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Decision theory without finite standard expected value

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Decision Theory without Finite Standard Expected Value

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We address the question, in decision theory, of how the value of risky options (gambles) should be assessed when they have no finite standard expected value (i.e. where the sum of the probability-weighted payoffs is infinite or not well defined). We endorse, combine, and extend (1) the proposal of Easwaran (2008) to evaluate options on the basis of their weak expected value, and (2) the proposal of Colyvan (2008) to rank options on the basis of their relative expected value.

Our goal is to outline a framework rather than to give a compelling defense of it. We shall motivate, through the use of examples, the plausibility of principles that go beyond standard expected value. Although the principles we endorse leave some options incomparable, we believe that they are, at least roughly speaking, the strongest plausible extensions of standard decision theory.

1. The Problem

We address the question in decision-theory of how (risky) options should be evaluated when they have no finite standard expected value. In this section, we shall define these terms and explain the problem.

Throughout, we restrict our attention to cases where an option determines a countable set $\{\langle p_1,v_1\rangle, \langle p_2,v_2\rangle, \ldots, \langle p_k,v_k\rangle,\ldots\}$, where p_k is the probability of receiving a payoff of v_k. We thus do not address cases where there are uncountably-many distinct finite payoffs with non-zero probability. We further assume that the values of outcomes are independently specified (e.g., monetary values or wellbeing values) on an interval scale. Thus, we assume that there is a fact about whether the difference in value between two outcomes is less, equal, or greater than the difference in value between two other outcomes. We leave open what kind of value (e.g., prudential vs. moral) is used, since that may vary based on the nature of the decision problem.

We further assume that the evaluation of options is risk neutral. Thus, we assume that, in the standard cases, risky options are evaluated on the basis of their standard expected value, that is, on the basis of the probability-weighted sum $p_1v_1 + p_2v_2 + \ldots + p_nv_n + \ldots$ The problem, as we shall explain, is that sometimes this sum is not well defined. The usual assumption is that such options cannot be evaluated. We shall endorse principles proposed by Easwaran (2008) and Colyvan (2008) and then combine and extend those principles.

The sum of a set of numbers is well-defined just in case the order of summation does not matter, that is, just in case each order of summation results in the same total. A crucial mathematical fact is that the *sum of a (countable) set of numbers is well defined* if and only if either the sum of the positive terms is finite (or there are no such terms) or the sum of the negative terms is finite (or there are no such terms). If both sums are finite, the sum of the entire set is simply the sum of these two finite numbers. If one sum is finite, and the other sum is (positively or negatively) infinite, the sum of the entire set is well defined and correspondingly infinite. If both sums are infinite, then there is no well-defined sum. For such sets the order of summation matters.

For example, for the set {… –1/32, –1/8, –1/2, 1, 1/4, 1/16, …}, the sum of the positive numbers (1+1/4+1/16+…) is 4/3 and the sum of the negative numbers (–1/2–1/8–1/32–…) is –2/3. This establishes that the above set of values has a well-defined sum equal to 2/3 (=4/3–2/3). For the set {–1/2, 1, 1/3, 1/5, …, 1/(2n+1), …}, the sum of the negatives is finite (–1/2) and the sum of the positives is infinite. Hence, this set has a well-defined sum, which is positive infinity. By contrast, for the set { …–1/(2n),…, –1/6, –1/4, –1/2, 1, 1/3, 1/5, …, 1/(2n+1), …}, the sum of the negatives is infinite, as is the sum of the positives, and hence this set has no well-defined sum.

Consider now the Pasadena game, introduced by Nover and Hájek (2004)[1]: a fair coin is flipped until a heads comes up, and one wins something if the number of flips is odd and loses something if the number of flips is even. More precisely, the payoffs, along with the associated probabilities are described as follows:

Pasadena

Positive payoffs: < 1/2, 2/1>, < 1/8, 8/3>, …, < $1/2^{2n-1}$, $2^{2n-1}/(2n-1)$>, ….

Negative payoffs: < 1/4, –4/2>, < 1/16, –16/4>, …, < $1/2^{2n}$, $-2^{2n}/2n$>, …

The set of probability-weighted payoffs is thus {…–1/2n, ….–1/6, –1/4, –1/2, 1, 1/3, 1/5, … 1/(2n–1) …}. Given that the sum of the negatives is infinite and the sum of the positives is infinite, this set has no well-defined sum. Nonetheless, it seems natural to add these terms in the order of how many flips it takes for a heads to occur and thereby realize the payoff: 1 –1/2 + 1/3 –1/4, …+1/(2n–1) –1(1/2n) …. Added in this order, the sum is log(2), or approximately .69. The problem is that, where a set of values has no well-defined sum, adding the terms in a different order can give a different sum. For example, adding the very same terms above in the following order (1+1/3–½)+(1/5+1/7–¼)+(1/9+1/11–1/6)+ … produces a total of 1.5 log(2), or approximately 1.04. The sum is not well-defined precisely because there is no order-independent fact about what the terms sum to.

The general problem that we address is the evaluation of risky options for which there is no standard expected value, defined as follows:

Standard Expected Value (definition)**:** The *standard expected value* of an option exists and has value v (which can be infinite) if and only if the sum of the (countable many) probability-weighted payoffs is well defined and equals value v.

Throughout, we allow that the value of an option can be infinite. Consider, for example, the well-known St. Petersburg game, where a fair coin is flipped until it lands on heads, and, if that takes n flips, the payoff is 2^n. This determines the probability-payoff set {<1/2, 2>, <1/4.>, ….< $1/2^n$, 2^n>, …}. This has a standard expected value equal to positive infinity, since the sum of the probability-weighted positive payoffs is infinite, and there are no negative payoffs.

It is important to remember that having an infinitely positive value only establishes that the option is more valuable than any option with a finite value. Two options with infinite positive value need not be equally valuable. For example, if the payoffs of the St. Petersburg game are increased by one unit, the result is an infinitely valuable option that is arguably more valuable than the original, and certainly not equally valuable with it.

