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Consumer information networks

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Abstract

This paper examines the informativeness of consumer information networks and their effect on price competition between firms. Under the proposed information mechanism, consumers share their initial information with the members of their network and as such become better informed. The main result of this paper shows how informative such networks are by characterizing how many *different* pieces of information a network is likely to contain. This informativeness is crucial for the degree of competition, as consumers comparing more prices induce firms to compete more fiercely. We find that larger networks imply better information transmission, which intensifies competition and decreases all the percentiles of the price distribution. An increase in the number of firms makes networks more informative, and decreases all the percentiles as well. Our results are robust to the introduction of sequential search and network segregation, but an increase in segregation decreases information transmission and increases all percentiles.

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1 Introduction

Modern society is characterized by an abundance of marketplace information. The extent to which consumers benefit from this information availability depends on how well they are able find information that is valuable to them. They obtain this information through a variety of channels, but in many types of situations they rely on personal contacts, or their "social network".[1] Such networks might contain valuable information as individuals tend to differ in terms of the initial information available to them. By creating and accessing an information network, consumers pool their initial information and as such become better informed.

Often it is of importance how *many* pieces of information a consumer manages to acquire through this information network. Consider for instance a consumer looking for the lowest price for a particular product. The more prices such a consumer knows, the more prices he can compare and the lower the price he will eventually pay. In such a situation the value of a network is clearly determined by the number of different prices contained in the network, but exactly how informative are they? This paper addresses this question by developing a simple, yet highly tractable, model of consumer information networks. The informativeness of the network has important consequences for the degree of competition between firms, as better informed consumers are more likely to buy from the lowest-priced firm. We therefore explore these consequences for firm price-setting behavior in the context of an oligopolistic market.

The basic framework considers a setting in which there are N potentially different "objects" that consumers can become informed about. Each consumer is initially informed about one randomly chosen object, but they can become more informed by accessing their information network. In particular, we follow the model of interpersonal communication found in Ellison and Fudenberg (1995) and Galeotti (2010), and assume that each consumer has access to the initial information of k other consumers. By combining this information with the consumers' own initial information they can therefore become informed about up to $k + 1$ different objects.

The main result of this paper is a characterization of the informativeness of consumer networks. In particular, we derive an explicit formula for the proportion of consumers that will be informed about m of the N different objects. This formula turns out to be very straightforward and uses Stirling numbers of the second kind, a series of numbers that is well-known in the field of combinatorics. Given this characterization we then show how the informativeness varies in the network

[1] See Galeotti (2010) for a recent survey of empirical papers documenting this phenomenon.

size k and the number of objects N. We derive the intuitive comparative static result that as the network grows larger, or the number of objects increases, consumers will become better informed. These intermediate results turn out to be crucial for analyzing the impact of an increase in k or N on firms' pricing behavior.

To investigate the impact of consumer networks on firm price setting behavior the model of information is combined with the canonical oligopoly model of imperfect information introduced in Armstrong, Vickers and Zhou (2009) and Lach and Moraga-González (2009, 2012). All consumers are initially informed about the price of 1 of the N firms, but also have access to the prices known by their network connections. We show that in the presence of consumer information networks, prices tend to be dispersed. The reason is that some consumers will not learn any additional prices through their network, whereas other consumers do. This discrepancy in consumer informedness creates a tendency for firms to have periodic "sales", as one the one hand they want to charge high prices to extract rent from the uninformed consumers, but they also want to charge low prices to attract the informed consumers. They balance these two incentives by randomizing over prices, generating intertemporal price dispersion as an equilibrium phenomenon, as in Varian (1980).

Recent ICT-developments, such as the spectacular growth in mobile internet devices and social media, tend to have increased consumer connectivity. Our model predicts that such an increase in the network size improves information transmission among consumers and forces firms to compete more fiercely. This in turn reduces prices for all consumers by decreasing all percentiles of the price distribution. A more connected world will thus tend to be a more competitive world. We also consider the effect of an increase in the number of firms on prices. It is well-known that in markets with imperfect information, an increase in the number of firms can have surprising effects. One of the most striking findings is that an increase in the number of firms might actually increase the average price (see, e.g., Rosenthal, 1980; Varian, 1980), contradicting the predictions of standard Cournot and Bertrand oligopoly models. We show that in the presence of consumer information networks an increase in the number of firms always decreases all percentiles of the price distribution, so consumers are always better off when there are more firms competing. The reasoning behind this result is following: As the number of firms increases, consumers become more informed as their connections are more likely to know a different price. This induces consumers to compare more prices and firms to compete more fiercely.

In a first extension we introduce costly sequential search as in the seminal paper by Stahl (1989). In particular, consumers can search sequentially for the prices they did not become informed about through their network. We show that costly searchers choose not to search, and are only informed by information pooling through their network. This generates a price dispersion equilibrium without imposing ex ante heterogeneity in search cost as in Stahl (1989). Prices are shown to be lower in the presence of consumer search, as firms have to charge prices that are sufficiently low to prevent search activities. Unlike in the original model, an increase in the number of firms unambiguously decreases prices. This is due to the fact that such an increase improves the informedness of all consumers, whereas in the original model costly searchers did not become more informed. The introduction of information networks therefore restores the prediction of standard oligopoly models that competition tends to decrease prices.

In the benchmark model each of the k consumers can be informed about any of the N objects. In reality agents have a tendency to associate themselves with others similar to themselves (see, e.g., Jackson and López-Pintado, 2013). In a second extension of the model we allow for such network segregation and show that our main results continue to hold if networks are not too segregated. In that case an increase in k or N still improves the information transmission, and continues to decrease all percentiles of the price distribution. We also show that an increase in the degree of network segregation tends to decrease information transmission and increase all percentiles.

This paper is related to several strands of economic literature. Firstly, it is related to the literature on social and economic networks (see, e.g., Jackson, 2008) which studies the implications of network structure on outcomes. Part of this literature studies how network structure matters for the "diffusal" of information, and identifies conditions on the network structure such that information spreads (see, e.g., Rogers and Rogers, 2003). The models used are typically dynamic to capture explicitly the diffusion of information, and the focus is on the convergence properties of the information diffusal process. This paper focuses on a more static framework in which information diffusal occurs immediately, and in which the network structure is very simple. These simplifying assumptions allow us to highlight the informativeness of the network by being able to calculate explicitly how many different objects the network contains. Another part of this literature focuses on "social learning" and studies how networks can be used to aggregate information of individual agents. In these models there is typically uncertainty about players' payoffs from different actions but they can learn about them over time by listening to the experiences of other agents. The network structure that we use is based on one such paper, Ellison and Fudenberg (1995), in which agents hear about the experiences regarding two products from a sample of k other agents in the context of repeated interaction. We consider a more static environment but generalize their setup to allow for more than two products.

Secondly, it is related to the literature on oligopoly under imperfect information.[2] A large part of this literature has explored the consequences on price-setting behavior of imposing a particular information gathering mechanism, such as consumer search (Burdett and Judd, 1983; Stahl, 1989), advertising (Butters, 1977) and information clearinghouses (Baye and Morgan, 2001). Recently some papers have also started to explore networks as an information mechanism. For instance, Lever (2011) and Nermuth, Pasini, Pin and Weidenholzer (2013) consider the implications of a network between firms and consumers. The current paper considers a network of information between consumers, as in Galeotti (2010), who studies the impact of such networks on consumer search behavior. Galeotti's setup is however limited to a duopoly, which restricts the informativeness of the network. The current paper considers an fully oligopolistic setup without consumer search. As such we are able to highlight the informativeness of the network and its relation to market structure.

The remainder of this paper is structured in the following way. In section 2 we introduce the benchmark model without search or network segregation. We characterize the informativeness of the network and derive implications for firms' pricing behavior. Section 3 covers two extensions of the model: In a first extension we allow consumers to search sequentially for prices they did not learn through their network. In a second extension we allow the network to be segregated. We also briefly discuss two other extensions which are not covered explicitly. In the last section we summarize our results and discuss directions for further research.

2 The Benchmark Model

2.1 Model Setup

The basic setup of the model is roughly identical to the model presented in Armstrong, Vickers and Zhou (2009) and Lach and Moraga-González (2009, 2012). They consider a market in which the supply side consists of $N \geq 2$ identical firms who compete in prices to sell a homogeneous good. Each firm faces a constant marginal cost $c \geq 0$ and there are no fixed costs.

