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Product diversity, demand structures and optimal taxation

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Product Diversity, Demand Structures and Optimal Taxation*

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Abstract

This paper studies optimal taxation in a general equilibrium model with endogenous entry. We compare the constant elasticity of substitution (CES) model to three alternative demand structures: oligopolistic competition in prices, oligopolistic competition in quantities, and translog preferences. Our economy is characterized by two distortions: a labor distortion due to the misalignment of markups on goods and leisure, and an entry distortion due to the misalignment of the consumer surplus effect and the profit destruction effect of entry. The two distortions interact in determining the wedge between the market-driven and optimal level of product diversity. We show how optimal labor and entry taxes depend upon the prevailing demand structure, the nature and size of entry costs, and the degree of substitutability between goods.

Keywords: product diversity, entry, oligopolistic competition, translog preferences, optimal taxation

JEL classification: E22, E61, E62

1 Introduction

How should taxes be set to obtain an optimal number of firms and products? This paper sheds light on the role of different demand structures for product diversity and the resulting implications for optimal taxation.

We consider a general equilibrium model with endogenous firm and product entry. Two distortions are present in our economy. First, there is an 'entry distortion'. This arises from the misalignment of two opposing effects of entry of a new firm and a differentiated product on welfare, which are not internalized by an individual entrant. On the one hand, more product diversity is welfare-enhancing ('consumer surplus effect'). On the other hand, a new entrant steals business from his competitors, who see demand for their products fall ('profit destruction effect'). The misalignment of these two effects gives rise to an inefficient number of entrants. The sizes of the

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consumer surplus effect and the profit destruction effect depend on the market structure and consumer preferences, both of which determine the shape of the demand function that a firm faces. Under constant elasticity of substitution (CES) preferences and entry costs in terms of a labor requirement, the consumer surplus and profit destruction effects offset each other.[1] As a result, the entry distortion is absent. However, if entry requires materials or if the firm's perceived demand function deviates from the CES structure, for instance in the case where entry compresses markups by intensifying competition, this knife-edge prediction no longer holds. Empirical evidence suggests that entry has a negative impact on markups in several industries.[2]

Second, insofar as firm entry is costly, markups on goods prices are efficient and indeed necessary for firms to cover entry costs and start to produce. As a consequence, a distortion of the leisure-consumption tradeoff arises from the absence of a tax on leisure (or a labor subsidy). In line with the literature, we call this the 'labor distortion'.

We contrast four different demand structures that have been used extensively in the literature: CES demand, oligopolistic competition in quantities, oligopolistic competition in prices, and translog preferences. While the CES demand structure is ubiquitous in dynamic macroeconomics since Dixit and Stiglitz (1977), the more recent literature on endogenous firm and product entry has considered alternative market structures or preferences. One branch assumes oligopolistic competition with strategic interactions between firms, e.g. Jaimovich and Floetotto (2008), Colciago and Etro (2010), and Colciago (2013). Another branch puts forward translog preferences to generate effects of product entry on demand elasticities, see in particular Bilbiie, Ghironi and Melitz (2008, 2012).[3] In addition to different demand structures, we consider two entry cost specifications that have been discussed in the aforementioned literature, a labor requirement and a materials requirement.

For the four demand structures and the two entry cost specifications, we derive the competitive market allocations without government interventions and contrast them with the First Best allocations. We then compute the optimal labor and entry taxes that implement the First Best allocation in the decentralized market economy. We show that optimal taxation crucially depends upon the nature of demand, the entry cost specification and the calibration of the model parameters, in particular the degree of competition/substitutability between different firms and goods.

Our main results are the following. The number of firms is inefficiently low under CES demand and inefficiently high under translog preferences. Under oligopolistic competition, entry is below

[1] See, Bilbiie, Ghironi, and Melitz (2008).

[2] See Domowitz, Hubbard and Petersen (1988), Bresnahan and Reiss (1991), and Campbell and Hopenhayn (2005).

[3] Lewis and Poilly (2012) compare oligopolistic competition with translog preferences in the context of monetary policy transmission.

(above) optimum if the substitution elasticity between goods is low (high). If firm creation is subject to a labor requirement, it is however always optimal to subsidize labor and to tax entry. This is because the labor subsidy by itself leads to too much entry, increasing consumer surplus by too little relative to the reduction in producer surplus through lower markups. If entry costs instead consist of materials (i.e., final output), entry is below its efficient level and must be subsidized in industries with highly differentiated goods, even after the labor distortion has been removed with an appropriate subsidy. The intuition for this result is that some of the additional output is used up in the creation of new firms and does not enter the consumption basket directly.

Our paper is related to a recent literature which considers optimal taxation in general equilibrium models with endogenous firm (or product) entry. This literature has analyzed different demand structures in isolation. Bilbiie, Ghironi and Melitz (2008) and Chugh and Ghironi (2012) consider demand-side complementarities, contrasting CES demand with translog preferences. Lewis (2010) – in an earlier version of this paper – and Colciago (2013) discuss supply-side complementarities, contrasting CES demand with Cournot and/or Bertrand competition.[4] Here, we analyze both demand- and supply-side complementarities and compare their policy implications. Our focus on the two static distortions removes a layer of complexity that is present in other contributions, such as Bilbiie, Ghironi and Melitz (2008) and Colciago (2013), where intertemporal distortions arising in a dynamic setting are taken into account. In addition, Chugh and Ghironi (2012) show the optimality of tax smoothing in the endogenous-entry framework of Bilbiie, Ghironi and Melitz (2012), which suggests that the inter-temporal distortions are less important than the static ones. We provide a comprehensive sensitivity analysis of our results to the nature of entry costs, demand structures and product substitutability that is absent in the above papers.

Our work is also related to Mankiw and Whinston (1986), who study optimal entry in a partial equilibrium framework, as well as a large trade literature surveyed in Mrázová and Neary (2013) and a vast endogenous growth literature, see for instance Grossman and Helpman (1991). In contrast to those contributions, the labor supply decision is endogenous in our model, giving rise to a labor distortion.

The remainder of the paper is structured as follows. In Section 2, we present the model. We first describe household and production choices in a general setup without any concrete specification of entry costs or of the demand function a firm faces. We then describe alternative specifications for entry costs and demand structures. In Section 3, we derive – in our general setup – the First Best allocation, contrast it with the competitive allocation without government interventions and present the optimal tax policy mix that implements the First Best allocation. Section 4 discusses

[4] Coto-Martinez, Garriga and Sanchez-Losada (2007) employ a model with CES demand.

the role of demand structures, the nature of entry costs, and the degree of substitutability across goods for the competitive and First Best allocations and for optimal taxation. In Section 5, we discuss the sensitivity of our findings with respect to parameters variations. We discuss the effects of changes in the size of the entry costs and the elasticity of labor supply. In Section 6, we suppose that only a restricted set of instruments is available to the policy-maker. First, we consider the case where lump-sum taxes are unavailable, such that a labor subsidy must be financed with an entry tax and vice versa. We find that the optimal entry tax is robust to this restriction. Second, we assume that only one distortionary tax instrument, an entry tax or a labor tax, is available. Section 7 concludes.

2 Model

We first describe household and production choices in a general setup. Second, we describe two different specifications for entry costs and four different specifications for the demand structure that the firm faces. The latter depends on the prevailing market structure and on the nature of preferences.

2.1 Households and Production

Households choose consumption C and hours worked L to maximize utility $U(C, L)$, subject to the budget constraint $C = (1 - \tau_L) WL + T$. Households receive labor income WL taxed at rate τ_L and lump-sum transfers T from the government. The utility function is increasing and concave in consumption, $U_C > 0$, $U_{CC} < 0$; decreasing and concave in labor, $U_L < 0$, $U_{LL} \leq 0$, and separable in its two arguments, $U_{CL} = 0$. Utility maximization leads to equality between the after-tax real wage $(1 - \tau_L)W$ and the marginal rate of substitution between labor and consumption,

$$(1 - \tau_L) W = -\frac{U_L}{U_C}. \tag{1}$$

The number of producers and differentiated goods is denoted by $N > 0$. The good j is produced with labor, $y_j = Zl_{Y,j}$, where Z is an economy-wide productivity index. Firm j maximizes profits $\left(\rho_j - \frac{W}{Z}\right) y_j - (1 + \tau_F) F$, where ρ_j is the price of good j relative to the price of a basket of goods, y_j is the demand for good j and marginal costs $\frac{W}{Z}$ are taken as given by the firm. The term $(1 + \tau_F) F$ captures per-period fixed production costs or entry costs, where τ_F is a tax on entry. Alternatively, we can consider a tax, τ_P, on operating profits $\left(\rho_j - \frac{W}{Z}\right) y_j$. All our results go through if $(1 + \tau_F)$ is replaced with $\frac{1}{1-\tau_P}$.

The firm's optimal price is set as a multiplicative markup μ_j over marginal cost,

$$\rho_j = \mu_j \frac{W}{Z}. \tag{2}$$

The markup depends on the market structure and the nature of preferences, as shown below. Because marginal costs are the same for all producers, relative prices are equal across firms in equilibrium and so are firm output and labor input per firm. We can therefore drop the j-subscript and write $\rho_j = \rho$, $y_j = y$ and $l_{Y,j} = l_Y$.

Intermediate goods are bundled into a final good Y. Under symmetry, the final goods bundle is the product of total output of all firms, Ny, and the relative price $\rho(N)$, i.e. $Y = \rho(N) Ny$. We allow the relative price to depend upon the number of competing firms and products. The functional form of $\rho(N)$ depends on the specific demand structure and will be derived for four alternative structures below. Using the production function to substitute out firm output y, we can express the aggregate production function as follows,

$$Y = \rho(N) Z N l_Y. \tag{3}$$

One unit of aggregate labor Nl_Y is transformed into ρZ units of the final good; this is the aggregate marginal rate of transformation. The marginal rate of transformation for an intermediate firm is Z. The relative price ρ drives a wedge between the aggregate and firm-level marginal rate of transformation. The aggregate production function (3) shows that if $\frac{\partial \rho}{\partial N} = \rho'(N) > 0$, there are increasing returns to product diversity.[5] This effect of entry on aggregate output is not internalized by an individual entrant. If $\rho'(N) = 0$, the aggregate final goods production function is linear in the number of products and the aforementioned effect disappears.

2.2 Entry

Free entry requires that average revenue equals average costs, such that profits are zero, i.e., formally, $\left(\rho(N) - \frac{W}{Z}\right) y = (1 + \tau_F) F$. Under symmetry, the zero-profit condition can be combined with price setting (2) and multiplied by the number of firms N to yield the aggregate free entry condition

$$\left(1 - \frac{1}{\mu(N)}\right) Y = (1 + \tau_F) NF. \tag{4}$$

Notice that the markup may depend on the number of producers. Entry costs can take two different forms. In the first specification, F is measured in terms of a labor requirement. More specifically, setting up a firm requires l_E labor units. Let F_L denote the exogenous entry cost in terms of effective labor units Zl_E, such that the pre-tax entry cost in terms of final output is $\frac{WF_L}{Z}$. In the second specification, entry costs per new firm are given by F_Y units of final output Y. Under the

[5] This feature has also been named 'love of variety' or 'increasing returns to specialization' in the literature.

two entry cost specifications, the aggregate free entry condition reads as, respectively,

$$\left(1-\frac{1}{\mu\left(N\right)}\right)Y=\left(1+\tau_F\right)N\frac{WF_L}{Z}, \tag{5a}$$

$$\left(1-\frac{1}{\mu\left(N\right)}\right)Y=\left(1+\tau_F\right)NF_Y. \tag{5b}$$

Henceforth, an equation labeled 'a' refers to the labor entry cost specification and an equation labeled 'b' refers to the materials entry cost specification. The aggregate zero profit condition (5) determines the number of entrants N, which in our static setup is equivalent to the number of producers and products.

2.3 Market Clearing, Government, Aggregate Resource Constraint

The labor market clearing conditions in the two specifications are, respectively,

$$L=\frac{NF_L}{Z}+Nl_Y, \tag{6a}$$

$$L=Nl_Y. \tag{6b}$$

The government finances lump-sum transfers with taxes on labor income and on entry. In Section 6 we drop the assumption that a lump-sum tax instrument is available. Our main insights remain robust. The government budget constraint is thus given by $\tau_L WL+\tau_F NF=T$. Using the firm's optimal pricing equation (2) to substitute out W and inserting the two specifications for entry costs F, we can express the government budget constraint as

$$\tau_L\frac{\rho(N)}{\mu(N)}ZL+\tau_F NF_L\frac{\rho(N)}{\mu(N)}=T, \tag{7a}$$

$$\tau_L\frac{\rho(N)}{\mu(N)}ZL+\tau_F NF_Y=T. \tag{7b}$$

The final goods market clearing conditions in the two specifications are, respectively,

$$Y=C, \tag{8a}$$

$$Y=C+NF_Y. \tag{8b}$$

Combining the aggregate production function (3) with the market clearing conditions for labor and goods (6) and (8), we obtain the aggregate resource constraint for the two specifications,

$$C=\rho\left(N\right)\left(ZL-NF_L\right), \tag{9a}$$

$$C=\rho\left(N\right)ZL-NF_Y. \tag{9b}$$

In the labor entry cost specification, entry costs are subtracted from the total labor input in the aggregate resource constraint (9a). In the materials entry cost specification, entry costs are instead subtracted from total output, see (9b).