Throughout, we shall assume the following relatively uncontroversial claim, given our assumption of risk-neutrality:

Standard Expectations: The value of an option is its standard (finite or infinite) expected value, if it exists.

We shall now formulate and endorse some principles of evaluation when options have no standard expected value. More specifically, we shall combine and extend the weak expectations principle advocated by Easwaran (2008) and the relative expectations principle advocated by Colyvan (2008).

2. Weak and Strong Expectations

Kenny Easwaran (2008) has tentatively suggested that the Pasadena game (above) has a value of log(2) on the following basis: (1) He distinguishes between weak expected value and strong expected value (defined below). (2) He establishes that the Pasadena game has a weak expected value of log(2). (3) He tentatively suggests that, in general, risky options can be assessed on the basis of their weak expected value.

Consider an option, X, and let E(X) be its standard expected value. Easwaran appeals to two versions of the law of large numbers to define weak and strong expected value for an option. Let Ave(X,n) be the average value of X for n independent trials. The two laws are:

Strong Law of Large Numbers: For any option, X for which E(X) exists, there is a probability of 1 that the limit, as n goes to infinity, of |Ave(X,n)–E(X)| is 0.

Weak Law of Large Numbers: For any option, X for which E(X) exists, for any positive number, e, the limit, as n goes to infinity, of the probability that |Ave(X,n)–E(X)| < *e* is 1.

Both laws concern the probability of the standard expected value and the sample average value being arbitrarily close to each other. The difference between the two laws concerns whether the limit, as the sample size goes to infinity, is internal to the probability assignment (the strong law) or external to it (the weak law).

These two laws define the weak and strong expected value as follows:

Finite Strong Expected Value (definition): The *strong expected value* of an option, X, exists and has finite value v if and only if there exists a real number v such that, with probability 1 the limit, as n goes to infinity, of |Ave(X,n) – v| is 0.

Finite Weak Expected Value (definition): The *weak expected value* of an option, X, exists and has finite value v if and only if there exists a real number v such that, for each positive number *e*, the limit, as n goes to infinity, of the probability that |Ave(X,n) – v| < *e* is 1.

In each of these definitions, the stochastic behavior of the sample average, Ave(X,n), is used to define a value for X. The strong expected value of X is the value v for which there is *a probability of 1 that, for large enough sample sizes,* the average value Ave(X,n) will be arbitrarily close to v. The weak expected value, by contrast, is the value v for which*, for large enough sample sizes, there is a probability arbitrarily close to 1* that the average value will be arbitrarily close to v. Whenever the strong expected value is defined, the weak expected value is also defined and has the same value.

It turns out that, in the finite case, strong expected value just is standard expected value (although we shall see that this is not so in the infinite case):

Finite Strong Expected Value Lemma (based on Durrett 2005, Ch. 1, sec. 8)**:** An option has a finite strong expected value of v if and only if it has a standard expected value of v.

Thus, the following is equivalent to the uncontroversial Finite Standard Expectations:

Finite Strong Expectations: The value of an option is its strong expected value, if it exists and is finite.

Because Pasadena has no finite standard expected value, it has no finite strong expected value. Although the existence of a finite strong expected value entails the existence of a finite weak expected value, the reverse entailment does not hold. Indeed, as Easwaran shows, drawing on Feller (1971) and Durrett (2005), Pasadena has a weak expected value of log(2), even though it has no finite strong expected value.

More generally, necessary and sufficient conditions for the existence of finite weak expected value is provided by the following lemma, where X_n is the same as X, except that all payoffs with absolute values above n are set equal to zero. For example, for Pasadena, X, X_2 sets all absolute payoffs above 2 equal to 0 and thus its probability-payoff pairs are <1/2, 2>, <1/4, –4/2>, and <$1/2^n$, 0>, for n>2. The standard expected value of X_n, $E(X_n)$, is always well defined, given that truncated payoffs are bounded.

Finite Weak Expected Value Lemma (Feller 1971, chapter VII, Thm. 1 and Durrett 2005, Chapter 1, 5.5)**:** Option X has a finite weak expected value, v, if and only if (1) X has *thin tails*, i.e., x multiplied by Pr(|X|>x) converges to zero as x goes to positive infinity, and (2) the limit of $E(X_n)$, as n goes to positive infinity, is v.

The first condition requires that the limit of x multiplied by Pr(|X|>x) go to zero, as x goes to infinity, where Pr(|X| > x) is the probability that the absolute value of the payoff, X, is greater than x. For example, for Pasadena, Pr(|X| >2) = ¼ (since Pr(X=2)=1/2 and Pr(X=–4/2) = ¼, and all other payoffs have an absolute value greater than 2), and 2 multiplied by [Pr(|X|>2)] is 2x1/4, or ½.

To determine the weak expected value on the basis of this lemma, for each payoff level n, one truncates the option X at level n, determines the strong expected value of this truncated variable, and then takes the limit as n goes to infinity. Although the Pasadena game has no standard expected value, it has a weak expected value of log(2). To see this, note that the absolute payoffs of the Pasadena have the form $2^n/n$, and thus condition (1) is equivalent to the requirement that $(2^n/n)\Pr(|X|>2^n/n)$ converge to zero, as n goes to infinity. For Pasadena, $\Pr(|X|>2^n/n) = 1/2^n$, and thus (1) requires that $(2^n/n)x(1/2^n)$, or 1/n, converge to zero as n goes to infinity. Hence, condition (1) of the Lemma is satisfied. To see that condition (2) also is met, note that E(Xn) = 1–1/2+1/3+…+ $(-1)^n/n$.[2] Thus, the limit of E(Xn) is log(2) as n goes to infinity. Hence, condition (2) of the definition is met, and Pasadena has a weak expected value equal to log(2). It's worth noting that the lemma ensures that the weak expected value is based on the summation of probability-weighted payoffs in increasing order the absolute value of the payoff. This is in contrast to Nover and Hájek (2004), who suggest that no particular order of summation is privileged.

