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Abstract

This paper studies the political economy of public transport pricing and quality decisions in urban areas. We consider a hypothetical two-region federation. In each region there is a demand for public transport and for car use, and the group mainly using public transport may be a majority or minority in the region; moreover, part of the users of both the public transport system and the road network may come from outside the region. In this setting, we compare regional and federal decision making on public transport fares and supply characteristics. Under regional decision-making we find that, first, the political process may result in very low public transport fares, even if car owners are a large majority of the population. The fare preferred by car owners is increasing in the toll on car use. Cost recovery always improves with the share of outside users. Second, imposing a zero deficit constraint on regional public transport operators implements the second-best welfare optimum, independent of whether car owners or non-car owners have the political majority. Third, compared to centralized decision making, decentralized decision making leads to higher fares and better cost recovery. Our findings are consistent with the lack of opposition to very large public transport subsidies in Europe, and they provide a potential explanation for the tendency towards decentralization of public transport policy-making observed in many countries over the last decades.

JEL codes: H23, D62, R41, R48

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1. Introduction

Almost all public transport activities are heavily subsidized by the government: cost recovery rates of local public transport vary between as little as 10% and almost 100% (see, for example, Van Goeverden, Rietveld, Koelemeijer en Peeters (2006) and Savage (2010))[1]. The subsidies come from the local, the regional or the federal government. It is rare to find local public transport systems that are almost fully funded by higher level governments, but especially when metro or light rail services are offered, important subsidies are provided (Reynolds-Fegan, Durkan and Durkan (2000)). This type of public transport is characterized by strong economics of density, and it can only be cost effective for big cities with high density (Tirachini, Hensher and Jara-Diaz (2010a), Wang (2011)). Efficient pricing then requires a large subsidy also for operation costs (Parry and Small (2009)).

Who is in charge of public transport decision-making seems to widely differ across countries and, although there are exceptions, there has been a shift towards decentralized decision making in several countries. For example, in the Netherlands, in the 1990's all investment in local public transport was funded by the central government. More recently, the pricing and operation of local public transport was decentralized to 19 regions that each received a lump-sum subsidy in function of their geographical characteristics. In the UK, all local public transport outside of London is privatized, and local governments can subsidize discounts for special categories of users. Sweden, Germany and France also have decentralized local public transport towards the regions in the last decennia. In Belgium, local public transport was first organized by each sub-region (province), but more recently decision making was centralized at the regional level (Flanders, Wallonia). The result in Flanders was a uniform service level and a uniform pricing system for all cities in the region, but individual cities could finance discounts and even offer free public transport for particular categories of citizens (retired people, students).

There is an extensive literature on the relative efficiency of different management structures for public transport operations. Surprisingly, although public transport absorbs an important part of the regional or city budget in many cities (in the Brussels Region, for example, this comes close to half of the budget), almost no literature exists on the political decisions determining the constraints for the management structure, such as the level of the fares, the level of subsidies and the minimum quality of service.

The purpose of this paper is to better understand the political economy of public transport pricing and quality decisions in urban areas. One intriguing question is why in many

[1] For detailed information on cost recovery rates, also see the database of the *UITP*.

countries very large public transport subsidies seem to get fairly wide political support, even when car owners are by far the majority. Another puzzle is why so many countries opt for decentralization of local public transport (often combined with privatization). The literature has intensively studied the efficiency implications of privatization and contracting out but, surprisingly, the reasons for the increasing decentralization have received much less attention.

We start from the framework developed for congestible local public goods (see De Borger and Proost (2013)). Their model predicts that, when the authority is given to the local government and users of the local public good have a majority, the user charge will be higher the more non-local users there are, and the higher the proportion of the population that is a user. More non-local users allow tax exporting, and a higher proportion of users implies that an individual voter receives a higher share of the revenues (or pays a smaller part of the deficit). The analysis also found that centralized provision may give rise to price and capacity discrimination between regions. Different institutional safeguards can be put in place to avoid this discrimination (for example, uniform prices). Finally, the analysis suggested that when only a minority of voters uses the local public good, the majority of non-users may well decide to have very high prices and low capacities.

Public transport has some peculiarities that require an adaptation of the model just discussed. First, public transport provision has increasing returns to scale, and both frequency and service quality are important characteristics of services supply. Second, there are few regions where people not owning a car (hence, they use only public transport) have a majority. Instead, in many regions car owners have a majority. Third, public transport subsidies can be a second-best response to too low prices for car use, once external costs of congestion and pollution are taken into account.

We find some interesting results. Suppose regions decide on public transport policy fares and quality characteristics. We show that, if non car owners have the political majority in the region, fares will be below the marginal social cost if there are few outside users of the public transport system and there is no toll on road use. On the contrary, with many outside users and a road toll in place, the fare wanted by the median voter may be much higher than the marginal social cost. If car owners are in charge at the regional political level, the fare is increasing in the toll on car use and declining in road congestion; the fare also decreases if car owners get a larger share of the public transport users surplus. The implication is that the political process may result in low public transport fares and high subsidies, even if non car owners are a small minority of the population. Not surprisingly, cost recovery always improves with the share of outside users. We further show that imposing a zero deficit restriction on public transport operations implements the second-best welfare optimum, independent of who is in charge at the regional political level. Comparing decentralized outcomes with policy decisions under centralized decision-making, it is found that under plausible conditions higher fares and improved cost recovery rates will result under decentralized decisions. Overall, our findings are consistent with the lack of opposition to very large public transport subsidies in Europe, and they provide a potential explanation for the tendency towards decentralization of public transport policy making observed over the last decades.

In section 2 we discuss briefly the literature and some empirical evidence. In section 3 we present the analytical model. The socially optimal fare, frequency and quality of public transport provision are derived in section 4, considering both the case with and without budgetary restrictions, and emphasizing the relation with un-priced external costs on the market for car use. In section 5 we analyze the outcome we can expect from political decisions by individual regions. Section 6 discusses public transport decisions by a federal government and presents a brief comparison between centralized and decentralized decisions. Finally, section 7 concludes.

2. Literature review

At least three strands of literature on the economics of public transport operations can be distinguished. First, pricing and service provision of public transport have been analyzed in detail from a normative, welfare economic viewpoint. Second, a large literature has developed on the role of the organizational form and the relative efficiency of private versus public operators. Third, a few papers have studied the political economy underlying public transport decisions.

Starting with the seminal paper by Mohring (1972) the literature has generated a large number of studies analyzing optimal pricing and service provision of public transport modes. Relevant extensions include, among many others, Jansson (1980), Frankena (1983) and Kraus (1991)). An early survey is provided in Jara-Diaz and Gschwender (2003); more recently, Small and Verhoef (2007) discussed the main theoretical insights. A series of numerical models have also been used to illustrate the implications of the theory in practice. For example, numerical examples of the second-best optimal fare for agglomerations like London can be found in Parry and Small (2009). They show that in 2001 the optimal second-best subsidy for most forms of public transport was higher than the actual subsidy observed. The main justification for the very high optimal subsidy (amounting to 90% of operating costs) was the important un-priced road congestion externality, combined with the strong assumption that for every two attracted public transport users, one would be a former car driver. Both De Borger and Wouters (1998) and Proost and Van Dender (2008) found similar qualitative results – low optimal fares and high subsidies -- for Belgium and the city of Brussels, respectively. More recently, Kilani, Proost and Van der Loo (2014) discuss a joint pricing reform of the congested road and public transport networks in Paris. They found that even at current road charges, it is best to increase prices of public transport on some links. Moreover, fares should be increased in general whenever road pricing is put in place. Finally, the relation between fares, frequency and information provision to passengers was studied by De Borger and Fosgerau (2012), focusing on various types of interaction between a national government and a local operator. They show that, if a private operator is responsible for setting frequency and information provision to passengers, welfare optimal fare regulation by a government agency implies setting the fare above the socially optimal level; both frequency and information provision to passengers are lower than socially optimal.

There has been intensive research on the effect of the organizational form on operational decisions and on overall performance. Issues such as the efficiency of private versus public provision, the effect of competition between different suppliers of bus services and the implications of different types of contracts have been analyzed (see, e.g., Van de Velde (2007) and Roy and Yvrande-Billon (2007)). Most recently, Gagnepain, Ivaldi and Martimort (2013) find that the type of contract is responsible for efficiency differences of 5 to 20%. However, the focus on the type of contract may make us forget that the total budget and the major supply characteristics are still decided by the political level. These decisions are the topic of the current paper.