2.4 Demand Structures

We consider four different setups regarding the market structure and consumer preferences that lead to different demand functions that an individual firm faces. First, monopolistic competition between many small firms à la Dixit and Stiglitz (1977), producing goods that are bundled in a constant elasticity of substitution (CES) aggregator; second, oligopolistic competition in prices or Bertrand competition; third, oligopolistic competition in quantities or Cournot competition. Fourth, we assume a translog preference structure as in Feenstra (2003).

Under a CES aggregator, the final goods bundle is given by $Y = (\int_0^N y_j^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ is the constant price-elasticity of demand in absolute value. The net markup is also constant at $\mu - 1 = \frac{1}{\theta-1}$. The relative price is $\rho(N) = N^{\frac{1}{\theta-1}}$.

We model oligopolistic competition within a two-layer production economy as in Devereux and Lee (2001). Final output is a CES bundle of many differentiated industry goods indexed by i on the unit interval, $Y = (\int_0^1 Y_i^{\frac{\omega-1}{\omega}} di)^{\frac{\omega}{\omega-1}}$, where $\omega > 1$. Industry goods Y_i, in turn, are a bundle of finitely many differentiated intermediate goods $y_{i,j}$ as follows,

$$Y_i = \left(\sum_{j=1}^{N} y_{i,j}^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}, \quad \lambda > 1. \tag{10}$$

Within each industry, there are N firms, each producing a differentiated intermediate good. Firms and intermediate goods carry the index i, j, where $j = 1, \ldots, N$. Let ω denote the elasticity of substitution between industry goods and λ the elasticity of substitution between goods within an industry. The demand for intermediate goods is

$$y_{i,j} = \left(\frac{P_{i,j}}{P_i} \right)^{-\lambda} \left(\frac{P_i}{P} \right)^{-\omega} Y, \tag{11}$$

where P, P_i and $P_{i,j}$ are the prices of final goods, industry goods and intermediate goods, respectively.

Under *Cournot* competition, intermediate goods firms set output to maximize profits subject to demand given by (11). Each firm takes into account how its production choice affects industry output, while taking as given the production levels of other firms in the industry and the output levels of other industries. The optimal net price markup under symmetry is

$$\mu(N) - 1 = \frac{\omega N + (\lambda - \omega)}{(\lambda - 1)\omega N - (\lambda - \omega)}. \tag{12}$$

Under *Bertrand* competition, intermediate goods firms set prices to maximize profits subject to demand given by (11). Each firm takes into account how its price setting affects its own industry's price, while taking as given the price choice of other firms in the industry and the price levels of other industries. The optimal net price markup under symmetry is

$$\mu(N) - 1 = \frac{N}{(\lambda - 1)N - (\lambda - \omega)}. \tag{13}$$

Under both Bertrand and Cournot competition, the markup is decreasing in the degree of substitutability λ and in the number of producers N for $\lambda > \omega$. If the between- and within-industry substitution elasticities are equal, $\omega = \lambda$, the negative effect of entry on markups under oligopolistic competition disappears and we revert to the CES structure. Broda and Weinstein (2006) estimate substitution elasticities between goods for different levels of aggregation. As they disaggregate product categories, goods varieties appear to be more substitutable to each other. This suggests that $\lambda > \omega$ is a reasonable assumption and we will make this assumption throughout the remainder of the paper. In a symmetric equilibrium in both the Cournot and Bertrand models, the industry price index P_i is equal to the final goods price P. The relative price is $\rho(N) = P_{i,j}/P = N^{\frac{1}{\lambda-1}}$.

Under a *translog* preference structure as in Feenstra (2003), the price-elasticity of demand is increasing in the number of differentiated goods, $1+\gamma N$, where $\gamma > 0$ measures the price-elasticity of the spending share on an individual good and both price-elasticities are expressed in absolute value. Thus, the optimal net price markup is decreasing in the number of goods and reads as

$$\mu(N) - 1 = \frac{1}{\gamma N}. \tag{14}$$

The relative price is related to the number of products through $\rho(N) = \exp(-\frac{1}{2}\frac{\tilde{N}-N}{\gamma \tilde{N} N})$, where $\tilde{N} > N$ is the (constant) mass of all conceivable goods.

The different demand structures are characterized by three key variables: the price markup $\mu(N)$, the relative price $\rho(N)$ and the benefit of variety $\zeta(N)$. The latter is defined as the elasticity of the relative price with respect to the number of firms, $\zeta(N) = \frac{\rho'(N)N}{\rho(N)}$. Table 1 shows, in the form of elasticities, how these three variables depend on the number of firms/products N, and on the parameter $\Theta \in \{\theta, \lambda, \gamma\}$, which captures – loosely speaking – the substitutability between the goods.

[insert Table 1 here]

First, we analyze the characteristics of the markup under the alternative demand setups. Consider the effect on the markup of a change either in competitive pressures due to firm entry or in the substitutability between goods due to product entry. This effect is measured by the elasticity of the markup with respect to the number of producers, $\varepsilon_{\mu,N}$. It is zero under CES demand and negative in the three other cases. We now turn to the elasticity of the markup with respect to substitutability $\varepsilon_{\mu,\Theta}$, holding the number of firms and goods constant. Under all four demand structures, this elasticity is negative. Intuitively, the more alike are the various goods, the lower is the market power of any individual producer (in the oligopolistic competition model), and the more price-elastic is the share of spending on any individual product (in the translog model). Hence, the higher is the degree of substitutability, the lower is the price markup.

Second, we look at the characteristics of the relative price and the benefit of variety under the four demand structures. Several results stand out. First, the benefit of variety, ζ, is positive in all cases. Second, in the CES, Bertrand and Cournot models, the benefit of variety is independent of product diversity, while under translog preferences, ζ instead depends negatively on N. Third, under CES demand, as well as Bertrand and Cournot competition, the relative price and the benefit of variety decrease with the degree of substitutability. Under translog preferences, the relative price and its elasticity with respect to the number of goods instead *increases* with Θ.

3 Distortions and Optimal Tax Policy

In the following, we first derive the First Best allocation and contrast it with the equilibrium allocation of the decentralized market economy. Second, comparing the two allocations allows us to characterize the distortions that fiscal policy should address. Third, we solve the optimal taxation problem and derive the tax rates which eliminate the distortions.

3.1 First Best Allocation

The First Best allocation is the solution to a social planner problem that maximizes household utility subject only to technological constraints. A formal definition is given next.

Definition 1. *The First Best allocation is a set* $\{C, L, N\}$*, which, given the exogenous variables* Z*,* F_L*, or* F_Y*, maximizes utility* $U(C, L)$*, subject to the resource constraint (9). First order conditions to the problem are, first, the intrasectoral efficiency condition,*

$$\rho(N)Z = -\frac{U_L}{U_C}, \tag{15}$$

and, second, the intersectoral efficiency condition,

$$\rho'(N)(ZL - NF_L) = \rho(N)F_L, \text{ or} \tag{16a}$$

$$\rho'(N)ZL = F_Y, \tag{16b}$$

if entry costs are specified in terms of labor units or in terms of materials, respectively. The variables $\rho(N)$ *and* $\rho'(N) = \frac{\partial \rho(N)}{\partial N}$ *are determined by the respective demand structure.*

The intrasectoral efficiency condition (15) states that the marginal rate of substitution between labor and consumption must equal the marginal rate of transformation of labor into final output.

The intersectoral efficiency condition (16) states that the cost of producing one additional firm must equal the benefits brought about by this extra firm, measured in final output units. The marginal cost of producing one additional firm (in terms of foregone consumption output) is given by, respectively, $\rho(N)F_L$ or F_Y if the creation of a firm requires labor input or final goods. The benefit of one extra firm is given by the rate at which a firm is transformed into final goods, that is $\rho'(N)(ZL - NF_L)(= \rho'(N)ZNl_Y)$ or $\rho'(N)ZL$, respectively.

Using the definition of the benefit of variety, $\zeta(N) = \frac{\rho'(N)N}{\rho(N)}$, the intersectoral efficiency condition (16) can be written as:

$$\zeta(N)\frac{C}{N} = \rho(N)F_L, \tag{17a}$$

$$\frac{\zeta(N)}{1-\zeta(N)}\frac{C}{N} = F_Y. \tag{17b}$$

Equation (17) shows that the term $\zeta(N)$ or $\frac{\zeta(N)}{1-\zeta(N)}$ scales the average consumption utility of each product and thus captures the consumer surplus effect of product diversity.[6]

3.2 Competitive Allocation

Next, we define the competitive allocation which prevails when, for a given set of tax policies, households maximize utility, firms maximize profits, and all markets clear. When using the term 'competitive equilibrium' we refer to a decentralized market equilibrium; we do not mean to imply that markets are perfectly competitive.

Definition 2. *A competitive allocation is a set* $\{C, L, N\}$*, which, given the exogenous variables* Z*,* F_L *or* F_Y *and the policies* τ_L*,* τ_F*, satisfies the following equilibrium conditions:*

$$\frac{1-\tau_L}{\mu(N)}\rho(N)Z = -\frac{U_L}{U_C}, \tag{18}$$

$$(\mu(N)-1)\frac{C}{N} = (1+\tau_F)\,\rho(N)F_L, \text{ or} \tag{19a}$$

$$(\mu(N)-1)\frac{C}{N} = (1+\mu(N)\tau_F)\,F_Y, \tag{19b}$$

and the resource constraint (9), where the prices $\rho(N)$ *and* $\mu(N)$ *are determined by the respective demand structure.*

Equation (18) is obtained by combining the optimal labor supply condition (1) with the firm's optimal pricing equation (2). Equation (19) are obtained by combining the aggregate free entry

[6] Notice that, if entry costs are in terms of materials, values for the benefit of variety equal to or greater than unity are not admissible, since they would violate the condition that average consumption utility must be strictly positive, $\frac{C}{N} > 0$. Therefore, we restrict attention to allocations that satisfy $\zeta(N) < 1$.

condition (5) with goods market clearing (8). In order to compare the competitive equilibrium without government interventions to the First Best allocation, we set taxes to zero, $\tau_F = \tau_L = 0$, in (18) and (19). The allocations differ through two distortions, the 'labor distortion' and the 'entry distortion'. We discuss these two distortions in turn.

First, comparing (15) and (18) shows that the markup $\mu(N)$ drives a wedge between the marginal rate of substitution between labor and consumption and the marginal rate of transformation. This wedge has been called the 'labor distortion', see e.g. Bilbiie, Ghironi and Melitz (2008) for an extensive discussion. It derives from the fact that goods are priced at a markup, while leisure is not. In the presence of entry costs, price markups do not per se represent an inefficiency, but are instead required to cover these costs. It is rather the misalignment between markups on goods and on leisure that leads to an inefficiently high consumption of leisure (i.e. too little labor supply) in equilibrium. The optimal policy response is a labor subsidy equal to the size of the net markup on goods prices.

Second, comparing the intersectoral optimality conditions (17) and (19), we see that there is a wedge between the cost and the benefit of setting up an additional firm. This 'entry distortion' derives from a misalignment between the 'profit destruction effect' and the 'consumer surplus effect'. The latter describe two countervailing effects of a change in the number of firms and products on welfare, which the individual entrant does not internalize. First, firm entry affects markups and thus profits negatively. This is known as the 'profit destruction' or 'business stealing' effect and is captured by the net markup $\mu(N) - 1$. Second, product entry raises consumer surplus due to the preference for product diversity. This 'consumer surplus effect' is captured by the benefit of variety $\zeta(N)$ in the first specification and by $\frac{\zeta(N)}{1-\zeta(N)}$ in the second specification. Absent the labor distortion, the number of firms is optimal if the profit destruction and consumer surplus effect just offset each other, as shown by several authors, including Bilbiie, Ghironi and Melitz (2008) and Colciago (2013). If the profit destruction effect is smaller than the consumer surplus effect, the number of firms is inefficiently low, and vice versa.