If a risky option has a finite standard expected value, then it has a finite weak expected value, and both values coincide. This follows from the fact that the standard expected value is finite only if the sum of the probability-weighted payoffs is finite for the both the positive, and the negative, payoffs. That in turn implies that (1) X has thin tails and (2) the sum of the probability-weighted payoffs does not depend upon the order of summation (which ensures the sum in increasing order of the absolute value of the payoffs gives the correct answer).Thus, the concept of weak expected value is a strengthening of standard expected value.

Easwaran tentatively proposes that Finite Strong Expectations be strengthened to the following principle:

Finite Weak Expectations: The value of an option is its weak expected value, if it exists and is finite.

As Easwaran notes, a player who plays a game a very large number of times at a price that is slightly higher (respectively: lower) than the weak expectation has a very high probability of ending up behind (respectively: ahead). Indeed, by repeating the game enough times, that probability can be made as close to 100% as one likes.

Although Finite Weak Expectations is not uncontroversial[3], we find it compelling.[4] In the remainder of the paper we shall strengthen it in various ways.[5]

3. Infinite Expectations

Consider the following option:

Squared St. Petersburg–Pasadena

Positive payoffs: < 1/2, 4>, <1/8, 2^6>, <1/32, 2^{10}>,…, <$1/2^{2n-1}$, 2^{4n-2}>, ….

Negative payoffs: <1/4, –2>, <1/16, –4>, <1/64, –64/6>, …, <$1/2^{2n}$, $-2^{2n}/2n$>, …

Here, the positive payoffs are the squares of the St. Petersburg payoffs, and the negative payoffs are the same as Pasadena. Both the positive and the negative parts have infinite totals. Hence, there is no standard expected value. Nevertheless, this option has infinite strong expected value, defined as follows:

Infinite Strong Expected Value (definition): Option X has infinitely positive (respectively: negative) strong expected value if and only if, for each positive number, *e*, with probability 1, for all sufficiently large n, Ave(X,n) > *e* (resp. < –*e*).

This is the same as the definition of *finite* strong expected value, except that it requires that Ave(X,n) become arbitrarily large in absolute value rather than arbitrarily close to some finite value.

An application of the following lemma establishes that the above option has an infinite strong expected value, even though it has no standard expected value:

Infinite Strong Expected Value Lemma (Derman and Robbins, 1955). An option, X, has a positively (respectively: negative) infinite strong expected value if: (1) for some a and b, 0 < a <b < 1, and some c, c > 0, for all sufficiently large x, [x^a multiplied by Pr(X+ > x)] > c (resp.: [x^a multiplied by Pr(|X–| > x)] > c), and (2) E($|X–|^b$) (resp.: E($|X+|^b$) is finite.

We state this lemma because it is useful for understanding when the strong expected value is infinite, but we won't appeal to it below. In order to avoid the need to explain some technical complexities, we simply assert, without explanation, that the lemma entails that the above option has infinite strong expected value, even though it has no standard expected value.

The crucial point here is that, although finite standard expected value and finite strong expected value always are the same, infinite strong expected value can exist when there is no standard expected value (but not vice-versa).

We believe that Standard Expectations can be strengthened to the following plausible principle:

Strong Expectations: The value of an option is its strong (finite or infinite) expected value, if it exists.

Indeed, we believe that a further strengthening is plausible. Consider:

Weakly Infinite Pasadena

Positive payoffs: <1/2, (2/1)x1.01>, <1/8, (8/3)x1.01>, <1/32, (32/5)x1.01>,…, $<1/2^{2n-1}, (2^{2n-1})\text{x}1.01/(2n-1)>$, ….

Negative payoffs: <1/4, –4/2>, <1/16, –16/4>, <1/64, –64/6>, …, $<1/2^{2n}, -2^{2n}/2n>$, …

This is the same as Pasadena, except that the positive payoffs are multiplied by 1.01.

Corollary 1 of Erickson (1973) establishes that the above option has no strong expected value. Showing this, however, would be complex, and we omit that demonstration.[6]

Although Weakly Infinite Pasadena has no strong expected value, it has infinite weak expected value, defined as follows:

Infinite Weak Expected Value (definition): Option X has infinite weak expected value if and only if, for each positive number, *e*, and each positive number strictly between 0 and 1, d, for all sufficiently large n, the probability that Ave(X,n) > *e* (resp. < –*e*) > 1–d.

This is the same as the definition of *finite* weak expected value, except that it requires that Ave(X,n) become arbitrarily large in absolute value rather than arbitrarily close to some finite value. It is like the definition of infinite *strong* expected value, except that it requires that, for all sufficiently large sample sizes, the probability of the average being greater than any given value *converges to 1* rather than that there be *probability 1* that, for all sufficiently large sample sizes, the average is greater than any given value. Whenever the strong expected value (finite or infinite) exists, the weak expected value exists and has the same value. Weak expected value (finite or infinite), however, can exist without strong expected value existing.

To see that the above option has infinite weak expected value, we can appeal to the following lemma:

Infinite Weak Expected Value Lemma 1 (Durrett 2005, Ch. 1, sec. 8)**:** An option, X, has a positively (respectively: negative) infinite weak expected value if: (1) X has *thin tails*, i.e., x multiplied by Pr(|X|>x) converges to zero as x goes to positive infinity, and (2) the limit of $E(X_n)$ is positively (resp: negatively) infinite.

This lemma is similar to the corresponding one for *finite* weak expected value, except (1) it requires that the limit of $E(X_n)$ be infinite, and (2) it supplies only a sufficient condition for weak expected value.

The above option has thin tails and the limit of $E(X_n)$ is infinite. Thus, it has infinite weak expected value.

It's worth noting that this lemma can be supplemented with:

Infinite Weak Expected Value Lemma 2 (Baum 1963, Theorem 2). An option, X, has a positively (respectively: negative) infinite weak expected value if: for some a, $0 < a < 1$, (1) x^a multiplied by Pr(X+ > x) [resp. x^a multiplied by Pr(|X–| > x)] goes to infinity, as x goes to infinity, and (2) $E(|X–|^a)$ (resp.: $E(|X+|^a)$ is finite.