The scarce literature dealing with the political decisions that drive public transport pricing and supply is easily summarized. Brueckner and Selod (2006) develop a political economy model with endogenous location in which the type of transport system is endogenous. Cities can opt for different systems offering a different money cost/time cost combination. The combinations vary from express highways (or metro) to slow bus systems. It is shown that in a homogenous city voters will make the socially optimal system choice, whatever the land ownership and associated rent effects of better urban transport systems. In a heterogeneous city, however, results are less clear. Under simple majority voting there could be a bias against capital-intensive, fast systems, resulting in a preference for bus systems rather than fast expressways. The main mechanism driving this bias is that highly skilled (high income) people live in the suburbs and vote for a fast system, but the median voter has not the same preference for speed. As a consequence, the resulting system is too slow from a welfare perspective. In a recent paper, Borck and Wrede (2008) specifically focused on the role of public transport subsidies for commuting. They show, for instance, that rich automobile drivers may suffer from transit subsidies, whereas poor transit users may benefit from subsidies to automobiles. Taking location as given, a small literature focuses on quality aspects of public transport services. For example, Glazer and Proost (2010) found that rent seeking mechanisms at the federal level may lead to too wide service provision of public transport. Finally, an empirical analysis by Holian and Kahn (2013) used voting outcomes on a number of transit-related ballot propositions from state-wide elections in California for the period 1990-2010. Controlling for demographic, socio-economic and political ideological factors, their results suggest that suburbanization is a possible factor in determining public support for public transit investment.

When we consider the pricing of a local public service in a majority voting system, we expect that several forces come into play (see De Borger and Proost (2013)). First, whenever users can rely on non-users to pay for a public service via general taxes, they will do so. Second, whenever non-users (or only part of the users) decide on the fares and have to contribute to paying the operational deficit, they will try to charge prices above the marginal social cost. Third, whenever non-locals use the service, the local government will try to extract a maximum of revenue from non-local users, resulting in tax-exporting behavior(see Arnott and Grieson (1981) for an early paper on tax exporting in a different setting). On top of these general tendencies, in the case of public transport the interaction with car use, congestion and possible road pricing cannot be ignored. Finally, if wages in the central business district are endogenous, neighboring regions that export labor may set too high transport fares and make too small public transport investments (Vandyck and Proost (2012)).

3. Specification of the model

In this section, we present the general structure of the model. After a brief overview of the general setting, we first discuss the specification of both the user cost and the operator cost of public transport in a given region. These specifications are based on an extensive literature (see, among many others, Mohring (1972), Jansson (1983), and Tirachini, Hensher and Jara-Diaz (2010)); however, given our interest in strategic policy decisions rather than in the details of public transport supply and costs, we use simple specifications that capture the main drivers of public transport costs. Next we specify the cost of car use. Finally, we present the demand structure of the model. Note that we focus throughout on a single public transport line in each region.

3.1. General setting

We use a model with only 2 regions that have the same size; each region has *R* voters. Within each region, we distinguish between two categories of voters. We assume some voters don't have a car[2] and can only use public transport; the number of such voters is denoted as $U_i \ (i=1,2)$. Other voters are car owners, but given the availability of public transport services these people may also use public transport; their number is denoted as $C_i \ (i=1,2)$. Of course, we have $U_i + C_i = R \ (i=1,2)$. The proportion of both types of voters can differ between the two regions. The total number of voters in the federation composed of the two regions is, of course, *2R*.

To describe the demand structure, let us start with the demand for public transport. There are three groups of public transport users in region 1:

(i) The group of U_1 local inhabitants that can only use public transport; their demand in region 1 is denoted as Y_1^U.

(ii) The group of C_1 inhabitants of region 1 that demand both public transport and private car trips. Their public transport demand in region 1 is denoted Y_1^C.

(iii) People living in region 2 that also demand some public transport services in region 1. Public transport demand in region 1 by these outsiders is denoted Y_1^T. Note that public transport demand in region 1 by inhabitants from region 2 comes from the group U_2 that can only use public transport (demand $Y_1^{T^U}$) as well as from the C_2 car owners in region 2 that use both their car and public transport services (demand $Y_1^{T^C}$). This distinction is not important until section 6 below; there it plays an important role.

Denoting total public transport demand in region 1 by Y_1, we therefore have

[2] Our model ignores the car ownership decision. We assume that individuals either can have a car or not, either depending on whether they can afford it or not (income, garage available or not, too old or too young for a drivers' licence, etc.).

$$Y_1 = Y_1^U + Y_1^C + Y_1^T \text{, where } Y_1^T = Y_1^{T^U} + Y_1^{T^C}$$

Next consider the demand for car use in region 1. We take into account two groups of car users:

(i) The demand for car use in region 1 that comes from the group of C_1 inhabitants of this region that can use both car and public transport. Their demand for car use in the region is denoted X_1^C.

(ii) Furthermore, the group of C_2 inhabitants from region 2 makes car trips in region 1. This demand from outsiders is given as X_1^T.

As a consequence, total demand for car trips in region 1 is

$$X_1 = X_1^C + X_1^T$$

The composition of demand for the two transport modes is summarized in Table 1 for region 1. Of course, similar definitions apply for region 2.

	Inhabitants of region 1 using public transport only	Inhabitants of region 1 using car and public transport	Inhabitants of region 2 using public transport only	Inhabitants of region 2 using car and public transport	Total
Size of the group	U_1	C_1	U_2	C_2	*2R*
Public transport use in region 1	Y_1^U	Y_1^C	$Y_1^{T^U}$	$Y_1^{T^C}$	Y_1
Car use in region 1	0	X_1^C	0	X_1^T	X_1

Table 1. Transport use in region 1

Our formulation allows for a heterogeneous composition of each region (car owners versus non car owners) as well as for spillovers in the use of public transport and car use between regions. As our focus is on public transport decisions and to simplify the technical analysis without losing essential insights, we will impose additional structure on demand, see below. Moreover, a congestion toll or tax on car use may be in place; however, unless otherwise noted, this tax is assumed to be exogenously fixed.

3.2. The user cost of public transport

The public transport literature referred to before (Jansson (1980), Tirachini et al. (2010), Jara-Diaz and Gschwender (2003)) argues that the user cost includes the fare, waiting time costs (at the bus or rail stop), access time costs (to go to bus or rail stop) and in-vehicle time costs. The latter include both the time cost when the vehicle is moving as well and the cost due to time losses at stops; the value of in-vehicle time depends on crowding. In general, one could specify the user cost as

$$p + v_w \frac{1}{2f} + v_a t_a + v_i t_i$$

Here p is the fare. The second term captures waiting time costs at the bus stop, assuming that users come to the stop ad random; v_w is the value of waiting time at the stop, f is the frequency offered. The third term are access costs. The value of access time v_a depends on things like weather conditions, but is independent of the policy variables. Access time t_a may depend on investment. However, as our model does not have much spatial detail, we assume access costs to be constant and normalize them at zero. Finally, the last term captures in-vehicle time costs. Let the value of in-vehicle time v_i depend on demand per vehicle (crowding) and on investment K_v in vehicle capacity or quality (vehicle size, number of seats and standing capacity, etc.); in-vehicle time t_i depends on investment in speed-related characteristics . We have

$$v_i(\frac{Y}{f}, K_v); \quad t_i(K_s)$$

Here Y is the total number of public transport passengers demanded in the region, and f is the frequency offered (per hour, per day, etc.).

The above discussion leads to the user cost

$$p + v_w \frac{1}{2f} + \left[v_i(\frac{Y}{f}, K_v) \right] \left(t_i(K_s) \right) \tag{1}$$

Now note that the sign of the effect of investment in speed and in vehicle capacity on total in-vehicle time costs is the same: investing in vehicle capacity or quality reduces the cost of the time to be spend in the vehicle, investing in speed reduces the time itself. We have

$$\frac{\partial \left\{ \left[v_i(\frac{Y}{f}, K_v) \right] \left(t_i(K_s) \right) \right\}}{\partial K_v} < 0; \quad \frac{\partial \left\{ \left[v_i(\frac{Y}{f}, K_v) \right] \left(t_i(K_s) \right) \right\}}{\partial K_s} < 0$$

We therefore summarize the in-vehicle time cost as

$$\left[v_i(\frac{Y}{f},K_v)\right]\left(t_i(K_s)\right)=C_i(\frac{Y}{f},K)$$

where K captures both types of investment. Specifying $C_i(.)$ linearly, the total generalized user cost (1) per public transport trip, denoted $g^Y(.)$, becomes

$$g^Y(Y,p,f,K)=p+\alpha\left(\frac{1}{f}\right)+\beta_0+\beta\frac{Y}{f}-\gamma K \qquad (2)$$

Here α is (half) the value of waiting time, β_0 is the in-vehicle time cost at normal operating speed (it does not play much of a role in the model and will be normalized at zero in what follows), β captures the crowding cost effect of higher demand per vehicle, and γ captures the effect on user costs of investing in speed, quality, information, vehicle capacity, etc.. Note that access time costs are ignored. User cost formulation (2) is the reduced form specification that will be used in the rest of this paper.

