FACULTY OF ECONOMICS AND
APPLIED ECONOMIC SCIENCES
CENTER FOR ECONOMIC STUDIES
ENERGY, TRANSPORT & ENVIRONMENT

KATHOLIEKE
UNIVERSITEIT
LEUVEN

WORKING PAPER SERIES
n° 2002-13

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December 2002

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Barring consumers from the electricity network might improve welfare

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December 6, 2002

Abstract

A monopolist supplies a homogenous good to two geographically separated markets. Production costs and demand conditions are different in each market. A line with a limited transport capacity connects both markets.

The paper compares two institutional frameworks: (1) exclusive access to the line is granted to the monopolist (2) access to the line is auctioned to the monopolist and consumers. It derives the monopolist's strategy, and illustrates the result with examples.

In general, it is not clear-cut which regime gives the highest total surplus. For linear demand functions exclusive access is superior to auctioning, if transport capacity is small, cost differences are large and demand conditions similar.

JEL: D42, L12, L42, L9, R41

1 Introduction

This paper considers a monopolist who supplies a homogenous product to two geographically separated markets. Production costs and demand conditions are different in each market. A line with a limited transport capacity connects both markets. Two regimes for the use of the line are considered.

In a first regime, the monopolist owns the transmission line. The monopolist price discriminates, charging a different price in each region. This regime is called *integration* in the remainder of the text.

In a second regime, transport capacity is auctioned. The monopolist and the consumers can buy part or all the capacity. Price discrimination is still possible, but costly for the monopolist, as consumers arbitrate on the price difference. This is called *unbundling*.

The monopolist's strategy is derived under both regimes. Numerical examples are used to clarify the results. Shifting from integration to unbundling decreases regional price differences: the price increases in the low priced region, and decreases in the high priced region. Consumers in the high priced region

*I would like to thank Stef Proost, Ben Hobbs, Guido Pepermans, Edward Calthrop, Bruno De Borger, Patrick Van Cayseele and Frank Verboven.

are better of under unbundling, and consumers in the low priced region are worse of. The monopolist prefers integration instead of unbundling. The effect on total surplus is ambiguous.

As a special case, the paper considers linear demand functions. Total surplus is compared under both regimes. It is shown that for small transmission capacities the results depend on the similarity of cost and demand functions in the two regions. If costs are different and demand similar, integration is better. If cost are similar and demand different, unbundling is better. For large transmission capacities, unbundling is always better.

The last part of the paper shows that there are two reasons why a monopolist would withhold transmission capacity, i.e. buying transmission capacity and not using it. First, it makes it possible to set a high price in the importing region without de-congestioning the transmission line. Second, it makes it cheaper to congest a line in the opposite direction of cost differences, i.e. congestion from the cheap region to the expensive region.

The paper extends the models on third degree price discrimination of for instance Varian (1985) and Tirole (1988). They assume no restriction on transmission capacity, which simplifies the model in two ways: (1) Under unbundling, arbitrage cancels regional price differences, and leads to a uniform price in both regions. (2) Goods are only produced in the low cost country, as it costs nothing to export them from the low to the high cost region.

With limited transmission capacity this is no longer true: unbundling does not eliminate price discrimination and cost differences do matter.

The model in this paper is especially relevant to study congestion in electricity markets. The existing transmission lines that interconnect countries, were designed to transport only emergency power during a contingency. With liberalization, interconnectors are also used to arbitrate on regional price differences. As a result some of the lines are highly congested.

There are two approaches to study the effect of congestion in electricity markets. One uses game theory to study a small network and solves algebraically for a Nash equilibrium. That is the approach followed here. Other authors followed the same approach but used a different set-up. Joskow and Tirole (2000) study a monopolist which has generation capacity in one region, whereas I assume production capacity in two regions. Borenstein et al. (1998) discuss a Cournot generation duopoly in a network with two regions. They put a generator in each region and show that insufficient transmission capacity decreases competition. Both models look only at one allocation mechanism: unbundling, and do not compare it with integration.

Another approach uses a simpler equilibrium concept than Nash and solve it numerically. Day et al. (2002) build oligopoly models with conjectured supply functions, both under integration and unbundling. They assume that generators are price takers in the transmission market, which allows them to use the Generalized Nash as defined by Harker (1991). See also Willems (2002).

Both approaches are combined by Hobbs et al. (2000). They use the standard Nash Equilibrium for a network of thirty nodes, which is solved numerically. For each player a mathematical program with equilibrium constraints is solved with a penalty interior point algorithm.

Outline of the paper. Section 2 describes the model and solves for the welfare optimum. Sections 3 and 4 look at integration and unbundling. Both regimes are compared in the fifth section. Section 6 discusses some policy implications.

2 Model

2.1 Set up

Two regions ($i \in \{1, 2\}$) are connected with a transmission line with a capacity k. In each region there are price-taking consumers, and generation plants. Consumers in region i are represented by a downward sloping and concave demand function $q_i = h_i(p)$.[1] The gross consumers surplus in region i is:

$$GCS_i(p_i) = \int_{p_i}^{\bar{p}_i} h_i(t)dt + p_i h_i(p) \tag{1}$$

with $\bar{p}_i$ the reservation price in region i :

$$\bar{p}_i \equiv h_i^{-1}(0) \tag{2}$$

The monopolist produces Q_i units of electricity in region i at constant marginal costs. The marginal production cost c_H in region 1 is larger than the cost c_L in region 2 ($c_H - c_L = \Delta c > 0$). Total production cost is $c_H Q_1 + c_L Q_2$. The innocuous assumption is made that the high production cost is lower than the reservation prices, $\bar{p}_i > c_H$.

The network transports x units of electricity from region 2 to region 1. There are no transmission costs. A negative x denotes transport from region 1 to region 2. In each region, generation, consumption and import (or export) over the transmission line are balanced.

$$h_2(p_2) + x = Q_2 \tag{3}$$

$$h_1(p_1) - x = Q_1 \tag{4}$$

The total production cost is:

$$C(p_1, p_2, x) = h_1(p_1)c_H + h_2(p_2)c_L - x\Delta c \tag{5}$$

When x is positive, expensive production is replaced with cheap production.

Welfare is the sum of gross consumers surplus minus the production cost:

$$W(p_1, p_2, x) = GCS_1(p_1) + GCS_2(p_2) - C(p_1, p_2, x) \tag{6}$$

2.2 Social optimum

The social optimum maximizes welfare W subject to three types of constraints. Consumption and generation should be positive ($h_i(p_i) \geq 0$ and $Q_i \geq 0$). Positive generation implies that local consumption

[1] Because the demand for electricity is rather inelastic, a monopoly model predicts large prices for electricity. By assuming that there are competitive fringe generators in both regions, the monopolist faces a more elastic demand. Such competitive fringe generators are easily included in the model. Assume competitive fringe generators with marginal cost function $c_i^f(q_i)$, and consumers with demand function $q = h_i^c(p)$ in region i. The aggregated demand function is then $h_i(p) = h_i^c(p) - c_i^{f\,-1}(p)$.

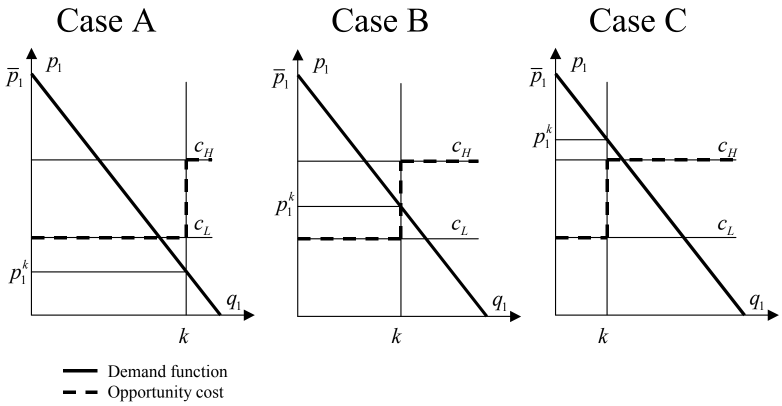

Figure 1: Equilibrium in market 1, for three different capacities of the transmission line.

is larger than import. This follows directly from equation 3 and 4. The *generation and consumption constraints* are

$$\max\{x, 0\} \le h_1(p_1) \tag{7}$$

$$\max\{-x, 0\} \le h_2(p_2) \tag{8}$$

The *transmission constraints* are :

$$-k \le x \le k \tag{9}$$

The paper does not assume an upper limit the production level of the generators.

The opportunity cost for electricity in region 2 is always c_L. Therefore, the socially optimal price is

$$p_2 = c_L \tag{10}$$

In the social optimum electricity is transported from the region with cheap production to the region with expensive production, with other words $x \ge 0$. Figure 1 presents the market equilibrium in region 1 for three transmission capacities. The full line is the demand function and the dotted line is the marginal opportunity cost given the import from the cheap region. The equilibrium marginal opportunity cost for electricity in region 1 depends upon the transmission capacity. For small capacities it is c_H (case C), for high capacities it is c_L (case A), and for intermediate capacities it is somewhere in between (case B). The socially optimal price in region 1, p_1, is

$$p_1 = \begin{cases} c_L & \text{if} \quad p_1^k < c_L \quad \text{Case A} \\ p_1^k & \text{if} \quad c_L \le p_1^k \le c_H \quad \text{Case B} \\ c_H & \text{if} \quad c_H \le p_1^k \quad \text{Case C} \end{cases} \tag{11}$$

with $p_i^k \equiv h_i^{-1}(k)$. It is the price at which demand is equal to k.

2.3 Rewriting the welfare function

For further reference, it is useful to rewrite welfare as a function of total production $h = h_1(p_1) + h_2(p_2)$, the regional price difference $\tau = p_1 - p_2$, and the quantity x transported: $W^*(h, \tau, x)$. Using the inverse function theorem the marginal effects of these variables can be written out as: (see appendix C) [2]

$$\frac{\partial W^*}{\partial h} = (p_1 - c_H)\sigma_1 + (p_2 - c_L)\sigma_2 \quad (12)$$

$$\frac{\partial W^*}{\partial \tau} = \rho\ [\Delta c - \tau] \quad (13)$$

$$\frac{\partial W^*}{\partial x} = \Delta c \quad (14)$$

with $\sigma_i = \frac{h_i'(p_i)}{h_1'(p_1)+h_2'(p_2)}$ and $\rho = -(h_1'(p_1)^{-1} + h_2'(p_2)^{-1})^{-1}$.

They express the marginal welfare effect of a variable, keeping the two other variables fixed. The logic behind the equations is as follows.

The first equation shows the marginal welfare effect of increasing total consumption h. σ_i is the marginal change of consumption in region i when total output increases, while keeping the price difference between the regions constant.

$$\sigma_i = \left.\frac{\partial h_i}{\partial h}\right|_{\tau,x} \quad (15)$$

The welfare effect of an consumption increase in region 1 is equal to $(p_1 - c_H)$, because it has to be produced locally, given that the transport x is not changed. The marginal welfare effect is decreasing with total production quantity, i.e. $\frac{\partial W^*}{\partial h}(h)$ is downward sloping.

