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The risk-return tradeoff in international stock markets: one-step multivariate GARCH-M estimation with many assets

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We study international asset pricing in a large-dimensional multivariate GARCH-in-mean framework. We examine different estimation methods and find that the two-step estimation method proposed by Bali and Engle (2010) tends to underestimate the risk-return coefficient and the corresponding standard error. We also show that the estimate is improved by one-step estimation and by increasing the cross-sectional dimension. Using stock index returns for up to 24 countries and 4 major currencies in the period 2001–2015, one-step estimation gives a market risk-return coefficient of around 6. The estimate is robust to variations in model specification, data frequency, and the number of stock markets considered.

Keywords: international asset pricing, currency risk, multivariate GARCH-M

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1 Introduction and background

We test a conditional Solnik-Sercu international capital asset pricing model (InCAPM) using a GARCH-in-mean framework on a rather wide range of assets (24 stock market indices and 4 currencies). On the methodology side, we find that the two-step estimation method of Bali and Engle (2010), the only procedure applied hitherto to such a broad menu of assets, delivers risk-return coefficient estimates and standard errors that have substantial downward bias. One-step estimation, although computationally more demanding, mitigates these problems. On the empirical side, using various model specifications, data frequencies, and markets considered, we obtain one-step market risk-return estimates on the order of 6. We also find that, while currency risk is priced—at least for the bigger currencies—the estimated prices of risk do not fit the static model well.

In the static InCAPM, aggregation of asset demand proceeds from the national level to the 'world' level, the economic space that encompasses all countries with mutually open financial markets. Every investor's asset demand is of the usual mean-variance optimum type: the portfolio is efficient in terms of the investor's own real units. But as relative purchasing power parity does not hold, the real returns realized on a given portfolio by agents from different countries are generally not equal. To be able to aggregate, the means and covariances that show up in the demand equations for investors from various countries must be translated into a common real unit, a procedure that introduces covariances with real exchange rates into the total-demand equation (Solnik 1974; Sercu 1980). Assuming that local inflation can be ignored—for instance, if it is uncorrelated with asset returns—these covariances simplify to covariances with percentage changes in the nominal exchange rates of all foreign countries.

For simplicity, consider the case where countries have equal relative risk aversions, θ. The price of currency risk, in the static model, then equals $w_j(1-\theta)$, where w_j is country j's share in the world's total invested wealth. Since even the share of the U.S. in the world's wealth is at most one fourth and θ is well above unity, the price of currency risk should be (i) negative, (ii) much smaller in magnitude than the price of market risk, and (iii) closer to zero the less wealthy the country. It must be realized, however, that in a dynamic model these characteristics are likely to be obscured by other effects. Notably, the exchange rates then play a role not only as variables that pick up country-specific changes in purchasing power, but also as state variables. A falling Euro, for example, may drastically affect the risk-return prospects of erstwhile marginal exporters in Europe and their competitors elsewhere. If investors regard such a shift in the investment opportunity set as undesirable, it must be offset by an extra risk premium on Euro over and above $w_j(1-\theta)$. Obviously, in that case the total price of currency risk is less easily characterized.

There is, by now, an impressive amount of literature on InCAPM tests. To situate our paper, we relate it to two seminal papers: Dumas and Solnik (1995) and De Santis and Gerard (1998). Dumas and Solnik (1995) let returns be partly predictable and test a conditional CAPM estimated by GMM. Their study covers the stock markets of 4 large countries—Germany, Japan, the United Kingdom, and the United States—and the corresponding 3 exchange rates relative to the U.S. dollar. They find that currency risk premia are needed to fit the data, and that the price of currency risk is not constant. De Santis and Gerard (1998) adopt a GARCH-M framework. For the sake of feasibility they limit their study to the same 7 assets. Their conclusions are similar: the prices of currency risk are significant, provided that they are allowed to change over time.

Estimating a multivariate GARCH-M model with a substantially larger dimension in one step is computationally challenging. Bali and Engle (2010) note that a large cross-section of assets becomes more manageable if estimation is carried out in two steps. To our knowledge of current literature, no other work exists on large-dimensional GARCH-M models.

In this paper, we examine the InCAPM coupled with a multivariate GARCH-M model involving stock indices of 24 countries (all with developed financial markets) and 4 major currencies, and we carry out the estimation in one step. In order to make one-step estimation faster, we use analytical log-likelihood derivatives. Our one-step analysis is motivated by simulations in which we compare the one-step estimator and the two-step estimators of Bali and Engle (2010). The latter come in two variants, depending on whether the stage-two SUR panel regression is estimated using unweighted or weighted least squares. Our Monte Carlo analysis reveals that the unweighted estimator tends to underestimate the risk-return coefficient. The bias can be interpreted as errors-in-variables bias, arising from the step-two regressors being generated and, therefore, subject to error. The corresponding standard errors, ignoring that the regressors are generated, are also too small. The weighted two-step estimation variant reduces both problems, but only less than half of the way. The one-step estimates, although not perfect, are found to be much less biased and deliver approximately correct standard errors. Finally, our simulations demonstrate that the estimation of the risk-return coefficient is greatly improved by increasing the cross-section dimension.

In the data we find a positive trade-off between market risk and return. In line with the simulation evidence, the one-step estimate is bigger (around 6) than the two-step estimates (around 3 to 4). Additionally, we detect a significant price of currency risk with some of the larger currencies, like the euro and the British pound. Inconsistently with the static model, though, the premium for the British pound is positive; and while the premium for the euro is negative, it is too large in magnitude to make sense as $w_j(1-\theta_j)$. So if this is to be compatible with an InCAPM at all, it must reflect that currencies additionally act as state variables, with a rising British pound being associated with a deteriorating investment opportunity set that necessitates a risk premium, and vice versa for the euro.

The paper is organized as follows. Section 2 presents some testable hypotheses about the asset pricing models we consider (the InCAPM and variants thereof). One-step and two-step estimation of multivariate GARCH-M models is discussed in Section 3. Section 4 studies their performances in simulations and examines the effect of increasing the cross-section dimension. Section 5 presents the empirical results. Section 6 concludes.

2 Asset pricing models

Following standard practice, we assume that the conditional InCAPM has no additional state variables relative to the static Solnik-Sercu model or that the additional state variables are orthogonal to the market and currency factors, so that they can be omitted. The exchange rates may or may not act as state variables, and the return expectations and covariances change over time.

Let n_c denote the number of countries considered and $n_e < n_c - 1$ the number of bilateral exchange rates vis-à-vis an arbitrarily chosen home country. Let r_t be the vector of excess returns in period t, with $n = n_c + n_e + 1$ elements arranged as follows. The first n_c elements are the excess returns on the market portfolios in the n_c countries.[1] The next $n_f = n_e + 1$ elements are returns on assets that also act as priced factors: the n_e excess returns on short-term deposits denominated in local currencies and measured in U.S. dollars, and the excess return on the world portfolio of all traded stocks.

