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Stable marriage with and without transferable utility: nonparametric testable implications

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Stable marriage with and without transferable utility: nonparametric testable implications[*]

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Abstract

We show that transferable utility has no nonparametrically testable implications for marriage stability in settings with a single consumption observation per household and heterogeneous individual preferences across households. This completes the results of Cherchye, Demuynck, De Rock, and Vermeulen (2017), who characterized Pareto efficient household consumption under the assumption of marriage stability without transferable utility. First, we show that the nonparametric testable conditions established by these authors are not only necessary but also sufficient for rationalizability by a stable marriage matching. Next, we demonstrate that exactly the same testable implications hold with and without transferable utility between household members. We build on this last result to provide a primal and dual linear programming characterization of a stable matching allocation for the observational setting at hand. This provides an explicit specification of the marital surplus function rationalizing the observed matching behavior.

JEL classifications: C14, D11, C78.

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1 Introduction

A most popular assumption in theoretical and empirical models of stable marriage markets is that utility is transferable between spouses.[1] Even though this assumption has been argued to impose a restrictive structure on the individual utilities, it is often used because it has several highly desirable implications.[2] Most notably, if individual utilities are transferable, the marriage market equilibrium can be characterized as maximizing the total marital surplus. In particular, we can define the equilibrium as the solution of a linear program, and this has been shown to be a powerful vehicle to analyze the properties of stable matching allocations.[3]

In a recent paper, Cherchye, Demuynck, De Rock, and Vermeulen (2017) (henceforth CDDV) established a revealed preference characterization of Pareto efficient household consumption when the marriage is stable. They characterized stable marriage with intrahousehold (consumption) transfers but without assuming transferable utility between household members. The characterization generates testable conditions even with a single consumption observation per household and with fully heterogeneous individual preferences across households. The characterization is of the revealed preference type and is intrinsically nonparametric, which means that its empirical implementation does not require a parametric/functional specification of the individual utilities.[4] Interestingly, the conditions are linear in unknowns, which makes them easy to use in practical applications.

The current paper complements the study of CDDV by characterizing marital stability under the same observational assumptions but now specifically including the case with transferable utilities. To begin our argument, we establish a first main result for the general case with potentially non-transferable utilities. For this case, CDDV introduced their conditions as necessary requirements for rationalizing the observed household consumption by a stable marriage matching. In particular, they proved that, if the conditions are

[1]See, for example, Browning, Chiappori, and Weiss (2014) and Chiappori (2017) for recent overviews of the literature. In what follows, we will distinguish between (1) the transferable utility case and (2) the case with (consumption) transfers but general (i.e. potentially non-transferable) utilities. Our case of transferable utility is sometimes also referred to as the "perfectly" transferable utility case in the literature, whereas our case with transfers and general utilities is referred to as the "imperfectly" transferable utility case. Chiappori (2017) provides a detailed discussion of these perfectly and imperfectly transferable utility notions in a household context.

[2]See Chiappori (2010), Cherchye, Demuynck, and De Rock (2015) and Chiappori and Gugl (2015) for detailed discussions on the transferable utility hypothesis in a household consumption context. These authors also outline the implicit assumptions that underlie the hypothesis as well as the associated testable implications for household consumption demand (but without using the assumption of marital stability).

[3]In this respect, a particularly motivating study is the one of Chiappori, McCann, and Nesheim (2010), who build on the linear programming formulation of the stable matching model (with transferable utilities) to explore the equivalence with hedonic price equilibria and optimal transportation problems. These authors state (on p. 318) that "due to the wide body of knowledge about linear programming in general, and optimal transportation in particular [...], the reduction of the model to this form seems not only conceptually clearer, but better adapted to bringing powerful methods of theoretical and computational analysis to bear on the question."

[4]Particularly, the characterization follows the revealed preference approach of Afriat (1967), Diewert (1973) and Varian (1982).

violated, then the observed household consumption is not rationalizable. In the current note, we complete this necessity result by showing that the conditions are also sufficient for such rationalizability: as soon as the data satisfy the conditions, we can construct individual utility functions that represent the observed household behavior in terms of a stable marriage matching.

Next, our second main result shows that exactly the same testable implications are necessary and sufficient for rationalizability under the additional assumption of transferable utility between household members. In other words, transferable utility has no separate nonparametric implications for the cross-sectional setting under study. Building on this, we also define a linear programming characterization of a stable matching allocation in our observational setting. Attractively, this formulation of our empirical conditions will have an analogous primal and dual structure as the standard theoretical characterization of a stable matching allocation under transferable utility. In particular, it reflects the notion that a stable matching allocation maximizes the total surplus over all possible assignments on the marriage market. In our case, the marital surplus function that rationalizes the observed matching behavior will have a specific interpretation in terms of "real income" differences.

Section 2 introduces our notation and formalizes our concept of stable matching. Section 3 sketches the basic intuition of the necessary conditions for rationalizability by a stable matching. We will summarize these conditions in terms of Axioms of Revealed Stable Matchings. Section 4 establishes that these axioms provide not only necessary but also sufficient conditions for such rationalizability. We show that this result holds for general utilities but also for transferable utilities of the individuals on the marriage market. Section 5 presents the linear programming formulation of our nonparametric characterization of a stable matching allocation. Section 6 concludes. The proofs of our main results are in the Appendix.

2 Preliminaries

We start this section by explaining the dataset that we assume for a given marriage market. Then, we introduce our condition for rationalizing this dataset by a stable marriage allocation. This will set the stage for presenting our main results in the following sections.

Dataset. We consider a marriage market with a finite set of men M and a finite set of women W. Married couples are defined by the matching function $\sigma : M \cup W \to M \cup W$, such that (with some abuse of notation)

- for all men $i \in M$, $\sigma(i) \in W$,
- for all women $r \in W, \sigma(r) \in M$,
- and $\sigma(i) = r$ if and only if $\sigma(r) = i$.

