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Coordination Efficiency in Multi-Output Settings: a DEA Approach*

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Abstract

We extend a recently developed methodology for measuring the efficiency of Decision Making Units (DMUs) in the case of multiple inputs and outputs. The methodology accounts for economies of scope through the use of joint inputs, and explicitly includes information about the allocation of inputs to particular outputs. We focus on possible efficiency gains by reallocating inputs across outputs. We introduce a measure of coordination efficiency, which captures these efficiency gains. We demonstrate the practical usefulness of our methodology through an efficiency analysis of education and research conducted at US universities.

Keywords: DEA, input allocation, decentralized efficiency, centralized efficiency, coordination efficiency, university education and research.

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1 Introduction

Data Envelopment Analysis (DEA) is a widely used approach to evaluate the efficiency of Decision Making Units (DMUs). In particular, DEA evaluates a DMU's efficiency by comparing its input and output quantities to those of other DMUs operating in a similar technological environment.[1] An attractive feature of DEA is that it is intrinsically nonparametric: DEA efficiency evaluations need not assume a specific functional/parametric form for the production technology. Instead, the production possibility set is reconstructed on the basis of the observed input-output combinations, using standard production axioms. A DMU's efficiency is then measured as the distance from its input-output combination to the frontier of this production possibility set.

Traditional DEA methods typically treat DMUs' production processes as a black box: they only use information on the aggregate amounts of inputs and outputs, and not on how the inputs and outputs are exactly linked to each other. Nevertheless, information on the allocation of inputs to outputs is often available in empirical research settings. Including this information can substantially increase the discriminatory power of the efficiency analysis, i.e. it creates considerably more potential to identify inefficient production behavior. Cherchye et al (2013) have put this idea into practice by developing a novel DEA-based methodology for measuring the efficiency of DMUs characterized by multiple inputs and outputs. Their methodology accounts for joint inputs in the production process, and explicitly includes information on how inputs are allocated to outputs.[2]

The current paper takes this multi-output methodology one step further. We propose an extension that quantifies possible efficiency gains by reallocating inputs over outputs. We capture these efficiency gains by a new measure of coordination efficiency. The measure takes a value of one when the input allocation over outputs is efficient, while a value below unity reveals that the productive efficiency can further increase by reallocating the inputs more optimally. Interestingly, our method also provides concrete guidelines on how to achieve the better input allocation, which is especially attractive from a practical point of view.

We also show the empirical usefulness of our methodology through an application that evalu-

[1]See, for example, Färe et al (1994), Cooper et al (2007), Fried et al (2008) and Cook and Seiford (2009) for extensive reviews of DEA.

[2]The treatment of multi-output production is partly inspired on recent work regarding the modeling of multi-person household consumption. See Cherchye et al (2007, 2011a,b).

ates the efficiency of US universities. We believe that university performance forms an interesting application area because universities typically have a two-fold assignment, i.e. education and research. In our application, we consider a university as a DMU that consists of an education division and a research division. Given our specific methodological contribution, a primal focus will be on the (efficient) allocation of university budgets over education and research outputs.

The remainder of our paper is organized as follows. Section 2 motivates our research question in more detail and relates it to the relevant literature. Section 3 formally introduces our measure of coordination efficiency. As we will explain, this measure essentially captures the difference between so-called centralized and decentralized efficiency. Section 4 discusses the practical implementation of our theoretical efficiency measures. Section 5 shows that our distinction between centralized and decentralized efficiency also bears an interesting dual representation. Section 6 presents our empirical application to US universities. Finally, Section 7 concludes.

2 Multi-output production and input allocation

To set the stage, we first provide a verbal explanation of the ideas that we formalize in the following sections. Next, we also discuss the relationship between our approach and alternative approaches that have appeared in the DEA literature.

2.1 Centralized, decentralized and coordination efficiency

Multi-output production is often motivated by the presence of economies of scope, which originate from joint use of inputs (Cherchye et al (2008)). Economies of scope occur if the average production cost decreases when the number of outputs increases (Baumol et al (1982)). Scope economies typically originate from jointly (or "publicly") used inputs, i.e. inputs that simultaneously benefit the production of multiple outputs. The DEA-based method of Cherchye et al (2013) accounts for economies of scope by explicitly modeling the presence of joint inputs in the production process.

We assume that a DMU consists of several divisions, where each division is responsible for the production of one or more outputs. On the input side, we distinguish joint inputs from division-specific inputs. Division-specific inputs differ from joint inputs in that they can be allocated to the outputs produced by a particular division.[3] As we will formalize in the next section, we assume that each division is characterized by its own production technology, while accounting for interdependencies between the different technologies through joint inputs.

As mentioned before, we are particularly interested in the way that a DMU allocates the available inputs among the divisions. More precisely, we consider whether a reallocation of the division-specific inputs within a DMU can lead to efficiency gains. To examine these gains from reallocation, we distinguish between centralized and decentralized efficiency measurement.

Essentially, the measure proposed by Cherchye et al (2013) considers efficiency from a decentralized perspective. In particular, the allocation of the division-specific inputs is considered to be predetermined and taken for granted in the efficiency analysis. Therefore, we will refer to Cherchye et al's original measure as decentralized efficiency in the sequel.

Our following analysis will complement this measure of decentralized efficiency with an alternative (novel) measure of centralized efficiency. Intuitively, this measure assumes that a DMU's central management can reallocate inputs over output divisions. Clearly, such reallocation can give rise to new gains of productive efficiency. We quantify these additional efficiency gains by our measure of coordination efficiency, which we calculate as the ratio of centralized over decentralized efficiency.

In Sections 3 and 4, we will introduce our measures of centralized and decentralized efficiency as measures of input technical efficiency. Interestingly, our distinction between centralized and decentralized efficiency also has an intuitive dual interpretation in terms of cost efficiency. In dual terms, the distinction relates to the (shadow) input prices that are used to evaluate a DMU's cost efficiency, which are defined differently in the decentralized and centralized cases. This will be explained more in detail in Section 5.

[3]Cherchye et al (2013) assume that a DMU is organized in such a way that each division is responsible for just one output. In that case, the notion output-specific input is used, to be distinguished from joint input. However, perfect information about the allocation of inputs to each individual output is often not available. We therefore consider a division structure in this paper. A main advantage of such a division structure is that it suffices to have data on the amount of inputs each division uses for the production of its outputs. Cherchye et al (2014) adopted a similar division structure in a setting that is formally close to the one that we consider here.

2.2 Related literature

At this point, it is worth indicating that there is a close link between the approach that we develop here and other approaches that have been presented in the DEA literature. Most notably, our set-up bears direct connections to earlier work on network DEA and centralized DEA models. In a sense, our method is situated on the intersection of these two existing approaches, by combining elements that are specific to each of them.

Firstly, there is clear relation with network DEA. Network models also add additional structure to the transformation process from inputs to outputs in the DEA assessment (see Färe and Grosskopf (2000), Färe et al (2007) and Cook and Zhu (2014)). Moreover, network DEA can be used to analyze the allocation of resources across various uses. For example, Färe et al (1997) consider the use of land to produce corn, wheat and soybeans.

Our approach has in common with network DEA that it explicitly incorporates information about the allocation of inputs to specific outputs. However, a crucial difference pertains to our dealing with joint inputs in the production model, which is not considered in the existing network DEA models. As indicated above, by including joint inputs in the analysis, we effectively model the presence of economies of scope, which forms a prime economic motivation for simultaneously producing multiple outputs.

Secondly, Lozano and Villa (2004) introduced so-called centralized DEA models. These models assume that there is a centralized decision maker who "owns" or supervises the DMUs. The centralized decision maker is interested in maximizing the efficiency of each individual unit, but is also concerned about the total input consumption. Lozano and Villa (2004) assume that inputs can be reallocated across DMUs and seek for an optimal allocation of the inputs among DMUs.

Following Lozano and Villa (2004), we also use the term "centralized" efficiency for a setting where input reallocation is possible. However, a main distinguishing feature of our approach is that we allocate inputs among various uses *within* the same DMU, i.e. across alternative production technologies (associated with different output divisions). This contrasts with centralized DEA models, which reallocate inputs across DMUs that are characterized by identical production technologies.

3 Theoretical efficiency measures

After introducing some necessary notation and terminology, we will formally define our measures of centralized, decentralized and coordination efficiency. To structure our discussion, we will first consider a theoretical set-up in which the production technology is known. The next section will consider the practical implementation of our theoretical efficiency measures.

