, the congestibility α_2 of route 2 is fixed, μ>0 and Δa-Δc>0

It is clear from the figure that it is possible to achieve the First Best demand by changing the congestability (capacity) of a given route for a given level of consumer preferences for diversity, when both routes are free or both are privately operated [21].

[19] At some point the generalised cost of travel on each route becomes too high compared to consumer income and demand drops to zero, as the participation constraint of a consumer on a given route is exceeded ($p_i + \alpha_i N P_i \leq \Gamma + a_i$). Exactly when this occurs will clearly depend on Γ and the other model parameters.

[20] For the deterministic case, there is a minimum level of congestion on route 1, below which all N users will take this route. There is no such minimum in the stochastic case.

[21] A corresponding set of results are obtained if α_1 is fixed and α_2 is varied.

4.3 Some second best results in the case government cannot control pricing

From a government policy perspective, it may be the case that the government is unable to implement the First Best on a duopoly network by any of the means we have explored in this paper: imposing tolls or subsidies or changing the quality or capacity of one of the routes. But if the government does have the option to allow one of the routes to be privately operated, should it allow this, and on which route?

Proposition 2

Assume the total demand is fixed and that route 1 is intrinsically better than route 2, ($\Delta a - \Delta c \geq 0$*), so that there exists a unique, stochastic user equilibrium, and assume government cannot price any of the two routes, then:*

a) if there is no congestion, the First Best solution always entails no pricing at all. - privatisation of a route is always second best and in that case it is socially preferable to price route 2 rather than route 1.

b) if preferences are deterministic and there is congestion, then if route 1 is the more congestible one $(\alpha_1 > \alpha_2)$*, it is preferable to toll route 1. However, if route 2 is also quite congestible, then it is better not to toll either route.*

c) if preferences are stochastic and there is congestion, then if route 1 is the more congestible $(\alpha_1 > \alpha_2)$*, or consumer preferences for diversity are small, it is preferable to toll route 1*.

Proof: See Appendix C available upon request

When there is no congestion, then all users can take their preferred route based on the objective quality of the route and their own intrinsic preferences. Tolling route 1 would raise more tax revenue as there is more demand for this route but this would not compensate for the loss in consumer surplus of those individuals who could no longer afford to use the better route. Hence, if a route must be privatised, then it should be route 2, although this may not be very attractive to a private operator. Adding congestion changes the trade-off faced by the user when deciding which route to take. When all users value the routes in the same way, the superior route 1 should be privatised as long as there is sufficient capacity on route 2. In this case, while some users will be forced to take the inferior route, the remaining users of route 2 will have improved travel times and the toll revenues also increase welfare. However, if route 2 is also prone to congestion, the overall positive welfare contribution may no longer hold and it is better to leave both routes untolled. In the stochastic setting, a more complex relationship between congestibility on the two routes and the strength of consumer preferences determines whether it is welfare enhancing to privatise one of the routes. In general, the arguments from the deterministic case for privatising route 1, if it is the more congestible route, also hold, but with the caveat that consumer preferences for diversity should not be too large.

5. CONCLUSIONS

In this paper we have studied in detail the role of stochastic consumer preferences and congestion in the allocation of demand on parallel networks and more particularly the performance of different combinations of operators that price their link in the network. In general, when compared with the deterministic case, demand on the intrinsically better route is found to be lower when preferences are stochastic. For the simple parallel duopoly, it is the difference in prices between the two routes that determine demand. Hence it is possible for a government to achieve the First Best demand allocation if it can impose tolls on one of the two routes. The presence of congestion on the network leads to market power for private operators, who are able to charge a mark-up over marginal external cost. In the stochastic case, additional market power results from the intrinsic preferences of the users for a given route. For a private monopoly, this means that prices are always too high and demand too low and it can be preferable to leave both routes untolled. However, on a congested network, if one route must be privately operated, it is generally better that the intrinsically superior route is selected. When there is a private duopoly or both routes are unpriced, it is possible that the First Best demand allocation can be achieved, depending on the relative magnitudes of the model parameters, including congestion and the degree of consumer preferences for diversity. Increasing the capacity of an alternative or improving its quality then become potential policy options when pricing is not available.