Once a man and a woman are married, they consume a set of n private goods and a set of N public goods. The (column) vector $Q \in \mathbb{R}^N_+$ represents the public consumption quantities. Similarly, the (column) vector $q \in \mathbb{R}^n_+$ represents the private consumption quantities, with q^m the private consumption of the man in the couple and q^w the private consumption of the woman.

Budget conditions are specific to (potential) couples $(i,r) \in M \times W$. First, the (row) vector $p_{i,r} \in \mathbb{R}^n_{++}$ denotes the prices for private consumption and the (row) vector $P_{i,r} \in \mathbb{R}^N_{++}$ the prices for public consumption. Next, a potential couple (i,r) can spend income $y_{i,r}$.[5] The couple's consumption possibilities are defined by the associated budget set.

Definition 1 (Budget set)**.** *The* ***budget set*** *for a couple (i,r) is given by the set of bundles (q^m, q^w, Q) that can be bought by budget $y_{i,r}$, i.e.*

$$B_{i,r} = \left\{ (q^m, q^w, Q) \in \mathbb{R}^{2n+N}_+ | p_{i,r}(q^m + q^w) + P_{i,r}Q \leq y_{i,r} \right\}.$$

In what follows, we will make two simplifying assumptions regarding what is observed by the empirical analyst. First, we will disregard singles, i.e. the empirical analyst only observes married individuals (and, thus, $|M| = |W|$). Next, we will assume that the empirical analyst can observe the public consumption Q as well as the individuals' private consumption q^m and q^w for the married couples (but not for other potential (unmarried) couples). As explained by CDDV, it is actually fairly easy to relax each of these assumptions. However, this would only complicate our notation and not lead to additional insights.[6]

Summarizing, for a given marriage market we assume the **dataset**

$$S = \left\{ \sigma, \{q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)}\}_{i \in M}, \{p_{i,r}, P_{i,r}, y_{i,r}\}_{i \in M, r \in W} \right\},$$

which consists of a matching function σ, observed intrahousehold allocations

$$(q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)}),$$

for all married couples $(i, \sigma(i))$, and couple-specific prices $(p_{i,r}, P_{i,r})$ and incomes $y_{i,r}$ for all potential couples (i,r), such that

$$p_{i,\sigma(i)}(q^m_{i,\sigma(i)} + q^w_{i,\sigma(i)}) + P_{i,\sigma(i)}Q_{i,\sigma(i)} = y_{i,\sigma(i)}$$

for every married couple $(i, \sigma(i))$.

[5] The couple-specific prices are especially relevant when the spouses' leisures are contained in the modelled commodities. In this case, the price of an hour of leisure of a spouse equals that individual's wage while the couple's income equals full income.

[6] We can add single females (males) as (virtual) couples with the male (female) consuming nothing (i.e. the private consumption $q^m = 0$ ($q^w = 0$)). Next, unknown individual quantities q^m and q^w can be treated similarly as the unknown individual prices P^m and P^w in the ARSM condition captured by our following Definition 3.

Rationalizability. We say that a dataset is rationalizable if there exist individual preferences for which the observed intrahousehold allocation is utility maximizing and such that the matching is stable, which means that no married individual wants to divorce. Generally, stability of the marriage market requires both "individual rationality" and "no blocking pairs". Individual rationality requires that no individual wants to become single and, similarly, no blocking pairs means that no two currently unmarried individuals prefer to remarry each other.

In our following analysis, we will solely consider the no blocking pairs condition explicitly. Again, it is easy to extend our arguments to also include the individual rationality condition (along the lines of CDDV), but this would complicate the exposition without adding new insights.[7]

Individual preferences depend on private and public consumption quantities. Formally, we assume that every man $i \in M$ is endowed with a utility function $u^i : \mathbb{R}^{n+N} \to \mathbb{R}$, and every women $r \in W$ with a utility function $u^r : \mathbb{R}^{n+N} \to \mathbb{R}$. Throughout, we will assume that the functions u^i and u^r are strictly monotone and concave. For a given dataset S, our rationalizability condition requires that there must exist such individual utility functions that make the observed household allocations consistent with marriage stability (i.e. no blocking pairs).

Definition 2 (Rationalizability)**.** *The dataset S is rationalizable by a stable matching if, for each man $i \in M$ and woman $r \in W$, there exist utility functions $u^i : \mathbb{R}^{n+N} \to \mathbb{R}$ and $u^r : \mathbb{R}^{n+N} \to \mathbb{R}$ such that, for all couples $(i, r) \in M \times W$ and allocations (q^m, q^w, Q), if*

$$u^i(q^m, Q) \geq u^i(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)}) \text{ and}$$
$$u^r(q^w, Q) \geq u^r(q^w_{\sigma(r),r}, Q_{\sigma(r),r}),$$

with at least one strict inequality, then $(q^m, q^w, Q) \notin B_{i,r}$.

Thus, rationalizability imposes a separate (no blocking pair) restriction for each potential couple (i, r): any consumption allocation (q^m, q^w, Q) that gives greater utility to both individuals than in their current match (i.e. $u^i(q^m, Q) \geq u^i(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ and $u^r(q^w, Q) \geq u^r(q^w_{\sigma(r),r}, Q_{\sigma(r),r})$, with at least one strict inequality) must be infeasible for the given budget set (i.e. $(q^m, q^w, Q) \notin B_{i,r}$). Indeed, if this last condition were not met, then both individuals would be better off by exiting their current marriage and remarrying each other, which would make the given matching allocation unstable.