3.3. Operator costs of public transport supply

Total operator costs per hour are the frequency times the costs associated with one driving cycle (doing the route once). The latter consist of cycle costs C_m when the vehicle moves plus costs C_s when the vehicle has to stop:

$$TC=f*\left(C_m+C_s\right)$$

The cycle costs when moving depend on investment in speed (the cycle time goes down) or in vehicle capacity (for example, a two-wagon tram has higher cycle costs than a large bus and than a small bus). The cycle cost per vehicle at stops depends on the number of passengers that have to board or leave per vehicle. We can formulate these ideas by writing

$$TC=f*\left(C_m+C_s\right)=f*\left[C_m(K)+C_s\left(\frac{Y}{f}\right)\right]$$

Specifying the first cost function linearly ($C_m(K)=c_0+c_KK$) and the second one linearly with constant marginal and average cost ($C_s(\frac{Y}{f})=c_Y\frac{Y}{f}$), we can write

$$TC=f*\left[c_0+c_KK\right]+c_YY \qquad (3)$$

In this expression, c_Y reflects the marginal cost of an extra passenger (loading, unloading) for the provider. Total costs of public transport further depend on the frequency f offered and the quality of service *K*. Interpretation of (3) is that a trip of minimum quality service costs c_0 (we normalize the quality of such a service at *K=0* for simplicity). If one wants quality above the minimum level (better seats, less noise, higher speed, etc,), this raises *K* above the minimum level, increasing the cost per trip an arbitrary vehicle makes. Note that specification (3) implies that we neglect the fixed cost component of increasing quality say in bus or metro stations. However, adding a fixed cost to total cost does not alter our results as long as the fixed cost does not depend on quality or frequency. Again formulation of total cost (3) is a reduced cost formulation that will allow us to study the political choices on the essential price and quality characteristics of public transport.

3.4. The generalized cost of car use

We assume the road network in a given region is congestible. Moreover, it is assumed that a toll per kilometer may be imposed on drivers, possibly to capture congestion externalities. Denoting the toll by τ° and remembering that car transport demand was denoted by *X*, we specify the generalized cost of car transport $g^X(X,\tau^{\circ})$ in the simplest possible way as follows (note that ν, the slope of the congestion function, reflects the severity of extra traffic for congestion costs):

$$g^X(X,\tau^{\circ}) = \tau^{\circ} + \nu X \tag{4}$$

3.5. Reduced-form demand for public transport and for car use

In this section we describe the assumptions underlying the demand side of the model, and we derive reduced-form demands that relate demand for public transport and car use in a region as a function of all exogenous variables and parameters in the model.

Denote the generalized prices (including all user-relevant costs discussed above) of public transport and car use in a region by P^Y, P^X, respectively. The demand functions by the different groups *U*, *C* and *T* can then be specified in general as (taking into account that the group of local inhabitants *U* only uses public transport):

$$Y^U(P^Y), Y^C(P^Y,P^X), Y^T(P^Y,P^X)$$
$$X^C(P^Y,P^X), X^T(P^Y,P^X)$$

Total demands are obtained by aggregating demands of the relevant groups

$$Y(P^Y,P^X) = Y^U(P^Y) + Y^C(P^Y,P^X) + Y^T(P^Y,P^X)$$
$$X(P^Y,P^X) = X^C(P^Y,P^X) + X^T(P^Y,P^X)$$

To simplify the exposition without affecting the qualitative results, we impose one additional assumption on the model. We will assume that for voters that use both their private car and public transport, the two modes are perfect substitutes. Although not crucial for the results, this drastically simplifies the derivations. Of course, it is a strong assumption, but it is not that unrealistic once one realizes that many non-monetary costs (waiting times at stops, access costs, congestion, etc.) have been included in the definition of the generalized costs. The assumption of perfect substitutability allows us to invoke Wardrop's principle: given an internal solution where both the private car and public transport are used, in a user equilibrium there will be equality of the generalized prices of car and public transport trips.

To see the implications, note that equality of equilibrium generalized prices allows us to write the demand system $X(P^X, P^Y), Y(P^X, P^Y)$ as

$$Y(P), X(P)$$

where $P = P^Y = P^X$. In essence, a single price for a transport trip exists in equilibrium. Denoting *P(Y+X)* as the inverse total transport demand function, we derive 'reduced-form' demand functions for car use and public transport use by solving the following set of equilibrium conditions:

$$\begin{aligned} P(Y+X) &= g^Y(Y, p, f, K) \\ P(Y+X) &= g^X(X, \tau°) \end{aligned} \tag{5}$$

The generalized costs on the right hand side are given by (2) and (4), respectively. The equilibrium conditions (5) impose Wardrop's condition of equal generalized prices. The reduced-form demand functions that solve this system of two equations for Y and X yield demand for car and public transport use as a function of the public transport fare, frequency and quality, and of the road toll:

$$\begin{aligned} &Y^r(p, \tau°, f, K) \\ &X^r(p, \tau°, f, K) \end{aligned} \tag{6}$$

The equilibrium generalized price of a transport trip is then given as $P\left(Y^r(.) + X^r(.)\right)$.

To determine the effect of an increase in the public transport fare on demand for public transport and car use, we totally differentiate (5) and use (2) and (4) to find:

$$\frac{dY}{dp}=\frac{\partial Y^r}{\partial p}=\frac{\dfrac{\partial P(Y+X)}{\partial (Y+X)}-v}{\Delta}<0$$
$$\frac{dX}{dp}=\frac{\partial X^r}{\partial p}=\frac{-\dfrac{\partial P(Y+X)}{\partial (Y+X)}}{\Delta}>0 \tag{7}$$

where $\Delta=\frac{\beta}{f}\left(v-\frac{\partial P(Y+X)}{\partial (Y+X)}\right)-v\frac{\partial P(Y+X)}{\partial (Y+X)}>0$. As expected, a higher fare reduces public transport demand and raises car use.

Using similar derivations, we find the effect of frequency on demand:

$$\frac{dY}{df}=\frac{\partial Y^r}{\partial f}=\frac{\left[\dfrac{\partial P(Y+X)}{\partial (Y+X)}-v\right]\left[\dfrac{-(\alpha+\beta Y)}{f^2}\right]}{\Delta}>0$$
$$\frac{dX}{df}=\frac{\partial X^r}{\partial f}=\frac{-\left[\dfrac{\partial P(Y+X)}{\partial (Y+X)}\right]\left[\dfrac{-(\alpha+\beta Y)}{f^2}\right]}{\Delta}<0 \tag{8}$$

Finally, better quality affects demand as follows:

$$\frac{dY}{dK}=\frac{\partial Y^r}{\partial K}=\frac{\left[\dfrac{\partial P(Y+X)}{\partial (Y+X)}-v\right](-\gamma)}{\Delta}>0$$
$$\frac{dX}{dK}=\frac{\partial X^r}{\partial K}=\frac{-\left[\dfrac{\partial P(Y+X)}{\partial (Y+X)}\right](-\gamma)}{\Delta}<0 \tag{9}$$

Frequency and quality of public transport supply raise public transport demand and reduce car use.

To conclude the description of the model, note that in principle corner solutions are possible (car owners using only their car or using only public transport). These will briefly be considered below.

4. The social optimum

In this section, we briefly study the social optimum. As this will prove useful for later comparisons, we first consider the simplified case without any car use at all. Next, we introduce car use, accounting for road congestion and an exogenous road toll.

4.1 The socially optimal solution: public transport only

We want the optimal values of the fare, frequency and quality investment. Given the absence of car traffic, the welfare maximization problem can simply be specified as the sum of net consumer surplus and fare revenues minus operating costs:

$$\underset{p,f,K}{Max} \quad \left[\int_0^Y P^Y(y)dy - g^Y(.)Y \right] + pY - f\left(c_0 + c_K K\right) - c_Y Y$$

Here P^Y *(y)* denotes the inverse total demand function for public transport; the generalized user cost of public transport use $g^Y(.)$ was defined by (2) above.

The first-order condition for the optimal fare can be written as (using equality between generalized price and generalized cost):

$$-Y\frac{dg^Y(.)}{dp} + p\frac{dY}{dp} + Y - c_Y\frac{dY}{dp} = 0$$

Note by differentiating (2) that

$$\frac{dg^Y(.)}{dp} = 1 + \frac{\beta}{f}\frac{dY}{dp} \tag{10}$$

Substitute this expression in the first-order condition; this gives after simple algebra

$$p = c_Y + \frac{\beta}{f}Y \tag{11}$$

The optimal fare rule (11) makes sense. The optimal fare is the marginal cost of an extra passenger plus the external cost of crowding: an extra passenger raises the generalized cost because of crowding, from which all users suffer.

Next consider the first-order condition for frequency; again using the equality between generalized price and generalized cost, it can be written as:

$$-Y\frac{dg^Y(.)}{df} + p\frac{dY}{df} - \left(c_0 + c_K K\right) - c_Y\frac{dY}{df} = 0 \tag{12}$$

Differentiating (2) with respect to frequency, we have

$$\frac{dg^Y(.)}{df} = -\frac{\alpha}{f^2} + \beta\left[\frac{f\dfrac{dY}{df} - Y}{f^2}\right] = \frac{\beta}{f}\frac{dY}{df} - \left(\frac{\alpha + \beta Y}{f^2}\right) \tag{13}$$

Using this expression in (12) and working out we find

$$\left(p - c_Y - \frac{\beta Y}{f}\right)\frac{dY}{df} - \left(c_0 + c_K K\right) + \left(\frac{\alpha + \beta Y}{f^2}\right)Y = 0$$

The optimal pricing rule (11) allows us to rewrite this expression as

$$\left(\frac{\alpha+\beta Y}{f^{2}}\right)Y=\left(c_{0}+c_{K}K\right)$$

The left-hand side is the marginal benefit of a frequency increase (reduction in waiting time and crowding costs), the right hand side is the marginal cost of raising frequency for a vehicle of given quality (speed, capacity, comfort) *K*. Solving for frequency finally yields

$$f=\sqrt{\frac{(\alpha+\beta Y)Y}{c_{0}+c_{K}K}} \tag{14}$$

Frequency rises when crowding is more important and when the value of waiting time increases; moreover, it rises at higher demand. Frequency declines with higher costs of providing more frequent service or when offering higher quality. Of course, (14) is not a closed-form solution because demand itself depends on frequency, and the right hand side further depends on the fare and the quality offered. Also note that in the absence of crowding costs ($\beta=0$) we get the standard square root formula often used in the early public transport literature, whereby optimal frequency rises in the square root of demand (see, for example, Mohring (1972), Jansson (1979)).

