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Production with storable and durable inputs: nonparametric analysis of intertemporal efficiency

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PRODUCTION WITH STORABLE AND DURABLE INPUTS: NONPARAMETRIC ANALYSIS OF INTERTEMPORAL EFFICIENCY

LAURENS CHERCHYE, BRAM DE ROCK, AND PIETER JAN KERSTENS

Abstract. We propose a nonparametric methodology for intertemporal production analysis that accounts for durable as well as storable inputs. Durable inputs contribute to the production outputs in multiple consecutive periods. Storable inputs are non-durable and can be stored in inventories for use in future periods. We explicitly model the possibility that firms use several vintages of the durable inputs, i.e. they invest in new durables and scrap older durables over time. Furthermore, we allow for production delays of durable inputs. We characterize production behavior that is dynamically cost efficient, which allows us to evaluate the efficiency of observed production decisions. For cost inefficient behavior, we propose a measure to quantify the degree of inefficiency. An attractive feature of this measure is that it can be decomposed in period-specific cost inefficiencies. We demonstrate the usefulness of our methodology through an application to Swiss railway companies.

Keywords: cost minimization, storable inputs, durable inputs, production delay, dynamic efficiency

JEL Classification: D21, D24, D92

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1. Introduction

Many production decisions have long term consequences for production and are capital-intensive: should a firm merely buy new machines to replace older machines that have reached (physical) end of life? Or should the firm invest in new machines to expand its production capacity? These capital-intensive investments are "durable" by nature, because they have a long term impact on production. Furthermore, firms often buy far larger quantities of inputs then they currently need. This can be economically rational for a number of reasons: there are discounts on bulk purchases of inputs or firms expect input prices to rise in the near future. These "storable" inputs can be stored in inventories and are used over several time periods. These durable and storable inputs used in production limit the flexibility of a firm in adjusting its input mix. In this paper, we introduce a novel methodology for economic (cost) efficiency analysis that explicitly takes these intertemporal aspects of firms' production behavior into account. This obtains a more realistic modeling of intertemporal relations in production situations where storability and durability of inputs are relevant, which is often the case in real-life settings.

1.1. **Intertemporal efficiency and regulation.** In regulated industries it is particularly vital that the regulator takes intertemporal dependencies of production into account in the regulation exercise. However, regulators generally do not incorporate these interdependencies in practice. This is sometimes motivated by a lack of panel data, which forces regulators to limit the analysis to cross-sectional data (Pollitt, 2005). However, also the used definition of capital costs can be contested (Haney and Pollitt, 2013; Shuttleworth, 2005). Clearly, not taking these dependencies into account can lead to erroneous cost reduction targets. Shuttleworth (2005) reports a case where Ofgem, a UK electricity distribution regulator, imposed a too strong target for one distributor (Seeboard) while imposing a too loose target on another (Southern). This discrepancy was due to the fact that Ofgem only considered operational expenses, while disregarding capital expenses. And it happened that Southern was characterized by high capital expenses and low operational expenses, while the opposite applied to Seeboard.

The relation between regulatory regime and investment has received a lot of attention (see Guthrie (2006) for a discussion). Focusing on cross-sectional data can lead to penalization of firms that invest while rewarding those that delay investments. Nick and Wetzel (2015) conclude that firms have an incentive to cut investments when the regulator uses a static benchmarking model. Our empirical application to Swiss railway companies will show that the resulting dynamic efficiency conclusions may significantly differ from the ones that are based on a static efficiency analysis. In our opinion, this directly motivates the practical relevance of our methodology, as these differences may substantially affect the regulatory policies that are based on the efficiency assessment.

1.2. **Efficiency analysis with durable and storable inputs.** The existing literature has devoted much attention to the analysis of dynamically efficient production behavior from a technical perspective (see Fallah-Fini et al. (2013) for a recent review). Such technical efficiency analysis then focuses, for example, on the modeling of production delays, inventories, capital (quasi-fixed factors in general), adjustment costs and learning. By contrast, far less work has tackled the issue from an economic perspective.[1] Importantly, however, the distinction between economic and technical efficiency analysis becomes particularly relevant in dynamic decision settings.

For durable inputs, it has long been known that firms do not scrap old (durable) capital equipment the moment new equipment becomes available. The process of replacing capital equipment is rather gradual. Firms deciding on new capital equipment face different substitution possibilities between inputs before (ex ante) and after (ex post) the purchase: once capital equipment is installed, it remains in use until the end of its predetermined lifetime (Forsund and Hjalmarsson, 1974; Johansen, 1959). Thus, while firms might seem inefficient from a technical perspective, they may actually be efficient from an economic perspective.[2]

Similarly, when deciding upon storable inputs, firms typically plan their production in advance for a certain time horizon. They form expectations on prices and demand and then decide on the amount of necessary inputs to acquire. Clearly, if prices of storable inputs vary over time, this can again generate significant discrepancies between technical and economic efficiency analysis.

In this paper, we present a unifying framework to analyze intertemporal cost minimizing behavior with both durable and storable inputs. For durable inputs, our framework explicitly models the possibility that firms use several vintages: they invest in new durables and scrap older durables over time. Furthermore, we allow for production delays of durable inputs. We also show how our framework can incorporate alternative hypotheses such as degressive write-off of durables over time.

A main distinguishing feature of our methodology is that it is intrinsically nonparametric (in the spirit of Afriat (1972); Banker and Maindiratta (1988); Varian (1984)): it can analyze production behavior without imposing any (usually nonverifiable) functional structure on the production technology. We characterize production behavior that is intertemporal cost efficient, which allows us to evaluate the efficiency of observed production decisions. For cost inefficient behavior, we propose a measure that quantifies the degree of inefficiency. This intertemporal inefficiency measure has the attractive property that it can be decomposed in period-specific cost inefficiencies.

[1] Notable exceptions include Nemoto and Goto (1999, 2003); Ouellette and Yan (2008); Silva and Stefanou (2003). We discuss the relation between our framework and this existing work in Section 2.

[2] Wibe (2008) coined the term "rational inefficiency" to mark this difference.

1.3. **Outline.** The remainder of this paper unfolds as follows. In Section 2, we discuss the connection between our work and the closely related literature on both intertemporal production models and efficiency analysis. Sections 3 to 5 formally introduce our methodology. After introducing our general set-up in Section 3 , we first consider the case where one has full information on allocations of storables and write-offs of durables in Section 4, to subsequently present the case where limited or no such information is known in Section 5. Section 6 presents some extensions to the basic framework. Section 7 contains the empirical application of our methodology to Swiss regional railway companies. Specifically, this analysis will demonstrate the relevance of accounting for the intertemporal (durable) nature of capital expenses in a regulated production environment. Finally, Section 8 concludes and points out a number of interesting extensions.

2. Related literature

Our framework for intertemporal production analysis bears close connections with a number of existing studies on the analysis of efficient production behavior. Most of this earlier work appeared under the label Data Envelopment Analysis (DEA), which is often used to refer to the nonparametric analysis of production efficiency. In what follows, we discuss the relation with earlier literature on network DEA, efficiency analysis with quasi-fixed inputs, and DEA with lagged input effects. In turn, this will allow us to articulate the specificities of our own contribution.

First, our work is closely related to the literature on network DEA (Färe and Grosskopf, 2000). In an early contribution to this literature, Färe (1986) showed how to measure output efficiency by allowing for inputs that are allocatable over time, which are similar in nature to what we call storable inputs. He makes a distinction between inputs for which the allocation over time is known and inputs for which (only) the total amount is known but not how this amount is allocated over time. Importantly, however, he does not consider the intermediate case with new inputs in every period that are to be allocated over multiple time periods. In a similar fashion, Färe et al. (1997) model fixed but allocatable inputs over outputs and develop an output efficiency measure that locates potential efficiency gains due to the reallocation of inputs over the outputs. Finally, inventories are also explicitly modeled in Hackman and Leachman (1989)'s general framework of production.

Similarly to our use of durable inputs, Färe et al. (2007) construct a network DEA model with durable and instantaneous inputs to model technology adoption, where one of the technologies is vintage. Durable inputs are vintage-specific, and the adoption of a new technology is accomplished by diverting instantaneous inputs away from the vintage technology to the new technology.

All these network DEA models have in common that they measure technical efficiency (without price information) and not economic efficiency (with price information). Such technical efficiency analysis requires specific assumptions regarding the nature of the production technology.[3]

Furthermore, our concept of durable inputs is also related to the notion of quasi-fixed inputs. Nemoto and Goto (1999, 2003) model adjustment costs due to quasi-fixed inputs and develop an efficiency measure. They treat quasi-fixed inputs as intermediate outputs which are used as inputs in subsequent periods. Their model was extended by Ouellette and Yan (2008) by weakening the restrictions on capital investment. Similarly, Silva and Stefanou (2003) develop nonparametric tests for investment in quasi-fixed inputs with internal adjustment costs in the spirit of Varian (1984).

Next, Chen and van Dalen (2010) incorporate lagged effects of inputs on outputs in DEA efficiency measurement. The relation between output and delayed inputs is fixed parametrically. Thus, they assume that these productive effects are known a priori and estimate these by a fixed effect panel vector autoregressive model in their empirical application. This makes their efficiency measure highly dependent on their parametric specification of the productive effects.

Basically, our contribution is that we present a unifying framework to nonparametrically analyze economic (cost) efficiency in intertemporal production with both storable and durable inputs. We explicitly model the fact that these two types of inputs are used over several time periods: storable inputs are allocated over multiple periods, and durable "vintage" inputs are not immediately replaced by newer durable inputs (thus following Johansen (1959) and Forsund and Hjalmarsson (1974)). In addition, we also allow for production delays of durable inputs over time. Next, we propose a cost inefficiency measure that can be decomposed in per-period inefficiencies.[4] Finally, as compared to the literature on quasi-fixed inputs, we do not focus on the issue of adjustment costs, but rather consider the replacement of vintages of durables over time from a cost perspective (see also the introduction of Section 3).