As a final note, we remark that CDDV also explicitly assumed Pareto efficient within-household allocations, in addition to marriage stability. In our set-up, this explicit assumption is redundant, as our rationalizability condition in Definition 2 automatically guarantees within-household Pareto efficiency. In particular, for each married couple (i.e. $r = \sigma(i)$) the condition imposes that there cannot exist a consumption allocation that makes both spouses better off (and at least one spouse strictly better off) than the given allocation $(q^m_{i,\sigma(i)}, q^w_{i,\sigma(i)}, Q_{i,\sigma(i)})$, which effectively excludes the possibility of Pareto improvements.

[7] Formally, the individual rationality requirement coincides with the no blocking pairs requirement when using "individuals pairing with nobody" as potentially blocking pairs.

3 Revealed stable matchings

We next introduce CDDV's testable conditions for Pareto efficient household behavior under marriage stability. The conditions are of the revealed preference type and intrinsically nonparametric, which means that their empirical implementation does not require an explicit parametric specification of the individuals' utilities. We provide the main intuition for the necessary nature of these rationalizability conditions, which we summarize in terms of the Axiom of Revealed Stable Matchings (ARSM). We say that an observed matching allocation consistent with ARSM is "revealed stable", to indicate that the associated dataset S does not allow us to reject stability.

In what follows, we will also introduce the Weak Axiom of Revealed Stable Matchings (WARSM), which provides a marginally weaker empirical requirement than the ARSM. We will explain that WARSM consistency is easier to verify empirically than ARSM consistency. In addition, the theoretical connection between the ARSM and WARSM conditions will be relevant for our discussion in Section 4.

Axiom of Revealed Stable Matchings. To sketch the necessity argument for the Axiom of Revealed Stable Matchings (ARSM) condition, we consider a currently unmatched couple (i, r). For this couple, we further assume a (Pareto efficient) allocation $(q^m_{i,r}, q^w_{i,r}, Q_{i,r}) \in B_{i,r}$ such that the man i is indifferent between the consumption bundle $(q^m_{i,r}, Q_{i,r})$ and the bundle $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ in his current marriage.

For this allocation $(q^m_{i,r}, q^w_{i,r}, Q_{i,r})$, the slope of i's indifference curve at the bundle $(q^m_{i,r}, Q_{i,r})$ is given by the price vectors $(p_{i,r}, P^m_{i,r})$, where the individual prices $P^m_{i,r}$ represent man i's willingness-to-pay for the public quantities. For convex preferences (following from concavity of u^i), a simple revealed preference argument shows that man i's indifference between $(q^m_{i,r}, Q_{i,r})$ and $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ implies

$$p_{i,r}q^m_{i,r} + P^m_{i,r}Q_{i,r} \leq p_{i,r}q^m_{i,\sigma(i)} + P^m_{i,r}Q_{i,\sigma(i)}, \tag{1}$$

i.e. the hyperplane with slope $(p_{i,r}, P^m_{i,r})$ through $(q^m_{i,r}, Q_{i,r})$ must be situated below the bundle $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$.

Next, the indifference curve of the woman r at the allocation $(q^w_{i,r}, Q_{i,r})$ will have the slope $(p_{i,r}, P^w_{i,r})$. In this case, the individual prices $P^w_{i,r}$ give woman r's willingness-to-pay for the public consumption in the allocation $(q^m_{i,r}, q^w_{i,r}, Q_{i,r})$. We can use that Pareto efficiency implies $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, i.e. the individual prices $P^m_{i,r}$ and $P^w_{i,r}$ must add up to the actual price $P_{i,r}$ and can be interpreted as "Lindahl prices" associated with the efficient consumption of public goods.

For (i, r) not to be a blocking pair, we must have that woman r prefers the bundle $(q^w_{\sigma(r),r}, Q_{\sigma(r),r})$ over the bundle $(q^w_{i,r}, Q_{i,r})$ (because man i is indifferent between $(q^m_{i,r}, Q_{i,r})$ and $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$). In other words, the indifference curve of woman r through $(q^w_{i,r}, Q_{i,r})$ should lie below the indifference curve through $(q^w_{\sigma(r),r}, Q_{\sigma(r),r})$, which implies

$$p_{i,r}q^w_{i,r} + P^w_{i,r}Q_{i,r} \leq p_{i,r}q^w_{\sigma(r),r} + P^w_{i,r}Q_{\sigma(r),r}. \tag{2}$$

By combining the inequalities (1) and (2), we obtain the requirement

$$y_{i,r} \leq p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r}, \tag{3}$$

where we use that $y_{i,r} = p_{i,r}(q^m_{i,r} + q^w_{i,r}) + (P^m_{i,r} + P^w_{i,r})Q_{i,r}$ and $P^m_{i,r} + P^w_{i,r} = P_{i,r}$.

The ARSM condition states that this necessary requirement for marriage stability applies to any potential couple (i, r).[8] An observed marriage matching is revealed stable if it satisfies ARSM.

Definition 3 (ARSM). *A dataset S satisfies the Axiom of Revealed Stable Matchings (ARSM) if, for all couples (i, r), there exist price vectors $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$, with $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, such that*

$$y_{i,r} \leq p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r}.$$

Essentially, the axiom complies with CDDV's requirement for consistency of a dataset S with the no blocking pairs requirement of marriage stability (i.e. statement (ii) in their Proposition 1).

Weak Axiom of Revealed Stable Matchings. The Weak Axiom of Revealed Stable Matchings (WARSM) provides an alternative necessary requirement for rationalizability. It also follows from a basic revealed preference argument. As we will explain below, it is computationally easier to verify but imposes marginally weaker empirical restrictions on S than the ARSM condition in Definition 3.