The general InCAPM equilibrium equation is

$$r_t = \omega + H_t \, L \, B + \epsilon_t, \qquad E_{t-1}(\epsilon_t) = 0, \qquad \mathrm{Var}_{t-1}(\epsilon_t) = H_t, \tag{1}$$

where $E_{t-1}(\cdot)$ and $\mathrm{Var}_{t-1}(\cdot)$ denote conditional moments formed in period $t-1$; ω is a vector of intercepts (predicted by the model to be zero); H_t is the matrix of conditional variances and covariances among all returns in r_t; $L = (0_{n_f \times n_c} : I_{n_f})'$, so that $H_t \, L$ is the matrix of covariances between r_t and the n_f potentially priced factors (n_e exchange rates and the world market); and $B = (B_1, \ldots, B_{n_e}, B_m)'$, where $B_1, \ldots, B_{n_e}$ are the currency risk prices and B_m is the market risk price. In the InCAPM logic, B_m is the wealth-weighted harmonic mean of the countries' relative risk aversions. That is, if all risk aversions are equal, $B_m = \theta$; otherwise, the price of market risk is $B_m = \left(\sum_{c=1}^{n_c} w_c \theta_c^{-1}\right)^{-1}$, where θ_c is the relative risk aversion coefficient of investors from country c and w_c is country c's share in the world's total invested wealth, i.e., $w_c = W_c / \sum_{c=1}^{n_c} W_c$, with W_c representing country c's invested wealth. In the static model, the price of risk of holding deposits denominated in the currency of country c is $B_c = w_c B_m (\theta_c^{-1} - 1)$.

In any CAPM, the vector of intercepts, ω, should be equal to zero (i.e., there should be no pricing errors). Whether to include ω in the regressions is a recurrent issue in the literature on risk-return relation tests. Several authors point out that if the CAPM holds, it is likely to uncover statistically significant prices of risk if ω is excluded; see, e.g., Lanne and Saikkonen (2006) and Guo and Neely (2008). In case there are pricing errors, however, excluding ω typically leads to biased estimates of B and incorrect testing results. Therefore we include ω in the InCAPM and variants thereof, and test for non-zero pricing errors.

In our empirical study we include 24 countries and 4 major currencies: the Japanese yen, the British pound, the euro and the U.S. dollar. Not including more currencies may introduce omitted-variable bias in the estimates, but the bias is likely to economically negligible because a large portion of invested wealth is denominated in one of these currencies; the other countries have wealth weights of a most a few percent, which makes their price of risk almost undetectable. Switzerland, for instance, represents a market of less than one-tenth the size of the U.S. market.[2]

The last asset in r_t is the world market of stocks. If all countries in the world were included, this could be problematic: the world portfolio is a linear combination of these markets, so conditionally on a set of weights, the covariance matrix of the returns on the country portfolios and the world portfolio would be singular in each period. However, in practice it is simply infeasible to include all national markets in the first place, and while in our study the omitted markets represent only a small fraction of world capitalization, the

[1] It is acceptable to work with country portfolios, at least in a static setting, as the model predicts that stocks are held in proportion to an internationally common 'fund', the portfolio held by a log-utility investor. As everyone should hold the world market portfolio of stocks, nothing is gained by introducing individual share returns.

[2] We use market capitalization as a rough proxy for invested wealth. Alternative data like in the IMF's comprehensive portfolio survey make no distinction between amounts invested by (or on behalf of) Swiss residents versus capital managed in Switzerland for non-residents.

weights of the national portfolios in the world index still vary over time. In short, there is always some extra information in the world market index.

In addition to the general InCAPM in equation (1), we also investigate two variants: the one-factor/one-world CAPM and a version with zero slopes in the equations for the currencies. The CAPM is the familiar equilibrium model between financial risk and return in which currency risk is absent or at least not priced:

$$r_t = \omega + H_t \, L \, B_m + \epsilon_t, \qquad E_{t-1}(\epsilon_t) = 0, \qquad \mathrm{Var}_{t-1}(\epsilon_t) = H_t, \tag{2}$$

where r_t is now a vector with $n = n_c + 1$ elements (i.e., without the returns on foreign deposits) and $L = (0_{1\times n_c} : 1)'$. The second variant, which we label InCAPMC, is a departure from the InCAPM logic rather than a special case. In the standard InCAPM, equity and currency markets are assumed to be fully integrated, so in the equations for stocks and currencies there is a common set of prices $B_1, \ldots, B_{n_e}$ and B_m ruling all expected returns. In practice, however, currency traders take positions that are short-lived relative to what happens in stocks; they act more like day traders than portfolio managers. In addition, they tend to deal in a single currency or at most a few closely related ones, and work independently from the equity desks and the other currency desks. Therefore, we also examine a model where the set of risks is considered in the equity markets only, while in the equations for currencies the slopes on all risk factors are set to zero. So this model specifies

$$r_t = \omega + W \, H_t \, L \, B + \epsilon_t, \qquad E_{t-1}(\epsilon_t) = 0, \qquad \mathrm{Var}_{t-1}(\epsilon_t) = H_t, \tag{3}$$

which is identical to (1) except for the presence of W, a diagonal matrix with the elements of $(1_{1\times n_c} : 0_{1\times n_e} : 1)$ along the diagonal, which wipes out the risk factors for the returns on currencies.

3 Multivariate GARCH-in-mean

Since the matrix H_t is not directly observable, estimating any of the models (1)–(3) requires a specification of how H_t evolves over time. Following Bollerslev et al. (1988) and many others, we adopt a multivariate GARCH-in-mean approach. We use the specification

$$H_t = H - (aa' + bb') \odot H + aa' \odot \epsilon_{t-1}\epsilon_{t-1}' + bb' \odot H_{t-1}, \tag{4}$$

where $H = E(\epsilon_t \epsilon_t')$ is the unconditional variance matrix of the errors, a and b are parameter vectors, and $\odot$ denotes the Hadamard product.

To reduce the number of parameters to be estimated simultaneously, we estimate H as the sample variance matrix of r_t. In principle, one should iterate back and forth until convergence between estimating H as the sample variance matrix of $\widehat{\epsilon}_t$, the vector of residuals for a given estimate of $\theta = (\omega', B', a', b')'$, and estimating θ for a given estimate of H. We experimented with the iterative procedure using simulated and empirical data and found that it gives almost the same estimate of H as the sample variance matrix of r_t. Therefore we settle for the latter, which speeds up the computation time.

Given an estimate of H, the remaining parameter, θ, can be estimated by maximizing the Gaussian quasi log-likelihood function, which is (up to an inessential constant)

$$l = -\frac{1}{2} \sum_{t=1}^{T} (\log |H_t| + \epsilon_t' H_t^{-1} \epsilon_t), \tag{5}$$

given the observed excess returns $r_1, \dots, r_T$.

To ensure stationary and positive definiteness of H_t, all elements of $aa' + bb'$ must be smaller than 1 in modulus. We impose this constraint by writing a_i and b_i in polar form

$$a_i = r_i \cos\vartheta_i, \qquad b_i = r_i \sin\vartheta_i,$$

with $r_i \in [0, 1)$ and $\theta \in [0, \pi/2)$. Then $a_i^2 + b_i^2 = r_i^2 < 1$ and

$$|a_i a_j + b_i b_j| = |r_i \cos\vartheta_i r_j \cos\vartheta_j + r_i \sin\vartheta_i r_j \sin\vartheta_j| = |r_i r_j \cos(\vartheta_i - \vartheta_j)| < 1,$$

satisfying the constraint automatically.

3.1 One-step estimation

One-step estimation of θ proceeds by maximizing l. The one-step estimator is consistent and asymptotically normal provided that the estimator of R is consistent and the first two moments of r_t are correctly specified (Bollerslev and Wooldridge 1992). To make the maximization reasonably fast and stable, we use a quasi-Newton method with analytical derivative $\partial l/\partial\theta$, which we now derive. The derivation parallels that in Lucchetti (2002) for the BEKK model.