3. Set-up

We assume a panel setting with K firms that are observed T times. For each firm k and time period t, we observe the S-dimensional output $\mathbf{y}_{k,t} \in \mathbb{R}^S_+$, the N-dimensional storable input $\mathbf{q}_{k,t} \in \mathbb{R}^N_+$, the M-dimensional durable input $\mathbf{Q}_{k,t} \in \mathbb{R}^M_+$ and the corresponding discounted input prices $\mathbf{p}_{k,t} \in \mathbb{R}^N_{++}$ and $\mathbf{P}_{k,t} \in \mathbb{R}^M_{++}$

[3] In our concluding Section 8 we will indicate the possibility to conduct a technical efficiency analysis in the intertemporal framework (for economic efficiency analysis) that we develop in the following sections. These technical efficiency formulations could subsequently establish a formal link with the existing network DEA models.

[4] We note that Kao (2013) proposed a similar decomposition of efficiency scores in per-period efficiencies for a DEA model with quasi-fixed inputs.

respectively. For every $k = 1, \ldots, K$, this defines the dataset

$$\mathcal{S}_k = \left\{ (\mathbf{p}_{k,t}, \mathbf{q}_{k,t}, \mathbf{P}_{k,t}, \mathbf{Q}_{k,t}, \mathbf{y}_{k,t}) | t = 1, \ldots, T \right\}.$$

To keep our exposition simple, we assume that firms have perfect foresight, i.e. they exactly anticipate the future prices. In fact, it is fairly easy to extend our method to account for predicted prices that deviate from the prices that are realized ex post.[5] But this would only complicate our reasoning without really adding new insights. Also, the fact that we evaluate a firm's cost efficiency in terms of realized prices makes that we may interpret measured inefficiencies as (ex post) prediction errors.

Storable inputs are divisible and we assume they are used over J periods: a fraction is used in each period, while the remaining part is stored for the next periods. Storable inputs can only be used once and are nondurable. Durable inputs are indivisible and usable in multiple periods before reaching end of life status. This is where they differ from storable inputs. Durable inputs are related to quasi-fixed inputs in that they have an effect over multiple periods, but differ from quasi-fixed inputs because they may also be adjusted instantaneously (e.g. one can stop using a laptop or company car immediately). In that sense, we can see quasi-fixed inputs as a subset of durable inputs. In general, durable inputs are seen as investments: a firm intends to use the durable input for a number of periods and writes off the cost of investment over these periods. Examples of durable inputs include machines, equipment, company cars, etc. To keep the exposition simple, we also assume they are used over J periods. We show in Section 6 how this assumption can be relaxed.

Our behavioral hypothesis is that firms are intertemporally cost minimizing. To formalize this assumption, we represent firm technologies in terms of input requirement sets $\mathcal{I}_t(\mathbf{y}_{k,t})$ for the output of firm k produced at time period t. These sets are defined in the usual way, i.e.

$$\mathcal{I}_t(\mathbf{y}_{k,t}) = \left\{ (\mathbf{q}, \mathbf{Q}) \in \mathbb{R}_+^{N+M} | (\mathbf{q}, \mathbf{Q}) \text{ can produce } \mathbf{y}_{k,t} \right\}.$$

Next, we make use of quantity allocations $(\mathfrak{q}_t^1, \ldots, \mathfrak{q}_t^J)_{t=1}^T$ of storable inputs and price write-offs $(\mathfrak{P}_t^1, \ldots, \mathfrak{P}_t^J)_{t=1}^T$ of durable inputs. These allocations and write-offs will be used to distribute firm k's input costs over the J relevant time periods, and are subject to the adding-up restrictions $\mathbf{q}_{k,t} = \sum_{j=1}^J \mathfrak{q}_{k,t}^j$ and $\mathbf{P}_{k,t} = \sum_{j=1}^J \mathfrak{P}_{k,t}^j$.[6] Then, we say that firm k minimizes its total production costs over the time horizon

[5]For example, a simple solution consists of verifying the cost efficiency conditions that we define below for alternative specifications of (anticipated) prices, as a robustness analysis.

[6]In Appendix A, we explain the economic intuition of the write-offs $(\mathfrak{P}_t^1, \ldots, \mathfrak{P}_t^J)_{t=1}^T$ as representing (in monetary terms) marginal productivities of the durable inputs.

$[J, \dots, T]$ if it chooses the allocation $(\mathfrak{q}_t^1, \dots, \mathfrak{q}_t^J)_{t=1}^T$ and write-off $(\mathfrak{P}_t^1, \dots, \mathfrak{P}_t^J)_{t=1}^T$ that solves

$$\min_{\substack{(\mathfrak{q}_t^1,\dots,\mathfrak{q}_t^J)_{t=1}^T \\ (\mathfrak{P}_t^1,\dots,\mathfrak{P}_t^J)_{t=1}^T}} \sum_{t=J}^{T} \sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j}\mathfrak{q}_j^{t-j+1} + \mathfrak{P}_j^{t-j+1}\mathbf{Q}_{k,j}\right) \tag{1a}$$

$$\text{s.t.} \quad \left(\sum_{j=1}^{J} \mathfrak{q}_{t-j+1}^{j}, \sum_{j=1}^{J} \mathbf{Q}_{k,t-j+1}\right) \in \mathcal{I}_t(\mathbf{y}_{k,t}) \qquad \forall t = J, \dots, T, \tag{1b}$$

where the last feasibility constraint states that the allocation of storables and durables effectively admits the production of the output $\mathbf{y}_{k,t}$ for the given technology. The fact that the storable and durable input quantities are summed over J present and past periods reveals the intertemporal dependency of firm k's production decisions.

Table 1 sharpens the intuition of the above concepts through a simple example that shows a firm's observed costs and production costs over time for $J = 2$. The table illustrates two crucial points. First, observed costs and production costs generally differ. Thus, any efficiency comparison using observed costs instead of production costs is potentially overly pessimistic. Second, the lack of information on allocations and write-offs beyond the observed time frame limits any test of (1) to the time period $[2, \dots, T]$ when $J = 2$ and $[J, \dots, T]$ in general. This explains why we only consider the period $[J, \dots, T]$ in (1) instead of $[1, \dots, T]$ in our minimization program.

t	observed cost	production cost	
		storable inputs	durable inputs
1	$\mathbf{p}_1\mathfrak{q}_1$	$\mathbf{p}_1\mathfrak{q}_1^1 + ?$	$\mathfrak{P}_1^1\mathbf{Q}_1 + ?$
2	$\mathbf{p}_2\mathfrak{q}_2$	$\mathbf{p}_2\mathfrak{q}_2^1 + \mathbf{p}_1\mathfrak{q}_1^2$	$\mathfrak{P}_2^1\mathbf{Q}_2 + \mathfrak{P}_1^2\mathbf{Q}_1$
3	$\mathbf{p}_3\mathfrak{q}_3$	$\mathbf{p}_3\mathfrak{q}_3^1 + \mathbf{p}_2\mathfrak{q}_2^2$	$\mathfrak{P}_3^1\mathbf{Q}_3 + \mathfrak{P}_2^2\mathbf{Q}_2$
4	$\mathbf{p}_4\mathfrak{q}_4$	$\mathbf{p}_4\mathfrak{q}_4^1 + \mathbf{p}_3\mathfrak{q}_3^2$	$\mathfrak{P}_4^1\mathbf{Q}_4 + \mathfrak{P}_3^2\mathbf{Q}_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$\mathbf{p}_t\mathfrak{q}_t$	$\mathbf{p}_t\mathfrak{q}_t^1 + \mathbf{p}_{t-1}\mathfrak{q}_{t-1}^2$	$\mathfrak{P}_t^1\mathbf{Q}_t + \mathfrak{P}_{t-1}^2\mathbf{Q}_{t-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$T-1$	$\mathbf{p}_{T-1}\mathfrak{q}_{T-1}$	$\mathbf{p}_{T-1}\mathfrak{q}_{T-1}^1 + \mathbf{p}_{T-2}\mathfrak{q}_{T-2}^2$	$\mathfrak{P}_{T-1}^1\mathbf{Q}_{T-1} + \mathfrak{P}_{T-2}^2\mathbf{Q}_{T-2}$
T	$\mathbf{p}_T\mathfrak{q}_T$	$\mathbf{p}_T\mathfrak{q}_T^1 + \mathbf{p}_{T-1}\mathfrak{q}_{T-1}^2$	$\mathfrak{P}_T^1\mathbf{Q}_T + \mathfrak{P}_{T-1}^2\mathbf{Q}_{T-1}$
$T+1$	?	$? + \mathbf{p}_T\mathfrak{q}_T^2$	$? + \mathfrak{P}_T^2\mathbf{Q}_T$

TABLE 1. Overview of production costs with storable and durable inputs for $J = 2$

4. Complete information

We next turn to deriving operational conditions for cost minimizing behavior as defined in (1). As indicated in the Introduction, we derive nonparametric conditions in the spirit of Afriat (1972); Banker and Maindiratta (1988); Varian (1984), which make minimal assumptions regarding the production technology. To set the stage, we first consider the limiting case that is characterized by full information on the quantity allocations of the storable inputs and the price write-offs of the durable inputs, that is, for each firm k the empirical analyst observes the allocations $(\mathfrak{q}^1_{k,t}, \mathfrak{q}^2_{k,t}, \ldots, \mathfrak{q}^J_{k,t})$ and the write-offs $(\mathfrak{P}^1_{k,t}, \mathfrak{P}^2_{k,t}, \ldots, \mathfrak{P}^J_{k,t})$ at every time period $t = 1, ..., T$.

Such complete information greatly simplifies matters. From (1), it is easy to verify that, for a given specification of storable allocations and durable write-offs, the production costs for any time period t are defined independently of the production costs for other time periods. As an implication, firm k behaves consistently with (1) if and only if it solves, for every $t = J, ..., T$,

$$\min_{\substack{(\mathfrak{q}^1_j,\ldots,\mathfrak{q}^J_j)^t_{j=t-J+1} \\ (\mathfrak{P}^1_j,\ldots,\mathfrak{P}^J_j)^t_{j=t-J+1}}} \sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j}\mathfrak{q}^{t-j+1}_j + \mathfrak{P}^{t-j+1}_j \mathbf{Q}_{k,j}\right) \tag{2a}$$

$$\text{s.t.} \left(\sum_{j=1}^{J} \mathfrak{q}^j_{t-j+1}, \sum_{j=1}^{J} \mathbf{Q}_{k,t-j+1}\right) \in \mathcal{I}_t(\mathbf{y}_{k,t}) \tag{2b}$$

Putting it differently, dynamically cost minimizing behavior under complete information can be represented as statically cost minimizing behavior for every period t. Varian (1984) developed the nonparametric characterization of such static cost minimization.[7] Thus, we can obtain our empirical condition for dynamic cost efficiency by translating Varian's reasoning to our particular setting.