The demand side is characterized by a unit mass of consumers with inelastic demand: Consumers wish to purchase one unit of the good as long as the price does not exceed their valuation $v > 0$. They purchase the good from the firm with the lowest price to their knowledge, but are heterogeneous in the number of prices they are informed about: A fraction $\theta_m \geq 0$ of consumers

[2] See Baye et al. (2006) for an excellent survey of this literature.

is informed about m prices, where $m \in \{1, \ldots, N\}$. The distribution of price information in the market is then summarized by the vector $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_N\}$. The most important difference between the model in Lach and Moraga-González (2009, 2012) and the current paper lies in $\boldsymbol{\theta}$: The authors do not explicitly specify $\boldsymbol{\theta}$ and identify sufficient conditions on $\boldsymbol{\theta}$ under which competition decreases prices for all consumers. The current model assumes an explicit information mechanism which is inspired by the literature on word-of-mouth communication and social networks (see, e.g., Ellison and Fudenberg, 1995; Galeotti, 2010).

The consumer information network mechanism takes on the following form: Consumers are initially imperfectly informed and know only the price of one randomly chosen firm. As in Galeotti (2010), they can become more informed through an information network: Each consumer has a network of k other consumers, who share their initial price information with the consumer. Some of these connections know the same price as the consumer, but others might know prices previously unknown to the consumer, causing the latter to become more informed. Unlike in Galeotti's paper, we do not allow for consumers to gain additional information through search activities in the benchmark model. This is because we want to focus explicitly on the properties of the network information mechanism. In an extension we explore the consequences of the information network when consumers search sequentially.

All consumers are assumed to have the same network size k, but some consumers will end up knowing more prices than others as their network contained a larger variety of prices. In the best case all of a consumer's connections will know a different price, and the consumer will learn a great deal from his network. In the worst case he might not learn any new prices at all, which happens if all of his connections tell him the price he already knew. How likely each of these cases is to occur is one of the main questions that this paper will provide an answer to. The network is also assumed to be exogenous in size and costless to access. One rationale for these assumptions could be that the network was formed for a more general purpose (e.g. social network), and not for gathering price information per se.

Multiple interpretations can be given to the information network. First of all, it could be thought of as an actual social network. If friends or colleagues are interested in similar products, they are likely to have additional information relevant for the consumer's purchase. This could for instance be the case because the members of the network are geographically dispersed. Secondly, it could be thought of as a type of "passive search". For some products consumers do not actively search, but they observe firms' prices during their day-to-day movements. A firm's price will then only be observed by consumers that cross the firm on their path. The network parameter k can in this case be seen as an intensity of movement: Consumers who move around a lot are more likely to observe a firm's price. One could also observe the purchases of his friends, who will tell what they paid for the good.

The timing of the game is the following: First, all firms simultaneously and independently set prices. After prices are set, each consumer observes the price of one randomly chosen firm. Consumers then consult their network and purchase from the lowest price known to them. We look for symmetric Nash equilibrium.

2.2 Information

As is well-known, a main determinant of the price distribution in imperfect information models is the degree and composition of information that is available to consumers. As consumers are typically considered to sample the firms in a random order, the consumers' degree of information is summarized by the *number* of firms they sample. In this section we will therefore derive the fraction of consumers that is informed about the prices of a certain number of firms.

As is standard in the literature, we assume that consumers are initially informed about the price of one randomly chosen firm; the so-called initial price. This assumption guarantees that all consumers are informed about at least one price and will always buy the product.[3] Upon observing the initial price, consumers access their network, through which they might learn additional prices. This happens if the initial price of the members of the network is different from the consumer's initial price. Since each consumer's initial price is random, the number of prices that will be known by consumers after accessing their network is a random variable. All consumers have the same network size k, so the probability that any consumer is informed about m different prices is also the overall fraction of consumers that is informed about m prices. If we denote this fraction by $\theta_m(k, N)$, then the total amount of information in a market with N firms is summarized by the information vector $\boldsymbol{\theta}(k, N) = \{\theta_1(k, N), \ldots, \theta_N(k, N)\}$. It turns out that the elements of this information vector have a rather simple expression, which is summarized in Theorem 1.

[3] Alternatively, one could assume that consumers have no prior information and only acquire information through the network. This is equivalent to reducing the network size by one, and hence does not change our results in any significant way.

Theorem 1 *In a market with N firms, the fraction of consumers with a network of size k that is informed about the prices of m firms, denoted by $\theta_m(k, N)$, is given by:*

$$\theta_m(k,N) = \begin{cases} \frac{N!}{N^{k+1}(N-m)!} S(k+1,m) > 0 & \text{if } m \le M^k \\ 0 & \text{if } m > M^k \end{cases} \tag{1}$$

where $S(.,.)$ are Stirling numbers of the second kind and $M^k = \min\{k+1, N\}$.[4]

Proof. By pooling his initial price quote with the price quotes obtained through the network, the consumer obtains a set of $k+1$ random price quotes. To be informed about m prices, this set of price quotes should contain the prices of exactly m firms, where the identities of the firms do not matter. Stirling numbers of the second kind $S(a,b)$ are useful in this context as they count the number of ways one can partition a set of a objects into b non-empty subsets. In the case at hand the set of objects are the price quotes and the subsets are m specific firms. To give an example, consider the case where the specific firms are firm 1 and 2 and the number of price quotes is 3. Denote the i-th price quote by s_i and denote allocations of these price quotes to the firms as sets, where the j-th subset contains the sample allocated to firm j. We then have that $S(3,2) = 3$, which counts the following allocations:

Firm	1	2		
(i)	$\{\{s_1\},$	$\{s_2, s_3\}\}$		$\{p_1, p_2, p_2\}$
(ii)	$\{\{s_1, s_2\},$	$\{s_3\}\}$	$\Leftrightarrow$	$\{p_1, p_1, p_2\}$
(iii)	$\{\{s_1, s_3\},$	$\{s_2\}\}$		$\{p_1, p_2, p_1\}$

Stirling numbers only consider partitions, so the identities of the subset do not matter. In the present context these identities do matter (as the j-th subset contains the price quotes of firm j). We therefore multiply by $N!/(N-m)!$ to take into account all possible ways in which m firms could be assigned to the subsets (when the order matters). The final step is to recognize that there

[4] The Stirling numbers of the second kind are given by:

$$S(a,b) = \frac{1}{b!}\sum_{j=0}^{b}(-1)^{b-j}\binom{b}{j} j^a$$

where $\binom{b}{j}$ is the binomial coefficient. These numbers are commonly used in combinatorics and represent the number of ways to partition a set of a objects into b non-empty subsets. The author would like to thank Tom Potoms for bringing these numbers to his attention.

are N^{k+1} possible sets of price quotes which the consumer could have obtained. ■

From these probabilities, two properties of the information vector are immediately clear: First of all, a consumer with a network of size k can never know more than $M^k = \min\{k+1, N\}$ prices: Clearly a consumer cannot know more prices than the number of potentially different prices, but consumers are also bounded by the size of their network, as they cannot learn more new prices as there are members in the network. This will have important implications for the equilibrium price distribution, as firms can never face competition from more than $M^k - 1$ other firms. Secondly, any number of prices equal or below M^k will be known with strictly positive probability. For some consumers firms therefore do not compete with other firms, whereas for others they face $m \leq M^k - 1$ competitors.

As Theorem 1 shows, calculating these probabilities require the use of Stirling numbers. A table containing the first few rows and columns can be found in the Appendix. For small m these numbers are very tractable. For instance, we have that $S(k+1, 1) = 1$ and $S(k+1, 2) = 2^k - 1$. Straightforward calculations now yield the following Corollary:

Corollary 2 *The fraction of consumers observing one price is given by*

$$\theta_1(k, N) = \frac{1}{N^k} \tag{2}$$

which is decreasing in N and k. The fraction of consumers observing two prices is given by

$$\theta_2(k, N) = \left(2^k - 1\right) \frac{(N-1)}{N^k} \tag{3}$$

which is decreasing (increasing) in N if $k > 1$ ($k = 1$) and decreasing (increasing) in k if $N > 2$ ($N = 2$).

The intuition behind the expression for θ_1 is clear: The only way a consumer does not learn any new prices from his network, is if all of his network members know the same price as he did. Since the initial prices are random, this occurs with probability N^{-k}. Note that when $k \geq 1$ we have that $0 < \theta_1 < 1$. In that case some consumers will not be comparing prices, a condition which is necessary and sufficient for a price dispersion equilibrium to exist. If $k = 0$ then clearly all consumers will only be informed about their initial price and we have that $\theta_1 = 1$. The fraction of consumers observing a single price is also decreasing in both N and k. Consumers with a larger network, or consumers in markets with more firms will thus always be more likely to know more than one price.

The fraction of consumers observing two prices is also decreasing in the number of firms if the network contains at least two members, and decreasing in the network size if there are at least three firms in the market. Clearly if there are only two firms in the market, then a decrease in θ_1 must increase θ_2 (as probabilities have to sum up to one). With more than two firms, consumers with a larger network are less likely to know only two prices, and more likely to know more than two prices. This is intuitive since in that case there are more prices to be learnt. Similarly, if the network contains only one connection, then consumers can never know more than two prices. An increase in the number of firms will then only make it more likely that a consumer learns two different prices. If the network size is larger than one, an increase in the number of firms will cause the consumer to be more likely to learn a number of prices higher than two.