Finally, the optimal quality investment satisfies the first-order condition

$$-Y\frac{dg^{Y}(.)}{dK}+p\frac{dY}{dK}-fc_{K}-c_{Y}\frac{dY}{dK}=0$$

We have, by differentiating (2)

$$\frac{dg^{Y}(.)}{dK}=\frac{\beta}{f}\frac{dY}{dK}-\gamma$$

Using this in the first-order condition and substituting (11), we find

$$fc_{K}=\gamma Y \tag{15}$$

As should be the case, the marginal cost of investment in quality equals the marginal benefit (the reduction in generalized cost for all users).

Expressions (11), (14) and (15) describe the social optimum. To find out whether first-best policies in the absence of car use guarantee cost recovery, we use (11) and (14) in the definition of net revenues to find

$$pY-f\left(c_{0}+c_{K}K\right)-c_{Y}Y=\left(c_{Y}+\frac{\beta}{f}Y\right)Y-\left(\frac{\alpha+\beta Y}{f}\right)Y-c_{Y}Y\ . \tag{16}$$

Working out and using (15) yields

$$pY - f\left(c_0 + c_K K\right) - c_Y Y = -\frac{\alpha c_K}{\gamma} < 0 \tag{17}$$

There is no full cost recovery. This is no surprise, because the stringent restrictions that guarantee exact cost recovery (restrictions on the user cost function and the capacity cost function) are not satisfied for the model specified above (see Mohring and Harwitz (1962), de Palma and Lindsey (2007) and De Borger, Dunkerley and Proost (2009) on cost recovery theorems). The loss will be larger the larger the value of waiting time, the larger the marginal cost of vehicle capacity and quality *K*, and the smaller the beneficial effect of service quality *K* on the generalized user cost.

The implications of imposing full cost recovery are easily explored. Requiring $pY - f\left(c_0 + c_K K\right) - c_Y Y \geq 0$ yields, after simple algebra, the following optimal fare:

$$p = c_Y + \frac{\beta}{f} Y - \frac{\lambda}{1+\lambda} \frac{Y}{\frac{\partial Y}{\partial P^Y}}$$

where λ is the multiplier associated with the constraint. For frequency and quality we find, using similar derivations as before, the first-best rules (14)-(15).

Using these results and substituting the (binding) budget constraint we can eliminate λ to find:

$$p = c_Y + \frac{\beta}{f} Y + \frac{\alpha}{f} \tag{18}$$

The higher fare guarantees cost recovery. The fare is the same as in the social optimum only if the cost of waiting time were zero. This makes sense, because in that case the first-best deficit equals zero, see (17) above.

4.2. Second-best optimal welfare with un-priced car externalities

One of the main economic justifications for subsidies to public transport is the second-best argument that lower prices of transit indirectly address un-priced car congestion. We introduce it in our model in a very simple way, because in the analysis below we want to avoid having to deal simultaneously with the political economy of road pricing and public transport policies. However, we do capture the dependency of policies towards public transport and the level of road charges, if any. We assume the generalized cost of road use to be linearly increasing in total road use in the region, as specified by (4): $g^X(\tau°, X) = \tau° + \nu X$. It is assumed that the road toll $\tau°$ is exogenous.

As argued before, the assumption of perfect substitutability between the two transport modes implies that in equilibrium a single generalized price prevails for both car and public transport use. It is easy to show that in that case we can formulate the problem of determining the second-best optimal public transport fare, frequency and quality as follows (assuming an interior solution so that both modes are used in equilibrium):

$$\underset{p,f,K}{Max} \quad \int_0^Y P^Y(y)dy + \int_0^X P^X(x)dx - g^Y(.)Y - g^X(.)X + \left[pY - f\left(c_0 + c_K K\right) - c_Y Y + \tau^\circ X \right]$$

In this expression, $P^Y(Y),\ P^X(X)$) are the inverse demands for public transport and car use, and Y and X depend on the policy variables (p,f,K) through the reduced-form demands $Y = Y^r(p,\tau^\circ, f, K), X = X^r(p,\tau^\circ, f, K)$ identified above.

The optimization problem generates the following first-order condition for the second-best public transport fare

$$-Y\left[\frac{dg^Y(.)}{dp}\right] - X\left[\frac{dg^X(.)}{dp}\right] + \left[Y + p\left(\frac{dY}{dp}\right) - c_Y\left(\frac{dY}{dp}\right) + \tau^\circ\frac{dX}{dp}\right] = 0$$

Note from (2) and (4) that

$$\frac{dg^Y}{dp} = 1 + \frac{\beta}{f}\frac{dY}{dp}; \quad \frac{dg^X}{dp} = v\frac{dX}{dp} \tag{19}$$

Using these relations in the first-order condition, we obtain

$$\left(p - c_Y - \frac{\beta Y}{f}\right)\left(\frac{dY}{dp}\right) + \left(\tau^\circ - vX\right)\frac{dX}{dp} = 0$$

Substituting the total derivatives as given by (7) in this expression and multiplying by (Δ) we obtain after simple algebra:

$$p = c_Y + \frac{\beta}{f}Y + (\tau^\circ - vX)\left(\varepsilon\right) \tag{20}$$

where $\varepsilon = -\dfrac{\frac{dX}{dp}}{\frac{dY}{dp}} = \dfrac{\frac{\partial P(Y+X)}{\partial(Y+X)}}{\frac{\partial P(Y+X)}{\partial(Y+X)} - v} > 0$. This parameter reflects the degree to which an increase in the public transport fare induces users to switch from public transport to car use.

As expected, the optimal second-best public transport fare equals the first-best fare with a downward correction when there is an un-priced externality on car use. The correction will be larger when a price decrease of the fare makes more car users switch to public transport.

Straightforward algebra shows that the second best frequency and quality follow the first-best rules we derived earlier. To see this, take as an example the first-order condition with respect to f. Note by differentiating (2) and (4) that

$$\frac{dg^Y}{df} = \frac{\beta}{f}\frac{dY}{df} - \left(\frac{\alpha + \beta Y}{f^2}\right); \quad \frac{dg^X}{df} = v\frac{dX}{df} \tag{21}$$

It then easily follows that this condition can be written as

$$\left(p - c_Y - \frac{\beta Y}{f}\right)\left(\frac{dY}{df}\right) + \left(\tau^\circ - vX\right)\frac{dX}{df} + \frac{\alpha + \beta Y}{f^2} - \left(c_0 + c_K K\right) = 0$$

Now observe from (7)-(8) that the Wardrop conditions imply

$$\frac{dY}{df} = -\left(\frac{\alpha + \beta Y}{f^2}\right)\frac{dY}{dp} > 0; \quad \frac{dX}{df} = -\left(\frac{\alpha + \beta Y}{f^2}\right)\frac{dX}{dp} < 0 \tag{22}$$

Substituting (22) in the first order condition and using the optimal fare rule (20) we immediately find

$$f = \sqrt{\frac{(\alpha + \beta Y)Y}{c_0 + c_K K}}$$

Following the same steps for K also produces the first-best rule. It suffices to take the first-order condition and to note (see (2) and (4))

$$\frac{dg^Y}{dK} = \frac{\beta}{f}\frac{dY}{dK} - \gamma; \quad \frac{dg^X}{dK} = v\frac{dX}{dK} \tag{23}$$

Then use (see (7) and (9))

$$\frac{dY}{dK} = -\gamma\frac{dY}{dp} > 0; \quad \frac{dX}{dK} = -\gamma\frac{dX}{dp} < 0 \tag{24}$$

This immediately yields the first-best rule obtained before. These findings are no surprise, as the optimal frequency and quality come down to minimizing the generalized price of public transport just as in first best.

Finally, imposing a no-loss restriction on public transport and assuming that the constraint is binding at the optimum, we can write the optimal fare after simple algebra as

$$p = c_Y + \frac{\beta}{f}Y + \frac{\lambda}{1+\lambda}Yv\left(\varepsilon\right) + \frac{1}{1+\lambda}(\tau^\circ - vX)\left(\varepsilon\right)$$

Again, results for frequency and quality are the same as before. Interestingly, using the budget constraint we can eliminate λ and substitute the result to find

$$p = c_Y + \frac{\beta}{f}Y + \frac{\alpha}{f}$$

This is the same as (18). The externality has disappeared from the price rule. In other words, the budget restriction on the public transport firm prevents the fare from correcting for the external cost of road use.