Throughout, we will adopt the next two axioms regarding the production technology (given by $\mathcal{I}_t(\mathbf{y}_{k,t})$):

Axiom 1 (observability means feasibility)**.** *For all* $t = 1, \ldots, T$ *and* $k = 1, \ldots, K$: $(\mathbf{p}_{k,t}, \mathfrak{q}^1_{k,t}, \ldots, \mathfrak{q}^J_{k,t}, \mathfrak{P}^1_{k,t}, \ldots, \mathfrak{P}^J_{k,t}, \mathbf{Q}_{k,t}, \mathbf{y}_{k,t}) \in \mathcal{S}_k \Rightarrow \left(\sum_{j=1}^J \mathfrak{q}^j_{k,t-j+1}, \sum_{j=1}^J \mathbf{Q}_{k,t-j+1}\right) \in \mathcal{I}_t(\mathbf{y}_{k,t})$.

Axiom 2 (nested input sets)**.** *For all* $t = 1, \ldots, T$ *and* $k, s = 1, \ldots, K$: $\mathbf{y}_{s,t} \geq \mathbf{y}_{k,t} \Rightarrow \mathcal{I}_t(\mathbf{y}_{s,t}) \subseteq \mathcal{I}_t(\mathbf{y}_{k,t})$.[8]

[7] Varian (1984) characterized cost minimizing production behavior in terms of the so-called Weak Axiom of Cost Minimization (WACM). Basically, Proposition 1 will state this WACM criterion for our intertemporal setting.

[8] Throughout $\mathbf{y}_{s,t} \geq \mathbf{y}_{k,t}$ should be interpreted as vector inequalities, implying that the inequality needs to hold for all components.

In words, Axiom 1 says that there are no significant measurement errors in the data.[9] Axiom 2 says that, for a given time period t, input requirement sets are nested: if firm s produces at least the same output as firm k (i.e. $\mathbf{y}_{s,t} \geq \mathbf{y}_{k,t}$), then the input set for s must be contained in the set for k (i.e. $\mathcal{I}_t(\mathbf{y}_{s,t}) \subseteq \mathcal{I}_t(\mathbf{y}_{k,t})$).[10] Intuitively, this means that outputs are freely disposable. These are the only two production axioms that we will assume in the sequel of this paper.

Then, we define

$$c_{k,t} = \min_{s \in D_k^t} \left\{ \sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j} \mathfrak{q}_{s,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1} \mathbf{Q}_{s,j} \right) \right\}. \tag{3}$$

for

$$D_k^t = \left\{ s | \mathbf{y}_{s,t} \geq \mathbf{y}_{k,t} \right\}, \tag{4}$$

i.e. the set of observed firms s that produce at least the same output as firm k in period t (i.e. $\mathbf{y}_{s,t} \geq \mathbf{y}_{k,t}$). By construction, we have $k \in D_k^t$, so that $D_k^t \neq \emptyset$. In words, $c_{k,t}$ represents the minimal cost over this set D_k^t. Obviously, we can compute $c_{k,t}$ by simply enumerating over all $s \in D_k^t$ if the allocations $(\mathfrak{q}_{k,t}^1, \mathfrak{q}_{k,t}^2, \ldots, \mathfrak{q}_{k,t}^J)$ and write-offs $(\mathfrak{P}_{k,t}^1, \mathfrak{P}_{k,t}^2, \ldots, \mathfrak{P}_{k,t}^J)$ are given.

We can now state the following result.

Proposition 1. *Firm k solves (1) for a production technology that satisfies Axioms 1 and 2 if and only if, for all $t = J, ..., T$,*

$$\sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j} \mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1} \mathbf{Q}_{k,j} \right) = c_{k,t}. \tag{5}$$

Proof. We use the equivalence between (1) and (2). Then, the result follows from Theorem 1 (statements (1) and (2)) of Varian (1984). □

[9]Clearly, this axiom may often be problematic in practical situations. In such instances, we can use alternative techniques to explicitly account for errors. For example, one may adjust our methodology by integrating it with the probabilistic method which Cazals, Florens, and Simar (2002) and Daraio and Simar (2005, 2007) originally proposed in a DEA context. To focus our discussion, we do not consider this extension here.

[10]We remark that this assumes that different firms s and k face the same technology in period t. Obviously, we can also use other hypotheses regarding technological homogeneity/heterogeneity across firms and time periods. For example, we may assume homogeneous technologies (only) for subsets of firms (e.g. defined on the basis of observable firm characteristics), or firm-specific technologies that are constant over time. For compactness, we will again not explicitly implement this.

This results directly suggests the next measure of cost inefficiency for every period t:

$$CE_k^t \equiv \sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right) - c_{k,t}, \tag{6}$$

Obviously, firm k meets the empirical cost minimization criterion (5) in Proposition 1 if and only if $CE_k^t = 0$. More generally, we have $CE_k^t \geq 0$, and the value of CE_k^t indicates how much firm k deviates from cost minimizing behavior at time t.

When aggregating over all $t = J, \ldots, T$, we can similarly define an overall cost inefficiency measure as

$$CE_k \equiv \sum_{t=J}^{T} \sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right) - \sum_{t=J}^{T} c_{k,t}. \tag{7}$$

By construction, we have

$$CE_k = \sum_{t=J}^{T} \left(\sum_{j=t-J+1}^{t} \left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right) - c_{k,t} \right) = \sum_{t=J}^{T} CE_k^t, \tag{8}$$

which yields the next result.

Proposition 2. $CE_k = 0 \Leftrightarrow CE_k^t = 0\ \forall t = J, \ldots, T$.

Proof. The result follows from (7) and the definitional fact that $CE_k^t \geq 0$. □

In words, firm k minimizes its total production costs over the full period $[J, \ldots, T]$ if and only if its production costs are minimal in every single period t. Essentially, this result shows that our overall cost inefficiency measure CE_k satisfies the aggregate indication axiom of Blackorby and Russell (1999).

5. Incomplete information

The previous section assumed an ideal scenario in which the empirical analyst had full knowledge of the allocations $(\mathfrak{q}_{k,t}^1, \mathfrak{q}_{k,t}^2, \ldots, \mathfrak{q}_{k,t}^J)$ and the write-offs $(\mathfrak{P}_{k,t}^1, \mathfrak{P}_{k,t}^2, \ldots, \mathfrak{P}_{k,t}^J)$. In practice, however, only very limited information on allocations and write-offs is often available. It may even happen that such information is completely absent. This section shows how to proceed in such (more realistic) instances.

Formally, we will assume that the available information is captured by the polyhedron

$$\Theta(\mathbf{A}, \mathbf{b}) \equiv \left\{ \boldsymbol{\rho} \in \mathbb{R}_+^{TJ(N+M)} : \mathbf{A}\boldsymbol{\rho} \geq \mathbf{b} \right\}, \tag{9}$$

which represents L restrictions on the allocations of storable inputs and on the write-offs of durable inputs. Specifically, $\mathbf{A}$ is a $L \times TJ(N+M)$ matrix and $\mathbf{b}$ a

$L \times 1$ vector, and $\boldsymbol{\rho}$ represents all vectors that satisfy the constraints imposed by $\mathbf{A}$ and $\mathbf{b}$.

To structure our discussion, we will first consider the limiting case in which we cannot use any information on firms' allocations and write-offs, which corresponds to $\Theta = \mathbb{R}_+^{TJ(N+M)}$. Subsequently, we will discuss the intermediate scenario where some information is available, i.e. $\Theta \subset \mathbb{R}_+^{TJ(N+M)}$.

5.1. **No information on allocations and write-offs.** In the absense of full information on storable allocations and durable write-offs, we can no longer check the condition (2) independently for every single time period t. In this case, we verify if there exists at least one possible specification of $(\mathfrak{q}_{k,t}^1, \mathfrak{q}_{k,t}^2, \ldots, \mathfrak{q}_{k,t}^J)$ and $(\mathfrak{P}_{k,t}^1, \mathfrak{P}_{k,t}^2, \ldots, \mathfrak{P}_{k,t}^J)$ that makes firm k's behavior consistent with the overall cost minimization condition (1). More specifically, we define feasible allocations and write-offs that present firm k as efficient as possible. This evaluates firm k in the most favorable light and, thus, gives this firm the benefit-of-the-doubt in the absence of full information.[11]

The following linear program operationalizes this idea:

(10a)
$$\min_{\substack{c_{k,t}\geq 0,\\ (\mathfrak{q}_{s,t}^1,\ldots,\mathfrak{q}_{s,t}^J)_{t=1}^T\geq 0,\\ (\mathfrak{P}_{k,t}^1,\ldots,\mathfrak{P}_{k,t}^J)_{t=1}^T\geq 0}} \sum_{t=J}^{T} \left(\sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j} - c_{k,t} \right)$$

(10b)
$$\text{s.t. } c_{k,t} \leq \sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathfrak{q}_{s,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{s,j} \qquad \forall s \in D_k^t, \quad \forall t = J, \ldots, T,$$

(10c)
$$\sum_{j=1}^{J} \mathfrak{q}_{s,t}^{j} = \mathbf{q}_{s,t} \qquad \forall s \in D_k^t, \quad \forall t = 1, \ldots, T,$$

(10d)
$$\sum_{j=1}^{J} \mathfrak{P}_{k,t}^{j} = \mathbf{P}_{k,t} \qquad \forall t = 1, \ldots, T,$$

In this program, the objective minimizes firm k's cost inefficiency (as defined in (7)) in terms of the chosen allocation and write-off schemes. The first constraint imposes that $c_{k,t}$ effectively represents the minimal cost to produce the output $\mathbf{y}_{k,t}$ (over the set D_k^t). The second and third constraints impose the

[11]This benefit-of-the-doubt idea is intrinsic to DEA efficiency evaluations. See, for example, Cherchye et al. (2007) for a detailed discussion of the benefit-of-the-doubt interpretation of DEA models in the specific context of composite indicator construction.

adding-up restrictions that apply to feasible specifications of $(\mathfrak{q}^1_{k,t}, \mathfrak{q}^2_{k,t}, \ldots, \mathfrak{q}^J_{k,t})$ and $(\mathfrak{P}^1_{k,t}, \mathfrak{P}^2_{k,t}, \ldots, \mathfrak{P}^J_{k,t})$. Intuitively, cost inefficiency occurs as soon as some other firm s is characterized by a lower production cost than firm k no matter what allocations and write-offs are used.