This analysis provides, in a simplified network, useful insights for government policy options on transport networks with alternatives that differ in a complex way (congestability, quality and unobserved heterogeneity). The simple duopoly results could also be generalised to the setting where users first choose their mode of transport and then select one alternative among an identical set of alternatives within that mode. It would then be possible to show under what conditions, the First Best can still be achieved. As future work it would be valuable to apply the model to an empirical example with more asymmetry. A further interesting theoretical extension would be to model a more complex network including serial links.

REFERENCES

Acemoglu, D., Ozdaglar A.,(2007): Competition and Effciency in congestion markets, Mathematics of Operations Research, 32,1, 1-31

Anderson, SP., de Palma, A. and Thisse, J.F. (1992): Discrete Choice Theory of Product Differentiation. *MIT Press*.

Arnott, R. and Kraus, M. (1998): When are anonymous congestion charges consistent with marginal cost pricing? *Journal of Public Economics, **67**, 45-64*.

De Borger B., Dunkerley F., Proost S. (2008), The interaction between tolls and capacity investment in serial and parallel transport networks, *Review of Network Economics, 7, 136-158.*

De Borger, B., Proost, S. and K. Van Dender.(2005): Congestion and tax competition on a parallel network, *European Economic Review*, **49**, 2013-2040.

De Borger, B. and K. Van Dender. (2006): Prices, capacities and service qualities in a congestible Bertrand duopoly. *Journal of Urban Economics,* **60**, 264-283.

de Palma, A and Leruth, L (1989): Congestion and game in capacity: A duopoly analysis in the presence of network externalities. *Annales d'Economie et de Statistique, **15/16**, 389-407.*

de Palma, A and Lindsey, R (2000): Private toll roads: Competition under various ownership regimes. *The Annals of Regional Science, **34**, 13-35.*

de Palma, Andre and Stef Proost. 2006. "Imperfect competition and congestion in the city." *Journal of Urban Economics, 60 (2), p185-209.*

Dunkerley F, de Palma A, Proost S (2009): Spatial asymmetric duopoly with an application to Brussels' airports, *Journal of Regional. Science*, **49**, 529-553.

Economides, N. and Salop, S.C. (1992):Competition and integration among complements and network market structure. *Journal of Industrial Economics*, **XL**, 105-123.

Liu Y, Guo, X and Yang, H (2009): Pareto-improving and revenue-neutral congestionpricing schemes in two-mode traffic networks. *Netnomics*, **10**, 123-40.

Melo E (2014), Price competition, free entry, and welfare in congested markets, *Games and economic behavior*, 83, 53-72.

Mun, S and Ahn, K (2008): Road Pricing in a Serial Network. *Journal of Transport Economics and Policy*, **42**, 367-95.

Santos G., Verhoef E., (2011): Road congestion pricing, Ch23 in A.de Palma, R.Lindsey, E.Quinet, R.Vickerman (eds.): *A Handbook of transport economics* Edward Elgar.

Scotchmer, S.(1985):.Profit-maximizing clubs. *Journal of Public Economics, 27, 25-45.*

Van Dender, K. (2005):. Duopoly prices under congested access, *Journal of Regional Science*, 45, 2, 343-362.

Small, K. and. Yan J. (2001):The value of "Value Pricing" of roads: second best pricing and product differentiation, *Journal of Urban Economics*, 49, 2, 310-336.

Verhoef E.T., P. Nijkamp and P. Rietveld. (1996): Second-best congestion pricing: the case of an untolled alternative, *Journal of Urban Economics*, 40, 3, 279-302.

Verhoef, E.T. and Small, K (2004): Product Differentiation on Roads - Constrained Congestion Pricing with Heterogeneous Users. *Journal of Transport Economics and Policy*,**38**, 127–156.

