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Should buses still be subsidized in Stockholm?

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Abstract
Many public transport services are heavily subsidized. One of the main justifications of this is the expected beneficial effect on road congestion. Stockholm introduced congestion pricing in 2006 and the effects on car and public transport demand were carefully monitored. This change in prices provides unique estimates on price- and cross-price elasticities. This paper uses these data to model the optimal pricing, frequency, bus size and number of bus lanes for a corridor in the presence of congestion pricing of cars. Results show that the subsidies for peak bus trips are indeed too high. However, the major welfare benefits of the reform are due to a decrease in frequencies during the off-peak period and the use of larger buses.

Keywords: Public transport, bus fares, bus lanes, bus frequency, subsidies, congestion pricing

JEL Codes: D61, H54, R41, R43, R48.

1 INTRODUCTION

Subsidies to public transport are a well-known and frequent example of second-best policy. As car use during peak periods has large external congestion costs, attracting car drivers into buses, metro or rail via low prices is an obvious second-best recipe. However, a pricing policy for buses should also take into account dimensions other than just substitution away from cars. First, pricing of public transport requires attention to the positive economies of density: more users allow higher frequency, implying decreased waiting costs. This is the so-called Mohring effect concerning the trade-off between waiting costs and bus operation costs (Mohring, 1972). Second, there are also discomfort and crowding effects associated with a more intensive use of existing bus supply (de Palma et al, 2015). Third, there is the optimal procurement of bus services. The bus service is subsidized, but the way in which the bus company is subsidized determines the efficiency of the bus services (Gagnepain et al, 2013). Fourth, to the extent that buses are more intensively used by lower income groups, reduced bus prices could be justified as income redistribution policy. Fifth, the average production cost of public transport is often decreasing due to large fixed costs. This is important for metro and rail services but less so for bus systems.

Previous studies (Parry & Small, 2009) have concluded that subsidization of public transport is primarily justified by the reduction of car use, indicating that the optimal level of subsidization is sensitive to the cross-elasticity of public transport price on car use, but also to the pricing of car use. In this paper we derive the optimal subsidization and frequency of buses for a corridor leading into the city of Stockholm. Stockholm differs from most other cities in that congestion charges are levied on the corridors leading into the city. Due to the extensive monitoring program that was put in place when the charges were first introduced in 2006, data regarding traffic flows, elasticities and cross-elasticities are also well documented in Stockholm. Such data are both scarce and crucial, given that the second-best argument for bus subsidies can only be addressed for cities where such data are available. We focus on the efficiency aspects of bus pricing, bus frequency, bus lanes and bus size and leave the procurement and redistribution dimensions aside.

Public transport subsidies raise several research questions. We will focus on three issues. The first is whether subsidies are still justified when road pricing is put in place such as in Stockholm. The second question is whether, in analogy to peak and off-peak toll differentiation for cars, how important it is to differentiate bus fares between peak and off-peak. Third, as bus fares depend strongly on the bus transport technology employed, we discuss, in addition to fares, the choice in bus frequency and size.

The bus corridor under study reaches from the inner-city Södermalm and south-east neighborhoods to the suburban areas of Nacka and Värmdö. The population of Nacka and Värmdö combined is 134000 and the number of round trips in the bus corridor is around 10000 per day. The corridor is served by approximately 200 buses in one direction during rush hour. The road network in the corridor is also heavily congested and is a candidate for metro extension (Cats et al, 2015).

Section 2 reviews the literature. Section 3 presents the theoretical model, and section 4 describes the main parameters used as well as the model calibration. Section 5 uses the model to analyze the main research questions. Section 6 discusses caveats.

2 LITERATURE REVIEW

The extensive literature on optimal public transport pricing has been reviewed in recent papers by Parry & Small (2009) and Basso & Silva (2014). We add our paper to the following Basso & Silva comparison:

Paper	Peak and off-peak periods	Total demand elasticity	Transit design optimization	Cross-congestion effects	Marginal cost of public funds	Transport policies analyzed	Welfare analyses and comparisons	Distributional analyses
Borjesson et al. (201x)	✓	✓	×	× (dedicated bus lanes)	×	Transit subsidies, congestion pricing, bus fares (uniform + time differentiating), bus lanes, bus sizes, bus frequencies	✓	×
Kilani et al. (2014)	✓	✓	×	✓	×	Zonal pricing, cordon pricing, transit fares, capacity extensions in public transport	✓	✓
Basso and Silva (2014)	✓	✓	✓	✓	✓	Transit subsidies, congestion pricing, bus lanes	✓	✓
Parry and Small (2009)	✓	✓	×	✓	×	One public transport fare at the time	×	×

Parry & Small set up a generic model to determine the optimal subsidy rate for public transport that is calibrated to London (pre-congestion tolling), Los Angeles and Washington DC. Subsidies to bus as well as to rail services are studied. For bus and rail services, the optimal second-best subsidy rate for operation costs turns out to be very high: 90% or more.

For buses, there are three main motivations for bus subsidies in the peak period. First there is the decreasing average cost of an additional passenger because the frequency of buses increases less than proportionally to the number of passengers, at least when buses are not full. Second, the car congestion costs reduce when a subsidy shifts car drivers to bus transport. Third, there are the savings in waiting time for existing users when the bus frequency increases, even though the increase may be less than proportional (this is the Mohring effect).

In the off-peak period, the car congestion reduction motive disappears, while both the savings in waiting time as well as the decreasing average cost of supplying an extra passenger (buses have lower load factor) become the main justifications for subsidizing bus services. As some two thirds of the PT passengers travel during the peak, car congestion cost savings becomes the most important motivation for subsidized PT. Whenever car congestion is priced or whenever the subsidy is less able to attract car drivers into public transport (Parry & Small assume that for every two passengers attracted into public transport, one is a former car user), the optimal subsidy rate in the peak decreases strongly for buses.

While Parry & Small study optimal bus and rail subsidies for given car taxes, Basso and Silva only focus on bus subsidies but also look into a wider set of policy interventions than simply for bus subsidies. They also analyze congestion pricing of cars, dedicated bus lanes and the role of peak differentiation for bus fares. Focusing on their results for London (pre-congestion tolling), they find that congestion pricing and dedicated bus lanes (with buses breaking even) are far more efficient policies than subsidizing bus fares. The additional contribution of subsidized bus fares would therefore be small.

Basso & Silva also analyze a policy of cross subsidization between peak and off-peak bus use, where overall bus operations must break-even but where off-peak bus users subsidize peak bus users. This policy improves welfare but only marginally.

Kilani et al (2014) find rather different results for Paris. They look into the effect of price discrimination for peak and off-peak public transport in the absence of congestion pricing for cars but without a budget constraint for public transport. They find that higher prices for peak bus users are welfare-improving. The main reason is the high level of congestion in PT, a factor that is absent in Basso & Silva and less important in Parry & Small.

An important difference between Parry & Small (2009), Kilani et al (2014) and our paper is that in our model, frequency is explicitly optimized and not determined as a rule of thumb for the way in which additional PT demand is met.

3 STYLIZED MODEL

In this model, we study one corridor that links the suburban areas of Nacka and Värmdö to the city center of Stockholm. Passengers can use either the car or the bus and can do this in either the peak or off-peak period. All transport is from either the suburb to the CBD or back. In this corridor only buses are available as public transport, and at present, there is a dedicated bus lane. Given the distance, the bike mode may also be considered but since it uses a separate bike path, there is not much interaction with the other modes. For this reason we do not consider cycling in this paper.

We first present the model components; next we set up the optimization problem that is used to compute equilibria.

3.1 Model components

For the corridor, the welfare consists of (1) the gross utility derived from car trips and public transport trips (in euros), (2) the user cost of these trips, (3) the cost of public transport supply and (4) the external costs other than congestion. We first look into each of these components below.

Gross utility derived from trips

Preferences of travelers are represented by a quasi-linear utility function U. It consists of the utility derived from other goods (money m) and the sub-utility function for transport trips. The sub-utility function B is a quadratic function. Since we assume that all individuals travelling in a given OD pair are homogeneous, we have the following utility function for a representative individual travelling in the OD corridor under study:

$$U(m, q_c^p, q_c^o, q_b^p, q_b^o) = m + B(q_c^p, q_c^o, q_b^p, q_b^o) \quad (1)$$

$$\begin{aligned} &B(q_c^p, q_c^o, q_b^p, q_b^o) = \\ &\left[a_c^p q_c^p - 0.5 b_c^p (q_c^p)^2\right] + \left[a_c^o q_c^o - 0.5 b_c^o (q_c^o)^2\right] + \left[a_b^p q_b^p - 0.5 b_b^p (q_b^p)^2\right] + \left[a_b^o q_b^o - 0.5 b_b^o (q_b^o)^2\right] \\ &- i_c^{po} q_c^p q_c^o - i_b^{po} q_b^p q_b^o - i_{cb}^p q_c^p q_b^p - i_{cb}^o q_c^o q_b^o - i_{cb}^{po} q_c^p q_b^o - i_{bc}^{po} q_b^p q_c^o \end{aligned} \quad (2)$$

q_j^i stands for the number of trips demanded by the representative individual in period i using mode j. The superscripts p and o represent the peak and off-peak period respectively and the subscripts c and b represent the car mode and the bus mode, respectively. Similarly, a_j^i and b_j^i are the parameters for period *i* and mode *j* in the sub-utility function *B*.

Since generalized prices are used, m is the generalized income, which includes money and all time spent on activities other than transport. Moreover, the quasi-linear utility formulation implies the absence of income effects. This is justifiable due to the fact that transport is only a small share (around 10 to 20 percent) of total expenditure. i represents the interaction terms between modes and/or periods, for instance i_{bc}^{po} for the interaction between the bus mode in the peak period and the car mode in the off-peak period. These terms are symmetric as required by consumer theory (the symmetry of the Slutsky matrix). This formulation allows us to derive the willingness to pay for the four transport goods (inverse demand functions)

$$
\begin{aligned}
\frac{\partial U}{\partial q_c^p} &= a_c^p - b_c^p q_c^p - i_c^{po} q_c^o - i_{cb}^p q_b^p - i_{cb}^{po} q_b^o \\
\frac{\partial U}{\partial q_c^o} &= a_c^o - b_c^o q_c^o - i_c^{po} q_c^p - i_{cb}^o q_b^o - i_{bc}^{po} q_b^p \\
\frac{\partial U}{\partial q_b^p} &= a_b^p - b_b^p q_b^p - i_b^{po} q_b^o - i_{cb}^p q_c^p - i_{bc}^{po} q_c^o \\
\frac{\partial U}{\partial q_b^o} &= a_b^o - b_b^o q_b^o - i_b^{po} q_b^p - i_{cb}^o q_c^o - i_{cb}^{po} q_c^p
\end{aligned}
\quad . \tag{3}
$$

User cost of trips

In our model, buses and cars use the same road infrastructure. We start with a separate bus lane, which is the current condition in the chosen corridor. As a result, for a given infrastructure, the time needed for a standardized trip by car is a function of car traffic volume q_c, while the time for a standardized trip by bus is a function of the frequency of buses f_b and the vehicle equivalent of buses (thus the size of buses) $\sigma(s_b)$. The user costs of car use and bus use per trip before taxes and charges are

$$
\begin{aligned}
uc_c^i &= c_c^i + [\alpha_c + \beta(\frac{Nq_c^i}{n^i cap_c} + \frac{out}{cap_c})] \cdot VOT_c^{in} \\
uc_b^i &= ac_b^i + [\alpha_b + \beta(\frac{\sigma(s_b)\cdot f_b^i}{cap_b})] \cdot VOT_b^{in} \left[1 + discom_b \left(\frac{Nq_b^i / n^i}{cs_b(s_b^i)\cdot f_b^i} - 1 \right)\right] + VOT_b^w \frac{60}{2 f_b^i} \quad .(4) \\
(i &= p, o)
\end{aligned}
$$

The user cost of one car trip is the sum of the monetary cost c_c and time cost of the trip. The latter is the in-car value of time VOT_c^{in} multiplied by the travel time $\alpha_c + \beta(\frac{Nq_c^i}{n^i cap_c} + \frac{out}{cap_c})$, where α_c is the free flow trip time, N is the number of potential users in the corridor (number of individuals living along the corridor), n is the number of hours, cap_c is the number of lanes for cars and out is the sum of commercial vehicles and private vehicles per hour whose destination is not the city centre. It is assumed to be a constant.