The second equation shows that when total production is kept fixed, the marginal effect of the price difference is proportional to $\Delta c - \tau$, keeping production quantity and transport of electricity fixed. Also the marginal effect of the price difference $\frac{\partial W^*}{\partial \tau}(\tau)$ is downward sloping. In the optimum, consumption should be allocated among the two regions such that the price difference is equal to the cost difference. The proportionality factor is ρ :

$$\rho = -\left.\frac{\partial h_1}{\partial \tau}\right|_{h,x} = \left.\frac{\partial h_2}{\partial \tau}\right|_{h,x} \quad (16)$$

which is an averaged slope of the two demand functions. For a constant level of consumption, it is a measure of how much demands shifts from region 1 to region 2 if the price difference between the regions increases.

The last expression shows that if extra electricity is transported from the low cost region to the high cost region, welfare increases with Δc. This effect is always positive.

If no electricity is produced in region 1, $x = h_1(p_1)$, x can be eliminated from the welfare function. Welfare $\hat{W}^*(h, \tau)$ is a function the total production h and the regional price difference τ. The first order derivatives are given by:

$$\frac{\partial \hat{W}^*}{\partial h} = p_1\sigma_1 + p_2\sigma_2 - c_L \quad (17)$$

$$\frac{\partial \hat{W}^*}{\partial \tau} = -\rho\tau \quad (18)$$

These two equations show that the opportunity cost for electricity is equal to c_L in both regions and that the marginal effect the price difference is proportional to the price difference τ.

[2] Note that these expressions do not take into account transmission, generation or consumption constraints.

3 Integration

Electricity generation and network operation are integrated in a single firm. (Integration = index I). The firm owns all transmission capacity and sets electricity prices[3]. His profit is equal to revenue minus production costs

$$\pi^I = p_1 h_1(p_1) + p_2 h_2(p_2) - C(p_1, p_2, x) \tag{19}$$

Define $\pi_i(p, c)$ as the profit of a generator who sells electricity in market i at a price p, and has production cost c.

$$\pi_i(p, c) = (p - c)\, h_i(p) \tag{20}$$

Marginal profit $\frac{\partial \pi_i(p,c)}{\partial p}$ increases with cost c and is assumed to decrease with price p.[4]

$$\frac{\partial^2 \pi_i(p, c)}{\partial p\, \partial c} > 0 \text{ and } \frac{\partial^2 \pi_i(p, c)}{\partial p^2} < 0 \tag{21}$$

The profit of the monopolist is the sum of three terms:

$$\pi^I = \pi_1(p_1, c_H) + \pi_2(p_2, c_L) + x \cdot \Delta c \tag{22}$$

The first two terms are the profits in region 1 and 2 when all electricity is produced locally. The third term is the profit increase, when expensive production is replaced by cheap production. The monopolist maximizes his profit subject to the generation and consumption constraints (7 and 8) and the transmission constraint (9).

Except for the objective function the problem of the integrated monopolist and the social planner are equal. The optimal solution for the monopolist is the intersection of the marginal revenue function and the opportunity cost of electricity.

Define p^m_{ij} as the local monopoly price in region i when the production cost is c_j.

$$p^m_{ij} \equiv \arg\max_p \pi_i(p, c_j) \quad i = 1, 2 \text{ and } j = L, H \tag{23}$$

This price is determined by the standard inverse elasticity rule:

$$\frac{p^m_{ij} - c_j}{p^m_{ij}} = \frac{1}{\varepsilon_i(p^m_{ij})} \quad i = 1, 2 \text{ and } j = L, H \tag{24}$$

with $\varepsilon_i(p_i) = -p_i \frac{h'_i(p_i)}{h_i(p_i)}$ the demand elasticity in region i.

[3] As in most models on price discrimination, it is assumed that the monopolist sets prices. This is of course equivalent with setting production quantities. Though, both approaches give different conditions for the objective to be concave, and the constraints to be convex.

[4] The objective function in prices is concave when $\frac{\partial^2 \pi_i(p,c)}{\partial p^2} = h''_1(p)(p - c_H) + 2h'_1(p) < 0$. In general, this function does not need to be concave when p becomes smaller than the costs. ($p < c_H$). If the monopolist has only one production plant this means that he is selling at a loss, what he of course will never do. In our problem the monopolist can set a price in region 1 below the marginal costs in region 1 without making a loss, as he can import electricity from low cost region 2. A price below marginal cost can not be ruled out. In the rest of the paper it is assumed that in the relevant price range the objective function is concave.

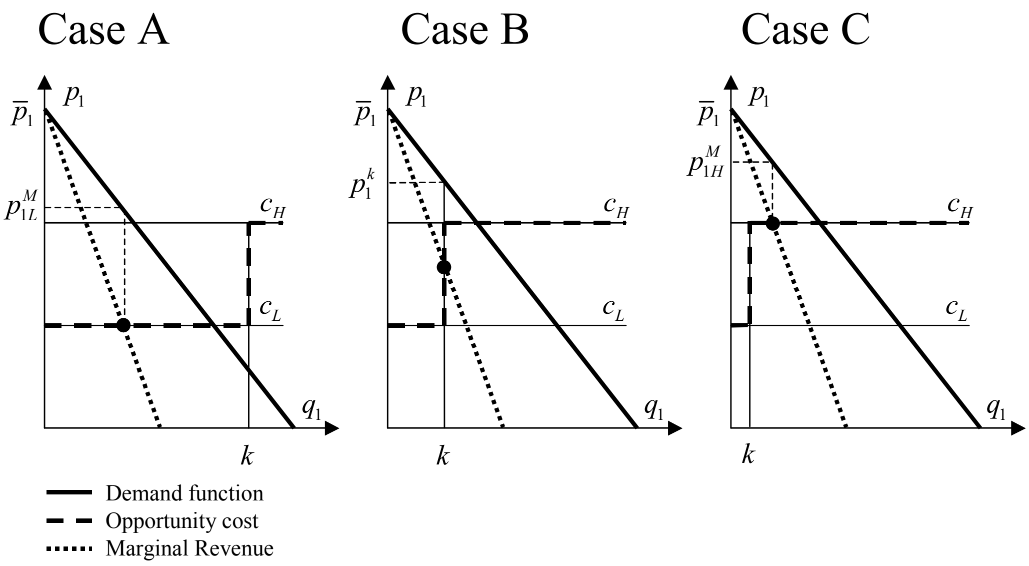

Figure 2: Marginal revenue and marginal opportunity cost of electricity in region 1.

Because the opportunity cost of electricity is c_L in region 2, the price in region 2 is the local monopoly price.[5]

$$p_2^I = p_{2L}^m$$

The opportunity cost of electricity in region 1, and hence the optimal price p_1^I, depends upon the size of the transmission line. Figure 2 presents the marginal opportunity cost and marginal revenue of selling electricity in region 1. Their intersection is the optimal value.

If k is small, the opportunity cost is c_H. (Case C) The optimal price is p_{1H}^m. For high transmission capacity, the opportunity costs is c_L, and the optimal price is p_{1L}^m. (Case A) For intermediate capacities the opportunity cost is somewhere in between. (Case B).

$$p_1^I = \begin{cases} p_{1L}^m & \text{if} \quad p_1^k < p_{1L}^m \quad \text{Case A} \\ p_1^k & \text{if} \quad p_{1L}^m \leq p_1^k \leq p_{1H}^m \quad \text{Case B} \\ p_{1H}^m & \text{if} \quad p_{1H}^m \leq p_1^k \quad \text{Case C} \end{cases} \tag{25}$$

The price in the high cost country can be higher or lower than in the low cost country. This depends on the shape of the demand functions and the cost difference. The generator takes a higher markup in the region where demand is less elastic. If demand in the high cost region is more elastic than in the low cost region, the price can be lower in the high cost region. In other words, the electricity price in a region can be high because it is expensive to generate electricity or because the monopolist sets a high markup.

As a special case, suppose that the two regions have the same demand for electricity: $h_1(\cdot) = h_2(\cdot) = \tilde{h}(\cdot)$ Define the local monopoly price $p^M(c) = \arg\max_p \tilde{h}(p)(p - c)$. For a concave demand function, the monopolist shifts a change in costs through to his consumers, but not completely: $0 < \frac{\partial p^M}{\partial c} < 1$.

[5] The assumption that $\bar{p}_i > c_H$ implies directly that the zero consumption constraint in region 2 is not binding.

	Example 1		Example 2	
	Region 1	Region 2	Region 1	Region 2
$a_1[MW^3h^2€^{-2}]$	-2.6401	-3.333	-3.333	-1.032
$a_2[MW^2h€^{-1}]$	-16.306	-33.333	0	-29.365
$a_3[MW]$	13 000	14 000	12 000	11 000
$c[€MW^{-1}]$	40	15	35	25

Table 1: Parameters of the two problems.

As a result, the high cost region has the highest price, but the price difference is smaller than the cost difference :

$$0 < p^M(c_H) - p^M(c_L) < \Delta c. \tag{26}$$

3.1 Numerical illustration

Throughout the paper, two numerical examples will be used to clarify the results. Demand function in region i is represented by a polynome of second order: $h_i(p) = a_1^i p^2 + a_2^i p + a_3^i$. Table 1 gives the coefficients a_j^i and production costs of both examples. Figure 3 and 4 show the prices that are set by the monopolist.

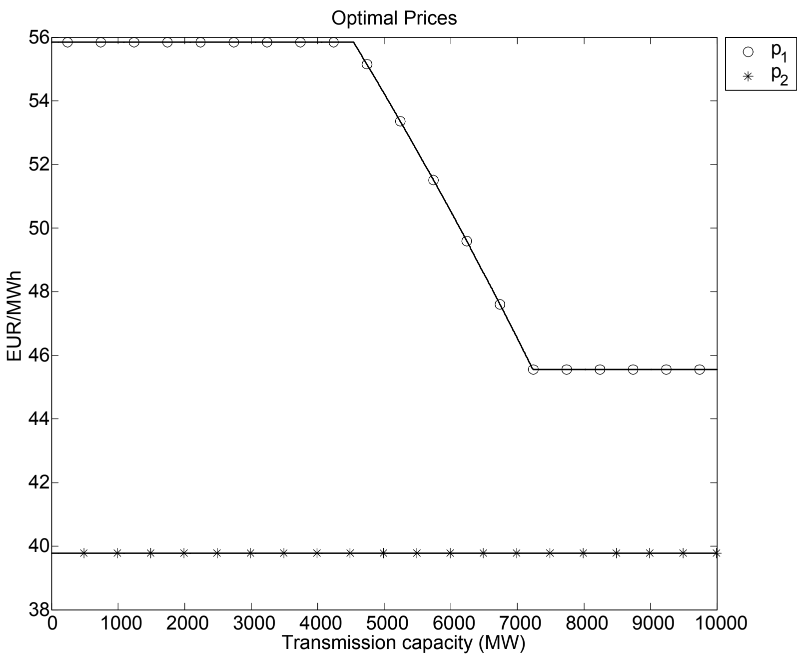

Figure 3: Example 1: optimal prices in both regions under integration.

Example 1 (Figure 3)

The monopolist sets a constant price $p_2^I = 40\ EUR/MWh$ in region 2. The price does not depend on the transmission capacity because the opportunity cost in region 2 is always $c_L = 15\ EUR/MWh$. In region 1 the price drops from 56 EUR/MWh to 46 EUR/MWh. This happens because the opportunity cost of electricity drops from $c_H = 40\ EUR/MWh$ to $c_L = 15\ EUR/MWh$.