3.1 Preliminaries

Efficiency analysis starts from a data set with T observed DMUs that produce N outputs. We assume that each DMU is subdivided into M divisions, where each division m $(1 \leq m \leq M)$ produces one or more outputs. Let d^m represent the number of outputs produced by division m. Thus, we have $N = d^1 + \ldots + d^M$. For each DMU t $(1 \leq t \leq T)$, we observe the vector of produced outputs $\mathbf{y}_t \in \mathbb{R}^N_+$, where

$$\mathbf{y}_t = (\mathbf{y}^1_t, \ldots, \mathbf{y}^M_t), \tag{1}$$

so that $\mathbf{y}^m_t \in \mathbb{R}^{d^m}_+$ denotes the vector of outputs produced by division m. Similarly, we also observe the division-specific and joint inputs. For any output division m, the vector $\mathbf{q}^m_t \in \mathbb{R}^{Nspec}_+$ contains the division-specific inputs. We let $\mathbf{q}_t = \sum_{m=1}^{M} \mathbf{q}^m_t$, i.e. $\mathbf{q}_t$ represents the total division-specific inputs of DMU t. Finally, $\mathbf{Q}_t \in \mathbb{R}^{Njoin}_+$ represents the joint inputs of DMU t. We note that these joint inputs $\mathbf{Q}_t$ cannot be allocated to particular divisions; they are simultaneously used in the production process of all output divisions. Taken together, the empirical analysis starts from the following data set:

$$S = \{(\mathbf{y}_t, \mathbf{q}^1_t, \ldots, \mathbf{q}^M_t, \mathbf{Q}_t) \mid t = 1, \ldots, T\}. \tag{2}$$

Next, we consider a separate production technology for each output division. Importantly, we account for interdependencies between the different technologies through jointly used inputs. More formally, we characterize the production technology of a division m by input requirement sets $I^m(\mathbf{y}^m)$, which contain all combinations of division-specific and joint inputs $(\mathbf{q}^m, \mathbf{Q})$ that can produce the output quantities $\mathbf{y}^m$, i.e.

$$I^m(\mathbf{y}^m) = \{(\mathbf{q}^m, \mathbf{Q}) \in \mathbb{R}_+^{Nspec} \times \mathbb{R}_+^{Njoin} | (\mathbf{q}^m, \mathbf{Q}) \text{ can produce } \mathbf{y}^m\}. \tag{3}$$

Finally, as explained above, our centralized efficiency measure assumes that the central management of a DMU coordinates the production process of all divisions. More specifically, the central management determines how much of the total amount $\mathbf{q}$ of division-specific inputs goes to every division. To formalize this idea, we need to consider input requirement sets $\mathbf{I}(\mathbf{y})$ for the "aggregate" output vector $\mathbf{y}$ (defined over all divisions simultaneously). These input requirement sets are constructed from the division-specific sets $I^m(\mathbf{y}^m)$, as follows:

$$\mathbf{I}(\mathbf{y}) = \{(\mathbf{q}, \mathbf{Q}) \mid \exists\ \mathbf{q}^1, \ldots, \mathbf{q}^M \text{ such that } \sum_{m=1}^{M} \mathbf{q}^m = \mathbf{q} \text{ and } \forall m : (\mathbf{q}^m, \mathbf{Q}) \in I^m(\mathbf{y}^m)\}. \tag{4}$$

Thus, each set $\mathbf{I}(\mathbf{y})$ contains all combinations of division-specific and joint inputs $(\mathbf{q}, \mathbf{Q})$ that can produce the output $\mathbf{y} = (\mathbf{y}^1, \ldots, \mathbf{y}^M)$. In particular, we say that $(\mathbf{q}, \mathbf{Q})$ can produce $\mathbf{y}$ if the input $\mathbf{q}$ can be allocated among the divisions such that every division can produce the associated output $\mathbf{y}^m$.

3.2 Efficiency measures

Throughout, we will consider input-oriented efficiency measurement, which identifies the maximum possible input reduction while keeping the output fixed. In doing so, we adopt radial (or Debreu-Farrell) efficiency measures, which are most popular in applied DEA work. Essentially, for some evaluated DMU and a given technology (represented by input requirement sets), these radial measures seek the maximum equiproportionate input reduction for the given output. Attractively, the measures have a natural degree interpretation: they are situated between 0 and 1, with an efficiency score of unity indicating technically efficient production, while lower values reflect greater productive inefficiency. Finally, and importantly, radial measures also have an interesting dual representation in terms of cost efficiency, which we will illustrate in Section 5.

Let us first introduce our radial measure of decentralized efficiency TE_t^d, which -to recall- is the one that was originally proposed by Cherchye et al (2013). In formal terms, we have

$$TE_t^d = \min\{\theta \mid \forall m : (\theta \mathbf{q}_t^m, \theta \mathbf{Q}_t) \in I^m(\mathbf{y}_t^m)\}. \tag{5}$$

This measure captures the maximum equiproportionate reduction of the inputs (captured by θ) that is feasible, for the given (observed) allocation of division-specific inputs.

By contrast, our (novel) centralized efficiency measure TE_t^c no longer takes the observed input allocation for granted. It quantifies the maximum input reduction that is feasible while accounting for possible (optimal) reallocation of the division-specific inputs. We define

$$TE_t^c = \min\{\theta \mid (\theta \mathbf{q}_t, \theta \mathbf{Q}_t) \in \mathbf{I}(\mathbf{y}_t)\}. \tag{6}$$

Basically, the measure considers not only the production process of the individual divisions (like the measure TE_t^d) *but also* the coordination among divisions. In other words, in contrast to our measure of decentralized efficiency, our centralized efficiency measure takes into account the inefficiencies that result from a suboptimal allocation of the inputs to the output divisions.

This directly provides the basic intuition of our following result, which states that decentralized efficiency is never lower than centralized efficiency.[4]

Proposition 1. *We have that $TE^c \leq TE^d$.*

In turn, this motivates the following ratio measure C^oE_t as a natural measure for the efficiency of input coordination among divisions:

$$C^oE_t = \frac{TE_t^c}{TE_t^d}. \tag{7}$$

By construction, this coordination efficiency measure is situated between 0 and 1. A coordination efficiency value of unity indicates that the inputs are allocated in an optimal way across the divisions. By contrast, a lower efficiency value reveals that DMU t's productive efficiency can be further increased by input reallocation. More specifically, for the given output DMU t can achieve additional input reduction by adjusting the input mix over output divisions.

[4] The proofs of our results appear in Appendix B.

As a final note, we can also write

$$TE_t^c = C^o E_t \times TE_t^d, \tag{8}$$

which provides an intuitive decomposition of centralized efficiency as the product of coordination efficiency and decentralized efficiency.

4 Practical implementation

In practice, the true input requirement sets $I^m(\mathbf{y}^m)$ (used for the decentralized measure TE_t^d) and $\mathbf{I}(\mathbf{y})$ (used for the centralized measure TE_t^c) are typically not observed. The DEA approach proceeds by defining empirical approximations of these input sets on the basis of some standard production axioms. In turn, this defines operational efficiency measures that can be computed by means of standard linear programming techniques.

4.1 Empirical efficiency measures

In what follows, we use the same production axioms as Cherchye et al (2013):[5]

Axiom 1 (Nested input sets). $\boldsymbol{y}^m \geq \boldsymbol{y}^{m*} \Rightarrow I^m(\boldsymbol{y}^m) \subset I^m(\boldsymbol{y}^{m*})$

Axiom 2 (Monotone input sets). $(\boldsymbol{q}^m, \boldsymbol{Q}) \in I^m(\boldsymbol{y}^m)$ *and* $(\boldsymbol{q}^{m*}, \boldsymbol{Q}^*) \geq (\boldsymbol{q}^m, \boldsymbol{Q}) \Rightarrow (\boldsymbol{q}^{m*}, \boldsymbol{Q}^*) \in I^m(\boldsymbol{y}^m)$

Axiom 3 (Convex input sets). $(\boldsymbol{q}^m, \boldsymbol{Q}), (\boldsymbol{q}^{m*}, \boldsymbol{Q}^*) \in I^m(\boldsymbol{y}^m) \Rightarrow \forall \lambda \in [0,1] : \lambda(\boldsymbol{q}^m, \boldsymbol{Q}) + (1-\lambda)(\boldsymbol{q}^{m*}, \boldsymbol{Q}^*) \in I^m(\boldsymbol{y}^m)$

Axiom 4 (Observability means feasibility). $(\boldsymbol{y}_t, \boldsymbol{q}_t^1, \ldots, \boldsymbol{q}_t^M, \boldsymbol{Q}_t) \in S \Rightarrow \forall m : (\boldsymbol{q}_t^m, \boldsymbol{Q}_t) \in I^m(\boldsymbol{y}_t^m)$.