Another consequence of Theorem 1 is that the shape of the information vector can only take on one of two forms:

Corollary 3 *$\theta_m(k,N)$ is single-peaked and is either:*

(i) (weakly) increasing in m, (if $k \leq k^*(N)$ or $k \geq k^{**}(N)$)

(ii) first increasing and then decreasing in m (if $k^*(N) < k < k^{**}(N)$)

where $k^*(N) < k^{**}(N)$ and both of these thresholds are (weakly) increasing in N.

Proof. see Appendix ■

When information networks are relatively small or relatively large compared to the number of firms, then the probability of knowing m prices is (weakly) increasing in m. Only very little consumers know only one price; most of them will know M^k prices. The logic behind this result is the following: If the network is very small, the members of the network are very likely to all know different prices, which makes it very likely that the consumer will learn k new prices. If on the other hand the network is very large, it becomes very likely that all firms' prices are shared on the network. In that case a consumer is very likely to be informed about all N prices. For information networks of intermediate size, the probability mass will be centered around some central value of m: Most consumers know an intermediate number of prices, and the share of consumers that know either a very small or a very large number of prices is small.

An important finding is how the information vector changes as either the network or the number of firms increases. The effect of an increase in the number of firms on the information vector will be important as it will be the driving force of competition. The effect of an increase the network size shows how information flows in more socially connected markets. Both effects are similar and contained in the following Corollary:

Corollary 4 *An increase in the number of firms N causes an upward shift in the information mass: there exists an m_N^* s.t. $\theta_m(k,N)$ weakly decreases (increases, resp.) for all m smaller (greater, resp.) than $m_N^*(k,N)$. This critical $m_N^*(k,N)$ is furthermore weakly increasing in k and N. As the number of firms increases without bound, all consumers eventually become informed about M^k prices. The effect of an increase in the network size k is similar to that of an increase in N. The critical $m_k^*(k,N)$ is also weakly increasing in k and N.*

Proof. see Appendix ■

As the network becomes larger or the number of firms increase, consumers will thus be more (less) likely to be informed about a high (low) number of prices. The intuition for this result is clear: If there is more to be learnt or there are more opportunities to learn, consumers will be better informed. The result in Corollary 4 is stronger than first-order stochastic dominance, so the expected number of prices observed by a consumer must increase in k and N as well. The next Corollary demonstrates this property by showing that this expectation has a convenient expression:

Corollary 5 *The expected number of prices observed by a consumer with a network of size k is given by:*

$$E_m(k,N) = \sum_{m=1}^{N} m \cdot \theta_m(k,N) = N\left(1-\left(1-\frac{1}{N}\right)^{k+1}\right) \tag{4}$$

which is concave and increasing in N and k.

Proof. The expected value can be found rather easily by expressing it as a sum of indicator variables and exploiting the linearity of the expectation operator. Define I_i as the indicator variable which takes on the value 1 if firm i's price is known by a consumer (after consulting the network), and 0 otherwise. The number of different prices known by the consumer is now given by the sum $\sum_{i=1}^{N} I_i$. By linearity of the expectation operator we have that

$$E_m(k,N,p_I) = E\left[\sum_{i=1}^{N} I_i\right] = \sum_{i=1}^{N} E[I_i] = NE[I_i] \tag{5}$$

where the last equality follows from the fact that firms are symmetric. The expected value $E[I_i]$ is the probability that firm i's price is drawn, and is the inverse of the probability that firm i's price is not drawn. The probability that i's price is not drawn is the probability that none of the $k+1$ samples contains firm i's price, which occurs with probability

$$p_{\sim i} = \left(1 - \frac{1}{N}\right)^{k+1}$$

The probability that firm i's price is known is thus given by $E\left[I_i\right] = 1 - p_{\sim i}$. Hence, the overall expectation is given by $N\left(1 - p_{\sim i}\right)$. ■

Before we proceed to discuss the firm side, we provide some examples of information probabilities to illustrate the main results of this section.

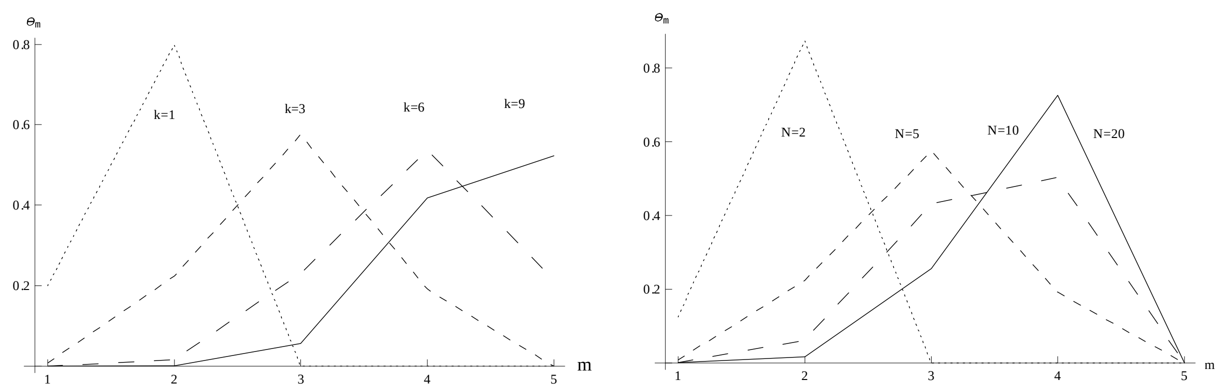
Table 1: Information probabilities

(a) For different values of k

k	θ_1	θ_2	θ_3	θ_4	θ_5
3	0.008	0.224	0.576	0.192	0.000
6	0.000	0.016	0.231	0.538	0.215
9	0.000	0.001	0.057	0.419	0.522
∞	0.000	0.000	0.000	0.000	1.000

(b) For different values of N

N	θ_1	θ_2	θ_3	θ_4	θ_5
5	0.008	0.224	0.576	0.192	0.000
10	0.001	0.063	0.432	0.504	0.000
20	0.000	0.017	0.266	0.727	0.000
∞	0.000	0.000	0.000	1.000	0.000



(a) For different values of k (N=5)

(b) For different values of N (k=3)

Figure 1: Information probabilities

Table 1 (a) and Figure 1 (a) list the information probabilities for various network sizes when there are five firms in the market. In such a market, consumers with three network connections will never know all five prices; they will most likely know two to four prices. As the network grows larger, the probability mass is shifted towards higher numbers: Consumers with nine network connections will know at least four out of five prices with probability 0.95. As the network size becomes increasingly large, consumers will eventually know the prices of all five firms with a probability approaching one. Table 1 (b) and Figure 1 (b) list the information probabilities for various market structures when consumers have a network with three connections. If there are more than 4 firms, consumers will never know all prices as they are limited by the size of their network. When there are five firms in the market, most consumers will know the prices of three firms. As the number of firms increases, the probability mass is again shifted towards higher numbers: In a market with twenty firms, consumers will know at least three prices with probability 0.98, and know exactly four prices with probability 0.73. As the number of firms becomes increasingly large, consumers will eventually know the prices of four firms with a probability that is approaching one as they are bounded by the network size.

2.3 Equilibrium Price Distribution and Comparative Statics

We now turn to firms' price setting behavior, taken as given the level of consumer informedness which we derived in the previous section. Throughout this section we will assume that the network size is strictly positive (i.e. $k \geq 1$), in which case we have by Corollary 2 that $\theta_1 \in (0,1)$.

As is well-known, a pure-strategy price equilibrium does not exist if $\theta_1 \in (0,1)$.[5] In that case firms both have an incentive to charge low prices to attract all price-comparing consumers (i.e. the business stealing effect) and an incentive to charge high prices to extract surplus from consumers that do not compare prices (i.e. the surplus extraction effect). Since firms cannot price discriminate, they balance these different incentives by randomizing over prices according to a cumulative price distribution $F(p)$. This distribution should be atomless since otherwise firms would have an incentive to undercut each other at the atom. When all other firms randomize according to $F(p)$, firm i's expected profits are given by

$$\pi_i\left(p_i, F(p)\right) = \left(p_i - c\right)\left[\sum_{m=1}^{M^k} \theta_m \left(\frac{m}{N}\right)\left(1 - F\left(p_i\right)\right)^{m-1}\right] \tag{6}$$

where the arguments for θ_m have been dropped for notational simplicity. When charging a price p_i, firm i's expected demand of consumers who are informed about m prices, is given by the joint probability that (a) such a consumer is informed about firm i's price, $\theta_m \cdot (m/N)$, and (b) firm i is the lowest price among all other prices known to this consumer, $\left(1 - F\left(p_i\right)\right)^{m-1}$. For the consumers

[5] It is well-known that if $\theta_1 = 1$ the Diamond paradox (Diamond, 1971) of monopoly pricing occurs ($p = v$), whereas if $\theta_1 = 0$ the Bertrand paradox of marginal cost pricing occurs ($p = c$).

who only know one price (i.e. θ_1) the firm does not face any competition, and it will attract an equal share of $1/N$ of these uninformed consumers regardless of its price.