Consider the model defined by (1) and (4). The differential of ϵ_t follows from (1) as

$$\mathrm{d}\epsilon_t = -(B'L' \otimes I_n)\mathrm{d}h_t - \mathrm{d}\omega - H_t L \mathrm{d}B, \tag{6}$$

where $h_t = \mathrm{vec} H_t$. The log-likelihood summand is

$$l_t = -\tfrac{1}{2}(\log|H_t| + \epsilon_t' H_t^{-1} \epsilon_t),$$

with differential

$$\begin{aligned} \mathrm{d}l_t &= \tfrac{1}{2}(u_t' \otimes u_t' - p_t')\mathrm{d}h_t - u_t' \mathrm{d}\epsilon_t \\ &= W_t' \mathrm{d}h_t + Q_t' \mathrm{d}\theta \end{aligned} \tag{7}$$

where $u_t = H_t^{-1}\epsilon_t$, $p_t = \mathrm{vec} H_t^{-1}$, and

$$\begin{aligned} W_t &= \mathrm{vec} u_t B'L' + \tfrac{1}{2}(\mathrm{vec} u_t u_t' - p_t), \\ Q_t &= (u_t' : u_t' H_t L : 0_{1\times n} : 0_{1\times n})'. \end{aligned}$$

Further,

$$\begin{aligned} \mathrm{d}h_t &= \mathrm{dvec}(aa') \odot (\mathrm{vec}(\epsilon_{t-1}\epsilon_{t-1}') - h) + \mathrm{vec}(aa') \odot \mathrm{dvec}(\epsilon_{t-1}\epsilon_{t-1}') \\ &\quad + \mathrm{dvec}(bb') \odot (h_{t-1} - h) + \mathrm{vec}(bb') \odot \mathrm{d}h_{t-1}, \end{aligned}$$

where $h = \mathrm{vec} H$. Since $\mathrm{dvec}(aa') = Q(a)\mathrm{d}a$ with $Q(a) = a \otimes I_n + I_n \otimes a$, we can rewrite $\mathrm{d}h_t$ as

$$\begin{aligned} \mathrm{d}h_t &= (\epsilon_{t-1} \otimes \epsilon_{t-1} - h) \odot Q(a)\mathrm{d}a + (a \otimes a) \odot Q(\epsilon_{t-1})\mathrm{d}\epsilon_{t-1} \\ &\quad + (h_{t-1} - h) \odot Q(b)\mathrm{d}b + (b \otimes b) \odot \mathrm{d}h_{t-1}. \end{aligned}$$

Using (6), we obtain the recursion

$$\mathrm{d}h_t = F_t \mathrm{d}\theta + G_t \mathrm{d}h_{t-1}$$

where

$$
\begin{aligned}
F_t &= (M_{1t} : M_{2t} : M_{3t} : M_{4t}), \\
G_t &= \mathrm{diag}(b \otimes b) + B'L' \otimes M_{1t}, \\
M_{1t} &= -(a \otimes a \otimes \iota_n') \odot Q(e_{t-1}), \\
M_{2t} &= M_{1t} H_{t-1} L, \\
M_{3t} &= ((e_{t-1} \otimes e_{t-1} - h) \otimes \iota_n') \odot Q(a), \\
M_{4t} &= ((h_{t-1} - h) \otimes \iota_n') \odot Q(b)
\end{aligned}
$$

and ι_n is an $n \times 1$ vector of ones. Therefore,

$$\mathrm{d}h_t = J_t \mathrm{d}\theta$$

where J_t is recursively defined as

$$J_0 = 0, \qquad J_t = F_t + G_t J_{t-1}.$$

From (7), we obtain $\mathrm{d}l_t = (W_t' J_t + Q_t')\mathrm{d}\theta$ and the required derivative follows as

$$\frac{\partial l}{\partial \theta} = \sum_{t=1}^{T} (J_t' W_t + Q_t).$$

For the other models $\partial l / \partial \theta$ is obtained using the formulas above with minor modification.

3.2 Two-step estimation

In view of the difficulty of estimating multivariate GARCH-M models in one step, Bali and Engle (2010) proposed a two-step estimation procedure. The method is based on the DCC model of Engle (2002) instead of (4). In the first step, the conditional variance matrices, H_t, are estimated with a bivariate DCC model fitted to each pair of returns, ignoring that the expected returns are time-varying. In the second step, the expected return equation is estimated as a panel with the estimated conditional variance matrices, $\widehat{H}_t$, as regressors, assuming a seemingly unrelated regression (SUR) error structure. As an alternative, they also suggested to use weighted least squares in the second step, dividing each equation by its estimated conditional standard deviation prior to estimating the panel by SUR. We refer to this as weighted two-step estimation.

In our setting, with (4) instead of the DCC specification of H_t, the first step consists of estimating a and b in (4) directly from the demeaned returns, i.e., assuming $E_{t-1}[(r_t - E(r_t))(r_t - E(r_t))'] = H_t$. This gives $\widehat{H}_t$ and $\widehat{D}_t = (I_n \odot \widehat{H}_t)^{1/2}$, the diagonal matrix with the estimated conditional standard deviations of the returns on its diagonal. Now, considering (1), write the expected return equation as

$$r_t = X_t \vartheta + \epsilon_t, \qquad X_t = (I_n : H_t L), \qquad \vartheta = (\omega' : B')',$$

and let $\widehat{X}_t = (I_n : \widehat{H}_t L)$. The unweighted two-step estimator of ϑ is

$$\widehat{\vartheta}_{\mathrm{u}} = \left(\sum\nolimits_t \widehat{X}_t' \widehat{\Sigma}^{-1} \widehat{X}_t \right)^{-1} \sum\nolimits_t \widehat{X}_t' \widehat{\Sigma}^{-1} r_t,$$

where $\widehat{\Sigma} = T^{-1} \sum_t \widehat{\epsilon}_t(\widehat{\vartheta}_{\mathrm{u}})\widehat{\epsilon}_t(\widehat{\vartheta}_{\mathrm{u}})'$ and $\widehat{\epsilon}_t(\vartheta) = r_t - X_t\vartheta$. The weighted two-step estimator is

$$\widehat{\vartheta}_{\mathrm{w}} = \left(\sum\nolimits_t \widehat{X}_t' \widehat{D}_t^{-1} \widehat{R}^{-1} \widehat{D}_t^{-1} \widehat{X}_t\right)^{-1} \sum\nolimits_t \widehat{X}_t' \widehat{D}_t^{-1} \widehat{R}^{-1} \widehat{D}_t^{-1} r_t,$$

where $\widehat{R} = T^{-1} \sum_t \widehat{D}_t^{-1} \widehat{\epsilon}_t(\widehat{\vartheta}_{\mathrm{w}})\widehat{\epsilon}_t(\widehat{\vartheta}_{\mathrm{w}})' \widehat{D}_t^{-1}$.