Compared with the first lemma, this strengthens the first condition by appealing to x^a rather than to x, and it replaces the requirement that the limit of $E(X_n)$ be infinite with the requirement that $E(|X–|^a)$ be finite. We mention this lemma because it is useful for understanding when weak expected value can be infinite, but we won't appeal to it below.

We believe that Strong Expectations can plausibly be strengthened to:

Weak Expectations: The value of an option is its weak (finite or infinite) expected value, if it exists.

Of course, those who reject Finite Weak Expectations will also reject the infinite version, as will those who reject infinite value even for standard expected value. We believe, nonetheless, that Weak Expectations is plausible and shall assume it below.

Not all options, of course, have a (finite or infinite) weak expected value. The following is an example of one that does not.

Symmetric St. Petersburg (SP*)

Positive payoffs: $< 1/4, 4>, < 1/8, 8>, \ldots, <1/2^{n+1}, 2^{n+1}>, \ldots.$

Negative payoffs: $< 1/4, -4>, < 1/8, -8>, \ldots, <1/2^{n+1}, -2^{n+1}>, \ldots.$

This is like the St. Petersburg, except that it has negative payoffs defined to be symmetric with the positive payoffs.

The Finite Weak Expected Value Lemma above establishes that this does not have *finite* weak expected value (because it does not have thin tails). Moreover, for each n, the states s_{2n-1} and s_{2n} have the very same probability but the opposite value (-2^{n+1} versus 2^{n+1}). It follows that, for any positive n and k, the probability that Ave(SP*,2n) is larger than k is equal to the probability that Ave(SP*,2n) is smaller than –k. Hence, the option SP* does not have an infinitely positive, or infinitely negative, weak expected value.

Weak Expectations makes cardinal assessments of options when both options have a weak expected value with at least one value being finite. We shall now introduce some further principles for assessing options where (1) both options have infinite value, or (2) at least one option has no weak expected value.

4. Indeterminate Expectations

The weak expected value of an option has finite value k just in case the probability that its average is arbitrarily close to k converges to one, as the sample size goes to infinity. We shall now generalize this notion to include *interval* assessments of weak expected value (e.g., a value of between 2 and 6 units).

To see the need for strengthening the weak expectations principle, consider the following option, O, which is the same as Pasadena except that, for n>1, the sign of the payoff for s_n is (1) the same as the preceding payoff when this is compatible with $E_n(O)$ being inclusively between 0 and 1, and (2) the opposite sign when this the same sign not so compatible. Thus, for example:

Oscillating Weak Expected Value

Probability:	1/2	1/4	1/8	1/16	1/32	1/64	1/128	1/256	…
State:	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	…
O	2	–4/2	–8/3	16/4	32/5	64/6	128/7	–256/8	…

Option O has no (finite or infinite) weak expected value. Although O has thin tails, the sequence of the truncated expected values, $E(O_n)$, oscillates, as n goes to infinity, between 0 and 1. (The specification of the signs of the payoff ensures exactly this.) Thus, Weak Expectations is silent. Nonetheless, it is plausible, we claim, that O has a value of at least 0 and at most 1. We shall suggest that it has an "interval value" of [0,1].

In order to introduce the concept of interval value, we need to first introduce the following two standard mathematical concepts. The *greatest lower bound* of a set of numbers is (a) the largest (finite) real number that is a lower bound for (i.e., smaller or equal to) all members of the set, if a lower bound exists, and (b) negative infinity, if there are no lower bounds. For example, 2 is the greatest lower bound for {2,3,4, ,, n,….}, and –∞ is the greatest lower bound for {…, –n, …–4, –3, –2}. The *least upper bound* of a set of numbers is (a) the smallest (finite) real number that is an upper bound for (i.e., greater or equal to) all members of the set, if there is an upper bound, and (b) positive infinity, if there are no upper bounds. For example, –2 is the least upper bound for {…, –n, …–4, –3, –2}, and ∞ is the least upper bound for {2,3,4, ,, n,….}.

We can now define the weak expected *interval* value of an option:

Weak Expected Interval Value (definition): The weak expected interval value of option X is the closed interval $[x_1,x_2]$, where: (1) x_1 is the greatest lower bound on the value of k for which lim pr(Ave(X,n) ≥ k) = 1, and (2) x_2 is the smallest upper bound on the value of k for which lim pr(Ave(X,n) ≤ k) = 1.

All options have a weak expected interval value. An option with finite weak expected value of k has an interval value of [k,k], and an option with infinite weak expected value has an interval value of [∞,∞]. An option with a completely indeterminate weak expected value (e.g., Symmetric St. Petersburg, from the previous section) has an interval value of [–∞,∞]. Option O, above, has an interval value of [0,1]. It is thus worth more than any negative value, and no more than 1 unit of value.

This provides the basis for the evaluation of options on the basis of the following plausible principle:

Ordinal Interval Weak Expectations: Option, X, with weak expected interval value $[x_1,x_2]$, is at least as valuable as option, Y, with weak expected interval value $[y_1,y_2]$, if $x_1 \geq y_2$.[7]

Here we stipulate, as is standard for the extended reals, that (1) for any finite number, n ∞ > n > –∞, and (2) neither ∞ ≥ ∞, nor –∞ ≥ –∞.

Thus, if X's greatest lower bound is at least as great as Y's least upper bound, then X is at least as valuable as Y. For example, X with [3,4] is at least as valuable as Y with [2,3], but Y is not at least as valuable as X (since it's not the case that $2 \geq 4$). Of course, for some options, Ordinal Interval Weak Expectations is silent. For example, it makes no comparative assessment of W with [3,10] with Z with [4,6].

This principle can be plausibly strengthened to say *how much* more valuable one option is than another. Consider then the following principles where it is stipulated that (1) $\infty+n=\infty$, for n finite or ∞, (2) $-\infty+n=-\infty$, for n finite or $-\infty$, and (3) $\infty+-\infty$ is not defined:

Cardinal Interval Weak Expectations: For option, X, with weak expected interval value [x1,x2], and option, Y, with weak expected interval value [y1,y2], X is *not less than* (x1–y2) units, *and not more than* (x2–y1) units, more valuable than Y.