APPENDIX A: Proof of Proposition 1

Part (i) then follows directly from Section 3.1. When the government operates one route, route 1 say, it still maximises welfare according to (5), taking the price on route 2 as given. Hence, when route 2 is unpriced, then this enters equation (5) as $p_2 = c_2$ and the optimal government toll is then $p_1 - c_1 = \alpha_1 N \mathrm{P}_1 - N\alpha_2 \left(1 - \mathrm{P}_1\right)$. This is the PUB-FREE scenario in

Table 1. Similarly, a private operator of route 2 maximises his profit, taking the price on route 1 as given, so that $p_2 - c_2 = \dfrac{\mu}{\mathrm{P}_1} + N(1-\mathrm{P}_1)(\alpha_1 + \alpha_2)$. This corresponds to the PUB-PRIV set-up. The government then sets a toll of $p_1 - c_1 = \dfrac{\mu}{\mathrm{P}_1} + N\alpha_1$ to satisfy (5). Since in both cases, the difference in price is the First Best one, the First Best demand follows automatically[23]. QED.

The existence of a demand equilibrium (P_1) can be established using the Brouwer Fixed Point Theorem. The function $Z(\Delta \mathrm{p}, \xi) = \left[1 + \exp \Psi\right]^{-1}$ in equation (3) has a fixed point if the set of probability demands $\mathrm{P}_1 \in A$ is a compact set. It can be shown that there is a one to one correspondence between the probability demand and the difference in price between the two routes, such that

$$\frac{d\mathrm{P}_1}{d\Delta p} = -\frac{1}{\left(\dfrac{\mu}{\mathrm{P}_1\left(1-\mathrm{P}_1\right)} + N\left(\alpha_1 + \alpha_2\right)\right)} < 0 \qquad \text{(A1)}$$

Further, if the set of prices available to the firms, $p_i \in S_i$, are compact, convex sets, then any linear combination of these prices is also a compact, convex set. Then the candidate demand equilibrium (3) exists.

It is reasonable to assume that each firm sets his price $p_i + \alpha_i N \mathrm{P}_i \leq \Gamma + a_i$ so that the consumer's total transport costs are less than income and utility gain from use of the route, to avoid zero demand, where Γ is consumer income. Similarly, the firm will choose $p_i \geq c_i$, to cover marginal costs. We know from (A1) that there is a one-to-one relationship between Δp and P_1. Hence, $\Delta p \in \left\{\Delta c, \max_i (\Gamma + a_i - \alpha_i N \mathrm{P}_i)\right\}$ is a compact, convex set. Thus we need only show that the welfare function of the government and the profit function of the private operator is quasi-concave (see Anderson et al 1992) for the candidate equilibrium to be the unique Nash solution. The quasi concavity of the profit function has been shown elsewhere (e.g. de Palma and Proost, 2006) and is not repeated here. Turning to the welfare function

[23] It can be shown that there is no first mover advantage for either party. Since total demand is fixed and each player knows that the other will set his price optimally, the government can always choose a toll to achieve the First Best demand allocation. Similarly, since the government always sets its toll as a function of the private operator's toll regardless of whether the tolls are set simultaneously or one operators acts first, the private operator can do no better than to behave like a monopolist and maximise his profit taking the government's behaviour as given.

$$W = \mu \log\left\{ \sum_{j=1,2} \exp\left(-\frac{p_j + \alpha_j N \mathrm{P}_j - a_j}{\mu} \right) \right\} + \sum_{j=1,2} \left[(p_j - c_j)\mathrm{P}_j - \frac{F_j}{N} \right]$$

And assuming without loss of generality that the government controls at least route 1 and takes the price on route 2 as given, then we know that, at the equilibrium:

$$\frac{\partial W}{\partial p_1} = \left[(p_1 - c_1) - (p_2 - c_2) - \left(N\mathrm{P}_1\alpha_1 - N(1-\mathrm{P}_1)\alpha_2 \right) \right] \frac{\partial \mathrm{P}_1}{\partial p_1} = 0 \tag{A2}$$