For zero transmission capacity the price difference between the regions is $\tau = 16\ EUR/MWh$ this is smaller than the cost difference $\Delta c = 25\ EUR/MWh$. The monopolist takes thus a lower margin in region 1 ($56-40 = 16\ EUR/MWh$) than in region 2 ($40-15 = 25\ EUR/MWh$). For large transmission capacities, the relevant cost difference is $\Delta c = 0\ EUR/MWh$ while the price difference decreases to $\tau = 6\ EUR/MWh$. The markup in region 1 ($46-15 = 31\ EUR/MWh$) is now larger than the markup in region 2 ($40-15 = 25\ EUR/MWh$).

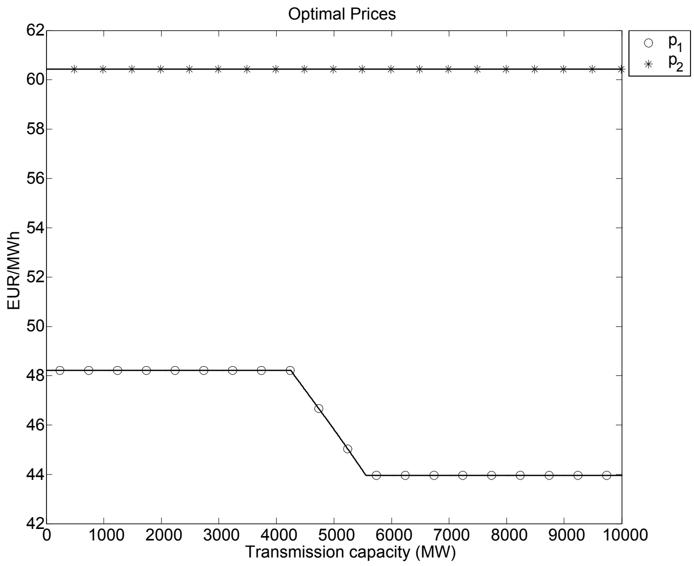

Figure 4: Example 2: optimal prices in both regions under integration.

Example 2 (Figure 4)

The monopolist sets a constant price $p_2^I = 61\ EUR/MWh$ in region 2. The price does not depend on the transmission capacity because the opportunity cost in region 2 is always $c_L = 25\ EUR/MWh$. The monopolist takes a mark-up of 36 EUR/MWh. In region 1 the prices drops from 48 EUR/MWh to 44 EUR/MWh when transmission capacity increases. This happens because the opportunity cost of electricity drops from $c_H = 35\ EUR/MWh$ to 25 EUR/MWh. The mark-up in region 1 is 13 EUR/MWh for low transmission capacities and 19 EUR/MWh for high capacities. The monopolist sets a higher mark-up in region 2 than in region 1. For zero transmission capacity the price difference between the regions is $\tau = -12\ EUR/MWh$. The low cost region 2 has a higher price than the low cost region, as the difference in mark-ups outweighs the cost differences.

Example 2 shows that when the transmission capacity is increased, the price difference between the regions might increase. This is more likely when demand in the high cost region (region 1) is more elastic then demand in the low cost region 2, and when cost differences are small.

4 Unbundling

The provision of electricity implies two activities: generation and transmission. In the decentralized transmission rights system (Chao and Peck, 1986) these two activities are separated. This is called

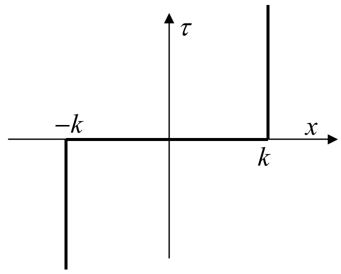

Figure 5: Supply function of transmission capacity.

unbundling (index U). In each region there is an energy market, and for transmission, there is a market for transmission rights. The transmission rights are sold to consumers and to the monopolist.

Joskow and Tirole (2000) explicitly model the microstructure of the transmission rights market.[6] Here, a simpler approach is used. It is assumed that consumers arbitrate perfectly on the regional price difference. In equilibrium, the transmission price τ equals the price difference between the regions (*arbitrage condition*)[7]

$$\tau = p_1 - p_2 \tag{27}$$

The total number of transmission rights sold for transport of electricity from region 2 to region 1 is x.[8] The rights are directed, i.e. a negative x indicates a transport in the opposite direction. If you buy a transmission right, you have the right *and the obligation* to use the transmission line. Buying capacity and withholding it from the market is not allowed. In section 6.4 this assumption will be relaxed.

The equilibrium in the transmission market is summarized as: (*transmission market equilibrium*)

$$\tau = 0 \Rightarrow -k \leq x \leq k \tag{28}$$

$$\tau > 0 \Rightarrow x = k \tag{29}$$

$$\tau < 0 \Rightarrow x = -k \tag{30}$$

The equations define the short run competitive supply function of transmission capacity $\tau(x)$. See figure 5. As long as demand for transmission is smaller than supply, its price is zero. When all transmission capacity is used, the transmission price can differ from zero.

The monopolist sets three variables: p_1 and p_2, the price for electricity in regions 1 and 2; and x_G, the number of transmission rights that he buys. Given these variables the consumers buy $x_C = x - x_G$ transmission rights such that there is an equilibrium in the transmission market and that the arbitrage condition is satisfied.

[6] They consider one or several auctioneers who sell these rights. Because in their model, the generators have an increasing marginal valuation of transmission rights, the auctioneers have the incentive to wait for selling their rights. They try to Free Ride on other players who sell their rights.

[7] If consumers do not participate in the transmission market there is no arbitrage. In this case, auctioning of transmission rights gives the same result as integration. A possible reason for the lack of arbitrage, is that arbitrageurs can not enter the market, or that consumers can not resell their electricity.

[8] This is a slight abuse of notation. Before, τ and x were merely the price difference and the quantity transported. Now, they are the price of transmission rights, and the number that is sold to consumers and generators.

The monopolist's profit equals revenue minus production and transmission costs.

$$\pi = \text{Revenue} - \text{Production Cost} - \text{ Transmission Cost} \tag{31}$$

In region 1, consumers consume $h_1(p_1)$ units of electricity. As they import x_C units of electricity from region 2, they buy locally $h_1(p_1)-x_C$. The monopolist's revenue in region 1 is $(h_1(p_1)-x_C)p_1$. The monopolist himself imports an amount x_G. In region 1, he produces $h_1(p_1)-x_C-x_G$, and his production cost is $(h_1(p_1)-x_C-x_G).c_L$. A similar reasoning is made for region 2. Revenue and production cost are $(h_2(p_2)+x_C)p_2$ and $(h_2(p_2)+x_C+x_G)c_L$. The transmission cost is equal to $x_G\,\tau$.

The objective function of the monopolist is:

$$\begin{aligned}\pi = {} & [p_1 \cdot (h_1(p_1) - x_C) + p_2 \cdot (h_2(p_2) + x_C)] \\ & - [c_H \cdot (h_1(p_1) - x_C - x_G) + c_L \cdot (h_2(p_2) + x_C + x_G)] - x_G \cdot \tau\end{aligned} \tag{32}$$

Lemma 1 *If there is perfect arbitrage, the monopolist does not care who buys the transmission rights. Only the total amount of rights sold, x, matters to him.*

The proof is straightforward: substitute the arbitrage condition $\tau = p_1 - p_2$ in the objective function 32 and recollect the different terms. The objective function rewrites as:

$$\max_{p_1,p_2,x,\tau} \pi_1(p_1, c_H) + \pi_2(p_2, c_L) + x \cdot \Delta c - x \cdot \tau$$

which does only depend on x, and not on x_C or x_G, separately.

The first three terms of the objective function are the same as in the previous model. The fourth term is new and can be interpreted in two ways. First, if the monopolist buys all transmission rights ($x_C = 0$, $x_G = x$), it is the cost of buying the rights: $x_G\,\tau$. Second, if the consumers buy all transmission rights ($x_C = x$, $x_G = 0$), it is the revenue forgone to consumers who do not buy their electricity locally, but import it from the low priced region: $x_C p_1 - x_C p_2$.

The monopolist maximizes his objective function subject to the generation and consumption constraints (7 and 8), the arbitrage condition (27) and, the transmission market equilibrium (28-30).[9]

This is a Mathematical Program with Equilibrium Constraints (MPEC, See Luo et al., 1996). It is hard to solve as the conditions for transmission market equilibrium are highly non-convex.

The monopolist maximizes his profit

$$\max_{p_1,p_2,x} \pi(p_1, p_2, x) \tag{33}$$

subject to

$$(p_1, p_2, x) \in S(p_1, p_2) \tag{34}$$

[9]Adding the consumption constraint to the problem of the monopolist is not completely correct. The monopolist can set a price higher than the reservation price, in which case demand is zero. Demand in region i should be defined as $q_i = \begin{cases} h_i(p_i) & p_i \leq \bar{p}_i \\ 0 & p_i > \bar{p}_i \end{cases}$. As it is not optimal for the monopolist to do so, this option is neglected.

with $S(p_1, p_2)$ the feasible set of prices and transmission. This problem is not trivial because the feasible set S is discontinuous in prices.

$$S = \begin{cases} S^{>} & \text{if} \quad \tau > 0 \quad \text{price region I} \\ S^{=} & \text{if} \quad \tau = 0 \quad \text{price region II} \\ S^{<} & \text{if} \quad \tau < 0 \quad \text{price region III} \end{cases} \tag{35}$$

with

$$S^{>} = \left\{ (p_1, p_2, x) \left| \begin{array}{c} h_1(p_1) \geq k, \\ h_2(p_2) \geq 0, \\ x = k \end{array} \right. \right\}, \tag{36}$$

$$S^{=} = \left\{ (p_1, p_2, x) \left| \begin{array}{c} h_1(p_1) \geq \max\{x, 0\} \\ h_2(p_2) \geq \max\{-x, 0\} \\ -k \leq x \leq k \end{array} \right. \right\}, \tag{37}$$

$$S^{<} = \left\{ (p_1, p_2, x) \left| \begin{array}{c} h_1(p_1) \geq 0 \\ h_2(p_2) \geq k \\ x = -k \end{array} \right. \right\} \tag{38}$$

4.1 Maximizing the monopolist's profit

In order to solve the problem of the monopolist three simpler problems are solved: Profit is maximized for each set of constraints $S^{=}$, $S^{<}$ and $S^{>}$. Define for $\omega \in \{=, >, <\}$ the highest profit $\Pi^{\omega}(\tau)$ that is reached in constraint set S^{ω} if the transmission price is τ:

$$\Pi^{\omega}(\tau) = \max_{p_1, p_2, x} \pi(p_1, p_2, x) \tag{39}$$

$$\text{s.t. } (p_1, p_2, x) \in S^{\omega} \tag{40}$$

$$p_1 - p_2 = \tau \tag{41}$$

Three such functions are sketched in figure 6. As the constraint set $S^{=}$ is less strict than $S^{<}$ and $S^{>}$, the profit function $\Pi^{=}$ lies above $\Pi^{<}$ and $\Pi^{>}$. The validity of constraint set S^{ω} depends on the transmission price τ.(See equation 35) The true constraint set is $S(p_1, p_2)$.