For example, for X with weak expected interval value [4,5] and Y with weak expected interval value [2,3], X is not less than 1 unit more valuable than Y and not more than 3 units more valuable. Moreover, for Z with weak expected interval value of [2,6], X is at least –2 units more valuable (i.e., 2 units less valuable) than Z and not more than 3 units more valuable. That is, X is not determinately more valuable than Z, but there are limits on how much worse (–2) and how much better (3) it is.

It's important to keep in mind here that Cardinal Interval Weak Expectation sets lower and upper bounds on the difference in value between two options, but it does not claim that these bounds are the *greatest* lower bounds or the *least* upper bounds. Thus, for example, the claim that X is at least –2 units more valuable and at most 3 units more valuable than Z is compatible with the claim that X is exactly 1 unit more valuable than Z (but not with the claim that it is exactly 4 units more valuable). Indeed, many of the relatively indeterminate assessment of

Cardinal Interval Weak Expectations will be strengthen to more precise assessment by the principles of the next section.

With respect to options with a weak expected value, Cardinal Interval Weak Expectation has the same implications as Weak Expectations. An option has a weak expected value just in case its weak expected value interval has the form [k,k], for some k (finite or infinite). For any two such options, say with intervals [k,k] and [l,l] the first is k–l to k–l units at least as valuable as the second, which is to say k–l units more valuable, as required.

Cardinal Interval Weak Expectations makes, however, many comparative judgments that Weak Expectations does not. For example, as noted above, it judges X, with weak expected interval value [4,5], to be at least 1 unit, and not more than 3 units, more valuable than Y, with weak expected interval value [2,3], even though neither option has a weak expected value.

In the following sections, we examine some principles for addressing two kinds of cases for which Cardinal Interval Weak Expectations makes no non-empty comparative assessment. One kind of case is where the subtractions involved are not well defined. This occurs where the interval values are [x1,x2] and [y1,y2], where both x1 and y2, or both y1 and x2, are positively infinite, or both negatively infinite (e.g., [–1,∞] compared with [∞,∞], or [–∞,1] compared with [–∞,–∞]. The second kind of case is where the assessment is radically indeterminate in the sense that the assessment is that one option is at least –∞, and at most ∞, units more valuable than the other. This may be true, but it is completely uninformative. It occurs where (1) both options have ∞ as their greatest interval value but not as their lowest interval value, or (2) both options have –∞ as their lowest interval value but not as their highest interval value (e.g., [2,∞] compared with [1,∞]). As we shall see, some additional principles make comparative assessments possible in some of these cases.

5. An Interlude on State Spaces

The weak, strong, and standard expected values of an option are determined by its payoff distribution, $\{<p_1,v_1>, <p_2,v_2>, \ldots, <p_k,v_k>,\ldots\}$, where p_k is the probability of receiving a payoff of v_k. Options, however, have more structure than this, and the principles that we introduce below (unlike those introduced above) depend on this structure. We shall now make that structure explicit.

Options are defined over state spaces. A *state space* is a set of basic states, along with an associated probability function over certain specified subsets of those states (the events). Because we restrict our attention to *countable* state spaces (i.e., with countably-many basic states) and *countably additive* probability functions (i.e., the probability of a union of a countable number of basic states is the sum of their individual probabilities), a state space can be represented by a countable set of couples $\{<s_1,p_1>, <s_2,p_2>,\ldots, <s_i,p_i>, \ldots\}$, where s_i is a basic state, p_i is the positive probability with which s_i occurs, and the p_i sum to one.[8]

An option is a real-valued function that assigns to each basic state (of its associated state space) the value of the outcome of the option, under that state, expressed in units of the relevant value (here left open).[9] An option can thus be represented by a countable set of triples $\{<s_1,p_1,v_1>, <s_2,p_2,v_2>, \ldots, <s_i,p_i,v_i>, \ldots\}$, with s_i and p_i as above, and with v_i the value (or payoff) of the outcome of the option under state s_i.

Above, in discussing weak and strong expected value, we simplified this by combining states with the same payoff (and adding together the associated probabilities). The remaining principles, however, require the fuller state-space specification.

We shall now address some additional principles for assessing options defined on the same state space.

6. Relative Expectations

If two options each have weak expectations, with at least one of them finite in value, then Weak Expectations determines how much more valuable one is compared with the other. If the two options do not have any weak expectation but each has a *weak expected interval value*, then Cardinal Interval Weak Expectations will typically give an assessment of some lower and upper limits on how much more valuable one option is compared with the other (e.g., at least 2 and at most 4 units more valuable).

As indicated above, however, there are two cases where this is not so. One is where both options have positively infinite, or both have negatively infinite, weak expected value. In this case, Cardinal Interval Weak Expectations is silent, for example, because comparing [∞, ∞] with [∞, ∞] involves ∞–∞ and that is undefined. The second kind of case is where both options have a weak expected interval values of the form [n,∞], or both have the form [–∞,n], where n can be finite or infinite and need not be the same for the two options. In this case, Cardinal Interval Weak Expectations is judges each option to be –∞ to ∞ units more valuable than the other, and that is true but uninformative. We shall now add a principle, appealing to the state space structure, that covers some (but not all) of these cases.

A very plausible principle, which we shall strengthen below, is:

Strong Dominance: If, (1) X and Y are options defined on the same state space, (2) for each basic state, X has a payoff that is at least as great as that of Y, and (3) for some basic states, X has a greater payoff than Y, then X is more valuable than Y.

Strong Dominance is highly plausible but very weak. Consider SP (St. Petersburg) and SP':

Relative Expectations

Probability:	1/2	1/4	1/8	1/16	1/32	…	$1/2^n$	…
State:	s_1	s_2	s_3	s_4	s_5	…	s_n	…
SP	2	4	8	16	32	…	2^n	…
SP'	1.8	5	9	17	33	…	2^n+1	…
SP'–SP	–.2	1	1	1	1	…	1	

For each state other than s_1, SP' has payoff that is one unit greater than SP, but for s_1 SP' has a payoff that is .2 lower. Thus, there is no strong dominance here. Nonetheless, it is extremely plausible that SP' is more valuable than SP.