3.3 Optimal Tax Policy

We now define the optimal combination of labor and entry taxes that prevails when lump-sum taxes are available to the government in order to balance its budget. The case without lump-sum taxation is examined in Section 6.

Definition 3. *The optimal tax policy mix is a set $\{\tau_L, \tau_F\}$, which, given the exogenous variables Z, F_L or F_Y, maximizes utility $U(C, L)$, subject to the resource constraint (9) and the equilibrium conditions of the market economy (18), (19). Lump-sum taxes, T, are adjusted appropriately to* *satisfy the government budget constraint (7).*

The following proposition states that the tax policies defined above are Pigovian in the sense of achieving the efficient or First Best allocation.[7]

Proposition 1. *The policy maker can implement the First Best allocation by subsidizing labor at a rate equal to the net goods price markup, i.e.*

$$1 - \tau_L = \mu(N). \tag{20}$$

and by setting the entry tax equal to

$$1 + \tau_F = \frac{\mu(N) - 1}{\zeta(N)}, \text{ or} \tag{21a}$$

$$1 + \tau_F = \frac{\mu(N) - 1}{\mu(N)\zeta(N)}, \tag{21b}$$

depending on the prevailing entry cost specification.

Proof. Inserting the optimal tax rates (20) and (21) in the equilibrium conditions of the competitive economy (18) and (19) yields the efficiency conditions (15) and (16). □

Two distortionary tax instruments are needed because there are two wedges: the markup misalignment between consumption and leisure, which is eliminated through a labor income subsidy, and the entry distortion due to complementarities between different firms/goods, which is addressed through an entry tax or subsidy. According to equation (21), the sign of τ_F depends on the markup, $\mu(N)$, and the benefit of variety, $\zeta(N)$, which in turn both depend on the prevailing demand structure.

4 The Role of Demand Structures

This section discusses the role of demand structures for the competitive and the First Best allocations and for optimal taxation. Table 2 exhibits the competitive equilibrium conditions without taxes and the First Best efficiency conditions, under a particular assumption about the utility function $U(C, L)$. More specifically, we consider logarithmic consumption utility such that $U_C = 1/C$ and set $U_L = -L^{\eta}$, where $\eta \geq 0$ is the inverse Frisch elasticity of labor supply to the real wage.

[insert Table 2 here]

To start with, consider labor and consumption. A positive value of the benefit of variety $\zeta(N)$ implies that labor is lower in the competitive allocation than in the First Best allocation for a

[7] Variants of Proposition 1, applied to different model environments, can be found in Bilbiie, Ghironi and Melitz (2008), Chugh and Ghironi (2012) and Colciago (2013).

positive and finite labor supply elasticity η. To be compensated for the higher disutility of work, households must consume more in the First Best compared to the competitive allocation. Thus, labor and consumption in the competitive allocation fall short of their optimal levels. This result is independent of the demand structure, the calibration and the type of entry costs.

In contrast, the discrepancy between the numbers of firms in the competitive equilibrium without taxes and in the First Best allocation depends on the nature of entry costs, the prevailing demand structure and the degree of substitutability between goods. We proceed by analyzing how the degree of substitutability as measured by the parameter $\Theta \in \{\theta, \lambda, \gamma\}$, embedded in $\mu(N)$, $\rho(N)$, and $\zeta(N)$, influences optimal product diversity, deviations therefrom in the competitive equilibrium, and optimal tax policies. We do this analytically where possible and numerically otherwise.

4.1 Labor Entry Costs

We first consider entry costs in terms of a labor requirement. The number of firms in the competitive equilibrium is lower in industries where the substitutability between goods is higher.[8] The reason is that in those industries, markups and profits are lower, resulting in a diminished incentive to enter the market. To see this, notice from Table 2 that N depends positively on the markup, which in turn is a decreasing function of the substitutability parameter Θ. The First Best number of firms is a positive function of the benefit of variety, see Table 2. Recall that the latter depends negatively on the degree of substitutability across goods. For highly differentiated goods, the benefit of variety and thus the effect of product entry on consumer surplus is strong; it is therefore efficient to have many firms. As goods become more substitutable, the benefit of variety falls and with it the optimal number of firms. To summarize, the number of firms in both allocations is high in industries where goods are very substitutable.

Next, we investigate whether the number of firms in the competitive allocation is higher or lower than in the First Best. Under CES demand, the number of firms in the competitive equilibrium is always inefficiently low. This is due to the labor distortion alluded to above. The more substitutable are the goods, the smaller is the markup and hence the smaller is the labor distortion. In this demand specification, the profit destruction and consumer surplus effect cancel out, $\mu - 1 = \zeta$, since both are equal to $\frac{1}{\theta - 1}$.

In contrast, under oligopolistic competition, the number of firms in the competitive equilibrium may be higher or lower than in the First Best. The reason is that the labor and entry distortion affect the discrepancy between the two in opposite directions. On the one hand, the profit destruction effect is always larger than the consumer surplus effect, which – in isolation –

[8] In the appendix we compute the elasticity of the number of firms to the degree of substitutability Θ.

implies that the number of firms in the competitive equilibrium is too high. To see this, compare the net markups in (12) and (13) with the benefit of variety $\zeta = \frac{1}{\lambda - 1}$. On the other hand, the labor distortion (the net markup $\mu - 1$) depresses entry compared to the First Best. Which of the two distortions dominates depends on the elasticity of substitution λ. One can show that there exists a threshold for λ in both the Cournot and Bertrand models below which the number of firms in the competitive equilibrium is lower than in the First Best.[9] The reason is that for highly differentiated goods, the markup and hence the labor distortion is large and dominates the misalignment between the profit destruction and consumer surplus effects. For very substitutable goods, though, this misalignment outweighs the labor distortion, resulting in excessive entry.

Under translog preferences, the profit destruction effect is $\mu(N) - 1 = \frac{1}{\gamma N}$ and the consumer surplus effect is $\zeta(N) = \frac{1}{2\gamma N}$. Therefore, similarly to oligopolistic competition, the profit destruction effect is always larger than the consumer surplus effect, leading – in isolation – to excessive entry. The misalignment between the two effects (the entry distortion) is constant and equal to 2. In contrast, the labor distortion depresses entry compared to the First Best. Recall that, the smaller is the price elasticity of the spending share γ, the larger is the labor distortion. This suggests that insufficient entry obtains for small values of γ.

The upper block of Table 3 shows the optimal tax rates under the alternative demand structures in the case of labor entry costs. From the table, the following result emerges.

Result 1. *Under labor entry costs, the optimal labor tax is negative under all demand structures, given a positive net markup. At the same time, the optimal entry tax is positive, except in the CES case, where it is zero.*

Notice that the net markup and with it the optimal labor subsidy ($-\tau_L$) is decreasing in the degree of substitutability $\Theta \in \{\theta, \lambda, \gamma\}$. In the CES model, the labor distortion is the only inefficiency; thus, the only instrument needed is the labor subsidy, and the optimal entry tax is zero. In the Cournot and Bertrand models, entry would be above its optimal level after correcting for the labor distortion, i.e. setting $\tau_L = 1 - \mu(N)$ and $\tau_F = 0$. Consequently, entry has to be taxed. Finally, under translog preferences, the entry tax is constant and equal to unity. This is due to the constancy of the misalignment between the profit destruction and the consumer surplus effect under that preference structure.

To sum up, the qualitative policy implications are similar under the four demand structures: labor should be subsidized and entry should be taxed (or rather, neither taxed nor subsidized under CES demand). This conclusion is, at a first glance, surprising if one looks only at the number of firms relative to its optimal level, which is very different across the four model variants. However,

[9] See appendix for details.

once the labor tax is set to its optimal value, zero entry taxes would lead to inefficiently high firm entry if a firm's perceived demand function deviates from the CES structure.

To illustrate how the model-specific parameters θ, λ and γ affect the distortions, we solve the model numerically and graph its predictions. Besides the graphical illustration of our findings, the numerical analysis is motivated by the fact that in the alternative case of materials entry costs, an analytical solution does not always exist. Therefore, for the sake of comparison, we solve the model numerically under both entry cost specifications.

We calibrate the parameters as follows. Suppose that the across-industry substitution elasticity ω is normalized to unity as in Devereux and Lee (2001) and Colciago and Etro (2010). For the elasticity of substitution θ in the CES model and within-industry substitution elasticity λ in the oligopoly models, we follow Broda and Weinstein (2006), who estimate elasticities between 1.2 (footwear) and 17 (crude oil). The price-elasticity of the spending share γ in the translog model is assumed to lie in the unit interval as in Bilbiie, Fujiwara and Ghironi (2011). Productivity Z is normalized to unity and the ratio $\frac{F_L}{Z} = 0.0038$ is set to match the value of legal entry fees for the US as a fraction of output per worker, see Barseghyan and DiCecio (2011).[10] The entry cost parameter F_Y is calibrated to generate equality of the number of entrants in the competitive equilibrium under CES demand with $\theta = 6$ across the two entry costs specifications. The resulting value is $F_Y = 0.0081$. Finally, we set the Frisch elasticity of labor supply to $1/\eta = 1$.

Figure 1 confirms our analytical results graphically.

[insert Figure 1 here]

The top row shows the First Best number of firms in logarithms, $n^* = \ln N^*$, and in the competitive equilibrium, $n = \ln N$, as a function of the model-specific parameters θ, λ and γ. The number of firms is a declining function of substitutability. Under oligopolistic competition, the n-curve lies below (above) the n^*-curve for small (large) values of λ. The parameter range characterized by insufficient entry is greater under Bertrand than under Cournot competition. Under translog preferences, the n-curve lies above the n^*-curve for the whole admissible range of γ, given our calibration. The middle row depicts the consumer surplus effect and the profit destruction effect. The bottom row in Figure 1 shows how the optimal tax rates depend upon the parameter Θ. The more similar the goods, the greater is the required entry tax in the oligopolistic competition models. The reason is that the misalignment between the profit destruction and consumer surplus effects increases in the degree of substitutability λ.

[10]For empirical estimates of entry costs, see Barseghyan and DiCecio (2011) and the references therein.

4.2 Materials Entry Costs

Let us now consider entry costs in terms of materials. Under CES demand, as well as Bertrand and Cournot competition, the number of firms is unambiguously decreasing in the degree of product substitutability. Under translog preferences, the sign of the effect of γ on the number of goods cannot be determined analytically; however, for reasonable parameter values, the competitive number of firms falls as γ increases. The First Best number of firms also falls as Θ increases. This can be shown analytically in the CES, Bertrand and Cournot model. Under translog preferences, this is true for reasonable parameter values.[11]

We now investigate how taxes should be set optimally. The lower block of Table 3 shows the optimal tax rates.

Result 2. *Under materials entry costs, the optimal labor tax is negative under all demand structures, given a positive net markup. Under a CES demand structure, an entry subsidy is optimal. Under the alternative demand structures, the sign of the entry tax is ambiguous and depends on the degree of substitutability across goods.*

In order to understand this result, we investigate whether the number of firms in the competitive allocation without taxes is higher or lower than in the First Best. Under CES demand, the number of firms in the competitive equilibrium is inefficiently low. As in the alternative entry cost specification, the discrepancy decreases in θ. Notice, however, that now the number of firms in the competitive allocation is suboptimally low even absent the labor distortion. The reason is that the profit destruction effect $\mu - 1 = \frac{1}{\theta - 1}$ is smaller than the consumer surplus effect $\frac{\zeta}{1-\zeta} = \frac{1}{\theta-2}$. This implies that, even when labor is subsidized appropriately, there remains an entry distortion leading to too few firms. Hence, an entry subsidy is needed for all admissible values of θ.

A closed form solution for the number of firms in the competitive equilibrium under the demand structures Cournot, Bertrand, and translog as well as the First Best allocation under a translog demand structure cannot be derived.[12] It is also not possible to determine analytically the sign of τ_F. To analyze whether the competitive number of firms is above or below its efficient level and whether entry should be taxed or subsidized, we resort to numerical solutions, which are shown in Figure 2.

[insert Figure 2 here]

Qualitatively, many results are similar to those in the labor entry cost specification. Under translog preferences, the profit destruction effect is again stronger than the consumer surplus effect over the

[11]The elasticity of the number of firms with respect to the generic substitutability parameter Θ is again derived in the appendix.

[12]The solutions for the number of firms in the competitive and the First Best allocation under the alternative demand structures are shown in Table A in the appendix.

whole admissible range for the price-elasticity of the spending share γ. This misalignment leads, in isolation, to excessive entry, while the labor distortion instead implies, in isolation, insufficient entry. The misalignment distortion dominates the labor distortion. In contrast to the labor entry cost specification, the misalignment between the profit destruction and the consumer surplus effect is not constant but increases in the degree of substitutability γ. The dominance of the profit destruction effect under our calibration gives rise to positive optimal entry taxes under translog preferences which now increases in γ (see lower right panel of Figure 2).

