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Does revenue equalisation mitigate tax competition? Ad valorem taxation in a federation

Willem SAS

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Does revenue equalisation mitigate tax competition?

Ad valorem taxation in a federation

Willem Sas*

Center for Economic Studies, KU Leuven

Abstract

In this paper, we model a federal economy where perfectly mobile labour supply is taxed on an *ad valorem* basis by the federal as well as lower-level (state) governments. We find that either under- or overtaxation occurs, under similar conditions as in Keen and Kotsogiannis (2002, 2004). However, the neat trade-off between *positive* horizontal externalities and *negative* vertical externalities breaks down entirely in our ad valorem setting. Precisely because of this ambiguity, decentralising the unitary outcome via revenue equalisation becomes far more complex than under unit taxation. Only when the marginal valuation of public provision is on par with private consumption, can we replicate the clear-cut, efficiency-enhancing equalisation formulas given by Kotsogiannis (2010) and Bucovetsky and Smart (2006).

JEL Classification: H71, H77, H23.

Keywords: Tax competition, Ad valorem taxation, Fiscal equalisation, Vertical and horizontal externalities, Commuting, Decentralisation, Federalism.

*Center for Economic Studies (CES), KU Leuven, Naamsestraat 69, 3000 Leuven, Belgium. Tel: +32 472 26 09 89. E-mail: Willem.Sas@kuleuven.be.

I. Introduction

In most federations today, both federal and state governments raise taxes on the same -or at least interdependent- tax bases. Inevitably then, tax policies of one government have an impact on revenues raised by other governments, as well as on the welfare of residents living in other states.[1] Now, whether strategically or unwittingly, governments often ignore such effects of own taxation on other parties, thereby misjudging the 'true' social cost of public provision. In this paper, we revisit these inefficiencies and their implications for revenue equalisation which, as we will argue, are more complex than commonly understood.

Take tax base mobility between states for one, where *horizontal* externalities bring about suboptimal levels of taxation and public provision. The textbook 'race to the bottom' scenario is often given as the example here,[2] but other outcomes are certainly possible.[3] If the tax base is co-occupied by the federal and state governments moreover, *vertical* externalities enter the fray. Here the externality works through the *shared* tax base, which may contract as states ratchet up tax levels. When the resulting negative effect on federal tax revenues is overlooked by the states, regional taxes as well as public provision come out inefficiently *high* compared to the second-best unitary optimum.[4] However, when taxes are levied on an *ad valorem* basis, such vertical externalities can just as well have the opposite effect as shown by Dahlby and Wilson (2003).

Also, since both types of externalities often take effect at the same time -and can work in opposite directions- the question becomes what their joint effect will be. Indeed, if horizontal externalities work against their vertical counterparts, the kind of welfare losses described above start wearing thin or could even fully cancel out. First tackled by Keen and Kotsogiannis (2002) with a focus on capital mobility, tax base elasticities and the relative size of the federal government are shown to be crucial factors in this trade-off.[5] In what follows, and since earlier work mainly stuck to unit taxation, we study the same trade-off in a setting where *labour* is taxed *ad valorem*.

Moreover, and unlike most studies[6] where inter-state *migration* drives horizontal labour tax externalities,[7] we focus on commuting. Wages are set in an integrated, national labour market, so that changes in one state are felt throughout the entire federal system even when household migration does not occur. In other words, we translate the

[1] Throughout this paper, we focus on the regional tier within a federation: the *state* level. Our findings would also apply to the municipal level however.

[2] Wilson (1986) and Zodrow and Mieszkowski (1986) provide the seminal formal derivations. For a survey of the empirical literature on horizontal interactions, see Brueckner (2003).

[3] Keen and Marchand (1997) discuss the overprovision of *productive* inputs. McLure (1967), Krelove (1992) or De Borger et al. (2007) consider tax exporting. See also Lockwood (2001) for a theoretical synthesis of commodity tax competition.

[4] See Dahlby (1996) or Boadway and Keen (1996). Esteller-Moré and Solé-Ollé (2001) and Andersson et al. (2004) deliver empirical evidence.

[5] Wilson and Janeba (2005) add one more dimension, by endogenising the *degree* of decentralisation. Brülhart and Jametti (2006) empirically confirming vertical externalities as more than 'theoretical curiosities', Devereux et al. (2007) focus on cross-border shopping.

[6] The last section in Boadway and Keen (1996) forms a notable exception and considers commuting, yet also here clearing *regional* labour markets drive wage formation.

[7] To the best of our knowledge, only Boadway and Keen (1996) and Andersson et al. (2004) have studied *both* types of externalities in a coherent framework of labour migration.

model of Keen and Kotsogiannis (2002) to a labour market setting, and extend it by introducing an ad valorem, *residence* based tax.

We find that when the tax base is shared by the federal and state governments, either under- or overtaxation occurs. This happens under similar conditions as in Keen and Kotsogiannis (2002, 2004), or Kotsogiannis (2010). However, the neat trade-off between *positive* horizontal externalities and *negative* vertical externalities breaks down entirely in an ad valorem setting. Inter-state mobility of the labour tax base produces the usual kind of positive externalities, but also gives rise to negative externalities operating through eroding rents. Vertical interaction can lead to a shrinking federation-wide tax base, but also brings about the opposite as rising gross wages boost federal revenues. It is precisely this ambiguity which renders decentralising the unitary outcome by means of revenue equalisation far more complex.

Such an equalisation mechanism usually corrects for differences in fiscal capacity across the various states, levelling out (a degree of) the divergence. And even though the underlying principle here is mainly one of horizontal equity -ensuring each state has sufficient revenues at its disposal to provide a minimum level of public services- efficiency arguments in favour of equalisation are indeed on the rise.