The following principle from Colyvan (2008) captures this assessment, where X–Y is an option that, for each state, has a payoff equal to the payoff of X less the payoff of Y:

Relative Expectations (Sufficiency Version): If (1) X and Y are options defined on the same state space, and (2) X–Y has a standard expected value that is nonnegative, then X is at least as valuable as Y.[10]

SP and SP' each have infinite standard expected value. Nonetheless, the standard expectation of (SP'–SP) is positive (.4 = –.1 + .5 =1/2 x –.2 + ¼ x 1 + 1/8 x1 …). Thus, Relative Expectations rightly says that SP' is more valuable than SP.

Colyvan (2008) endorses a stronger version of this principle, which also holds that X is at least as valuable as Y *only if* the standard expected value of X–Y exists and is non-negative. We believe that this is too strong. We believe that the relative value of two options can be assessed in many cases where this condition fails. We show this below by showing that stronger versions of the principles are plausible.

First, it is plausible to endorse a cardinal version of the principle:

Cardinal Relative Expectations: If (1) X and Y are options defined on the same state space, (2) the standard expected value of X–Y is n (which can be infinite), then X is n units more valuable than Y.

This is just like the previous principle, except that it specifies *how much* more valuable on option is than the other. It says, for example that SP' is .4 units more valuable than SP.

It may seem confused to hold that one infinitely valuable option is n units more valuable than another infinitely valuable option, but it is not. For *finitely valuable* options, X and Y, if X is n units more valuable than Y, then the value of X is n units higher than the value of Y. This relationship, however, does not hold for infinitely valuable options. To say that an option is infinitely valuable is not to assign it a specific value. It is merely to say that it is more valuable than any finitely valuable option. Infinitely valuable options need not be equally valuable (and usually are not). Often they are incomparable, but sometimes one is more valuable another (e.g., when one dominates the other). Moreover, it is sometimes possible to say how much more valuable one infinitely valuable option is compared with another. To say that X is n units more valuable than Y is just to say that X is equally valuable with the option obtained by increasing, for each state, Y's payoffs by n units. For example, above, SP' is .4 units more valuable than SP, in the sense that it would be worth paying .4 units to exchange SP for SP'. There is no claim that adding .4 to infinity is somehow greater than infinity. Once that is understood, there is no incoherence.

To see the need for the second strengthening, consider:

Relative WEV

Probability:	1/2	1/4	1/8	1/16 …	$1/2^{2n-1}$	$1/2^{2n}$ …
State:	s_1	s_2	s_3	s_4 …	s_{2n-1}	s_{2n} …
SP	2	4	8	16 …	2^{2n-1}	2^{2n} …
Q	2+2	4–4/2	8+8/3	16–16/4 …	$2^{2n-1}+(2^{2n-1})/(2n-1)$	$2^{2n}-(2^{2n})/2n$ …
Q–SP	2	–4/2	8/3	–16/4 …	$(2^{2n-1})/(2n-1)$	$-(2^{2n})/2n$ …

In this example, for Q–SP (which is just Pasadena), the sum of the probability-weighted payoffs is infinite for both the positive payoffs and for the negative payoffs. Thus, Q–SP has no standard expected value, and Cardinal Relative Expectations is silent. Note, however, that Q–SP has a *weak expected value* of log(2). We believe that that is sufficient to evaluate Q as being log(2) units more valuable than SP.

More generally, we believe that the following principle is plausible:

Cardinal Relative WEV: If (1) X and Y are options defined on the same state space, and (2) the *weak expected value* of X–Y is n, then X is n units more valuable than Y.

This principle replaces the appeal to the standard expected value of X–Y with an appeal to the weak expected value of X–Y. It rightly assesses Q as log(2) units more valuable than SP. (Cardinal Relative WEV will often give more determinate assessments of the comparative value of two options than does Cardinal Interval Weak Expectations. This is not a conflict. It is simply a case of being more determinate.)

In the above example, the two options each have infinite weak expected value. A similar example could be given where neither option has a weak expected value but their difference does (e.g., the same difference as Q–SP above).

To see the need for one final strengthening of the relative expectations principle, consider the following example:

Oscillating Differences

Probability:	1/2	1/4	1/8	1/16	1/32	1/64	1/128	1/256	… $1/2^n$ …
State:	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	… s^n …
SP	2	4	8	16	32	64	128	256	… 2^n …
U	$2+a_1$	$4+a_2$	$8+a_3$	$16+a_4$	$32+a_5$	$64+a_6$	$128+a_7$	$256+a_8$	… 2^n+a_n …
U–SP	2	$-2^2/2$	$-2^3/3$	$2^4/4$	$2^5/5$	$2^6/6$	$2^7/7$	$-2^8/8$	… a_n …

Here (1) SP is St. Petersburg, (2) U is defined to have the same payoffs as SP but increased by a_n for state s_n, and (3) the a_i are the same as the Pasadena payoffs for state s_i, except that, for n>1, the sign of the payoff for a_n is (1) the same as a_{n-1}, when this is compatible with $E([U–SP]_n)$ being inclusively between 0 and 1, and (2) the opposite sign when the same sign not so compatible. This ensures that $E([U–SP]_n)$ oscillates between 0 and 1, as n goes to infinity. (The set up here is similar to that of Oscillating Weak Expected Value in the section on indeterminate value.)

Here, SP and U each have infinite weak expected value, but SP–U has no weak expected value, given that $E([U–SP]_n)$ does not converge as n goes to infinity. Thus, Cardinal Relative WEV is silent. Nonetheless, it is plausible, we claim, that U is 0 to 1 units more valuable than SP. Indeed, note that, although U–SP has no weak expected value, it does have a weak expected interval value of [0, 1], where (as defined above) this means that 0 is the greatest lower bound on the value of k for which lim pr(Ave(X,n) ≥ k) = 1, and (2) 1 is the smallest upper bound on the value of k for which lim pr(Ave(X,n) ≤ k) = 1.