Define $\Pi(r)$ as the maximum profit that can be reached in the constraint set $S(p_1, p_2)$ for a transmission price τ

$$\Pi(\tau) = \max_{p_1, p_2, x} \pi(p_1, p_2, x) \tag{42}$$

$$\text{s.t. } (p_1, p_2, x) \in S(p_1, p_2) \tag{43}$$

$$p_1 - p_2 = \tau \tag{44}$$

Given the defintions above, this can be written as:

$$\Pi(\tau) = \begin{cases} \Pi^{>}(\tau) & \tau > 0 \\ \Pi^{=}(\tau) & \tau = 0 \\ \Pi^{<}(\tau) & \tau < 0 \end{cases} \tag{45}$$

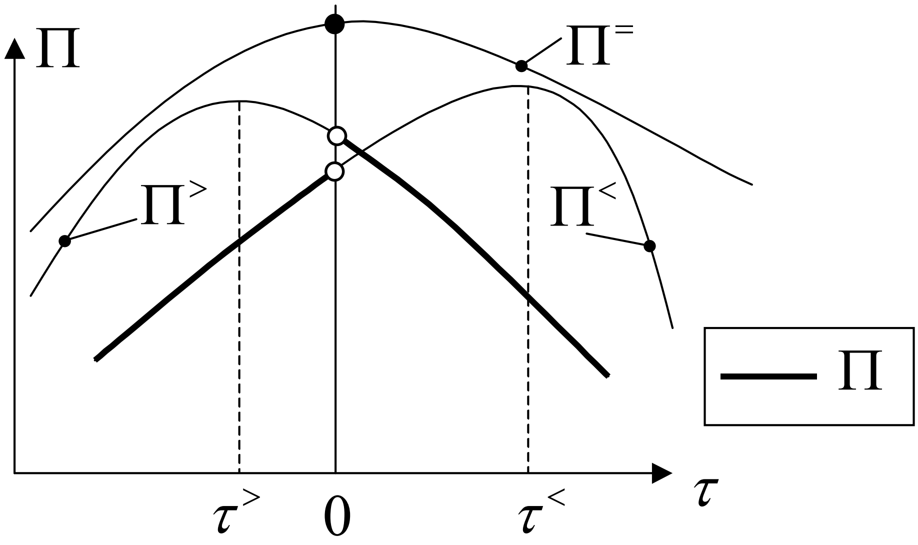

Figure 6: Maximal profit $\Pi(\tau)$ for a given transmission price τ.

which is presented by the thick line in figure 6.

The monopolist chooses the transmission price τ^U that maximizes profit.

$$\tau^U = \arg\max_{\tau} \Pi(\tau) \tag{46}$$

The function $\Pi(\tau)$ is discontinuous and can have several local optima. The shape of $\Pi(\tau)$ is determined by the location of $\tau^>$ and $\tau^<$, which are the maximizers of $\Pi^>(\tau)$ and $\Pi^<(\tau)$.

$$\tau^\omega = \arg\max_{\tau} \Pi^\omega(\tau) \quad \omega \in \{<, >\} \tag{47}$$

See figure 7.

Given that $\tau^> < \tau^<$ (see later), there are three possibilities for the location of $\tau^<$ and $\tau^>$:

$$\text{case } 1 : 0 < \tau^> < \tau^> \tag{48}$$

$$\text{case } 2 : \tau^> < 0 < \tau^> \tag{49}$$

$$\text{case } 3 : \tau^> < \tau^> < 0 \tag{50}$$

For each case, the local optima of $\Pi(\tau)$ are indicated in Figure 7. If the local optimum is unique (case 2) it is the global optimum. If there are several local optima (case 1 and case 3), they need to be calculated and compared explicitly. In case 1, $\tau^>$ and $\tau = 0$, need to be compared, in case 2, $\tau^<$ and $\tau = 0$.

The remaining of the section derives the three possible local optima of $\Pi(\tau)$: $\tau = 0$, $\tau = \tau^>$ and $\tau = \tau^<$. The last subsection illustrates the results numerically.

4.2 Uniform price in the regions ($S^=$)

The monopolist sets a uniform price p in both regions. Assume that at the optimal price, both regions are served. A sufficient condition for this is that at the monopoly price for one market, the consumption

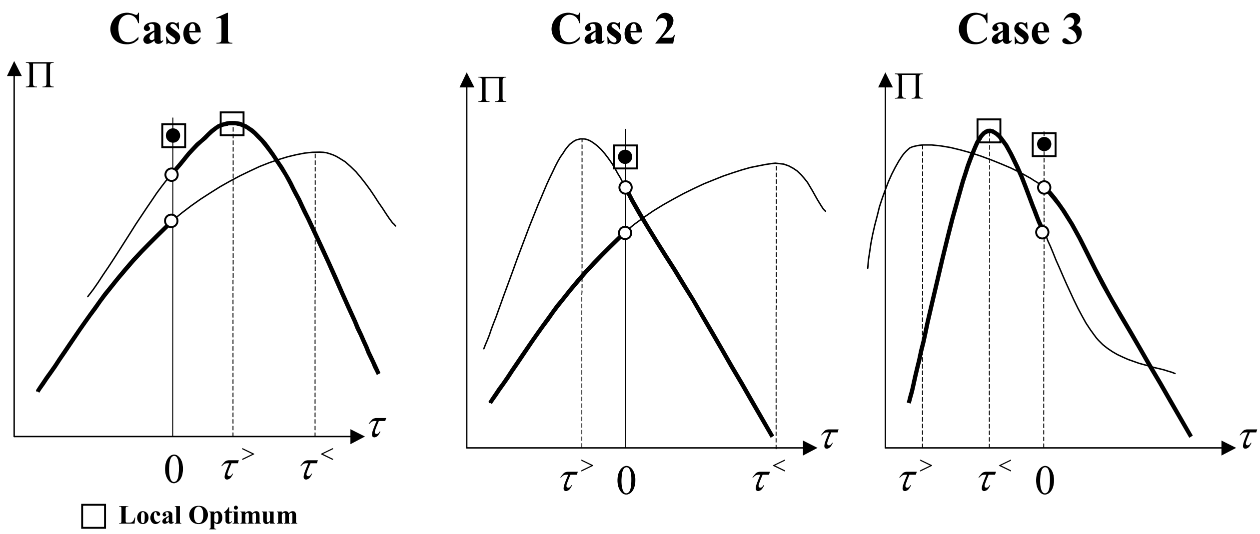

Figure 7: Local optima of $\Pi(\tau)$.

in the other market is positive:

$$p_{2H}^m < \bar{p}_1 \quad \text{and} \quad p_{1L}^m < \bar{p}_2 \tag{51}$$

The monopolist uses an amount x of the transmission capacity at a zero price. His problem is:

$$(p^=, x^=) = \arg\max_{p,x} \ \pi_1(p, c_H) + \pi_2(p, c_L) + x \cdot \Delta c \tag{52}$$

subject to the transmission constraint and the generation constraint in region 1[10]:

$$x \leq k \tag{53}$$

$$h_1(p) - x \geq 0 \tag{54}$$

Define p_{totH}, and p_{totL} as

$$p_{totH} \equiv \arg\max_{p} \ \ \pi_1(p, c_H) + \pi_2(p, c_L) \tag{55}$$

$$p_{totL} \equiv \arg\max_{p} \ \ \pi_1(p, c_L) + \pi_2(p, c_L) \tag{56}$$

They are the optimal uniform prices when the opportunity costs in region 1 and 2 are (c_H, c_L) and (c_L, c_L). Using demand elasticities these prices can also be written as:

$$\frac{p_{totH} - \sigma_1 c_H - \sigma_2 c_L}{p_{totH}} = \frac{1}{s_1 \varepsilon_1 + s_2 \varepsilon_2}$$
$$\frac{p_{totL} - c_L}{p_{totL}} = \frac{1}{s_1 \varepsilon_1 + s_2 \varepsilon_2}$$

with $s_i = \frac{h_i(p)}{h_1(p)+h_2(p)}$ the market share, and $\sigma_i = \frac{h_i'(p)}{h_1'(p)+h_2'(p)}$ the relative production increase.

The optimal price $p^=$ is

$$p^= = \begin{cases} p_{totL} & \text{if} \quad p_1^k < p_{totL} \quad \text{Case A} \\ p_1^k & \text{if} \quad p_{totL} \leq p_1^k \leq p_{totH} \quad \text{Case B} \\ p_{totH} & \text{if} \quad p_{totH} < p_1^k \quad \text{Case C} \end{cases} \tag{57}$$

[10] Due to the assumptions made above, the consumption constraint in region 2 is not binding.

For low transmission capacities (case C), the opportunity cost in region 1 is c_H. Electricity is produced in both regions. For large transmission capacities (case A), the opportunity cost of electricity in region 1 is c_L. Electricity is only produced in the low cost country, the transmission capacity is not fully used. For intermediate transmission capacities (Case B), opportunity cost in region 1 is between c_H and c_L electricity is only produced in the low cost country, and the line is used at its capacity.

4.3 Trade from low to high cost region ($S^>$)

In constraint set $S^>$ all transmission capacity is used from region 2 to region 1: $x^> = k$. The monopolist solves the following problem:

$$(p_1^>, p_2^>) = \arg\max_{p_1,p_2} \pi_1(p_1, c_H) + \pi_2(p_2, c_L) + k \cdot (\Delta c - \tau) \tag{58}$$

subject to the generation and consumption constraints

$$k \leq h_1(p_1) \tag{59}$$

$$0 \leq h_2(p_2) \tag{60}$$

The objective function of the monopolist depends on p_1 and p_2. If the monopolist changes the price p_1, it has an influence on local profit in region 1, $\pi_1(p_1, c_H)$ and on the transmission cost $k\,\tau$.

Define for $i = 1, 2$ and $j = L, H$ the prices p_{ij}^+, p_{ij}^- as

$$p_{ij}^+ \equiv \arg\max_p \{\pi_i(p, c_j) + kp\} \tag{61}$$

$$p_{ij}^- \equiv \arg\max_p \{\pi_i(p, c_j) - kp\} \tag{62}$$

which can be interpreted as the prices set by a monopolist with financial obligations. p_{ij}^+ is optimal for a monopolist with production cost c_j in market i, who has a long position on the price p_i i.e. he sold k forward contracts on the price p_i. The price p_{ij}^- is optimal when he has a short position, i.e. he bought forward contracts.

If $k = 0$, there are no financial obligations, and the monopolist sets the monopoly price $p_{ij}^+ = p_{ij}^- = p_{ij}^m$. For positive transmission capacities, p_{ij}^+ increases and p_{ij}^- decreases.

$$p_{ij}^+ > p_{ij}^m > p_{ij}^- \tag{63}$$

Using the demand elasticity these prices can be written as:

$$\frac{p_{ij}^+ - c_j}{p_{ij}^+} = \frac{1 + k/h_i(p_{ij}^+)}{\varepsilon_i(p_{ij}^+)} \tag{64}$$

$$\frac{p_{ij}^- - c_j}{p_{ij}^-} = \frac{1 - k/h_i(p_{ij}^-)}{\varepsilon_i(p_{ij}^-)} \tag{65}$$

In region 2, the opportunity cost of electricity is c_L. The optimal price in region 2 is p_{2L}^+ as long as it is lower than the reservation price in market 2.

$$p_2^> = \min(p_{2L}^+, \bar{p}_2) \tag{66}$$

The price in region 1 is p_{1H}^{-} as long as demand in region 1 is larger than the transmission capacity.

$$p_1^{>} = \min(p_{1H}^{-}, p_1^k) \tag{67}$$

The monopolist can not set a price in region 1 above p_1^k. Otherwise demand in region 1 would be smaller than the transmission capacity k. There would be no congestion, and the price difference between the regions would become zero.

If the price in region 1 is equal to p_{1H}^{-}, the opportunity cost in region 1 is equal to c_H. Electricity is produced in both regions.