We have a similar formulation for the user cost of a bus trip but we use the access cost to bus stops ac_b instead of monetary cost, plus the in-vehicle time cost $\alpha_b + \beta(\frac{\sigma(s_b) f_b^i}{cap_b})$ that is augmented with a crowding or discomfort cost as a function of the number of travellers on a bus over the maximum capacity where a traveller can stand or sit comfortably, cs_b, plus the waiting cost $VOT_b^w \frac{60}{2 f_b^i}$. σ is vehicle equivalent depending on the size of the bus s_b, and f_b is the frequency of buses per hour. The user cost of a bus trip is defined as a piecewise function, setting $\frac{Nq_b^i / n^i}{cs_b^i f_b^i} - 1$ to zero when the number of passengers has not reached the "discomfort threshold" (cs_b).[1] This is to ensure that no extra gain from comfort condition can be captured by lowering bus occupancy when the bus occupancy is already low.

Cost of public transport supply

For buses, the total operating costs for a corridor per day is a function of total frequency f and the size s of the buses

$$\begin{aligned} C^b &= FI^b + n^p f_b^p(\mathrm{k}_{b1}^p + \mathrm{k}_{b2}^p(s_b - 45)) + n^o f_b^o(\mathrm{k}_{b1}^o + \mathrm{k}_{b2}^o(s_b - 45)) \\ &= C^b\left[F_b^p(\mathrm{q}_b^p), \mathrm{S}_b(\mathrm{q}_b^p), F_b^o(\mathrm{q}_b^o), \mathrm{S}_b(\mathrm{q}_b^o)\right] \end{aligned} \quad (5)$$

The fixed cost including the maintenance of bus stops is represented by FI^b. One bus can have a decreasing average cost per seat offered when fixed cost k_{b1} per vehicle is large. k_{b2} is the cost of increasing the size of the vehicle beyond the standard of 45 seats.[2] Vehicle costs also differ between peak and off-peak since the size of vehicles and the fleet is typically determined by the needs during the peak - peak load pricing then requires that capacity costs are allocated to the peak period. Returns to scale in public transport are the result of high fixed costs FI and high fixed costs k_{b1} per vehicle.

This type of formulation has been used by Parry & Small (2009), but as we keep the bus routes fixed, we do not need to model the access costs and tradeoff between user access cost and supplier cost. This results in the simpler formulation used by De Borger & Proost (2015), where the access cost cannot be changed. The choice of frequency and capacity of public transport will ultimately be a function of the volumes of public transport. The optimal supply of frequencies is denoted by $F_b^i(\mathrm{q}_b^i), \mathrm{S}_b(\mathrm{q}_b^i)$ so that (5) becomes a function of the quantities of public transport in optimum.[3]

---

[1] $discom_b(x)$ $equals$ 0 $if x < 0$ and x $otherwise$.

[2] The fixed cost per bus k_1 is assumed for a bus with 45 seats.

[3] In the analysis the same size of buses is used for peak and off-peak. It could be optimal to use a combination of small and large buses, but this is an extra complication and probably less important for welfare optimization.

The external costs other than congestion

For cars, external costs other than congestion consist of the external accident and climate costs. We assume that they hold a constant charge per vehicle km, which is equal to the existing fuel taxes, so that these external costs are already internalized in the fuel taxes.

For buses, we have climate costs, accident costs and wear and tear of the road. Most buses are diesel buses and again these are internalized in diesel taxes.

3.2 The welfare optimum

When we can use lump-sum taxes (head taxes) to finance any deficits, and in the absence of any other distortions in the economy, we can formulate the welfare function of each corridor as the total user surplus (before taxes or charges) minus the total costs of public transport provision

$$\begin{aligned}\Omega &= N \cdot B(q_c^p, q_c^o, q_b^p, q_b^o) - N[q_c^p \,\mathrm{uc}_c^p + q_b^p \,\mathrm{uc}_b^p + q_c^o \,\mathrm{uc}_c^o + q_b^o \,\mathrm{uc}_b^o] \\ &- C^b\left[F_b^p(\mathrm{q}_b^p), \mathrm{S}_b(\mathrm{q}_b^p), F_b^o(\mathrm{q}_b^o), \mathrm{S}_b(\mathrm{q}_b^o)\right]\end{aligned} \quad . \qquad (6)$$

Our problem formulation makes sure that the selected quantities are in a user equilibrium. The pricing of trips of different modes, bus frequencies and bus sizes are chosen to maximize welfare, given the current allocation of road space over car and bus lanes.

Optimal pricing, frequencies and bus size

We maximize this social welfare function subject to the constraints of user equilibrium in each mode and period, where τ_j^i is the toll or fare on mode j in period i

$$\begin{aligned}\Omega &= N \cdot B(q_c^p, q_c^o, q_b^p, q_b^o) - N[q_c^p \,\mathrm{uc}_c^p + q_b^p \,\mathrm{uc}_b^p + q_c^o \,\mathrm{uc}_c^o + q_b^o \,\mathrm{uc}_b^o] \\ &- C^b\left[F_b^p(\mathrm{q}_b^p), \mathrm{S}_b(\mathrm{q}_b^p), F_b^o(\mathrm{q}_b^o), \mathrm{S}_b(\mathrm{q}_b^o)\right] \\ &+ \lambda_c^p\left[uc_c^p + \tau_c^p - \frac{dB}{dq_c^p}\right] + \lambda_c^o\left[uc_c^o + \tau_c^o - \frac{dB}{dq_c^o}\right] + \lambda_b^p\left[uc_b^p + \tau_b^p - \frac{dB}{dq_b^p}\right] + \lambda_b^o\left[uc_b^o + \tau_b^o - \frac{dB}{dq_b^o}\right]\end{aligned} \qquad (7)$$

Differentiating with respect to λ and setting them equal to zero, we have

$$\begin{aligned} uc_c^p + \tau_c^p &= \frac{dB}{dq_c^p}, uc_c^o + \tau_c^o = \frac{dB}{dq_c^o} \\ uc_b^p + \tau_b^p &= \frac{dB}{dq_b^p}, uc_b^o + \tau_b^o = \frac{dB}{dq_b^o}\end{aligned} \qquad (8)$$

Differentiating the objective Ω with respect to the number of trips, setting them to zero, and rearranging gives[4]

---

[4] Since there is a separate bus lane, $\frac{\partial uc_b}{\partial q_c} = \frac{\partial uc_c}{\partial q_b} = 0$, that is the user cost of car (bus) users is not affected by the change in the number of bus (car) users.

$$
\begin{aligned}
\tau_c^p &= q_c^p \frac{\partial uc_c^p}{\partial q_c^p} \\
\tau_c^o &= q_c^o \frac{\partial uc_c^o}{\partial q_c^o} \\
\tau_b^p &= q_b^p \frac{duc_b^p}{dq_b^p} + \frac{1}{N} \frac{\partial C^b(F_b^p(q_b^p), S_b(q_b^p))}{\partial q_b^p} \\
\tau_b^o &= q_b^o \frac{duc_b^o}{dq_b^o} + \frac{1}{N} \frac{\partial C^b(F_b^o(q_b^o), S_b(q_b^o))}{\partial q_b^o}
\end{aligned} \tag{9}
$$

The resulting equations show that the optimal tolls for cars equal the marginal external congestion cost imposed on car users. The optimal bus fares consist of the marginal external costs imposed on bus users (taking into account the marginal benefits of higher frequency and crowding costs) and the marginal cost of supplying bus services.

Differentiating the objective Ω with respect to the public transport frequencies in the peak and off-peak periods, we hold the number of standardized trips q and the size of the vehicle s constant to derive the optimal public transport frequencies f

$$
-Nq_b^i \frac{\partial uc_b^i}{\partial f_b^i} = n^i(k_{b1}^i + k_{b2}^i(s_b - 45))
$$

$$
(i = p, o)
$$

$$
\frac{\partial uc_b^i}{\partial f_b^i} = VOT_b^{in} \frac{\beta\sigma(s_b^i)}{cap_b}(1 + discom_b(\frac{Nq_b^i}{n^i cs_b(s_b^i)\cdot f_b^i} - 1))
$$

$$
-[\alpha_b + \beta(\frac{\sigma(s_b^i) f_b^i}{cap_b})]\cdot VOT_b^{in}\cdot(discom_b \frac{Nq_b^i}{n^i cs_b(s_b^i)\cdot(f_b^i)^2}) - VOT_b^w \frac{60}{2(f_b^i)^2}
$$

(10)

From (10), we see that an increase in bus frequency introduces three effects[5]

(i) the decrease in bus waiting time $Nq_b^i \cdot VOT_b^w(\frac{60}{2(f_b^i)^2})$,

(ii) the decrease in the cost of the in-vehicle time (reduced crowding in the buses and increased congestion on the road which increases the in-vehicle travel time)

$$
Nq_b^i[\alpha_b + \beta(\frac{\sigma(s_b^i)\cdot f_b^i}{cap_b})]\cdot VOT_b^{in}\cdot(discom_b \frac{Nq_b^i}{n^i cs_b(s_b^i)\cdot(f_b^i)^2})
$$

$$
-Nq_b^i \cdot VOT_b^{in} \frac{\beta\sigma(s_b^i)}{cap_b}(1 + discom_b(\frac{Nq_b^i}{n^i cs_b(s_b^i)\cdot f_b^i} - 1)),
$$

(iii) the increase in operating costs of bus $-n^i(k_{b1}^i + k_{b2}^i(s_b - 45))$.

Rearranging the first order condition with respect to bus frequency gives the optimal bus frequency in terms of the number of standardized trips of different modes as

[5] The part of the piecewise function accounting for the number of bus passengers being below the "discomfort threshold" is ignored here.

$$f_b^i = \sqrt{\frac{\frac{(Nq_b^i)^2}{n^i cs_b(s_b^i)} \cdot discom_b \cdot VOT_b^{in} \cdot \alpha_b + 30 \cdot Nq_b^i \cdot VOT_b^w}{Nq_b^i \cdot VOT_b^{in} \frac{\beta\sigma(\mathrm{s}_b)}{cap_b}(1-discom_b) + n^i(k_{b1}^i + k_{b2}^i(s_b - 45))}} \tag{11}$$

It is not surprising that the optimal bus frequency increases with the value of waiting time and decreases with the operating cost per bus. Regarding the value of in-vehicle time for buses, the direction of change of bus frequency depends on the discomfort coefficient.