To introduce the WARSM condition, we again consider a currently unmatched pair (i, r), with the man consuming the bundle $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ and the woman consuming the bundle $(q^w_{\sigma(r),r}, Q_{\sigma(r),r})$ in their current marriages. Assume that the budget conditions that apply to this couple are such that

$$y_{i,r} > p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P_{i,r} \max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\}, \tag{4}$$

where the max function is element-wise. In words, the budget $y_{i,r}$ available to the couple (i, r) strictly exceeds the cost of buying the private quantities ($q^m_{\sigma(i),i}$ and $q^w_{\sigma(r),r}$) and the maximal public quantities ($\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\}$) that are consumed in the given marriages. This would mean that the couple can afford a bundle $(q^m_{i,r}, q^w_{i,r}, Q_{i,r}) \in B_{i,r}$ such that

$$(q^m_{i,r}, Q_{i,r}) \geq (q^m_{i,\sigma(i)}, Q_{i,\sigma(i)}) \text{ and}$$
$$(q^w_{i,r}, Q_{i,r}) \geq (q^w_{\sigma(r),r}, Q_{\sigma(r),r}),$$

with a strict inequality for at least one component of the quantity vectors. Because the utility functions u^i and u^r are strictly monotone, this obtains

$$u^i(q^m_{i,r}, Q_{i,r}) \geq u^i(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)}) \text{ and}$$
$$u^r(q^w_{i,r}, Q_{i,r}) \geq u^r(q^w_{\sigma(r),r}, Q_{\sigma(r),r}),$$

[8] Observe that, for a married couple $(i, \sigma(i))$, the inequality restriction in the ARSM definition becomes an equality restriction that simply reproduces the associated budget constraint.

with at least one strict inequality. But this implies that the rationalizability condition in Definition 2 is violated.

Thus, we can conclude that rationalizability by a stable matching requires that the inequality (4) does not hold for any couple (i, r). This is captured by our WARSM condition.

Definition 4 (WARSM)**.** *A dataset S satisfies the Weak Axiom of Revealed Stable Matchings (WARSM) if, for all couples (i, r),*

$$y_{i,r} \leq p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P_{i,r} \max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\},$$

where the max *function is element-wise.*

Technically, the sole difference between the ARSM concept in Definition 3 and the WARSM concept in Definition 4 pertains to the fact that the term

$$P^m_{i,r} Q_{i,\sigma(i)} + P^w_{i,r} Q_{\sigma(r),r}, \tag{5}$$

in the ARSM requirement, is replaced by the term

$$P_{i,r} \max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\}, \tag{6}$$

in the WARSM requirement. As an implication, checking the WARSM condition does not require searching for unknown individual prices $P^m_{i,r}$ and $P^w_{i,r}$. This makes it easier to verify WARSM than ARSM and, thus, the WARSM condition is more attractive from a computational point of view.

To conclude, we formalize the theoretical connection between the ARSM and WARSM requirements. This connection will also be relevant for our discussion in the following section.

Proposition 1. *If the ARSM is satisfied, then the WARSM is also satisfied. If the WARSM inequality holds as a strict inequality for all unmatched couples, then the ARSM is also satisfied with strict inequality for all unmatched couples.*

Proposition 1 states the ARSM condition is marginally stronger than the WARSM condition. However, under an empirically mild and easy-to-verify condition, the WARSM requirement coincides with the ARSM requirement.

4 Necessary and sufficient conditions

In this section, we first show that the above (W)ARSM requirements provide testable conditions that are not only necessary but also sufficient for rationalizability by a stable matching allocation. This completes the argument of CDDV, who only proved necessity of the empirical stability requirements. In addition, we establish that exactly the same necessary and sufficient conditions for rationalizability hold under transferable utility. This implies that transferable utility does not generate separate testable implications for the cross-sectional setting under study.

General utilities. Our first main result shows that, as soon as the data satisfy the the ARSM, we can reconstruct individual utility functions that represent the observed household behavior in terms of a stable marriage matching. These functions are strictly monotone and concave, as required.

Theorem 1. *A dataset S is rationalizable by a stable matching if and only if it satisfies the ARSM.*

We can combine this result with Proposition 1, which showed that the WARSM requirement is (only) marginally weaker than the ARSM requirement.

Corollary 1. *If a dataset S is rationalizable by a stable matching, then it satisfies the WARSM. If the WARSM inequality holds as a strict inequality for all unmatched couples, then the dataset S is rationalizable by a stable matching.*

This is a useful result from a practical perspective. As indicated above, WARSM consistency is easier to verify than ARSM consistency (see our discussion of (5) and (6)). In fact, we can expect the WARSM characterization to be a sufficient characterization in many practical instances, as the empirical difference between ARSM and WARSM (defined in Proposition 1) will generally be very weak. In such cases, explicitly using the individual Lindahl prices $P^m_{i,r}$ and $P^w_{i,r}$ for publicly consumed quantities does not generate specific testable implications. As soon as a dataset S satisfies the necessary and sufficient (ASRM) characterization that explicitly accounts for these individual prices, it also satisfies the necessary and sufficient (WASRM) characterization that does not include these unobserved prices.

Transferable utilities. Our second main result shows that the (W)ASRM condition characterizes stable matching allocations not only in terms of general utilities but also in terms of transferable utilities. Specifically, as soon as a given dataset S is rationalizable by a stable matching allocation, it is also rationalizable with individual utilities that are transferable, and vice versa.

To formalize this point, we treat one of the private goods as a numeraire good. We let x represent the quantities of this good and we define the normalized prices such that the price of the numeraire good is one for all (potential) couples.[9] Then, the ARSM inequality in Definition 3 can be rephrased as

$$\begin{aligned} y_{i,r} \leq & p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) \\ & + P^m_{i,r} Q_{i,\sigma(i)} + P^w_{i,r} Q_{\sigma(r),r} + x^m_{i,\sigma(i)} + x^w_{\sigma(r),r}, \end{aligned} \tag{7}$$

and the WARSM inequality in Definition 4 as

$$\begin{aligned} y_{i,r} \leq & p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) \\ & + P_{i,r} \max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} + x^m_{i,\sigma(i)} + x^w_{\sigma(r),r}. \end{aligned} \tag{8}$$

[9] We remark that, because we use this price normalization, we can arbitrarily select any observed good as the numeraire good to make our following argument. Thus, assuming the existence of a numeraire good is without loss of generality.