5.2. **Partial information on allocations and write-offs.** In many practical situations, it is possible to put some additional restrictions on the feasible allocation and write-off schemes. Such partial information can be incorporated by suitably specifying $\Theta(\mathbf{A}, \mathbf{b})$. Correspondingly, we can append the restriction

$$(\mathfrak{q}^1_{s,t}, \ldots, \mathfrak{q}^J_{s,t}, \mathfrak{P}^1_{k,t}, \ldots, \mathfrak{P}^J_{k,t})^T_{t=1} \in \Theta(\mathbf{A}, \mathbf{b}) \tag{11}$$

to program (10), and solve the resulting (linear) problem. Clearly, by using restriction (11) we constrain the solution space, which will generally result in higher values of the computed cost inefficiencies.

To take a specific instance, let $(\mathfrak{q}^{u,A}_{k,v})_{u \in U \subseteq [1,\ldots,J]}$ represent lower bounds on the quantity allocations of the storable inputs for firm k and time period(s) $v \in V \subseteq [J, \ldots, T]$. Similarly, let $(\mathfrak{P}^{w,A}_{k,z})_{w \in W \subseteq [1,\ldots,J]}$ be known lower bounds on the price write-offs of the durable inputs for time period(s) $z \in Z \subseteq [J, \ldots, T]$. We then define

$$\Theta = \Big\{ \mathfrak{q}^u_{k,v} \geq \mathfrak{q}^{u,A}_{k,v},\ \forall u \in U,\ \forall v \in V$$
$$\mathfrak{P}^w_{k,z} \geq \mathfrak{P}^{w,A}_{k,z},\ \forall w \in W,\ \forall z \in Z \Big\}.$$

As a limiting case, instantaneous input consumption complies with $\mathfrak{q}^{u,A}_{k,v} = (\mathbf{q}_{k,v}, 0, \ldots, 0)$ or, equivalently, $\mathfrak{P}^{w,A}_{k,z} = (\mathbf{P}_{k,z}, 0, \ldots, 0)$.

5.3. **Write-off hypotheses.** By using this approach, we can actually include (and check) alternative hypotheses regarding the allocation of the durable costs to individual time periods (i.e. specific write-off schemes). For example, it might often be reasonable to assume that the firm's valuation of a durable input diminishes over time. In our framework, this corresponds to

$$\mathfrak{P}^1_{k,t} \geq \mathfrak{P}^2_{k,t} \geq \ldots \geq \mathfrak{P}^J_{k,t}, \tag{12}$$

which complies with a degressive write-off of investment costs. From our above explanation, it follows that this is also consistent with the assumption of technological improvement, where older machines are scrapped and replaced by newer -technologically improved- ones over time.

Alternatively, a linear write-off of investment corresponds to

$$\mathfrak{P}^1_{k,t} = \mathfrak{P}^2_{k,t} = \ldots = \mathfrak{P}^J_{k,t}, \tag{13}$$

implying that $\mathfrak{P}^j_{k,t} = \mathbf{P}_{k,t}/J\ \forall j = 1, \ldots, J$. Both hypotheses can be tested by adding (12) or (13) for $t = J, \ldots, T$ to $\Theta(\mathbf{A}, \mathbf{b})$.

6. Extensions

We next focus on a number of extensions of our basic framework set out in the previous section. These extensions highlight the versatility of our framework and, of course, are not exhaustive. First, we show how to convert our (difference) cost inefficiency measures (6) and (7) in ratio form. Then, we discuss the extension of our framework to allow for heterogeneous input lifetime and production delays. Finally, we indicate how to proceed in the absence of input price information by applying shadow pricing. Here, we will also explain the decomposition of cost inefficiency as defined above in terms of technical and allocative inefficiency. We will illustrate the different extensions in our empirical application in Section 7.

6.1. **Ratio measures of inefficiency.** A downside of our cost inefficiency measure in (7) is that it is not invariant to rescaling of prices and inputs. However, one can turn this difference measure into a ratio measure of cost inefficiency by an appropriate normalization. In principle, a multitude of normalizations are possible. A natural choice is to divide by the actual cost, i.e.

$$RCE_k \equiv \frac{CE_k}{\sum_{t=J}^{T}\sum_{j=t-J+1}^{t}\left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right)} \tag{14}$$

This relative measure is situated between 0 and 1 and expresses the proportion of total production costs that can be saved by minimizing total production costs over the periods $[J,\ldots,T]$.[12]

Analogously to (8), we can decompose this overall ratio measure in terms of per-period measures.[13] In this case, we have that RCE_k equals a weighted sum of per-period cost inefficiencies in ratio form RCE_k^t. Specifically, it uses the period-specific weights

$$w_k^t \equiv \frac{\sum_{j=t-J+1}^{t}\left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right)}{\sum_{t=J}^{T}\sum_{j=t-J+1}^{t}\left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right)}, \tag{15}$$

which represent the proportions of total production costs allocated to every period t. This obtains

[12] This normalization mirrors the one used by Chambers et al. (1998) for profit efficiency.

[13] The following decomposition parallels Färe and Zelenyuk (2003)'s decomposition of industry revenue efficiency as a weighted sum of firms' revenue efficiency.

$$
\begin{aligned}
RCE_k &= \sum_{t=J}^{T} w_k^t \frac{CE_k^t}{\sum_{j=t-J+1}^{t}\left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right)} \\
&= \sum_{t=J}^{T} w_k^t \left(1 - \frac{c_{k,t}}{\sum_{j=t-J+1}^{t}\left(\mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{Q}_{k,j}\right)}\right) \\
&= \sum_{t=J}^{T} w_k^t RCE_k^t, \qquad (16)
\end{aligned}
$$

As a final note, we indicate that $1 - RCE_k$ and $1 - RCE_k^t$ give the conventional cost efficiency measures, i.e. minimal cost divided by actual cost.

6.2. **Heterogeneous input lifetime and production delays.** Until now, we have assumed that all durable inputs have the same lifetime J. Admittedly, this may sometimes be a too strong assumption. In addition, our current specification does not allow for production delays. As we show next, we can solve both issues by making use of the concept of delay matrices.

Specifically, let $\mathbf{D}^D = (\mathbf{d}_1^D, \ldots, \mathbf{d}_J^D) \in \{0,1\}^{M\times J}$ denote a binary delay matrix, where each row represents a durable input. For example, for the durable input m we may use one of the following specifications:

- $(1, \underbrace{0, \ldots, 0}_{J-1})$ if input m is an instantaneous input;
- $(1, \ldots, 1)$ if input m is a durable input with lifetime J;
- $(\underbrace{0, \ldots, 0}_{U}, \underbrace{1, \ldots, 1}_{J-U})$ if input m is a durable input usable after a delay of $U < J$ periods with a lifetime of $J - U$ periods;
- $(\underbrace{1, \ldots, 1}_{U}, \underbrace{0, \ldots, 0}_{J-U})$ if input m is a durable input usable over $U < J$ periods.

Similarly, let $\mathbf{D}^S = (\mathbf{d}_1^S, \ldots, \mathbf{d}_J^S) \in \{0,1\}^{N\times J}$ represent a binary delay matrix for the storable inputs. Then, we can formulate the next modified optimization problem of firm k:

(17a)
$$
\min_{\substack{(\mathfrak{q}_t^1,\ldots,\mathfrak{q}_t^J)_{t=1}^T \\ (\mathfrak{P}_t^1,\ldots,\mathfrak{P}_t^J)_{t=1}^T}} \sum_{t=J}^{T} \sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathbf{d}_{t-j+1}^S \mathfrak{q}_j^{t-j+1} + \mathfrak{P}_j^{t-j+1}\mathbf{d}_{t-j+1}^D \mathbf{Q}_{k,j}
$$

$$
\text{s.t.} \quad \left(\sum_{j=1}^{J} \mathbf{d}_j^S \mathfrak{q}_{t-j+1}^j, \sum_{j=1}^{J} \mathbf{d}_j^D \mathbf{Q}_{k,t-j+1}\right) \in \mathcal{I}_t(\mathbf{y}_{k,t}) \qquad \forall t = J, \ldots, T \qquad (17b)
$$

Closer inspection reveals that $\mathbf{d}_j^S \mathfrak{q}_{t-j+1}^j$ and $\mathbf{d}_j^D \mathbf{Q}_{k,t-j+1}$ only selects those inputs that are usable in period j. In this case, J stands for the maximum lifetime over all durable and storable inputs.