Similarly, the optimal bus size can be computed given the number of trips and bus frequencies. (For simplicity, cs_b has the value of 30, which is two-thirds of the bus size, and $\sigma = S_b/18$ in the following expression.)

$$S_b^i = \sqrt{\frac{\frac{(Nq_b^i)^2}{n^i f_b^i}\frac{3}{2} \cdot discom_b \cdot VOT_b^{in}\alpha_b}{Nq_b^i \cdot VOT_b^{in} \frac{\beta f_b^i}{cap_b}\frac{1}{18}(1-discom_b) + n^i(k_{b1}^i + k_{b2}^i(s_b - 45))}} \tag{12}$$

4 MODEL CALIBRATION

4.1 Values of parameters

We calibrate our model and solve the optimization problem numerically, using data from the corridor connecting the suburban areas of Nacka and Värmdö to the Stockholm city centre. We use incoming trips to the city centre and assume that outgoing trips are symmetrical. The average travel distance of 14 km is taken from the travel surveys conducted before and after the introduction of congestion charges. Table 1 lists the values of parameters used in the calibration. All monetary values in Swedish Krona (SEK) are converted to Euro using an exchange rate of 1 SEK = 0.1 €.

The own- and cross-price elasticities for car and public transport are derived from the observed changes in the number of car and public transport passages over the toll cordon (which the corridor under study passes) before and after the Stockholm congestion charges were introduced in 2006 (see Appendix A). According to these calculations the own-elasticities for car use are -0.54 and -0.85 in the peak and off-peak, respectively. These elasticities have remained roughly stable over time, but have perhaps increased slightly since 2006. Note that these elasticities are higher than what is normally observed for car cost elasticity, and in particular for fuel cost elasticity (see for instance (Goodwin et al., 2004)). This is because fuel costs make up around half the marginal cost of driving in Sweden but also because the elasticity of traffic in a particular corridor with respect to the charge is higher than a general price increase in the system, since there are more adaptation mechanisms available such as changing destinations. For public transport we assume an own-elasticity of -0.4. This is what is found in many studies (Litman, 2004) and in the Swedish transport model (Börjesson, 2014).

The cross-price elasticity between car and public transport can be computed from observed data since we know how the price increase for cars influences the public transport volume. Our calculations indicate that the cross-price elasticity between car and public transport $\frac{dq_b}{dP_c}\frac{P_c}{q_b}$ is 0.13 and 0.11 for peak and off-peak, respectively, where P_b is the public transport fare and P_c is the car toll. The symmetry of the Slutsky matrix implies that $\frac{dq_c}{dP_b}\frac{P_b}{q_c}$ equals 0.48 and 0.17 for peak and off-peak. These cross-price elasticities are substantially lower than the cross-price elasticity that would result from the assumption made by Parry & Small (2009) - that 50 percent of the total passenger increase on public transport as a result of a reduced ticket fare are drivers diverting to public transport - given the public transport market share in the given corridor. (The peak market share of public transport is 81% in the corridor under study as well as for all trips to and from the inner city of Stockholm.)

To see this let the own-price elasticity for public transport be $\varepsilon = \frac{dq_b}{dP_b}\frac{P_b}{q_b}$. Parry and Small's assumption implies $dq_c = 0.5dq_b$. If $\frac{q_c}{q_b} = \frac{0.19}{0.81}$ and $dq_c = 0.5dq_b$ the implied cross-price elasticity is $\frac{dq_c / q_c}{dP_b / \mathrm{P}_b} = -\frac{0.5dq_b / (\frac{0.19}{0.81}q_b)}{dP_b / \mathrm{P}_b} = -0.5\frac{0.81}{0.19}\varepsilon = 0.85$.

This is a much higher elasticity than we find for Stockholm.

The cost functions for buses (see Appendix B) are based on data from Public Transport Stockholm (SLL), taking into account the capital and operating costs of buses (cost of drivers and fuel and maintenance). The free flow travel time for cars is computed by assuming 40 km/hour for cars. The free flow time for buses is assumed to be 1.6 times that of cars. The values of time are taken from the Swedish study on this topic (Börjesson & Eliasson, 2014).

We take the bus equivalent as 2.5 cars and the maximum capacity of a bus that can sit everyone comfortably (discomfort threshold) as 30 when the bus size is 45 seats. However, both the bus equivalent and the discomfort threshold are functions of bus size when bus size is not fixed, as is the case in later sections. The discomfort coefficient for bus trips is calibrated from the observation that the in-vehicle bus trip time cost is increased by 50% if the bus is full.[6]

Parameter	Notation	Value
Number of peak hours per d	n^p	4 hours/day
Number of off-peak hours per day	n^o	9 hours/day
Length of a standardized trip (one-way)	L	14 km
Number of representative individuals in the corridor	N	134207 individuals[7]
Own generalized price elasticity, peak car trips	e_c^p	-0.54
Own generalized price elasticity, off-peak car trips	e_c^o	-0.85

[6] This is consistent with values in Wardman & Whelan (2011) since we assume that most travellers in the corridor are commuters and that some passengers stand even if some seats are unoccupied. Also, we experiment with the case that the in-vehicle bus trip time cost is increased by 100% due to discomfort, see appendix E.

[7] Population in the municipalities of Nacka and Värmdö, Statistics Sweden, http://www.scb.se/

Own generalized price elasticity, peak bus trips	e_b^p	-0.4
Own generalized price elasticity, off-peak bus trips	e_b^o	-0.4
Cross generalized price elasticity between peak and off-peak car trips	e_{cc}^{po}	0.1
Cross generalized price elasticity between peak car trips and peak bus trips	e_{cb}^{pp}	0.13
Cross generalized price elasticity between peak car trips and off-peak bus trips	e_{cb}^{po}	0.05
Cross generalized price elasticity between off-peak car trips and peak bus trips	e_{cb}^{op}	0.05
Cross generalized price elasticity between off-peak car trips and off-peak bus trips	e_{cb}^{oo}	0.11
Cross generalized price elasticity between peak and off-peak bus trips	e_{bb}^{po}	0.1
Fixed cost of bus services (bus stops)	FI_b	0 euro/trip
Cost of supplying bus frequency, independent of bus size, in the peak period	k_{b1}^p	144.6 euro/trip
Cost of supplying bus frequency, dependent of bus size, in the peak period	k_{b2}^p	1.37 euro/extra passenger per trip
Cost of supplying bus frequency, independent of bus size, in the off-peak period	k_{b1}^o	70 euro/trip
Cost of supplying bus frequency, dependent of bus size, in the off-peak period	k_{b2}^o	0 euro/extra passenger per trip
Free flow time for a standardized car trip	α_c	20.88 minutes/trip
Free flow time for a standardized bus trip	α_b	33.41 minutes/trip
Bus equivalent to number of cars	σ	2.5 cars/bus
Value of in-vehicle time, car	VOT_c^{in}	0.2 euro/minute
Value of in-vehicle time, bus	VOT_b^{in}	0.12 euro/minute
Value of waiting time for a bus	VOT_b^w	0.204 euro/minute
Monetary cost per car trip before toll	c_c	2.1 euro/trip
Access cost to bus	ac_b	2 euro/trip
Discomfort coefficient for bus trips	$discom_b$	1
Maximum capacity of a bus that can sit everyone comfortably (discomfort threshold)	cs_b	30

Table 1: Value of parameters

4.2 The baseline case

There are three lanes in each direction including one dedicated bus lane. Therefore, cap_c and cap_b are set to 2 and 1 respectively. We calibrate the parameter in the congestion function, β, of the lanes such that they are consistent with the observation that a car trip in the peak has twice (208%) the travel time of the off-peak (See calculation in Appendix C).[8] The numbers of standardized trips by car and public transport are shown in Table 2 and are taken from the large travel survey conducted in Stockholm in 2005 before the introduction of congestion pricing. Since the traffic has increased since then, we uniformly adjust trip frequencies for both car and public transport upward by 5%. Taking into account the response to congestion charges

[8]Source tomtom: http://news.cision.com/se/tomtom/r/kotider-for-dagspendlarna-i-stockholmsomradet-har-okat---brommaplan-ny-varsting,c9222853

introduced after the survey, we reduce the number of standardized car trips during both peak and off-peak periods by 30% and increase the number of standardized bus trips in both periods by 5%.

The bus frequencies are calibrated to match the observations that buses are fully occupied in the peak period and only 20% of seats are occupied in the off-peak period, giving an overall occupancy of around 40% throughout the day. The car tolls and bus fares used in calibration are the current tolls and bus fares.[9]

At this point, we take the number of lanes allocated to different modes and the size of buses as given.

Parameter in the base case	Notation	Value
Additional time needed for an extra standardized trip	β	0.01975
Number of peak car trips per representative individual per day	q_c^p	0.0105 trips per person per day
Number of off-peak car trips per representative individual per day	q_c^o	0.0150 trips per person per day
Number of peak bus trips per representative individual per day	q_b^p	0.0453 trips per person per day
Number of off-peak bus trips per representative individual per day	q_b^o	0.0288 trips per person per day
Toll per peak car trip	τ_c^p	1.8 euro/trip
Toll per off-peak car trip	τ_c^o	1.0 euro/trip
Fare per peak bus trip	τ_b^p	2.175 euro/trip
Fare per off-peak bus trip	τ_b^o	2.175 euro/trip
Bus frequency, peak	f_b^p	67.49 buses/hour
Bus frequency, off-peak	f_b^o	47.72 buses/hour

Table 2: The baseline case

5 RESULTS

We proceed in three steps. In section 5.1 we first look into the effects of marginal changes in the policy parameters (frequencies, tolls, fares) separately. This helps in understanding the main inefficiencies in the baseline equilibrium. In section 5.2 we look for optimal combinations of policy parameters, exploring different second-best equilibria. In section 5.3 we explore the impact of bus lane allocation and bus size. In section 5.4 we present some sensitivity studies.

5.1 Direction of marginal changes

In this section we explore the gains and losses associated with a marginal increase in frequencies, tolls and fares, assuming the current allocation of road space over car and bus lanes.

[9] It is assumed that 90% of trips are made by passengers using monthly tickets (800SEK/month, 40 trips) and the rest using single tickets (37.5SEK/trip).

Frequency

A first exercise is to keep all tolls, fares and traffic volumes fixed and explore the gains and losses associated with a marginal increase of the bus frequencies in the peak and off-peak periods. We report results, in terms of benefits and costs per day, in Table 3. An increase in frequency will reduce waiting time and crowding in the bus. It will also increase the cost of supplying extra vehicles: driver, fuel, maintenance and capital costs. During the peak period, a marginal increase in the bus frequency (i) reduces bus waiting time cost by 8.16 euros, (ii) reduces in-vehicle time costs (reduced discomfort but increased trip time due to more road congestion) by 541.18 euros, and (iii) increases the operating cost by 578.35 euros. This means that there is no obvious welfare gain in adjusting the frequency in the peak – given the present volumes of passengers.

In the off-peak it makes even less sense to increase the frequency: there is less congestion and the passenger load on a bus has not reached the discomfort threshold. So the gain from increasing frequency is limited to the decrease in waiting time, 10.39 euros. The cost of increasing frequency is the increase in operating cost, 630 euros, and the increase in road congestion, 22.90 euros. With little gain from the reduction in waiting time but high operating cost and congestion from the extra frequency, welfare would increase substantially if the bus frequency was reduced in the off-peak.