As argued by Bucovetsky and Smart (2006) or Kotsogiannis (2010), horizontal as well as vertical externalities can be exactly offset by the conventional equalisation system, or at least by a simple and intuitive adjustment of the main model.[8] We show that in an ad valorem setting -where we'll most commonly come across equalisation in practice- these results can only be replicated when the marginal valuation of public provision is on par with private consumption. However, even when this condition fails to hold, not all is lost. The pre-requisite for equalisation grants to successfully nudge state politicians towards second-best policies, remains analytically tractable. The extent to which it is operationally attainable in the field on the other hand, becomes a different question.

The remainder of this paper is organized as follows. Section 2 introduces the model. Employing a unitary country focus, the second-best optimum is characterised in section 3. In the following section, the federal and state governments share the labour tax base. Section 5 introduces a fiscal equalisation mechanism to our economy, after which section 6 concludes.

II. The Model

Our federal economy consists of a limited number $n > 1$ of states, where ad valorem taxes are levied on labour incomes of *immobile* households. Importantly, and although their *residence* is fixed as a result, members of each household are free to work in any other state of the federation. To simplify notation as much as possible, we normalize the mass of households in each state to unity.[9]

[8] Aside from internalising both types of tax externalities by decentralising the second-best outcome, an equalisation mechanism is liable to rectify other, locational inefficiencies as well. See Boadway and Shah (2009) for an overview.

[9] A common assumption in the literature focusing on similar *efficiency* issues, see e.g. Keen and Kotsogiannis (2002, 2004), Lucas (2004), Brülhart and Jametti (2006), Aronsson and Blomquist (2008) or Kotsogiannis (2010).

Output in each state is given by technology $F_i(L_{D_i})$, where L_{D_i} denotes the amount of labour demanded by firms in state i, with $F' > 0 > F''$.[10] The private sector maximises profits, given by

$$\pi_i = F_i(L_{D_i}) - w_i L_{D_i} \tag{1}$$

with w_i the gross wage in state i. As a result, labour demand $L_{D_i}(w_i)$ is implicitly defined by $F_i'(L_{D_i}) = w_i$, with $L'_{D_i}(w_i) = \frac{1}{F''} < 0$. Production can be used interchangeably for private and public consumption, at a marginal rate of transformation of 1. Profits accrue entirely to the representative household living in the state where rents are realised. Turning then to the consumer side of our economy, the representative household of state i derives utility from private consumption, public provision, and leisure. That is,

$$U_i(C_i, L_{S_i}, G_i, G_i^F) = u_i(C_i, L_{S_i}) + \Gamma_i(G_i, G_i^F) \tag{2}$$

with C_i the consumption of a composite (numeraire) private good, L_{S_i} labour supply, and $G_i^{(F)}$ state and federal publicly provided *private* goods.[11] Sub-utility $u_i(C_i, L_{S_i})$ is concave, increasing in C_i and decreasing in L_{S_i}. $\Gamma_i(G_i, G_i^F)$ is concave and increasing in both G_i and G_i^F respectively. As in Kotsogiannis and Martínez (2008), public provision is financed by an ad valorem tax on labour income, which is levied according to the *residence* principle. Denoted by t_i for the states and T for the federal government, the consolidated labour tax for state i becomes $\tau_i = t_i + T$.

Each household then maximises (2) subject to its budget constraint: $C_i = \bar{w}_i L_i + \pi_i$, with $\bar{w}_i = (1 - \tau_i) w_i$ the net wage. As a result, labour supply $L_{S_i}(\bar{w}_i)$ is implicitly defined by $u_{C_i}(.)\bar{w}_i + u_{L_i} = 0$ and assumed increasing, so that $L'_{S_i}(\bar{w}_i) > 0$. Indirect utility is then given by

$$V_i(\bar{w}_i, \pi_i, G_i, G_i^F) = v_i(\bar{w}_i, \pi_i) + \Gamma_i(G_i, G_i^F) \tag{3}$$

Crucially, households in each state i are immobile, but are free to work in any state of their choosing. A common inter-state labour market thus allows for commuting, where labour itself is costlessly mobile and commuting flows equilibrate *gross* wages across all states. Since state populations are normalised to 1, this implies that states where labour supply L_{S_i} outweighs labour demand L_{D_i} are marked by commuting outflows, and vice versa.[12]

As a result, and denoting the n-vector of consolidated tax rates by $\boldsymbol{\tau} \equiv (\tau_1, ..., \tau_n)$, the gross wage $w(\boldsymbol{\tau})$ clearing the common labour market is implicitly defined by

$$\sum_i^n L_{S_i}(\bar{w}_i(\boldsymbol{\tau})) = \sum_i^n L_{D_i}(w(\boldsymbol{\tau})) \tag{4}$$

[10] A subscript denotes the derivative of a function of several variables whereas a prime denotes the derivative of a function of one variable.

[11] Note that since we have normalised population to 1, these could just as well be pure public goods as in Kotsogiannis and Martínez (2008).

[12] Here, normalised state populations keep us from introducing a commuting cost. Since such a cost would not change our core results however, we can safely omit it.