As discussed in Chiappori (2010) and Cherchye et al. (2015), a sufficient condition for transferable utility between household members is that the agents have quasi-linear utility functions. That is, for all men $i \in M$ and women $r \in W$ the utility functions take the form

$$u^i(q, Q, x) = v^i(q, Q) + x,$$
$$u^r(q, Q, x) = v^r(q, Q) + x.$$

For these utility structures, the spouses can use the numeraire good to transfer utilities at a fixed rate of exchange. As we show in the Appendix, we can fairly easily adapt the construction of the utility functions in the proof of Theorem 1 to extend the sufficiency argument to the case with quasi-linear utilities as defined above. In turn, this implies the next result.

Theorem 2. *A dataset S is rationalizable by a stable matching with transferable utilities if and only if it satisfies the ASRM with inequality* (7)*.*

A direct implication of Theorems 1 and 2 is that transferable utility has no separately testable implications for the observational setting at hand. Or, putting it differently, it is not possible to nonparametrically identify whether individual utilities are transferable (or not) if the observed matching allocation is stable.

Analogous to before, we can formulate the following corollary.

Corollary 2. *If a dataset is rationalizable by a stable matching with transferable utilities, then it satisfies the WARSM with inequality* (8)*. If the WARSM inequality* (8) *holds as a strict inequality for all unmatched couples, then the dataset S is rationalizable by a stable matching with transferable utilities.*

5 Linear programming formulation

From the previous section, we conclude that any dataset S that is rationalizable by a stable matching allocation (i.e. S satisfies the (W)ARSM condition) can also be represented *as if* it is stable with transferable individual utilities. Building on this, we next show that our conditions for a stable matching allocation (with or without transferable utility) can be rephrased in terms of the standard linear programming characterization of stable matchings under transferable utility. As motivated in the Introduction, this linear programming formulation of stable marriage allocations with transferable utilities has shown to be a powerful vehicle to analyze the properties of these allocations. Our results show that, for the cross-sectional setting that we study here, the same linear programming tools can be used for the empirical analysis of stable marriages, without needing to assume transferable utilities or specific parametric structures for the individual preferences.

The linear programming formulation reflects the notion that a stable matching allocation maximizes the total surplus over all possible assignments on the marriage market. In our case, the marital surplus function that rationalizes the observed behavior will have a specific representation in terms of "real income" differences. Intuitively, it implies that any rationalizable dataset S can always be represented *as if* it follows from "matching on real income" (i.e. marriage and divorce decisions are determined by the aggregate real income of the two partners). Interestingly, we can relate this finding to the argument of Chiappori, Iyigun, and Weiss (2007, 2015) that such matching on real income holds under (i) transferable utility between spouses under marriage, (ii) transferable utility between spouses under divorce and (iii) invariance of the exchange rate of the utilities of the two partners to changes in marital status. These conditions are satisfied under quasi-linearity of individual preferences.[10] In Section 4, we have shown that quasi-linearity is not testable under stable marriage in our observational setting: as soon as the dataset is rationalizable by a stable matching, it is rationalizable under the additional assumption of quasi-linearity. In the current section, we add that such a stable matching allocation can equally be represented as maximizing the total surplus expressed in real income terms.

Primal and dual programs. We begin our argument by briefly recapturing the primal and dual linear programming characterization of a stable matching allocation under transferable utility.[11] To focus our discussion, we will only consider the linear programs for the WARSM condition. But the following reasoning readily extends to the ARSM condition for rationalizability.[12]

The primal formulation describes the stable marriage matching in terms of an optimal assignment problem (as in Shapley and Shubik (1972)). Formally, let $\chi_{i,r}$ represent the marital surplus if male i were married to female r. Further, the assignment indicators $\pi_{i,r}$ are defined such that $\pi_{i,r} = 1$ if i is married to r and $\pi_{i,r} = 0$ otherwise. Then, a stable marriage assignment solves

$$\max_{\pi_{i,r}} \sum_{i \in M} \sum_{r \in W} \pi_{i,r} \chi_{i,r} \tag{9}$$

$$\textit{such that } \pi_{i,r} \geq 0 \textit{ and}$$

$$\sum_{r \in W} \pi_{i,r} \leq 1 \text{ for all } i \in M,$$
$$\sum_{i \in M} \pi_{i,r} \leq 1 \text{ for all } r \in W.$$

[10] Quasilinearity is crucial for conditions (i), (ii) and (iii) to hold simultaneously. For example, Chiappori, Iyigun, and Weiss (2007, 2015) showed that "generalized" quasi-linearity (as defined by Bergstrom and Cornes (1981, 1983)) guarantees (i) but not (ii) and (iii).

[11] For a more detailed discussion of the primal and dual problems (9) and (10), we refer to Chapter 7 of Browning, Chiappori, and Weiss (2014).

[12] Specifically, for the linear programming formulation of the ASRM we need to modify the surplus function (11) in accordance with Definition 3 (for unknown $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^n_{++}$ that satisfy $P^m_{i,r} + P^w_{i,r} = P_{i,r}$). The primal and dual programs (9) and (10) must be adjusted correspondingly.