Period\components	Waiting cost	Time cost (in-vehicle) (discomfort + congestion)	Operating cost	Welfare effect
Peak (4 hours)	-8.16	-541.18	+578.35	Small Loss
Off-peak (9 hours)	-10.39	+22.90	+630.00	Large Loss

Table 3: Cost and benefit of increasing bus frequency (in euros per day)

Tolls and fares

We know that in the first-best scenario, when all charges can be varied, the generalized prices, including tolls and fares, equal the marginal social costs. The first step in analyzing the potential reform is therefore to compute the marginal external cost of increasing the volumes with one passenger each time. Table 4 presents these marginal external costs.

At the baseline, we find that the marginal external congestion cost of a car trip is higher than the current tolls during both the peak and off-peak periods. The reason for this is that the commercial vehicles in the corridor are not sensitive to the tolls and the vehicles in the corridor that do not have the centre as their destination are not charged.

For a bus passenger, things are different. Adding one more passenger (keeping frequency constant) in the peak has an external discomfort cost of 6.61 euro. In the off-peak the marginal external cost of one more passenger is practically zero, as the discomfort threshold has not been reached.

Mode and period\components	MECC& discomfort cost	Current charges	Change needed
Peak car	+4.51	1.80	Up
Off-peak car	+3.56	1.00	Up
Peak bus	+6.61	2.18	Up
Off-peak bus	-	2.18	Down

Table 4: Components of optimal tolls or fares (for given frequencies)

5.2 Optimal pricing and frequency

We have 6 +1 +1 policy variables: two car tolls, two bus fares, two bus frequencies, the size of the bus and the road space allocated to the bus lane. In this section we keep the bus size and allocation of road space fixed and analyze different combinations of tolls, charges and frequencies. Table 5 shows which variables in each scenario are optimized, as well as the resulting tolls, fares, frequencies and occupancies. The variables that are not optimized are kept at baseline values. In Figure 1, welfare results are reported as deviations from the baseline, and subsidies are reported as percentages of the total operating costs. Bus revenues and operating costs are also reported in Figure 1.

When car tolls in the peak and off-peak periods are optimized (scenario (2)), only a small improvement in welfare is observed when compared to the baseline case. This shows that the current car tolls do not leave much room for improvement as long as bus pricing and frequencies are unchanged. Given the high marginal external congestion cost (4.51 euros as reported in Table 4) one would expect a higher optimal peak car toll than 1.1 euro. However, the toll in Table 5 is a second-best toll: increasing the car toll above 1.1 euro sends too many passengers to bus transport during peak periods where the external discomfort cost is also high. Therefore, as long as bus fares and bus frequencies are not optimized, one should refrain from increasing the peak car toll. On the other hand, the same problem does not exist during the off-peak period, where the discomfort threshold has not been reached. In this case, sending more passengers to buses increases welfare, and therefore we observe an optimal off-peak car toll which is close to the marginal external congestion cost.

Optimizing bus fares in both the peak and off-peak (scenario (3)) gives a larger welfare gain than optimizing car tolls, as there are more bus users than car users. Crowding in the buses is the main cause of the optimal bus fare being higher during the peak. On the other hand, there is ample space in off-peak buses and prices during these periods may therefore be zero.

Scenario (4) shows that increasing peak frequency increases welfare because this lowers the discomfort on buses. The welfare gain is small, however, as discomfort gains and higher operation costs almost balance each other out (cfr. Table 3).

Scenario (5) shows that a decrease in off-peak bus frequency increases welfare substantially, because maintaining a high bus frequency with a low number of passengers is costly. Bus occupancy increases from 0.20 in the baseline situation to 0.67 when off-peak frequency is optimized. The welfare gain from optimizing the off-peak bus frequency is much larger than the welfare gain from the optimisation of peak frequency, car tolls or bus fares. Additionally, when the off-peak frequency is optimized, optimization of the peak frequency increases welfare further (scenario (6)).

Once the optimal bus frequencies are in place, scenarios (7) and (8) show that a higher peak toll is optimal because with the increased peak frequency, more passengers can be accommodated without an excessively high discomfort cost. The peak bus fare is lower than in scenario (3) due to lower marginal external discomfort with higher peak frequency. However, the additional instruments of bus fares or car tolls barely bring any welfare increases.

Scenario (9) simultaneously optimizes tolls, fares and frequencies; this is identical to the scenario denoted S(a) in Section 5.3. During the peak, we see higher fares and tolls compared to the baseline (but lower off-peak bus fare). The most important for welfare is an increase in bus frequency during the peak and a strong decrease in bus frequencies during the off-peak. Over 90 percent of the welfare gain in the full optimization (for given bus size and allocation of road space) has already been achieved by adjusting the bus frequencies.

Scenario (10) is an extreme scenario that we present for illustrative purposes only, since it returns to the pre-toll world and looks for the optimal bus fare. It is optimal to have peak load pricing on buses. Despite the higher level of road congestion, it still pays off to discourage passengers from using peak buses and send them to off-peak buses. This is the result of high external discomfort in peak-period buses and a relatively low cross-elasticity to car use as in scenario (3).

Scenario (3) computes the optimal subsidy for the current prevailing situation in Stockholm. Hence, this scenario is of particular interest for one of the key issues of this paper: whether buses should still be subsidized considering that Stockholm has congestion charges. The results indicates that the subsidy could indeed be reduced to 14666 €/day corresponding to 20% of the operating costs (compared to 25859 €/day or 37.5% of the operating costs in the current situation).

Scenario	τ_c^p	τ_c^o	τ_b^p	τ_b^o	f_b^p	f_b^o	$occup_p$	$occup_o$	Operating cost	Subsidy	Subsidy
		€/trip			buses/h		$\frac{passengers}{capacity}$		€/day	€/day	as % of operating cost
(1)Baseline	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	69095	25859	37.43
(2)Car toll	**1.15**	**3.03**	2.18	2.18	67.49	47.72	1.00	0.20	69095	25499	36.90
(3)Bus fare	1.80	1.00	**4.93**	**0.00**	67.49	47.72	0.91	0.22	69095	14666	21.23
(4)Peak freq	1.80	1.00	2.18	2.18	**94.64**	47.72	0.75	0.20	84797	40377	47.62
(5)Off-peak freq	1.80	1.00	2.18	2.18	67.49	**14.25**	1.00	0.67	48010	4840	10.08
(6)Peak + off-peak freq	1.80	1.00	2.18	2.18	**94.94**	**14.04**	0.75	0.67	63751	19391	30.42
(7)Car toll + freq	**1.98**	**3.11**	2.18	2.18	**95.38**	**14.36**	0.75	0.67	64207	19201	29.90
(8) Bus fare +freq	1.80	1.00	**3.76**	**0.00**	**92.16**	**15.70**	0.73	0.67	63189	17883	28.30
(9)Car toll + bus fare + freq (S(a))	**4.44**	**3.48**	**4.96**	**0.87**	**92.11**	**15.78**	0.73	0.67	63209	-3836	-6.07
(10) Bus fare + zero toll	**0.00**	**0.00**	**4.30**	**0.00**	67.49	47.72	0.91	0.22	69095	21578	31.23

Table 5: Optimization of various combinations of tolls, fares and frequencies for given bus size (45 seats)

The total operating cost when optimizing peak and off-peak frequencies is similar to the baseline (scenario (6)) because operating cost savings from lowering off-peak frequency is offset by the increase in operating cost due to higher peak frequency.

In summary we find that bus frequencies are currently highly suboptimal: the bus frequency during off-peak periods is much too high. Adjusting that frequency downwards strongly improves the efficiency of the system. The additional welfare gain that could be realized from pricing instruments is small in comparison to that from off-peak frequency adjustment. Optimizing bus frequencies, despite its importance, neither entails a large increase in operating costs nor a subsidy increase.

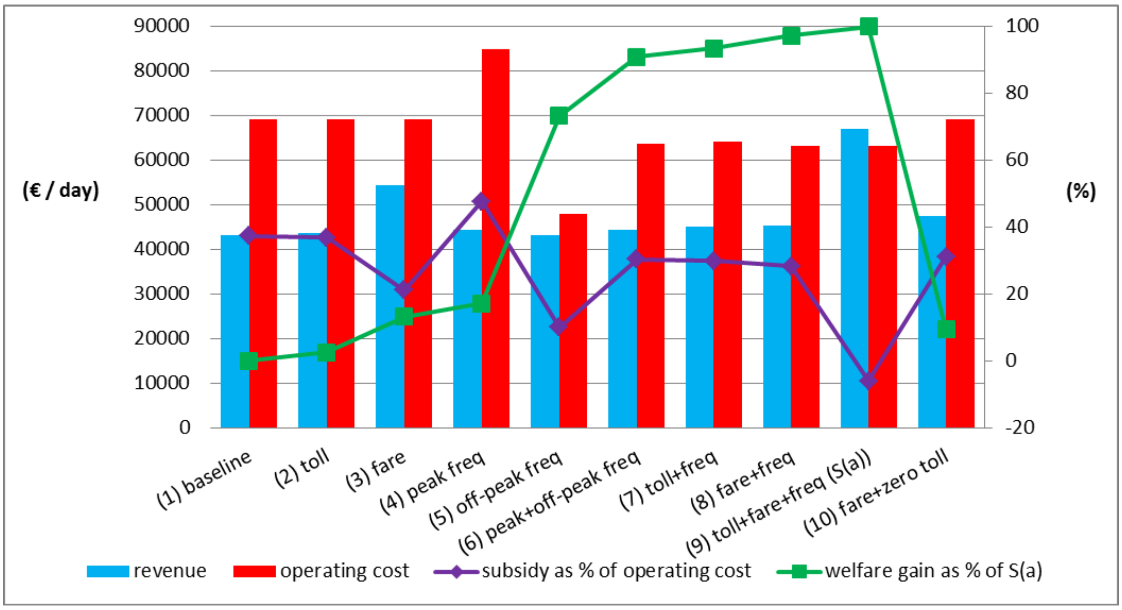

Figure 1: Welfare, bus revenue, operating cost and subsidy values from various combinations of tolls, fares and frequencies for a given bus size (45 seats) for all scenarios presented in Table 5.

5.3 Bus size and bus lane allocation

In this section we compare a few second-best scenarios with potential policy combinations that are commonly observed or can be adopted in reality. The policies involve (i) different bus sizes, (ii) differentiated bus fares, and (iii) endogenous allocation of lanes. An upper bound of five euros is placed on all tolls and fares. Given these constraints, tolls, fares and frequencies are optimized.

We compare the welfare levels in these scenarios and look for policy implications. Table 6 summarizes these scenarios (a)-(d). In all scenarios under study, the buses serving the corridor may have either 45 or 84 seats, and the bus size is the same during both the peak and off-peak. Each of the four scenarios (a)-(d) therefore appears in two versions, with large buses (L) and small buses (S).

Table 7 reports the tolls, fares, frequencies, bus occupancy rates, and allocation of bus and car lanes for each of the 8 scenarios. Note that scenario (a) for small buses (S(a)) corresponds to scenario (9) in Table 5 of the previous section.

Figure 2 reports the bus revenue and operating cost in euros, welfare gain as a percentage of maximum welfare gain (reached in scenario (c) with large buses), and subsidy as a percentage of total operating cost in each scenario.