In words, Axiom 1 says that, if some input can produce the output $\mathbf{y}^m$, then it can also produce any lower output $\mathbf{y}^{m*}$. Essentially, this means that outputs are freely disposable. Similarly, Axiom 2 defines free input disposability, i.e. more input never reduces the output. Next, Axiom 3 states that, if two inputs can produce the output $\mathbf{y}^m$, then any convex combination of these

[5]Of course, other axioms can be used as well. For example, such axioms may impose alternative assumptions regarding the nature of returns-to-scale (constant, decreasing or increasing) that underlie the production technology.

inputs can also produce the same output. Finally, Axiom 4 says that the observed input-output combinations are certainly feasible.[6]

Cherchye et al (2013) have shown that, if Axioms 1-4 hold, an empirical inner bound approximation of the true (but unobserved) set $I^m(\mathbf{y}_t^m)$ is defined as

$$\hat{I}(\mathbf{y}_t^m) = \{(\mathbf{q}^m, \mathbf{Q}) | \sum_{s\in D_t^m} \lambda_s^m \mathbf{q}_s^m \leq \mathbf{q}^m, \sum_{s\in D_t^m} \lambda_s^m \mathbf{Q}_s \leq \mathbf{Q}, \sum_{s\in D_t^m} \lambda_s^m = 1, \lambda_s^m \geq 0\}, \tag{9}$$

with

$$D_t^m = \{s | \mathbf{y}_t^m \leq \mathbf{y}_s^m\}. \tag{10}$$

By construction, we have $\hat{I}^m(\mathbf{y}_t^m) \subseteq I^m(\mathbf{y}_t^m)$ under Axioms 1-4. The set $\hat{I}(\mathbf{y}_t^m)$ is the smallest production set that is consistent with Axioms 1-4 for a given data set S and, therefore, it defines a useful empirical approximation of the true set $I^m(\mathbf{y}_t^m)$.

By combining (4) and (9), we obtain the following empirical approximation of the set $\widehat{\mathbf{I}}(\mathbf{y})$:

$$\widehat{\mathbf{I}}(\mathbf{y}) = \{(\mathbf{q}, \mathbf{Q}) \mid \exists\ \mathbf{q}^1, \ldots, \mathbf{q}^M \text{ such that } \sum_m \mathbf{q}^m = \mathbf{q} \text{ and } \forall m : (\mathbf{q}^m, Q) \in \hat{I}(\mathbf{y}^m)\}. \tag{11}$$

Clearly, because $\hat{I}^m(\mathbf{y}_t^m) \subseteq I^m(\mathbf{y}_t^m)$ we also have that $\hat{\mathbf{I}}(\mathbf{y}_t) \subseteq \mathbf{I}(\mathbf{y}_t)$.

Using the empirical approximations $\hat{I}^m(\mathbf{y}_t^m)$ and $\hat{\mathbf{I}}(\mathbf{y}_t)$, we can define the empirical counterparts of TE_t^d and TE_t^c as, respectively,

$$\widehat{TE_t^d} = \min\{\theta | \forall m : (\theta \mathbf{q}_t^m, \theta \mathbf{Q}_t) \in \hat{I}^m(\mathbf{y}_t^m)\}, \tag{12}$$

and

$$\widehat{TE_t^c} = \min\{\theta \mid (\theta \mathbf{q}_t, \theta \mathbf{Q}_t) \in \widehat{\mathbf{I}}(\mathbf{y}_t)\}, \tag{13}$$

which have a readily analogous interpretation as the theoretical measures. Interestingly, the empirical measures $\widehat{TE_t^d}$ and $\widehat{TE_t^c}$ can be computed by solving simple linear programming problems, which we discuss in more detail below.

[6]Essentially, this assumes that the input and output data are not contaminated by measurement errors. The DEA literature has forwarded alternative proposals to deal with errors-in-the-data in practical applications. To compactify our exposition, we will abstract from measurement issues in what follows, but these existing methodologies are fairly easily integrated in the framework that we set out here. See Cherchye et al (2013) for related discussion.

Before doing so, we point out two properties of the measures that are relevant for our following exposition. Firstly, because $\hat{I}^m(\mathbf{y}_t^m) \subseteq I^m(\mathbf{y}_t^m)$ and $\hat{\mathbf{I}}(\mathbf{y}_t) \subseteq \mathbf{I}(\mathbf{y}_t)$, we naturally obtain

$$TE_t^d \leq \widehat{TE_t^d} \quad \text{and} \quad TE_t^c \leq \widehat{TE_t^c}, \tag{14}$$

i.e. the empirical efficiency measures define natural upper bounds for the theoretical measures. Secondly, just as in the theoretical case, the centralized efficiency measure $\widehat{TE_t^c}$ never exceeds the decentralized measure $\widehat{TE_t^d}$.

Proposition 2. *We have that* $\widehat{TE^c} \leq \widehat{TE^d}$.

Analogous to before, the result in Proposition 2 motivates the empirical measure for coordination efficiency

$$\widehat{C^oE_t} = \frac{\widehat{TE_t^c}}{\widehat{TE_t^d}}, \tag{15}$$

which has a similar meaning as the theoretical measure C^oE_t. Again, a low coordination efficiency reveals that efficiency gains are possible by reallocating the inputs in a more optimal way.

4.2 Linear programming formulation

We conclude this section by discussing the linear programming formulation of $\widehat{TE_t^d}$ and $\widehat{TE_t^d}$. By using (9) and (11), it is straightforward to verify that

$$\begin{array}{llll}
\textbf{(LP-1)} & \widehat{TE_t^d} & = & \min\limits_{\theta_t \geq 0, \lambda_s^m \geq 0} \theta_t \\
& s.t. & & \\
& \text{(D-1)} & & \forall m : \sum\limits_{s \in D_t^m} \lambda_s^m \mathbf{Q}_s \leq \theta_t \mathbf{Q}_t \\
& \text{(D-2)} & & \forall m : \sum\limits_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m \leq \theta_t \mathbf{q}_t^m \\
& \text{(D-3)} & & \forall m : \sum\limits_{s \in D_t^m} \lambda_s^m = 1;
\end{array}$$

$$
\begin{array}{llll}
\textbf{(LP-2)} & \widehat{TE}_t^c & = & \min_{\theta_t \geq 0, \lambda_s^m \geq 0, \mathbf{q}^m \geq 0} \theta_t \\
& s.t. & & \\
& \text{(D-0)} & & \sum_m \mathbf{q}^m = \theta_t \mathbf{q}_t \\
& \text{(D-1)} & & \forall m : \sum_{s \in D_t^m} \lambda_s^m \mathbf{Q}_s \leq \theta_t \mathbf{Q}_t \\
& \text{(D-2)'} & & \forall m : \sum_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m \leq \mathbf{q}^m \\
& \text{(D-3)} & & \forall m : \sum_{s \in D_t^m} \lambda_s^m = 1.
\end{array}
$$

Both **(LP-1)** and **(LP-2)** compute efficiency measures that quantify the maximum possible input reduction (captured by θ_t) for DMU t, while keeping the output fixed. This maximum input reduction is defined by reference input vectors that are constructed from other observed DMUs. In particular, for each division m this reference vector corresponds to a convex combination of all DMUs s that produce at least the output $\mathbf{y}_t^m$ (i.e. $s \in D_t^m$). Each variable λ_s^m then represents the weight of every DMU s in this convex combination. These variables are also called "intensity parameters" in the DEA literature.

The essential difference between **(LP-1)** and **(LP-2)** pertains to the variables $\mathbf{q}^m$ that appear in **(LP-2)**. Specifically, in **(LP-2)** the input quantities $\mathbf{q}^m$ can be chosen freely (except from the non-negativity requirement and the adding-up constraint (D-0), i.e. $\sum_m \mathbf{q}^m = \theta_t \mathbf{q}_t$), whereas in **(LP-1)** the division-specific inputs are fixed at their observed level $\mathbf{q}_t^m$. This additional freedom to choose the quantities $\mathbf{q}^m$ in **(LP-2)** also directly explains the inequality $\widehat{TE}^c \leq \widehat{TE}^d$ in Proposition 2.

Interestingly, the linear programming problem **(LP-2)** not only defines an empirical measure of centralized efficiency. It also returns reference values for the division-specific inputs (i.e. the solution values for $\mathbf{q}^m$) that correspond to an optimal input allocation. This is especially attractive from a practical point of view. For DMUs that have a low coordination efficiency, it provides specific guidelines on how to improve the input allocation over output divisions.

One final note pertains to the possibility that input reallocations may be restricted in practice.

For example, it may well be that some inputs are simply not adjustable and/or that changing the input mix is very expensive. Conveniently, it is fairly easy to adapt our framework to take such considerations into account. In particular, as long as the associated restrictions can be formulated in linear form, they can simply be added to **(LP-2)** without interfering with the linear nature of the resulting programming problem. For compactness, and because we believe this type of extension is relatively straightforward, we will not further elaborate on this in the current paper.