In the case of unit demand, the upper bound of the price distribution $F(p)$ should equal the consumer's reservation price v. No firm should ever charge a price above v, as no consumer would be willing to buy at such a price. If the upper bound is below v, all firms' expected profits could be increased by raising the upper bound, which contradicts it being part of a Nash equilibrium. We therefore have that

$$\overline{p} = v \tag{7}$$

This yields an expected profit of $\overline{\pi}(v, F(p)) = (v - c)(\theta_1/N)$, which is also the profit level every other price of the mixed strategy should yield (as to make the firm indifferent). The equilibrium price distribution can then be found by equating (6) to this common profit level:

$$F(p) \text{ solves } \pi_i(p_i, F(p)) = \overline{\pi}(v, F(p)) \tag{8}$$

As the profit function is a polynomial of order $M^k - 1$, for which there is no general algebraic solution (i.e. a solution in radicals), it is not possible to solve explicitly for the price distribution $F(p)$. By Descartes' rules of sign, a unique positive solution $F(p')$ exists for each p' and can be calculated using numerical methods. Even though we cannot solve for $F(p)$, we can easily find the inverse price distribution by solving for p:

$$p(x) = c + \frac{v - c}{\sum_{m=1}^{M^k} m\left(\frac{\theta_m}{\theta_1}\right) x^{m-1}} \tag{9}$$

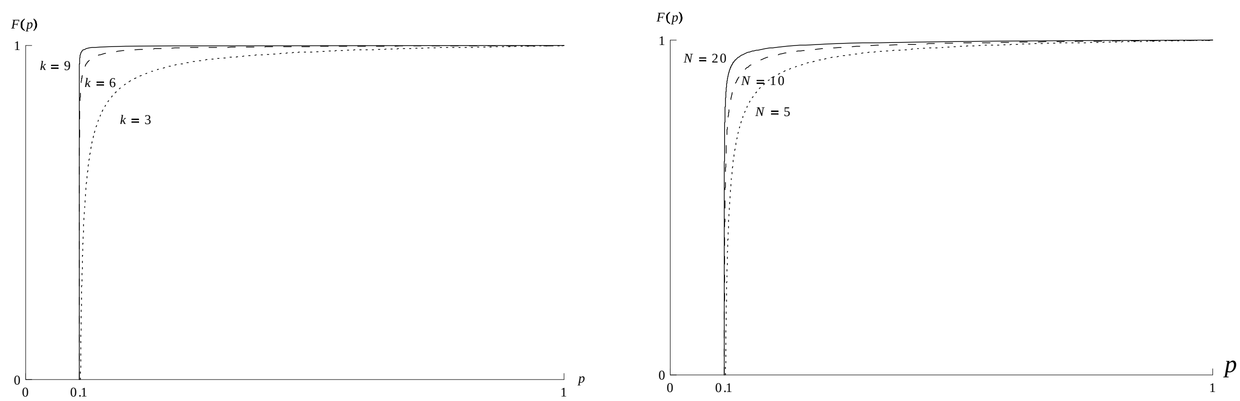
where $x = 1 - F(p)$. The lower bound is then found by setting $x = 1$, which yields

$$\underline{p} = c + \frac{v - c}{\sum_{m=1}^{M^k} m\left(\frac{\theta_m}{\theta_1}\right)} \tag{10}$$

We are now ready to characterize the welfare consequences of an increase in k and N. We start by showing how such an increase affects the percentiles of the price distribution, as is summarized in Proposition 6.

Proposition 6 *All percentiles of $F(p)$ are decreasing in the number of firms N and the network size k. As the number of firms or the network size grows larger pricing eventually becomes perfectly competitive.*

Proof. see Appendix ■



(a) For different values of k (N=5)

(b) For different values of N (k=3)

Figure 2: Equilibrium price distribution (v=1, c=0.1)

By Corollary 2 and 4 we know that there will be an upward shift in the information mass, and that the share of consumer not comparing prices strictly decreases. All firms therefore face a smaller share of uninformed consumers, which is the source of their market power. One might be tempted to think that such a decrease must induce firms to reduce all prices in the distribution, but this is not necessarily the case. For instance, in a market with only three firms, a sharp decrease in θ_2 accompanying the decrease in θ_1 (so that θ_3 increases) can actually *increase* the upper percentiles of the price distribution.[6] Lach and Moraga-González (2009, 2012) point out that a sufficient condition for *all* percentiles to decrease is that the ratio (θ_m/θ_1) should (weakly) increase. In that case each share of consumers comparing at least two prices becomes relatively more important compared to the non-comparing consumers. In the Appendix we show that an increase in k or N raises this ratio. Both an increase in the number of firms and the network size thus decreases all percentiles of the price distribution, including the lower bound (which is the lowest percentile). Figure 2 (a) and (b) demonstrate this result by plotting $F(p)$ for a number of values of k and N. Higher values of k and N are associated with a cumulative price distribution that is shifted

[6] Consider for instance the following change: $\{\theta_1, \theta_2, \theta_3\} : \{0.3, 0.45, 0.25\} \rightarrow \{0.225, 0.225, 0.55\}$. In that case it is easy to see from equation (9) that all percentiles above 0.793 actually *increase*.

upwards (so the percentiles *decrease*). As k or N increases without bound, the price distribution becomes degenerate and all mass will be centered around the marginal cost (i.e. a pure strategy equilibrium).

Consumer welfare depends negatively on the expected price paid by a consumer. The next Corollary shows how an increase in N or k affects these expected prices.

Corollary 7 *The expected price paid by consumers is decreasing in the number of firms N and the network size k.*

Proof. The expected price a consumer pays depends on the number of prices he is informed about: A consumer who is informed about m firms' prices will pay the minimum of the m prices he is informed about. The expected price paid by such a consumer is $E_{\min}^{(m)}[p] = \int_0^1 p dF_{\min}^{(m)}$, where $F_{\min}^{(m)} = 1 - (1 - F(p))^m$ is the distribution of the minimum price of m draws from $F(p)$. Since all percentiles are decreasing, $E_{\min}^{(m)}[p]$ is also decreasing in k and N. The expected price paid by a consumer with a network of size k is a weighted average of the expected minimum prices $E_{\min}^{(m)}[p]$, where the weights are determined by the probabilities of observing m prices. More specifically, the expected price paid by a consumer with a network of size k is $E^k[p] = \sum_{m=1}^{M^k} \theta_m(k, N) \cdot E_{\min}^{(m)}[p]$. Since $E_{\min}^{(m)}[p]$ is decreasing in N and k, and since an increase in N or k causes an upward shift in the information mass (cf. Corollary 4), such an increase will also decrease $E^k[p]$. $\blacksquare$

In line with the predictions of perfect information oligopoly models, we therefore have that competition unambiguously decreases (increases, resp.) all consumers' expected prices (utility, resp.). With unit demand, prices are just transfers from consumers to firms. If all prices in the distribution are decreasing then individual and aggregate firm profits must also be decreasing in k and N.

3 Extensions

3.1 Sequential Search

In this section we apply our model of information networks to the sequential search oligopoly model by Stahl (1989). In the original model consumers varied in terms of their initial information and search costs, and could only become more informed by engaging in costly sequential search. We incorporate information spillovers by allowing consumers to pool their initial information with the members of their network before deciding whether they want to engage in costly search. The introduction of consumer information networks generates price dispersion as an equilibrium and restores the standard prediction that an increase in the number of firms unambiguously decreases prices.

In Stahl's seminar paper on sequential search oligopoly, price dispersion was obtained as an equilibrium phenomenon in a model in which both firms and sequentially searching consumers behaved optimally. Firms find it optimal to randomize over prices as there is heterogeneity in consumer informedness. In that case randomization balances the business stealing and surplus extraction effects discussed earlier on. The heterogeneity in informedness is rationalized by introducing heterogeneity in search costs: A share of "shoppers" has a zero search cost and always finds it optimal to compare all prices; the remaining share of "non-shoppers" has a positive search cost s and does not find it optimal to search as the expected price decrease from searching is too low compared to the search cost.