The associated linear program is

(18a)
$$\min_{\substack{c_{k,t}\geq 0,\\ (\mathfrak{q}_{s,t}^1,\ldots,\mathfrak{q}_{s,t}^J)_{t=1}^T \geq 0,\\ (\mathfrak{P}_{k,t}^1,\ldots,\mathfrak{P}_{k,t}^J)_{t=1}^T\geq 0}} \sum_{t=J}^{T}\left(\sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathbf{d}_{t-j+1}^S \mathfrak{q}_{k,j}^{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{d}_{t-j+1}^D \mathbf{Q}_{k,j} - c_{k,t}\right)$$

(18b)
$$\text{s.t. } c_{k,t} \leq \sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathfrak{q}_{s,j}^{t-j+1}\mathbf{d}_{t-j+1}^S + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{d}_{t-j+1}^D \mathbf{Q}_{s,j} \qquad \forall s \in D_k^t, \quad \forall t = J,\ldots,T,$$

(18c)
$$\sum_{j=1}^{J} \mathfrak{q}_{s,t}^j \mathbf{d}_j^S = \mathbf{q}_{s,t} \qquad \forall s \in D_k^t, \quad \forall t = 1,\ldots,T,$$

(18d)
$$\sum_{j=1}^{J} \mathfrak{P}_{k,t}^j \mathbf{d}_j^D = \mathbf{P}_{k,t} \qquad \forall t = 1,\ldots,T,$$

(18e)
$$(\mathfrak{q}_{s,t}^1,\ldots,\mathfrak{q}_{s,t}^J,\mathfrak{P}_{k,t}^1,\ldots,\mathfrak{P}_{k,t}^J)_{t=1}^T \in \Theta(\mathbf{A},\mathbf{b}).$$

It is easy to verify that this program reduces to (10) for $\mathbf{D}^S = \mathbb{1}_{N\times J}$ and $\mathbf{D}^D = \mathbb{1}_{M\times J}$. Furthermore, any zero values in $\mathbf{D}^S$ and $\mathbf{D}^D$ immediately imply zero values for the corresponding allocations and write-offs. In other words, the use of delay matrices allows us to impose a priori restrictions on the storable allocations and durable write-offs. We also remark that, in principle, we can specify firm-specific delay matrices if this seems desirable.

6.3. **Shadow prices and technical inefficiency.** So far, we have focused on economic (cost) efficiency, which requires price information for the relevant inputs. By contrast, technical efficiency analysis does not require such price information and, thus, can be used if limited price information is available.

Generally, technical efficiency criteria/measures can be characterized as economic efficiency criteria/measures evaluated at so-called "shadow prices".[14] Thus, by establishing the shadow price representation of our dynamic efficiency concepts,

[14] In DEA terminology, this shadow price characterization of technical efficiency corresponds to the "multiplier" formulation of DEA models. Practical applications often make use of DEA models in "envelopment" form, which is dual to this multiplier formulation. In our set-up, the

we can define technical efficiency notions that explicitly account for the dynamic (storable and durable) nature of the inputs.

It is easy to use shadow pricing if the exact allocation of storable inputs over time periods (i.e. $(\mathfrak{q}^1_{s,t}, \ldots, \mathfrak{q}^J_{s,t})^T_{t=1}$) is known to the empirical analyst. In that case, it suffices to solve (18) with the input prices $(\mathbf{P}_{k,t})^T_{t=1}$ and $(\mathbf{p}_{k,t})^T_{t=1}$ as additional free variables that are subject to a negativity constraint and the normalization

$$\sum_{t=J}^{T}\left(\sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathfrak{q}_{k,j}^{t-j+1}\mathbf{d}^S_{t-j+1} + \mathfrak{P}_{k,j}^{t-j+1}\mathbf{d}^D_{t-j+1}\mathbf{Q}_{k,j}\right) = 1.$$

We remark that this price normalization implies $CE_k = RCE_k$.

Let TE_k represent the "technical inefficiency" measure that is obtained as the solution of the resulting linear program. By construction, we have $TE_k \leq RCE_k$. The difference between TE_k and RCE_k gives us a measure AE_k of allocative inefficiency, i.e.

$$TE_k + AE_K = RCE_k \Leftrightarrow AE_k = RCE_k - TE_k. \tag{19}$$

Interestingly, because $RCE_k = CE_k$ under our price normalization and using (7), we can also decompose AE_k in period-specific allocative inefficiencies, as follows:

$$AE_k \equiv \sum_{t=J}^{T}\left(RCE^t_k - TE^t_k\right) = \sum_{t=J}^{T} AE^t_k. \tag{20}$$

Finally, matters are more complicated when the allocation $(\mathfrak{q}^1_{s,t}, \ldots, \mathfrak{q}^J_{s,t})^T_{t=1}$ is unobserved. In that case, the analogue of the programming problem (18) becomes nonlinear in unknown prices and quantities. We can restore linearity by making specific assumptions regarding the storable input allocation. For example, if we are willing to assume that all DMUs allocate their storable inputs in the same way over time, then we can use a similar procedure as outlined in Cook et al. (2000) and Cherchye et al. (2013).

7. Empirical illustration

We apply our model to a panel dataset of Swiss regional railway companies that was also studied by Farsi et al. (2005).[15] The original panel is unbalanced and contains yearly information on 50 railway companies over the period 1985 − 1997. From this dataset, we constructed a balanced panel that covers the 13-year period for 37 companies, and which contains all input and output information needed to apply our methodology. The constructed balanced panel contains 481 (= 13 × 37) firm observations.

envelopment formulation can be obtained as the dual of the linear program that we define below (to compute TE_k).

[15] The data are available at http://people.stern.nyu.edu/wgreene/Econometrics/PanelDataSets.htm, which also contains a detailed description of all variables.

In what follows, we first motivate our selection of outputs and inputs, where we will use capital expenses as a durable input. Then, we present the results of our empirical analysis, which will mainly focus on overall and per-period efficiencies as well as technical and allocative inefficiency. We conclude by a number of sensitivity checks.

7.1. **Output and input specification.** The original dataset contains information on total expenses, labor and energy expenses, as well as the total number of employees, electricity consumption, network length, total number of available seats, total number of train-kilometers, passenger-kilometers and ton-kilometers. Capital expenses are defined as the residual after deducting labor and energy from the total expenses. Prices for labor and energy are found by dividing (labor and energy) expenses by total quantity (number of employees and total electricity consumption in kWh, respectively). The price of capital is defined by dividing capital expenses by total number of seats.

Following Farsi et al. (2005), we use total passenger-kilometers and ton-kilometers as our two outputs in our following analysis. Next, we have three inputs: labor, energy and capital expenses. We refer to Farsi et al. (2005) for more details on the data. Table 2 presents summary statistics. The most important observation is that labor and capital expenditures are the major costs (52.82% and 43.41% on average), while energy expenditures represents only a small fraction of the total cost (i.e. 3.77% on average).

	mean	std	median	min	max	share (in %)
Passenger output in passenger kilometers (Q2) ($\times 10^8$)	0.2843	0.5192	0.0919	0.0041	3.1100	95.27
Goods output in ton kilometers (Q3) ($\times 10^7$)	0.2135	0.8158	0.0226	0.0000	5.9400	4.73
Length of railway network in km (NETWORK)	39.5340	61.0973	22.8200	3.8980	376.9970	n.a.
Number of stations on the network (STOPS)	20.8274	20.1613	15.0000	4.0000	121.0000	n.a.
Labor price adjusted for inflation (PL) ($\times 10^5$)	0.8550	0.0602	0.8557	0.6093	1.0493	n.a.
Number of employees (STAFF) ($\times 10^3$)	0.1401	0.2517	0.0520	0.0120	1.6410	n.a.
Labor expenditures (LABOREXP) ($\times 10^7$)	1.2189	2.1945	0.4406	0.0985	14.6988	52.82
Price of electricity in CHF per kWh (PE)	0.1574	0.0240	0.1580	0.0763	0.2652	n.a.
Total consumed electricity in kWh (KWH) ($\times 10^7$)	0.5775	1.0317	0.1980	0.0082	6.5849	n.a.
Energy expenditures (ELECEXP) ($\times 10^5$)	8.4849	12.9842	3.0220	0.1400	81.0408	3.77
Capital price per seat (PK) ($\times 10^6$)	0.2182	0.3644	0.0872	0.0212	2.4105	n.a.
Quantity of Capital (CAPITAL)	43.4092	9.4026	41.7978	23.8892	77.3315	n.a.
Capital expenditures ($\times 10^6$)	8.7849	13.4185	3.9922	0.6119	87.9753	43.41
Total costs adjusted for inflation (CT) ($\times 10^8$)	0.2182	0.3644	0.0872	0.0212	2.4105	100

TABLE 2. Summary statistics of the railway data (481 observations)

As indicated above, capital expenditures form a prime example of durable inputs. Therefore, while we consider labor and energy expenditures as instantaneously consumed (i.e. not storable or durable), we will treat capital expenditures as durable. For the general model specification (with capital usable in J years), this obtains the $3 \times J$ delay matrix

$$\mathbf{D}^D = \begin{pmatrix} 1 & 0 \dots 0 \\ 1 & 0 \dots 0 \\ 1 & \underbrace{1 \dots 1}_{J-1} \end{pmatrix},$$

with labor, energy and capital corresponding to rows 1, 2 and 3, respectively.

Table 3 reports summary statistics on our durable input. We see that, in nominal terms, capital costs are steadily increasing until 1991. Within individual years, we also observe considerable variation across firms. The magnitude of this variation is fairly stable over time.

year	mean	std	min	max
85	7821.9918	13415.8909	675.5952	79364.5942
86	7947.3623	13476.0897	611.8980	79606.2277
87	8226.9291	13474.9261	614.8458	78815.1313
88	8769.3795	13483.8216	725.2748	79002.7419
89	9293.5348	13890.6071	1104.4175	79672.0210
90	9531.1382	14310.4937	833.1193	82232.3410
91	9683.8843	14759.9689	991.5967	86268.9002
92	9541.3585	14988.2197	949.5515	87975.3422
93	8868.5766	13414.5760	940.9871	78469.9559
94	8709.7330	13665.2525	704.6633	80731.6209
95	8707.0591	12669.9599	691.9920	73890.1362
96	8396.3872	12086.1047	780.2419	70634.0593
97	8706.7027	12561.9671	873.0000	73087.0000

TABLE 3. Capital costs: summary statistics per year (in 1000 CHF)

It follows from our discussion in the previous sections that treating capital expenses as a durable input requires us to use discounted prices, and to specify the lifetime of capital (i.e. J). In our application all provided prices are adjusted for inflation with respect to 1997 prices. Next, capital costs are related to equipment as well as materials. This makes it hard to specify the exact lifetime of this durable input. For this reason, and to clearly demonstrate the potential impact of intertemporal dependencies between inputs, we will mainly focus on a minimalistic scenario with $J = 2$. As an additional exercise, we will also consider alternative values for J, to check robustness of our main conclusions.