The two-step estimators are subject to errors-in-variables bias. Write $\widehat{X}_t = X_t + \zeta_t$, where ζ_t arises from the difference between $\widehat{H}_t$ and H_t. Then, as $T \to \infty$,

$$\mathrm{plim} \widehat{\vartheta}_{\mathrm{u}} = \vartheta + A^{-1} b$$

where

$$\begin{aligned} A &= \mathrm{plim} T^{-1} \sum\nolimits_t \widehat{X}_t' \widehat{\Sigma}^{-1} \widehat{X}_t, \\ b &= \mathrm{plim} T^{-1} \sum\nolimits_t \widehat{X}_t' \widehat{\Sigma}^{-1} (\epsilon_t - \zeta_t \vartheta). \end{aligned}$$

If ζ_t vanishes as $T \to \infty$, we have $b = 0$. But since the first-step estimation ignores the time-varying mean of r_t, in general ζ_t does not vanish asymptotically, and neither does b. Hence, $\widehat{\vartheta}_{\mathrm{u}}$ is expected to be asymptotically biased, and similarly for $\widehat{\vartheta}_{\mathrm{w}}$ and the corresponding two-step estimators in the other models (CAPM and InCAPMC, in conjunction with (4)). The simulations reported in the next section examine the magnitude of the bias.

4 Simulations

In this section, we compare the performance of the three estimation methods discussed: one-step, unweighted two-step, and weighted two-step estimation. We also examine the effect of the cross-sectional dimension, n, on the performance of the estimators; the quality of the standard errors; and the rejection rates of the Wald test of zero pricing errors.

We generated excess returns $r_1, \ldots, r_T$ for $T = 1000$ periods, corresponding to around 19 years of weekly returns, according to the one-factor CAPM model

$$\begin{aligned} r_t &= \omega + H_t \, L \, B_m + \epsilon_t, \qquad \epsilon_t \sim N(0, H_t), \\ H_t &= H - (aa' + bb') \odot H + aa' \odot \epsilon_{t-1}\epsilon_{t-1}' + bb' \odot H_{t-1}, \end{aligned}$$

with $L = (0_{1\times(n-1)} : 1)'$, $B_m = 3$, $\omega = 0_{n\times 1}$, $a = 0.15\iota_n$, $b = 0.98\iota_n$, and H equal to the sample variance matrix of the weekly return data studied further in the next section. We varied the number of assets as $n = 1, 5, 14, 25$. For $n = 1$ the world index is the sole asset. For $n = 5$ we add the stock indices of Germany, Japan, the U.K., and the U.S.; for $n = 14$ we add nine more indices with market weights exceeding 1% in market capitalization, namely Australia, Canada, France, Hong Kong, Italy, The Netherlands, Spain, Sweden, and Switzerland; and, lastly, for $n = 25$ we further add the eleven indices for the other developed markets, Austria, Belgium, Denmark, Finland, Greece, Ireland, Israel, New Zealand, Norway, Portugal, and Singapore, giving 24 countries in total.

We set the number of Monte Carlo replications equal to 200, given that one-step estimation is still time-consuming when n is large (currently around one hour per replication when $n = 25$). The parameters to be estimated are R and $\theta = (\omega', B_m, a', b')'$. The main parameter of interest is B_m, the coefficient of relative risk aversion. This parameter plays a key role in financial economics, but estimation has proved to be rather elusive, with wildly diverging estimates. Our simulations seek to shed light on this issue.

We first present Monte Carlo results relating to the distribution of $\widehat{B}_m$, the estimator of B_m. Table 1 gives the mean, median, and other quantiles, and Figure 1 shows its frequency distribution, with the bins being centered at the odd integers.

Table 1: Distribution of $\widehat{B}_m$ when $B_m = 3$

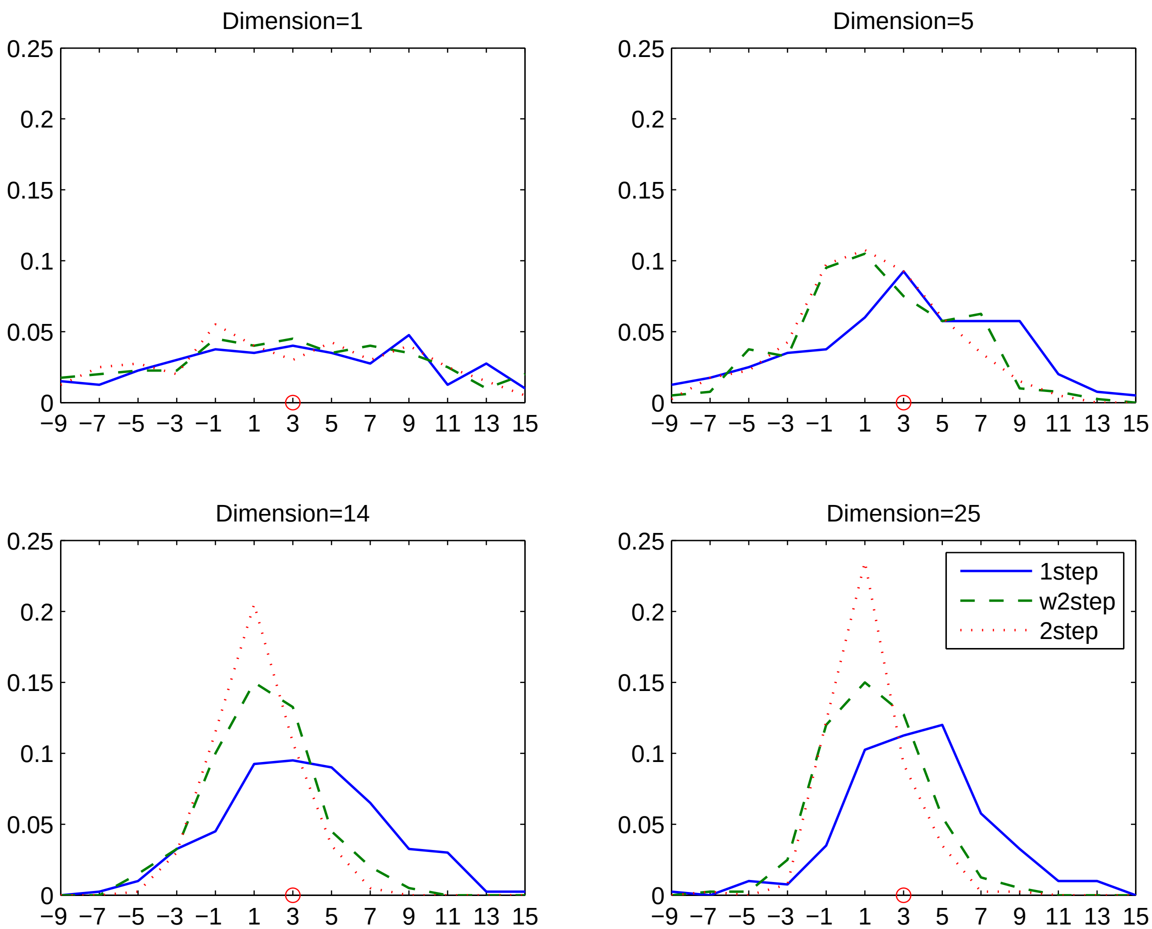
			quantiles				
n	method	mean	minimum	0.25	median	0.75	maximum
25	1step	3.7000	−8.3801	1.2796	3.7514	5.8112	12.7090
25	2step	1.0479	−7.0628	−0.1228	0.8224	2.1623	9.4162
25	w2step	1.4298	−6.8984	−0.1641	1.2375	2.9917	8.8357
14	1step	3.4948	−6.8378	1.0531	3.2433	6.1393	15.9022
14	2step	1.0179	−4.0555	−0.3419	0.8447	2.2492	7.2790
14	2step	1.3931	−5.5461	−0.3703	1.4423	2.9081	8.3919
5	1step	2.9813	−11.3930	−0.6071	2.9848	6.7386	22.3852
5	2step	1.2739	−14.0426	−0.8928	1.2181	3.8376	11.9378
5	w2step	1.5711	−14.8448	−0.7631	1.4538	4.7184	13.4167
1	1step	−28.7639	−4852, 38	−4.0484	3.5987	9.8994	3893, 561
1	2step	3.5874	−1015, 63	−3.3113	3.1172	9.8810	758, 7478
1	w2step	3.6628	−1018, 41	−2.6869	3.2494	9.9554	761, 0504

Note: 200 Monte Carlo replications.

