FACULTY OF ECONOMICS AND
APPLIED ECONOMIC SCIENCES
CENTER FOR ECONOMIC STUDIES
ENERGY, TRANSPORT & ENVIRONMENT

KATHOLIEKE
UNIVERSITEIT
LEUVEN

WORKING PAPER SERIES
n°2004-11

Imperfect Competition and Congestion in a City with asymmetric subcenters

A. de Palma (Member of the Institut Universitaire de France, THEMA, Université de Cergy-Pontoise, France and CORE, Belgium)

F. Dunkerley (K.U.Leuven – CES)

S. Proost (K.U.Leuven – CES; CORE - UCLouvain)

August 2004

secretariat:
Isabelle Benoit
KULeuven-CES
Naamsestraat 69, B-3000 Leuven (Belgium)
tel: +32 (0) 16 32.66.33
fax: +32 (0) 16 32.69.10
e-mail: Isabelle.Benoit@econ.kuleuven.ac.be

Imperfect Competition and Congestion in a City with asymmetric subcenters

Andre de Palma, Fay Dunkerley, Stef Proost[1]

August 2004

Abstract

This paper develops a model for the monopolistic competition of subcenters for the shoppers and workers of a central city. The model is an extension of the de Palma & Proost (2004) model that is limited to the symmetric case. Inhabitants of a CBD can choose one of the subcenters to buy a differentiated product and choose one of the subcenters to supply differentiated labour. The subcenters compete in prices and wages and the access to the subcenters can be congested. The short term and free entry equilibria are studied. As general properties are rare in the non-symmetrical monopolistic competition case, this paper draws more on numerical examples than on hard theorems. Starting from a symmetric base case, the paper explores the effects on welfare and number of subcenters of introducing diversity in the distances to the subcenter, quality of the subcenters, congestion and attractiveness of the subcenter as workplace. The paper shows cases where asymmetry can increase welfare and where the order in which firms enter the market matters for the equilibrium outcome.

JEL codes: L13, D43, R41, R13,

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[1] Andre de Palma (Institut Universitaire de France, THEMA, Univ de Cergy Pontoise, CORE), Fay Dunkerley (CES-KULeuven), Stef Proost (CES-KULeuven and CORE) .We thank P.Van Cayseele for comments on an earlier version of this paper.

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1. Introduction

Congestion on a Friday night or Saturday morning on your way to the shopping mall is a well-known problem. Surprisingly, it has not been on the research agenda for many economists. Fujita and Thisse (2002) looked into the economics of shopping malls. Shopping malls reduce search costs for the customers but also reduce the profitability of the firms located in the subcenter if they offer products that are easily substitutable. This explains the presence of very different shops in one shopping mall. We are interested in the competition between shopping malls and in the effects of congestion on their profitability and ultimately on the number of shopping malls. In fact we are interested in subcenters that may be shopping malls or sell any other product. Important is that the product they offer is diversified and that customers are all located in a center and can choose what subcenter to go to for their shopping. There will be different roads to each of the subcenters so that we can study the role of congestion on the competition between subcenters and on the number of subcenters.

De Palma and Proost (2004) developed a monopolistic competition model for a city with subcenters where the city inhabitants can shop and work in the subcenters. They focussed on symmetric Nash equilibria and looked into the existence and properties of these equilibria. The real world is very often non symmetric and this paper studies non-symmetric equilibria. As general properties are rare in the non-symmetrical monopolistic competition case, this paper draws more on numerical examples than on hard theorems.

Section 2 describes the model. Section 3 describes briefly the numerical solution algorithm as non-symmetrical monopolistic competition offers non trivial computation problems. Section 4 presents one transparent numerical base case without congestion. This symmetric base case with 5 subcenters will be the starting point for exploring the effect of diversity in parameters as there are distances to the subcenter, quality of the subcenters and attractiveness of the subcenter as workplace. Section 5 discusses the welfare implications of the free entry equilibrium. We show in this section that asymmetry can sometimes increase welfare. We also show that the order in which firms enter the market matters for the equilibrium outcome. We leave the most difficult part, the role of congestion, to section 6. Congestion is introduced via the bottleneck model and we show how congestion interacts with quality differentials and how this affects welfare. Results appear much less generalisable. Section 7 concludes and offers some ideas for further research in this area.