An undesirable property of the Stahl model is the lack of information spillovers between consumers and the associated informational unresponsiveness of non-shoppers to the number of firms in the market. In equilibrium, non-shoppers are only informed about the price of a single firm and shoppers are always fully informed, irrespective of the number of firms. This asymmetric informational effect of competition has surprising effects on prices: Stahl noted that for any downward sloping demand curve an increase in the number of firms would eventually lead to monopolistic pricing. Janssen, Pichler, and Weidenholzer (2009) show that for the case of unit demand the expected price is strictly increasing in the number of firms. The driving force behind these strange results is exactly the asymmetric informational effect. As the number of firms increases, firms find it more difficult to compete for the informed consumers as they are less likely to be the lowest-priced firm. This induces them to shift their focus towards the non-comparing consumers and increase their prices. By introducing information spillovers non-shoppers become more informed which prevents firms from raising prices.

We incorporate information spillovers in the Stahl model by combining it with information networks. In particular, we assume that prior to engaging in costly sequential search activities consumers pool their initial information with the members of their network. If consumers are not fully informed after accessing their network then they can still decide to search. Contrary to the original model we do not introduce any heterogeneity in search costs among consumers. All consumers are assumed to be of the non-shopper type and face a positive search cost s. In the original model this heterogeneity was necessary in order for there to be heterogeneity in consumer informedness such that price dispersion could arise. In our model this heterogeneity will also manifest due to the fact that some consumers will have more informative networks than others. As such we do not explicitly need the somewhat artificial construct of “shoppers” to generate price dispersion.

After accessing the network, the share of consumers that is informed about m prices is given by the expression in Theorem 1. Consumers that do not become fully informed can now become more informed by engaging in costly sequential search activities. It is well-known that for such a consumer the optimal sequential search strategy is characterized by a price threshold ρ, which is given by:

$$\rho = \min\{r, v\} \tag{11}$$

If the lowest price quote known is below ρ, the consumer will stop searching and buy from the firm offering that price quote. Clearly ρ should be weakly smaller than the willingness to pay v, since he must obtain a positive utility in order to be willing to buy. The price should also be smaller than the reservation price r. The latter is such that if it is the lowest price known by a particular consumer, the expected price decrease from searching for another price, which we denote by $\triangle$, would exactly be offset by the search cost s. For any price above (below, resp.) r consumers will thus (not) find it worthwhile to continue searching. Hence the consumer's reservation price r satisfies:

$$\triangle(r) \equiv \int_{\underline{p}}^{r} (r-p) f(p)\, dp = s \tag{12}$$

where $f(p)$ is the density function of the price distribution $F(p)$.

In equilibrium all firms again randomize according to a nondegenerate price distribution $F(p)$ with an upper bound now equal to $\overline{p} = \rho$. Why firms should not price above v is obvious, but firms will also not charge a price above r, since this will cause consumers to continue to search and buy from another firm. Since $\overline{p} = \rho$, the lowest price known to each consumer after accessing the network will be sufficiently low such that no consumer finds it worthwhile to engage in costly search. Consumers thus choose not to become more informed and the information probabilities are therefore unchanged and given by Theorem 1.

The firm's profit function is unchanged compared to the case without sequential search. Using similar arguments as before we can therefore derive the inverse price distribution, which is now given by

$$p(x) = c + \frac{\rho - c}{\sum\limits_{m=1}^{M^k} m \left(\frac{\theta_m}{\theta_1}\right) x^{m-1}} \tag{13}$$

where ρ is given by equation (11) and $x = 1 - F(p)$.

The equilibrium of the sequential search model is now implicitly defined by equation (11), (12) and (13). Due to the assumption of unit demand we can solve explicitly for the equilibrium reservation price r^*. To see this, note that since $\overline{p} = \rho$, we have that whenever $r < v$, equation (12) can be rewritten as:

$$r = E[p] + s \tag{14}$$

where $E[p] = \int_{\underline{p}}^{\overline{p}} pf(p)\,dp$. The reservation price equals the expected overall price $E[p]$, which is what the consumer would pay on average if he would search again, augmented by the search cost s. The expected price decrease from searching once more, $r - E[p]$, is then exactly offset by the search cost s. Using the expression for the inverse price distribution $p(x)$ we can rewrite the expected price more conveniently as

$$\begin{aligned} E[p] &= \int_{\underline{p}}^{\overline{p}} pdf(p)\,dp = \int_0^1 p(x)\,dx \\ &= c + \alpha(k, N) \cdot (r - c) \end{aligned} \tag{15}$$

where

$$\alpha(k, N) = \int_0^1 \left[\sum_{m=1}^{M^k} m \left(\frac{\theta_m}{\theta_1} \right) x^{m-1} \right]^{-1} dx$$

with α decreasing in k and N, $0 \leq \alpha \leq 1$, and $\lim_{N\to\infty} \alpha = \lim_{k\to\infty} \alpha = 0$.

The overall expected price charged by firms is thus equal to the firms marginal cost plus a markup which is proportional to the difference in the reservation price and the marginal cost. For a given r and c this markup is decreasing in the network size k and the number of firms N. The equilibrium value for the reservation price can now be found by plugging in the expression for $E[p]$ into equation (14) and solving for r, which yields:

$$r^* = c + \frac{s}{1 - \alpha} \tag{16}$$

which is decreasing (increasing, resp.) in k and N (c and s, resp.), and $\lim_{N\to\infty} r^* = \lim_{k\to\infty} r^* = c + s$. The mechanics behind these results are discussed below.

Given our characterization of the equilibrium we now proceed to discuss the main results of the model with sequential search. We start by comparing the model with sequential search to our benchmark model without search.

Proposition 8 *All percentiles of the distribution are weakly lower when consumers have the opportunity to search sequentially.*

Both in the model with and without search consumers are equally informed, as consumers choose not to search. When consumers are able to search, firms might however not be able to charge the same maximum price as before. Firms never charge a price higher than the consumer valuation v, since otherwise no consumer would ever buy. In the model with search, firms must also take into account that charging a price above the reservation price r^* induces consumers to start searching. Firms refrain from doing so as this would lead them to lose all their customers to rivals. Whenever the equilibrium reservation price r^* is below v, the upper bound of the price distribution is therefore strictly lower when consumers have to opportunity to search. By equation (13) a lower value for the upper bound ρ decreases all percentiles of the price distribution.

Next, we consider the comparative statics of an increase in N and k. The comparative statics of an increase in c and s are not discussed explicitly as they do not change compared to the original Stahl model. In particular, both increases cause prices to increases unambiguously.

Proposition 9 *The reservation price r^* and all percentiles of $F(p)$ are decreasing in the number of firms N and the network size k. As the number of firms or the network size grows larger pricing eventually becomes perfectly competitive.*

An increase in the number of firms N or the network size k decreases the equilibrium reservation price and all percentiles of the distribution. This is because such an increase improves information transmission among consumers and causes them to become more informed, just like in the model without search. This in turn induces firms to compete more fiercely and decreases prices. Now there is also be a secondary effect, as consumers reoptimize their reservation price. By equation (14) a decrease in the expected price of the distribution causes consumers to decrease their reservation price with the same amount. Whenever $r^* < v$ such a decrease reduces the upper bound of the price distribution and further reduces prices. An increase in N or k can therefore have a larger impact on prices when consumers have the opportunity to search. As the number of firms or the network grows larger, consumers become fully informed and firms have to compete fiercely. For a given upper bound this again decrease all percentiles of the price distribution. All mass of the price distribution is concentrated at the marginal cost level c, and the expected price of this degenerate distribution equals $E[p] = c$. By equation (14) the reservation price converges to $c + s$, and not to c, as consumers are willing to stop searching at a higher-than-average price due to search frictions.

Figure 3 demonstrates the effect of an increase in N or k on r^* by plotting it for various values of k and N.

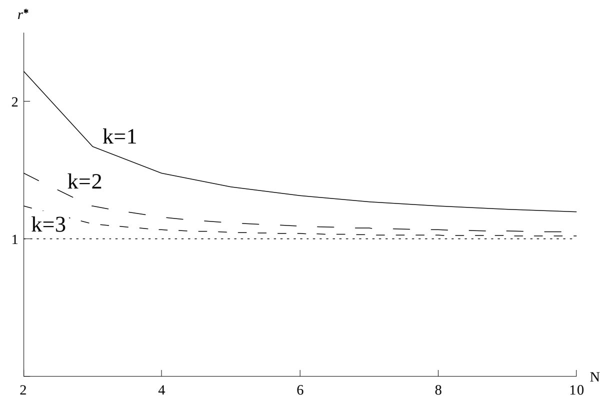


Figure 3: Equilibrium reservation price (c=0; s=1)

3.2 Network Segregation

In the benchmark model each individual on a consumer's network is assumed to have information on one randomly chosen firm. The information between the different members of a network and the consumer using the network is therefore independent. In that case an increase in the number of firms or the network size increases the likelihood that the consumers on the network have price information about a different firm than the one already known by the consumer, thus enhancing information transmission. The literature on networks with heterogeneous agents has shown however that agents have a tendency to associate with others similar to themselves (see, e.g., Jackson and López-Pintado, 2013). In that case the information between the network members is no longer independent. A larger network might therefore no longer contribute to the information diffusal process, as the extra network members have information that is similar to the members already on the network. We explore the effects of network segregation on information transmission in two different ways and show that our main results are robust if networks are not too segregated/integrated. In that case an increase in N or k continues to improve the information transmission and decreases all percentiles of the price distribution. We also show that an increase in the degree of network segregation tends to decrease information transmission and increase all percentiles.