Figure 3 and Figure 4 show the number of trips and the mode split for both the peak and off-peak, respectively.

	Bus size	Bus fares	Allocation of capacities
(a)	fixed at 45 or 84	differentiated in peak and off-peak	fixed at 1 bus lane + 2 car lanes
(b)	fixed at 45 or 84	Uniform	fixed at 1 bus lane + 2 car lanes
(c)	fixed at 45 or 84	differentiated in peak and off-peak	variable, $cap_b + cap_c = 3$
(d)	fixed at 45 or 84	Uniform	variable, $cap_b + cap_c = 3$

Table 6: Second-best scenarios (a)-(d)

Fares, tolls, welfare and subsidies and the allocation of road space

In all scenarios, optimal frequencies, tolls and bus fares are all lower for large buses. Bus fares are lower because the marginal cost of discomfort is lower. Since more bus passengers can be served using large buses, there is less car congestion and car tolls can therefore be slightly lower.

In scenario (a) welfare is 62% higher with large buses. The situation for the subsidy percentage is similar due to the fact that the revenue almost covers the entire operating cost in the large bus scenario.

In scenario (b) the bus fare is uniform, i.e. the same fare applies for peak and off-peak. For both small and large buses there is only a small negative impact on welfare compared to scenario (a). The optimal peak toll in scenario (b) is lower than in (a) to prevent car users from diverting to bus transport, which would cause discomfort due to crowding. This is less of a concern in the off-peak period, and the off-peak tolls are therefore similar in scenario (a) and (b). Moreover, subsidies are much higher in scenario (b) due to the low optimal uniform fare (see Figure 2). The welfare is higher in scenario L(b) than in scenario S(b), and large buses should thus be used if there is no budget concern for the bus operating cost.

In scenario (c) bus fares are differentiated between the peak and off-peak, and the allocation of road capacity is endogenous. We maximize welfare by optimizing for peak and off-peak car tolls, bus fares, bus frequencies and allocation of capacity to buses and cars. We find that the current allocation of road space to buses is suboptimal and it is welfare-improving to allocate more space to cars, in both scenarios of large and small buses (scenario S(c) and L(c) in Figure 2). The re-allocation of capacity is welfare-improving, despite optimal pricing and bus frequencies, because car lanes are heavily congested by traffic types that are insensitive to tolls (commercial vehicles and vehicles having destinations other than the city centre). Non-integer allocation of bus lanes can be implemented by reducing the proportion of dedicated bus lanes in the corridor.

The welfare for the large bus scenario L(c) is the maximum welfare that can be achieved with the combination of policies that we have, and is therefore the reference point in welfare comparison.

In scenario (d) the bus fare is again uniform. As in scenario (b), we find a small welfare loss, but the differentiation of bus fares between peak and off-peak increases welfare marginally. It is welfare-improving to give more road space to cars as in scenario (c).

Scenario	τ_c^p	τ_c^o	τ_b^p	τ_b^o	f_b^p	f_b^o	$occup_p$	$occup_o$	cap_c	cap_b	Operating cost	Subsidy	Subsidy
		€/trip			buses/h		$\frac{passengers}{capacity}$		number of lanes		€/day	€/day	as % of operating cost
BL	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	2.00	1.00	69095	25859	37.43
Small bus (45)													
S(a)	4.44	3.48	4.96	0.87	92.11	15.78	0.73	0.67	2.00	1.00	63209	-3836	-6.07
S(b)	2.56	3.35	2.59	2.59	95.16	14.20	0.75	0.67	2.00	1.00	63982	10843	16.95
S(c)	2.63	2.42	5.00	0.72	84.92	15.47	0.73	0.67	2.62	0.38	58862	-3027	-5.14
S(d)	1.38	2.56	2.98	2.98	87.80	13.67	0.76	0.67	2.61	0.39	59393	1627	2.74
Large bus (84)													
L(a)	4.37	3.47	4.04	0.00	55.83	8.55	0.67	0.67	2.00	1.00	49579	-989	-2.00
L(b)	2.48	3.34	1.82	1.82	59.68	8.53	0.67	0.60	2.00	1.00	52606	14209	27.01
L(c)	3.50	2.72	4.84	0.13	51.35	8.40	0.67	0.67	2.60	0.40	45934	-10803	-23.52
L(d)	1.27	2.56	2.16	2.16	55.93	7.78	0.67	0.63	2.59	0.41	49168	6016	12.24

Table 7: Results in second-best scenarios including different bus sizes and allocation of bus lanes

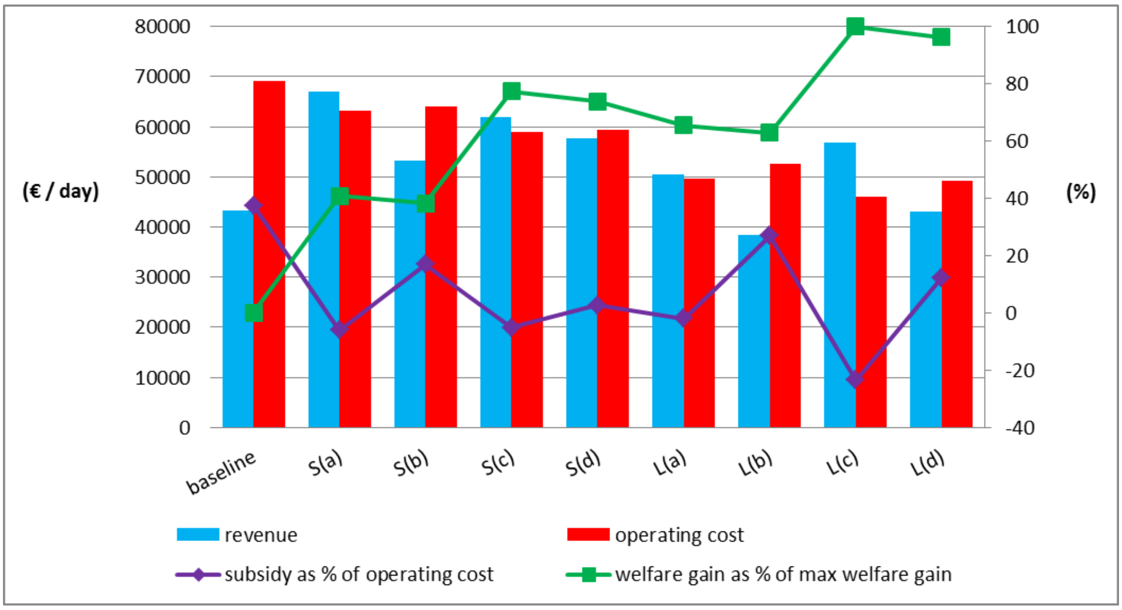

Figure 2: Welfare, bus revenue, operating cost and subsidy values in second-best scenarios

Frequency, crowding and modal split

In this section we explore the frequencies for the peak and off-peak more closely, and discuss crowding and modal split. Table 7 shows that when the peak and off-peak bus fares are uniform (scenarios (b) and (d)) there is a larger difference between peak and off-peak bus frequencies. Uniform fares result in more bus trips during the peak and less trips during the off-peak compared to the scenarios with time-differentiated fares, and the frequencies are adjusted accordingly.

The optimal bus frequency is lower when large buses are used. The benefit of increasing bus frequency is smaller when the large buses are used, because there is less crowding. Moreover, the cost of increasing frequency is higher for large buses due to (i) a higher operating cost and (ii) a higher congestion cost since it takes up more road space than a small bus. This applies to both peak and off-peak periods.

According to Table 7 the occupancy ratio during the peak is 0.73-0.76 for small buses and 0.67 for large buses. These peak occupancy ratios mean that not all seats are filled, while the discomfort levels at these ratios implies an increase in the time cost of a bus ride. In the off-peak period, the occupancy ratio is 0.67 for small buses and 0.60-0.67 for large buses. As the discomfort thresholds are set at two-thirds the number of seats on a bus (30 for a small bus and 56 for a large bus), there is no discomfort in off-peak bus trips.

Figure 3 and Figure 4 show the number of trips per hour and the modal split for peak and off-peak, respectively. (See appendix D for these numbers in greater detail.) Scenarios (b) and (d) are constrained by uniform bus fares, resulting in more bus trips (and a higher market share for bus transport) during the peak and less bus trips (and a lower market share for bus transport) during the off-peak. Large buses result in a higher market share for bus transport during the peak, due to lower fares and less crowding.

To summarize, it increases welfare to use large buses and allocate less road space to buses, regardless of whether fares are uniform or not. Time-differentiated fares have only a small impact on welfare. Uniform bus fares result in a larger difference between optimal peak and off-peak bus frequencies and the resulting market shares for bus transport. The optimal frequency for large buses is lower than that for small buses, in both the peak and off-peak. The resulting bus market share for the peak is still higher for large buses, due to less crowding and lower fares. Optimal frequency and prices result in occupancy ratios in the range of 0.6-0.8.