5. The political economy of decentralized decisions

In this section, we turn to the political decisions with respect to public transport if the regions autonomously decide on fares, frequencies and quality. As mentioned before, we denote the number of voters that can only use public transport by U and let the population be given by R. We assume regional political decisions are taken by simple majority voting. As we have only two groups of voters, either car owners or people not owning a car will have the majority. We start by looking at the region's decisions if it controls all public transport policy instruments in an unregulated environment. Next we consider the region's behavior under a budgetary restriction.

5.1 The unregulated regional political optimum

Suppose first that those who can only use public transport (members of the group U) have a majority so that the median voter belongs to this group; he or she is decisive in making decisions. Assuming that government surpluses or deficits are equally shared among the whole population, the median voter's decision is the solution of the following problem:

$$\underset{p,f,K}{Max} \quad \frac{1}{U}\left[\int_0^{Y^U} P^{Y^U}(y)dy - g^Y(.)\,Y^U\right] + \frac{\left[pY + \tau^\circ X - f\left(c_0 + c_K K\right) - c_Y Y\right]}{R}$$

In this expression, $P^{Y^U}(Y^U)$ is the inverse demand function for public transport use in the own region by people only using public transport. The first term in the objective function captures the user surplus for the local median voter; it is expressed on a per person basis. The second term gives the net revenues of public transport operations plus the toll revenues on road use, expressed per person of the population. Observe that voters who only use public transport do not care about the reduction of road congestion, as they don't suffer from the time losses congestion implies for other road users.

The first-order condition for the fare p can be written as

$$\frac{\theta^U}{U}\left[-Y\frac{dg^Y}{dp}\right]+\frac{1}{R}\left[(p-c_Y)\frac{dY}{dp}+Y+\tau^\circ\frac{dX}{dp}\right]=0$$

where we have introduced the notation θ^U for the share of demand by group U in total public transport demand:

$$\theta^U=\frac{Y^U}{Y} \tag{25}$$

Of course, this share is endogenous, as both overall demand and demand by group U depend on the fare and the quality characteristics. Similar algebraic steps as before show that the rule describing the optimal fare is given by:

$$p=\left(c_Y+\frac{\beta Y}{f}\right)+\left(1-\frac{\theta^U}{\eta}\right)Yv(\varepsilon)+\tau^\circ(\varepsilon) \tag{26}$$

In this expression, we have defined

$$\eta=\frac{U}{R}.$$

This parameter captures the share of the population in the region that can only use public transport.

Expression (26) is the optimal fare rule if the majority of voters in the region are people that can only use public transport (hence $0.5<\eta\leq 1$) and decisions are taken by majority voting. The rule consists of three terms. The first term between brackets is the first-best rule. The second term is the 'bias' due to decentralized political decisions. If voters that can only use public transport do not have a large majority (η well below 1) but their share in total public transport use in the region is large (θ^U is close to one), then public transport fares will be below marginal social cost. Fares wanted by the regional decision maker may in that case be very low, possibly zero. However, even though he himself is a public transport user, the same decision maker wants public transport to be much more expensive -- with fares above marginal social cost -- if the system is used to a large extent by people from outside the region and by local car owners (so that θ^U is close to zero). This is a direct application of a well-known general insight brought forward in a seminal paper by Arnott & Grieson (1981). Finally, the third term captures the idea that the public transport user wants higher fares if an increase in the public transport fare implies larger road toll revenues (distributed to all inhabitants): the higher fare induces people to switch from public transport to car, as captured by ε.

To conclude the discussion of the rule for the public transport fare we point out that slightly rewriting (26) yields yet another insight. To see this, note from (7) that

$$\frac{\partial Y^r}{\partial p}=\frac{\dfrac{\partial P(Y+X)}{\partial(Y+X)}-v}{\Delta} \text{ where } \Delta=\frac{\beta}{f}\left(v-\frac{\partial P(Y+X)}{\partial(Y+X)}\right)-v\frac{\partial P(Y+X)}{\partial(Y+X)}$$

Using this result, we can write

$$\frac{Y}{\dfrac{\partial Y^r}{\partial p}}=\frac{\dfrac{\beta Y}{f}\left(v-\dfrac{\partial P(Y+X)}{\partial(Y+X)}\right)-vY\dfrac{\partial P(Y+X)}{\partial(Y+X)}}{\dfrac{\partial P(Y+X)}{\partial(Y+X)}-v}=-\frac{\beta Y}{f}-vY\varepsilon$$

Hence, we have

$$vY\varepsilon=-\frac{\beta Y}{f}-\frac{Y}{\dfrac{\partial Y^r}{\partial p}}$$

Substituting this in (26) gives

$$p=c_Y+\left(\frac{\theta^U}{\eta}\right)\frac{\beta Y}{f}-\left[1-\frac{\theta^U}{\eta}\right]\frac{Y}{\dfrac{\partial Y^r}{\partial p}}+\tau^\circ(\varepsilon) \tag{27}$$

This alternative formulation shows that the voter who only uses public transport internalizes only part of the crowding externality (captured by $\frac{\beta Y}{f}$) in the fare. Specifically, he accounts for crowding only to the extent that it affects members of his group (θ^U) and in as far as he shares in the fare revenues $(1/\eta)$.

The voter that can only use public transport who is in charge in the region will set frequency and quality of service investment according to the first-best rules given before. To see this, take the first-order conditions with respect to *f* and *K*, respectively. They can be written as:

$$\frac{\theta^U}{U}\left[-Y\frac{dg^Y}{df}\right]+\frac{1}{R}\left[(p-c_Y)\frac{dY}{df}-(c_0+c_K K)+\tau^o\frac{dX}{df}\right]=0$$

$$\frac{\theta^U}{U}\left[-Y\frac{dg^Y}{dK}\right]+\frac{1}{R}\left[(p-c_Y)\frac{dY}{dK}-fc_K+\tau^o\frac{dX}{dK}\right]=0$$

Substituting expressions (21)-(24) in the first-order conditions and using the optimal fare rule (26) then shows, after straightforward algebra, that the optimal frequency and capacity rules are as in the first-best:

$$f = \sqrt{\frac{(\alpha + \beta Y)Y}{c_0 + c_K K}}$$

$$fc_K = \gamma Y$$

Will public transport costs be fully recovered? Using the optimal rules for the decision variables in the definition of net revenue, we easily show

$$pY - f(c_0 + c_K K) - c_Y Y = -\frac{\alpha c_K}{\gamma} + \left(1 - \frac{\theta^U}{\eta}\right) Y^2 v(\varepsilon) + \tau^{\circ}(\varepsilon) Y$$

In general, cost recovery can be positive (for example, if road tolls are high and the public transport system is mainly used by people from outside the region or by car owners, and waiting time costs are small) or negative (if mainly locals use the public transport system and no road toll is implemented). Note that these observations provide an empirically testable hypothesis: we expect cost recovery to be better when road charges are high and the public transport system is highly used by local car owners and people from outside the region. Also observe that cost recovery may be better or worse as compared to the social optimum.

We summarize our findings in Proposition 1.

Proposition 1. Suppose regions decide on public transport fares, frequency and quality of service by majority voting. Let <u>voters that can only use public transport have a majority.</u>

- ***a.*** ***If there are no other users of the public transport system and there is no toll on road use, public transport fares are below the marginal social cost, because part of the costs are shifted to non-users and the benefits of reduced road congestion are not taken into account.***
- ***b.*** ***Public transport fares may well be much higher than marginal social cost if public transport is widely used by car owners and people from outside the region, and if road use is tolled.***
- ***c.*** ***The frequency and quality of service are first-best, conditional on the number of users. Hence, frequency and quality may be too low or too high.***

d. ***Depending on the share of users from outside the region, revenues may or may not be sufficient to cover the public transport operator's costs. Cost recovery always improves with the share of outside users.***

Next, let us assume – which is probably more realistic in many regions -- that <u>a voter who is a car owner is decisive</u> in the region (hence, a member of group *C*). He considers the following problem:

$$\underset{p,f,K}{Max} \quad \frac{1}{R-U}\left[\int_0^{Y^C} P^{Y^C}(y^C)dy - g^Y(.)Y^C\right] + \frac{1}{R-U}\left[\int_0^{X^C} P^{X^C}(x)dx - g^X(.)X^C\right] +$$
$$+\frac{1}{R}\left[pY - f\left(c_0 + c_K K\right) - c_Y Y + \tau^\circ X\right]$$

A car owner's welfare consists of three components. The first component is, expressed per person, his direct net surplus from public transport use ($P^{Y^C}(Y^C)$ is the inverse demand function for public transport by people from group *C*). The second term captures his indirect net user benefit from car use in his own region; this is also affected by changes in public transport fares, as the fare induces changes in the number of local car trips made. The final term is the voter's share in net revenues.