3.2.1 Segregation I: Information Clustering Around a Subset of Firm Prices

A first but quite extreme way to include network segregation would be to assume that all network connections of a particular consumer only know the prices of a random *subset* of $n \leq N$ firms, which is common across the network and includes the firm whose price the consumer already knows. For example, when $n = 2$ all connections of a consumer initially knowing the price of firm 1 might only know the prices of firms 1 and 2, but never know the prices of the other firms. This might be the case because the consumer and its network connections are geographically concentrated around a particular set of firms. The value of n relative to N can be thought of as an inverse measure of network segregation, in the sense that a lower value of n induces information to become clustered around a smaller set of firms. In the extreme case that $n = 1$ the network is *fully segregated*, and becomes uninformative to the consumer as all connections only know the same firm's price.

In this first setting of segregation all connections now randomly receive price information from one of the n firms, so we can reinterpret Theorem 1 in terms of n: The consumer's probability of learning m different prices is now given by $\theta_m(k, n)$ instead of $\theta_m(k, N)$. Using this reinterpretation it is easy to see that an increase in the network size k only has an impact on θ_m if the network is not fully segregated (i.e. $n \geq 2$). In that case the effect of an increase in k is qualitatively the same as in the benchmark model: A larger network implies a higher probability of learning a new price, thus increasing information transmission and decreasing all percentiles of the price distribution. With fully segregated networks (i.e. $n = 1$) all connections know the same price as the consumer, so the latter does not learn any new prices from his network, making the network completely uninformative. An increase in the actual number of firms N on the other hand will only have an impact on θ_m to the extent that n changes. As long as n increases in N, networks become more informative as N increases, and the results of the benchmark model continue to hold: More firms imply more informative networks inducing more competition and decreasing all percentiles. If on the other hand n is invariant to N, θ_m will not change and more firms do not increase competition. It is however more likely that n is increasing in N as the newly entering firm will have to locate somewhere and as such be close to at least some of the consumers. A last comparative static of interest is that of a decrease in n. For a given number of firms N, such a decrease can be interpreted as an increase in the degree of network segregation. By the reinterpretation of Theorem 1 such a decrease is equivalent to a decrease of N in the benchmark model, thus decreasing information transmission and increasing all percentiles of the price distribution.

3.2.2 Segregation II: Information Clustering Around the Own Price

A second, less extreme, way of including network segregation is to maintain the assumption that the connections can be informed about all N prices, but to vary the likelihood that any of these connections knows the *same* price as the consumer. In particular, denote the probability that a consumer's network connection knows the same price as he does by p_I. If the network connection knows a different price, which occurs with probability $1-p_I$, we assume that he is equally likely to be informed about any of $N-1$ remaining firms' prices. In the benchmark model all prices known by the network were completely random, so the probability that any connection knew the same price as the consumer was $1/N$. The benchmark model is therefore the special case in which $p_I = 1/N$, and a higher (lower, resp.) value of p_I increases (decreases, resp.) the degree of segregation and makes it less (more, resp.) likely that a new price is learnt through the network. If $p_I = 0$, the network is *fully integrated* and all connections will know a different price than the consumer's, so the consumer will always learn at least one additional price. If on the other hand $p_I = 1$, the network is *fully segregated* and all connections know the same price as the consumer's, so the consumer will not learn any new prices.

Generalizing Theorem 1 to allow for the second type of network segregation yields the following Proposition:

Proposition 10 *In a market with N firms and a network segregation level of p_I, the fraction of consumers with a network of size k that is informed about the prices of m firms, denoted by $\widetilde{\theta_m}(k, N, p_I)$, is given by:*

$$\widetilde{\theta_m}(k, N, p_I) = \begin{cases} \sum_{l=1}^{k} \binom{k}{l} (p_I)^{k-l} (1-p_I)^l \theta_{m-1}(l-1, N-1) & \text{if } m \leq M^k \\ 0 & \text{if } m > M^k \end{cases} \tag{17}$$

where $\theta_{m-1}(l-1, N-1)$ is given by (1) and $M^k = \min\{k+1, N\}$.

Proof. A consumer with a network of size k will only learn additional prices if some of these k connections are not informed about the same price. The probability that l out of k consumers know a different price is given by $\binom{k}{l} (p_I)^{k-l} (1-p_I)^l$. Since these l connections know one or more prices of the remaining $N-1$ prices, the consumer learns $m-1$ new prices with probability $\theta_{m-1}(l-1, N-1)$. The reason that we write $l-1$ instead of l is that we have to exclude the price known by the consumer as we are conditioning on the fact that the connections know a different price than the consumer's. To conclude we sum over all possible realizations of l, ranging from 1 to k. ■

Note that whenever $p_I = 1/N$, the benchmark case is obtained and we have that $\widetilde{\theta_m}(k, N, 1/N) = \theta_m(k, N)$. If networks are fully integrated ($p_I = 0$), we have that $\widetilde{\theta_m}(k, N, 0) = \theta_{m-1}(k-1, N-1)$ where $\widetilde{\theta_1}(k, N, 0) = 0$. In that cases a consumer would learn new prices as if he had a smaller network ($k-1$), but whose connections knew only prices he did not already know ($N-1$). If on the other hand networks are fully segregated ($p_I = 1$) consumers do not learn any new prices from their network and we have that $\widetilde{\theta_1}(k, N, 1) = 1$.

Intuitively we would expect that as long as networks are not fully integrated or segregated, an increase in the number of firms N or the network size k should have a similar effect on the information probabilities as in the benchmark model. Moreover, an increase in the degree of network segregation p_I should have a similar effect as a decrease in the network size in the benchmark model. The reason is that the degree of network segregation can be seen as adding noise to the information diffusal process. In a sense, a more segregated network can therefore be seen as a decrease of the "effective" network size in the baseline case. This is because in a more segregated network more connections know the same price as the consumer's initial price, making these connections uninformative. Unfortunately we have not been able to provide a generalization of Corollary 4 for this second type of network segregation. We do have an expression for the expected number of prices, which provide evidence for our claim that the result of Corollary 4 should continue to hold if $0 < p_I < 1$.

Corollary 11 *The expected number of prices observed by a consumer with a network of size k and network segregation parameter p_I is given by:*

$$\widetilde{E_m}(k, N, p_I) = \sum_{m=1}^{N} m\widetilde{\theta_m}(k, N, p_I) = N\left(1 - \left(1 - \frac{1}{N}\right)\left(1 - \frac{1-p_I}{N-1}\right)^k\right) \tag{18}$$

which is concave and increasing in N and k, and convex and decreasing in p_I

Proof. see Appendix ■

Note that whenever $p_I = 1/N$ this reduces to $E_m(k, N)$. For the two other special cases we have that when $p_I = 1$ it reduces to $\widetilde{E_m}(k, N, 1) = 1$, whereas if $p_I = 0$ we have that $\widetilde{E_m}(k, N, 0) = 1 + E_m(k-1, N-1)$. These results are again intuitive since with fully segregated networks consumers do not learn any additional prices and are thus always informed about a single price. When networks are fully integrated consumers always learn additional prices, as if they were sampling k prices from the $N-1$ remaining firms.

To conclude we note that the welfare consequences of an increase in k and N also continue to hold if $0 < p_I < 1$, as summarized in the Proposition presented below. Moreover, an increase in the degree of network segregation increases all percentiles.

Proposition 12 *When networks are fully integrated or segregated, an increase in N or k does not affect $F(p)$. If networks are not fully integrated or segregated, an increase in N or k decreases all percentiles of $F(p)$. An increase in the degree of network segregation p_I increases all percentiles of $F(p)$.*

Proof. see Appendix ■

The intuition behind the effect of an increase in N and k is unchanged compared to the benchmark model The reason that an increase in the degree of network segregation p_I increases all percentiles is exactly because it tends to clutter the information transmission. It should therefore have a similar effect as an decrease in the network size.

3.3 Other Extensions

In this section we briefly discuss two other ways in which the model may be further extended: Product differentiation and network heterogeneity.