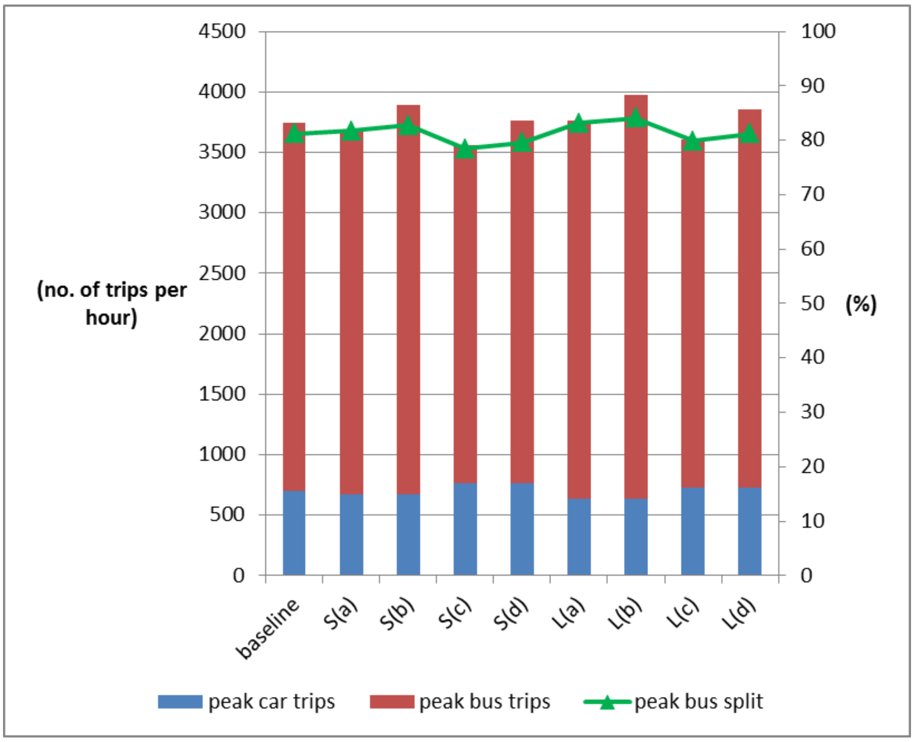

Figure 3: Number of peak trips and modal splits

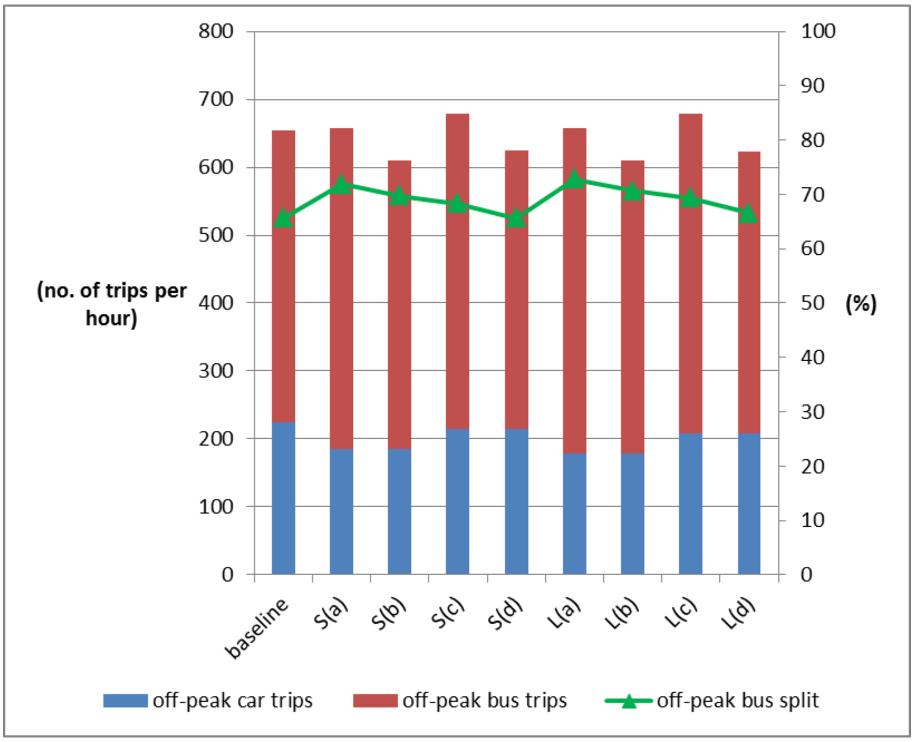

Figure 4: Number of off-peak trips and modal splits

5.4 Sensitivity analysis

We perform sensitivity tests on two key parameters: the discomfort penalty (coefficient) and the cross-price elasticity between the car mode and the bus mode. First, we increase the discomfort coefficient such that a full bus increases the in-vehicle value of time by 200% (instead of 100%). Next, we experiment with the cross-price elasticity between bus and car modes. We increase the cross-price elasticity $\frac{dq_b}{dP_c}\frac{P_c}{q_b}$ from, in the base case, 0.48 in the peak and 0.17 in the off-peak to (i) 0.75 in the peak and 0.30 in the off-peak [10] and (ii) 1.49 in the peak and 0.61 off-peak[11].

The results are presented in detail in Appendix E, and overall the same types of mechanisms are at work. Some main conclusions can be drawn from the sensitivity analyses. First, higher discomfort from crowding implies a higher optimal bus frequency during the peak. It also implies lower subsidies during the peak when the car toll remains unchanged (in particular when car toll is zero) and more road space allocated to bus lanes. Higher cross-price elasticity implies a lower optimal peak car toll, and a higher optimal peak frequency. It also implies a higher optimal subsidy, particularly when car toll is zero. In scenario 9 (optimizing fare, tolls and frequency) and the highest cross-price elasticity, the optimal subsidy is higher than in any other scenario (but the original analysis in Table 5 results in a subsidy that is even negative for scenario 9!).

[10] The symmetry of Slutsky matrix implies that $\frac{dq_c}{dP_b}\frac{P_b}{q_c}$ equals 0.2 for both the peak and the off-peak.

[11] The symmetry of Slutsky matrix implies that $\frac{dq_c}{dP_b}\frac{P_b}{q_c}$ equals 0.4 for both the peak and the off-peak.

5.5 Policy lessons for Stockholm

In section 5.2 and 5.3 we have analyzed 16 scenarios. The scenario with the highest welfare is scenario L(c): large buses, a bus fare differentiated between peak and off-peak, and less road space allocated to bus lanes. Table 8 summarizes the welfare effects of the different scenarios. The following policy results emerge for the corridor under study:

1. Targeting pricing only is insufficient, and simply optimizing bus frequencies for given prices increases welfare significantly. The main reform needed is a decrease in off-peak frequencies and a slight increase in the peak frequency. Optimal pricing only adds a relatively small welfare gain. Improving bus frequencies becomes even more important when discomfort is higher because of the huge losses associated with it.

2. Cost recovery is not really a concern for bus frequency optimization if the baseline deficit is acceptable. The additional operating costs for higher peak frequency is covered by savings during lower off-peak frequency. Scenarios with uniform bus fares require more subsidies in general because uniform fares lead to more peak passengers and less off-peak passengers, which increases operating costs.

3. *The best pricing reform consists of higher peak and off-peak tolls combined with higher peak bus fares and free off-peak bus services.* Higher peak and off-peak car tolls increase welfare given the current road congestion. The optimal peak bus fare is higher than the baseline due to the high operating costs from increased capacity and high crowding discomfort during the peak. The off-peak bus fare should be low or zero since crowding in the off-peak buses is low. However, the welfare increase from better pricing schemes is small compared to the welfare gain that can be obtained from an optimal service frequency.

4. It is beneficial to switch from the existing bus fleet of smaller 45-seaters to larger buses with 84 seats. Increasing bus size lowers user costs (less crowding and lower congestion on the road due to the lower market share for car transport) and operating costs (per seat) of the bus service. Using large buses increases welfare gain considerably.

5. Giving more road space for car use is welfare-improving because existing car lanes are congested during both the peak and off-peak periods. This re-allocation could bring a further welfare gain comparable to the switch to larger buses. This conclusion, however, is dependent upon the manner in which we modelled car congestion[12].

6. *The optimal subsidy* is sensitive to cross-price elasticity. In the current road tolling and bus frequency regime (scenario (3)), optimal fares point to a lower subsidy rate (21%) than the current one of 37%. Increasing the cross-price elasticity, which could be valid for other corridors, increases the optimal subsidy to 26% and 45%. The effect of the cross-price elasticity on the optimal subsidy is even higher when frequencies are also optimized.

[12] Using a static model assuming a volume delay function rather than a dynamic bottleneck model, taking into account that queues build up upstream of a bottleneck, etc.

Scenarios that optimize	
(1) Baseline	0.0
(2) Toll	1.1
(3) Fare	5.4
(4) Peak frequency	7.0
(5) Off-peak frequency	30.0
(6) Peak and off-peak frequencies	37.2
(7) Toll + frequencies	38.2
(8) Fare + frequencies	39.8
(9) Toll + fare + frequencies	40.9
(10) Fare + zero toll	9.5
Second-best scenarios	
S(a) – small bus + differentiated peak and off peak bus fares + fixed allocation of lanes	40.9
S(b) – small bus + uniform peak and off peak bus fares + fixed allocation of lanes	38.2
S(c) – small bus + differentiated peak and off peak bus fares + variable allocation of lanes	77.3
S(d) – small bus + uniform peak and off peak bus fares + variable allocation of lanes	73.9
L(a) – large bus + differentiated peak and off peak bus fares + fixed allocation of lanes	65.5
L(b) – large bus + uniform peak and off peak bus fares + fixed allocation of lanes	62.8
L(c) – large bus + differentiated peak and off peak bus fares + variable allocation of lanes	100.0
L(d) – large bus + uniform peak and off peak bus fares + variable allocation of lanes	96.3

Table 8: Welfare gains as percentages of maximum welfare gain (L(c))

5.6 Do these results carry over to other cities?

The operating costs and the differences in demand between peak and off-peak applies to commuting corridors in many cities, and high service frequency in the off-peak along with uniform pricing schemes is commonly observed. The policy results above are thus relevant for many cities, at least in Europe.

The optimal subsidy is sensitive to cross-price elasticity, which probably does vary substantially across cities and corridors due to large differences in the car-public transport modal split. The optimal rate of subsidization (20 to 40%) we find for Stockholm is substantially lower than those found for pre-toll London by Parry & Small (2009), who find a subsidy rate of 90% or more for buses in the peak.

The major reason for the different subsidy rate is the large beneficial effect of bus subsidies on car congestion in Parry & Small. This beneficial effect explains 42% of the high subsidy rate, since there was a high external congestion cost per car before the toll. In addition, their high cross-price elasticity implies that one out of two additional passengers attracted by lower fares means that lower bus fares are an effective way to reduce car congestion. In Stockholm, we analyze the optimal subsidy rate in the presence of a toll on cars and we use a lower cross-price elasticity, which implies that only 28% of new bus passengers are former car users.

We can also compare with the results of Basso & Silva (2014) for London. Like Parry & Small (2009), Basso & Silva have a much higher cross-price elasticity than we do. A second major difference with our results is that two of their best policies (dedicated bus lane and congestion toll) are already in place in Stockholm. We discuss subsidization when optimal policies are almost all in place and then the additional power of higher subsidies becomes marginal and is supplied in the correct amount (concerning frequencies and bus size) to offer the most potential for welfare gains.

6 CAVEATS AND IDEAS FOR FURTHER RESEARCH

This paper models the optimal pricing, frequency, bus size and share of road space allocated to bus lanes for a corridor in the presence of congestion pricing of cars. The model is used to look into the reform of bus operations in one representative congested corridor in Stockholm.

There are two simplifications that warrant further discussion. First, when calculating the discomfort from crowding in buses we assume that travellers are evenly distributed across all buses within the peak and off-peak periods. This is a strong assumption for several reasons. First, many bus systems in crowded corridors are subject to bus bunching, which implies that the service frequencies are not regular, leading to an uneven distribution of passengers. Second, travellers may not arrive evenly to the bus stops. Third, there are in fact several parallel bus lines operating in the corridor under study. These bus lines have slightly different starting points within Nacka, but they were simplified to one bus line in the analysis. This will also lead to an uneven distribution of passengers, because some bus lines are more used than others. The effect of this assumption of evenly distributed passengers is that the crowding within buses is underestimated in the model. The possible effect of this underestimation is somewhat covered by the fact that the sensitivity analysis assigns a higher discomfort penalty for crowding. This leads to higher optimal bus frequencies in the peak and more road space allocated to bus lanes.

The second simplification is related to the way we model road congestion. We apply a static model based on a volume delay function, which responds to an increase in traffic volume by predicting a longer travel time in the road segment under study. In such a model the car volumes can exceed the maximum capacity of the road segment; the model responds by predicting a long travel time. However, if the reason for the congestion in the corridor is a city center bottleneck, then congestion can only be modelled in a dynamic way, capturing that the travel times increase due to queues building up upstream of the bottleneck. If this is the case, the travel time for cars will not be reduced even if more space is allocated to them, since the capacity of the bottleneck has not changed.

Of course, one also needs to consider the longer-term effects of public transport policies on labor supply and more generally on agglomeration economics. Labor supply and agglomeration economics is mostly driven by travel costs and accessibility during the peak period (Proost & Thisse, 2016). If fine-tuned road pricing implies only small shifts in working hours, then the agglomeration externalities are not affected significantly by changes in travel costs (Arnott, 2007). Moreover, Anderstig et al. (2016) have shown that road pricing helps in a better match of highly skilled workers. Finally, we point to lower rather than higher subsidies for buses, and therefore there will be a smaller negative feedback effect via labor taxes.