The objective (9) reflects that a stable matching allocation maximizes the total surplus on the marriage market, which is the defining property of a stable allocation under transferable utility. Intuitively, we can interpret each variable $\pi_{i,r}$ as the probability that male i (re)matches with female r. Because of the specific linear structure of the problem, there always exists at least one solution of program (9) with all $\pi_{i,r}$ equal to either zero or one.

Next, let ν_i and μ_r be the dual variables associated with the constraints of program (9). Then, we can define the dual program

$$\min_{\nu_i,\mu_r} \sum_{i\in M} \nu_i + \sum_{r\in W} \mu_r \tag{10}$$

$$\textit{such that } \nu_i \geq 0,\ \mu_r \geq 0 \quad \textit{and}$$

$$(\nu_i + \mu_r) \geq \chi_{i,r} \text{ for all } i \in M \text{ and } r \in W.$$

In this program, the variables ν_i and μ_r represent potential remarriage gains of males i and females r. They can be interpreted as the shadow prices of the constraints of the primal maximization problem. By construction, we have that $\nu_i + \mu_r = \chi_{i,r}$ when a marriage is formed (i.e. $\pi_{i,r} = 1$ is an optimal solution of (9)) and $\nu_i + \mu_r \geq \chi_{i,r}$ otherwise. In the literature, this is referred to as the "complementary slackness condition".

Marital surplus. For a given dataset S, we can derive the linear programming characterization of a stable matching allocation (satisfying the WARSM condition) when specifying the marital surplus as

$$\chi_{i,r} = y_{i,r} - y^B_{i,r}, \tag{11}$$

i.e. the difference between income $y_{i,r}$ of the (potential) couple (i, r) and the benchmark income

$$y^B_{i,r} = p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P_{i,r} \max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\}, \tag{12}$$

In words, the benchmark income $y^B_{i,r}$ corresponds to the value of total (public and private) consumption of the male i and female r in their current marriages. Because we evaluate this benchmark income in terms of the prices that apply to the couple (i, r), we can interpret the surplus specification (11) as representing the "real income" difference between the potential decision situation (i, r) and the actual consumption situations $(i, \sigma(i))$ for male i and $(\sigma(r), r)$ for female r. It gives the additional consumption possibilities for the potentially blocking pair (i, r) when choosing to remarry each other. The fact that we use the actual consumption allocations $(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)})$ and $(q^m_{\sigma(r),r}, Q_{\sigma(r),r})$ for the benchmark real income (12) follows naturally from our empirical set-up: these are the (only) consumption bundles that are effectively observed for the given males i and females r.

When using the specification (11) for the marital surplus $\chi_{i,r}$, we easily obtain that the dataset S satisfies the WARSM condition (i.e. S is rationalizable by a stable matching) if and only if the dual problem program (10) has an optimal objective value that equals zero. Intuitively, there is no consumption (i.e. real income) gain associated with remarriage. By contrast, if the optimal objective value of program (10) is strictly positive, then the associated optimal values of the dual variables $\nu_i > 0$ and $\mu_r > 0$ represent the possible consumption gains for males i and females r in case of remarriage.

Because of the Dual Theorem of linear programming, rationalizability of S equally requires that the optimal objective value of the primal program (9) is zero for the surplus $\chi_{i,r}$ defined in (11). In this case, we will have $\pi_{i,\sigma(i)} = 1$ and $\pi_{\sigma(r),r} = 1$ (for all i and r) as an optimal solution, which means that there is no remarriage gain for the observed matchings in S. On the contrary, if the optimal objective value is above zero, the optimal values of the variables $\pi_{i,r}$ will characterize the most profitable rematches (in consumption terms) for the observed males i and females r. This allows us in turn to further analyse the stability of the new matching allocation.

As a final remark, when using this primal formulation (9), we can also give a specific interpretation to the optimal matching allocation in the case where prices are the same in all possible decision situations, i.e. $p_{i,r} = p$ and $P_{i,r} = P$ for all (i, r). In such an instance, the objective function becomes

$$\max_{\pi_{i,r}} \sum_{i \in M} \sum_{r \in W} \pi_{i,r} y_{i,r} - C, \tag{13}$$

for the constant

$$C = \sum_{i \in M} \sum_{r \in W} [p(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P \max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\}].$$

From (13), it follows directly that under constant prices the optimal marriage assignment (defined by the variables $\pi_{i,r}$) is the one that maximizes the total (nominal) income. Conversely, it also implies that variation in relative prices over marriages and differences in individual preferences can lead to stable matching allocations that do not correspond to a maximal total (nominal) income on the marriage market.

6 Conclusion

We have shown that transferable utility has no nonparametrically testable implications for marriage stability in settings with a single consumption observation per household and heterogeneous individual preferences across households. To establish this conclusion, we have completed the results of Cherchye, Demuynck, De Rock, and Vermeulen (2017). First, we have shown that these authors' conditions for rationalizability by a stable matching are not only necessary but also sufficient. We characterized a stable matching allocation in terms of the Axiom of Revealed Stable Matchings (ARSM) and the Weak Axiom of Revealed Stable Matchings (WARSM). We argued that the ARSM condition and the (marginally weaker) ASRM condition will be empirically equivalent in many practical instances. This is convenient from a practical point of view, as WARSM consistency is easier to check than ARSM consistency.

Next, we have shown that exactly the same testable conditions hold under the additional assumption of transferable utility between household members. Thus, transferable utility does not generate separate testable implications for the cross-sectional setting at hand. We built on this result to provide a linear programming formulation of our empirical conditions for a stable matching allocation that parallels the standard theoretical characterization of stable marriage under transferable utility. Attractively, this obtains that the powerful linear programming tools that have been used for the analysis of stable matching allocations can actually be used for the empirical analysis of stable marriages, even without assuming transferable utilities or particular parametric forms for the individual preferences. In addition, it gives a specific interpretation to a stable matching allocation as maximizing the total surplus expressed in real income terms.