Taking the total differential with respect to τ_i of (4) yields

$$\frac{\partial w}{\partial \tau_i} = \frac{w\eta_i \frac{L_{s_i}}{\bar{w}_i}}{\left(\sum_i^n \left((1-\tau_i)\eta_i \frac{L_{s_i}}{\bar{w}_i}\right) - \left(\sum_i^n \varepsilon_i \frac{L_{D_i}}{w}\right)\right)} > 0 \tag{5}$$

with $\eta_i > 0$ labour supply, and $\varepsilon_i < 0$ labour demand elasticities in state i. Throughout the analysis we limit our attention to symmetric equilibria, in which all states set the same tax rate ($t_i = t$, $\forall i$). The gross wage in such an equilibrium then becomes $w(\boldsymbol{\tau}) \equiv w(\tau, ..., \tau)$, with

$$w'(\boldsymbol{\tau}) = \frac{w\eta}{(1-\tau)(\eta - \varepsilon)} > 0 \tag{6}$$

So that, still in symmetric equilibrium and using (5), we also get

$$\frac{\partial w}{\partial \tau_i} = \frac{1}{n} w'(\boldsymbol{\tau}) \tag{7}$$

Logically, since marginally increasing the *common* state tax rate τ has a federation-wide impact, the gross wage will respond differently compared to a state-specific tax hike. Indeed, from (7) we find the latter to be smaller. As the gross wage starts to rise in the state raising its taxes, more and more workers from other states will flock to this region, mitigating the gross wage increase. If n were to go to infinity, the gross wage effect would be fully countered by the commuting response, as can be seen in expression (5) or (7). The marginal tax burden then falls entirely on the state in question through the drop in net wages, as $\frac{\partial \bar{w}_i}{\partial \tau_i} = (1-\tau_i)\frac{\partial w}{\partial \tau_i} - w$, thus reducing to $-w$.
The effects of taxation on net wages can then be written as

$$\bar{w}'(\boldsymbol{\tau}) = \frac{w\varepsilon}{(\eta - \varepsilon)} < 0 \tag{8}$$

and

$$\frac{\partial \bar{w}}{\partial \tau_i} = \frac{w\left(n\varepsilon - (n-1)\eta\right)}{n(\eta - \varepsilon)} < 0 \tag{9}$$

Also, the effect of marginally increased taxation on profits -which is the same for state as well as federal taxation- is given by

$$\frac{\partial \pi_i}{\partial \tau_{(i)}} = \frac{\partial\left(F_i(L_{D_i}) - wL_{D_i}\right)}{\partial \tau_{(i)}} = -L_{D_i}\frac{\partial w}{\partial \tau_{(i)}} < 0 \tag{10}$$

Lastly, and for reasons of simplicity, we assume states to be perfectly homogeneous in what follows.

III. Second-best optimum in a 'unitary' country

We start with the benchmark case of a unitary country, where states are given no taxing or spending powers and the federal level makes all the calls. Here, the federal government sets a uniform tax rate τ to finance consolidated public provision. Since in this case tax externalities do not arise, the second-best outcome under distortionary taxation is attained.

We furthermore assume the federal government can tailor public provision ($G_i^{(F)}$) to the preferences of the representative household living in each state i, thus ruling out inefficiencies at the federal level working through *policy* uniformity. Other arguments in favour of more decentralisation, such as enhanced accountability, cost-effectiveness or innovation, are also omitted. This way, the second-best optimum as we describe it in what follows, will always be welfare superior to the decentralised outcomes discussed in the following sections. It hence serves as an ideal benchmark. The federal government then maximises a Utilitarian welfare function given by[13]

$$Max_{G_i,G_i^F,\tau} \sum_i^n V_i\left(\bar{w}_i(\boldsymbol{\tau}), \pi_i(\boldsymbol{\tau}), G_i, G_i^F\right) \tag{11}$$

subject to its budget constraint given by

$$\sum_i^n \left(G_i + G_i^F\right) = \tau \sum_i^n L_{S_i}\left(\bar{w}_i(\boldsymbol{\tau})\right) w(\boldsymbol{\tau}) \tag{12}$$

where the values of τ, G_i^F and G_i are chosen by the government. Using symmetry, (6), and (12), the first-order conditions readily reduce to

$$\frac{\frac{\partial V}{\partial G^F}}{\lambda} = \frac{\frac{\partial V}{\partial G}}{\lambda} = \frac{1}{\left(1 - \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)} = MCPF \quad \forall i = 1,..,n \tag{13}$$

with λ the marginal utility of income. Equations (13), together with the budget constraint (12), characterize the second-best optimum denoted by (τ^*, G^*, G^{F*}) for each state i. At the unitary optimum, τ is set such that the marginal rate of substitution (MRS) between both the publicly provided good and the private good must be equal to the Marginal Cost of Public Funds (MCPF). As is well known, the MCPF is the efficiency cost of raising revenue with a distortionary tax (Dahlby, 2008). Indeed, equations (13) are the usual optimality conditions for public provision in a distorted economy, being simplified versions of the standard Atkinson and Stern (1974) rule.[14]

IV. Inefficiency of state taxation

We now move on to the case where both the federal and state governments levy taxes on labour, giving rise to horizontal *and* vertical tax externalities. The federal government sets a uniform tax rate T to tailor public provision (G_i^F) to the preferences of each representative household as before, thus maximising (11) subject to:

$$\sum_i^n G_i^F = T \sum_i^n L_{S_i}\left(\bar{w}_i(\tau_i, \boldsymbol{\tau})\right) w(\boldsymbol{\tau}) \tag{14}$$

[13] Since we focus solely on efficiency issues, further redistributional concerns can safely be omitted from the welfare function.

[14] The Atkinson-Stern rule includes an additional term in the numerator, capturing the effect of changes in public provision on tax revenues. Because utility is additively seperable in our setting, this relation does not bite here.