Solving the above problem using similar techniques as before, we find the fare preferred by voters who are car owners:

$$p = c_Y + \left[\frac{\theta^C}{1-\eta}\right]\frac{\beta Y}{f} - \left[1 - \frac{\theta^C}{1-\eta}\right]\frac{Y}{\frac{\partial Y^r}{\partial p}} + (\tau^\circ - \frac{\rho^C vX}{1-\eta})\left(\varepsilon\right) \tag{28}$$

In this expression, θ^C is the fraction of demand by group *C* in total public transport demand in the region:

$$\theta^C = \frac{Y^C}{Y}$$

The notation ρ^C is in a similar way used to denote the fraction of car demand in the region that comes from this group of local car owners (the remainder comes from drivers from outside the region):

$$\rho^C = \frac{X^C}{X}$$

Finally, the share of the people living in the region that demand both car and public transport trips as fraction of the region's population is given by

$$1-\eta=\frac{C}{R}.$$

To interpret the result given in (28), note that the fare will be lower the larger the share of public transport demand by car owners and the lower the road toll. Assuming the toll is below the marginal external cost borne by car owners, the fare rises if in response more people use their car. Car owners prefer that public transport fares charge for crowding in public transport in as far as they use public transport (θ^C) and in as far as they can share in the fare revenues $(1/1-\eta)$. Car owners who use both modes only pay attention to road congestion by lowering the fare to the extent that they suffer from it, as captured by the fraction ρ^C, and in as far as they do not have to pay the full cost of it (1-η).

Differentiating the objective function with respect to frequency and quality, using earlier expressions and the optimal fare rule (28), we again find the first-best rules for the two supply variables. Of course, these are evaluated for a lower demand due to the higher fare.

In sum, when car owners have a majority, we expect fares and cost recovery to increase when there are many external public transport and car users, when regions have a higher toll in place, or when regions are not strongly congested.

To find out whether costs are fully recovered, profit can be rewritten as

$$pY-f\left(c_0+c_K K\right)-c_Y Y=-\frac{\alpha c_K}{\gamma}+v\varepsilon Y^2\left[1-\frac{\theta^C}{1-\eta}\right]+(\tau^\circ-\frac{\rho^C vX}{1-\eta})\varepsilon Y$$

Again, cost recovery is not guaranteed if waiting time and marginal capacity costs are large.

Finally, consider the possibility of corner solutions, when either car owners use only their car ($\theta^C=0$) or use only public transport ($\rho^C=0$). If they do not use public transport at all, they only care about public transport to the extent that it yields revenues. On the contrary, if they only use public transport, they want fares to be low if there is little crowding on public transport and the road toll is low. However, high road tolls and crowding in public transport will induce them to want higher public transport fares.

We summarize our findings in the following proposition:

Proposition 2. Suppose regions decide on public transport fares, frequency and quality of service by majority voting. Let <u>car owners have a majority.</u>

a. The fare wanted by car owners is increasing in the toll on car use. It decreases if car owners have a larger share in total public transport use.

b. ***The frequency and quality of service are first-best, conditional on the number of users.***

c. ***Depending on the share of users from outside the region, revenues may or may not be sufficient to cover the public transport operator's costs. Cost recovery always improves with the share of outside users.***

The policy implication from proposition 2 is clear. It implies that voters that use both transport modes may agree on setting low public transport fares and allowing large subsidies. They will do so if there is substantial congestion in the urban area and there is no road toll (or a toll that is too low compared to marginal external cost), a set of conditions that holds in many cities.

The analysis of the desired public transport policies by voters from groups *U* and *C* yields some interesting conclusions. To see this, consider expressions (27) and (28). There are two major differences between the policies wanted by voters that only use public transport and those also driving their car. One is that each group is concerned with their own share in the net consumer surplus of public transport use, so that they only care about crowding in public transport to the extent they are directly affected. The other difference is that the first group *U* only cares about the toll revenues from road use, whereas group *C* is also concerned about reducing the road congestion from which it suffers. This leads to the following insights. First, if relative to their voting majority the user shares of groups *U* and *C* are equal (in essence, if $\frac{\theta^U}{\eta}=\frac{\theta^C}{1-\eta}$), then car owners in fact want a lower public transport fare than people always using public transport. To see this it suffices to look at the final terms on the right hand sides of (27 and (28). The reason is that car owners care about reducing congestion; this can be achieved by lower public transport fares. Second, assume more plausibly that the public transport user share of group *U* (relative to the voting majority) would be higher if a member of group *U* was decisive than if a car owner was (hence $\frac{\theta^U}{\eta}>\frac{\theta^C}{1-\eta}$) . The second terms on the right hand side of (27) and (28) suggest that this would reduce the fare wanted by non-car owners *U* relative to those desired by car owners *C*. We summarize in the following Proposition.

Proposition 3.

a. ***Non car owners (group U) will opt for lower public transport fares than car owners (group C) if their share in public transport use is large and there is little car congestion (or mainly outsiders suffer from congestion).***

b. ***Car owners (group C) will want lower public transport fares than non-car owners (group U) if they suffer substantially from congestion and the exogenous road toll is low.***

c. ***When in charge, both car owners and non-car owners may opt for low public transport fares.***

5.2. Regional decisions under a budgetary restriction on public transport

We found that the political decisions at the regional level deviate from the social optimum. Moreover, cost recovery is not guaranteed, although under some specific conditions net revenue may be positive. In this section, we study the role of imposing a formal budget constraint on the political process.

If voters that <u>only use public transport have a majority</u>, the problem to be studied is

$$\underset{p,f,K}{Max} \quad \frac{1}{U}\left[\int_0^{Y^U} P^{Y^U}(y)dy - g^Y(.)Y^U\right] + \frac{\left[pY - f\left(c_0 + c_K K\right) - c_Y Y + \tau^\circ X\right]}{R}$$

$$s.t. \quad \frac{\left[pY - f\left(c_0 + c_K K\right) - c_Y Y\right]}{R} = 0$$

Associating a multiplier λ with the constraint, we find the following rule for the optimal fare (using the same method as in the absence of a constraint on revenues):

$$p = \left(c_Y + \frac{\beta Y}{f}\right) + \left(1 - \frac{\theta^U}{\eta}\frac{1}{1+\lambda}\right) vY\left(\varepsilon\right) + \tau^\circ \frac{1}{1+\lambda}\left(\varepsilon\right) \tag{29}$$

It is easy to show that frequency and quality are again determined according to the first-best rules given before. Using these results and substituting the budget restriction in the fare rule (29) the latter can be rewritten, after rearrangement, as:

$$p = \left(c_Y + \frac{\beta Y}{f}\right) + \frac{\alpha}{f}$$

Note that the fare rule is identical to the rule in the second-best welfare optimum given before (see (18)). This implies that, if users have a majority, decentralized decision making under a zero profit constraint implements the second best welfare optimum.

If a <u>car owner is decisive</u> in the region, the problem is

$$\underset{p,f,K}{Max} \quad \frac{1}{R-U}\left[\int_0^{Y^C} P^{Y^C}(y)dy - g^Y(.)Y^C\right] + \frac{1}{R-U}\left[\int_0^{X^C} P^{X^C}(x)dx - g^X(.)X^C\right] +$$

$$+\frac{1}{R}\left[pY - f\left(c_0 + c_K K\right) - c_Y Y + \tau^\circ X\right]$$

$$s.t. \quad \frac{1}{R}\left[pY - f\left(c_0 + c_K K\right) - c_Y Y\right] = 0$$

Solving the above problem, we find the fare preferred by voters who also rely on car driving:

$$p = c_Y + \frac{\beta Y}{f} + \left[1 - \frac{\theta^C}{1-\eta}\frac{1}{1+\lambda}\right]vY\left(\varepsilon\right) + (\tau^\circ - \frac{\rho^C vX}{1-\eta})\frac{1}{1+\lambda}\left(\varepsilon\right) \tag{30}$$

As several times before, the rules for frequency and quality are the first best rules derived before. Substituting the budget restriction for the public transport operator, we again find the second-best optimal fare (18):

$$p = \left(c_Y + \frac{\beta Y}{f}\right) + \frac{\alpha}{f} \tag{31}$$

We summarize our findings as follows.

Proposition 4. Suppose regions decide on public transport fares, frequency and quality of service by majority voting, but they face a zero net revenue constraint. Then car owners and non car owners implement the second-best welfare optimum.

Note that Proposition 4 is a strong result. It implies that the (second-best) budget constrained welfare optimum can be implemented by decentralized decision making, independent of who is in charge at the political level. One of the important conditions for this result to hold is that no price-discrimination is allowed, otherwise non-locals would always be discriminated against and we lose the second best property. The no price-discrimination rule forces local governments to do as well as possible for their local voter, so they are forced to also optimize the conditions for non-local users.

6. Centralized decisions

One may wonder whether it is a good idea to let cities, urban areas or regions within a country decide on public transport fares and quality attributes of the regional public transport system. In fact, there are good reasons to believe that this will indeed be the case. For example, regional decisions may better reflect local demand and congestion conditions than when a central government agency determines fares, frequencies and quality aspects of public transport operations for the different regions or urban areas. Of course, a disadvantage of decentralized decisions is that regions may engage in tax exporting behavior (see Arnott and Grieson (1981)). Hence, it is a priori unclear which political system performs best.

As noted in the introduction, in many countries a tendency has been observed over the past decades towards decentralized decision making for public transport services. To understand this tendency, we study in this section what the outcomes of centralized decision making would be in our simplified setting, and we compare with our findings under decentralization (see Section 5).