As a final note, we remark that these conclusions hold for cross-sectional datasets containing (only) a single consumption observation per household. As soon as multiple (time series) household observations are available, the assumption of Pareto efficiency will generate additional testable implications, which will also allow one to identify individuals' willingness-to-pay for public consumption. See, for example, Cherchye, De Rock, and Vermeulen (2007, 2011) for a nonparametric revealed preference analysis. In a similar vein, transferable utility will have separate testable implications when more consumption observations for one and the same household can be used. Cherchye, Demuynck, and De Rock (2015) derived the associated revealed preference characterization.

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Appendix

Proof of Proposition 1

Proof. Consider any two price vectors $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$ such that $P^m_{i,r} + P^w_{i,r} = P_{i,r}$. Then,

$$\begin{aligned} P_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} &= P^m_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} + P^w_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} \\ &\geq P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r}. \end{aligned}$$

As such, if the ARSM is satisfied, then

$$\begin{aligned} y_{i,r} &\leq p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) + P^m_{i,\sigma(i)}Q_{i,\sigma(i)} + P^w_{\sigma(r),r}Q_{\sigma(r),r}, \\ &\leq p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) + P_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\}, \end{aligned}$$

which gives the WARSM. For the second part, let

$$\bar{\zeta} = p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) + P_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} - y_{i,r} > 0.$$

Then, by Lemma 1 we know that there exists a $\zeta > 0$, with $\zeta < \bar{\zeta}$, and prices $P^m_{i,r}, P^w_{i,r} \in \mathbb{R}^N_{++}$, with $P^m_{i,r} + P^w_{i,r} = P_{i,r}$, such that

$$P_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} = P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r} + \zeta.$$

This implies

$$\begin{aligned}\bar{\zeta} = &\quad p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) + P_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} - y_{i,r} \\ > &\quad P_{i,r}\max\{Q_{i,\sigma(i)}, Q_{\sigma(r),r}\} - P^m_{i,r}Q_{i,\sigma(i)} - P^w_{i,r}Q_{\sigma(r),r} \quad = \zeta.\end{aligned}$$

Or, equivalently,

$$y_{i,r} < p_{i,r}\left(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}\right) + P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r},$$

as was to be shown. □

Lemma 1. *Let $P \in \mathbb{R}^N_{++}$ be a vector of prices and $Q_1, Q_2 \in \mathbb{R}^N_{+}$ two vectors of quantities. For every $\bar{\zeta} > 0$, there exists a number $\zeta \in [0, \bar{\zeta}]$ and vectors of prices $P_1, P_2 \in \mathbb{R}^N_{++}$, with $P_1 + P_2 = P$, such that*

$$P\max\{Q_1, Q_2\} = P_1Q_1 + P_2Q_2 + \zeta.$$

Proof. Throughout our proofs, $[z]_j$ denotes the $j - th$ component of some given vector z. Then, take a number $\varepsilon > 0$ small enough such that

$$\varepsilon\left(\sum_{j=1}^{N}([Q_2]_j - [Q_1]_j)1[[Q_2]_j > [Q_1]_j] + \sum_{j=1}^{N}([Q_1]_j - [Q_2]_j)1[[Q_1]_j \geq [Q_2]_j]\right) \equiv \zeta < \bar{\zeta},$$

where $1[.]$ stands for the indicator function. Take $[P_1]_j = [P]_j - \varepsilon$ if $[Q_1]_j \geq [Q_2]_j$ and

$[P_1]_j = \varepsilon$ otherwise. Let $P_2 = P - P_1$. Then, we have

$$
\begin{aligned}
P\max\{Q_1, Q_2\} =& P_1 \max\{Q_1, Q_2\} + P_2 \max\{Q_1, Q_2\} \\
=& \sum_{j=1}^{N} (P^j - \varepsilon)[Q_1]_j 1[[Q_1]_j \geq [Q_2]_j] + \sum_{j=1}^{N} \varepsilon [Q_2]_j 1[[Q_1]_j < [Q_2]_j] \\
&+ \sum_{j=1}^{N} \varepsilon [Q_1]_j 1[[Q_1]_j \geq [Q_2]_j] + \sum_{j=1}^{N} ([P]_j - \varepsilon)[Q_2]_j 1[[Q_1]_j < [Q_2]_j] \\
=& \sum_{j=1}^{N} ([P]_j - \varepsilon)[Q_1]_j 1[[Q_1]_j \geq [Q_2]_j] + \sum_{j=1}^{N} \varepsilon [Q_1]_j 1[[Q_1]_j < [Q_2]_j] \\
&+ \sum_{j=1}^{N} \varepsilon([Q_2]_j - [Q_1]_j) 1[[Q_1]_j < [Q_2]_j] \\
&+ \sum_{j=1}^{N} \varepsilon([Q_1]_j - [Q_2]_j) 1[[Q_1]_j \geq [Q_2]_j] \\
&+ \sum_{j=1}^{N} (P^j - \varepsilon)[Q_2]_j 1[[Q_1]_j < [Q_2]_j] + \sum_{j} \varepsilon [Q_2]_j 1[[Q_1]_j \geq [Q_2]_j] \\
=& P_1 Q_1 + P_2 Q_2 + \sum_{i} \varepsilon([Q_2]_j - [Q_1]_j) 1[[Q_1]_j < [Q_2]_j] \\
&+ \sum_{j=1}^{N} \varepsilon([Q_1]_j - [Q_2]_j) 1[[Q_1]_j \geq [Q_2]_j] \\
=& P_1 Q_1 + P_2 Q_2 + \zeta.
\end{aligned}
$$