On top of this, each state raises additional taxes to finance own public provision (G_i). Following the literature, we assume the federal government sees through states' budget constraints, so there will be no 'top-down' vertical externality. We also assume that all governments behave non-cooperatively -taking each other's policies as given- and that states are identical in every way. Since under these assumptions federal decision making is equivalent to the unitary outcome derived in section 3, we can jump straight to the state level. The government of state i maximises (3) subject to

$$G_i = t_i L_{S_i}\left(\bar{w}_i(\tau_i, \boldsymbol{\tau})\right) w(\boldsymbol{\tau}) \tag{15}$$

Using (7), (9) and (15), and evaluated in symmetric equilibrium, we arrive at the following characterisation:[15]

Lemma 1 *With households immobile, but workers commuting to other states, the MCPF of a state government sharing its labour tax base with the federal government, is given by*

$$\frac{\frac{\partial V}{\partial G}}{\lambda} = \frac{1}{\left(1 - \frac{t\eta(n\varepsilon-(n-1)\eta)-T\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)} = MCPF_i \tag{16}$$

Expression (16), together with the budget constraint (15), defines the Nash equilibrium, denoted by (t^*, G^*).

Again, public provision continues until the marginal rate of substitution between both the publicly provided good and the private good is equal to the marginal cost of public funds (MCPF). Moreover, not only does this efficiency cost overlook all horizontal effects of own taxation on other states, it also fails to internalise the *vertical* effects.

As touched upon in the introduction, these vertical effects can be positive as well as negative. Negative, since a state tax hike induces a higher federation-wide gross wage, which curtails federation-wide labour demand. And since the labour market clears at the national level, this brings about lower federation-wide labour supply, and by consequence, shrinking federal tax receipts. However, and crucially, since the labour income tax base is taxed ad valorem, the same gross wage increase also boosts the tax base. This then has a positive effect on federal coffers.

To find out which effect comes out on top, and thus to determine the sign of the *overall* externality, we compare the unitary MCPF with the state efficiency cost. Evaluated in the symmetric Nash-equilibrium, using lemma 1 and (13), we obtain:[16]

Proposition 1 *In a federation where workers can commute between identical states, where households as a whole do not migrate, and both the federal and state governments tax labour income, the overall externality:*

1. *Is positive when labour supply is highly responsive to changes in the net wage, and federation-wide labour demand is inelastic*

2. *Is negative when labour supply is relatively inelastic compared to labour demand, and the federal level accounts for the larger part of public provision*

[15] A Proof is given in appendix A.
[16] A Proof is given in appendix B.

3. *Disappears as soon as labour supply is unaffected by the net wage*

When the overall externality is negative, state over-taxation and over-provision ensue. In this case the upward pressure on the perceived state efficiency cost is more than undone by the negative effects. If, on the other hand, the state MCPF stays well above the second-best efficiency cost, we end up with under-provision and under-taxation.

Now, although earlier results by Keen and Kotsogiannis (2002) seem to carry through in our setting here, zooming in on the factors determining proposition 1 brings an important difference to the surface. True enough, tax base elasticities play a crucial part in our setting as well, yet ad valorem taxation renders their interplay considerably less clear cut than in Keen and Kotsogiannis (2002). To shed more light on this mechanism, and using (1), (3), (4) and (15), we write the welfare of the representative household in state i as

$$W_i(t_i, T, \boldsymbol{\tau}) = v_i\left(w(\boldsymbol{\tau}), \tau_i, \pi_i(w(\boldsymbol{\tau}))\right) + \mathit{\Gamma}_i\left(t_i L_S((\tau_i, w(\boldsymbol{\tau}))w(\boldsymbol{\tau}), G_i^F(T)\right) \tag{17}$$

Reformulating state optimisation expressed by (3) and (15), state i then chooses its tax rate t_i to maximize (17), taking all other tax rates as given. The necessary first order condition for this, evaluated in symmetric equilibrium and making use of (7) and (10), becomes

$$\frac{\partial W_i}{\partial t_i} = \lambda\left[L_S\left((1-\tau)\frac{w'}{n} - w\right) - L_D\frac{w'}{n}\right] + \mathit{\Gamma}_G\left[tL_S^{'}\left((1-\tau)\frac{w'}{n} - w\right)w + tL_S\frac{w'}{n}\right] = 0 \tag{18}$$

Condition (18) is nothing more than lemma 1 in rewritten form, implicitly defining the equilibrium state tax rate t^*, again given T^* set by the federal government and the number of states n.[17] The first term of (18) reflects the *direct* utility loss incurred by the representative household because of increased state taxation, working through the decreasing net wage and falling profits. The second term, involving $\mathit{\Gamma}_G$, expresses the effect of a state tax hike on state government revenues. Crucially, since taxes are levied ad valorem, this could either inflict utility losses *or* gains. A tax hike depresses the net wage and consequently labour supply, but also pushes up the federation-wide gross wage, so the effect on revenues is ambiguous.

To bring potential tax externalities to the surface by means of (18), we first write out welfare in symmetric equilibrium and under full information. Using (1), (3), (4), (14) and (15), we obtain[18]

$$W(t, T, \boldsymbol{\tau}) = v\left(w(\boldsymbol{\tau}), \tau, \pi(w(\boldsymbol{\tau}))\right) + \mathit{\Gamma}\left(tL_S(\tau, w(\boldsymbol{\tau}))w(\boldsymbol{\tau}), TL_D(w(\boldsymbol{\tau}))w(\boldsymbol{\tau})\right) \tag{19}$$

Differentiating (19) with respect to a *common* tax rate t, which is equivalent to a coordinated tax increase in all states, then yields

$$W_t = \lambda\left[L_S\left((1-\tau)w' - w\right) - L_D w'\right] + \mathit{\Gamma}_G\left[tL_S^{'}\left((1-\tau)w' - w\right)w + tL_S w'\right] + \mathit{\Gamma}_{G^F}\left[TL_D^{'}w'w + TL_D w'\right] \tag{20}$$

[17] Where the federal tax rate T^* comes in through the consolidated tax rate $\boldsymbol{\tau}$ defining $w(\boldsymbol{\tau})$.