Under oligopolistic competition, there are too few firms if goods are highly differentiated (the elasticity of substitution λ is low) and too many firms if goods are rather similar (λ is high). This can be explained by two effects. First, the labor distortion or net markup is declining in the degree of substitutability λ. Thus, the discrepancy is large for small values of λ. Second, the consumer surplus effect tends to be higher than the profit destruction for small values of the substitutability parameter λ, see middle row in Figure 2. Consequently, an entry subsidy (tax) is warranted for industries with highly differentiated (substitutable) goods, i.e. for low (high) λ.

To summarize, we have analyzed the competitive and efficient levels of product diversity under several demand structures. The discrepancy between the two was ascribed to either the labor or the entry distortion, or usually a combination of the two. From the discussion of the distortions followed a policy recommendation of how to set labor and entry taxes appropriately, given that both instruments and lump-sum taxes are available.

4.3 Discussion

The preceding analysis has shown that the size and sign of the optimal entry tax depends on three things: the nature of entry costs, the demand structure and the degree of product substitutability. In this regard, which model features are the most relevant empirically?

The literature provides several *entry cost* estimates, based on a variety of models and assumptions. For instance, the World Bank's Doing Business project (www.doingbusiness.org) estimates the number of days it takes to set up a business, and converts this number into a measure of entry costs using GDP data. That approach is closer to our interpretation of entry costs as a material requirement, since it uses production data rather than wage data to gauge the opportunity costs of an entrepreneur's time. Bollard, Klenow and Li (2013) however argue that the Doing Business data capture government-imposed entry costs and find that these represent only a small part of total entry costs. Empirical support for labor-related entry costs is provided in Domowitz, Hubbard and Petersen (1988), who find that a major share of fixed costs is related to overhead labor, advertising, and central office expenses. Bollard, Klenow and Li (2013) argue that "if the choice is between fixed entry costs in terms of labor or output, our evidence favors denominating entry costs in terms of labor". They reason that a labor entry cost specification is consistent with, and indeed the most plausible candidate explanation for, their key finding that entry costs rise with the level of development.

As for the *demand structure* facing a firm, we regard CES demand with its constant markups and knife-edge predictions as a theoretically appealing, but empirically less relevant case. Estimated price-cost margins vary over the business cycle. The jury is still out on the cyclicality of markups and their likely determinants. A large body of literature finds evidence for countercyclical markups, e.g. Bils (1987) and Rotemberg and Woodford (1999), while there is competing evidence of procyclicality, see Nekarda and Ramey (2013). Countercyclical responses of markups to technology shocks, government spending shocks and monetary policy shocks are reported, respectively, in Colciago and Etro (2010), Monacelli and Perotti (2008) and Lewis and Poilly (2012). While the CES model can be reconciled with countercyclical markups when prices are sticky, Lewis and Poilly (2012) show that price stickiness is not sufficient to generate the countercyclical response of the markup to a monetary policy shock that is observed in the data. Variations in demand structures of the type analyzed here might be more appealing in explaining markup fluctuations.[13] On the one hand, the procyclicality of entry has been documented in early work by Chatterjee and Cooper (1993) and Portier (1995) for firms and, more recently, by Broda and Weinstein (2010) for products. On the other hand, there is compelling evidence for a negative relationship between entry and markups. Domowitz, Hubbard and Petersen (1988) report a positive effect of concentration on markups. In addition, Bresnahan and Reiss (1991) and, more recently, Campbell and Hopenhayn (2005) find that markups depend negatively on the number of competitors in an industry.

Once we depart from the CES aggregator, which of the other demand structures has stronger empirical support: one based on translog preferences or one based on oligopolistic competition? Lewis and Poilly (2012) show that oligopolistic competition of the type examined here does not generate sufficient variation in the markup in response to a monetary policy shock, while translog preferences do a better job in this respect. As shown by Bilbiie, Ghironi and Melitz (2012), the endogenous-entry model with translog preferences succeeds at replicating key business cycle moments in US data, while generating procyclical profits and countercyclical markups. In contrast, the canonical real business cycle model with CES preferences fails in the latter two dimensions. Bilbiie, Ghironi and Melitz (2012) favor translog preferences as a demand-side explanation of countercyclical markups rather than strategic interactions between firms as a supply-side explanation. They argue, first, that the impact of firm entry and exit on industry-wide markups may be lim-

[13]There are a number of alternative explanations of markup countercyclicality, which will not be discussed here.

ited by the fact that such firms are typically small. Second, they appeal to evidence in Bernard, Redding and Schott (2010) and Broda and Weinstein (2010) by which new goods brought on the market by multi-product firms matter more for output fluctuations than those of entirely new firms.

We have shown the degree of *product substitutability* is crucial for the optimal entry tax rate. As mentioned above, our calibration range for the elasticity of substitution between goods θ is based on empirical evidence in Broda and Weinstein (2006). Given an interval for θ between 2.1 and 18, the corresponding net markup in the CES model ranges from close to 100 percent to 6 percent, which is broadly consistent with empirical estimates. Out of fifty sectors in the US, Christopoulou and Vermeulen (2008) estimate markups below 12 percent for seven sectors and markups above 100 percent for only three sectors. Hall (1986) reports estimates of substantial markups; in many US industries markups are above 50 percent, which is consistent with a high degree of product differentiation.

There is much disagreement on the correct specification of entry costs, which nonetheless matters greatly for our results. Recent evidence favors the labor entry cost specification. While both types of entry costs may matter, our conjecture is that, as production evolves from heavy industry to modern services, material inputs become less important and labor-related expenses matter more for startup costs. Empirical evidence for time-variation of price-cost markups reduces the appeal of the CES model, which predicts that markups are constant. If we model *product* entry with the translog framework and *firm* entry with the oligopolistic competition model, our findings suggest that product entry should be taxed, whereas firm entry in oligopolistic industries with low substitutability should be subsidized.

5 Sensitivity Analysis

Until now, we have discussed the sensitivity of optimal tax policies with respect to the nature of demand, the specification of entry costs and the degree of substitutability between goods. This section discusses the effect of variations in the two remaining key parameters of the model on our findings: first, the size of the entry costs F_L or F_Y, and second, the inverse labor supply elasticity η. Another parameter that affects our findings is the productivity shifter Z. However, as Table 2 shows, a change in Z affects the number of firms in both allocations, and thus the optimal conduct of tax policy, in the same way as a change in F_L or F_Y with the opposite sign.

5.1 Size of Entry Costs

The number of firms in both the First Best allocation and the competitive allocation without government interventions depends negatively on entry costs F_L or F_Y (and thus positively on productivity Z).[14]

Figure 3 shows how the model predictions change when *labor entry costs* rise.

[insert Figure 3 here]

Under all demand structures, a rise in the entry cost shifts the n- and n^*-curves down. Under CES demand, the curves shift down by the same amount. The profit destruction and consumer surplus effects are constant and thus the entry distortion is unaffected by the change in entry costs. Since the markup is constant, the labor distortion also remains unaltered. As a consequence, the optimal taxes are the same as before. Under oligopolistic competition, the n^*-curve shifts down by more than the n-curve, such that a rise in entry costs enlarges the parameter region characterized by excess entry. The markup rises through the fall in the number of firms and therefore, the labor distortion increases, see the middle row of Figure 3. The profit destruction effect shifts up, while the consumer surplus effect is constant under oligopolistic competition as the benefit of variety does not depend on the number of firms. Hence, the entry distortion, i.e. the gap between $\mu - 1$ and ζ, widens for high degrees of substitutability, increasing the optimal entry tax. The optimal labor subsidy increases since the competitive number of firms drops, which puts upward pressure on the markup and thus on the labor distortion. Under translog preferences, the n-curve shifts down by more than the n^*-curve. The profit destruction and consumer surplus effects shift up by the same amount; the entry distortion is constant in the translog demand structure. As a result, the optimal entry tax rate is unchanged. However, the optimal labor subsidy rises due to the increase in the labor distortion.

Next, we analyze the effect of a change in *materials entry costs* F_Y on the number of firms in equilibrium and on the optimal tax rates. Figure 4 shows the results.

[insert Figure 4 here]

The n- and n^*-curves shift up under all four demand structures. Under CES demand, the shift of both curves is identical and inversely proportional to the change in F_Y. As in the labor entry cost specification, the entry and labor distortions are unaffected, as are the optimal tax rates when entry costs change. Under oligopolistic competition, the n^*-curve shifts up by more than the n-curve,

[14] The appendix provides detailed analytical results on how a change in entry costs F_L or F_Y affects the number of firms in the competitive allocation without taxes and in the First Best allocation. More precisely, we compute the percentage change of the number of firms in the two allocations when entry costs change by one percent. Using these elasticities, we compare the effects of a change in entry costs on the number of firms across the two allocations.

implying that there is now a larger parameter range (for small elasticities of substitution) which is characterized by insufficient entry. In that range, entry must be subsidized. In the translog case, the n-curve shifts up by more than the n^*-curve, such that the necessary entry tax rises. For smaller values of the entry cost parameter, there exists a certain parameter range, for small γ, where we observe insufficient entry.

5.2 Labor Supply Elasticity

In order to discuss the effect of variations in the inverse labor supply elasticity η, consider again Table 2. The number of firms in the First Best allocation is higher, the more elastic is the supply of labor, i.e. the lower is η. The competitive number of firms is, however, unaffected by η. If labor supply is perfectly inelastic such that the inverse Frisch elasticity goes to infinity ($\eta \to \infty$), labor in the First Best allocation approaches the solution in the competitive equilibrium. In the opposite case of a perfectly elastic labor supply ($\eta = 0$), labor in the First Best and thus the gap with the competitive equilibrium is maximized. As labor supply becomes more elastic (η falls), labor in the First Best allocation rises and with it the efficient number of firms. Thus, as we decrease η, the n^*-curve shifts up in both Figures 1 and 2. In the case of oligopolistic competition, this implies a larger range of parameter values for which entry is below the optimum and should be subsidized.

To summarize the findings of the preceding sensitivity exercises, neither a change in entry costs nor a change in elasticity of labor supply affects our results qualitatively. A finding worth noting is that a rise in the labor supply elasticity or a fall in entry costs enlarges the parameter region for which the optimal policy is to subsidize labor *and* entry under oligopolistic competition when entry costs are specified as materials .

6 Restricted Instrument Set

Until now, we have assumed that two distortionary instruments, as well as lump-sum taxes, are available to the policy maker. Given that there are two distortions in our economy, the optimal policy can then always implement the First Best allocation. We now carry out an additional exercise in which we suppose that only two out of the three tax instruments are available.

6.1 No Lump-Sum Taxes

We first consider the case where lump-sum taxes are unavailable. While one of the distortionary taxes can be used to address one of the distortions, the other tax must be set to satisfy the government budget constraint.

Definition 4. *The optimal tax policy mix in the absence of lump-sum taxes is a set* $\{\tau_L, \tau_F\}$*, which, given the exogenous variables* Z*,* F_L *or* F_Y*, maximizes utility* $U(C, L)$*, subject to the resource constraint (9), the equilibrium conditions of the market economy (18), (19), the government budget constraint (7) and the restriction that* $T = 0$*.*

The optimal labor and entry taxes are displayed in Figure 5, again as a function of the substitutability across goods. By way of comparison, we also plot in the same figure the optimal tax rates of the three-instrument scenario. The most striking result, which we observe under both labor entry costs and materials entry costs, is that the optimal entry tax in the absence of lump-sum taxes is almost identical to the value in the three-instrument case. The optimal labor tax, however, differs substantially between the two scenarios.

[insert Figure 5 here]

What happens in the labor entry cost specification? The CES demand structure is again an exception; here both tax instruments are set to zero, the policy maker cannot improve upon the competitive allocation. Under oligopolistic competition and translog demand, an entry tax remains optimal. The revenue obtained from this entry tax is however, too limited to subsidize labor appropriately. The government implements a smaller than optimal labor subsidy.

In the materials entry cost specification, the optimal entry tax is negative in the CES case and under oligopolistic competition for low degrees of product substitutability. Under oligopolistic competition with high degrees of product substitutability, and under translog demand, the optimal entry tax is instead positive. This result is the same as in the three-instrument case.

The most notable result of this exercise is that the policy recommendations regarding entry taxes remain valid even if lump-sum taxes are not available. This finding is consistent with Chugh and Ghironi (2012) and Colciago (2013).