When $n = 25$, the distribution of the one-step estimate (in blue in Figure 1) is reasonably well centered around the true value; its mean and median are 3.70 and 3.75, and its standard deviation is 3.44 (see Table 2). The unweighted two-step estimates, in contrast, are systematically lower, with a mean and median of 1.05 and 0.82, and the distribution (in red in Figure 1) peaks at unity. The attenuation bias, of course, was to be expected given that the regressor $\widehat{H}_t$ is measured with noise. The noisier $\widehat{H}_t$, the more bias is introduced in $\widehat{B}_m$. If the noise is proportional to H_t (or, at least, correlates positively with H_t), one would expect the weighted two-step estimator, which downweights observations with large $\widehat{H}_t$, to reduce the bias. The simulations suggest that this is indeed the case. The mean and median of the weighted two-step estimator are 1.43 and 1.24. Although an improvement on the unweighted estimator, the bias is still large.

Given that the slope coefficient, B_m, is identical for all n assets and this is imposed in the estimation, expanding the set of assets increases the statistical information on B_m, which must benefit the estimators. The simulations results manifestly confirm this. When $n = 1$, all estimates are very poor and the log-likelihood function is extremely flat. Thus, there is very little information about B_m contained in data on the world index only, in line with the findings of Lundblad (2007), who used the U.S. index as the sole index. As n increases to 5, 14, and 25, all estimators considerably improve, as Table 1 and Figure 1 unambiguously show. The simulations nevertheless suggest that, even with nearly twenty years of index data from all developed markets and the world index, it remains very difficult to empirically pin down the relative risk aversion coefficient.

Figure 1: Distribution of $\widehat{B}_m$ when $B_m = 3$



Note: 200 Monte Carlo replications.

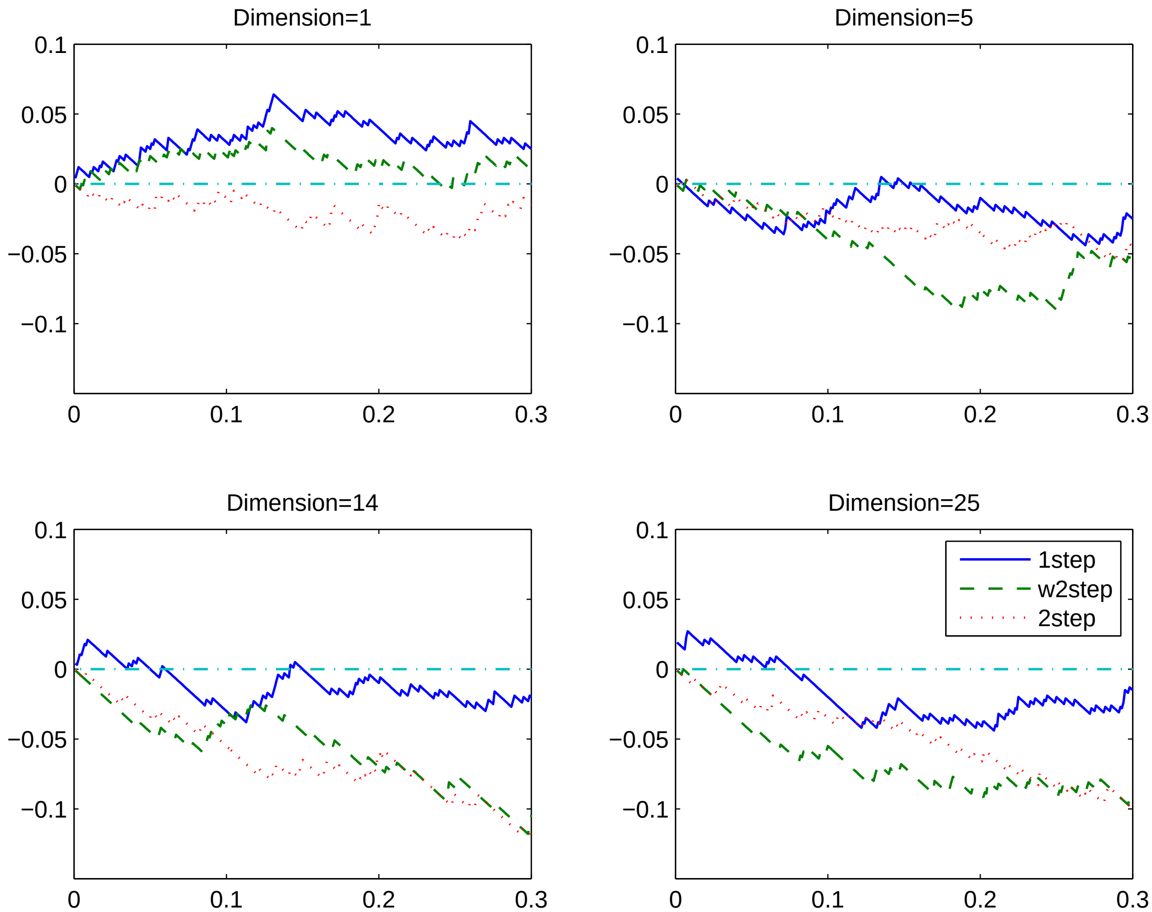
Apart from the coefficient estimates $\widehat{B}_m$ themselves, we also examine the reliability of the standard errors, se($\widehat{B}_m$), computed from the Hessian of the log-likelihood for the one-step estimator and from the SUR procedure for the two-step estimators. Theoretically, neither of these standard errors is fully correct; for the one-step estimator the effect of setting H equal to the sample variance of r_t is ignored, while for the two-step estimators the effect of the first step is ignored. Therefore, one may expect the standard errors to underestimate the true standard deviation of the estimates, given as std($\hat{B}_m$) in Table 2.

The average se($\widehat{B}_m$) across the simulations, one hopes, is close to std($\hat{B}_m$). Table 2 summarizes the distribution of se($\widehat{B}_m$). When $n = 25$, the average values of se($\widehat{B}_m$) are 3.24, 1.46, and 1.98 for the one-step, unweighted two-step, and weighted two-step estimates, with corresponding values of std($\hat{B}_m$) equal to 3.44, 1.91, and 2.46. Thus, all standard errors are too small on average. The distortion is mild for the one-step estimator (around 6%), while it is bigger for the two-step estimators (around 25%). We also note that the relative distortion of the standard errors tends to slightly increase with n.

Table 3 reports the mean of $\widehat{\omega}$. Except for $n = 1$, the one-step estimator tends to slightly underestimate ω, but its mean still remains very close to the true value, $\omega = 0$. The two-step estimates, in contrast, tend to substantially overestimate ω, with estimated values on average around 0.1 for the unweighted estimator and slightly higher for the weighted estimator.