7.2. **Cost efficiency analysis.** In summary, the dynamic nature of our empirical analysis relates to a single durable input, capital expenses. Moreover, this input represents a fairly large fraction of the total cost relative to the instantaneous inputs, labor and energy expenses (see our discussion of Table 2). In what follows, we will show that ignoring the intertemporal (durable) aspect of capital can substantially affect the efficiency analysis. In turn, referring to our discussion in the Introduction, this can considerably distort regulatory policies (in our case for Swiss railway companies) that are based on the efficiency results. Obviously, these distortions will generally be more pronounced in production settings where durable inputs form an even more important fraction of total costs, and in settings with storable inputs in addition to durable inputs.

We first consider the differences in cost inefficiencies between the dynamic and static setting. Figure 1 shows the differences between the CE_k^t-values for our dynamic model (with $J = 2$) and the static model (which corresponds to $J = 1$). The differences in per-period inefficiencies are quite substantial: in some years (such as 89, 92 and 94) ignoring intertemporal effects leads to an underestimation of productive inefficiency by as much as 4 million CHF, while in other years (e.g. 88, 90 and 93) it leads to an overestimation by no less than 6 million CHF. The differences are statistically significant: comparing the cumulative density functions of CE_k^t using a Kolmogorov-Smirnov test, we reject at the 1% significance level the hypothesis that both densities have the same underlying distribution.[16]

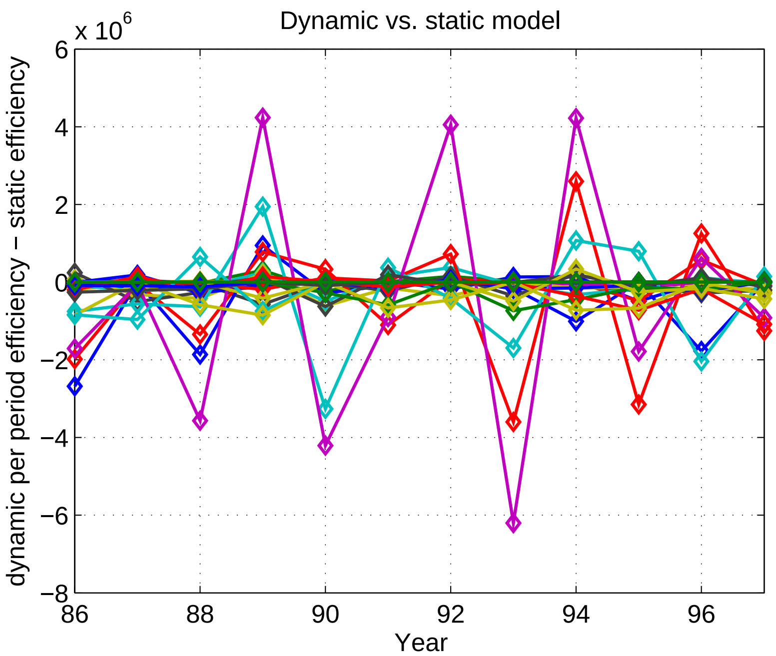

FIGURE 1. Dynamic ($J = 2$) vs. static ($J = 1$) CE_k^t

Next, we turn to the ratio measure RCE_k that we defined in (14). Comparing the results for $J = 1$ with those for $J = 2$ provides further insight into the severity

[16]The value of the test statistic is 0.1351, which corresponds to a p-value of 5.24×10^{-4}.

and frequency of disagreement between the dynamic and static inefficiency models. Figure 2 depicts a histogram of the differences in RCE_k^t-values. Both models agree in terms of RCE_k^t in 61.94% of all cases. For the cases where they do not agree, this histogram shows that inefficiency is more often overestimated by the static model: in 13.02% of all cases the static model overestimates inefficiency by at least 5%, while inefficiency is underestimated by at least 5% in only 4.14% of the cases.

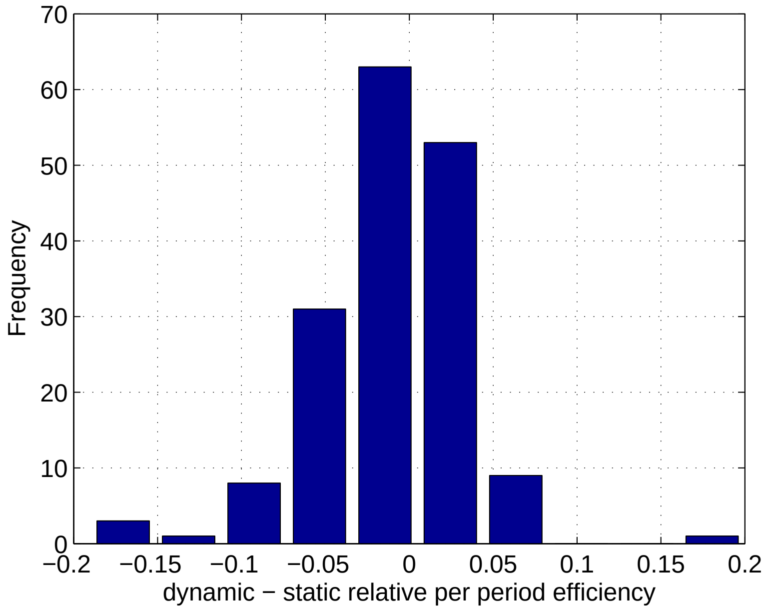

FIGURE 2. Histogram of dynamic - static RCE_k^t (> 0)

As a following exercise, we redo our analysis by using shadow prices. As explained above, this effectively computes the technical inefficiency measure TE_k, which we can further use to calculate the aggregate allocative inefficiency measure AE_k (in (19)) as well as per-period allocative inefficiencies AE_k^t (in (20)). Table 4 shows the TE_k and AE_k results for all firms. We find that technical inefficiency is rather negligible for the firms under study: the maximal TE_k-value amounts to no more than 4.899×10^{-9}. In contrast, the AE_k-values are quite high for a number of firms: the worst performing firm has an allocative inefficiency of as much as 0.3636. Figure 3 also shows that there is substantial variation in the AE_k^t-values over time.

7.3. **Robustness checks.** We conclude our empirical analysis by conducting a number of robustness checks. These additional exercises will further illustrate the versatility of our general framework, in terms of relaxing or imposing particular assumptions in the intertemporal efficiency assessment. First, we consider the effect of specifying J on our results. Second, we investigate the effects of different environmental variables on the efficiency scores. Finally, we compute efficiency

firm id	TE_k	rank	AE_k	rank
2	1.334e-13	26	0.02762	23
3	3.209e-14	24	0.0399	25
4	9.104e-15	16	3.726e-11	11
5	2.645e-17	8	-7.34e-18	5
6	1.608e-14	17	0.01319	19
7	2.781e-14	23	0.01699	21
8	1.86e-13	28	0.02674	22
9	2.508e-13	29	0.1993	34
10	-1.166e-15	5	1.612e-10	13
12	5.801e-14	25	0.04194	26
13	5.633e-11	36	0.3636	37
14	5.694e-12	33	-5.692e-12	2
15	1.437e-13	27	0.09381	30
16	4.899e-09	37	0.1528	31
17	2.359e-16	10	0.1605	32
18	1.42e-12	31	-1.412e-12	3
20	-4.066e-15	3	0.2009	35
21	5.967e-13	30	0.0883	28
22	-1.11e-16	6	0.2082	36
23	-1.535e-13	2	7.38e-13	8
24	1.976e-14	20	0.08463	27
26	2.57e-14	22	1.238e-12	9
27	2.22e-16	9	0.004088	16
30	4.287e-12	32	0.1862	33
31	-4.978e-13	1	3.269e-10	15
34	6.904e-12	35	2.034e-10	14
36	5.135e-16	11	0.0127	18
37	0	7	4.068e-14	6
39	5.551e-16	12	3.511e-13	7
41	5.878e-12	34	-5.878e-12	1
42	2.329e-14	21	3.881e-11	12
43	7.883e-15	15	0.0139	20
45	1.221e-15	13	-1.124e-15	4
46	-3.608e-15	4	0.0937	29
47	1.787e-14	18	0.00819	17
48	4.219e-15	14	0.03071	24
49	1.932e-14	19	3.268e-12	10

TABLE 4. TE_k and AE_k

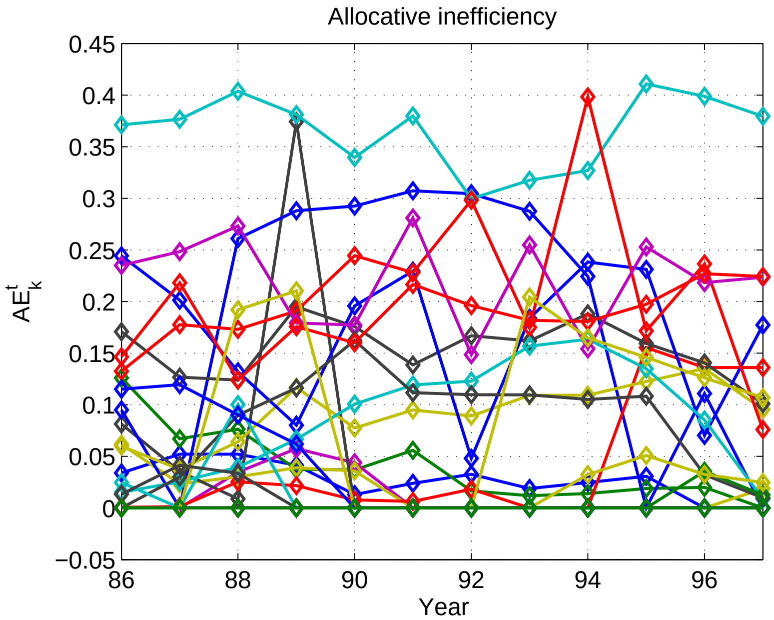

FIGURE 3. Allocative inefficiencies AE_k^t

results when imposing particular (degressive and linear) structure on the write-off schemes used by the evaluated firms.