7 NOTATION

Superscript	p = peak, o = off-peak (period)
Subscript	c = car, b = bus (mode)
n^p	number of peak hours (per day)
n^o	number of off-peak hours (per day)
N	number of potential travellers in the corridor
U	utility for a representative individual (per day)
B	sub-utility function from transport for a representative individual (per day)
m	the utility derived from other goods for a representative individual (per day)
q_j^i	flows per day for each individual
a_j^i, b_j^i	parameters in demand function
i_{cb}^{po}	interaction term between peak period car use and off-peak bus use
FP_j^i	full price of using mode *j* in period *i* per trip
C^b	total cost of public transport supply (per day)
FI^b	Fixed cost of the mode *b*
f_b^i	frequency of bus in period *i* (per hour)
s_b	size of buses
cs_b	"discomfort threshold", the number of passengers on a bus where passengers start to be affected by crowding
$discom_b$	discomfort coefficient
k_{b1}^i	cost of frequency supply independent of size of vehicle (per hour)
k_{b2}^i	cost of frequency supply dependent on size of vehicle (per hour)
α_c	free flow time needed for cars
α_b	free flow time needed for buses
β	parameter of congestion
$\sigma(s_b)$	bus equivalent to number of cars, depending on the size of buses
VOT_j^{in}	value of time (in-vehicle) of mode *j*
VOT_b^w	value of time (waiting) of buses
ac_b^i	access cost per trip of buses
c_c^i	money cost of driving (before tax) per trip
uc_j^i	user costs of mode j at period i per trip per individual (before charges)
τ_j^i	fare or tax of mode j at period i per trip per individual
Ω	welfare per day

8 LITERATURE

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Appendix A

Own- and cross-price elasticities

The own-elasticity for cars calculated as $\varepsilon^i = \frac{\ln(v^i{}_2/v^i{}_1)}{\ln(p^i{}_2/p^i{}_1)}$, where v_2 and v_1 are the volumes of non-exempt vehicles and p_2 and p_1 are total travel costs in real terms (adjusted for inflation, deductibility etc.). Index 1 is 2005 and index 2 is 2006. The index *i* refers to peak and off-peak. To get an estimate of the average total travel cost, we note that the median length of trips crossing the cordon was 13 km[13] in both 2005 and 2006 according to travel surveys conducted before and during the congestion pricing trial. Controlling for increases in fuel price, the average marginal driving cost was €0.15/km and thus the median trip cost excluding the charge is € 13·0.15 = € 1.95. The average charge in the peak is €1.8 and €1.0 in the off-peak. The cost thus increased 92% for the peak and 51% for the off-peak. The volume reduction of non-exempt vehicles were 29.7% in peak and off-peak, and the increase in public transport was 9% in the peak and 4.5% in the off-peak (the lower increase in public transport use in the off-peak happened because few non-commuting trips are adapting to public tansport). These numbers imply own-elasticities of -0.54 and -0.84 in the peak and off-peak, respectively. The cross-price elasticities between car and public transport are 0.13 and 0.11 (*log(1.09)/log(1.92)* and *log(1.045)/log(1.51)*) for the peak and the off-peak.

Appendix B

Bus cost functions

Seats in the bus	45
Bus purchasing cost €	350000
Life years of bus	12
Discount rate	0.04
Days of the year	250
Number of trips per bus in the peak	2
Capital cost per year per bus €	37293
Capital cost per peak trip €	74.59
Capital cost per extra seat per peak trip €	1.37
Marginal cost for maintenance and fuel €/km	1
Marginal cost for drivers €/km	1.5
Marginal cost per trip (maintenance, fuel, drivers) €	70
Marginal cost per trip peak €	144.59
Marginal cost per trip off-peak €	70

Table B1: Bus costs

An 84-seat bus costs €600000 and a 45-seat bus is €350000. Assuming 12 years of use and a discount rate of 4 percent, the cost per year of the 84-seat bus is 63931 euro per year, while for a 45-seat bus it is 37,293 per year. Hence the extra seat costs of each seat per trip in the peak (K_{b2}^p) comes to € (63931-37293)/(84-45)/250/2= 1.37.

[13] For trips crossing the cordon twice, this calculation divides trip length by 2.

Appendix C

Speed flow function

We assume the linear travel time function for cars:

$$T = \beta(V) + T_{free}$$

We know from measurements that the travel time in the corridor is 108 percent longer than the free flow travel time: 43.43 and 20.88 minutes, respectively (see reference in the main text). We know from traffic counts that the total car volume is 2076 vehicles, meaning 1038 vehicles for each of 2 lanes and per hour during the two peak hours.[14] This traffic consists of both private and commercial traffic as well as both traffic with a destination within the city and traffic with its destination outside the city center. From the travel survey we have that the total private traffic with destination within the city center is 704 per hour during the peak. Commercial traffic makes up 30 percent of the total traffic. As commercial vehicles may be of different sizes, we adjust the number upward to 40 percent of total traffic for car equivalence. Hence, the total car equivalence is 2283 (1141 per lane). Out of the 2283 car equivalents present on the road, 830 are commercial, 704 are private traffic to the inner city and 749 are private traffic with destinations outside the inner city. This implies

$$\beta = \frac{T - T_{free}}{(V)} = \frac{43.43 - 20.88}{2283/2} = 0.01975.$$

In our model we assume that commercial traffic is completely insensitive to congestion charges.

$$T = \beta(V_{Pin} + V_{Pout} + V_C) + T_{free},$$

where $T_{free} = 20.88$, $\beta = 0.01975$, and the (fixed) commercial traffic volume V_C in car equivalents = 830 (415 per lane). The volume of private traffic with a destination inside the city V_{Pin} varies with congestion charge and public transport fare. In the current situation V_{Pin} = 704 (352 per lane). The volume of private traffic with a destination outside the city is $V_{Pout} = 749$ (375 per lane). We assume that V_{Pout} is not affected by charges, fares or public transport supply, because these trips are not charged with the current charging scheme.

This means that the congestion function is

$$T = \beta(V_{Pin} + V_{Pout} + V_C) + T_{free}$$
$$T = 0.01975\,(V_{pin} + 375 + 415) + 20.88$$

[14] Source: http://www.trafikverket.se/contentassets/cb30927f4c4946658f50c84d0d4db54d/pm_trafik_kvarnholmen_111228.pdf

Appendix D

Additional tables on number of trips, modal shares and components in welfare are presented here.

Number of trips and modal shares

per day	Total number of trips	Peak car trips/h	Off-peak car trips/h	Peak bus trips/h	Off-peak bus trips/h	Peak car split	Peak bus split	Off-peak car split	Off-peak bus split
BL	13367	704	224	3037	429	0.19	0.81	0.34	0.66
Small bus (45)									
S(a)	13277	670	185	3007	473	0.18	0.82	0.28	0.72
S(b)	13278	671	185	3220	426	0.17	0.83	0.30	0.70
S(c)	13234	767	215	2793	464	0.22	0.78	0.32	0.68
S(d)	13149	767	215	2996	410	0.20	0.80	0.34	0.66
Large bus (84)									
L(a)	13439	633	179	3127	479	0.17	0.83	0.27	0.73
L(b)	13434	634	179	3342	430	0.16	0.84	0.29	0.71
L(c)	13313	726	209	2876	470	0.20	0.80	0.31	0.69
L(d)	13316	726	208	3132	414	0.19	0.81	0.33	0.67

Table D1: Number of trips and splits

Components of welfare

(euro per day)	Welfare	Utility	User cost	Operating cost	Bus fare revenue	Subsidy
BL	708027	1350433	960950	69095	43236	25859
Small bus (45)						
S(a)	735529	1344890	546151	63209	67045	-3836
S(b)	733743	1347176	549451	63982	53140	10843
S(c)	760045	1345108	526201	58862	61889	-3027
S(d)	757702	1346483	529389	59393	57766	1627
Large bus (84)						
L(a)	752060	1347380	545741	49579	50568	-989
L(b)	750278	1349040	546156	52606	38397	14209
L(c)	775287	1346606	525385	45934	56737	-10803
L(d)	772798	1349648	527682	49168	43153	6016

Table D2: Components of welfare

Appendix E

<u>Discomfort doubled</u>

	Δ Welfare €/day	τ_c^p €/trip	τ_c^o	τ_b^p	τ_b^o	f_b^p buses/h	f_b^o	$occup_p$	$occup_o$ $\frac{passengers}{capacity}$	Operating cost €/day	Subsidy €/day	Subsidy as % of operating cost
(1)Baseline	0	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	69095	25859	37.43
(2)Car toll	1218	**0.00**	**2.11**	2.18	2.18	67.49	47.72	0.99	0.20	69095	26040	37.69
(3)Bus fare	8314	1.80	1.00	**5.00**	**0.00**	67.49	47.72	0.93	0.22	69095	12510	18.11
(4)Peak freq	31623	1.80	1.00	2.18	2.18	**113.88**	47.72	0.67	0.19	95924	49945	52.07
(5)Off-peak freq	20131	1.80	1.00	2.18	2.18	67.49	**14.25**	1.00	0.67	48010	4844	10.09
(6)Peak and off-peak freq	52010	1.80	1.00	2.18	2.18	**113.94**	**13.76**	0.67	0.67	74568	28664	38.44
(7)Car toll + freq	52660	**1.19**	**2.93**	2.18	2.18	**114.23**	**14.01**	0.67	0.67	74887	28622	38.22
(8) Bus fare +freq	53880	1.80	1.00	**4.03**	**0.00**	**107.28**	**15.44**	0.67	0.67	71774	19839	27.64
(9)Car toll + bus fare + freq (S(a))	54573	**3.84**	**3.32**	**5.00**	**0.72**	**107.81**	**15.51**	0.67	0.67	72122	1384	1.92
(10) Bus fare + zero toll	8404	**0.00**	**0.00**	**5.00**	**0.00**	67.49	47.72	0.92	0.22	69095	13475	19.50

Table E1: Optimization of various combinations of tolls, fares and frequencies for given bus size (Bus size = 45)

	Δ Welfare €/day	τ_c^p €/trip	τ_c^o	τ_b^p	τ_b^o	f_b^p buses/h	f_b^o	$occup_p$	$occup_o$ $\frac{passengers}{capacity}$	cap_c number of lanes	cap_b	Operating cost €/day	Subsidy €/day	Subsidy as % of operating cost
BL	0	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	2.00	1.00	69095	25859	37.43
Small bus (45)														
S(a)	54573	3.84	3.32	5.00	0.72	107.81	15.51	0.67	0.67	2.00	1.00	72122	1384	1.92
S(b)	52661	1.30	2.97	2.25	2.25	114.16	13.98	0.67	0.67	2.00	1.00	74831	26979	36.05
S(c)	76857	2.02	2.27	5.00	0.56	101.65	15.21	0.67	0.67	2.59	0.41	68373	2793	4.08
S(d)	74448	0.02	2.17	2.55	2.55	107.93	13.48	0.67	0.67	2.58	0.42	70910	19306	27.23
Large bus (84)														
L(a)	72471	4.25	3.46	4.09	0.00	59.28	8.47	0.67	0.66	2.00	1.00	52257	-2000	-3.83
L(b)	70935	1.57	3.02	1.31	1.31	62.57	8.47	0.67	0.60	2.00	1.00	54862	26442	48.20
L(c)	93666	3.44	2.73	4.86	0.10	55.48	8.26	0.67	0.67	2.57	0.43	49119	-12056	-24.54
L(d)	91533	0.28	2.22	1.54	1.54	59.39	7.80	0.67	0.63	2.56	0.44	51918	19860	38.25