5 Dual representations

An interesting feature of our measures of centralized and decentralized efficiency is that they can be given a dual representation as measures of cost efficiency, evaluated at shadow input prices. The difference between the two measures relates to the input prices that are used for evaluating the division-specific inputs. As we will explain, this difference can be given a precise interpretation in terms of centralized versus decentralized decision making (i.e. with versus without input reallocations across output divisions).

5.1 Decentralized efficiency

We first consider our measure of decentralized efficiency. The dual version of the linear program **(LP-1)** can be written as[7]

$$
\textbf{(LP-3)} \qquad \widehat{TE}_t^d = \max_{\substack{c_t^m \geq 0, \mathbf{P}_t^m \in \mathbb{R}_+^{Njoin}, \\ \mathbf{P}_t \in \mathbb{R}_+^{Njoin}, \mathbf{p}_t^m \in \mathbb{R}_+^{Nspec}}} \sum_{m=1}^{M} c_t^m
$$

$$
\begin{aligned}
& s.t. \\
& \text{(C-1)} \qquad \sum_{m=1}^{M} \mathbf{P}_t^m = \mathbf{P}_t \\
& \text{(C-2)} \qquad \forall m: c_t^m \leq (\mathbf{p}_t^m)' \, \mathbf{q}_s^m + (\mathbf{P}_t^m)' \, \mathbf{Q}_s \ \forall s \in D_t^m \\
& \text{(C-3)} \qquad \sum_{m=1}^{M} (\mathbf{p}_t^m)' \, \mathbf{q}_t^m + \mathbf{P}_t' \mathbf{Q}_t = 1.
\end{aligned}
$$

[7]Appendix A gives specific details on how we obtain the dual problems **(LP-3)** and **(LP-4)**.

To explain the cost efficiency interpretation of this dual program, we need to interpret the vectors $\mathbf{P}_t^m$, $\mathbf{P}_t$ and $\mathbf{p}_t^m$ as shadow price vectors. For every division m, $\mathbf{p}_t^m$ and $\mathbf{P}_t^m$ contain the shadow prices for the division-specific and joint inputs, respectively. Similarly, $\mathbf{P}_t$ contains the prices for the joint inputs at the level of the aggregate DMU.

For the joint inputs, the division-specific prices $\mathbf{P}_t^m$ are related to the DMU-level prices $\mathbf{P}_t$ by the adding-up restriction (C-1). This adding-up restriction implies that the shadow prices $\mathbf{P}_t^m$ actually represent the fractions of DMU t's aggregate prices $\mathbf{P}_t$ that are borne by each division m. In a sense, they represent the "willingness-to-pay" (reflecting marginal productivities) of the different divisions for the jointly (or "publicly") consumed inputs. This parallels the interpretation of so-called Lindahl prices that correspond to the efficient provision of public goods.

Next, the constraint (C-3) defines a cost normalization for the evaluated DMU t. It specifies that the shadow prices must be such that the aggregate cost of DMU t equals unity.

Given all this, $\sum_{m=1}^{M} c_t^m$ can be interpreted as the minimum cost for producing the output of DMU t. Each variable c_t^m then represents the minimum cost for producing the output of DMU t's division m, while accounting for the interrelation with the output production of the other divisions (through joint inputs). In particular, restriction (C-2) imposes that c_t^m cannot exceed the cost level associated with any other DMU s that produces at least the output $\mathbf{y}_t^m$ (i.e. $s \in D_t^m$).

In this respect, we also note that, by construction, $t \in D_t^m$ for any m. Therefore, the normalization constraint (C-3) guarantees that

$$\sum_{m=1}^{M} c_t^m \leq 1 (= \sum_{m=1}^{M} (\mathbf{p}_t^m)' \, \mathbf{q}_t^m + \mathbf{P}'_t \mathbf{Q}_t)), \tag{16}$$

which implies that the sum of division-specific minimal costs $\sum_{m=1}^{M} c_t^m$ is situated between 0 (because of the non-negativity constraints) and 1. Conveniently, because DMU t's aggregate cost level is normalized at unity, this also makes that the sum $\sum_{m=1}^{M} c_t^m$ in the objective of problem **(LP-3)** can be interpreted as the ratio of minimal cost (for the aggregate output $\mathbf{y}_t$) over DMU t's actual cost. Putting it differently, the objective function value of **(LP-3)** expresses DMU t's cost efficiency in relative terms.

The max operator in the objective guarantees that the shadow prices $\mathbf{p}_t^m$, $\mathbf{P}_t^m$ and $\mathbf{P}_t$ are chosen such that this measure of cost efficiency is maximized. In a sense, this actually gives the "benefit-of-the-doubt" to the evaluated DMU. Most favorable prices are chosen, so putting DMU t in the best possible light.

One important final note is in order. In program **(LP-3)** the prices $\mathbf{p}_t^m$ for the division-specific inputs may vary depending on the division m at hand. Intuitively, the fact that different divisions can use different (shadow) prices for these inputs reflects that these inputs are not directly substitutable across divisions. This effectively relates to the very essence of our notion of decentralized efficiency, which assumes that input reallocations across divisions are impossible. It will also imply a crucial difference with the dual representation of our centralized efficiency measure.

5.2 Centralized efficiency

We next turn to our centralized efficiency measure. The dual of program **(LP-2)** is given as

$$\textbf{(LP-4)} \qquad \widehat{TE}_t^c = \max_{\substack{c_t^m \geq 0, \mathbf{P}_t^m \in \mathbb{R}_+^{Njoin}, \\ \mathbf{P}_t \in \mathbb{R}_+^{Njoin}, \mathbf{p}_t \in \mathbb{R}_+^{Nspec}}} \sum_{m=1}^{M} c_t^m$$

$$s.t.$$

$$\text{(C-1)} \qquad \sum_{m=1}^{M} \mathbf{P}_t^m = \mathbf{P}_t$$

$$\text{(C-2)} \qquad \forall m : c_t^m \leq \mathbf{p}_t' \mathbf{q}_s^m + (\mathbf{P}_t^m)' \, \mathbf{Q}_s \; \forall s \in D_t^m$$

$$\text{(C-3)} \qquad \sum_{m=1}^{M} \mathbf{p}_t' \mathbf{q}_t^m + \mathbf{P}_t' \mathbf{Q}_t = 1.$$

This linear programming problem has basically the same structure as problem **(LP-3)**. Therefore, it also has a directly similar cost efficiency interpretation. However, there is a subtle but important difference, which pertains to the shadow prices for the division-specific inputs. In the new problem **(LP-4)**, these prices are the same for all divisions m (i.e. $\mathbf{p}_t^m = \mathbf{p}_t$ for all m). The intuition directly relates to our concept of centralized efficiency: in contrast to the decentralized efficiency setting, we now assume that input reallocations over divisions are possible, which means that division-specific inputs are perfectly substitutable across divisions (and, thus, we use common division-specific shadow prices).

A last remark relates to the possibility to impose restrictions on input substitutability across divisions. As discussed at the end of Section 4, restrictions on input reallocations could be implemented as linear constraints added to the primal problem **(LP-2)**. In a similar vein, we could also include additional restrictions to the dual problem **(LP-4)**. It follows from our above discussion that particular constraints on the (non)substitutability of inputs can here be implemented in the form of shadow price restrictions. For example, one may want to impose that some division-specific inputs are perfectly substitutable across divisions (resulting in common shadow prices for all divisions) while other inputs are not (implying division-specific shadow prices).

6 Application: US university education and research

We illustrate the practical usefulness of our methodology through an empirical application that evaluates the efficiency of US universities. In particular, we are interested in the allocation of the university budget across education and research divisions. We measure inputs as expenditures, which we subdivide into division-specific and joint expenses. In this application, division-specific inputs are university expenses that are clearly directed towards either education or research. Next, joint inputs contain expenditures related to "public" services like libraries, museums, media, technology and administration. In what follows, we will first motivate our input and output data in more detail, and subsequently present our main efficiency results.

6.1 Input and output data

When it comes to evaluating university efficiency, the definition of the relevant input and output dimensions is all but straightforward. There is a lack of consensus in the literature on the most appropriate selection of inputs and outputs. To focus our discussion, and given that our objective here is mainly to illustrate the practical application of our methodology, we opt for a fairly basic specification of a university's production process.

We are interested in the joint production of education and research conducted by universities, and we will particularly concentrate on the allocation of the university budget across the education and research divisions. In this respect, our study here is close in spirit to Beasley (1995), who analyzed the teaching and research efficiency of chemistry and physics departments in the UK. This author also takes into account that some resources are shared between the different activities. However, an important difference is that Beasley (1995) considered the shared input as a division-specific input of which the allocation over research and teaching is unknown. By contrast, our specific methodology allows us to treat shared inputs as joint inputs that simultaneously benefit the production of both teaching and research.