If the price in region 1 is equal to p_1^k, there is no production in region 1. The price is kept low in order to keep the transmission line congested. The opportunity cost of electricity is lower than c_H.

Note that if $p_1^k < p_{1L}^{-}$, the opportunity cost in region 1 is lower than c_L. Even if there would be cheap generation in region 1, it would not be used, as it would eliminate the congestion on the network.

Both p_1^k and p_{1H}^{-} decrease when the transmission capacity increases. The decision to set p_1^k or p_{1H}^{-} does not have be monotonic in transmission capacity i.e. if the transmission capacity increases, the price in region 1 can switch several times between p_1^k and p_{1H}^{-}.

4.4 Trade from high to low cost region ($S^<$)

The solution for constraint set $S^<$ is very similar to constraint set $S^>$. Electricity is transported from the high cost region to the low cost region ($x^< = -k$), the optimal prices in region 1 and 2 are

$$p_2^{<} = \min(p_2^k, p_{2L}^{-}) \tag{68}$$

$$p_1^{<} = \min(\bar{p}_1, p_{1H}^{+}) \tag{69}$$

The price in region 1 is set according to a high opportunity cost ($= c_H$) as long as the price is below the reservation price in region 1. In region 2 the price of electricity is p_{2L}^{-}, unless the transmission line is not congested at this price. The price is than lowered to p_2^k. In this case, the opportunity cost of electricity is lower than c_L.

4.5 Numerical illustration

Consider the two examples discussed before. Figures 8 and 9 show the optimal prices under unbundling. To simplify a comparison, the optimal prices under integration are drawn as well.

Example 1 (Figure 8)

If the transmission capacity is equal to zero, the monopolist sets the local monopoly price in both regions p_{1H}^M and p_{1L}^M. The price in the high cost region is above the price in the low cost region. Prices under integration and unbundling are the same. For low transmission capacities the monopolist sets the prices $p_1^>$ and $p_2^>$. If the transmission capacity increases, price differentiation becomes more and more costly. The price difference gradually decreases until prices become equal (around 5500 MW). From that point on, a uniform price $p^=$ is set for both regions. For transmission capacities below 7000 MW, the prices is $p_{totH} = 46.5\ EUR/MWh$, as the opportunity cost in region 1 is c_H. For capacities between 7000 MW and 8000 MW, the price in equal to p_1^k. The opportunity cost in region 1 is between c_H and

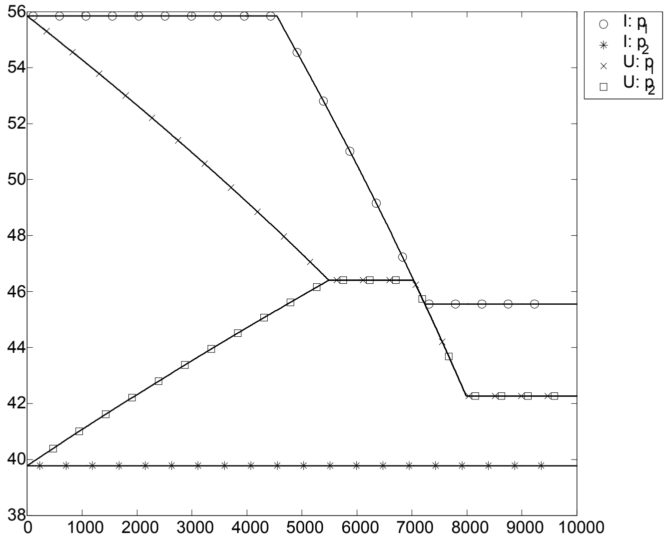

Figure 8: Example 1: prices in both regions

c_L. For capacities above 8000 MW the monopolist sets $p_{totL} = 42EUR/MWh$. The opportunity cost in region one is c_L.

Example 2 (Figure 9)

For zero transmission capacity, the monopolist sets the same prices under unbundling as under integration. (p_{1H}^M and p_{2L}^M). The monopolist sets a higher price in region 2 than in region 1. For low transmission capacities the monopolists sets $p_1^<$ and $p_2^<$. The price difference decreases with transmission capacity. Setting a higher price in the low cost region than in the high cost region is costly for the monopolist: he has to pay for the transmission capacity, and he has to transport electricity from the high cost region to the low cost. At a transmission capacity around 600 MW the monopolist switches strategy. He does no longer price discriminate, but charges a uniform price ($p^=$) in both regions. He does no longer pay for transmission capacity (gain = $(p_2 - p_1)\ k$) and produces electricity more efficiently (gain = $2\ \Delta c\ k$), but looses the profit of price discrimination. For capacities below 3000 MW, the opportunity cost in region 1 is c_H. The uniform price is $p_{tot}H = 52EUR/MWh$. For capacities between 3000 MW and 4000 MW, the price is p_1^k. And for capacities above 4000MW the opportunity cost is c_L and the uniform price is $p_{totL} = 49EUR/MWh$. .

5 Comparison of unbundling and integration

This section compares integration (index I) and unbundling (index U). Define the resulting level of welfare V under regime $l = U, I$ as

$$V^l(k) = W(p_1^l(k), p_2^l(k), x^l(k)) \quad l = I, U \tag{70}$$

with $p_i^l(k)$ the price in region i and $x^l(k)$ the transport of electricity that are optimal for the monopolist under regime l. Note that welfare W is equal to gross consumer surplus minus generation costs. It can

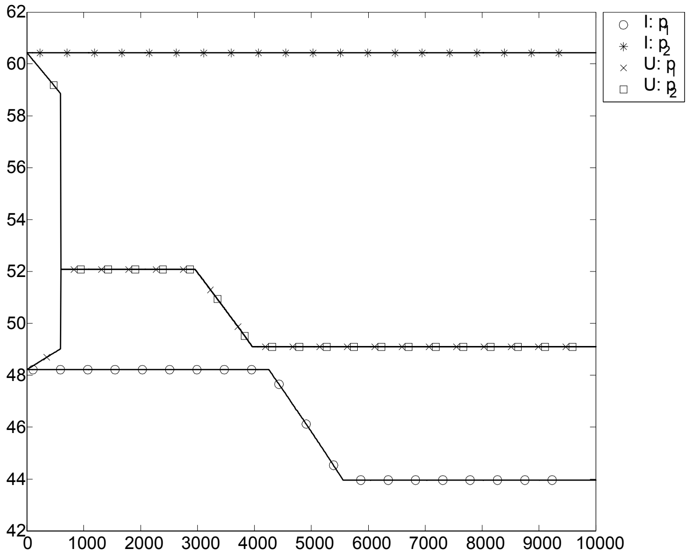

Figure 9: Example 2: prices in both regions

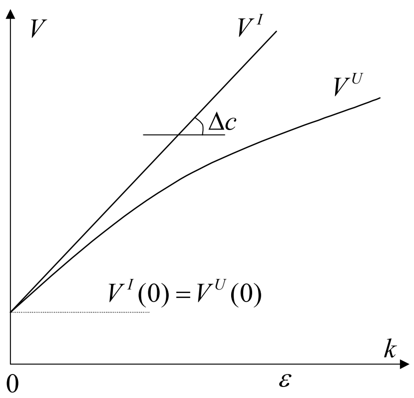

Figure 10: Welfare under integration and unbundling for small transmission capacities.

be rewritten as the sum of consumer surplus, monopoly profit, and revenue for the auctioneer of the transmission rights.

5.1 Small transmission capacity

If there is no transmission, $k = 0$, the monopolists sets the local monopoly price in both regimes: $p_1^l(k = 0) = p_{1H}^M$ and $p_2^l(k = 0) = p_{2L}^M$. Welfare is equal under both regimes and zero transmission capacities $V^U(0) = V^I(0)$. For transmission capacities close to zero welfare can be approximated as $V^l(\varepsilon) = V^I(0) + \varepsilon \cdot \frac{dV^I}{dk}(0)$. Integration is better than unbundling when $\frac{dV^I}{dk}(0) > \frac{dV^U}{dk}(0)$, as sketched in Figure 10.

Under integration and small transmission capacity, marginal welfare is equal to the gain in production efficiency.[11]

$$\frac{dV^I}{dk} = \Delta c \tag{71}$$

Under unbundling, the marginal effect is[12]

$$\frac{dV^U}{dk} = \left[\kappa_1(p_1) + \frac{2}{p_1 - c_H}\right]^{-1} - \left[k_2(p_2) + \frac{2}{p_2 - c_L}\right]^{-1} + \Delta c \tag{72}$$

with $\kappa_i(p)$ the curvature of the demand function

$$\kappa_i(p) = \frac{h_i''(p)}{h_i'(p)}$$

The last term is the gain in production efficiency. The first two terms reflect the effect on local surplus in each market, given the local production cost. If the sum of the first two terms is negative at $k = 0$, welfare is higher under integration than under unbundling.

5.2 Large transmission capacity

For large transmission capacities the comparison of integration and unbundling becomes that of third degree price discrimination and uniform pricing. See for instance Tirole, 1988. There are two reasons for this:

(1) Under unbundling, arbitrage makes the transmission price equal to zero ($\tau = 0$). A uniform price is charged in both regions.

(2) Cost differences do not matter: given the costless and unconstrained transmission, all electricity is produced in the low cost region at a cost c_L .

To my knowledge, no general conditions for the comparison of unbundling and integration exist.

5.3 Linear demand

As a special case, consider linear demand functions of the form $h_i(p) = \alpha_i - \beta_i p$. Define: $\chi = \frac{\alpha_1}{\beta_1} - \frac{\alpha_2}{\beta_2}$. It is measure of the difference of the demand functions.

Lemma 2 *For sufficiently large transmission capacities, the monopolist sets $p_1^I = p_{1L}^M$ and $p_2^I = p_{2L}^M$ under integration, and $p_1^U = p_2^U = p_{totL}$ under unbundling.*

See appendix B.

Theorem 3 *For sufficiently large transmission capacities, unbundling is better than integration.*

The monopolist's strategy is given by the previous lemma. It can be shown that total production h is equal under integration and unbundling

$$h = h^I = h^U = \frac{1}{2}[h_1(c_L) + h_2(c_L)] \tag{73}$$

[11] This assumes that the opportunity cost in region 1 is c_H. As a result, region one's price does not change.

[12] This can be calculated as $\frac{dV^U}{dk} = W^{p_1}\frac{dp_1^U}{dk} + W^{p_2}\frac{dp_2^U}{dk} + W^x\frac{dx^I}{dk}$. The derivation assumes that the transmission constraint is binding, and that the production and consumption constraints are not binding. This is the case for small transmission capacities. Further it assumes that the price in region 1 is higher than in region 2.

and that for large transmission capacity, the transmission constraint is not binding:

$$x^I = h_1(p_1^I). \quad (74)$$

$$x^U = h_1(p_1^U) \quad (75)$$

The opportunity cost for electricity is c_L in both regions.

Equation 18 rewrites for linear demand functions as:

$$W^I - W^U = \int_{\tau^U}^{\tau^I} -\rho t dt = -\rho \frac{\chi^2}{8} \quad (76)$$

with $\tau^I = \frac{1}{2}\chi$, $\tau^U = 0$ and $\rho = (\beta_1^{-1} + \beta_2^{-1})^{-1}$. Unbundling is clearly better than integration.

Lemma 4 *For sufficiently low transmission capacities and* $\chi < \Delta c$ *the monopolist sets* $p_1^I = p_{1H}^M$ *and* $p_2^I = p_{2L}^M$ *under integration, and* $p_1^I = p_{1H}^+$ *and* $p_2^I = p_{2L}^+$ *under unbundling.*

See appendix B.