Differentiating a second time,

$$\begin{aligned} \frac{\partial^2 W}{\partial p_1^2} &= \left[(p_1 - c_1) - (p_2 - c_2) - \left(N\mathrm{P}_1\alpha_1 - N(1-\mathrm{P}_1)\alpha_2 \right) \right] \frac{\partial^2 \mathrm{P}_1}{\partial p_1^2} \\ &+ \frac{\partial \mathrm{P}_1}{\partial p_1} \left[1 - N(\alpha_1 + \alpha_2) \frac{\partial \mathrm{P}_1}{\partial p_1} \right] \end{aligned} \tag{A3}$$

Evaluatiing equation A3 at the extremum, results in

$$\frac{\partial^2 W}{\partial p_1^2} = \frac{\partial \mathrm{P}_1}{\partial p_1} - N(\alpha_1 + \alpha_2) \left(\frac{\partial \mathrm{P}_1}{\partial p_1} \right)^2 \tag{A4}$$

The second term on the RHS is clearly negative. It remains to show that the first term is non-positiive.

By definition $\mathrm{P}_1(p_1, p_2; \xi) = \left[1 + \exp \Psi \right]^{-1}$

where $\Psi(\mathrm{P}_1, p_1, p_2; \xi) \equiv \left[p_1 - p_2 + N\mathrm{P}_1(\alpha_1 + \alpha_2) - N\alpha_2 + \Delta a \right] \mu^{-1}$.

Differentiating with respect to , we obtain

$$\begin{aligned} \frac{\partial \mathrm{P}_1}{\partial p_1} &= -\mathrm{P}_1(1-\mathrm{P}_1) \frac{\partial \Psi}{\partial p_1} \\ &= -\mathrm{P}_1(1-\mathrm{P}_1) \left[\mu + N\mathrm{P}_1(1-\mathrm{P}_1)(\alpha_1 + \alpha_2) \right]^{-1} < 0 \end{aligned}$$

since $\frac{\partial \Psi}{\partial p_1} = \left[1 + N(\alpha_1 + \alpha_2) \frac{\partial \mathrm{P}_1}{\partial p_1} \right] \mu^{-1}$.Hence $\partial \mathrm{P}_1 / \partial p_1$ is non- positive and the welfare function is quasi-concave as required. QED

APPENDIX B SUMMARY RESULTS FOR PERFECT SUBSTITUTION WITH CONGESTION AND FOR IMPERFECT SUBSITITUTION WITH AND WITHOUT CONGESTION FOR A PARALLEL NETWORK

Table A1 Results for perfect substitution (μ=0) with congestion (α>0)

scenario	Demand n_1	Price p_1	Price p_2
PUB-PUB	$\frac{2N\alpha_2 + \Delta a - \Delta c}{2(\alpha_1+\alpha_2)} = \tilde{n}_1^{FB}$	$p_1 = c_1 + \alpha_1 n_1$	$p_2 = c_2 + \alpha_2 n_2$
FREE-FREE	$\tilde{n}_1^{FF} = \tilde{n}_1^{FB} + \frac{\Delta a - \Delta c}{2(\alpha_1+\alpha_2)}$	$p_1 = c_1$	$p_2 = c_2$
FREE-PUB	$\tilde{n}_1^{FB}$	$p_1 = c_1$	$p_2 = c_2 + \alpha_2 n_2 - \alpha_1 n_1$
PRIV-FREE	$\tilde{n}_1^{PF} = \tilde{n}_1^{FB} - \frac{N\alpha_2}{2(\alpha_1+\alpha_2)}$	$p_1 = c_1 + (\alpha_1+\alpha_2) n_1$	$p_2 = c_2$
PRIV-PRIV	$\tilde{n}_1^{PP} = \tilde{n}_1^{FB} - \frac{\Delta a - \Delta c - 2N(\alpha_1-\alpha_2)}{6(\alpha_1+\alpha_2)}$	$p_1 = c_1 + (\alpha_1+\alpha_2) n_1$	$p_2 = c_2 + (\alpha_1+\alpha_2) n_2$
PRIV-PUB	$\tilde{n}_1^{FB}$	$p_1 = c_1 + (\alpha_1+\alpha_2) n_1$	$p_2 = c_2 + \alpha_2 N$