Table E2: Results in second-best scenarios including different bus sizes and allocation of lanes

Scenarios that optimizes	
(1) Baseline	0.0
(2) Toll	1.3
(3) Fare	8.9
(4) Peak frequency	33.8
(5) Off-peak frequency	21.5
(6) Peak and off-peak frequencies	55.5
(7) Toll + frequencies	56.2
(8) Fare + frequencies	57.5
(9) Toll + fare + frequencies	58.3
(10) Fare + zero toll	9.0
Second-best scenarios	
S(a) – small bus + differentiated peak and off peak bus fares + fixed allocation of lanes	58.3
S(b) – small bus + uniform peak and off peak bus fares + fixed allocation of lanes	56.2
S(c) – small bus + differentiated peak and off peak bus fares + variable allocation of lanes	82.1
S(d) – small bus + uniform peak and off peak bus fares + variable allocation of lanes	79.5
L(a) – large bus + differentiated peak and off peak bus fares + fixed allocation of lanes	77.4
L(b) – large bus + uniform peak and off peak bus fares + fixed allocation of lanes	75.7
L(c) – large bus + differentiated peak and off peak bus fares + variable allocation of lanes	100.0
L(d) – large bus + uniform peak and off peak bus fares + variable allocation of lanes	97.7

Table E3: Welfare gains as percentages of maximum welfare gain (L(c))

Cross-elasticities=0.2

	Δ Welfare €/day	τ_c^p €/trip	τ_c^o	τ_b^p	τ_b^o	f_b^p buses/h	f_b^o	$occup_p$ $\frac{passengers}{capacity}$	$occup_o$	Operating cost €/day	Subsidy €/day	Subsidy as % of operating cost
(1)Baseline	0	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	69095	25859	37.43
(2)Car toll	1366	**0.00**	**3.30**	2.18	2.18	67.49	47.72	0.99	0.21	69095	25643	37.11
(3)Bus fare	3596	1.80	1.00	**4.55**	**0.00**	67.49	47.72	0.92	0.22	69095	18218	26.37
(4)Peak freq	5007	1.80	1.00	2.18	2.18	**95.30**	47.72	0.75	0.20	85175	40767	47.86
(5)Off-peak freq	20137	1.80	1.00	2.18	2.18	67.49	**14.25**	1.00	0.67	48010	4840	10.08
(6)Peak and off-peak freq	25253	1.80	1.00	2.18	2.18	**95.59**	**14.03**	0.74	0.67	64124	19777	30.84
(7)Car toll + freq	26236	**0.44**	**3.21**	2.18	2.18	**95.26**	**14.53**	0.74	0.67	64243	19554	30.44
(8) Bus fare +freq	27124	1.80	1.00	**3.36**	**0.00**	**93.42**	**15.64**	0.73	0.67	63883	22893	35.84
(9)Car toll + bus fare + freq (S(a))	27503	**1.59**	**2.52**	**3.46**	**0.00**	**93.47**	**16.03**	0.73	0.67	64158	21933	34.19
(10) Bus fare + zero toll	2934	**0**	**0**	**3.64**	**0.00**	67.49	47.72	0.92	0.22	69095	28314	40.98

Table E4: Optimization of various combinations of tolls, fares and frequencies for given bus size (Bus size = 45)

	Δ Welfare €/day	τ_c^p €/trip	τ_c^o	τ_b^p	τ_b^o	f_b^p buses/h	f_b^o	$occup_p$ $\frac{passengers}{capacity}$	$occup_o$	cap_c number of lanes	cap_b	Operating cost €/day	Subsidy €/day	Subsidy as % of operating cost
BL	0	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	2.00	1.00	69095	25859	37.43
Small bus (45)														
S(a)	27503	1.59	2.52	3.46	0.00	93.47	16.03	0.73	0.67	2.00	1.00	64158	21933	34.19
S(b)	26412	0.00	2.63	1.33	1.33	95.81	14.81	0.75	0.67	2.00	1.00	64742	36970	57.10
S(c)	51992	0.28	1.60	4.05	0.00	84.81	15.55	0.73	0.67	2.62	0.38	58844	14018	23.82
S(d)	50231	0.00	2.34	2.22	2.22	87.82	14.02	0.75	0.67	2.61	0.39	59628	16282	27.31
Large bus (84)														
L(a)	44319	5.00	3.68	4.38	0.22	56.53	8.70	0.67	0.67	2.00	1.00	50220	-7242	-14.42
L(b)	43220	0.00	2.67	0.58	0.58	59.54	8.59	0.67	0.62	2.00	1.00	52541	40030	76.19
L(c)	67268	3.09	2.56	4.61	0.00	51.62	8.45	0.67	0.67	2.60	0.40	46176	-7095	-15.37
L(d)	65452	0.00	2.40	1.43	1.43	55.55	7.80	0.67	0.65	2.59	0.41	48882	20133	41.19

Table E5: Results in second-best scenarios including different bus sizes and allocation of lanes

Scenarios that optimizes	
(1) Baseline	0.0
(2) Toll	2.0
(3) Fare	5.3
(4) Peak frequency	7.4
(5) Off-peak frequency	29.9
(6) Peak and off-peak frequencies	37.5
(7) Toll + frequencies	39.0
(8) Fare + frequencies	40.3
(9) Toll + fare + frequencies	40.9
(10) Fare + zero toll	4.4
Second-best scenarios	
S(a) – small bus + differentiated peak and off peak bus fares + fixed allocation of lanes	40.9
S(b) – small bus + uniform peak and off peak bus fares + fixed allocation of lanes	39.3
S(c) – small bus + differentiated peak and off peak bus fares + variable allocation of lanes	77.3
S(d) – small bus + uniform peak and off peak bus fares + variable allocation of lanes	74.7
L(a) – large bus + differentiated peak and off peak bus fares + fixed allocation of lanes	65.9
L(b) – large bus + uniform peak and off peak bus fares + fixed allocation of lanes	64.2
L(c) – large bus + differentiated peak and off peak bus fares + variable allocation of lanes	100.0
L(d) – large bus + uniform peak and off peak bus fares + variable allocation of lanes	97.3

Table E6: Welfare gains as percentages of maximum welfare gain (L(c))

Cross-elasticities=0.4

	Δ Welfare €/day	τ_c^p €/trip	τ_c^o	τ_b^p	τ_b^o	f_b^p buses/h	f_b^o	$occup_p$ $\frac{passengers}{capacity}$	$occup_o$	Operating cost €/day	Subsidy €/day	Subsidy as % of operating cost
(1)Baseline	0	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	69095	25859	37.43
(2)Car toll	3572	**0.00**	**4.16**	2.18	2.18	67.49	47.72	0.97	0.22	69095	24960	36.12
(3)Bus fare	3456	1.80	1.00	**3.23**	**0.00**	67.49	47.72	0.96	0.22	69095	31377	45.41
(4)Peak freq	5878	1.80	1.00	2.18	2.18	**96.95**	47.72	0.73	0.20	86129	41845	48.58
(5)Off-peak freq	20097	1.80	1.00	2.18	2.18	67.49	**14.25**	1.00	0.67	48011	4838	10.08
(6)Peak and off-peak freq	26096	1.80	1.00	2.18	2.18	**97.24**	**14.01**	0.73	0.67	65066	20840	32.03
(7)Car toll + freq	28515	**0.00**	**3.70**	2.18	2.18	**95.71**	**15.29**	0.71	0.67	64983	20245	31.15
(8) Bus fare +freq	28471	1.80	1.00	**2.12**	**0.00**	**96.66**	**15.43**	0.73	0.67	65621	38803	59.13
(9)Car toll + bus fare + freq (S(a))	30420	**0.00**	**1.47**	**0.62**	**0.00**	**96.20**	**15.35**	0.73	0.67	65308	57546	88.11
(10) Bus fare + zero toll	4901	**0.00**	**0.00**	**1,52**	**0.00**	**67.49**	**47.72**	0.96	0.21	69095	51482	74.51

Table E7: Optimization of various combinations of tolls, fares and frequencies for given bus size (Bus size = 45)

	Δ Welfare €/day	τ_c^p €/trip	τ_c^o	τ_b^p	τ_b^o	f_b^p buses/h	f_b^o	$occup_p$ $\frac{passengers}{capacity}$	$occup_o$	cap_c number of lanes	cap_b	Operating cost €/day	Subsidy €/day	Subsidy as % of operati ng cost
BL	0	1.80	1.00	2.18	2.18	67.49	47.72	1.00	0.20	2.00	1.00	69095	25859	37.43
Small bus (45)														
S(a)	30420	0.00	1.47	0.62	0.00	96.20	15.35	0.73	0.67	2.00	1.00	65308	57546	88.11
S(b)	30343	0.00	1.37	0.00	0.00	96.82	15.22	0.73	0.67	2.00	1.00	65582	65582	100.00
S(c)	53805	0.00	0.79	2.10	0.00	86.14	14.76	0.73	0.67	2.61	0.39	59117	35506	60.06
S(d)	52944	0.00	0.53	0.03	0.03	88.48	14.34	0.75	0.67	2.60	0.40	60210	59539	98.89
Large bus (84)														
L(a)	47185	0.00	2.29	0.13	0.00	57.75	8.72	0.67	0.63	2.00	1.00	51202	49551	96.77
L(b)	47182	0.00	2.27	0.00	0.00	57.95	8.72	0.67	0.63	2.00	1.00	51365	51365	100.00
L(c)	68808	0.00	1.71	1.54	0.00	52.12	7.98	0.67	0.67	2.59	0.41	46278	28248	61.04
L(d)	68300	0.00	1.48	0.00	0.00	54.73	7.85	0.67	0.66	2.58	0.42	48265	48265	100.00

Table E8: Results in second-best scenarios including different bus sizes and allocation of lanes

Scenarios that optimizes	
(1) Baseline	0.0
(2) Toll	5.2
(3) Fare	5.0
(4) Peak frequency	8.5
(5) Off-peak frequency	29.2
(6) Peak and off-peak frequencies	37.9
(7) Toll + frequencies	41.4
(8) Fare + frequencies	41.4
(9) Toll + fare + frequencies	44.2
(10) Fare + zero toll	7.1
Second-best scenarios	
S(a) – small bus + differentiated peak and off peak bus fares + fixed allocation of lanes	44.2
S(b) – small bus + uniform peak and off peak bus fares + fixed allocation of lanes	44.1
S(c) – small bus + differentiated peak and off peak bus fares + variable allocation of lanes	78.2
S(d) – small bus + uniform peak and off peak bus fares + variable allocation of lanes	76.9
L(a) – large bus + differentiated peak and off peak bus fares + fixed allocation of lanes	68.6
L(b) – large bus + uniform peak and off peak bus fares + fixed allocation of lanes	68.6
L(c) – large bus + differentiated peak and off peak bus fares + variable allocation of lanes	100.0
L(d) – large bus + uniform peak and off peak bus fares + variable allocation of lanes	99.3

Table E9: Welfare gains as percentages of maximum welfare gain (L(c))