Theorem 5 *If demand functions are similar* ($-\Delta c < \chi < \Delta c$)*, integration is better than unbundling for small transmission capacities.*

The previous lemma gives the monopolist's strategy for low transmission capacities. It can be shown that the total production h is equal under unbundling and integration:

$$h = h^I = h^U = \frac{1}{2}[h_1(c_H) + h_2(c_L)] \quad (77)$$

The transmission line is congested, the total quantity transported is equal to the available transmission capacity:

$$x^U(k) = x^I(k) = k \quad (78)$$

As h and x are equal under both regimes, the welfare difference depends only on the price difference τ and by integrating equation 13

$$W^I - W^U = \int_{\tau^U}^{\tau^I} \rho(\Delta c - t)dt = \frac{k}{4}(\Delta c - \chi) + \frac{k^2}{8\rho} \quad (79)$$

with $\tau^U = \frac{1}{2}\left(\chi + \Delta c - \frac{1}{\rho}k\right)$ and $\tau^I = \frac{1}{2}(\chi + \Delta c)$. For small k, integration is better than unbundling when $\chi < \Delta c$.

5.3.1 Intuition

For large transmission capacities, the opportunity costs for electricity is the same in both regions. Welfare optimality requires that the price difference between the regions is zero. This is achieved by unbundling.

For small transmission capacities, the regions have a different opportunity cost for electricity. Welfare optimality requires that the price difference equals the cost difference. If the demand functions are similar, the monopolist sets a price difference is that is smaller than the cost difference.[13] Unbundling reduces the price difference and is thus welfare decreasing.

[13] Above it has been shown that this is the case if the demand functions are equal. See equation 26.

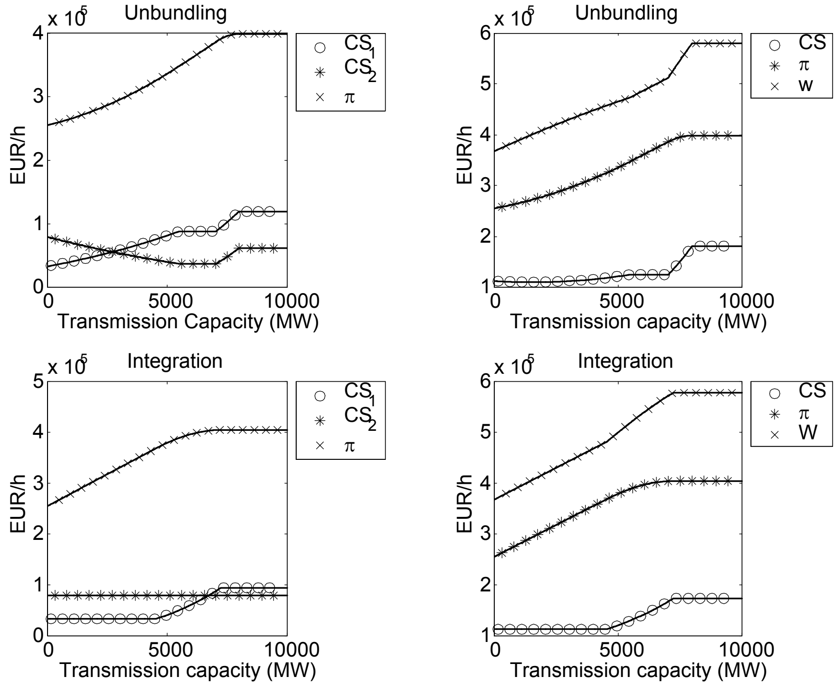

Figure 11: Example 1: Consumer surplus, monopolist's profit, and total welfare in both regimes.

5.4 Numerical illustration

Figures 11 and 13 present, as function of the transmission capacity, consumers surplus in both regions, monopoly profit and welfare. For the figures on the top there is unbundling. For the figures on the bottom, there is integration. In order to compare both regimes, figures 12 and 14 show the difference in consumer surplus, monopoly profit and welfare under both regimes.

Example 1 (Figure 12)

For transmission capacities smaller than 8000 MW welfare is higher under integration. The monopolist always prefers integration, while the consumers like unbundling for capacities between 3000 and 5200 MW and above 8000 MW.

Example 2 (Figure 14).

For transmission capacities below 600 MW, welfare is higher under integration than under unbundling. The monopolist never likes unbundling as he has less strict constraints. Consumers always prefer unbundling.

6 Extensions and Discussions

This section covers four topics related with policy problems. The first subsection uses the model with linear demand and small transmission capacities and assumes that one of the markets is perfectly competitive. The second subsection checks what the model in this paper can tell about the size of transmission line. The third subsection shows that a centralized pool system and an unbundled transmission rights market are equivalent. The fourth subsection extends the model and allows the monopolist to withhold transmission capacity.

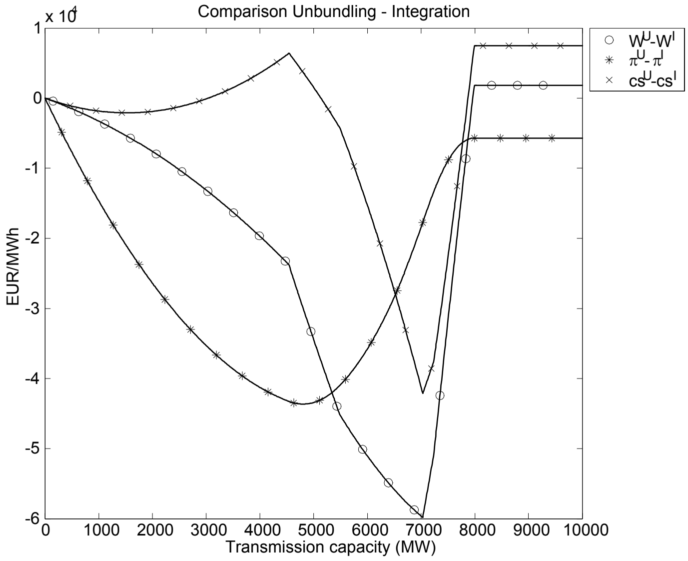

Figure 12: Example 1: Comparison of integration and unbundling.

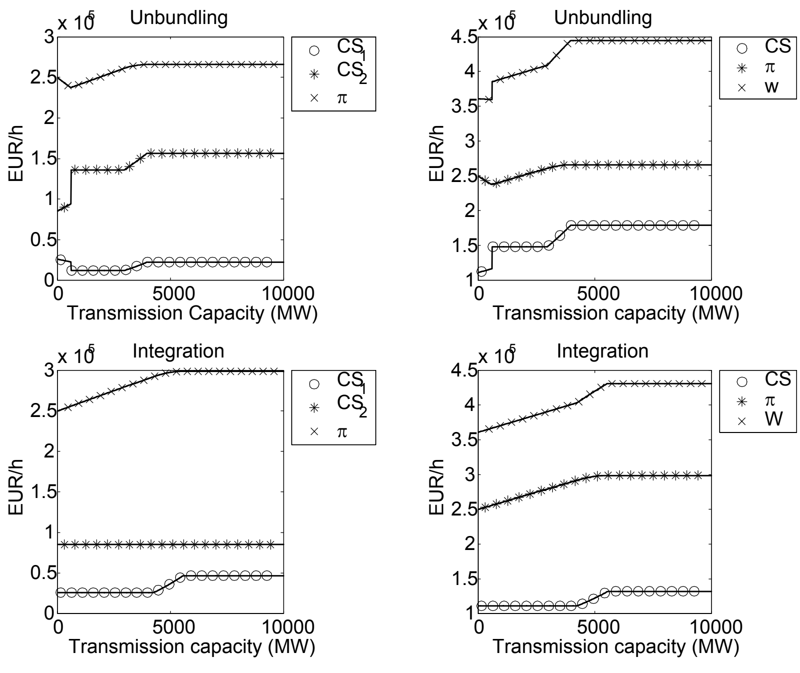

Figure 13: Example 2: Consumer surplus, monopolist's profit, network and total welfare in both regimes.

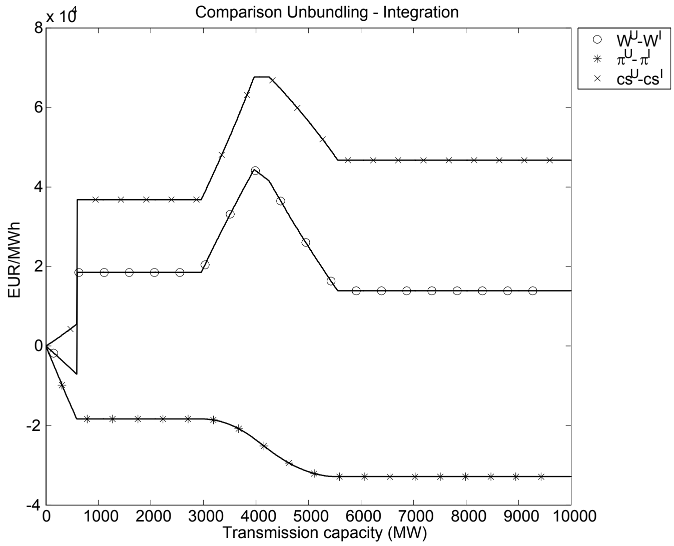

Figure 14: Example 2: Comparison of integration and unbundling.

6.1 Perfect competition in one market

This subsection looks at the impact of the competitiveness of the markets on the choice of the allocation mechanism for low transmission capacities. Suppose that a lot of competitive fringe generators enter one of the regions. And assume that they have the same production cost as the monopolist. Their entry will make the residual demand function for the monopolist perfectly elastic. As the competitive fringe generators all produce at their marginal costs, it does no longer matter whether the monopolist owns production capacity in that region or not.

First, assume perfect competition in the low cost region 2. The residual demand function for the monopolist is perfect elastic $\beta_2 \to \infty$ and the price in the low cost region is $\frac{\alpha_2}{\beta_2} = c_L$. The price in the high cost region 1 is always above c_L. Unbundling gives an incentive to decrease the price difference between the regions, and hence decreases the price in region 1, and is always optimal.

Second, assume perfect competition in the high cost region 1. ($\beta_1 \to \infty$ and $\frac{\alpha_1}{\beta_1} \to c_H$.) The price in region 2 is larger than c_H if c_L is large and the region 1 is not competitive. In this case unbundling gives the incentive to decrease the price difference between the regions, which is welfare improving, but could also lead to a inefficient flow of electricity opposite to the cost difference which is welfare decreasing. The price in region 2 is below c_H, if $c_L << c_H$ and market 2 is rather competitive. Unbundling will increase the price in region 2, which is not optimal.

Summarizing: access to a transmission line that connects a competitive low cost region should be auctioned. If the line connects a competitive high cost region, auctioning can lead to a decrease in welfare.