6.2 Only One Distortionary Tax

Consider now the scenario where lump-sum taxes are available but only one of the two distortionary tax instruments, the labor tax or the entry tax.[15]

Definition 5. *The optimal labor (entry) tax in the absence of entry (labor) taxes is a set* $\{\tau_L, T\}$ *(*$\{\tau_F, T\}$*), which, given the exogenous variables* Z*,* F_L *or* F_Y*, maximizes utility* $U(C, L)$*, subject to the resource constraint (9), the equilibrium conditions of the market economy (18), (19), the government budget constraint (7) and the restriction that* $\tau_F = 0$ *(*$\tau_L = 0$*).*

[15] In the terminology of Chugh and Ghironi (2012), this constitutes an *incomplete tax system*, since there is less than one tax instrument per independent margin of adjustment.

In general, policy cannot decentralize the First Best outcome. A special case is worth noting. Under CES demand and labor entry costs, the profit destruction and consumer surplus effects just offset each other, such that there exists no entry distortion. Only the labor income tax is needed and its optimal value is $\tau_L = -\frac{1}{\theta-1}$, as in the three-instrument case.

The resulting tax rates in the general case are plotted in Figure 6 as a function of the degree of product substitutability, for the different demand structures considered. By way of comparison, we also plot in the same figure the optimal tax rates of the three-instrument scenario. Let us first discuss the top row, which refers to the model with labor entry costs.

[insert Figure 6 here]

Consider the optimal labor tax that obtains when entry taxes are not available. Under all four demand structures, a labor subsidy is warranted on the entire range of values considered for the substitutability parameter Θ, as in the case with two distortionary tax instruments. Labor supply, which is suboptimally low in the absence of fiscal instruments, is boosted, and so is consumption and the number of entrants.

What if labor subsidies are not implementable? In the CES case, it is optimal to subsidize entry, which is inefficiently low because of the labor distortion. Recall that in the three-instrument case, the entry tax is optimally set to zero. Under Bertrand and Cournot competition, subsidizing entry is recommended for industries with low substitutability across goods (low λ). Recall that for low values of λ and in the absence of fiscal policy, the number of firms in the competitive equilibrium is too low compared to the First Best, see Figures 1 and 2. By subsidizing entry, the optimal policy brings the number of firms closer to efficiency. Notice the contrast with the three-instrument scenario with labor entry costs. There, entry is inefficiently *high* over the whole range of admissible values of the degree of substitutability Θ once we have corrected the misalignment between the markups on goods and leisure by taxing leisure appropriately. This makes an entry *tax* necessary. Finally, under a translog demand structure it is optimal to tax entry, which is suboptimally high in the no-taxes case since the entry distortion outweighs the labor distortion, see again Figures 1 and 2.

In the bottom panel of Figure 6, we depict the optimal tax rates in the restricted instruments scenario, under the materials entry cost assumption. Qualitatively, the policy recommendations are similar to the three-instrument scenario.

To summarize, we obtain an interesting result which says that, if labor subsidies are not implementable and entry costs are labor costs, firm creation should be subsidized in industries characterized by oligopolistic competition between firms producing highly differentiated goods. An appropriately set labor subsidy turns this policy recommendation on its head by making an entry tax necessary in this setting.

7 Conclusion

This paper analyzes optimal tax policies in a general equilibrium model with an endogenous number of firms and products. Market entry is subject to a cost, which can be specified as a labor requirement or in terms of materials (lost output). Due to the presence of this entry cost, the firm must charge a markup and generate profits in order to break even. The markup on goods prices and the absence of a markup on leisure distorts the consumption-leisure tradeoff, such that labor, consumption and the number of entrants are below their optimal levels. This 'labor distortion' is present regardless of the specifics of goods demand. A second distortion ('entry distortion') stems from the misalignment of the profit destruction effect and the consumer surplus effect. The nature and size of this distortion depends upon the demand structure faced by the firm. We consider four cases that have been discussed in the literature: constant elasticity of substitution (CES), Cournot competition, Bertrand competition, and translog preferences. In the labor entry cost specification, we find both insufficient and excess entry, depending on the demand structure and the degree of substitutability across goods. However, once labor supply is subsidized to remove the misalignment between the markups on goods and leisure, excess entry obtains, leading to an entry tax being optimal. An exception is the CES case, where no entry tax is required. Under oligopolistic competition, the optimal entry tax is increasing in the parameter measuring substitutability, while it is constant under translog preferences. The policy implications are rather different if entry costs are specified in terms of materials. Under CES demand and in the Cournot and Bertrand models, an entry subsidy is warranted in industries with highly differentiated goods. In the translog model, an entry tax remains optimal, but its value increases with the degree of product substitutability. In general, as goods become more substitutable, labor must be subsidized less and entry must be taxed more.

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Table 1: Characteristics of Demand Structures

	Markup	Relative Price	Benefit of Variety
Elasticity to Entry	$\varepsilon_{\mu,N} = \frac{\partial\mu/\mu}{\partial N/N} \leq 0$	$\zeta = \frac{\partial\rho/\rho}{\partial N/N} > 0$	$\varepsilon_{\zeta,N} = \frac{\partial\zeta/\zeta}{\partial N/N} \leq 0$
CES	0	$\frac{1}{\theta-1}$	0
Bertrand	$-\frac{\lambda-\omega}{[(\lambda-1)N-(\lambda-\omega)][\lambda N-(\lambda-\omega)]}$	$\frac{1}{\lambda-1}$	0
Cournot	$-\frac{\lambda-\omega}{(\lambda-1)\omega N-(\lambda-\omega)}$	$\frac{1}{\lambda-1}$	0
Translog	$-\frac{1}{1+\gamma N}$	$\frac{1}{2\gamma N}$	-1
Elast. to Substitutability	$\varepsilon_{\mu,\Theta} = \frac{\partial\mu/\mu}{\partial\Theta/\Theta} < 0$	$\varepsilon_{\rho,\Theta} = \frac{\partial\rho/\rho}{\partial\Theta/\Theta} \gtrless 0$	$\varepsilon_{\zeta,\Theta} = \frac{\partial\zeta/\zeta}{\partial\Theta/\Theta} < 0$
CES	$-\frac{1}{\theta-1}$	$-\frac{\theta\ln N}{(\theta-1)^2} < 0$	$-\frac{\theta}{\theta-1}$
Bertrand	$-\frac{\lambda(N-1)}{[(\lambda-1)N-(\lambda-\omega)][\lambda N-(\lambda-\omega)]}$	$-\frac{\lambda\ln N}{(\lambda-1)^2} < 0$	$-\frac{\lambda}{\lambda-1}$
Cournot	$-\frac{\omega(N-1)}{(\lambda-1)\omega N-(\lambda-\omega)}$	$-\frac{\lambda\ln N}{(\lambda-1)^2} < 0$	$-\frac{\lambda}{\lambda-1}$
Translog	$-\frac{1}{1+\gamma N}$	$\frac{\tilde{N}-N}{\gamma\tilde{N}N} > 0$	-1

We consider four demand structures. Each is characterized by the markup μ, the relative price ρ and the elasticity of the relative price to the number of firms ζ. The variable μ determines the profit destruction effect, while ρ and ζ determine the consumer surplus effect. The table derives the elasticities of these three measures to entry N and to the degree of substitutability Θ.

Table 2: Model

	Competitive	First Best
Labor Entry Costs	$N = \frac{Z}{F_L} \frac{\mu(N)-1}{\mu(N)}$	$N^{*} = \frac{Z}{F_L} \zeta(N^{*})(1+\zeta(N^{*}))^{-\frac{\eta}{1+\eta}}$
	$C = \frac{\rho(N)Z}{\mu(N)}$	$C^{*} = \rho(N^{*})Z(1+\zeta(N^{*}))^{-\frac{\eta}{1+\eta}}$
	$L = 1$	$L^{*} = (1+\zeta(N^{*}))^{\frac{1}{1+\eta}}$
Materials Entry Costs	$N = \frac{\rho(N)Z}{F_Y} \frac{\mu(N)-1}{\mu(N)}$	$N^{*} = \frac{\rho(N^{*})Z}{F_Y} \zeta(N^{*}) \left(\frac{1}{1-\zeta(N^{*})}\right)^{\frac{1}{1+\eta}}$
	$C = \frac{\rho(N)Z}{\mu(N)}$	$C^{*} = \rho(N^{*})Z \left(\frac{1}{1-\zeta(N^{*})}\right)^{-\frac{\eta}{1+\eta}}$
	$L = 1$	$L^{*} = \left(\frac{1}{1-\zeta(N^{*})}\right)^{\frac{1}{1+\eta}}$

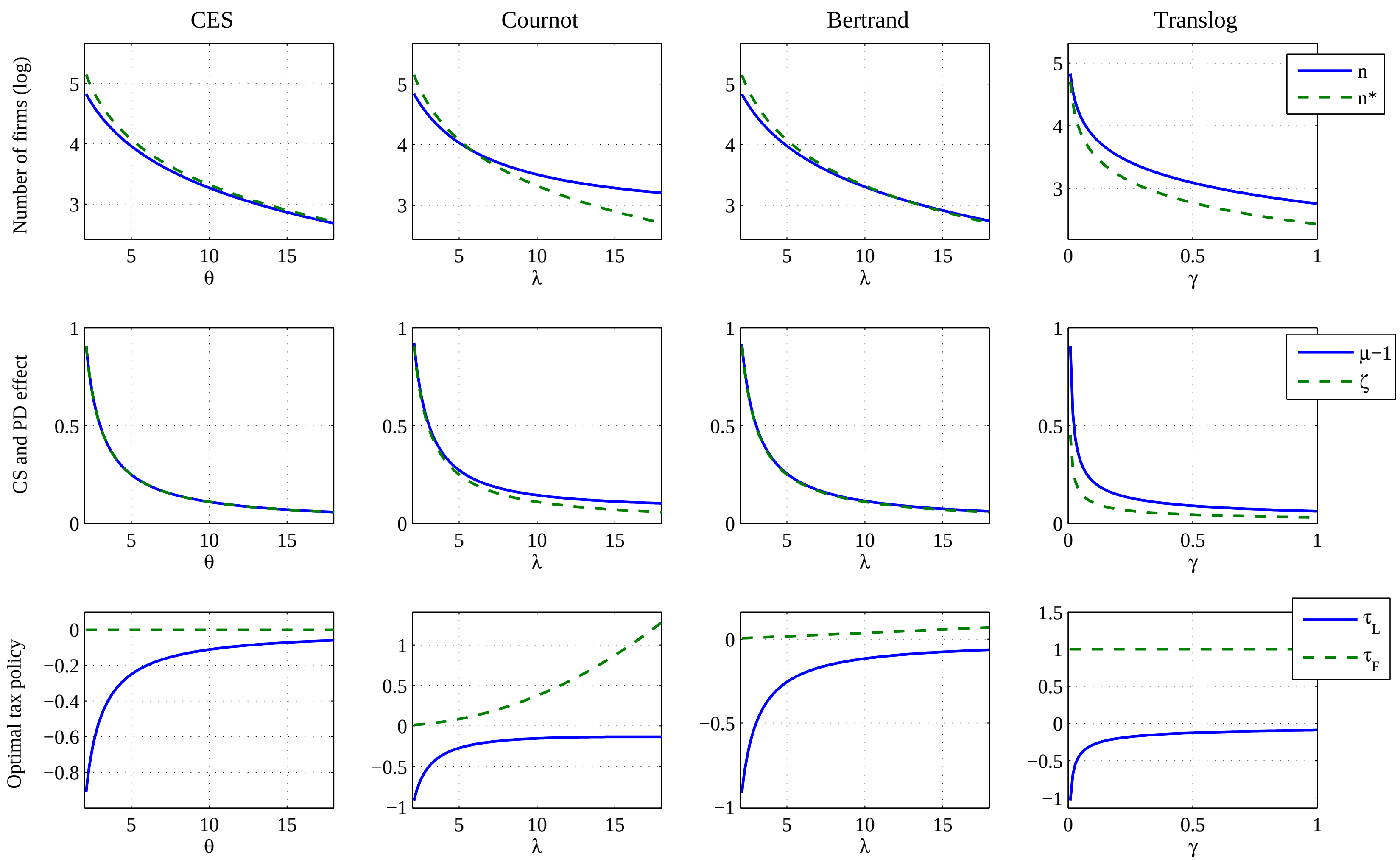
This table summarizes the equilibrium conditions in the competitive economy and the efficiency conditions of the First Best allocation, for the two entry cost specifications considered. We can solve for the number of firms, consumption and labor for a particular demand structure, which determines the functional forms for μ, ρ and ζ.