A complete analysis of centralized decision making requires looking at many different cases depending on which type of voter has a majority in each of the regions (car owners may have a majority in both regions, non-car owners may have a majority in both regions, or in one region there may be a car owner majority whereas this is not the case in the other region). Moreover, different political mechanisms to reach decisions exist at the central level. Two popular models assume that either a minimum winning coalition decides at the central level, or that decisions are the result of a bargaining process (Lockwood (2002), Besley and Coate (2003), Hickey (2013)). In our two-region model, decisions by a minimum winning coalition boils down to assuming that regions each delegate one representative to the central level to form a central parliament, and that each representative has an equal probability of being decisive at the central level. Bargaining would imply that elected regional representative negotiate in a central parliament to reach decisions. Finally, the central level may further operate under various constitutional constraints, including a uniform policy requirement across regions, or it may be subject to regional or global budgetary constraints.

It is impossible to work out centralized decisions for all possible cases that can be imagined. In this section, we focus on a specific but especially relevant case. First, we focus on the case where car owners have a majority in both regions. This seems the most realistic assumption for most countries. Moreover, the qualitative results when people that only use public transport are in charge (and both regions are symmetric) can be directly derived from the model analyzed in De Borger and Proost (2013): although they focused on road users and road pricing decisions, their findings can with minor adaptation be reinterpreted for public transport[3]. However, when in our setting car owners have a majority but decisions concern

[3] They compared the outcome of 4 institutions: decentralized decisions, centralized decision making, centralized decision making under bargaining, and centralized decision making with a uniform pricing constraint across

public transport fares and quality, the results of the previous paper provide little guidance. We therefore study this case in what follows. Second, although we briefly discuss the more general case in Appendix, here we focus on the symmetric case. In other words, we assume all parameters are identical in the two regions. The advantage of doing this is not only that the general case yields results that are difficult to interpret (unlike the symmetric case) but also that under symmetry we can show (see the Appendix) that the fare and quality rules for decision making by a minimum winning coalition and decisions obtained through bargaining are the same. Third, we assume that decision makers at the central level have to respect a uniformity constraint, in that fares have to be the same in both regions. Based on the comparison of centralized versus decentralized decisions in a different context, we expect centralized decisions without such constraints to produce inferior results as compared to decentralization (De Borger and Proost (2013)). It is mainly the potential exploitation of one region by another that makes centralized decision making worse than decentralized decision making. For example, in the absence of behavioral restrictions, regional representatives that can only use public transport in the own region have an incentive to favor imposing high fares in other cities but low fares in their own city. Imposing uniformity of fares across regions prevents such potential exploitation of regions. In addition, in cases where public transport decisions have been taken at the central level, it seems to have been the most commonly observed regime in practice.

Under our assumptions, we show in the Appendix that the rule for the fare decided by car owners can be written in the following two equivalent ways (given the symmetry assumption we have deleted the regional index (*i=1,2*)):

$$p = c_Y + \frac{\beta Y}{f} + \left[1 - \left(\frac{\theta^C + \theta^{T^C}}{1-\eta}\right)\right] vY(\varepsilon) + (\tau^\circ - \frac{vX}{(1-\eta)})(\varepsilon) \tag{32a}$$

$$p = c_Y + \left(\frac{\theta^C + \theta^{T^C}}{1-\eta}\right)\frac{\beta Y}{f} + \left[1 - \left(\frac{\theta^C + \theta^{T^C}}{1-\eta}\right)\right] \frac{Y}{\frac{\partial Y^r}{\partial p}} + (\tau^\circ - \frac{vX}{(1-\eta)})(\varepsilon) \tag{32b}$$

In these expressions, the parameter θ^{T^C} captures, in a given region, the share of total regional public transport demand that comes from car owners of the other region (see the discussion in Section 3, especially Table 1):

regions. They show that negotiated central decisions and uniform pricing restrictions may outperform decentralization only if non-car owners have a large political majority and there are substantial spill-overs (in the sense that the public transport system is to a large extent used by people from outside the region).

$$\theta^{T^C} = \frac{Y^{T^C}}{Y} = \frac{Y^{T^C}}{Y^U + Y^C + (Y^{T^U} + Y^{T^C})} \tag{33}$$

Note that this share is likely to be quite small in many regions. It may be substantial in some cases, such as in regions that attract many commuters or tourists.

Given our symmetry assumption, we show that both the frequency and quality rules follow the standard first best rules we have mentioned several times before in this paper.

Turn to the interpretation of (32a-32b). As car owners were assumed to have the majority in both regions (hence $\eta < 0.5 \rightarrow (1-\eta) > 0.5$), and a regional representative in charge at the central level will take into account that he uses some public transport in both regions, their shares in total public transport demand have the same effect on the fare. What matters is the total share of public transport use by people that own a car, independent of whether they are local or from the other region. The larger this share is, the lower the fare. Similarly, it is the total external cost of car use in the region that matters in the fare, independent of whether it is the local population or the car owners from the other region that suffer from congestion.

More insights can be derived from the comparison of (32) with the rule for the public transport fare we obtained under decentralized decisions. For the case where car owners had a majority, this was given by (28). Direct comparison is not obvious because $\theta^C, \theta^T, \rho^C$ are endogenous: they depend on the policy variables. However, interpretation in a qualitative sense is straightforward.

First, for the sake of the argument, suppose there are no spill-overs at all, in the sense that neither the regional public transport system nor the regional road system is used by outsiders. These assumptions imply

$$\theta^{T^C} = \rho^T = 0$$

It follows that under these special conditions centralized and decentralized decisions yield the same fare rule. This directly follows from comparing (28) and (32). This makes sense, as all public transport demand by car owners comes from within the region (and regions are symmetric) and only local drivers suffer from congestion.

Second, however, suppose that in a given region both public transport demand and car use by car owners from outside the region is quite important, so that

$$\theta^{T^C} > 0, \quad \rho^T > 0$$

The importance of public transport users from outside the region will lead to higher fares under decentralization (see (28)) than under centralized decisions (see (32)). Of course, this is due to tax exporting behavior in the former case. Similarly, let car use by outside users be important in a region, and assume that car transport is under-priced relative to the marginal external cost. Then comparing the final terms on the right hand sides of (28) and (32) implies that decentralized public transport fares will be higher than under centralization. Intuitively, if the region would raise public transport fares it would attract more car traffic, implying higher congestion. The burden of this heavier traffic -- assuming congestion is underpriced -- falls to a substantial extent on outsiders, so the region has an incentive to opt for high public transport fares. In sum, the importance of outside users of both the public transport and the road system imply that decentralized public transport fares are lower than under centralized decision-making.

Proposition 5.

a. **We expect higher fares under decentralized decisions as compared to central decision-making when public transport demand in a region comes to a large extent from outsiders.**
b. **Similarly, if car use is underpriced compared to its external cost and car use in a region comes to a large extent from outsiders, we expect higher fares for public transport under decentralized decision-making.**
c. **We expect better cost recovery rates under decentralization.**

Note that this proposition yields a prediction that is in principle – given data availability – empirically testable. It says that decentralized decision-making will under plausible conditions lead to better cost recovery rates, and that the degree to which decentralized systems cover costs depends on the importance of users of the public transport system that come from outside the region.

7. Conclusions

In this paper we developed a simple model that allows to better understand the political economy of public transport pricing and quality decisions in urban areas. The model we developed explains why in many countries very large public transport subsidies get fairly wide political support, even when car owners are by far the majority. Comparing decentralized outcomes with policy decisions under centralized decision-making, we find that under plausible conditions higher fares and improved cost recovery rates will result under decentralized decisions. Overall, our findings are consistent with the lack of opposition to very large public transport subsidies in Europe, and they provide a potential explanation for the tendency towards decentralization of public transport policy making observed over the last decades.

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Appendix: political decisions under centralisation

Throughout we assume car that owners have a majority in both regions. Moreover, we assume that fare and quality policies have to be uniform across regions so as to avoid exploitation of one region by the other (De Borger and Proost (2013)).