So far we have assumed that the product being sold by firms is homogeneous. In that case the only source of price dispersion is imperfect information. In general firms sell differentiated products, adding a second dimension of pricing differences. For the case of vertically differentiated products, the model could easily be extended by using the approach laid down in Wildenbeest (2011). That approach allows the consumer's valuation v to vary across firms, but firms offering a higher quality face a higher marginal cost c such that the value-to-cost margin $v_i - c_i$ is constant across firms. In equilibrium all firms still randomize over prices, but firms offering a higher quality charge a higher expected price that exactly offsets the quality difference. The comparative statics discussed earlier would then apply to each firm's price distribution: An increase in the number of firms or the network size would decrease all percentiles of each firm's price distribution, but price differences between firms due to quality differences would persist.

In the benchmark model we have also assumed that consumers are homogeneous ex ante, in the sense that they all have an equally large network size. Ex post, consumers are heterogeneous in terms of the number of prices they have learnt through this network, as the network of some consumers will contain more new prices than others. In reality, consumers are also heterogeneous in terms of their network size. The model could therefore be adjusted by considering a distribution of network sizes. As long as consumers' network sizes are sufficiently similar in size, the benchmark model should however provide a good approximation and all results should carry over. When there is extreme heterogeneity things will be different however. Consider for instance the case when there are only two types of consumers: A type without a network, who does not compare prices; and another type with an extremely large network, who always compares all N prices. In such a configuration of "shoppers" and "non-shoppers" it is well-known that an increase in the number of firms might actually increase prices (see Rosenthal, 1980). Lach and Moraga-González (2012) however provide a set of general conditions which ensure that all percentiles are decreasing in N, which can be checked for each network distribution at hand.

4 Conclusion

This paper introduces consumer information networks as an alternative information mechanism which is able to capture information spillovers between consumers in a tractable way. It is shown how larger networks and the entry of new firms increase information transmission, which results in firms competing more fiercely and as a result charge lower prices. These results are robust to the introduction of costly sequential search as in Stahl (1989). Network segregation tends to reduce information transmission, but unless networks are fully segregated, a significant amount of information can still be transmitted and prices will be decreasing in the network size and the number of firms. An increase in the degree of network segregation tends to increase prices by clogging the information transmission.

Future research might focus on the empirical implications of the model. For instance, over the last decade, an increasing number of papers has been dedicated to uncovering information mechanisms using structural methods (e.g. Hong and Shum, 2006; Moraga-González and Wildenbeest, 2008). Uncovering these mechanisms is essential for understanding how markets of imperfect information work and how policy changes will affect market outcomes. The literature so far has only focused on estimating search cost, by assuming that consumer information is obtained only by consumers' costly search effort. One direction for future research might therefore to be to focus on uncovering information networks instead of search costs, and try to estimate how large such networks might be. Alternatively, one might consider combining both information channels in a single empirical model. Galeotti (2010) already pointed out that in reality consumers are likely to obtain information through their networks as well, and that neglecting this additional channel of information would lead to a serious bias in search cost estimates. The setup in Galeotti (2010) is however restricted to a duopoly and thus neglects the fact networks become more informative when there are more firms. Our model provides a tool to explicitly model information networks in setting with more than two firms. Future research efforts might therefore go into creating an empirical model in which both consumer search as well as information networks occur simultaneously in a fully oligopolistic setting.

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5 Appendix

Stirling numbers of the second kind

Table 2: Stirling numbers of the second kind S(a,b)

$a \backslash b$	1	2	3	4	5	6	7	8	9	10
1	1	0	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	0	0	0	0
3	1	3	1	0	0	0	0	0	0	0
4	1	7	6	1	0	0	0	0	0	0
5	1	15	25	10	1	0	0	0	0	0
6	1	31	90	65	15	1	0	0	0	0
7	1	63	301	350	140	21	1	0	0	0
8	1	127	966	1701	1050	266	28	1	0	0
9	1	255	3023	7770	6951	2646	462	36	1	0
10	1	511	9330	34105	42525	22827	5880	750	45	1

Proofs

Throughout these proofs we assume that $N \geq 2$ and $k \geq 1$.

Proof of Corollary 3

First note that $S(k+1, m)$ and $N!/\left[N^{k+1}(N-m)!\right]$ are both log concave in m. The product of two log concave functions is also log concave. Strong unimodality or single-peakedness is implied by log concavity, and allows for three possible cases: (i) θ_m is (weakly) increasing in m, (ii) θ_m is first (weakly) increasing and then (weakly) decreasing in m, (iii) θ_m is (weakly) decreasing in m. We now show that the beginning of the sequence $\theta_1, \theta_2, \ldots, \theta_{M^k-1}, \theta_{M^k}$ is never decreasing, which excludes case (iii). From Theorem 1 we have that $\theta_2 < \theta_1$ if $(N-1)\left(2^k-1\right)-1<0$, which is never satisfied if $N \geq 2$ and $k \geq 1$. We therefore have that θ_m can never be decreasing in m, which yields case (i) and (ii) of the Corollary.

By unimodality we now only need to check the end of the sequence: If the end is decreasing, we have case (ii), whereas otherwise we have case (i). We therefore need to verify when $\theta_{M^k-1} > \theta_{M^k}$. There are two situations we need to consider: (a) $N \geq k+1$ (s.t. $M^k = k+1$), and (b) $N < k+1$ (s.t. $M^k = N$).

Starting with (a), we have that $\theta_k(k,N) > \theta_{k+1}(k,N)$ if and only if

$$\begin{aligned} \frac{N!}{N^{k+1}(N-k)!} S(k+1,k) &> \frac{N!}{N^{k+1}(N-(k+1))!} S(k+1,k+1), \text{ or} \\ \frac{S(k+1,k)}{(N-k)} &> S(k+1,k+1) \end{aligned} \tag{19}$$

Since $S(k+1,k+1) = 1$ and $S(k+1,k) = \binom{k+1}{2} = (k+1)k/2$ this reduces to

$$\begin{aligned} \frac{(k+1)k}{2} &> (N-k), \text{ or} \\ k &> k^*(N) \end{aligned} \tag{20}$$

where

$$k^*(N) = \frac{\left(\sqrt{9+8N}-3\right)}{2}, \text{ with } 1 \leq k^*(N) \leq N-1 \tag{21}$$

which is strictly increasing in N. This inequality is satisfied if k is sufficiently large. For any given N and k (where $N \geq k+1$) we therefore have that if $k \leq (>, \text{resp.}) k^*(N)$, then θ_m is weakly increasing in m (first increasing and then decreasing in m, resp.).

Now consider (b): We have that $\theta_{N-1}(k,N) > \theta_N(k,N)$ if and only if

$$\begin{aligned} \frac{N!}{N^{k+1}(N-(N-1))!} S(k+1,N-1) &> \frac{N!}{N^{k+1}(N-N)!} S(k+1,N), \text{ or} \\ S(k+1,N-1) &> S(k+1,N), \text{ or} \\ G(k+1,N) &\equiv \frac{S(k+1,N-1)}{S(k+1,N)} > 1 \end{aligned} \tag{22}$$

The ratio $G(k+1,N)$ is strictly decreasing in $k+1$ and strictly increasing in N.[7] Now define for each N the largest k that satisfies the above inequality (if such a k exists) as

$$k^{**}(N) = \max\{k \in \mathbb{N} : G(k+1,N) > 1\} \tag{23}$$

such that $\theta_{N-1} > \theta_N$ if $N-1 < k \leq k^{**}(N)$ and $\theta_{N-1} \leq \theta_N$ if $k > k^{**}(N)$. If this critical $k^{**}(N)$ exists, then it is unique and (weakly) increasing in N, as $G(k+1,N)$ is strictly decreasing (increasing, resp.) in $k+1$ (N, resp.). We now only need to verify that the interval $\{N, \ldots, k^{**}(N)\}$

[7] See Theorem 3.2 in Sibuya (1987) for a proof of this result.

is non-empty. To do this we consider the smallest value of k in this interval, i.e. $k = N$, and check whether $N \leq k^{**}(N)$. If this holds, then there is at least one k in the interval so it is non-empty. Plugging in $k = N$, we obtain

$$G(N+1,N) = \frac{S(N+1,N-1)}{S(N+1,N)} \tag{24}$$

If we now use that[8]

$$\begin{aligned} S(N+1,N) &= \binom{N+1}{2} = \frac{(N+1)N}{2} \\ S(N+1,N-1) &= \frac{1}{24}(N+1)N(N-1)(1+3(N-1)) \end{aligned} \tag{25}$$

we can rewrite $G(N+1,N)$ as

$$G(N+1,N) = \frac{1}{12}(N-1)(3N-2) \tag{26}$$

which is strictly increasing in N if $N \geq 2$. If we find that $G(N+1,N) > 1$ for a particular value of N, we have that $N \leq k^{**}(N)$ for that particular value of N, but also for all N exceeding that value. If $N = 2$ then $G(3,2) = 1/3 < 1$, so the interval is empty. In that case we never have that $\theta_1 > \theta_2$. If $N = 3$ then $G(4,3) = 7/6 > 1$, and the interval is non-empty. For all $N \geq 3$ we therefore have that if $N < k+1$ and $k >(\leq$, resp.$)k^{**}(N)$, then θ_m is weakly increasing in m (first increasing and then decreasing in m, resp.).