We begin by evaluating the overall dynamic inefficiencies of our 37 firms for alternative specifications of J (> 1). Table 5 shows the scores and the relative rankings of all firms for varying choices of J. An interesting observation is that, although we observe some changes for different J-values, the firm rankings are fairly robust in general. In a similar vein, for most firms the inefficiency scores do not change much with J. This is confirmed by 6 Kolmogorov-Smirnov tests that verify equality of distribution of the inefficiency scores: all p-values ranged between $0.4787 - 0.9995$, so that we cannot reject the null hypothesis that the inefficiencies come from the same underlying distribution.

Next, we examine whether differences in firms' (observable) environments impact the efficiency results. In this respect, our dataset contains information on the length of the railway network (NETWORK) and the number of stations in the network (STOPS) (see Table 2). A priori, one may expect both variables to have a negative effect on efficiency, as larger networks can give rise to higher costs due to maintenance and, similarly, because more stops imply additional (e.g. time-related) expenditures, all else equal. Next, the dataset contains a dummy variable indicating whether the network has a rack rail ("cremaillere"; represented by the binary variable RACK). Rack rails are special rails that are used to aid climbing of trains on steep terrain. Therefore, one may argue that the presence of rack rails effectively signals a less favorable operational environment.

firm id	J=2	rank	J=3	rank	J=4	rank	J=5	rank
2	0.028	23	0.021	23	0.011	22	0.0016	22
3	0.04	25	0.032	25	0.03	25	0.024	25
4	3.7e-11	11	1.5e-14	12	5.9e-13	12	1.4e-14	7
5	1.9e-17	1	2.4e-12	16	9.1e-13	13	-8.4e-17	1
6	0.013	19	0.013	22	0.011	23	0.0064	23
7	0.017	21	0.0091	21	0.0015	21	0.00036	21
8	0.027	22	0.023	24	0.02	24	0.022	24
9	0.2	34	0.19	35	0.19	35	0.16	34
10	1.6e-10	13	2.1e-10	18	4.3e-10	20	1.3e-10	19
12	0.042	26	0.041	26	0.04	26	0.041	26
13	0.36	37	0.37	37	0.37	37	0.36	37
14	1.6e-15	4	4.6e-15	9	3.2e-15	5	3e-11	16
15	0.094	30	0.081	28	0.066	27	0.057	27
16	0.15	31	0.14	32	0.14	32	0.14	32
17	0.16	32	0.14	31	0.13	31	0.11	31
18	7.7e-15	5	4.7e-16	7	2e-16	2	1.6e-10	20
20	0.2	35	0.18	33	0.17	33	0.16	33
21	0.088	28	0.085	29	0.081	29	0.067	28
22	0.21	36	0.21	36	0.21	36	0.21	36
23	5.8e-13	8	6.5e-15	10	8.5e-15	7	2.6e-15	6
24	0.085	27	0.08	27	0.078	28	0.068	29
26	1.3e-12	9	9.3e-17	6	2.7e-14	10	3.3e-14	8
27	0.0041	16	3.6e-15	8	2.5e-12	15	6e-16	5
30	0.19	33	0.19	34	0.18	34	0.18	35
31	3.3e-10	15	3.2e-17	3	1.9e-17	1	3.1e-13	10
34	2.1e-10	14	2.1e-12	15	5.1e-12	16	3.5e-14	9
36	0.013	18	-3.4e-15	2	2.4e-14	9	2.2e-12	13
37	4.1e-14	6	5.3e-17	5	5.9e-13	11	2.4e-16	4
39	3.5e-13	7	1.4e-14	11	3.2e-16	3	3.8e-12	14
41	2e-16	3	3.1e-13	13	5.7e-11	18	1.4e-16	3
42	3.9e-11	12	2.6e-10	19	2.1e-14	8	3.8e-11	17
43	0.014	20	2.8e-11	17	1.2e-12	14	-2.9e-17	2
45	9.7e-17	2	1.2e-12	14	1.9e-10	19	6.2e-13	11
46	0.094	29	0.092	30	0.096	30	0.09	30
47	0.0082	17	-1.3e-14	1	5.7e-15	6	1.2e-12	12
48	0.031	24	0.0032	20	3.9e-11	17	3.9e-11	18
49	3.3e-12	10	4.9e-17	4	3.8e-16	4	5.2e-12	15

TABLE 5. RCE_k for different J

We investigate how these variables affect our results by conducting, for each variable separately, an extra efficiency analysis in which we add the environmental variable $z_{s,t} \in \mathbb{R}_+$ as an additional output. Basically, this procedure implies that the dominating set (4) is modified to (only) include those peers that (1) produce at least the same output and (2) operate under the same, or worse, environmental conditions than the firm under examination. Following Ruggiero (1996), the modified dominating set is:

$$D_k^t = \left\{ s | \mathbf{y}_{s,t} \geq \mathbf{y}_{k,t} \right\} \cap \begin{cases} s|z_{s,t} = z_{k,t}\} & z_{s,t} \in \{0,1\}, \\ s|z_{s,t} \geq z_{k,t}\} & \text{Otherwise} \end{cases},$$

where $z_{s,t} \geq z_{k,t}$ implies s operates under worse conditions than k. By comparing these new efficiency results with the original ones for $J = 2$ (see Table 5), we can investigate the efficiency effect of the three contextual variables under study.

The results of these three exercises are summarized in Figures 4-5-6. Specifically, each of these figures sets out the firm ranks for the new exercises to the original firm ranks. Firms situated below the 45 degree line have a higher ranking (i.e. lower rank number) when the contextual variable (respectively, NETWORK, STOPS and RACK) is taken into account while, obviously, the opposite holds for firms above the 45 degree line. For each of our three environmental variables, we find that the firm ranks are fairly mildly affected, with the exception of a few firms. This is confirmed by Wilcoxon signed-rank tests, which check the statistical significance of the difference between the new and original rankings: it turns out that there is no significant difference for any of the three variables under evaluation.[17] We may thus conclude that none of our three contextual variables has a substantial effect on the efficiency patterns that we presented above.

Finally, at the end of Section 5 we indicated that an interesting feature of our methodology is that it allows for imposing specific hypotheses regarding the allocation of the costs of durables to individual time periods (i.e. putting structure on the write-off schemes). As a last robustness check, we compute efficiency results for the degressive scheme in (12) and the linear scheme in (13).

Our results are given in Table 6. We observe that, for a number of firms, the inefficiency values for the degressive write-off scheme are somewhat above the ones that we obtained in our original analysis (see the "Unconstrained" column), and the values for the linear scheme are always above those for the degressive scheme. Actually, this could be expected a priori, as the degressive and linear models put increasingly stringent structure on possible allocations of capital costs to successive

[17] More specifically, the Wilcoxon signed-rank test compares the ranking of individual firms by first taking the difference in ranking. Observations with zero difference are dropped. These differences are then ranked and these ranks are summed. If there is no difference in ranking then the test statistic is zero. For NETWORKS the statistic equals 269 and the associated p-value is 0.3136, for STOPS the statistic is 258 and the p-value 0.1576, and for RACK the statistic amounts to 236 and the p-value 0.1246.