To get started, note that we have the following definitions of generalized costs in the regions:

$$g_1^Y = p + \frac{\alpha_1}{f_1} + \frac{\beta_1 Y_1}{f_1} - \gamma_1 K_1; \quad g_2^Y = p + \frac{\alpha_2}{f_2} + \frac{\beta_2 Y_2}{f_2} - \gamma_2 K_2$$
$$g_1^X = \tau_1 + v_1 X_1; \qquad g_2^X = \tau_2 + v_2 X_2$$

The car owner from a given region uses four types of transport. For example, the one from region 1 demands public transport and car trips in the own region (Y_1^C, X_1^C, respectively) as well as public transport and car trips in the other region 2 (denoted $Y_2^{T^C}, X_2^T$, respectively). For details, see Table 1 in the main body of the paper. Further note the following definitions

$$\theta_1^U = \frac{Y_1^U}{Y_1}; \theta_1^C = \frac{Y_1^C}{Y_1}; \theta_1^{T^C} = \frac{Y_1^{T^C}}{Y_1}; \theta_1^{T^U} = \frac{Y_1^{T^U}}{Y_1} \qquad \theta_1^U + \theta_1^C + \theta_1^{T^C} + \theta_1^{T^U} = 1$$
$$\theta_2^U = \frac{Y_2^U}{Y_2}; \theta_2^C = \frac{Y_2^C}{Y_2}; \theta_2^{T^C} = \frac{Y_2^{T^C}}{Y_2}; \theta_2^{T^U} = \frac{Y_2^{T^U}}{Y_2} \qquad \theta_2^U + \theta_2^C + \theta_2^{T^C} + \theta_2^{T^U} = 1$$
$$\rho_1^C = \frac{X_1^U}{X_1}; \rho_1^T = \frac{X_1^T}{X_1} \qquad \rho_1^C + \rho_1^T = 1$$
$$\rho_2^C = \frac{X_2^U}{X_2}; \rho_2^T = \frac{X_2^T}{X_2} \qquad \rho_2^C + \rho_2^T = 1$$

We first assume decisions by a Minimum Winning Coalition, then we briefly deal with bargaining. As will become clear below, under symmetry and uniform fares, we have the same rules under both political systems.

Minimum Winning Coalition

Following, among others, Besley and Coate (2003), this can be modeled by assuming that each region delegates one representative to a federal parliament; at the central level, each representative has an equal probability of being in charge and being decisive.

Suppose the representative from region 1 is decisive (a similar analysis applies when the representative from region 2 is decisive). This can be modeled as the result to the following problem:

$$\underset{p,f_1,f_2,K_1,K_2}{Max} \quad \frac{1}{C_1}\left\{\left[\int_0^{Y_1^C} P^{Y_1^C}(y)dy - g_1^Y(.)\,Y_1^C\right] + \left[\int_0^{X_1^C} P^{X_1^C}(x)dx - g_1^X(.)\,X_1^C\right]\right\}$$

$$\frac{1}{C_1}\left\{\left[\int_0^{Y_2^{T^C}} P^{Y_2^{T^C}}(y)dy - g_2^Y(.)\,Y_2^{T^C}\right] + \left[\int_0^{X_2^T} P^{X_2^T}(x)dx - g_2^X(.)\,X_2^T\right]\right\}$$

$$+\frac{1}{2R}\left[(p-c_Y)(Y_1+Y_2)+\tau_1 X_1+\tau_2 X_2 - f_1\left(c_0+c_K K_1\right) - f_2\left(c_0+c_K K_2\right)\right]$$

The first two lines capture the net surplus of the car owner from region 1; he derives surplus from car use and public transport use in his own region (line 1) and in the other region (line 2). The representative further takes account of his share in the total net revenues of public transport and tolls on car use in the two regions.

The first order condition with respect to the fare can be written as follows:

$$-\frac{1}{C_1}\left\{Y_1^C\frac{dg_1^Y(.)}{dp}+X_1^C\frac{dg_1^X(.)}{dp}+Y_2^{T^C}\frac{dg_2^Y(.)}{dp}+X_2^T\frac{dg_2^X(.)}{dp}\right\}$$

$$+\frac{1}{2R}\left\{(p-c_Y)\frac{d(Y_1+Y_2)}{dp}+(Y_1+Y_2)+\tau_1\frac{dX_1}{dp}+\tau_2\frac{dX_2}{dp}\right\}=0$$

Then note the following relations:

$$\frac{dg_1^Y(.)}{dp}=1+\frac{\beta_1}{f_1}\frac{dY_1}{dp}; \quad \frac{dg_2^Y(.)}{dp}=1+\frac{\beta_2}{f_2}\frac{dY_2}{dp}$$

$$\frac{dg_1^X(.)}{dp}=v_1\frac{dX_1}{dp}; \quad \frac{dg_2^X(.)}{dp}=v_2\frac{dX_2}{dp}$$

Use these relations in the first order condition, and multiply by *2R*. Noting that

$$\frac{U_1}{R}=\eta_1 \quad \rightarrow \quad \frac{R}{R-U_1}=\frac{R}{C_1}=\frac{1}{1-\eta_1}$$

and using the definitions given above, the first-order condition can be rewritten as

$$\frac{dY_1}{dp}\left\{(p-c_Y)-2\frac{\theta_1^C}{1-\eta_1}\frac{\beta_1 Y_1}{f_1}\right\}+\frac{dY_2}{dp}\left\{(p-c_Y)-2\frac{\theta_2^{T^C}}{1-\eta_1}\frac{\beta_2 Y_2}{f_2}\right\}$$

$$+\frac{dX_1}{dp}\left\{\tau_1-2\frac{\rho_1^C}{1-\eta_1}v_1 X_1\right\}+\frac{dX_2}{dp}\left\{\tau_2-2\frac{\rho_2^T}{1-\eta_1}v_2 X_2\right\}$$

$$+Y_1\left[1-\frac{2\theta_1^C}{1-\eta_1}\right]+Y_2\left[1-\frac{2\theta_2^{T^C}}{1-\eta_1}\right]=0$$

Substituting the two-region equivalent of (7) in this expression and working out yields the fare as a complicated expression of all parameters and demand components. However, interpretation is not straightforward, and to save space the general result is not reported here.

Instead, to ease the interpretation we impose symmetry; in essence, all parameters, the exogenous toll levels etc. are assumed to be the same in both regions. This allows us to reformulate the first-order condition as (since $\rho^C + \rho^T = 1$):

$$\frac{dY}{dp}\left\{(p-c_Y)-\frac{\beta Y}{f}\left[\frac{\theta^C+\theta^{T^C}}{1-\eta}\right]\right\}+\frac{dX}{dp}\left\{\tau-vX\left[\frac{1}{1-\eta}\right]\right\}+\left[1-\frac{(\theta^C+\theta^{T^C})}{1-\eta}\right]Y=0$$

Working out this expression using (7), and solving for the fare yields the following result:

$$p=c_Y+\frac{\beta Y}{f}+\left[1-\left(\frac{\theta^C+\theta^{T^C}}{(1-\eta)}\right)\right]vY(\varepsilon)+(\tau^\circ-\frac{vX}{(1-\eta)})(\varepsilon)$$

This is expression (32a) discussed in the main body of the paper. Analogous calculations as those described in Section 5.1 then produce expression (32b).

A similar method is used to analyze the first order condition with respect to frequency. We imposing symmetry and use the following relations:

$$\frac{dg^Y}{df}=-\frac{\alpha+\beta Y}{f^2}+\frac{\beta}{f}\frac{dY}{df}$$

$$\frac{dg^X}{df}=v\frac{dX}{df}$$

Using (8) and the optimal rule for the fare just derived, it is easy to show that the first order condition for frequency can be manipulated to yield the first best rule for frequency:

$$f=\sqrt{\frac{(\alpha+\beta Y)Y}{c_0+c_K K}}$$

In a similar fashion we show that quality follows the first best rule.

Bargaining

Decisions by bargaining can be modeled as the result to the following problem (basically, assuming equal bargaining power this is just the sum of the objective functions of the car owners in the two regions):

$$
\begin{aligned}
\underset{p,f,K}{Max}\quad & \frac{1}{C_1}\left\{\left[\int_0^{Y_1^C} P^{Y_1^C}(y)dy - g_1^Y(.)\,Y_1^C\right] + \left[\int_0^{X_1^C} P^{X_1^C}(x)dx - g_1^X(.)\,X_1^C\right]\right\} \\
& \frac{1}{C_1}\left\{\left[\int_0^{Y_2^{T^C}} P^{Y_2^{T^C}}(y)dy - g_2^Y(.)\,Y_2^{T^C}\right] + \left[\int_0^{X_2^T} P^{X_2^T}(x)dx - g_2^X(.)\,X_2^T\right]\right\} \\
& \frac{1}{C_2}\left\{\left[\int_0^{Y_2^C} P^{Y_2^C}(y)dy - g_2^Y(.)\,Y_2^C\right] + \left[\int_0^{X_2^C} P^{X_2^C}(x)dx - g_2^X(.)\,X_2^C\right]\right\} \\
& \frac{1}{C_2}\left\{\left[\int_0^{Y_1^{T^C}} P^{Y_1^{T^C}}(y)dy - g_1^Y(.)\,Y_1^{T^C}\right] + \left[\int_0^{X_1^T} P^{X_1^T}(x)dx - g_1^X(.)\,X_1^T\right]\right\} \\
& + 2 * \frac{1}{2R}\left[(p-c_Y)(Y_1+Y_2) + \tau_1 X_1 + \tau_2 X_2 - f_1\left(c_0 + c_K K_1\right) - f_2\left(c_0 + c_K K_2\right)\right]
\end{aligned}
$$

The first two lines capture the net surplus of the car owners from region 1; he derives surplus from car use and public transport use in his own region (line 1) and in the other region (line 2). Similarly, the third and fourth lines give the net surplus of the car owner from region two. Bargaining power is assumed to be equal for both regions. Finally, each car owner takes account of his share in the total net revenues of public transport and tolls on car use.

Using analogous methods as before, it easily follows that for symmetric regions we find the same expression for the fare as in the previous case of a minimum winning coalition.