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Addendum to: "Ranking opportunity sets on the basis of similarities of preferences: a proposal"

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Abstract. Vázquez (2014) proposes to rank opportunity sets on the basis of the similarities of the elements within each set. The ranking rule, denoted by *SL*, is lexicographic and takes into account the indirect utility, the number of elements that are similar to the best element, and finally

the utilities of representatives of subsequently lower similarity classes.

This note corrects a gap in the characterization of this rule.

We recall the setup, argue that the rule *SL* violates one of the axioms, propose a modified version of this axiom, and restore the characterization result.

The set X of alternatives is equipped with an asymmetric preference relation P. A non-empty subset of X is called an opportunity set, Z collects all opportunity sets, and $\succsim$ is an ordering on Z. A similarity relation S on X induces a partitioning of the opportunity sets into homogeneous partition classes. As such, the opportunity set A partitions into

$$A \;=\; A_1 \cup A_2 \cup \cdots \cup A_k,$$

with $A_1 P A_2 P \cdots P A_k$. Elements in the same partition class are sufficiently close (in terms of preferences). The best element of the homogeneous class A_i is denoted by a_i.

Axiom (S-C) introduces a three-clause similarity-composition condition. (S-C,a) considers four homogeneous opportunity sets A, B, C, D with $A \cap C = B \cap D = \varnothing$, $A \sim B$, APc_1, BPd_1, and $a_1 S d_1$; and imposes that $B \cup D \succsim A \cup C$. This axiom is used in the proof of Lemma 1 and of Theorem 1 (case 3).

We now argue that the rule *SL* does not satisfy (S-C,a). Split up a large homogeneous set into two disjoint parts A and D with APD and $|D| \geq 2$. Let $B = A$ and $C = \{d_1\}$. Then, $c_1 = d_1$, APc_1, and $a_1 S d_1$. According to (S-C,a), $B \cup D \succsim A \cup C$. The sets $A \cup C$ and $B \cup D$ are both homogeneous and have the same indirect utility (equal to

$u(a_1)$). According to *SL*, the smaller set $A \cup C$ is strictly better than $B \cup D = A \cup D$ (cf. Lemma 2). Conclude that *SL* violates (S-C,a).

The following modified version (S-C,a*) of (S-C,a) restores the characterization result.

(S-C,a*). Let A, B, C, D be homogeneous, $D = \{d_1\}$, $A \cap C = B \cap D = \varnothing$, APc_1, BPd_1, $A \sim B$, and $a_1 S d_1$. Then, $B \cup D \succsim A \cup C$.

The rule *SL* satisfies axiom (C-S,a*). We now reconsider the proofs of Lemma 1 and Theorem 1.

Lemma 1. Both (S-C,a) and (S-C,a*) turn the indifference $\{a_1\} \sim \{b_1\}$ into $\{a_1, \alpha_2\} \sim \{b_1, \beta_2\}$. No further changes are needed: continue the proof and conclude that homogeneous sets with the same indirect utility and the same cardinality are equally good.

Theorem 1 (case 3)**.** Here, (S-C,a) is used to obtain that $B_1 \cup \{b_2\} \succsim B$. In this case, however, the use of (S-C,a) can be avoided. We distinguish two cases.
Case 1. $B = B_1 \cup \{b_2\}$. Then, $B_1 \cup \{b_2\} \sim B$.
Case 2. B is a strict superset of $B_1 \cup \{b_2\}$. Now, let $z \neq b_2$ be the worst (in term of preferences) element in B and let $B' = B \setminus \{z\}$. Then, $B'_1 = B_1$. Lemma 3 entails $B_1 \succ B'$. Use (S-C,c) with B_1 (homogeneous), B' (not homogeneous), $B_1 \succ B'$, $C = \{b_2\}$ (homogeneous and $B_1 PC$), and $D = \{z\}$ (homogeneous and

$B'Pz$). Conclude that $B_1 \cup \{b_2\} \succ B' \cup \{z\}$.

Hence, in both cases, the desired conclusion $B_1 \cup \{b_2\} \succsim B$ is obtained.

Independency (page 26, first column, bottom, item 1)**.** The lexicographic ordering on Z satisfies (M). Indeed, for each pair a_1 and a_2 such that $a_1 P a_2$, the vector $(u(a_1), 2)$ dominates $(u(a_1), u(a_2))$ (as all utilities belong to the $[0, 1]$-interval). Hence, this rule does not entail that (M) is independent of the other axioms.

Here, we propose to consider the set $X = \{x, y\}$ with xPy. The ordering $\succsim$, defined by $\{x\} \sim \{x, y\} \succ \{y\}$, violates (M) and satisfies the other three axioms. Alternatively, one can change the value 2 (used to expand the vector of utilities) into 0 and use the lexicographic ordering.

Reference

Vázquez C (2014) Ranking opportunity sets on the basis of similarities of preferences: a proposal. Mathematical Social Sciences 67, 23-26.