□

Proof of Theorem 1

Proof. Necessity was shown in the main text. For the sufficiency part, let us assume that the ARSM is satisfied. Let the numbers $m, M \in \mathbb{R}_{++}$ satisfy

$$m < \min_{i,r,j} \{[p_{i,r}]_j, [P^w_{i,r}]_j, [P^m_{i,r}]_j\} \text{ and } M > \max_{i,r,j} \{[p_{i,r}]_j, [P^w_{i,r}]_j, [P^m_{i,r}]_j\}.$$

We can then define the piecewise linear function v:

$$v : \mathbb{R} \to \mathbb{R} : x \mapsto v(x) = \begin{cases} Mx \text{ if } x \leq 0, \\ mx \text{ if } x > 0. \end{cases}$$

This function v will be used to construct the two utility functions. More specifically, for a man $i \in M$, consider the utility function

$$u^i : \mathbb{R}^{n+N} \to \mathbb{R} : (q, Q) \mapsto u^i(q, Q) = \sum_{j=1}^{n} v([q]_j - [q^m_{i,\sigma(i)}]_j) + \sum_{J=1}^{N} v([Q]_J - [Q_{i,\sigma(i)}]_J).$$

Clearly, this obtains that the man has utility zero for the observed bundle $(q^m_{i,\sigma(i)}, Q)$ in his current match. As a direct implication, the man i needs positive utility to form a blocking pair. Similarly, for a woman $r \in W$ we consider the utility function

$$u^r : \mathbb{R}^{n+N} \to \mathbb{R} : (q, Q) \mapsto u^r(q, Q) = \sum_{j=1}^{n} v([q]_j - [q^w_{\sigma(r),r}]_j) + \sum_{J=1}^{N} v([Q]_J - [Q_{\sigma(r),r}]_J).$$

Let us assume that for these specific utility functions the dataset is not rationalizable by a stable matching. As explained above, this implies that there exists a couple $(i, r) \in M \times W$ and an allocation $(q^m, q^w, Q) \in B_{i,r}$ such that

$$u^i(q^m, Q) \geq u^i(q^m_{i,\sigma(i)}, Q_{i,\sigma(i)}) = 0,$$
$$u^r(q^w, Q) \geq u^r(q^w_{\sigma(r),r}, Q_{\sigma(r),r}) = 0,$$

with at least one strict inequality.
For man i, we have that if $[q]_j > [q^m_{i,\sigma(i)}]_j$ (or $[Q]_J > [Q_{i,\sigma(i)}]_J$), then by definition $m([q]_j - [q^m_{i,\sigma(i)}]_j) < [p_{i,r}]_j([q]_j - [q^m_{i,\sigma(i)}]_j)$ (or $m([Q]_J - [Q_{i,\sigma(i)}]_J) < P^m_{i,r}([Q]_J - [Q_{i,\sigma(i)}]_J)$. Conversely, if $[q]_j \leq [q^m_{i,\sigma(i)}]_j$ (or $[Q]_J \leq [Q_{i,\sigma(i)}]_J$), then by definition $M([q]_j - [q^m_{i,\sigma(i)}]_j) \leq [p_{i,r}]_j([q]_j - [q^m_{i,\sigma(i)}]_j)$ (or $m([Q]_J - [Q_{i,\sigma(i)}]_J) \leq P^m_{i,r}([Q]_J - [Q_{i,\sigma(i)}]_J)$. This shows that $u^i(q^m, Q) \geq 0$ implies

$$p_{i,r}q^m + P^m_{i,r}Q \geq p_{i,r}q^m_{i,\sigma(i)} + P^m_{i,r}Q_{i,\sigma(i)},$$

by replacing the positive terms by a higher positive number and the negative terms by a less negative number. The same reasoning holds for woman r:

$$p_{i,r}q^w + P^w_{i,r}Q \geq p_{i,r}q^w_{\sigma(r),r} + P^w_{i,r}Q_{\sigma(r),r},$$

where at least one of the last two inequalities has to be strict. Adding up these two inequalities gives

$$p_{i,r}(q^m + q^w) + P_{i,r}Q > p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r}.$$

Also, given that $(q^m, q^w, Q) \in B_{i,r}$, we have that the left hand side is less than or equal to $y_{i,r}$, which gives

$$y_{i,r} > p_{i,r}(q^m_{i,\sigma(i)} + q^w_{\sigma(r),r}) + P^m_{i,r}Q_{i,\sigma(i)} + P^w_{i,r}Q_{\sigma(r),r}.$$

This obtains a violation of ARSM, which finishes the proof. □

Proof of Theorem 2

Proof. The reasoning is exactly the same as in Theorem 1. The only difference is that we now need the following quasilinear utility functions for man i and woman r:

$$u^i : \mathbb{R}^{n+N} \to \mathbb{R} : (x, q, Q) \mapsto u^i(x, q, Q) = x - x^m_{i,\sigma(i)} + \sum_{j=1}^{n-1} v([q]_j - [q^m_{i,\sigma(i)}]_j) + \sum_{J=1}^{N} v([Q]_J - [Q_{i,\sigma(i)}]_J).$$

$$u^r : \mathbb{R}^{n+N} \to \mathbb{R} : (x, q, Q) \mapsto u^r(x, q, Q) = x - x^w_{\sigma(r),r} + \sum_{j=1}^{n-1} v([q]_j - [q^w_{\sigma(r),r}]_j) + \sum_{J=1}^{N} v([Q]_J - [Q_{\sigma(r),r}]_J).$$

Correspondingly, in this case the numbers $m, M \in \mathbb{R}_{++}$ must satisfy

$$m < \min_{i,r,j}\{[p_{i,r}]_j, [P^w_{i,r}]_j, [P^m_{i,r}]_j, 1\} \text{ and } M > \max_{i,r,j}\{[p_{i,r}]_j, [P^w_{i,r}]_j, [P^m_{i,r}]_j, 1\}.$$

□

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