Combining the results from (a) and (b) we have that: If $N = 2$, then $\theta_1 \leq \theta_2$. If $N \geq 3$, then θ_m is increasing in m if $k \leq k^*(N)$ or $k \geq k^{**}(N)$, and first increasing and then decreasing in m if $k^*(N) < k < k^{**}(N)$.

Proof of Corollary 4

From Theorem 1, we have that $\theta_m(k,N+1) - \theta_m(k,N) \geq 0$ if and only if

$$\frac{(N+1)!}{(N+1)^{k+1}(N+1-m)!} \geq \frac{N!}{N^{k+1}(N-m)!} \tag{27}$$

[8] See Abramowitz and Stegun (1972).

Solving for m yields

$$m \geq m_N^*(k,N) = (N+1)\left(1-\left(\frac{N}{N+1}\right)^k\right), \text{ where } 1 < m_N^*(k) < N+1 \tag{28}$$

This critical $m_N^*(k,N)$ is concave and increasing in both k and N.

From Theorem 1, we also have that $\theta_m(k+1,N) - \theta_m(k,N)$ if and only if

$$\begin{aligned} \frac{1}{N^{k+1}}S(k+2,m) &\geq \frac{1}{N^k}S(k+1,m), \text{ or} \\ \frac{1}{N} &\geq R(k+1,m) \equiv \frac{S(k+1,m)}{S(k+2,m)} \end{aligned} \tag{29}$$

Now note that the LHS of the inequality is positive, weakly smaller than 1/2, and independent of m. The RHS is strictly decreasing in m.[9] It also holds that $R(k+1,1) = 1$ and $R(k+1,k+2) = 0$. Consequently there exists a unique $1 \leq m_k^*(k,N) \leq k+2$ such that $\frac{1}{N} \geq R(k+1,m)$ if $m \geq m_k^*(k,N)$. As the RHS is independent of N and the LHS is decreasing in N, we have that $m_k^*(k,N)$ is (weakly) increasing in N. Since the LHS is independent of k, and the RHS is increasing in k we also have that $m_k^*(k,N)$ is increasing in k.[9]

Proof of Proposition 6

Restating equation (9), we have that

$$p(x) = c + \frac{v-c}{\sum_{m=1}^{M^k} m\left(\frac{\theta_m}{\theta_1}\right)x^{m-1}}$$

A sufficient condition for all percentiles to be increasing is if (θ_m/θ_1) is strictly increasing. We now show that this sufficient condition is indeed satisfied. From Theorem 1, we have that

$$\frac{\theta_m}{\theta_1} = \frac{(N-1)!}{(N-m)!}S(k+1,m) \geq 1 \tag{30}$$

Showing that this fraction is increasing in N is now straightforward. The result that the fraction is increasing in k follows from the fact that $S(k+1,m)$ is increasing in $k+1$.[10]

[9] See Theorem 3.2 and 3.3 in Sibuya (1987) for a proof of these results.

[10] This follows immedeatly from the fact that Stirling numbers of the second kind satisfy the recurrence relation $S(k+1,m) = mS(k,m) + S(k,m-1)$.

Proof of Corollary 11

Even though the probability distribution function of the number of different prices is fairly complicated (cf. Theorem 1), the expected value of this distribution,

$$E_m(k, N, p_I) = \textstyle\sum_{m=1}^{N} m \cdot \theta_m(k, N, p_I), \tag{31}$$

can be found rather easily by expressing it as a sum of indicator variables and exploiting the linearity of the expectation operator. To see this, define I_i as the indicator variable which takes on the value 1 if firm i's price is known by a consumer (after consulting the network), and 0 otherwise. The number of different prices known by the consumer is then given by the sum $\sum_{i=1}^{N} I_i$. By linearity of the expectation operator we have that

$$E_m(k, N, p_I) = E\left[\textstyle\sum_{i=1}^{N} I_i\right] = \textstyle\sum_{i=1}^{N} E[I_i] = NE[I_i] \tag{32}$$

where the last equality follows from the fact that firms are symmetric. The expected value of $E[I_i]$ is the probability that firm i's price is drawn, and is the inverse of the probability that firm i's price is not drawn. The probability that i's price is not drawn is the probability that firm i is not the consumer's initial sample, which happens with probability $1-(1/N)$, and the probability that none of the k network connections contains firm i's price. If l out of k network connections are in a different group (so they do not know the same price as the consumer's initial price), which occurs with probability $\binom{k}{l}(p_I)^{k-l}(1-p_I)^l$, the conditional probability that price i is not known by the l connections is $(1-1/(N-1))^l$. The probability that none of the k connections knows price i is then given by the following sum:

$$\textstyle\sum_{l=1}^{k} \binom{k}{l} (p_I)^{k-l}(1-p_I)^l \left(1-\frac{1}{N-1}\right)^l = \left(1-\frac{1-p_I}{N-1}\right)^k \tag{33}$$

where the equality follows from the Binomial theorem. The probability that firm i's price is known is thus given by

$$E[I_i] = 1-\left(1-\frac{1}{N}\right)\left(1-\frac{1-p_I}{N-1}\right)^k \tag{34}$$

The overall expectation is thus given by

$$E_m(k, N, p_I) = N\left(1-\left(1-\frac{1}{N}\right)\left(1-\frac{1-p_I}{N-1}\right)^k\right) \tag{35}$$

When $p_I = 1/N$ this reduces to

$$E_m(k, N, 1/N) = N\left(1 - \left(1 - \frac{1}{N}\right)^{k+1}\right) \tag{36}$$

Proof of Corollary 12

Since $\widetilde{\theta_1}(k, N, p_I) = (p_I)^k$, we have that

$$\frac{\widetilde{\theta_m}}{\widetilde{\theta_1}} = \sum_{l=1}^{k}\binom{k}{l}\left(\frac{1-p_I}{p_I}\right)^l \theta_{m-1}(l-1, N-1) \tag{37}$$

We now rewrite p_I as a function of the benchmark value $1/N$. More in particular, let

$$p_I(a) = \frac{1}{(1-a)+aN}, \text{ where } a \in [0, \infty) \tag{38}$$

which is strictly decreasing in a and where $p_I(0) = 1$, $p_I(1) = 1/N$ and $\lim_{a\to\infty} p_I(a) = 0$. The ratio $(1-p_I)/p_I$ is then given by $a(N-1)$, so we can rewrite equation (37) as

$$\begin{aligned}\frac{\widetilde{\theta_m}}{\widetilde{\theta_1}} &= \sum_{l=1}^{k}\binom{k}{l}a^l (N-1)^l \theta_{m-1}(l-1, N-1) \\ &= \sum_{l=1}^{k}\binom{k}{l}a^l (N-1)^l \frac{(N-1)!}{(N-1)^l (N-m)!} S(l, m-1) \\ &= \sum_{l=1}^{k}\binom{k}{l}a^l \frac{(N-1)!}{(N-m)!} S(l, m-1)\end{aligned} \tag{39}$$

where the second equality follows from Theorem 1. When $p_I \to 0$ we have that $\widetilde{\theta_m}/\widetilde{\theta_1} \to \infty$ for $m \geq 2$, whereas if $p_I = 1$ we have that $\widetilde{\theta_m}/\widetilde{\theta_1} = 0$ for $m \geq 2$. In both cases the ratio is independent of N or k. Whenever $0 < p_I < 1$ the ratio is again increasing in N and k. This is because each of the components of the sum are increasing. Similarly, the ratio is decreasing in p_I as each of the components of the sum is increasing in a.

Note that when $p_I = 1/N$ we have that $a = 1$ and ratio reduces to

$$\begin{aligned}\frac{\widetilde{\theta_m}(p_I = 1/N)}{\widetilde{\theta_1}(p_I = 1/N)} &= \frac{(N-1)!}{(N-m)!}\sum_{l=1}^{k}\binom{k}{l} S(l, m-1) \\ &= \frac{(N-1)!}{(N-m)!}\sum_{l=m-1}^{k}\binom{k}{l} S(l, m-1) \\ &= \frac{(N-1)!}{(N-m)!} S(k+1, m)\end{aligned} \tag{40}$$

where the second equality follow the fact that $S(a,b) = 0$ if $a < b$, and the third equality follows from a well-known recurrence relation that the Stirling numbers satisfy.