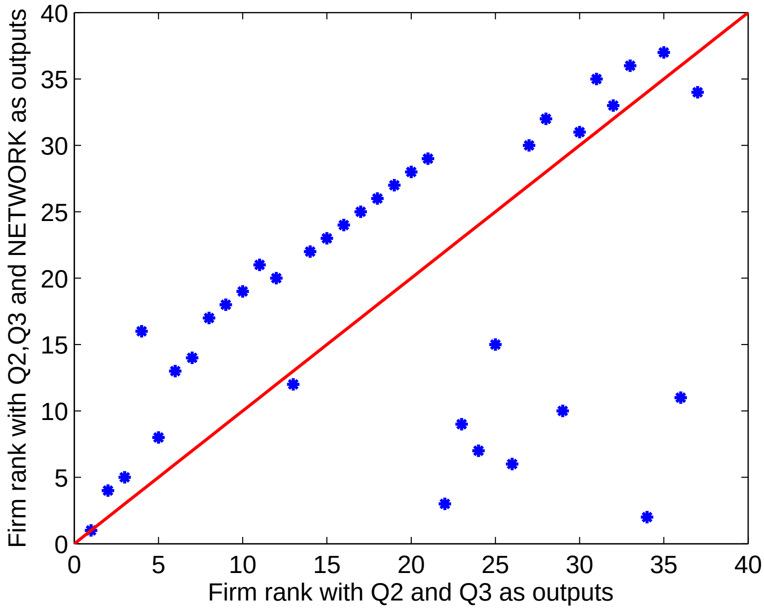

FIGURE 4. Firm rank comparison: with and without NETWORK as output

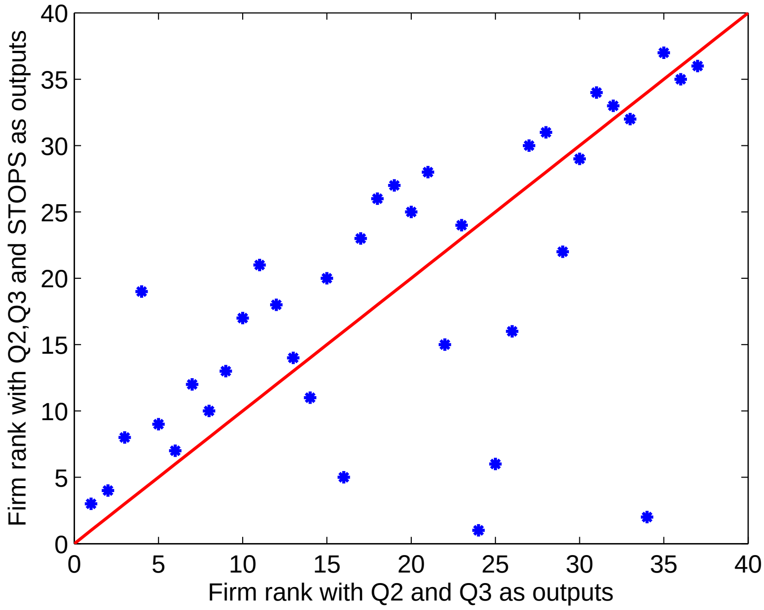

FIGURE 5. Firm rank comparison: with and without STOPS as output

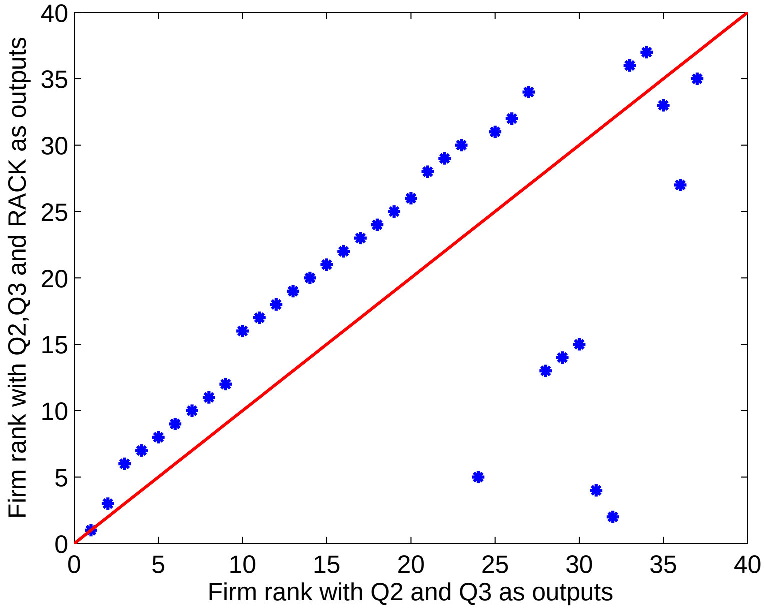

Figure 6. Firm rank comparison: with and without RACK as output

time periods. However, the firms' inefficiency differences are very small in general. In fact, the efficiency rankings hardly change. Thus, we may safely conclude that the efficiency results of our main analysis presented above are quite robust with respect to using degressivity or linearity for the write-off schemes of the durable capital input.

8. Conclusion

We have presented a methodology for intertemporal analysis of economically (cost) efficient production behavior that can account for intertemporal considerations related to the use of storable and durable inputs. The methodology is intrinsically nonparametric, which means that it does not require imposing (non-verifiable) functional structure on the production technology. The methodology is versatile in that it can account for production delays of durable inputs. In addition, it allows for defining a cost inefficiency measure that can be decomposed in period-specific inefficiencies. These cost inefficiencies can be computed through simple linear programming.

Our application to Swiss railway companies has shown the empirical usefulness of our methodology. Most notably, it showed that explicitly accounting for the dynamic nature of (in our case capital) inputs can significantly impact the efficiency results. For a considerable number of firms, we found that per-period inefficiencies for our model with capital investments as durable inputs differed substantially from the ones for the (static) model that ignores such intertemporal durability. At

firm id	Unconstrained	rank	Degressive	rank	Linear	rank
2	0.028	23	0.03	22	0.032	19
3	0.04	25	0.04	25	0.041	23
4	3.7e-11	11	4.7e-11	12	2.6e-16	1
5	1.9e-17	1	1.5e-10	13	0.0011	14
6	0.013	19	0.014	17	0.015	17
7	0.017	21	0.018	19	0.021	18
8	0.027	22	0.031	23	0.044	24
9	0.2	34	0.21	36	0.23	36
10	1.6e-10	13	4e-17	2	1.6e-15	4
12	0.042	26	0.043	26	0.047	25
13	0.36	37	0.36	37	0.36	37
14	1.6e-15	4	0.0014	15	0.0042	15
15	0.094	30	0.1	30	0.11	30
16	0.15	31	0.16	31	0.17	31
17	0.16	32	0.17	32	0.18	32
18	7.7e-15	5	3.5e-10	14	3.1e-12	10
20	0.2	35	0.21	35	0.22	35
21	0.088	28	0.091	28	0.096	28
22	0.21	36	0.21	34	0.21	34
23	5.8e-13	8	3.4e-12	11	1.5e-10	13
24	0.085	27	0.088	27	0.091	27
26	1.3e-12	9	1.2e-16	3	1.7e-13	6
27	0.0041	16	0.016	18	0.034	20
30	0.19	33	0.19	33	0.19	33
31	3.3e-10	15	1.2e-12	10	7.2e-12	12
34	2.1e-10	14	3.8e-18	1	1.3e-15	3
36	0.013	18	0.024	21	0.053	26
37	4.1e-14	6	1.1e-12	9	4.8e-16	2
39	3.5e-13	7	7.2e-15	4	6.4e-13	9
41	2e-16	3	3e-13	8	5.1e-12	11
42	3.9e-11	12	1.3e-14	6	1.6e-13	5
43	0.014	20	0.023	20	0.034	21
45	9.7e-17	2	1.5e-14	7	2e-13	7
46	0.094	29	0.096	29	0.1	29
47	0.0082	17	0.012	16	0.012	16
48	0.031	24	0.035	24	0.039	22
49	3.3e-12	10	1.2e-14	5	4.1e-13	8

TABLE 6. Dynamic efficiency scores RCE_k $(J = 2)$ under degressive and linear write-off

a more general level, these empirical findings demonstrate the practical relevance of our methodology for regulators: erroneously disregarding intertemporal aspects of firms' production decisions may substantially distort the efficiency assessment and, therefore, also the policy conclusions that are drawn from it.

We see multiple possible extensions. First, we have been considering a multi-output setting in the current paper but ignored any output interdependencies, mainly to simplify our exposition. In practice, however, interdependencies among outputs often exist in the form of joint inputs. From this perspective, it seems particularly interesting to combine our methodology for dynamic production analysis with the (nonparametric) methodology for multi-output production analysis that was recently developed by Cherchye et al. (2013, 2014). This multi-output framework accounts for interdependencies between different output production processes through jointly used inputs, which are formally similar in nature to the durable inputs on which we focus in the current paper (i.e. they capture inter-period interdependencies between production decisions). Combining the two methodologies will further enhance the realistic modeling of production interdependencies (across outputs as well as time periods).

Next, our cost and technical inefficiency measures can be used to measure productivity by combining it with various productivity measures such as the cost Malmquist index of Maniadakis and Thanassoulis (2004) or the Malmquist index of Caves et al. (1982), Bjurek (1996)'s Hicks-Moorsteen index or the Luenberger indicator of Chambers et al. (1996), among others. These productivity measures have been proposed in the context of nonparametric (DEA) analysis of productive efficiency and, therefore, are easily combined with our novel methodology. This combination will lead to richer productivity analyses because it explicitly accounts for intertemporal production interdependencies through storable and durable inputs.

Finally, Varian (1982) has developed a nonparametric approach to consumer demand analysis that is formally analogous to the nonparametric approach to production analysis to which we adhere here. Following this analogy, we may translate the insights developed in the previous sections towards a consumption setting, so to obtain a more realistic modeling of intertemporal aspects of consumer behavior.[18] Specifically, our concept of storable inputs corresponds to the notion of infrequent purchases in a consumption context, and durable inputs are similar in spirit to durable consumption goods (for example, cars, houses, etc.) in a demand setting.

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[18] See, for example Crawford (2010) and Crawford and Polisson (2014) for recent contributions to the nonparametric analysis of intertemporal consumer behavior.

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Appendix A. The economic meaning of write-off schemes

In this short section we clarify the economic intuition behind the write-off schemes for the durable inputs. For a moment, let us rewrite (1) by replacing input requirement sets with production functions:

$$\min_{\substack{(\mathfrak{q}_t^1,\dots,\mathfrak{q}_t^J)_{t=J}^T \\ (\mathfrak{P}_t^1,\dots,\mathfrak{P}_t^J)_{t=J}^T}} \sum_{t=J}^{T} \sum_{j=t-J+1}^{t} \mathbf{p}_{k,j}\mathfrak{q}_j^{t-j+1} + \mathfrak{P}_j^{t-j+1}\mathbf{Q}_{k,j} \tag{21a}$$

$$\text{s.t. } \mathbf{F}_t\left(\sum_{j=1}^{J}\mathfrak{q}_{t-j+1}^{j}, \sum_{j=1}^{J}\mathbf{Q}_{k,t-j+1}\right) \geq \mathbf{y}_{k,t} \qquad \forall t = J,\dots,T \tag{21b}$$

The first order conditions with respect to $\mathbf{Q}_{k,t}$, for all $t = J,\dots,T$, are

$$\sum_{j=1}^{J}\mathfrak{P}_{k,t}^{j} - \sum_{j=1}^{J}\lambda_{t+j-1}\frac{\partial \mathbf{F}_{t+j-1}}{\partial \mathbf{Q}_{k,t}} \geq 0 \Leftrightarrow \mathbf{P}_{k,t} - \sum_{j=1}^{J}\lambda_{t+j-1}\frac{\partial \mathbf{F}_{t+j-1}}{\partial \mathbf{Q}_{k,t}} \geq 0,$$

which holds with equality if $\mathbf{Q}_{k,t} > 0$. Rearranging shows that, when a durable input $\mathbf{Q}_{k,t}$ is purchased at time t, the discounted market prices reflect the expected marginal benefits to production of the durable inputs over their entire lifetime, i.e.

$$\mathbf{P}_{k,t} = \sum_{j=1}^{J}\lambda_{t+j-1}\frac{\partial \mathbf{F}_{t+j-1}}{\partial \mathbf{Q}_{k,t}}, \tag{22}$$

which reveals a write-off scheme that defines the valuation of the firm for durable inputs in terms of their marginal effects on productivity in periods t to $t+J-1$. It is as if the firm invests in this input and writes off this investment for J periods. We capture this interpretation by (implicit) period-specific prices

$$\mathfrak{P}_{k,t}^{j} = \lambda_j \frac{\partial \mathbf{F}_j}{\partial \mathbf{Q}_{k,t}}. \tag{23}$$

Intuitively, these period-specific prices attribute part of the cost of the durable inputs to different periods t in accordance to the inputs' marginal productivities.