Table 3: Optimal Tax Rates

	Optimal Labor Tax	Optimal Entry Tax
Labor Entry Costs	$\tau_L = -(\mu(N) - 1)$	$\tau_F = \frac{\mu(N)-1}{\zeta(N)} - 1$
CES	$-\frac{1}{\theta-1} < 0$	0
Bertrand	$-\frac{N}{(\lambda-1)N-(\lambda-\omega)} < 0$	$\frac{(\lambda-\omega)}{(\lambda-1)N-(\lambda-\omega)} > 0$
Cournot	$-\frac{\omega N+(\lambda-\omega)}{(\lambda-1)\omega N-(\lambda-\omega)} < 0$	$\frac{(\lambda-\omega)\lambda}{(\lambda-1)\omega N-(\lambda-\omega)} > 0$
Translog	$-\frac{1}{\gamma N} < 0$	1
Materials Entry Costs	$\tau_L = -(\mu(N) - 1)$	$\tau_F = \frac{\mu(N)-1}{\mu(N)\zeta(N)} - 1$
CES	$-\frac{1}{\theta-1} < 0$	$-\frac{1}{\theta} < 0$
Bertrand	$-\frac{N}{(\lambda-1)N-(\lambda-\omega)} < 0$	$-\frac{N-(\lambda-\omega)}{\lambda N-(\lambda-\omega)} \lesseqgtr 0$
Cournot	$-\frac{\omega N+(\lambda-\omega)}{(\lambda-1)\omega N-(\lambda-\omega)} < 0$	$-\frac{\omega N-(\lambda-1)(\lambda-\omega)}{\lambda\omega N} \lesseqgtr 0$
Translog	$-\frac{1}{\gamma N} < 0$	$\frac{\gamma N-1}{1+\gamma N} \lesseqgtr 0$

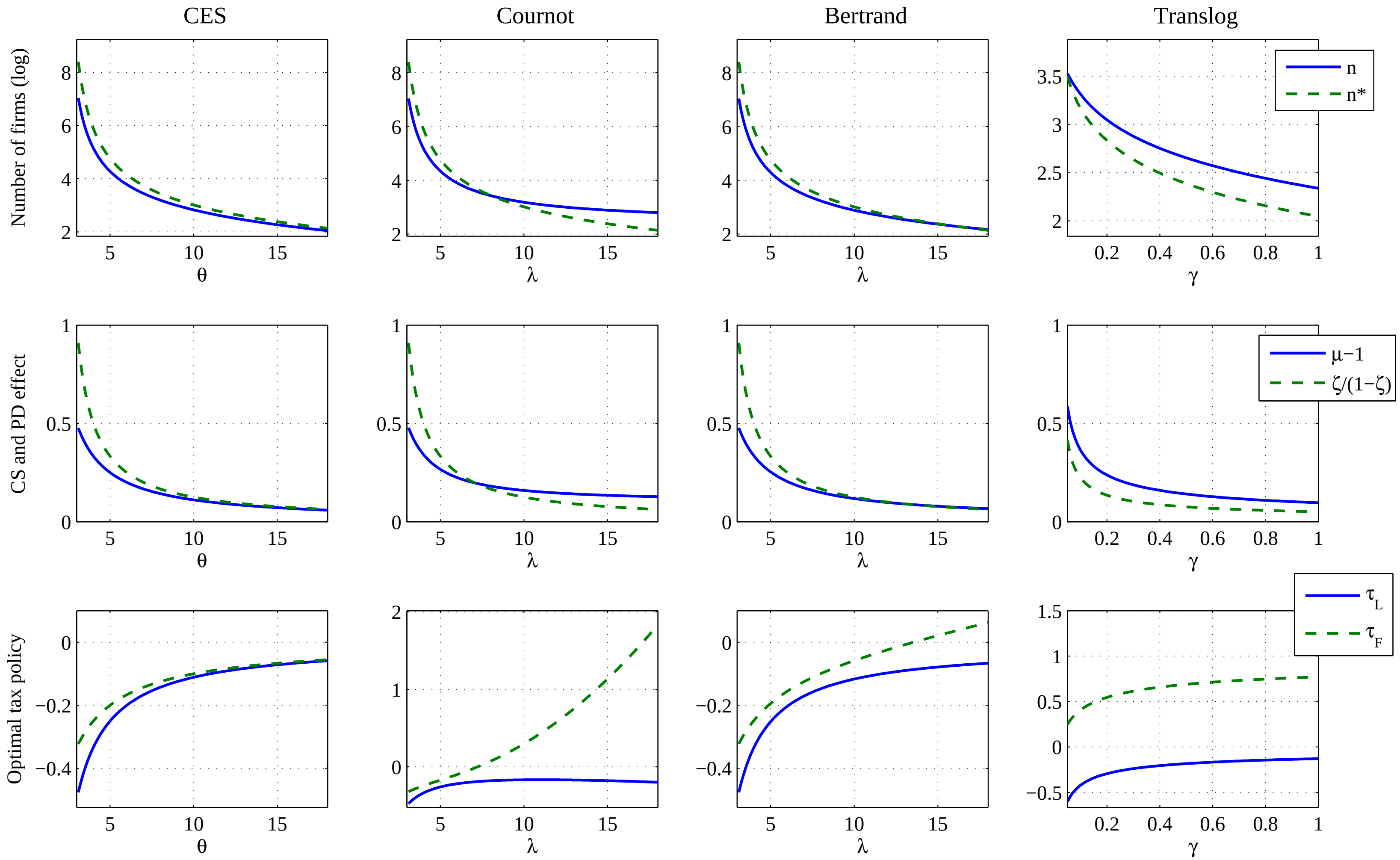
This table lists the optimal labor and entry tax rates for the two entry cost specifications, in terms of a general formula and under each specific demand structure. We assume $\lambda > \omega$.

Figure 1: Results under Labor Entry Costs



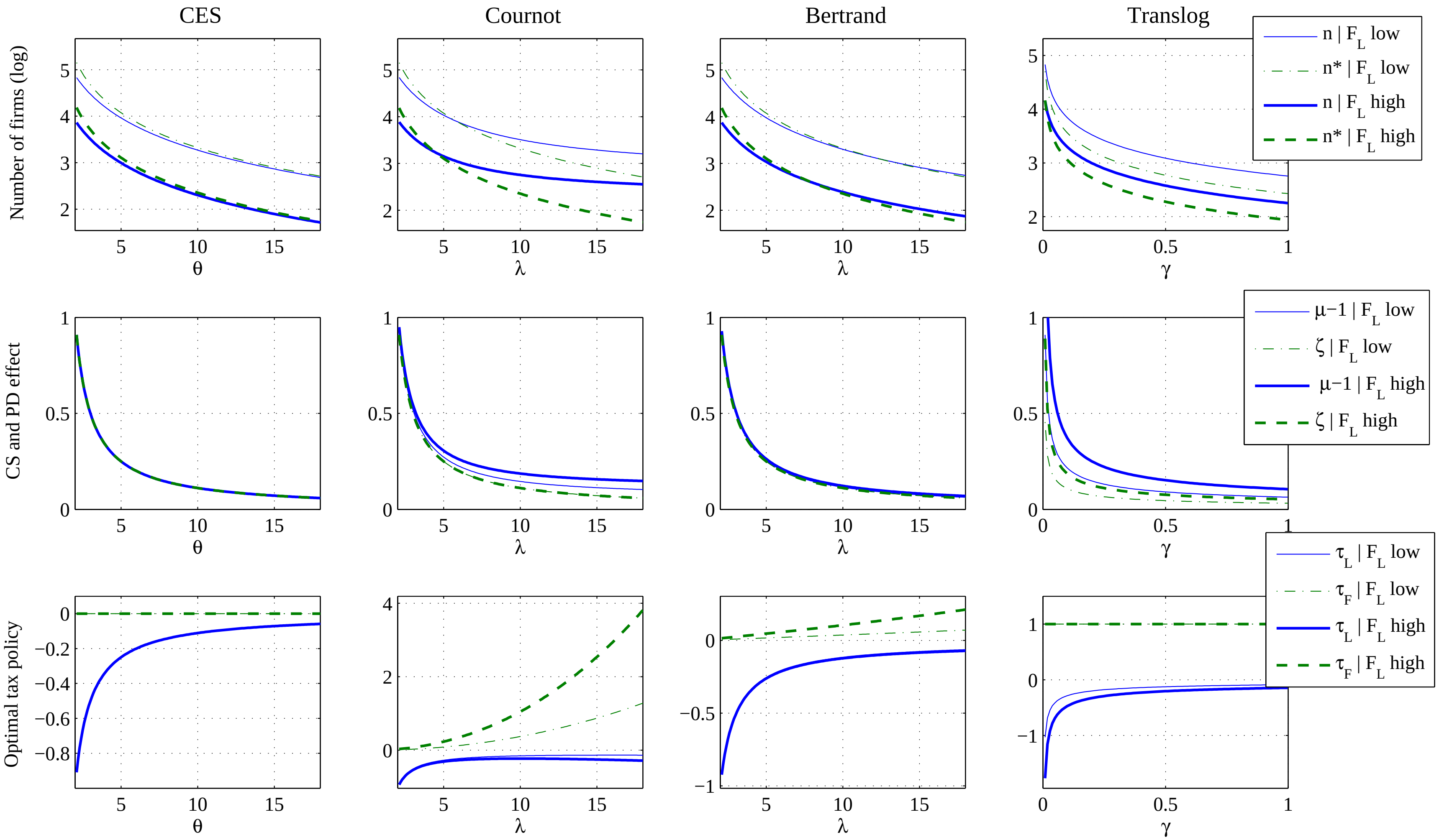
The figure plots various model variables as a function of the parameter measuring the substitutability between goods, $\Theta \in \{\theta, \theta_f, \gamma\}$. The top panel shows the competitive and efficient number of firms in logarithms, n and n^*; the middle panel depicts the profit destruction (PD) and consumer surplus (CS) effects, $\mu - 1$ and ζ, respectively; the bottom panel shows the optimal labor and entry tax rates.

Figure 2: Results under Materials Entry Costs



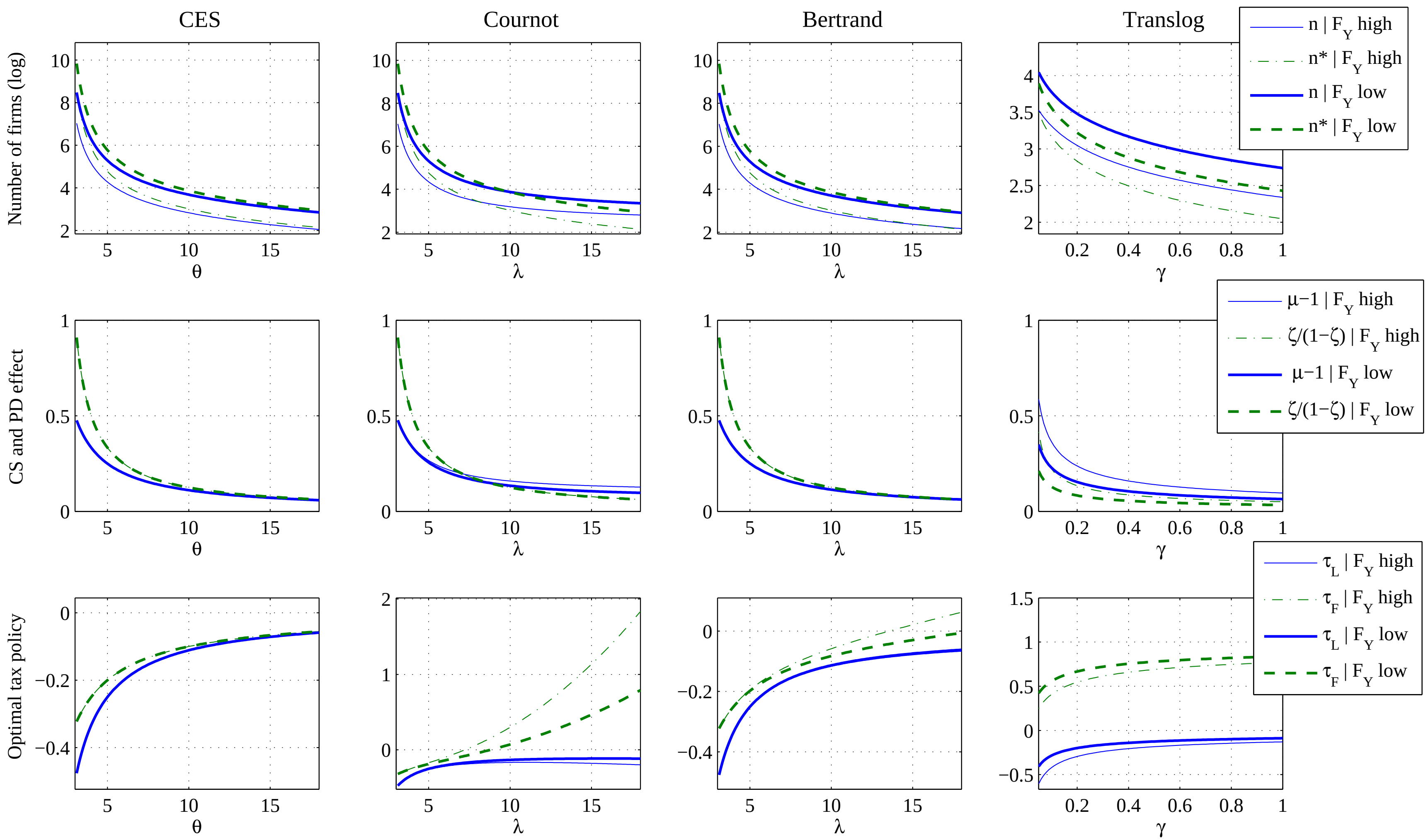
The figure plots various model variables as a function of the parameter measuring the substitutability between goods, $\Theta \in \{\theta, \theta_f, \gamma\}$. The top panel shows the competitive and efficient number of firms in logarithms, n and n^*; the middle panel depicts the profit destruction (PD) and consumer surplus (CS) effects, $\mu - 1$ and $\frac{\zeta}{1-\zeta}$, respectively; the bottom panel shows the optimal labor and entry tax rates τ_L and τ_F.