2. The Model

2.1. Model setting

De Palma and Proost (2004) have developed a model to study imperfect competition in a city both with and without congestion. Although they concentrate their analysis on the symmetric situation, the basic model set-up also applies in the more general asymmetric case. A brief description of the model setting is therefore presented here together with the relevant equations for household preferences and firms' profits. More details can of course be found in the original paper. From this starting point, we derive the asymmetric equilibrium solution: the no-congestion case being considered here, while the effects of congestion are discussed briefly.

Residents live in a city centre and travel to sub-centres to work and shop. Shopping and working decisions are made independently, so that trip chaining is excluded, and residents can only travel between the centre and each subcentre and not between subcentres (Figure 2-1). A homogeneous good is produced in the city centre and used as an intermediate input for the differentiated good, which is produced in the sub-centres. Thus, both firms and consumers incur travel costs. In this general equilibrium setting, the numeraire homogeneous good represents all production in the economy other than the differentiated good and all profits are returned to the households. The labour market is also considered separately and jobs in the differentiated industry are heterogeneous. Only one differentiated product variant is produced at each sub-centre by a single firm and each household will consume one unit of differentiated good and supply one unit of labour for its production. Hence, in the current formulation, demand for the differentiated good is inelastic and, if the labour market is assumed to be fully flexible, the product and labour markets will clear. All remaining labour and income is devoted to the homogeneous good and there is therefore no possibility of non-consumption or unemployment. Each sub-centre requires some road infrastructure, which is paid for by a levy on firms and head-tax on consumers.

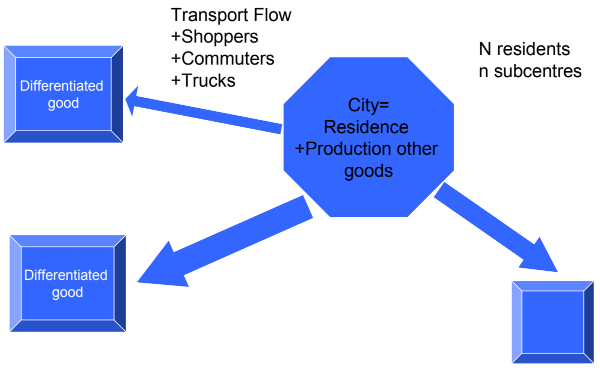

Figure 2-1 schematic of city layout

2.2. Household preferences

As households make discrete choices, an indirect conditional utility function can be used to express their preferences. In this case the utility function represents the preferences of a household that buys differentiated good *k* and supplies labour to sub-centre *i*:

$$U_{ik} = \tilde{h}_k - p_k - \alpha^d t_k + w_i - \tilde{\beta}_i - \alpha^w t_i + \theta(1-\beta) + \frac{1}{N}\sum_l \pi_l - T \tag{1}$$

There are N households, each of which is paid a wage, w_i, for working at sub-centre *i* and buys one unit of variant *k* at price, p_k. Both these variables will be determined by the model. In the following we will use household and consumer interchangeably as it is easier to consider the household as a single worker or shopper. Thus, the consumer's commuting and shopping travel costs, which are exogenous when there is no congestion, are given by $\alpha^w t_i$ and $\alpha^d t_i$ respectively. The remaining terms in (1) represent his utility from production of the homogeneous good, share of the profits and a head-tax, *T*. These are the same for all consumers.