Finally, we examine the properties of the Wald test of the null hypothesis of no pricing errors, i.e., $H_0 : \omega = 0$. The asymptotic distribution of the Wald statistic under H_0 is χ^2_n provided that $\widehat{\omega}$ is consistent, asymptotically normal, and its variance matrix estimate is consistent. Biases in $\widehat{\omega}$ or its estimated variance generally lead to deviations from the asymptotic distribution, which show up as non-uniformly distributed asymptotic p-values (i.e., computed from the χ^2_n as the reference distribution) under H_0. We computed the Wald statistics and their asymptotic p-values. Figure 2 presents the results in the form of p-value discrepancy plots (Davidson and MacKinnon 1998), which graph the level error of a test against its nominal level. Deviations from the zero line indicate level errors of the test: the test overrejects (or underrejects) at a given nominal level, given on the horizontal axis, when the ordinate of the p-value discrepancy plot is positive (or negative). Overall, we find that the level distortions of the Wald test are relatively mild: for nominal levels between 0 and 0.3, they are almost always less than 0.1 in magnitude, regardless of n and of the estimator considered. For the two-step estimators there is some underrejection for $n \geq 5$.

Figure 2: p-value discrepancy plots of Wald statistics of $H_0 : \omega = 0$



Notes: 200 Monte Carlo replications; horizontal axis: nominal level; vertical axis: level error.

5 Empirical results

5.1 Data

We use weekly and daily returns on MSCI stock indices for 24 markets and an MSCI value-weighted world index. The markets are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Israel, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States. Datastream provides two types of return indices: the price index (MSPI) and the total return index including dividends (MSRI). We use the MSRI indices from January 1, 2001 to February 27, 2015. It may be remarked that the MSCI world portfolio, as a proxy of the market portfolio, is biased towards larger stocks, but it is equally true that foreign shareholders are similarly focusing on bigger stocks, with better liquidity, more analyst following, and less of an information disadvantage vis-à-vis local investors. For the risk-free rate we take the 3-month U.S. T-Bill yield, labelled TRUS3MT in Datastream. We consider 4 currencies: the Japanese yen, the British pound, the euro and the U.S. dollar. Their Libor deposit rates are collected and returns are computed. To get closer to the national T-Bill rates, we then subtract the TED spread from all Libor rates. All returns are measured in U.S. dollars, so we have currency risks relative to the Japanese yen, the British pound, and the euro. Summarizing, we obtain returns on 28 assets: 24 stock indices, 1 world stock index, and 3 currencies.

5.2 Results

Table 4 presents the one-step estimates of the InCAPM based on daily returns. We find a statistically significant estimate of the market risk coefficient, B_m with a point estimate of 6.03 with a standard error of 2.01. In contrast, the estimates of the currency risk coefficients are statistically insignificant.

The estimates of B in the InCAPM, InCAPMC, and CAPM with daily returns are given in Table 5. The one-step estimates of B_m are found to be reasonably robust across models, being 6.63 and 3.71 in the InCAPMC and CAPM, again statistically significant. The two-step estimates of B_m, on the other hand, are disappointing: except for one instance they are all negative, although often statistically insignificant. The estimated currency risk coefficients are, as expected, negative most of the time, but they generally lack statistical accuracy. Some of these estimates are positive, suggesting that the currencies may pick up omitted state variables. This is line with the results of the Wald tests, which in all cases reject the hypothesis of no pricing errors.

Table 6 presents the estimates of B for the weekly return data. We find one-step estimates of B_m equal to 5.68, 5.37, and 5.54 in the InCAPM, InCAPMC, and CAPM. The statistical significance is lower than in the daily data, although the estimates remain significant at the 5% level in one-tailed tests. The two-step estimates of B_m are all positive now, but they are statistically insignificant and much lower than the one-step estimates. This is very much in line with the simulation findings discussed earlier, which were set up to mimic weekly data. A further observation is that now there is a clearer pattern for the currency risk coefficients. The estimates of B_{EUR} are all negative and those of B_{GBP} are all positive. According to the one-step estimates of the InCAPM and InCAPMC, B_{EUR} and B_{GBP} are both statistically significant. Regarding the Wald tests, the conclusion is the same as with the daily data: the joint hypothesis of zero pricing errors is rejected in all models by all estimates.

5.3 Results for models with fewer markets

De Santis and Gerard (1998) estimated a multivariate GARCH-M model using the stock indices and currencies of the 4 largest markets (Germany, Japan, the United Kingdom, and the United States), giving eight assets in total. We also estimated the models for this smaller set of countries; Tables 7 and 8 present the estimates of B using daily and weekly data. The results are broadly in line with those of the model with 28 assets. With daily returns, the one-step estimates of B_m are between 4.30 and 6.62; with weekly returns they are larger, all around 7.5; their statistical significance levels remain about the same as earlier. The two-step estimates of B_m remain troublesome, often being negative (for daily data) or statistically insignificant. As expected, the estimates of the currency risks remain inaccurate, even more so than before. Interestingly, the null hypothesis of no pricing errors is no longer rejected; the ensemble of p-values in Tables 7 and 8 does not give any indication of pricing errors.

Further, we estimated a system with the thirteen big and mid-sized markets whose market value is larger than 1% of the world total market value ('world' means the 24 countries with developed financial markets).[3] Tables 9 and 10 report the estimates of B, with results that are in line with the earlier ones, roughly between those with 4 and with 24 markets. The additional point of interest is that the Wald test now almost universally rejects (except for the weighted two-step estimator with daily data), suggesting that the mispricing concerns the mid-sized markets.

6 Conclusions

In this paper we study a conditional version of three global asset pricing models. In the InCAPM, currency risk is priced; in the CAPM it is absent; and in the InCAPMC it is taking into consideration only in the stock markets. Our empirical methodology is a GARCH-M framework, where we introduce a one-step estimation approach as an alternative to the two-step method of Bali and Engle (2010).

Using Monte Carlo simulations, we first investigate the performance of alternative estimation methods. The simulations reveal that two-step estimation tends to underestimate the risk-return coefficient; even though the bias of the weighted two-step method is less pronounced, it remains substantial. We then study the influence of the cross-section dimension and find that the estimate of the (single) price of risk considerably improves if more asset price series are used.

In the empirical part, we use weekly and daily data from 2001 to 2015 for up to 24 stock indices, a world index, and 4 currencies. We find statistically significant one-step estimates of the price of market risk of around 6. The estimate is relatively robust with respect to the model specification, data frequency, and the number of assets employed. The weighted two-step estimates, on the other hand, are around 2 for weekly returns but very messy for daily returns, while the unweighted two-step estimates are often between 0 and 1 for weekly returns but negative for daily returns. Regarding the prices of currency risk, the standard errors of the estimates are too large to draw meaningful conclusions.