Figure 3: Varying the Size of Labor Entry Costs



In this figure, we show how the number of firms/goods in the competitive equilibrium and in the First Best allocation, n and n^*, the profit destruction (PD) and consumer surplus (CS) effects, $\mu - 1$ and ζ, as well as the optimal tax rates τ_L and τ_F change when we vary the size of the entry cost F_L.

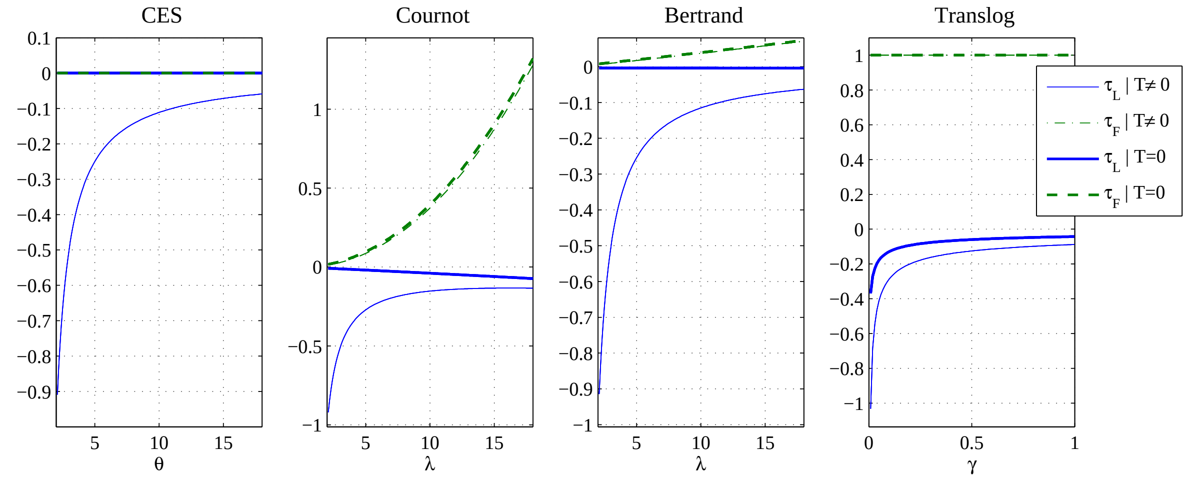
Figure 4: Varying the Size of Materials Entry Costs



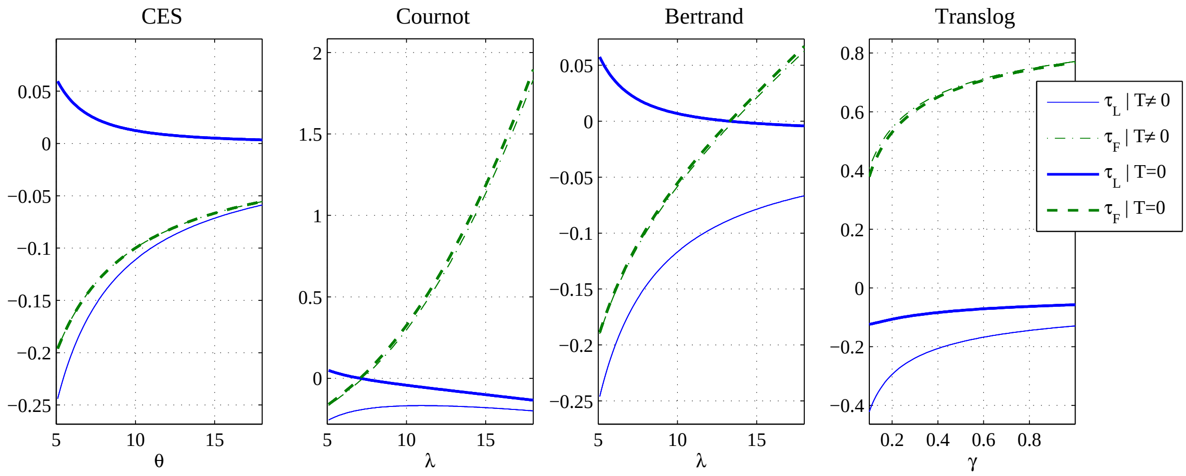
In this figure, we show how the number of firms/goods in the competitive equilibrium and in the First Best allocation, n and n^*, the profit destruction (PD) and consumer surplus (CS) effects, $\mu - 1$ and $\frac{\zeta}{1-\zeta}$, as well as the optimal tax rates τ_L and τ_F change when we vary the size of the entry cost F_Y.

Figure 5: Restricted Instrument Set: No Lump-Sum Taxes

(a) Labor Entry Costs



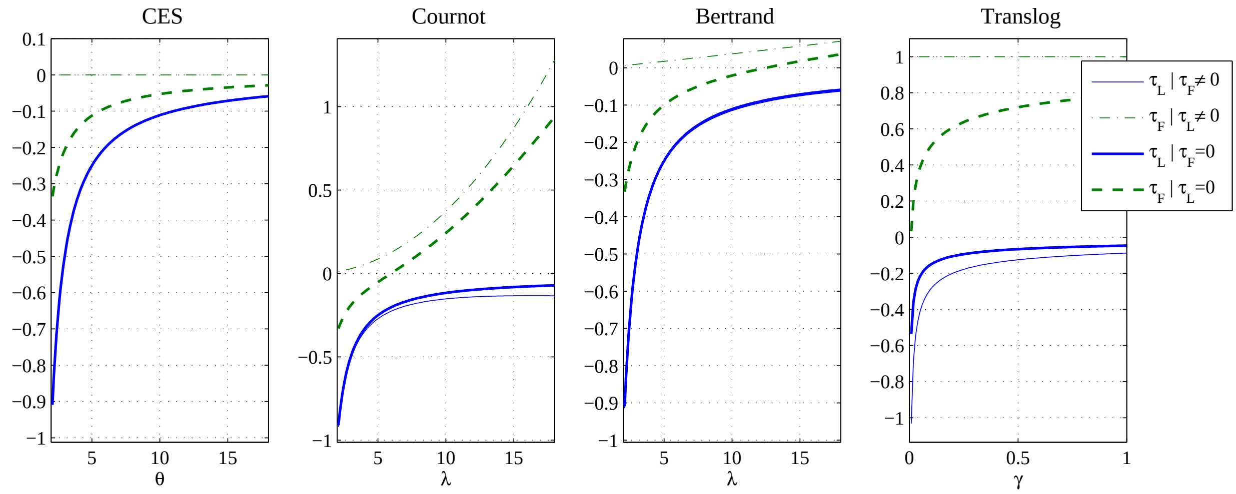
(b) Materials Entry Costs



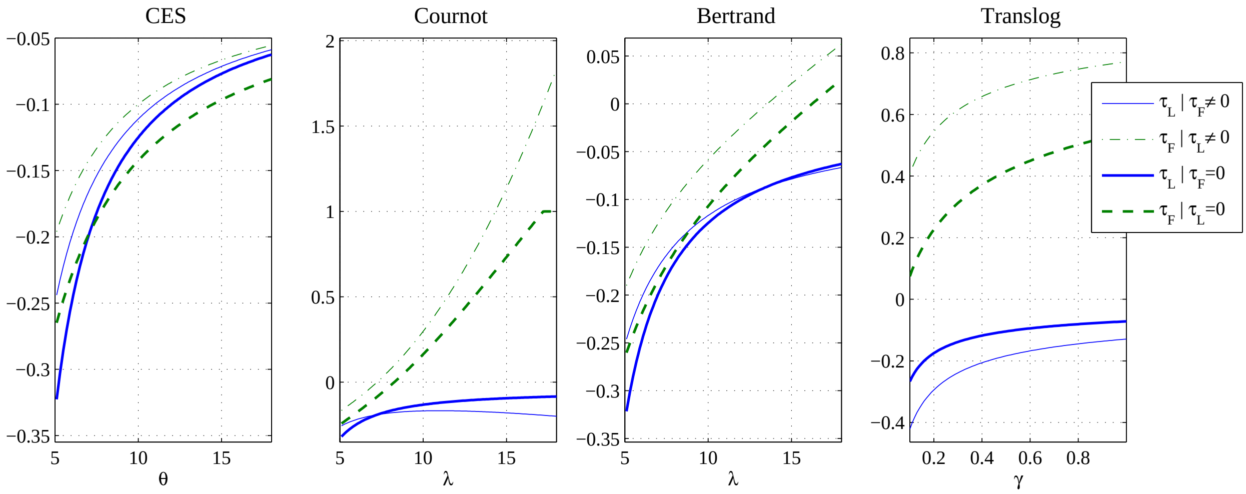
In this figure, we show how the number of firms/goods in the optimal tax rates τ_L and τ_F depend on the degree of product substitutability Θ. The optimal tax rates are computed under the assumption that only distortionary policy instruments and no lump sum taxes are available.

Figure 6: Restricted Instrument Set: Only one Distortionary Tax

(a) Labor Entry Costs



(b) Materials Entry Costs



In this figure, we show how the number of firms/goods in the optimal tax rates τ_L and τ_F depend on the degree of product substitutability Θ. The optimal tax rates are computed under the assumption that only one distortionary policy instrument is available.

Appendix

This appendix provides more detailed results on three parts of the paper. First, we show equilibrium conditions for the number of firms and goods in the competitive vs. the First Best allocation, under the different demand structures and the two entry cost specifications. Second, we derive elasticities of the number of firms/goods with respect to the degree of cross-product substitutability. Third, we derive elasticities of the number of firms/goods with respect to the size of entry costs.

Number of Firms/Goods: Competitive vs. First Best Allocation

Using Table A, we can show that under oligopolistic competition and labor entry costs the number of firms in the competitive equilibrium without government interventions may be higher or lower than the optimal number of firms.

[insert Table A here]

More specifically, under Cournot competition, the number of firms in the competitive equilibrium is lower than in the First Best if

$$1+\sqrt{1+4\frac{\lambda}{\omega}\left(\lambda-\omega\right)\frac{F_L}{Z}}<2\left(\frac{\lambda}{\lambda-1}\right)^{\frac{1}{1+\eta}}. \tag{A1}$$

Under Bertrand competition, the number of firms in the competitive equilibrium is too low if

$$1+\left(\lambda-\omega\right)\frac{F_L}{Z}<\left(\frac{\lambda}{\lambda-1}\right)^{\frac{1}{1+\eta}}. \tag{A2}$$

For a given ratio of entry costs to total productivity $\frac{F_L}{Z}$, both inequalities (A1) and (A2) are more likely to be satisfied, i.e. the number of firms in the competitive allocation is too small for small values of λ. To see this, note that the right hand side of the inequality falls with λ, while the left hand side rises with λ. Under translog preferences and with linear labor disutility, $\eta = 0$, the number of firms in the competitive equilibrium is lower than in the First Best if $\gamma < 2\frac{F_L}{Z}$. This inequality is more likely to be satisfied for small values of γ, i.e. for a low price-elasticity of the spending share.

Effect of Substitutability on Number of Firms/Goods

Table B shows the elasticity of the number of firms and goods to the degree of product substitutability $\Theta \in \{\theta, \lambda, \gamma\}$, in the competitive allocation and in the First Best allocation, under the different demand structures and for the two entry cost specifications.