scenario	Ψ	Ψ as a function of $\tilde{n}_1$	p1	p2	Δp
PUB-PUB (FB)	$\Psi_{FB}=\left[\Delta c-\Delta a+2n_1\left(\alpha_1+\alpha_2\right)-2N\alpha_2\right]\mu^{-1}$	$\left[2(\alpha_1+\alpha_2)\left(n_1^{FB}-\tilde{n}_1^{FB}\right)\right]\mu^{-1}$	$p_1=c_1+N\alpha_1\mathrm{P}_1$	$p_2=c_2+N\alpha_2\mathrm{P}_2$	$\Delta p_{FB}=\Delta c+N\left(\alpha_1+\alpha_2\right)\mathrm{P}_1-N\alpha_2$
FREE-FREE	$\left[\Delta c-\Delta a+n_1\left(\alpha_1+\alpha_2\right)-N\alpha_2\right]\mu^{-1}$	$\left[(\alpha_1+\alpha_2)\left(n_1^{FF}-\tilde{n}_1^{FF}\right)\right]\mu^{-1}$	$p_1=c_1$	$p_2=c_2$	$\Delta p_{FF}=\Delta c$
FREE-PUB	Ψ_{FB}	Ψ_{FB}	$p_1=c_1$	$p_2=c_2+N\alpha_2\mathrm{P}_2-N\alpha_1\mathrm{P}_1$	Δp_{FB}
PRIV-FREE	$\left(\Delta c-\Delta a+2n_1(\alpha_1+\alpha_2)-N\alpha_2\right)\mu^{-1}+\frac{1}{1-\mathrm{P}_1}$	$\left[2(\alpha_1+\alpha_2)\left(n_1^{PF}-\tilde{n}_1^{PF}\right)\right]\mu^{-1}+\frac{1}{1-\mathrm{P}_1^{PF}}$	$p_1=c_1+\frac{\mu}{\left(1-\mathrm{P}_1\right)}+N\left(\alpha_1+\alpha_2\right)\mathrm{P}_1$	$p_2=c_2$	$\Delta p_{PF}=\Delta c+\frac{\mu}{\left(1-\mathrm{P}_1\right)}+N\left(\alpha_1+\alpha_2\right)\mathrm{P}_1$
PRIV-PRIV	$\left[\Delta c-\Delta a+3n_1\left(\alpha_1+\alpha_2\right)-N\left(\alpha_1+2\alpha_2\right)\right]\mu^{-1}+\frac{2\mathrm{P}_1-1}{\mathrm{P}_1\left(1-\mathrm{P}_1\right)}$	$\left[3(\alpha_1+\alpha_2)\left(n_1^{PP}-\tilde{n}_1^{PP}\right)\right]\mu^{-1}+\frac{2\mathrm{P}_1^{PP}-1}{\mathrm{P}_1^{PP}\left(1-\mathrm{P}_1^{PP}\right)}$	$p_1=c_1+\frac{\mu}{1-\mathrm{P}_1}+N\mathrm{P}_1(\alpha_1+\alpha_2)$	$p_2=c_2+\frac{\mu}{\mathrm{P}_1}+N\mathrm{P}_2(\alpha_1+\alpha_2)$	$\Delta p_{PP}=\Delta c+\left(2\mathrm{P}_1-1\right)\left[\frac{\mu}{\mathrm{P}_1\left(1-\mathrm{P}_1\right)}+N\left(\alpha_1+\alpha_2\right)\right]$
PRIV-PUB	Ψ_{FB}	Ψ_{FB}	$p_1=c_1+\frac{\mu}{1-\mathrm{P}_1}+N\mathrm{P}_1(\alpha_1+\alpha_2)$	$p_2=c_2+\frac{\mu}{\left(1-\mathrm{P}_1\right)}+N\alpha_2$	Δp_{FB}

Table A2 Results for scenarios with imperfect substitution and congestion