Consider, then, the following principle, where to say that X is *m to n units more valuable* than Y is to say that (1) m is the greatest lower bound on the value by which the value of X exceeds that of Y, and (2) n is the least upper bound on the value by which the value of X exceeds that of Y:

Cardinal Relative Interval WEV: If X and Y are options defined on the same state space, and X–Y has a weak expected interval value of [m, n], then X is m to n units more valuable than Y.

As indicated above, when m is negative, to say that X is m units more valuable than Y is to say that X is at least as valuable as the option obtained by decreasing payoffs by |m| units.

Cardinal Relative Interval WEV is equivalent to Cardinal Relative WEV in the special case where X–Y has a weak expected interval value [k, k], for k finite or infinite. Moreover, Cardinal Relative Interval WEV is equivalent to Cardinal Interval WEV in the special case where X and Y each have a weak expected interval values of [x1,x2] and [y1,y2], where: (1) all the subtractions are well defined: x1 and y2 are not both positive infinity and not both negative infinity, and likewise for y1 and x2, and (2) the resulting interval value is not [–∞,∞]: x2 and y2 are not both positively infinite x1 and y1 are not both negatively infinite.

Cardinal Relative Interval WEV determines when one option is m to n units more valuable than another. It does not, however, assess the *non-relative* value of any option. It does not entail that Pasadena, for example, has value log(2). It thus does not entail Weak Expectations (assigning each option its weak expected value, if it has one). If, however, we add the following uncontroversial principle, then Weak Expectations will follow:

Zero Value for Zero Option: An option that has a payoff of zero for all basic states has value 0.

Given this principle, Cardinal Relative Interval WEV entails Weak Expectations, since if X has a weak expected value of n, then X is n units more valuable than the zero option. Given that the latter has value 0, the former has value n, as required.

Cardinal Relative Interval WEV provides a sufficient condition for one option being n units more valuable than another on the same state space. We shall now tentatively suggest that it also provides a necessary condition.

7. No Ordinal Ranking without Cardinal Relative Interval WEV

Consider:

Converse Cardinal Relative Interval WEV: If X and Y are options defined on the same state space, then X is m to n units more valuable than Y only if the weak expected interval value of X–Y is [m, n], that is, only if m is the greatest lower bound on the value of k for which the limit, as t goes to infinity, of pr(Ave(X–Y,t) ≥ k) = 1, and (2) n is the smallest upper bound on the value of k for which the limit, as t goes to infinity, of pr(Ave(X–Y,t) ≤ k) = 1.

This is simply the converse of the previous principle. It is, of course, a controversial condition, and we won't attempt to defend it here. The key claim, however, is that value assessment must be grounded in the behavior of the long run averages of the options. If the probability of the average for X–Y being at least k, or at most k, does not converge to 1, then, we claim, there is no basis for judging X to be at least k, or at most k, units more valuable than Y.

Let us illustrate the force of Converse Cardinal Relative Interval WEV by applying it to Symmetric St. Petersburg and Inverse Symmetric St. Petersburg (which is the same, but with payoffs multiplied by -1):

Symmetric St. Petersburg

Probability:	1/4	1/4	1/8	1/8	…	$1/2^{n+1}$	$1/2^{n+1}$	…
State:	s_1	s_2	s_3	s_4	…	s^{2n-1}	s^{2n}	…
SP	4	–4	8	–8	…	2^{n+1}	-2^{n+1}	…
–SP	–4	4	–8	8	…	-2^{n+1}	2^{n+1}	…
SP–(–SP)	8	–8	16	–16	…	2^{n+2}	-2^{n+2}	…

SP and –SP each has no weak expectations. Each has interval value weak expectations of [–∞,∞]. Still, one might be inclined to hold that they are equally valuable given that they are basic-state isomorphic in the following sense. Two options are *basic-state isomorphic* just in case (1) each of the basic states in the state space of the first option can be uniquely paired up with a basic state in the state space of the second option, (2) with all basic states in the second space being paired up with some basic state in the first space (i.e., there exists a 1-to-1 mapping from the set of basic states of the first option to the set of basic states of the second option), such that: (3) for each option, and each of its basic states, (a) the probability of the state is the same as its partner's, and (b) the first option's payoff under that state is the same as the second option's payoff under the state's partner. In the above example, SP and –SP are basic-state isomorphic. The only difference between the two is that SP has positive payoffs for the odd states and negative payoffs for the even states whereas the opposite is true for –SP. For example, the payoff for SP under s1 is the same as the payoff for –SP under s2, and s1 and s2 have the same probability, and so on.

Given that SP and –SP are basic-state isomorphic, one might be inclined to hold that they are equally valuable on the basis of the following principle:

Basic-State Isomorphism: If two options are defined on the same state-space and are basic-state isomorphic, then they are equally valuable.

This, however, is incompatible with Converse Cardinal Relative Interval WEV. For in order for SP to be equally valuable with –SP, it requires that SP–(–SP) have weak expected interval value of [0,0], but in fact that value is [–∞,∞]. It thus requires the rejection of Basic State Isomorphism.

Although we won't argue it here, we believe that the restriction imposed by Converse Cardinal Relative Interval WEV is appropriate. SP and –SP are not equally valuable.[11] Moreover, although Basic State Isomorphism is a sound principle when restricted to options *with finite weak expectations*, it is not, we suggest, a sound principle for options that do not have such expectations.