We use data on 130 US universities in 2012. Our sample contains both (87) public and (43) private non-profit institutions. We retrieved the main part of our data from the National Center for Education Statistics (NCES). The NCES collects and analyzes detailed information on education in the United States. In order to obtain a sample that is sufficiently comparable (i.e. homogeneous), we (only) selected universities that are classified as research universities, with high or very high research activity (according to the Carnegie classification). Moreover, our selected universities all appeared in the top 500 of the Shanghai ranking in the year 2012.[8]

Output selection. We consider two outputs for the education division and two outputs for the research division of each university. The two education outputs are undergraduate and graduate enrollments, expressed in full time equivalents. The advantage of considering enrollments (and not degrees granted) is that this measure also takes into account students who have not yet completed their studies but did receive education from the university. Our enrollment data come from the NCES and pertain to the academic year 2011-2012.

The two research outputs are number of doctor's degrees and a measure of publication output. Again, number of doctor's degrees is taken from the NCES and relates to the year 2012. Next, our measure of publication output is constructed such that it not only accounts for the quantity but also the quality of scientific publications, where quality is measured in terms of researchers' citations. Specifically, we quantify publication output as the mean of 3 scores (standardized between 0 and 100) that are also used for the Shanghai ranking: HiCi (i.e. highly cited researchers in 21 broad subject categories), N&S (i.e. papers published in Nature and Science) and PUB

[8]The Shanghai ranking is a widely used world ranking of universities that simultaneously accounts for several indicators of research performance. See www.shanghairanking.com for more details.

(i.e. papers indexed in Science Citation Index-expanded and Social Science Citation Index).

Input selection. We use university expenses as inputs. The public universities report expenses according to GASB (Governmental Accounting Standards Board), while the private universities report according to FASB (Financial Accounting Standards Board).[9] We distinguish between expenses that can be allocated directly to the research and eduction divisions and expenses that have a joint (or "public") nature.

Our division-specific inputs can be retrieved directly from the NCES data. Specifically, we use as education input all expenses that relate to activities that form part of the instruction program. Similarly, our research input contains all expenses related to activities specifically organized to produce research outcomes.

Next, our joint input contains expenses on institutional and academic support, which are vital to the proper "overall" functioning of a university (including both education and research). Institutional support includes, for example, the management, personnel administration and logistic activities. Academic support contains academic administration, libraries, museums and computer services.

We remark that our following analysis will not consider expenses on public services (e.g. community service programs, radio, television and consulting) or student services (e.g. student activities, newspapers, health services and athletics). Our motivation is that we believe these expenses are not directly related to the education and research outputs that we selected. However, it should be clear that these data could easily be included in the analysis if deemed appropriate. Our choice not to use this information is purely an empirical one and does by no means indicate a limitation of our methodology.

Data. Appendix C contains our input and output data for each individual university, while Table 1 reports some summary statistics. Expenses are reported in millions of dollars. We find substantial variation across our sample of universities for each of the variables that we selected.

[9] Our analysis implicitly assumes that GASB and FASB are comparable accounting standards. In this respect, we also conducted several robustness checks to ensure that our conclusions with respect to public and private universities cannot be attributed to differences in these accounting standards. In particular, we computed efficiency results with numbers of staff (instead of expenses) as inputs. Interestingly, these alternative exercises yielded the same qualitative conclusions.

See, for example, the high standard deviations and the large differences between minimum and maximum values.

The last two columns of Table 1 report the mean values for the public and private universities. A main difference between these two categories of universities relates to the number of undergraduate enrollments. On average, public universities have 22551 undergraduate enrollments, which is more than twice the average of 9177 enrollments for the private universities. In this respect, a remarkable observation is that average expenses on instruction for the public universities are nearly one third below those for their private counterparts.

On the basis of these figures, public universities seem to be more efficient than private universities in the provision of (undergraduate) education. However, an important remark is that student enrollments only measure the quantity but not the quality of this education output. For example, it may well be that private universities specialize in "excellent" education, while public universities rather focus on "standard" education. Unfortunately, we could not find data on the educational quality for our sample of universities, and so we are bound to ignore such quality considerations in our following efficiency analysis. While we will not repeat it explicitly, this qualification must be kept in mind when interpreting our efficiency results.

Variables	Full sample				Public	Private
	Mean	Std	Min	Max	Mean	Mean
Instruction and research expenses	766	560	142	2774	690	921
→ Instruction	458	343	88	1947	399	579
→ Research	308	259	4	1291	291	342
Academic and institutional expenses	242	186	50	1434	205	318
Undergraduate enrollment	18127	10065	968	55016	22551	9177
Graduate enrollment	5318	3189	757	16373	5271	5413
Doctor's degree	329	213	35	892	350	284
Publication	30	16	11	100	28	35

Table 1: Descriptive statistics for the sample of 130 US universities, including 87 public and 43 private universities. Expenses are reported in millions of dollars.

6.2 Efficiency results

We will first report summary statistics for our measures of decentralized, centralized and coordination efficiency, defined over our sample of 130 universities. Subsequently, we will consider efficiency differences between public and private universities. In a final step, we will take a closer look at the allocation of university budgets across education and research. In particular, starting from the observation that our sample is characterized by substantial coordination inefficiency, we use the results of our linear programs to identify possible strategies that can lead universities towards a more optimal input allocation.

Full sample results. Table 2 provides summary results for our different efficiency measures. Detailed information on the efficiency results of each university can be found in Appendix C.

If we first consider centralized efficiency for the full sample of universities, we find that about 40% of the universities is labeled as efficient. The average centralized efficiency amounts to 0.86, which suggests that the average university can reduce its expenses by 14% for the given levels of education and research outputs.

As indicated above, centralized efficiency can be decomposed into decentralized efficiency and coordination efficiency. We find that the average decentralized efficiency equals 0.92. This indicates that the possible input reduction amounts to only 8% if input reallocations over research and education are impossible.

The difference between centralized and decentralized efficiency is captured by our measure of coordination efficiency. The average coordination efficiency turns out to be 0.93, which reveals that optimal input reallocations can yield an additional efficiency gain of 7%. Interestingly, because the median coordination efficiency value is 0.98 (i.e. below unity), we conclude that such input reallocations can be beneficial to more than half of the universities in our sample.

		Min.	1^{st} Qu.	Mean	Median	3^{rd} Qu.	Max.
Centralized Eff.		0.40	0.76	0.86	0.92	1.00	1.00
	→ public	0.45	0.80	0.89	0.97	1.00	1.00
	→ private	0.40	0.66	0.80	0.85	0.99	1.00
Decentralized Eff.		0.50	0.88	0.92	1.00	1.00	1.00
	→ public	0.57	0.90	0.94	1.00	1.00	1.00
	→ private	0.50	0.75	0.88	1.00	1.00	1.00
Coordination Eff.		0.62	0.90	0.93	0.98	1.00	1.00
	→ public	0.64	0.91	0.94	1.00	1.00	1.00
	→ private	0.62	0.84	0.91	0.94	1.00	1.00

Table 2: Descriptive statistics for our efficiency results

Public versus private universities. When distinguishing between the public and private universities, we observe that the public universities operate on average more efficient than their private counterparts. This difference is more pronounced for our measures of decentralized and centralized efficiency than for our measure of coordination efficiency. This suggests that the better performance of public universities is not so much the result of a better input allocation per se (i.e. coordination efficiency), but rather follows from a more efficient input use for the given allocation of expenses over education and research (i.e. decentralized efficiency).

We also conducted Wilcoxon rank-sum tests to evaluate the statistical significance of these observed efficiency differences. We find that the difference is significant for the centralized efficiency measure (the null hypothesis that both subsamples achieve the same efficiency level has a p-value of only 0.004) and the decentralized efficiency measure (p-value of 0.024). By contrast, there is no significant difference in terms of coordination efficiency (p-value of 0.109).

Generally, we may conclude that we find a rather substantial difference in centralized efficiency between private and public universities, and decentralized efficiency seems to be a more important explanation of this difference than coordination efficiency. Interestingly, our outcome that public universities perform better than private universities conforms with earlier findings of Ahn et al (1988), who compared the relative efficiencies of public and private doctoral-granting universities in the US. These authors equally concluded that public universities prove to be more efficient than private universities when managerial and program inefficiencies are present in the data.

Reallocation strategies. Our above analysis indicates that only 49 of the 130 universities achieve a centralized efficiency score of one. The remaining 81 universities have a score below one, which means that efficiency improvements are possible. When zooming in on these 81 universities, we find that 73 of them exhibit coordination inefficiency. For these universities, reallocating the division-specific inputs over the education and research divisions effectively leads to efficiency gains.