[18] Note how, under symmetric equilibrium, the federal budget constraint (14) reduces to $G^F = TL_D(w(\boldsymbol{\tau})w(\boldsymbol{\tau})$, using the market clearing condition (4).

where the third term reflects the impact of state taxation on welfare through Γ_{G^F} and federal tax revenues. Now, since setting (20) equal to zero implicitly defines the socially optimal state tax rate, the sign of W_t evaluated at the Nash-equilibrium established in (18) is vital. If W_t turns out to be positive at this point, a slight increase in state tax rates would improve overall welfare. Hence, tax rates in the non-cooperative equilibrium defined by (18) -or lemma 1- were set inefficiently low from a social viewpoint. Conversely, when W_t is negative, state taxes were set too high. To investigate the sign of (20) at the Nash equilibrium (t^*, T^*) we subtract (18) from (20), to arrive at

$$W_t = \overbrace{\left[-\lambda\tau L_D + t^*\Gamma_G\left(L_S^{'}(1-\tau)+L_S\right)\right]\left(1-\frac{1}{n}\right)w'}^{Horizontal\,externality} + \overbrace{T^*\Gamma_{G^F}\left[wL_D^{'}+L_D\right]w'}^{Vertical\,externality} \quad (21)$$

Since w' is unambiguously positive, the sign of (21) hinges on the terms between square brackets.

On the right of (21) the vertical externality appears, spelling out the ignored impact of a state tax hike on the federal budget. As $L_D^{'} < 0$, shrinking federation-wide labour demand comes in via $L_D^{'}w$, whilst the positive twist of the gross wage increase is expressed by L_D.

On the left we find the omitted effects of state taxation on other states, reflecting the horizontal externality. Contrary to a scenario employing unit taxation, state taxation has multiple effects on the welfare of households living in other states. Due to the gross wage increase firstly -which is identical across states because of commuting- tax bases in other states rise alongside collected tax revenues, at a pace of L_S. Second, since other states keep their tax rates constant, net wages in these states follow gross wage hikes so that more labour is supplied. This also fattens state coffers via $L_S^{'}$, and would be the only effect under unit taxation. These higher net wages not only improve non-resident welfare through increased public provision furthermore, but also simply because purchasing power comes out reinforced. This latter effects is outmatched by the negative direct effect on non-resident welfare however, since collected profits fall due to the higher gross wage, at a rate of τL_D.

Now, we see that (21) turns positive when labour supply becomes more responsive to changes in the net wage ($L_S^{'} \uparrow$), and labour demand less sensitive to gross wage movements ($L_D^{'} \downarrow$). More so, this latter effect is strengthened by a higher state tax rate t^*, in itself a result of a higher appreciation of state public provision Γ_G. Inversely, and coming full circle on points (1) and (2) of proposition 2, inelastic labour supply and highly responsive labour demand amplify the negative effects in (21), as will a higher utility share of federal public provision Γ_{G^F}. Setting $L_S^{'} = 0$ in (6) lastly, also reduces w' and by consequence the whole of (21) to zero, as captured by point (3) of proposition 1.

As pointed out above, these results seem to chime well with Keen and Kotsogiannis (2002, 2004), or Kotsogiannis (2010). What we learn from (21) however, is that the strict demarcation between horizontal and vertical externalities is lost in our ad valorem setting. Indeed, the neat trade-off between *positive* horizontal externalities and *negative* vertical externalities breaks down entirely. Inter-state mobility of the labour tax base produces the usual kind of positive externalities, but also gives rise to negative externalities operating through eroding rents. Vertical interaction can lead to a shrinking federation-wide tax base, but also brings about the opposite as rising gross wages boost federal revenues.[19] It is precisely this ambiguity which renders decentralising the unitary outcome by means of revenue equalisation far more complex.

V. Revenue equalisation and ad valorem taxation

We assume an equalisation mechanism of the conventional sort. Here, the per capita equalisation grant ω_i to each state i is given by the difference between its fiscal capacity and a benchmark fiscal capacity, multiplied by a standard -usually average- federal tax rate:[20]

$$\omega_i = \bar{t}(\bar{B} - B_i) \tag{22}$$

where $\bar{t} = \frac{\sum_i^n t_i L_{S_i} w}{\sum_i^n L_{S_i} w}$ is the average lower-level government tax rate. B_i is the fiscal capacity of state i, captured by its tax base $L_{S_i} w$, and $\bar{B} = \frac{\sum_i^n L_{S_i} w}{n}$ is the benchmark fiscal capacity, being the average federation-wide tax base.