[insert Table B here]

The table shows that in the labor entry cost specification, the competitive and the First Best number of firms is unambiguously decreasing in the substitutability between goods. This is also true in the materials entry cost specification under CES demand and under oligopolistic competition. However, under translog preferences, the sign of the elasticity is ambiguous.

Effect of Entry Costs on Number of Firms/Goods

Table C shows the effects of a change in entry costs on the number of firms in elasticity form. Consider the competitive allocation under *labor entry costs*. From the general formula, we know that the number of firms is decreasing in the entry cost F_L. Under a CES demand structure, the number of firms is inversely proportional to the entry cost. Under oligopolistic competition or translog preferences, the number of firms falls less than proportionately as entry costs increase. Now consider the corresponding elasticity in the First Best allocation. The efficient number of firms falls by 1% if entry costs rise by 1%, under CES demand and oligopolistic competition. In the CES case, therefore, the number of firms in the competitive and First Best allocations both fall by equal proportions as entry costs rise. Under oligopolistic competition, an increase in entry costs leads to a one-for-one decline in the First Best number of firms, which is greater than the decline in the competitive allocation. Under translog preferences, we obtain analytical results in the limiting case where labor is perfectly elastic ($\eta = 0$), see Table C. An increase in entry costs reduces the number of firms more in the competitive allocation compared to the First Best.

[insert Table C here]

Next, we analyze the effect of a change in *materials entry costs* F_Y on the number of firms in equilibrium and on the optimal tax rates. The corresponding elasticities in the competitive allocation and in the First Best are given in the lower panel of Table C. In the CES case, the number of firms in the competitive and First Best allocations respond in the same way to a change in entry costs, while under Bertrand and Cournot competition, the number of firms in the competitive equilibrium rises by less than the number of firms in the First Best allocation when entry costs fall. Under translog preferences, the response coefficients are more complicated, since the benefit of variety depends on the number of firms. With perfectly elastic labor supply ($\eta = 0$), and if product diversity is not too far from its optimal level, we obtain that the number of firms reacts more to entry cost changes in the competitive equilibrium than in the First Best.

Table A: Number of Firms/Goods

	Competitive	First Best
Labor Entry Costs		
CES	$N = \frac{Z}{F_L \theta}$	$N^* = \frac{Z}{F_L} \frac{1}{\theta - 1} \left(\frac{\theta}{\theta - 1} \right)^{-\frac{\eta}{1+\eta}}$
Cournot	$N = \frac{Z}{F_L} \frac{1}{2\theta_f} \left(1 + \sqrt{1 + 4\lambda \left(\frac{\lambda - \omega}{\omega} \right) \frac{F_L}{Z}} \right)$	$N^* = \frac{Z}{F_L} \frac{1}{\lambda - 1} \left(\frac{\lambda}{\lambda - 1} \right)^{-\frac{\eta}{1+\eta}}$
Bertrand	$N = \frac{1}{\lambda} \left(\frac{Z}{F_L} + \lambda - \omega \right)$	$N^* = \frac{Z}{F_L} \frac{1}{\lambda - 1} \left(\frac{\lambda}{\lambda - 1} \right)^{-\frac{\eta}{1+\eta}}$
Translog	$N = \frac{1}{2\gamma} \left(\sqrt{1 + \frac{4\gamma Z}{F_L}} - 1 \right)$	$N^* = \frac{Z}{F_L} \frac{1}{2\gamma N^*} \left(1 + \frac{1}{2\gamma N^*} \right)^{-\frac{\eta}{1+\eta}}$
Materials Entry Costs		
CES	$N = \left(\frac{Z}{F_Y \theta} \right)^{\frac{\theta - 1}{\theta - 2}}$	$N^* = \left(\frac{Z}{F_Y} \frac{1}{\theta - 1} \left(\frac{\theta - 1}{\theta - 2} \right)^{\frac{1}{1+\eta}} \right)^{\frac{\theta - 1}{\theta - 2}}$
Cournot	$N^{2 - \frac{1}{\lambda - 1}} - \frac{Z}{F_Y \lambda} \left(N + \frac{\lambda - \omega}{\omega} \right) = 0$	$N^* = \left(\frac{Z}{F_Y} \frac{1}{\lambda - 1} \left(\frac{\lambda - 1}{\lambda - 2} \right)^{\frac{1}{1+\eta}} \right)^{\frac{\lambda - 1}{\lambda - 2}}$
Bertrand	$N^{\frac{\lambda - 2}{\lambda - 1}} - \frac{\lambda - \omega}{\lambda} N^{-\frac{1}{\lambda - 1}} - \frac{Z}{F_Y \lambda} = 0$	$N^* = \left(\frac{Z}{F_Y} \frac{1}{\theta_f - 1} \left(\frac{\lambda - 1}{\lambda - 2} \right)^{\frac{1}{1+\eta}} \right)^{\frac{\lambda - 1}{\lambda - 2}}$
Translog	$(\gamma N + 1) N \exp \left(\frac{\tilde{N} - N}{2\gamma \tilde{N} N} \right) - \frac{Z}{F_Y} = 0$	$(N^*)^2 \left(\frac{2\gamma N^* - 1}{2\gamma N^*} \right)^{\frac{1}{1+\eta}} \exp \left(\frac{\tilde{N} - N^*}{2\gamma \tilde{N} N^*} \right) - \frac{Z}{2\gamma F_Y} = 0$

This table shows the equilibrium conditions determining the number of firms and goods in the competitive allocation and in the First Best allocation, under the different demand structures and for the two entry cost specifications.

Table B: Effect of Substitutability on Number of Firms/Goods

	Competitive	First Best
Entry Costs Labor		
General Formula	$\varepsilon_{N,\Theta} = \frac{\varepsilon_{\mu,\Theta}}{(\mu(N)-1)-\varepsilon_{\mu,N}} < 0$	$\varepsilon_{N^*,\Theta} = \frac{\varepsilon_{\zeta,\Theta}\left(1-\frac{\eta}{1+\eta}\frac{\zeta(N^*)}{1+\zeta(N^*)}\right)}{1-\varepsilon_{\zeta,N}\left(1-\frac{\eta}{1+\eta}\frac{\zeta(N^*)}{1+\zeta(N^*)}\right)} < 0$
CES	-1	$-\frac{\theta}{\theta-1}\left(1-\frac{\eta}{1+\eta}\frac{1}{\theta}\right)$
Bertrand	$-\left(1-\frac{1}{N}\right)$	$-\frac{\lambda}{\lambda-1}\left(1-\frac{\eta}{1+\eta}\frac{1}{\lambda}\right)$
Cournot	$-\frac{\omega(N-1)}{\theta_i N+2(\lambda-\omega)}$	$-\frac{\lambda}{\lambda-1}\left(1-\frac{\eta}{1+\eta}\frac{1}{\lambda}\right)$
Translog	$-\frac{\gamma N}{1+2\gamma N}$	$-\frac{1-\frac{\eta}{1+\eta}\frac{1}{1+2\gamma N^*}}{1+\left(1-\frac{\eta}{1+\eta}\frac{1}{1+2\gamma N^*}\right)}$
Entry Costs Materials		
General Formula	$\varepsilon_{N,\Theta} = \frac{\varepsilon_{\mu,\Theta}+(\mu(N)-1)\varepsilon_{\rho,\Theta}}{(\mu(N)-1)(1-\zeta(N))-\varepsilon_{\mu,N}}$	$\varepsilon_{N^*,\Theta} = \frac{\varepsilon_{\rho,\Theta}+\varepsilon_{\zeta,\Theta}\left(1+\frac{1}{1+\eta}\frac{\zeta(N^*)}{1+\zeta(N^*)}\right)}{1-\zeta(N^*)-\varepsilon_{\zeta,N}\left(1+\frac{1}{1+\eta}\frac{\zeta(N^*)}{1+\zeta(N^*)}\right)}$
CES	$-\frac{\theta-2}{\theta-1}\left(\frac{\theta\ln N}{(\theta-1)^2}+1\right) < 0$	$-\frac{\theta-2}{\theta-1}\left[\frac{\theta\ln N}{(\theta-1)^2}+\frac{\theta}{\theta-1}\left(1+\frac{1}{1+\eta}\frac{1}{\theta}\right)\right] < 0$
Bertrand	$-\frac{(\lambda N-(\theta_f-\omega))\frac{\lambda\ln N}{(\lambda-1)^2}+\theta_f(N-1)}{(\lambda N-(\lambda-\omega))\frac{\lambda-2}{\lambda-1}+(\lambda-\omega)} < 0$	$-\frac{\lambda-2}{\lambda-1}\left[\frac{\lambda\ln N}{(\theta_f-1)^2}+\frac{\lambda}{\lambda-1}\left(1+\frac{1}{1+\eta}\frac{1}{\lambda}\right)\right] < 0$
Cournot	$-\frac{(\omega N+(\theta_f-\omega))\frac{\lambda\ln N}{(\lambda-1)^2}+\omega(N-1)}{(\omega N+(\theta_f-\omega))\frac{\lambda-2}{\lambda-1}+(\lambda-\omega)} < 0$	$-\frac{\lambda-2}{\lambda-1}\left[\frac{\lambda\ln N}{(\lambda-1)^2}+\frac{\lambda}{\lambda-1}\left(1+\frac{1}{1+\eta}\frac{1}{\lambda}\right)\right] < 0$
Translog	$\frac{\frac{\tilde{N}-N}{\gamma\tilde{N}N}-\frac{\gamma N}{1+\gamma N}}{\frac{2\gamma N-1}{2\gamma N}+\frac{\gamma N}{1+\gamma N}} \lesseqgtr 0$	$\frac{\frac{\tilde{N}-N}{\gamma\tilde{N}N}-\left(1+\frac{1}{1+\eta}\frac{1}{1+2\gamma N^*}\right)}{1-\frac{1}{2\gamma N^*}+\left(1+\frac{1}{1+\eta}\frac{1}{1+2\gamma N^*}\right)} \lesseqgtr 0$

This table shows the elasticity of the number of firms and goods to the degree of product substitutability Θ, in the competitive allocation and in the First Best allocation, under the different demand structures and for the two entry cost specifications.

Table C: Effect of Entry Costs on Number of Firms

	Competitive	First Best
Labor Entry Costs	$\varepsilon_{N,F_L} = \frac{\partial N/N}{\partial F_L/F_L}$	$\varepsilon_{N^*,F_L} = \frac{\partial N^*/N^*}{\partial F_L/F_L}$
General Formula	$-\frac{1}{1-\frac{\varepsilon_{\mu,N}}{\mu(N)-1}} < 0$	$-\frac{1}{1-\varepsilon_{\zeta,N}\left(1-\frac{\eta}{1+\eta}\frac{\zeta(N^*)}{1+\zeta(N^*)}\right)} < 0$
CES	-1	-1
Bertrand	$-1 < \varepsilon_{N,F_L} < 0$	-1
Cournot	$-1 < \varepsilon_{N,F_L} < 0$	-1
Translog ($\eta = 0$)	$-\frac{\mu(N)}{1+\mu(N)}$	$-\frac{1}{2}$
Materials Entry Costs	$\varepsilon_{N,F_Y} = \frac{\partial N/N}{\partial F_Y/F_Y}$	$\varepsilon_{N^*,F_Y} = \frac{\partial N^*/N^*}{\partial F_Y/F_Y}$
General Formula	$-\frac{1}{1-\zeta(N)-\frac{\varepsilon_{\mu,N}}{\mu(N)-1}} < 0$	$-\frac{1}{1-\zeta(N^*)-\varepsilon_{\zeta,N^*}\left(1+\frac{1}{1+\eta}\frac{\zeta(N^*)}{1-\zeta(N^*)}\right)} < 0$
CES	$-\frac{\theta-1}{\theta-2}$	$-\frac{\theta-1}{\theta-2}$
Bertrand	$-\frac{\lambda-1}{\lambda-2} < \varepsilon_{N,F_Y} < 0$	$-\frac{\lambda-1}{\lambda-2}$
Cournot	$-\frac{\lambda-1}{\lambda-2} < \varepsilon_{N,F_Y} < 0$	$-\frac{\lambda-1}{\lambda-2}$
Translog ($\eta = 0$)	$-\frac{1}{1-\zeta(N)-\frac{1}{\mu(N)}}$	$-\frac{1}{1-\zeta(N^*)+\frac{1}{1-\zeta(N^*)}}$

The table shows the elasticity of the number of firms to entry costs F_L and F_Y, in the competitive equilibrium and in the First Best allocation. All four elasticities are negative in all demand structures.