In this respect, we recall that a centralized efficiency score of θ indicates that all expenses (i.e. joint expenses as well as division-specific expenses) can be reduced by a fraction $(1-\theta)$. However, and importantly, these savings need not be equally distributed across the education and research divisions. It is even possible that one of the divisions receives additional (division-specific) budget

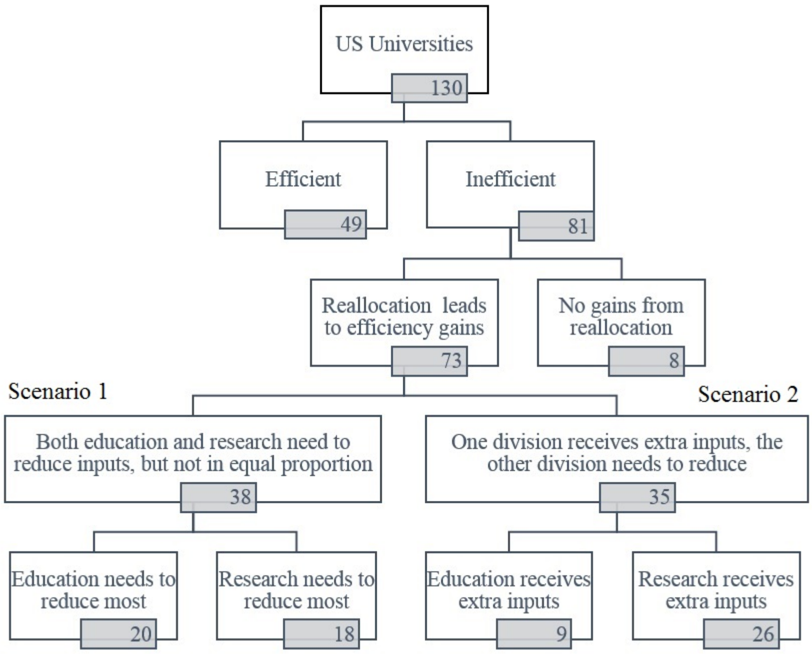

Figure 1: Possible strategies to achieve an optimal allocation of the budget.

as a consequence of the reallocation, at the expense of the other division.

As explained above (when discussing **(LP-2)**), our linear programs not only define an empirical measure of centralized efficiency, but also provide specific guidelines on how to enhance the input allocation over output divisions. Specifically, it returns reference values for the division-specific inputs that correspond to an optimal input allocation. Interestingly, this information enables us to conclude whether a division needs to reduce or increase the current (division-specific) input, in order to remedy the observed coordination inefficiency.

In Appendix C, we report for each university the changes in the budget directed towards education or research that are necessary to eliminate its coordination inefficiency. Figure 1 provides a schematic summary of the different strategies that can be followed, hereby also indicating to how many universities each strategy applies.

In the first scenario, both divisions need to reduce their inputs, but not in equal proportion. In percentage terms, one of the divisions needs to save considerably more than the other division. This scenario holds for 38 universities. For 20 universities, the education division needs to reduce the division-specific input more than the research division. For the remaining 18 universities, the opposite conclusion holds.

In the second scenario, which holds for 35 universities, one of the divisions receives additional division-specific input. Of course, in such a case the required input reduction for the other division is particularly substantial, since the input use at the aggregate DMU level (summed over the two divisions) needs to go down. For this scenario, we find that the research division should receive extra budget (at the expense of the education division) in 26 of the 35 universities, while the opposite conclusion applies to the other 9 universities.

7 Conclusion

We have extended the DEA-based methodology of Cherchye et al (2013) for multi-output efficiency measurement. Our extension exploits the specific feature of this methodology, which includes information on joint and division-specific inputs in the efficiency evaluation. In particular, we use this input information to develop a method that investigates whether input reallocations across output divisions can yield specific efficiency gains. We propose a measure of coordination efficiency to quantify these gains. Interestingly, for DMUs with low coordination efficiency our method also provides concrete guidelines to achieve a more optimal input allocation.

We have used our methodology to evaluate the productive efficiency of education and research conducted at US universities. We believe our methodology is particularly well-suited to analyze the joint production of education and research, as it can account for both joint inputs and inputs specifically allocated to education or research. Although our application was mainly intended to serve illustrative purposes, it did clearly reveal the potential of our new method. For example, our empirical results suggest that the universities under study can considerably enhance their productive efficiency by adopting a more optimal input allocation (over education and research outputs). In particular, we found that more than half of the universities suffer from coordination inefficiency.

A Dual formulations

In this section, we clarify the link between the primal and dual formulation of the LP-models. In the decentralized setting, the linear programming problem **(LP-3)** is dual to problem **(LP-1)**. Similarly, in the centralized setting, problem **(LP-4)** is dual to problem **(LP-2)**.

We start by considering the decentralized setting. The constraint (D-1) in **(LP-1)**, which constructs for every division m a reference vector for the joint inputs, corresponds to the shadow price vector $\mathbf{P}_t^m$ in the dual cost efficiency problem. Similarly, the constraint (D-2), which constructs a benchmark vector for the division-specific inputs, corresponds to the shadow price vector $\mathbf{p}_t^m$. Further, the constraint (D-3) corresponds to the (minimal) costs variable c_t^m for every division m.

Next, we focus on the centralized setting. The only difference between **(LP-3)** and **(LP-4)** is that **(LP-4)** uses the common price vector $\mathbf{p}_t$ for the division-specific inputs. To see that **(LP-4)** is dual to the linear programming problem **(LP-2)**, we first write the problem **(LP-2)** in a slightly different form. For this, we use lemma 1:

Lemma 1. *The statement*

$$\exists\ \boldsymbol{q}^1, \ldots, \boldsymbol{q}^M \quad \textit{such that} \sum_m \boldsymbol{q}^m = \theta_t \boldsymbol{q}_t \ \textit{and}\ \forall m: \sum_{s \in D_t^m} \lambda_s^m \boldsymbol{q}_s^m \leq \boldsymbol{q}^m \tag{17}$$

is equivalent to the statement

$$\sum_m \sum_{s \in D_t^m} \lambda_s^m \boldsymbol{q}_s^m \leq \theta_t \boldsymbol{q}_t. \tag{18}$$

As a consequence, the linear programming problem **(LP-2)** is equivalent to the following problem **(LP-2b)**:

$$
\begin{array}{lrcl}
\textbf{(LP-2b)} & \widehat{TE}_t^c & = & \min_{\theta_t \geq 0, \lambda_s^m \geq 0} \theta_t \\
 & s.t. & & \\
 & \text{(D-1)} & & \forall m : \sum_{s \in D_t^m} \lambda_s^m \mathbf{Q}_s \leq \theta_t \mathbf{Q}_t \\
 & \text{(D-2)"} & & \sum_m \sum_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m \leq \theta_t \mathbf{q}_t \\
 & \text{(D-3)} & & \forall m : \sum_{s \in D_t^m} \lambda_s^m = 1
\end{array}
$$

Note that this linear programming problem **(LP-2b)** is very similar to **(LP-1)**. The only difference is that left hand side of constraint (D-2)" contains a sum over all divisions m (whereas the constraint (D-2) specifies a separate constraint for every individual m). The constraint (D-2)" corresponds to the shadow price vector $\mathbf{p}_t$ in the dual problem. We conclude that problem **(LP-4)** is dual to problem **(LP-2b)** and, thus, also to the equivalent problem **(LP-2)**.

B Proofs

Proposition 1. For $\mathbf{q}_t = \sum_m \mathbf{q}_t^m$ we have

$$
\begin{array}{rcl}
\{\theta \mid \forall m : (\theta \mathbf{q}_t^m, \theta \mathbf{Q}_t) \in I^m(\mathbf{y}_t^m)\} & \subset & \left\{ \theta \;\middle|\; \begin{array}{c} \exists\, \mathbf{q}^1, \ldots, \mathbf{q}^M \text{ such that } \sum_m \mathbf{q}^m = \mathbf{q}_t \\ \text{and } \forall m : (\theta \mathbf{q}^m, \theta \mathbf{Q}_t) \in I^m(\mathbf{y}_t^m) \end{array} \right\} \\
 & = & \{\theta \mid (\theta \mathbf{q}_t, \theta \mathbf{Q}_t) \in \mathbf{I}(\mathbf{y}_t)\}
\end{array}
$$

Consequently,

$$
\min\{\theta \mid (\theta \mathbf{q}_t, \theta \mathbf{Q}_t) \in \mathbf{I}(\mathbf{y}_t)\} \leq \min\{\theta \mid \forall m : (\theta \mathbf{q}_t^m, \theta \mathbf{Q}_t) \in I^m(\mathbf{y}_t^m)\},
$$

which obtains that $TE^c \leq TE^d$. □

Proposition 2. We will prove that constraint (D-2) implies the constraints (D-0) and (D-2)'.