[3] These thirteen markets are: Australia, Canada, France, Germany, Hong Kong, Italy, Japan, The Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

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Table 2: Distribution of the standard errors of $\widehat{B}_m$

			quantiles					
n	method	mean	minimum	0.25	median	0.75	maximum	std($\hat{B}_m$)
25	1step	3.2474	2.0175	3.0063	3.2264	3.4173	7.8669	3.4444
25	2step	1.4616	0.6431	1.1657	1.4673	1.7320	2.3594	1.9064
25	w2step	1.9777	1.3902	1.7688	1.9676	2.1542	2.7750	2.4640
14	1step	3.8538	2.8106	3.5715	3.8008	4.1233	5.4912	3.9634
14	2step	1.7721	0.5202	1.4393	1.7108	2.1299	2.7929	2.0532
14	w2step	2.3766	1.2043	2.1107	2.3339	2.6333	3.4148	2.6684
5	1step	6.0858	3.1915	5.0802	5.7073	6.4615	55.6687	5.7942
5	2step	3.7504	1.6619	2.9822	3.6625	4.3456	9.6311	3.9756
5	w2step	4.1272	2.3078	3.3802	4.0667	4.6614	9.6091	4.2107
1	1step	10520	4.2459	7.6133	10.9098	17.2494	1976996	583.2198
1	2step	27.3892	3.9337	6.8871	9.1996	13.5883	1134.0073	110.6516
1	w2step	27.7851	4.1006	7.2746	9.5479	14.4940	1139.6343	110.8959

Note: 200 Monte Carlo replications.

Table 3: Mean of $\widehat{\omega}$ when $\omega = 0$

$n = 25$			$n = 14$		
1step	w2step	2step	1step	w2step	2step
-0.0380	0.1004	0.1211	-0.0353	0.1032	0.1209
-0.0458	0.1161	0.1400	-0.0326	0.1088	0.1261
-0.0382	0.1111	0.1326	-0.0397	0.1264	0.1478
-0.0307	0.1142	0.1327	-0.0397	0.1314	0.1547
-0.0328	0.0915	0.1105	-0.0251	0.0824	0.0953
-0.0360	0.1425	0.1680	-0.0360	0.1222	0.1435
-0.0436	0.1203	0.1443	-0.0196	0.0718	0.0819
-0.0479	0.1234	0.1483	-0.0360	0.1241	0.1458
-0.0495	0.1098	0.1316	-0.0357	0.1249	0.1447
-0.0296	0.0797	0.0951	-0.0392	0.1448	0.1712
-0.0452	0.1141	0.1360	-0.0295	0.0863	0.1014
-0.0295	0.0600	0.0737	-0.0343	0.1048	0.1227
-0.0444	0.1154	0.1395	-0.0292	0.0937	0.1090
-0.0253	0.0668	0.0789	-0.0305	0.0984	0.1148
-0.0423	0.1148	0.1384			
-0.0260	0.0660	0.0795	$n = 5$		
-0.0421	0.1301	0.1560	1step	w2step	2step
-0.0329	0.0860	0.1024	-0.0118	0.1054	0.1273
-0.0311	0.0838	0.1015	0.0023	0.0630	0.0764
-0.0439	0.1153	0.1380	-0.0140	0.0794	0.0969
-0.0464	0.1376	0.1642	-0.0071	0.0750	0.0903
-0.0323	0.0829	0.1000	-0.0076	0.0790	0.0954
-0.0311	0.1036	0.1241	$n = 1$		
-0.0248	0.1000	0.1164	1step	w2step	2step
-0.0294	0.1001	0.1181	1.7392	-0.0004	0.0033

Note: 200 Monte Carlo replications.

Table 4: One-step estimates of InCAPM with 24 markets and daily returns

market/currency	$100 \times \omega$	B	a	b
Australia	0.0642	–	0.1102	0.9925
Austria	0.0771	–	0.1158	0.9914
Belgium	0.0802	–	0.1196	0.9914
Canada	0.0412	–	0.1200	0.9917
Denmark	0.0911	–	0.1051	0.9925
Finland	0.0767	–	0.1148	0.9923
France	0.0681	–	0.1216	0.9914
Germany	0.0829	–	0.1172	0.9918
Greece	0.0600	–	0.1115	0.9926
Hong Kong	0.0442	–	0.0939	0.9944
Ireland	0.0738	–	0.1196	0.9910
Israel	0.0412	–	0.0840	0.9949
Italy	0.0563	–	0.1246	0.9911
Japan	0.0209	–	0.1205	0.9915
Netherlands	0.0674	–	0.1214	0.9914
New Zealand	0.0691	–	0.0901	0.9946
Norway	0.0845	–	0.1130	0.9918
Portugal	0.0611	–	0.1158	0.9915
Singapore	0.0528	–	0.1001	0.9937
Spain	0.0849	–	0.1245	0.9911
Sweden	0.0831	–	0.1168	0.9917
Switzerland	0.0630	–	0.1175	0.9915
U.K.	0.0592	–	0.1217	0.9914
U.S.	0.0274	–	0.1179	0.9920
world	0.0379	6.0279 (2.0126)	0.1177	0.9919
GBP	0.0284	−4.9149 (6.8891)	0.1061	0.9923
JPY	−0.0123	3.1290 (5.3176)	0.0939	0.9932
EUR	0.0256	−5.3178 (6.3408)	0.1006	0.9930

Note: Standard errors in parentheses.

Table 5: Estimates with 24 markets and daily returns

Model	Method	B_m	B_{GBP}	B_{JPY}	B_{EUR}	Wald
InCAPM	1step	6.0279	−4.9149	3.1290	−5.3178	193.5383
		(2.0126)	(6.8891)	(5.3176)	(6.3408)	(0.0000)
InCAPM	2step	−1.6974	−4.4754	−1.8823	1.8286	68.4450
		(0.5512)	(2.8229)	(1.9065)	(2.6365)	(0.0000)
InCAPM	w2step	−0.5740	−7.2953	−0.2349	7.3900	42.5344
		(1.5298)	(4.6941)	(2.9226)	(4.0286)	(0.0386)
InCAPMC	1step	6.6347	2.8058	2.1080	−10.8995	177.9743
		(1.8954)	(6.0092)	(4.9143)	(5.2565)	(0.0000)
InCAPMC	2step	−1.8546	−3.2544	−0.1238	2.3588	66.2307
		(0.5644)	(3.2832)	(2.0804)	(2.9209)	(0.0000)
InCAPMC	w2step	−0.0843	−4.5863	−0.4968	7.6417	34.9986
		(1.6189)	(5.8710)	(3.4250)	(4.7282)	(0.0882)
CAPM	1step	3.7110	–	–	–	186.4721
		(1.6622)		–		(0.0000)
CAPM	2step	−1.9828	–	–	–	65.5100
		(0.4407)		–		(0.0000)
CAPM	w2step	0.0826		–		40.3409
		(1.4730)		–		(0.0269)

Note: standard errors and p-values of Wald statistics of $H_0 : \omega = 0$ in parentheses.

Table 6: Estimates of with 24 markets and weekly returns

Model	Method	B_m	B_{GBP}	B_{JPY}	B_{EUR}	Wald
InCAPM	1step	5.6784	34.7439	3.5496	−48.9575	76.2772
		(3.2498)	(11.9776)	(13.4101)	(14.5455)	(0.0000)
InCAPM	2step	0.5804	4.3702	−1.2326	−3.6058	63.3403
		(0.4623)	(2.4858)	(1.3970)	(2.4510)	(0.0001)
InCAPM	w2step	1.4966	4.7546	0.4171	−0.9247	42.5480
		(1.4375)	(4.2593)	(2.7829)	(3.8325)	(0.0385)
InCAPMC	1step	5.3719	19.3104	1.5597	−28.3461	68.4601
		(2.9974)	(8.8497)	(10.8064)	(11.3079)	(0.0000)
InCAPMC	2step	0.4569	7.5601	0.0725	−4.8428	61.5084
		(0.4753)	(2.7215)	(1.4803)	(2.6519)	(0.0001)
InCAPMC	w2step	1.4698	10.8167	0.5750	−2.2470	38.1254
		(1.5380)	(4.9246)	(3.1677)	(4.3024)	(0.0450)
CAPM	1step	5.5373	–	–	–	62.6912
		(2.7200)		–	–	(0.0000)
CAPM	2step	0.8125	–	–	–	66.8579
		(0.3539)		–	–	(0.0000)
CAPM	w2step	2.4469	–	–	–	38.9244
		(1.4299)		–	–	(0.0375)

Note: standard errors and p-values of Wald statistics of $H_0 : \omega = 0$ in parentheses.