Now, to factor in the equalisation grant when states decide on taxation, we introduce it to state budgets as an extra source of income. The budget constraint of state i thus becomes $G_i = t_i L_{S_i} + \omega_i$, which, optimising for state taxation t_i, extends (18) as follows

$$\frac{\partial W_i}{\partial t_i} = \lambda \left[L_S \left((1-\tau)\frac{w'}{n} - w \right) - L_D \frac{w'}{n} \right] + \Gamma_G \left[t L_S' \left((1-\tau)\frac{w'}{n} - w \right) w + t L_S \frac{w'}{n} + \frac{\partial \omega_i}{\partial t_i} \right] = 0 \tag{23}$$

where the equalisation grant enters representative utility via state public provision. Now, deriving the equalisation grant w.r.t. t_i gives us

$$\frac{\partial \omega_i}{\partial t_i} = \frac{\partial \bar{t}}{\partial t_i}(\bar{B} - B_i) + \bar{t}\left(\frac{\partial \bar{B}}{\partial t_i} - \frac{\partial B_i}{\partial t_i}\right) \tag{24}$$

A tax hike in state i thus influences the equalisation grants via two channels: through the change in the average state tax rate, given fiscal capacity, and through a change in the actual fiscal capacities. In symmetric equilibrium, (24) becomes

$$\frac{\partial \omega_i}{\partial t_i} = t_i \left(\frac{n-1}{n} \right) L_S' w^2 > 0 \tag{25}$$

which confirms earlier results by Bucovetsky and Smart (2006) and Kotsogiannis (2010), who consider unit based capital taxation. Also under ad valorem taxation in other words, will the equalisation grant respond positively to a state tax increase. Here too state governments are partially compensated for raising taxes, and induced to set taxation at

[19] It is trivial to show that these results do not follow from the assumptions on utility, but are strictly due to taxation practices. Imposing quasi-linear preferences, as in Keen and Kotsogiannis (2002, 2004) simply sets λ equal to unity in (21), which doesn't alter our core insight.

[20] Since the fiscal capacity differences are entirely equalised in the proposed scheme, we have assumed 'full equalisation' here. Partial equalisation however, where only a fraction of the fiscal capacity divide is bridged, yields similar -but logically less pronounced- results.

higher levels than socially desirable. However, the way in which this result mitigates or even neutralises the externalities described above, is different here.

To see this, we first need to know under which conditions *any* equalisation formula would decentralise the unitary outcome derived in section 3, and characterised by (20) set to zero. Plugging (23) into (20), holding as equality, we distill the following precondition for the equalisation grant to internalise both horizontal and vertical externalities

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \left\{ \left[-\tau\lambda L_D + t^* \Gamma_G \left(L_S^{'}(1-\tau)w + L_S \right) \right] \left(1 - \frac{1}{n} \right) w' + T^* \Gamma_{G^F} \left[w L_S^{'} \left((1-\tau)w' - w \right) + L_S w' \right] \right\} \tag{26}$$

The question then becomes whether the equalisation scheme ω_i, expressed at the margin by (25), satisfies our prerequisite for second-best efficiency captured by (26). As a first step, and to focus ideas, we use the same starting case as studied by Bucovetsky and Smart (2006) and Kotsogiannis (2010). Here, the federation-wide tax base sensitivity defining the intensity of the vertical externality, L_D' in our setting, is set to zero. In this simplified setting, the gross wage effect expressed by (6) reduces to $\frac{w}{(1-\tau)}$. Using (25), we can then write (26) as

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \frac{\partial \omega_i}{\partial t_i} + \left(1 - \frac{1}{n} \right) \left[\left(t^* L_S - \tau \frac{\lambda}{\Gamma_G} L_D \right) + T^* \frac{\Gamma_{G^F}}{\Gamma_G} L_S \right] \frac{w}{(1-\tau)} \tag{27}$$

What emerges from (27) and our ad valorem setting, is a twofold divergence from earlier findings.[21] Contrary to Kotsogiannis (2010) first of all, the equalisation scheme characterised by (25) does not induce state politicians to replicate the unitary outcome. Indeed, whereas under unit taxation the vertical externality would disappear altogether as soon as labour demand is fully inelastic, ad valorem taxation throws in a second effect on gross wages. But even in the absence of federal spending secondly, with T set to zero so that $t = \tau$ in (27), the equalisation scheme (24) still fails to internalise the remaining horizontal externalities. Indeed, for (27) to collapse to $\frac{\partial \omega_i}{\partial t_i}$, marginal valuations of public provision have to be on par with private provision across the board, so that $\lambda = \Gamma_G = \Gamma_{G^F}$. Unlike Bucovetsky and Smart (2006) in other words, we find that conventional horizontal equalisation does not necessarily entice state governments to set taxes at their second-best optimal level. *Even* when the relevant federation-wide tax base is wholly inelastic, and tax autonomy is fully decentralised.

Unsurprisingly then, when labour demand *is* responsive to changes in the gross wage, but federal spending is still kept at zero, (27) also fails to reproduce the clear-cut results of Kotsogiannis (2010). Indeed, in this case we can write the marginal equalisation grant required for second-best efficiency as

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \left(\frac{\eta}{\eta - \varepsilon} \right) \left[\frac{\partial \omega_i}{\partial t_i} + t^* \left(1 - \frac{1}{n} \right) \left(L_S - \frac{\lambda}{\Gamma_G} L_D \right) \frac{w}{(1-\tau)} \right] \tag{28}$$

which, again, only collapses into the reduced form in function of labour elasticities when $\lambda = \Gamma_G$. We summarize, and generalise these findings in proposition 2:[22]

[21] Where we have assumed state governments can see through the part of the federal budget constraint which concerns them. Since Kotsogiannis (2010) and Bucovetsky and Smart (2006) maintain the same assumption, this allows for a full comparison here. We elaborate on these calculations in appendix C.

[22] A proof is given in appendix C.