Suppose that (D-2) holds. Define $\mathbf{q}^m = \theta_t \mathbf{q}_t^m$. We have found values for $\mathbf{q}^1, \ldots, \mathbf{q}^M$ such that (D-0) and (D-2)' hold. We conclude that the feasible region of **(LP-1)** is a subset of the feasible region of **(LP-2)**. Since **(LP-1)** and **(LP-2)** are both minimization problems, the optimal objective function value for **(LP-2)** cannot exceed the optimal value for **(LP-1)**. We conclude that $\widehat{TE^c} \leq \widehat{TE^d}$. $\square$

Lemma 1. It is straightforward that (17) implies (18). Furthermore, suppose that (18) holds. Define

$$\mathbf{q}^m = \sum_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m,$$

for $m = 1, \ldots, M-1$, and

$$\mathbf{q}^M = \theta_t \mathbf{q}_t - \sum_{m=1}^{M-1} \mathbf{q}^m.$$

Then, by construction $\sum_m \mathbf{q}_t^m = \theta_t \mathbf{q}_t$ and $\sum_{s \in D_t^m} \lambda_s^m \mathbf{q}_s^m \leq \mathbf{q}^m$, which implies that statement (17) is satisfied. $\square$

C Data and results

Universities	Inputs			Outputs				Efficiency scores			Reallocation	
	Expenses ($ million)			Enrollment (FTE)		Research					of budget for	
	Instr	Res	A&I	Undergr	Grad	Doc	Pub	TE^d	TE^c	C^oE	Instr	Res
Arizona State University	637	243	369	55016	10479	611	33,1	1,00	1,00	1,00	1,00	1,00
Auburn University	264	131	121	19777	3137	247	14	0,92	0,78	0,85	0,71	0,93
Boston College	243	37	171	9525	3573	149	14,6	1,00	0,76	0,76	0,64	1,58
Boston University	838	194	264	20951	8692	507	35,9	1,00	1,00	1,00	1,00	1,00
Brandeis University	133	47	51	3970	2644	82	15,3	1,00	1,00	1,00	1,00	1,00
Brigham Young University-Provo	424	43	145	31364	2775	92	15,5	1,00	0,87	0,87	0,78	1,72
Brown University	254	124	180	6109	1864	232	34,5	1,00	0,76	0,76	0,46	1,38
California Institute of Technology	203	281	118	968	1286	172	56,1	1,00	1,00	1,00	1,00	1,00
Carnegie Mellon University	337	221	199	5834	5048	284	27,5	0,75	0,62	0,83	0,46	0,86
Case Western Reserve University	270	395	113	4177	2469	186	26,6	0,82	0,82	1,00	0,72	0,89
Clemson University	211	144	76	16035	3297	220	16,5	1,00	1,00	1,00	1,00	1,00
Colorado State University-Fort Collins	241	218	103	21891	4014	235	26,2	1,00	1,00	1,00	1,00	1,00
Columbia University in the City of New York	1947	671	357	8163	14946	558	58,8	1,00	1,00	1,00	1,00	1,00
Cornell University	471	378	474	14887	5463	501	52,5	1,00	0,90	0,90	0,59	1,28
Dartmouth College	153	161	321	4237	1630	73	23,1	0,76	0,66	0,86	0,76	0,55
Drexel University	299	119	313	16433	5388	163	15,2	0,92	0,85	0,92	0,92	0,67
Duke University	901	899	464	8109	6159	450	50,4	0,74	0,62	0,83	0,31	0,93
Emory University	532	409	271	8495	5699	243	35,4	0,53	0,51	0,97	0,53	0,49
Florida State University	330	160	126	31800	6824	428	24,1	1,00	1,00	1,00	1,00	1,00
George Mason University	303	80	115	19594	6547	212	16	1,00	1,00	1,00	1,00	1,00
George Washington University	398	140	269	10206	8356	224	18,9	1,00	0,97	0,97	1,04	0,80
Georgetown University	416	172	288	7440	4911	116	17,1	0,50	0,41	0,82	0,37	0,50
Georgia Institute of Technology-Main Campus	282	658	137	14517	7076	483	29,8	1,00	1,00	1,00	1,00	1,00
Harvard University	1064	769	1434	9515	13315	691	100	1,00	1,00	1,00	1,00	1,00
Indiana University-Bloomington	530	113	205	32420	8514	468	29,1	1,00	1,00	1,00	1,00	1,00
Indiana University-Purdue University-Indianapolis	399	180	229	18794	8311	35	15,9	1,00	0,91	0,91	1,31	0,02
Iowa State University	257	195	191	24128	3203	376	27,5	1,00	1,00	1,00	1,00	1,00
Johns Hopkins University	1520	1254	438	6452	13979	479	53,9	0,92	0,85	0,93	1,14	0,50
Kansas State University	211	156	92	17593	2569	162	17,1	1,00	1,00	1,00	1,00	1,00
Kent State University at Kent	180	23	106	20660	4709	142	15,2	1,00	1,00	1,00	1,00	1,00
Lehigh University	130	34	87	5239	1227	101	14,1	1,00	1,00	1,00	1,00	1,00
Louisiana State University	274	275	150	23195	4094	322	23,1	0,83	0,83	1,00	0,83	0,83
Massachusetts Institute of Technology	643	1291	618	4364	6398	573	66,4	0,58	0,58	1,00	0,58	0,58
Michigan State University	620	380	232	35003	5912	491	34,9	0,95	0,95	1,00	0,95	0,94
Montana State University	88	127	50	11314	1039	53	13,1	1,00	1,00	1,00	1,00	1,00
New York University	1202	644	417	24402	16373	417	46,3	1,00	1,00	1,00	1,00	1,00
North Carolina State University at Raleigh	386	267	162	24394	5993	446	27,3	1,00	0,97	0,97	0,78	1,26
Northeastern University	327	91	196	17934	6653	125	16,7	0,94	0,89	0,94	0,88	0,94
Northwestern University	631	426	476	9017	8331	378	48,2	0,97	0,86	0,90	0,63	1,21

Universities	Inputs			Outputs				Efficiency scores			Reallocation	
	Expenses ($ million)			Enrollment (FTE)		Research					of budget for	
	Instr	Res	A&I	Undergr	Grad	Doc	Pub	TE^d	TE^c	C^oE	Instr	Res
Ohio State University-Main Campus	922	490	402	45479	10545	756	41,8	1,00	1,00	1,00	1,00	1,00
Oregon State University	219	197	119	19003	3138	197	26,7	0,84	0,78	0,93	0,93	0,62
Pennsylvania State University-Main Campus	1170	766	618	41350	5612	629	45,5	0,82	0,52	0,64	0,24	0,96
Princeton University	381	273	312	5240	2839	351	50,4	1,00	0,94	0,94	0,41	1,69
Purdue University-Main Campus	620	268	231	31592	7276	649	35,1	1,00	0,97	0,97	0,85	1,23
Rensselaer Polytechnic Institute	142	117	83	5622	1165	136	17,7	1,00	1,00	1,00	1,00	1,00
Rice University	251	89	90	3774	2614	190	25,3	1,00	1,00	1,00	1,00	1,00
Rutgers University-New Brunswick	824	342	280	32676	6246	414	36	1,00	0,87	0,87	0,72	1,25
Saint Louis University-Main Campus	235	44	107	9175	2442	205	12,9	1,00	0,87	0,87	0,66	2,01
San Diego State University	190	4	92	23874	3294	48	18	1,00	1,00	1,00	1,00	1,00
Southern Methodist University	153	22	142	6333	2759	67	10,7	1,00	1,00	1,00	1,00	1,00
Stanford University	1200	1023	549	7485	6749	764	76,8	1,00	1,00	1,00	1,00	1,00
Stony Brook University	426	123	225	16544	5515	263	25,9	0,92	0,72	0,78	0,65	0,94
SUNY at Albany	198	231	141	12475	2900	158	16,6	0,81	0,64	0,79	0,95	0,37
Syracuse University	312	73	197	16429	3776	150	15,9	1,00	0,67	0,67	0,58	1,08
Temple University	420	110	264	26598	3828	216	17,2	1,00	0,74	0,74	0,66	1,03
Texas A & M University-College Station	591	538	200	36801	7613	663	36,5	1,00	1,00	1,00	1,00	1,00
Texas Tech University	202	137	125	23897	4232	254	14,9	1,00	1,00	1,00	1,00	1,00
The University of Montana	97	45	58	11645	1196	44	14,9	1,00	1,00	1,00	1,00	1,00
The University of Tennessee	554	289	252	19668	5289	461	24,7	0,70	0,64	0,91	0,51	0,89
The University of Texas at Austin	772	549	358	35361	9049	867	45,4	1,00	1,00	1,00	1,00	1,00
The University of Texas at Dallas	155	86	82	10549	5487	181	17,1	1,00	1,00	1,00	1,00	1,00
The University of Texas at San Antonio	160	51	109	22898	3008	77	12,9	1,00	0,94	0,94	1,08	0,49
Tufts University	212	137	288	4728	4442	143	24,5	0,73	0,70	0,95	0,73	0,65
Tulane University of Louisiana	258	157	104	8257	2642	120	17,8	0,84	0,84	1,00	0,83	0,85
University at Buffalo	390	146	244	20380	4730	305	20,5	0,80	0,74	0,91	0,69	0,86
University of Alabama at Birmingham	273	268	279	9915	4661	174	25	0,57	0,45	0,79	0,57	0,33
University of Alaska Fairbanks	104	145	69	5749	757	50	15,7	1,00	1,00	1,00	1,00	1,00
University of Arizona	430	467	297	29451	5319	446	38,3	0,92	0,84	0,92	0,73	0,94
University of Arkansas	184	125	106	18154	2704	164	14,9	0,95	0,88	0,93	1,02	0,67
University of California-Berkeley	647	629	359	27737	9680	892	68,6	1,00	1,00	1,00	1,00	1,00
University of California-Davis	652	614	378	25315	4955	566	47,2	1,00	0,81	0,81	0,38	1,27
University of California-Irvine	507	317	261	23472	5013	413	37,6	1,00	0,96	0,96	0,68	1,40
University of California-Los Angeles	1451	833	670	27911	10929	725	60,5	0,76	0,54	0,71	0,41	0,76
University of California-Riverside	212	121	93	18182	2463	263	29,3	1,00	1,00	1,00	1,00	1,00
University of California-San Diego	631	829	459	24272	5306	523	59	0,78	0,78	1,00	0,78	0,78
University of California-Santa Barbara	235	203	108	19439	3392	346	38,4	1,00	1,00	1,00	1,00	1,00
University of California-Santa Cruz	144	132	87	16220	1607	172	31,4	1,00	1,00	1,00	1,00	1,00
University of Central Florida	276	115	164	45446	5852	229	19,1	1,00	1,00	1,00	1,00	1,00