6.2 Transmission Capacity

Until know the transmission capacity was assumed fixed. The two examples show that welfare is in general increasing in transmission capacity. As long as investment in transmission capacity is not too

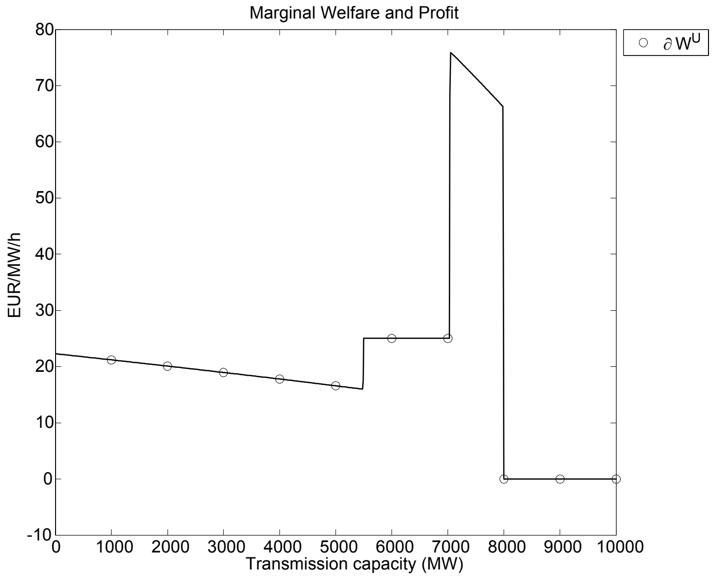

Figure 15: Example 1: marginal welfare effect of transmission capacity under unbundling

expensive, new capacity should be build.

Figures 15 and 16 show the marginal welfare effect of transmission capacity $\frac{dV^U}{dk}(k)$ under unbundling. It is the social demand function for transmission capacity. The figures show that it is not a nicely downward sloping and continuous function, but has jumps, and is negative in some regions. As a result it is difficult to make general conclusions about the optimal size of the transmission line.

Both regimes (unbundling and integration) give different incentives to build new transmission lines by private investors and by the monopolist. A long run comparison should take these incentives into account. This remains open for further research.

6.3 Centralized pool

Another mechanism for the allocation of transmission capacity than an auction, is a centralized (pool) system (Schweppe et al., 1988). Generators and consumers submit bids to the network operator for the production and consumption of electricity. The network operator solves an optimization problem, and sets quantities and prices of production and consumption. In this process transmission capacity is allocated implicitly. The centralized market yields identical prices, profits and total sales as the decentralized market when there is perfect arbitrage, equilibrium in the transmission market and no withholding of transmission capacity. Note that the sales in a region can be different. These results are generally valid under perfect competition (Boucher and Smeers, in press), Cournot competition (Metzler et al. in press) and conjectured supply functions (Day et al., 2001).

The reason for this equivalence result is that the monopolist is indifferent in transporting electricity himself, or allowing the network operator to transport electricity. The reasoning is similar as in equation 32.

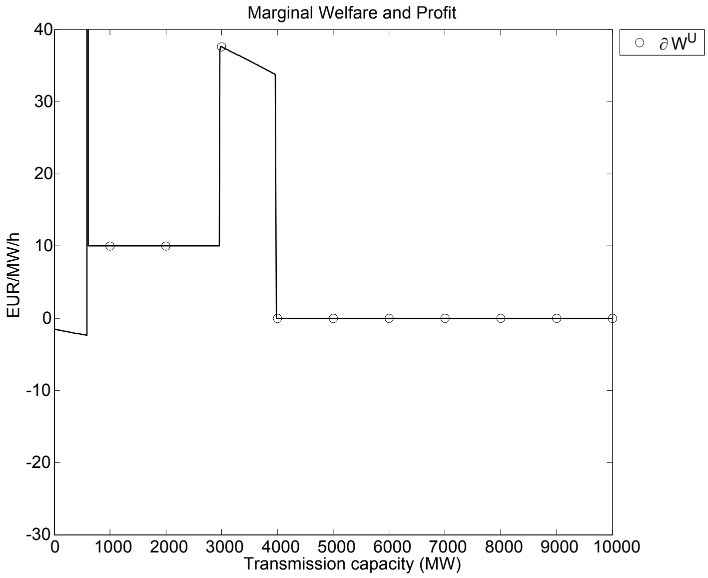

Figure 16: Example 2: marginal welfare effect of transmission capacity under unbundling

6.4 Withholding

This subsection introduces the possibility of withholding. By buying transmission capacity and not using it, the monopolist is able to reduce the quantity that is effectively transported over the network, without decongestioning the transmission line. This section does not derive a full set of solutions, but highlights when withholding will occur.

The monopolist withholds a fraction $(1-\gamma)$ of the transmission rights that are sold, x. He maximizes profit:

$$\max_{p_1,p_2,x,\gamma} \pi_1(p_1 - c_H) + \pi_2(p_2 - c_L) + x(\gamma\Delta c - \tau)$$

subject to the arbitrage condition (27), the transmission market equilibrium (28-30) and

$$0 \leq \gamma \leq 1 \tag{80}$$

$$\max\{\gamma x, 0\} \leq h_1(p_1) \tag{81}$$

$$\max\{-\gamma x, 0\} \leq h_2(p_2) \tag{82}$$

The last two constraints are generation and consumption constraints in region 1 and 2. If the generator sets $\gamma = 1$, he does not withhold transmission capacity. The problem reduces to the one discussed in the previous sections.

This optimization problem can be solved by looking at the three constraint sets valid for $\tau = 0$, $\tau > 0$ and $\tau < 0$.

For prices $\tau = 0$, the problem is identical to the one already solved. Withholding has no effect, as transmission price is equal to zero.

A positive transmission price implies that $x = k$. Withholding of transmission capacity increases production costs, as less electricity is transported from the cheap region to the expensive region. But it relaxes the consumption constraint in region 1. Without withholding, the opportunity cost in region

1 could drop below c_L (See the discussion above). This was the case if $p_1^k < p_{1L}^-$. The monopolist kept prices up in order to keep the line congested. In this case, if withholding is allowed, the monopolist can set a price p_{1L}^- in region 1, import $h_1(p_{1L}^-)$, and withhold capacity to keep the line congested: $(1-\gamma)k = h_1(p_{1L}^-)$.

A lower price in region 1 than in region 2 (price region III), implies that $x = -k$. The generator withholds all transmission capacity ($\gamma = 0$). It increases production efficiency and relaxes the consumption constraint in region 2. Withholding allows production cost to decrease with $x\ \Delta c$. The relaxation of the consumption constraint in region 2 allows the monopolist to set a price p_{2L}^- in region 1 even if consumption in region 2 is smaller than the transmission capacity: $h_2(p_{2L}^-) < k$.

Joskow and Tirole (2000) show that a monopolist can withhold transmission capacity in order to extract some of the inframarginal rents of a low cost fringe generator. Such behavior can lead to inefficient restricting the imports from a low cost region. This is very similar to the result found here: the monopolist will withhold capacity in order to increase the price difference, i.e. extract more rents of the consumers in the high price region.

Only in the second numerical example withholding occurs. Figure 17 gives the optimal price under unbundling when withholding is allowed (index W), and when not (index U). Withholding makes price discrimination cheaper for the monopolist, therefore he discriminates up to higher transmission capacities. For larger transmission capacities, a uniform price is set, unbundling with and without withholding give the same results. Figure 18 compares welfare under unbundling with and without unbundling and integration. For transmission capacities below 590 MW, $W^I > W^W > W^U$. For transmission capacities between 590 MW and 890 MW $W^U > W^I > W^W$. And for capacities above 890 MW :$W^U = W^W > W^I$.

7 Conclusion

This paper considers a monopolist who supplies a homogenous product to two geographically separated markets. Production costs and demand conditions are different in each market. A line with a limited transport capacity connects both markets. The paper derives the equilibrium strategy for the monopolist under two regimes for the use of the line: unbundling and integration.

As a special case, it considers linear demand functions. It shows that for low transmission capacities, different costs and similar demand conditions, integration gives a higher welfare than unbundling. For large transmission capacities, unbundling is always better.

The intuition for small transmission capacities is as follows. Welfare is determined by (1) total production quantity, (2) the allocation of production among consumers, and (3) the transport of electricity. For small transmission capacities and linear demand, only the second term is important to compare integration and unbundling. Allocational efficiency requires that the price difference is equal to the cost difference. Imperfect arbitrage makes price differentiation costly, and gives an incentive to the monopolist to decrease the price difference between the regions. This is welfare decreasing if the price difference was already smaller than the cost difference.

The last part of the paper shows that there are two reasons why a monopolist would withhold

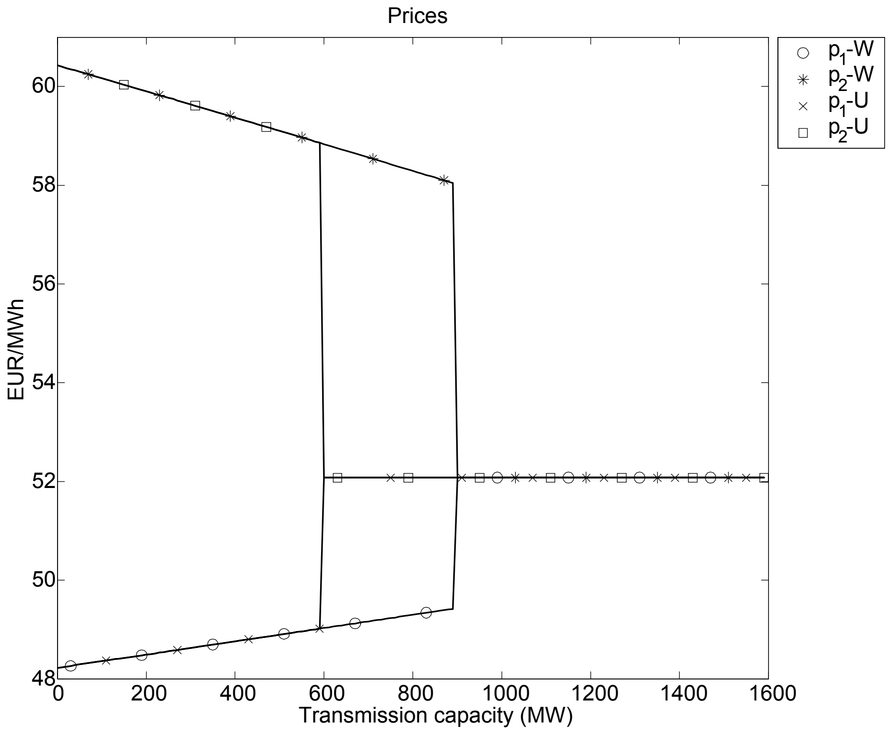

Figure 17: Optimal prices under unbundling with withholding (index W) and without (index U).