Table 7: Estimates with 4 largest markets and daily returns

Model	Method	B_m	B_{GBP}	B_{JPY}	B_{EUR}	Wald
InCAPM	1step	4.2998	1.1183	2.1123	−2.4323	9.1390
		(2.2841)	(6.9079)	(5.2115)	(6.3734)	(0.3307)
InCAPM	2step	−1.6965	−1.7188	−1.8239	−3.4306	10.6798
		(0.9287)	(3.7830)	(3.1594)	(3.8842)	(0.2205)
InCAPM	w2step	−0.4435	−2.6970	1.8501	3.3499	7.4082
		(1.9106)	(5.8194)	(3.9579)	(5.3298)	(0.4933)
InCAPMC	1step	6.6226	9.7857	7.3606	−14.4120	4.0890
		(2.1827)	(6.7355)	(5.3449)	(6.2393)	(0.5367)
InCAPMC	2step	−1.6814	6.8044	5.1258	−6.5574	5.6501
		(1.0644)	(5.5139)	(4.6715)	(5.0350)	(0.3418)
InCAPMC	w2step	1.4165	9.7373	4.2443	−4.9674	3.0163
		(2.2507)	(9.0739)	(5.8389)	(8.1705)	(0.6975)
CAPM	1step	4.7013	–	–	–	4.3417
		(1.9875)		–	–	(0.5013)
CAPM	2step	−2.2769	–	–	–	6.9453
		(0.7944)		–	–	(0.2247)
CAPM	w2step	1.6937	–	–	–	2.1779
		(1.9394)		–	–	(0.8240)

Note: standard errors and p-values of Wald statistics of $H_0 : \omega = 0$ in parentheses.

Table 8: Estimates with 4 largest markets and weekly returns

Model	Method	B_m	B_{GBP}	B_{JPY}	B_{EUR}	Wald
InCAPM	1step	7.7995	6.7543	1.8839	−10.9354	3.3388
		(3.0952)	(7.5157)	(9.8743)	(7.7379)	(0.9113)
InCAPM	2step	0.6586	1.6950	−2.0218	−5.2836	3.7595
		(0.9805)	(3.9356)	(2.9617)	(4.4226)	(0.8781)
InCAPM	w2step	2.7113	3.3217	2.4529	−2.7238	7.1877
		(1.8528)	(5.9192)	(3.9746)	(5.8155)	(0.5165)
InCAPMC	1step	7.4657	4.9860	2.8066	−13.0634	3.4325
		(2.9335)	(6.6131)	(9.2209)	(7.6631)	(0.6336)
InCAPMC	2step	0.7830	14.8634	2.2613	−13.5976	3.2700
		(1.1811)	(5.4984)	(4.1783)	(6.3972)	(0.6584)
InCAPMC	w2step	3.8298	19.7026	3.5315	−11.2005	9.4739
		(2.2703)	(8.6962)	(5.4137)	(8.8036)	(0.0916)
CAPM	1step	7.4791	–	–	–	1.7668
		(2.6489)		–	–	(0.8804)
CAPM	2step	0.4549	–	–	–	1.2459
		(0.8397)		–	–	(0.9404)
CAPM	w2step	4.5441	–	–	–	5.3746
		(1.9303)		–	–	(0.3719)

Note: standard errors and p-values of Wald statistics of $H_0 : \omega = 0$ in parentheses.

Table 9: Estimates with 13 largest markets and daily returns

Model	Method	B_m	B_{GBP}	B_{JPY}	B_{EUR}	Wald
InCAPM	1step	5.5713	−4.4294	2.5387	−0.2088	106.6071
		(2.0358)	(6.6694)	(5.0734)	(6.1046)	(0.0000)
InCAPM	2step	−2.4383	−2.9478	−0.3032	−0.2936	50.5968
		(0.6133)	(3.0563)	(2.1400)	(2.9329)	(0.0000)
InCAPM	w2step	−0.2381	−5.6359	−0.0730	7.7509	16.8504
		(1.6847)	(5.0607)	(3.2708)	(4.4775)	(0.4645)
InCAPMC	1step	6.2401	3.7631	1.0088	−8.7618	98.3932
		(1.9330)	(5.9839)	(4.8205)	(5.2631)	(0.0000)
InCAPMC	2step	−2.5995	−0.5071	2.3737	−0.5490	48.1041
		(0.6322)	(3.6648)	(2.4067)	(3.3330)	(0.0000)
InCAPMC	w2step	0.5938	−2.8310	−0.0559	9.1150	12.6590
		(1.8388)	(6.6316)	(4.0316)	(5.5782)	(0.5535)
CAPM	1step	3.8142	–	–	–	104.2661
		(1.6879)		–	–	(0.0000)
CAPM	2step	−3.0335	–	–	–	48.3004
		(0.4941)		–	–	(0.0000)
CAPM	w2step	1.2544	–	–	–	13.7529
		(1.6429)		–	–	(0.4683)

Note: standard errors and p-values of Wald statistics of $H_0 : \omega = 0$ in parentheses.

Table 10: Estimates with 13 largest markets and weekly returns

Model	Method	B_m	B_{GBP}	B_{JPY}	B_{EUR}	Wald
InCAPM	1step	7.5610	11.5812	−8.3651	−21.3127	50.9423
		(2.6873)	(8.6785)	(9.3296)	(10.0557)	(0.0000)
InCAPM	2step	0.6509	3.9356	−1.0667	−4.7916	41.5340
		(0.5505)	(2.9345)	(1.6750)	(2.9685)	(0.0008)
InCAPM	w2step	2.3634	3.8289	−1.5617	0.6090	29.2881
		(1.6291)	(4.9474)	(3.1677)	(4.6475)	(0.0320)
InCAPMC	1step	6.4161	10.8182	−10.0414	−20.9968	49.2824
		(2.5667)	(6.8071)	(8.1039)	(8.0312)	(0.0000)
InCAPMC	2step	0.4342	9.3475	0.3644	−8.1093	39.2431
		(0.5754)	(3.3639)	(1.8295)	(3.3763)	(0.0003)
InCAPMC	w2step	2.7464	13.4321	−1.8120	−2.6809	30.1362
		(1.8383)	(6.1261)	(3.7729)	(5.6594)	(0.0073)
CAPM	1step	6.5042	–	–	–	42.6526
		(2.2829)		–	–	(0.0001)
CAPM	2step	0.5184	–	–	–	41.2750
		(−0.0844)		–	–	(0.0002)
CAPM	w2step	4.3947	–	–	–	28.0531
		(5.7561)		–	–	(0.0140)

Note: standard errors and p-values of Wald statistics of $H_0 : \omega = 0$ in parentheses.