Here is a second example of the force of Converse Cardinal Relative Interval WEV. Consider two St. Petersburg-like options:

Relative Expectations

Probability:	1/4	1/4	1/8	1/8	1/16	1/16	…	$1/2^{n+1}$	$1/2^{n+1}$	…
State:	s_{1a}	s_{1b}	s_{2a}	s_{2b}	s_{3a}	s_{3b}	…	s_{na}	s_{nb}	…
SP1	4	0	8	0	16	0	…	2^{n+1}	0	…
SP2	0	4	0	8	0	16	…	0	2^{n+1}	…
SP1–SP2	4	–4	8	–8	16	–16	…	2^{n+1}	-2^{n+1}	…

SP1 and SP2 each have infinite standard expected value (and hence infinite weak expectations). This does not entail that they are equally valuable, of course. Still, they are exactly the same gambles except they have payoffs for different but equiprobable states. Given that they are basic-state isomorphic, one might be inclined to hold that they are equally valuable, but, again, Converse Cardinal Relative Interval WEV does not allow this. For their difference does not have any determinate interval wev. It is [–∞,∞].

We believe that SP1 and SP2 are not equally valuable.[12] Basic State Isomorphism is not, we suggest, a sound principle when applied to options that do not have finite weak expectations.

We should emphasize, however, that Converse Cardinal Relative Interval WEV entails that SP1 is not more valuable than SP2, and that SP2 is not more valuable than SP1. It just insists that they are incomparable rather than equally valuable. A similar point applies to SP and –SP above.

Cardinal Relative Interval WEV, and Zero Value for Zero Option, exhaust, we hypothesize, the sound ordinal and cardinal assessments that can be made for options.[13]

8. Conclusion

We have limited our attention throughout to state spaces with *countably-many* basic states and *countably additive* probability functions (i.e., the probability of a union of a countable number of basic states is the sum of their individual probabilities). Things are significantly more complex in the uncountable case.

We have developed, with motivation but not compelling argument, several principles for the evaluation of options that have no finite standard expected value. The case arises when the sum of the probability-weighted values of an option is either ill defined (because the sum depends on the order in which the terms are added together) or infinite. Standard decision theory is silent about the evaluation of such options. Our project has been to extend the domain of evaluation.

First, we endorsed the proposal of Easwaran (2008) to evaluate options on the basis of their finite *weak* expected value, if they have one. We then extended that to include *infinite* weak expected value.

Second, we extended the relevant notion of weak expected value to *interval value*, where this is defined as the closed interval [x1,x2] such that: (1) x1 is the greatest lower bound on the value of k for which lim pr(Ave(X,n) ≥ k) = 1, and (2) x2 is the smallest upper bound on the value of k for which lim pr(Ave(X,n) ≤ k) = 1. Many options have an interval value (e.g., [1,3]) even though they have no weak expected value (e.g., [3,3]). An option with an interval value of [4,6] is more valuable than an option with interval value [1,3]. Indeed it is 1 to 5 units more valuable.

Third, for options defined on the same state space, we endorsed the proposal of Colyvan (2008) to evaluate an option, X, as at least as valuable as an option Y, if the standard expected value of X–Y exists and is non-negative (where X–Y is defined over basic states) . We then extended that principle (1) to be a cardinal principle that states how many more units more valuable X is, (2) to base the evaluation on weak (rather than standard) expected value, and (3) to apply in certain cases where X–Y has no weak expected value but has an *interval* value. When complemented with the uncontroversial Zero Value for Zero Option, this entails the Cardinal Interval Weak Expectations (that options with interval weak expectations have that value).

Finally, we tentatively suggested, via Converse Cardinal Relative Interval WEV, that the above principles exhaust the sound ordinal and cardinal assessments that can be made of options (absolutely or relative to others).

Each of our principles is, of course, controversial. We hope that we have motivated them enough to be taken seriously.[14]

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[1] See also, Hájek and Nover (2006), Hájek and Nover (2008), and Hájek (2009).

[2] See, for example, http://en.wikipedia.org/wiki/Alternating_harmonic_series

[3] For additional discussion of Easwaran's approach, see Fine (2008), Sprenger and Heesen (2009), and Smith (forthcoming).

[4] Actually, only one of us (Vallentyne) finds Finite Weak Expectations compelling. The other

(Lauwers) finds it compelling for one-stage lotteries but not for compound lotteries. We discuss the problems for multi-stage lotteries in Lauwers and Vallentyne (in progress, 2014).

[5] Once the appeal to finite weak expectations is allowed, the Pizza vs. Chinese food problem, introduced by Smithson and Hájek and Smithson (2012) and discussed by Bartha (2014), disappears. In this problem, one has a choice between ordering Chinese food and ordering Pizza, but each also has some chance of a Pasadena lottery. Adding a chance of the Pasadena lottery, with its weak expected value of log(2), does not introduce any complications, once finite weak expectations are accepted.

[6] Roughly: For an option for which E(|X|) is infinite, the strong expected value is positively (respectively: negatively) infinite if and only if a certain measure, J_- (respectively: J_+) is finite. Given that Pasadena does not have an infinite strong expected value, both of these measures are both infinite. Multiplying the positive payoffs by 1.01 leaves both measures infinite. Thus, Weakly Infinite Pasadena does not have infinite strong expected value either.

[7] See Fishburn (1985) for more on interval orders.

[8] We assume here that the probabilities of states are independent of the options chosen.

[9] For simplicity, we take options to assign the value of their outcomes, rather the outcomes themselves.

[10] For a closely related principle in value theory (without probabilities), see Lauwers and Vallentyne (2004).

[11] For related discussion, see Alexander (2012).

[12] We thus agree with Seidenfeld et al. (2009) that two options with the same probability distribution over payoffs need not be equally valuable. We show this in the text for two options

that are basic-state isomorphic. Seidenfeld et al. brilliantly show that an option (with infinite standard expected value) can strictly dominate a second option with the same probability distribution over payoffs (but is not basic-state isomorphic).

[13] Thus, we believe that Peterson's Petrogradskij and Leningradskij options are incomparable, even though the two have identical payoffs except that (1) the former has an additional payoff of the St. Petersburg lottery under the state with a with a 1/8 probability and (2) the latter has an additional payoff of the St. Petersburg lottery under the state with a with a 1/4 probability. The interval wev of [Petrogradskij–Leningradskij] is [–∞,∞], and thus they are, we claim, incomparable.

[14] For extremely helpful comments, we thank [removed for anonymity].

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