Proposition 2 *In the presence of horizontal and vertical externalities, and with states being symmetric, the unitary optimum can be decentralised with a system of equalisation grants in the form of* $\hat{\omega}_i = z\omega_i$ *where*

1. *z is defined by* $\left(\frac{\eta}{\eta-\varepsilon}\right)\left\{1+\frac{G}{G}^F\frac{\varepsilon}{\eta}\right\}$
2. *ω_i is given by (22)*
3. *And* $\lambda = \Gamma_G = \Gamma_{G^F}$

In other words, when marginal valuations of public and private provision differ, we cannot simply adjust the popular equalisation mechanism in (22) by a factor of z - which simply accounts for tax base elasticities and the size of the vertical externality- to attain second-best efficiency. As a result, condition (3) of proposition 2 proves vital for the findings of Kotsogiannis (2010) to hold in a context of ad valorem taxation. When this condition fails, only an equalisation mechanism satisfying (27) will drive state governments to set taxation at second-best efficient levels.

VI. Concluding remarks

We presented a theoretical model based on a *common* labour market, where wages are endogenously determined as commuting flows equilibrate wages across all states of a federation. Policy changes in one state are felt throughout the entire federal system, even when household *migration* does not occur. We have thus modeled a situation where horizontal externalities are re-introduced to the analysis through commuting effects. In short, we translated the model of Keen and Kotsogiannis (2002) to a labour market setting, and extended it by introducing an ad valorem, *residence* based tax. Indeed, when the commuting labour force is taxed *ad valorem*, vertical externalities can be of either sign which re-routes the interplay with their horizontal antagonists. This adds to the novelty of our results.

When the tax base is co-occupied by the federal and state governments, either under- or overtaxation occured, and under similar conditions as in Keen and Kotsogiannis (2002, 2004) or Kotsogiannis (2010). However, the neat trade-off between *positive* horizontal externalities and *negative* vertical externalities breaks down entirely in an ad valorem setting. Inter-state mobility of the labour tax base produces the usual kind of positive externalities, but also gives rise to negative externalities operating through eroding rents. Vertical interaction can lead to a shrinking federation-wide tax base, but also brings about the opposite as rising gross wages boost federal revenues. It is precisely this ambiguity which renders decentralising the unitary outcome by means of revenue equalisation far more complex.

We showed that in an ad valorem setting -where we'll most likely come across equalisation in practice- influencing state political incentives by means of an equalisation system is less clear-cut than in earlier work. Only when the marginal valuation of public provision is on par with private consumption, can we replicate results by Kotsogiannis (2010) and Bucovetsky and Smart (2006). However, the pre-requisite for equalisation grants to successfully nudge state politicians towards second-best policies, remains analytically tractable even when this latter condition fails to hold. The extent to which it is operationally attainable in the field on the other hand, becomes a different question.

A Proof of lemma 1

The optimisation problem yields the same marginal rate of substitution as before in (13). Since in symmetric Nash equilibrium all identical states will set an identical tax rate t, and after plugging in the profit effect (10), we obtain

$$MCPF_i = -\frac{\left(L_S \frac{\partial \bar{w}}{\partial \tau_i} - L_D \frac{\partial w}{\partial \tau_i}\right)}{\left(L_S w + t L_S \frac{\partial w}{\partial \tau_i} + tw \frac{\partial L_S}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \tau_i}\right)}$$

Plugging in the wage effects (5), (9) and profit effect (10), with wages and labour supply and demand dropping out:

$$MCPF_i = -\frac{\left(\frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)} - \frac{\eta}{n(1-\tau)(\eta-\varepsilon)}\right)}{\left(1 + t\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{t\eta}{(1-\tau)} \frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)}\right)}$$

Keeping in mind that $\tau = t + T$. Rewriting yields:

$$MCPF_i = -\frac{\frac{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}{n(1-\tau)(\eta-\varepsilon)}}{\left(1 + t\frac{\eta}{n(1-\tau)(\eta-\varepsilon)} + \frac{t\eta}{(1-\tau)} \frac{(n\varepsilon-(n-1)\eta)}{n(\eta-\varepsilon)}\right)}$$

$$MCPF_i = \frac{1}{\left(\frac{n(1-\tau)(\varepsilon-\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} - \frac{\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} + \frac{\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta} - t\frac{\eta}{((1-\tau)(n\varepsilon-(n-1)\eta)-\eta)} - \frac{t\eta(n\varepsilon-(n-1)\eta)}{((1-\tau)(n\varepsilon-(n-1)\eta)-\eta)}\right)}$$

Hence,

$$MCPF_i = \frac{1}{\left(1 + \frac{-(1-t-T)\eta+\eta-t\eta-t\eta(n\varepsilon-(n-1)\eta)}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

Or,

$$MCPF_i = \frac{1}{\left(1 - \frac{t\eta(n\varepsilon-(n-1)\eta)-T\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)}$$

So that:

$$\frac{\frac{\partial V}{\partial G}}{\lambda} = \frac{1}{\left(1 - \frac{t\eta(n\varepsilon-(n-1)\eta)-T\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)} = MCPF_i \tag{29}$$

Which, together with (15), defines the symmetric Nash equilibrium. ■

B Proof of proposition 1

We compare the unitary $MCPF_u$ derived in section 3 which was unaffected by externalities, to the state $MCPF_i$ captured by lemma 1:

$$MCPF_i = \frac{1}{\left(1 - \frac{t^*\eta(n\varepsilon-(n-1)\eta)-T^*\eta}{(1-\tau)(n\varepsilon-(n-1)\eta)-\eta}\right)} \lesseqgtr \frac{1}{\left(1 - \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}\right)} = MCPF_u \qquad (30)$$

As soon as $MCPF_i$ outweighs the unitary outcome $MCPF_u$, positive externalities will result in undertaxation. Looking at the denominators in (30), and evaluating in symmetric Nash equilibrium so that $t^* + T^* = \tau^* = \tau$, this happens when:

$$\frac{t^*\eta\left(n\varepsilon-(n-1)\eta\right)-T^*\eta}{(1-\tau)\left(n\varepsilon-(n-1)\eta\right)-\eta} > \frac{\tau\eta\varepsilon}{(1-\tau)\varepsilon-\eta}$$