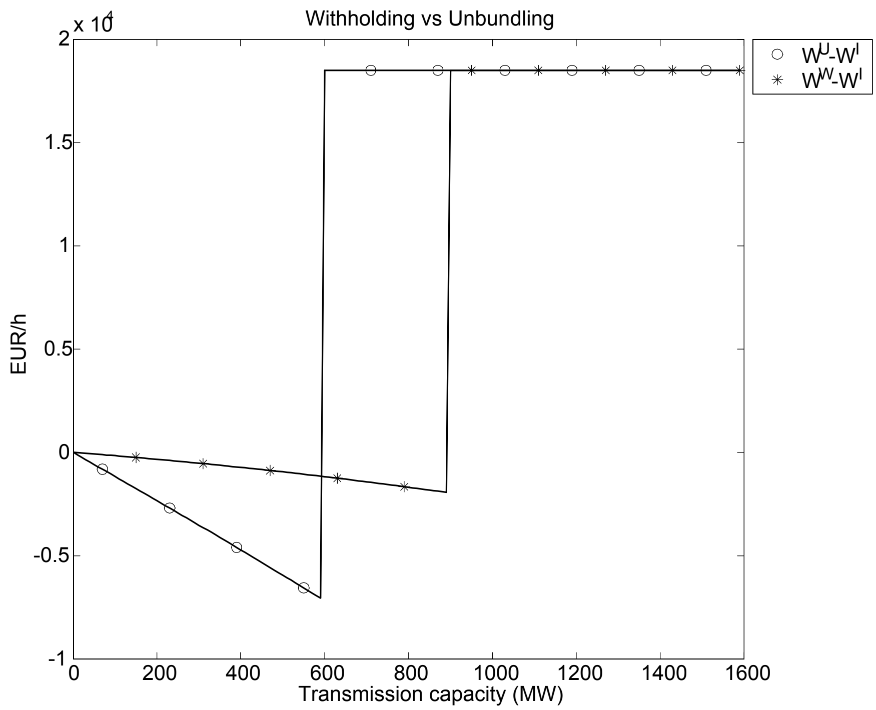

Figure 18: Comparison of welfare under unbundling with withholding (W) without (U) and integration (I). Integration is taken as the reference.

transmission capacity, i.e. buying transmission capacity and not using it. First, it makes it possible to set a high price in the importing region without de-congestioning the transmission line. Second, it makes it cheaper to congest a line in the opposite direction of cost differences, i.e. congestion from the cheap region to the expensive region.

The policy implications could be summarized as follows:

- If a transmission line is small and production costs are different between two regions, it is sometimes better to let the monopolist manage the transmission line.
- If transmission capacity is sufficiently large, auctioning of transmission is optimal. (linear demand functions.)
- A transmission lines that starts in a competitive low cost region should always be auctioned.
- Withholding transmission capacity can have positive and negative welfare effects.

The model made assumptions like perfect arbitrage in the transmission market, a one-link network, a single generator, and a strict transmission limit:

The model compares two extreme assumptions: perfect arbitrage and no arbitrage at all. In practice, the creation of a transmission market, does not always lead to perfect arbitrage.[14] Possible reasons could include that the electricity markets are not cleared at the same moment in time, the existence of asymmetric information, lack of liquidity in the electricity and the transmission markets. It is not clear how this aspects can be incorporated in the model. Joskow and Tirole look at non-competitive supply of transmission rights by several auctioneers. [15]

Extensions of the model to a more complex network are cumbersome, given the non-linear constraints in the optimization problem for the monopolist.

Extending the model from a monopoly to an oligopoly model requires an extra assumption on the variables that are set by the generators: quantities, supply functions, etc.. Two regimes can be compared: one with regional arbitrage and one without. See for instance Day et al 2002.

The paper assumes strict capacity limits for the transmission line, which is valid in the short run. This gives the supply function of transmission as in figure 5. In the long run, transmission capacity is not fixed. This can be incorporated in the model by assuming a flatter supply function. This remains for further research.

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A Uniqueness of the solution

This appendix discusses the uniqueness of the strategy of the monopolist. In order to do so, the decision variables of the monopolist are taken to be quantities.

A.1 Integration

The problem of the monopolist can be written in production quantities Q_1, Q_2, consumption quantities q_1, q_2 and transmission quantity x:

$$\max_{q_i, Q_i, x} \pi^I(q_1, q_2, Q_1, Q_2) = q_1\ p_1(q_1) - Q_1 c_H + q_2\ p_2(q_2) - Q_2 c_L$$

subject to

$$\begin{aligned} x &= q_1 - Q_1 \\ x &= Q_2 - q_2 \\ -k &\leq x \leq k \\ 0 &\leq q_i \\ 0 &\leq Q_i \end{aligned}$$

When $h_i()$ is decreasing and concave as assumed in the text, the objective function $\pi^I(q_1, q_2, Q_1, Q_2)$ is concave. The constraints form a convex, but unbounded set. The set is unbounded as local production and consumption can be increased simultaneously without violating any constraint. In order for a unique solution to exist, the marginal revenue of increasing local consumption, should be lower than the local production costs for large consumption quantities.

$$\begin{aligned} \lim_{q_1 \to \infty} p_1'(q_1)q_1 + p_1(q_1) &\leq c_H \\ \lim_{q_2 \to \infty} p_2'(q_2)q_2 + p_2(q_2) &\leq c_L \end{aligned}$$

A.2 Unbundling

This section studies the uniqueness of $(p_1^>, p_2^>, x^>)$. It is the maximum of the profit function in the constraint set $S^>$. The monopolist's problem is transformed in quantities and becomes:

$$\max_{q_1, q_2}(p_1(q_1) - c_H) \cdot (q_1 - k) + (p_2(q_2) - c_L) \cdot (q_2 + k) \tag{83}$$

subject to

$$q_1 \geq k \quad (z_1) \tag{84}$$

$$q_2 \geq 0 \quad (z_2) \tag{85}$$

The objective function is concave when

$$p_1''(q_1)(q_1 - k) + 2p_1'(q_1) < 0 \tag{86}$$

Given the constraint $q_1 \geq k$ this is always the case.

The constraints form an unbounded convex set. In order for a solution to exist, the following conditions should be satisfied:

$$\lim_{q_1 \to \infty} p_1'(q_1)\ (q_1 - k) + p_1(q_1) \quad < \quad c_H \tag{87}$$

$$\lim_{q_2 \to \infty} p_2'(q_2)\ (q_2 + k) + p_2(q_2) \quad < \quad c_L \tag{88}$$

A.3 The supremum of $\Pi^>(\tau)$

Define $\tilde{\tau}^>$ as the supremum of $\Pi^>(\tau)$ for positive τ:

$$\tilde{\tau}^> = \arg\sup_{\tau} \Pi^>(\tau) \ \text{ s.t. } \ \tau > 0, \tag{89}$$

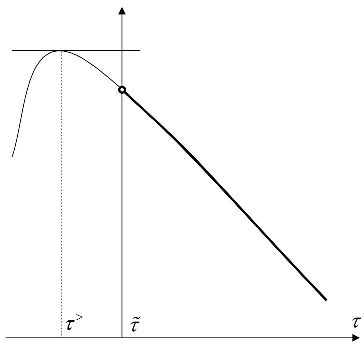

Figure 19: The supermum of the profit function for positive transmission prices $\tau > 0$.

Section 4.1 assumes that if $\tau^> < 0$, that the supremum lies at the boundary of the price region: $\tilde{\tau}^> = 0$. See figure 19.

This is not so obvious as it is not certain that $\Pi^>(\cdot)$ is a concave function. In order to rewrite 89 in quantities, the following constraint should be added to the optimization problem:

$$p_1(q_1) > p_2(q_2) \tag{90}$$

This condition is not convex in q_1 and q_2. However, as all other constraints are convex in q_1 and q_2, and the objective function is concave, the supremum should be located on the boundary if $\tau^> < 0$. But, there can be multiple local suprema on the boundary $p_1(q_1) = p_2(q_2)$.

B Linear demand functions

Lemma 6 *For sufficiently large transmission capacities, the monopolist sets $p_1^I = p_{1L}^M$ and $p_2^I = p_{2L}^M$ under integration, and $p_1^U = p_2^U = p_{totL}$ under unbundling.*

Take $k > \max\{\frac{1}{2}p_1(c_L), |\rho(\chi + \Delta c)|, \alpha_1 - \frac{1}{2}\beta_1\left(\frac{\alpha_1+\alpha_2}{\beta_1+\beta_2} + c_L\right)\}$.

Integration

If $k > \frac{1}{2}p_1(c_L)$, the opportunity cost in region 1 is equal to the c_L. (see equation 25)

Unbundling

If $k > |\rho(\chi + \Delta c)|$, the monopolist sets a uniform price in both regions: $(\pi(p_1^>, p_2^>, x^>) < \pi(p_1^=, p_1^=, x^=)$ and $\pi(p_1^<, p_2^<, x^<) < \pi(p_1^=, p_1^=, x^=))$. If $k > \alpha_1 - \frac{1}{2}\beta_1\left(\frac{\alpha_1+\alpha_2}{\beta_1+\beta_2} + c_L\right)$, the transmission constraint is not binding (equation 57).

Lemma 7 *For sufficiently low transmission capacities and $\chi < \Delta c$ the monopolist sets $p_1^I(k) = p_{1H}^M$ and $p_2^I(k) = p_{2L}^M$ under integration, and $p_1^I(k) = p_{1H}^+$ and $p_2^I(k) = p_{2L}^+$ under unbundling.*

Take $k < \min\{\frac{1}{2}h_1(c_H), h_2(c_L), \rho(\chi - \Delta c)\}$

Integration:

If $k < \frac{1}{2}h_1(c_H)$ the monopolists sets $p_1^I(k) = p_{1H}^M$ and $p_2^I(k) = p_{2L}^M$ under integration. The condition makes sure that the opportunity cost in region 1 is equal to c_H. This follows directly from equation 25.

Unbundling

For $k < \rho(\chi - \Delta c)$, it is optimal to set a higher price in region 1 than in region 2. The conditions $k < h_1(c_H)$ and $k < h_2(c_L)$ make sure that the production constrain in region 1 and the consumption constraint in region 2 are not binding.

C Welfare function

The marginal effects of welfare defined by equation 6 are:

$$\begin{aligned} W^{p_1} &= h_1'(p_1)(p_1 - c_H) && (91) \\ W^{p_2} &= h_2'(p_2)(p_2 - c_L) && (92) \\ W^x &= \Delta c && (93) \end{aligned}$$

Rewriting the welfare $W^*(h, \tau, x)$ as function of total production h, the regional price difference τ, and the transported capacity, the marginal effects are calculated as

$$\begin{bmatrix} W^{*h} \\ W^{*\tau} \\ W^{*x} \end{bmatrix} = \begin{bmatrix} \frac{\partial h}{\partial p_1} & \frac{\partial \tau}{\partial p_1} & 0 \\ \frac{\partial h}{\partial p_2} & \frac{\partial \tau}{\partial p_2} & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} W^{p_1} \\ W^{p_2} \\ W^x \end{bmatrix} \tag{94}$$

with $\frac{\partial h}{\partial p_i} = h_i'(p_i)$, $\frac{\partial \tau}{\partial p_1} = 1$, and $\frac{\partial \tau}{\partial p_2} = -1$.

If no electricity is produced in region 1, $x = h_1(p_1)$, and the objective function can be written as function of p_1 and p_2:

$$\hat{W}(p_1, p_2) = GCS_1(p_1) + GCS_2(p_2) - c_L(h_1(p_1) + h_2(p_2)) \tag{95}$$

The marginal welfare effects are:

$$\begin{aligned} \hat{W}^{p_1} &= h_1'(p_1)(p_1 - c_L) && (96) \\ \hat{W}^{p_2} &= h_1'(p_2)(p_2 - c_L) && (97) \end{aligned}$$

As before, welfare $\hat{W}^*(h, \tau)$ can be expressed as function of total production and the regional price difference:

$$\begin{bmatrix} \hat{W}^{*h} \\ \hat{W}^{*\tau} \end{bmatrix} = \begin{bmatrix} \frac{\partial h}{\partial p_1} & \frac{\partial \tau}{\partial p_1} \\ \frac{\partial h}{\partial p_2} & \frac{\partial \tau}{\partial p_2} \end{bmatrix}^{-1} \begin{bmatrix} \hat{W}^{p_1} \\ \hat{W}^{p_2} \end{bmatrix} \tag{98}$$

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