Universities	Inputs			Outputs				Efficiency scores			Reallocation	
	Expenses ($ million)			Enrollment (FTE)		Research					of budget for	
	Instr	Res	A&I	Undergr	Grad	Doc	Pub	TE^d	TE^c	C^oE	Instr	Res
University of Chicago	990	319	322	4988	7035	401	47,3	1,00	0,83	0,83	0,29	2,52
University of Cincinnati-Main Campus	304	209	223	20875	7592	242	23,5	0,94	0,78	0,83	0,94	0,55
University of Colorado Boulder	378	321	145	25523	3423	344	41,2	1,00	1,00	1,00	1,00	1,00
University of Colorado Denver	408	291	97	9746	5364	107	21,6	0,90	0,90	1,00	0,90	0,90
University of Connecticut	477	163	337	17998	4379	313	20,1	0,69	0,49	0,71	0,38	0,82
University of Delaware	346	135	141	18067	3510	228	22	0,88	0,74	0,84	0,68	0,89
University of Florida	708	609	288	32257	11226	696	39,3	1,00	1,00	1,00	1,00	1,00
University of Georgia	286	352	173	25346	7599	453	29,7	1,00	1,00	1,00	1,00	1,00
University of Hawaii at Manoa	295	350	92	13443	2811	196	29,5	1,00	1,00	1,00	1,00	1,00
University of Houston	248	115	220	26555	5017	301	22,7	1,00	1,00	1,00	1,00	0,98
University of Illinois at Chicago	664	296	173	16024	7758	342	28,7	1,00	1,00	1,00	1,00	1,00
University of Illinois at Urbana-Champaign	589	489	323	34331	12847	869	45,4	1,00	1,00	1,00	1,00	1,00
University of Iowa	389	338	235	22609	3471	437	32,6	0,96	0,89	0,94	0,52	1,32
University of Kansas	375	298	174	18874	4787	302	20,8	0,78	0,70	0,90	0,80	0,57
University of Kentucky	291	293	210	19363	3397	322	19,5	0,62	0,59	0,95	0,62	0,56
University of Louisville	272	161	135	13668	2847	185	13,3	0,68	0,66	0,96	0,70	0,58
University of Maryland-College Park	448	434	250	26510	6480	632	41,7	1,00	1,00	1,00	1,00	1,00
University of Massachusetts Amherst	349	133	138	22330	4849	268	28,4	1,00	0,96	0,96	0,91	1,09
University of Miami	459	233	278	10556	2645	181	28,4	0,51	0,40	0,80	0,35	0,51
University of Michigan-Ann Arbor	956	817	508	27287	13466	857	60,4	1,00	1,00	1,00	1,00	1,00
University of Minnesota-Twin Cities	668	739	609	30115	5621	734	49,9	0,85	0,64	0,76	0,41	0,85
University of Missouri-Columbia	319	161	137	24251	4532	367	20,1	0,98	0,98	1,00	0,99	0,97
University of Nebraska-Lincoln	206	197	118	17878	3216	246	21,9	0,88	0,79	0,89	0,96	0,62
University of New Hampshire-Main Campus	155	147	71	13351	1690	58	16,2	1,00	1,00	1,00	1,00	1,00
University of New Mexico-Main Campus	260	191	107	19167	3796	202	21,8	0,97	0,97	1,00	0,97	0,97
University of North Carolina at Chapel Hill	723	505	254	18078	6143	495	43,2	1,00	0,99	0,99	0,40	1,84
University of Notre Dame	332	116	240	9212	2828	210	21,1	0,98	0,61	0,62	0,47	1,01
University of Oklahoma Norman Campus	275	115	142	18367	3843	218	16	1,00	0,77	0,77	0,66	1,04
University of Oregon	261	90	121	20118	3323	170	18,7	0,99	0,87	0,88	0,82	0,99
University of Pennsylvania	1085	704	944	11871	10413	514	58,4	0,89	0,68	0,76	0,54	0,89
University of Pittsburgh-Pittsburgh Campus	466	690	323	18426	8473	479	42,2	1,00	0,85	0,85	1,06	0,71
University of Rhode Island	117	99	97	13280	1929	89	16,6	1,00	0,98	0,98	1,08	0,86
University of Rochester	314	316	157	6461	3473	265	30,6	0,69	0,69	1,00	0,69	0,69
University of South Carolina-Columbia	295	133	133	22619	4784	279	20,8	1,00	1,00	1,00	1,00	1,00
University of South Florida-Main Campus	310	278	169	28780	7104	270	21,1	1,00	1,00	1,00	1,00	1,00
University of Southern California	1436	392	460	18849	12851	634	38,3	1,00	0,91	0,91	0,84	1,15
University of Utah	342	285	183	21657	6114	339	34,3	0,87	0,81	0,93	0,90	0,70
University of Vermont	190	101	116	10987	861	62	15,6	0,83	0,68	0,83	0,46	1,10
University of Virginia-Main Campus	376	345	230	15513	6915	393	36	1,00	0,96	0,96	0,76	1,17

Universities	Inputs			Outputs				Efficiency scores			Reallocation	
	Expenses ($ million)			Enrollment (FTE)		Research					of budget for	
	Instr	Res	A&I	Undergr	Grad	Doc	Pub	TE^d	TE^c	C^oE	Instr	Res
University of Washington-Seattle Campus	1053	890	516	29247	12081	708	59,2	0,71	0,70	0,99	0,63	0,77
University of Wisconsin-Madison	512	931	252	27872	6270	813	52	1,00	1,00	1,00	1,00	1,00
University of Wyoming	165	90	83	9300	1462	72	14,6	0,90	0,85	0,94	0,94	0,69
Utah State University	167	134	87	20080	2015	94	13,7	1,00	1,00	1,00	1,00	1,00
Vanderbilt University	843	439	191	6814	3880	273	35,1	0,57	0,57	1,00	0,57	0,57
Virginia Commonwealth University	338	148	137	21703	5080	333	21	1,00	0,97	0,97	0,94	1,03
Virginia Polytechnic Institute and State University	306	330	138	24847	5887	469	25,1	1,00	1,00	1,00	1,00	1,00
Wake Forest University	184	171	619	4639	1625	57	17,4	0,63	0,58	0,91	0,63	0,52
Washington State University	234	226	151	21399	4487	203	21	1,00	0,89	0,89	1,24	0,52
Washington University in St Louis	1254	495	269	6934	3565	251	44,9	1,00	0,94	0,94	0,45	2,17
Wayne State University	316	187	167	15873	4821	229	19,2	0,90	0,81	0,91	0,92	0,63
Yale University	1288	506	513	6863	6370	390	60	1,00	0,70	0,70	0,45	1,35
Yeshiva University	221	257	159	2593	1463	129	21,1	0,55	0,55	1,00	0,55	0,55

Table 3: Data on US universities in 2012

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