Rewriting yields:

$$\frac{\tau\left(n\varepsilon-(n-1)\eta\right)-T^*\left(n\varepsilon-(n-1)\eta\right)-T^*}{(1-\tau)\left(n\varepsilon-(n-1)\eta\right)-\eta} > \frac{\tau\varepsilon}{(1-\tau)\varepsilon-\eta}$$

Or:

$$\frac{n\tau\varepsilon-(n-1)T^*\eta-(n-1)t^*\eta-T^*\left(n\varepsilon-(n-1)\eta\right)-T^*}{(1-\tau)\left(n\varepsilon-(n-1)\eta\right)-\eta} > \frac{n\tau\varepsilon}{n\left((1-\tau)\varepsilon-\eta\right)}$$

Which, after some more manipulation gives us

$$\frac{n\tau\varepsilon-t^*(n-1)\eta-T^*(1+n\varepsilon)}{n(1-\tau)\left(\varepsilon-\eta\right)+(1-\tau)\eta-\eta} > \frac{n\tau\varepsilon}{n\left((1-\tau)\varepsilon-\eta\right)}$$

Or,

$$\frac{n\tau\varepsilon-t^*(n-1)\eta-T^*(1+n\varepsilon)}{n\left((1-\tau)\varepsilon-\eta\right)+(n-1)\,\tau\eta} > \frac{n\tau\varepsilon}{n\left((1-\tau)\varepsilon-\eta\right)} \qquad (31)$$

Now, Keeping in mind that $\varepsilon < 0$ and $\eta > 0$, the denominators and the numerators on both sides of (31) will always be negative. A higher labour supply elasticity η then brings the denominator on the LHS down in absolute value, and this through $(n-1)\,\tau\eta$, thus pushing up the bias. Turning to the numerators, we see that η brings about the same effect, but via $t^*(n-1)\eta$. The labour demand elasticity ε on the other hand, pulls the perceived MCPF down through $T^*(1+n\varepsilon)$. Lastly, as the state tax rate t^* accounts for a smaller share of the total tax rate τ^*, this latter effect comes out reinforced. All four points combined gives us proposition 2. ■

C Proof of proposition 2

Using the federal budget constraint (14) in symmetric equilibrium, and under the assumption that state governments can see through the part of the federal budget constraint which concerns them, (18) can be written as

$$\frac{\partial W_i}{\partial t_i} = \lambda\left[L_S\left((1-\tau)\frac{w'}{n}-w\right)-L_D\frac{w'}{n}\right] + t\Gamma_G\left[L_S^{'}\left((1-\tau)\frac{w'}{n}-w\right)w+L_S\frac{w'}{n}\right] + T\Gamma_{G^F}\left[L_D^{'}\frac{w'}{n}w+L_D\frac{w'}{n}\right] = 0 \qquad (32)$$

Plugging in (6), this allows us to write (26) as

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \left(\frac{\eta}{\eta-\varepsilon}\right)\left\{t^* L_S^{'} w^2 + \left(1-\frac{1}{n}\right)\left(t^* L_S - \tau\frac{\lambda}{\Gamma_G} L_D + T^* \frac{\Gamma_{G^F}}{\Gamma_G}\left[L_S^{'}\left((1-\tau)-\frac{w}{w'}\right)w + L_S\right]\right)\frac{w}{(1-\tau)}\right\} \tag{33}$$

Setting $\lambda = \Gamma_G = \Gamma_{G^F}$, and using (24), yields

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \left(\frac{\eta}{\eta-\varepsilon}\right)\left\{\frac{\partial \omega_i}{\partial t_i} + \left(\frac{n-1}{n}\right)T^*\left[L_s^{'}\left((1-\tau)-\frac{w}{w'}\right)w\right]\frac{w}{(1-\tau)}\right\} \tag{34}$$

And since $nG^F = TnL_S w$ and $G = tL_s w$, we obtain

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \left(\frac{\eta}{\eta-\varepsilon}\right)\left\{\frac{\partial \omega_i}{\partial t_i} + \left(\frac{n-1}{n}\right)\frac{t_i w G^F}{G}\left[L_s^{'}\left((1-\tau)-\frac{w}{w'}\right)\right]\frac{w}{(1-\tau)}\right\} \tag{35}$$

Further manipulation then finally gives us

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = \left(\frac{\eta}{\eta-\varepsilon}\right)\frac{\partial \omega_i}{\partial t_i}\left\{1 + \frac{G^F}{G(1-\tau)}\left((1-\tau)-\frac{w}{w'}\right)\right\} \tag{36}$$

So that

$$\frac{\partial \hat{\omega}_i}{\partial t_i} = z\left[\frac{\partial \omega_i}{\partial t_i}\right] \tag{37}$$

Where the factor z can be written as

$$z = \left(\frac{\eta}{\eta-\varepsilon}\right)\left\{1 + \frac{G^F}{G(1-\tau)}\left((1-\tau)-\frac{w}{w'}\right)\right\} \tag{38}$$

Hence, using (6) once more, we arrive at

$$z = \left(\frac{\eta}{\eta-\varepsilon}\right)\left\{1 + \frac{G^F}{G} - \frac{G^F}{G}\frac{(\eta-\varepsilon)}{\eta}\right\} \tag{39}$$

Or,

$$z = \left(\frac{\eta}{\eta-\varepsilon}\right)\left\{1 + \frac{G^F}{G}\frac{\varepsilon}{\eta}\right\} \tag{40}$$

■

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