The utility of consumption of differentiated product variant *k* is given by an intrinsic quality component h_k and a stochastic component $\mu^d \varepsilon_k$:

$$\tilde{h}_k = h_k + \mu^d \varepsilon_k \tag{2}$$

and the disutility of labour at sub-centre *i* is similarly given by the following two components:

$$\tilde{\beta}_i = \beta_i - \mu^w \varepsilon_i \tag{3}$$

Hence, all households will value the quality of the product variant manufactured at a particular subcentre in the same way and will experience the same disinclination to work at a given subcentre; in both cases possibly assigning different values to different subcentres. However, the households will still vary in their tastes: the parameters ε_i and ε_k represent the intrinsic heterogeneity of consumer tastes and it is again assumed that they are double exponentially distributed. The parameters μ^w and μ^d determine the degree of heterogeneity of preferences.

When a household chooses where to work, this is independent of its shopping decision and vice versa. Substituting from (2) and (3) in (1), we obtain $U_{i|k} = \Omega_k + w_i - \beta_i - \alpha^w t_i + \mu^w \varepsilon_i$, where $\Omega_k = \theta(1-\beta) + \frac{1}{N}\sum_l \pi_l - T + h_k - p_k - \alpha^d t_k + \mu^d \varepsilon_k$ is assumed fixed for the choice of employment location. The probability that a consumer chooses to commute to sub-centre *i* of the *n* possible sub-centres is then $P_i^w = \Pr ob\{U_{i|k} \geq U_{j|k} \ \forall j = 1,...,n\}$, independent of *k* and can be written as a logit type probability

$$P_i^w = \frac{\exp\left(\dfrac{w_i - \beta_i - \alpha^w t_i}{\mu^w}\right)}{\sum_j \exp\left(\dfrac{w_j - \beta_j - \alpha^w t_j}{\mu^w}\right)}. \tag{4}$$

For the household choice of shopping location, we obtain $U_{k|i} = \Omega_i + h_k - p_k - \alpha^d t_k + \mu^d \varepsilon_k$, where $\Omega_i = \theta(1-\beta) + \frac{1}{N}\sum_l \pi_l - T + w_i - \beta_i - \alpha^w t_i + \mu^w \varepsilon_i$ is assumed constant for shopping decisions, and a similar expression for the probability is derived:

$$P_k^d = \frac{\exp\left(\dfrac{h_k - p_k - \alpha^d t_k}{\mu^d}\right)}{\sum_j \exp\left(\dfrac{h_j - p_j - \alpha^d t_j}{\mu^d}\right)} \tag{5}$$

Using the assumptions of inelastic demand for the differentiated good and fixed labour input for the differentiated good, a market clearing condition also applies at each sub-centre:

$$P_i^w = P_i^d \tag{6}$$

2.3. Profits of firms

There are n firms, each located at one of the subcentres. The profit of firm *i* is:

$$\pi_i(w,p) = (p_i - w_i - c^1 - \alpha^h t_i)D_i - (F_i + S_i) \tag{7}$$

where $c^1 + \alpha^h t_i$ is the marginal cost of the intermediate input, F_i is the fixed production cost and S_i is the government levy to pay for public infrastructure. The inelastic demand condition gives us $\sum_i^n D_i = N$ and from (7), we obtain demand $D_i = NP_i^w = NP_i^d$.

Each firm selects prices and wages to maximise his profits, given that his competitors do the same. Thus we look for a non-cooperative Nash equilibrium in these variables.

2.4. Equilibrium

The strategic variables of firm *i* are w_i and p_i. From the market clearing condition (6), substituting from (4) and(5), it is clear that the choice of w_i determines the choice of p_i (and vice versa), since all other prices and wages are taken as given. Thus, we can rewrite the profit condition (7) as:

$$\pi_i(w_i) = (p_i[w_i] - w_i - c^1 - \alpha^h t_i)NP^w{}_i[w_i] - (F_i + S_i) \tag{8}$$

Taking w_i as the only strategic variable, the best response of firm *i* is given by:

$$\frac{d\pi_i}{dw_i} = \left[\left(\frac{dp_i}{dw_i} - 1\right) + \left(p_i - w_i - c^1 - \alpha^h t_i\right)\left(\frac{1-P_i^w}{\mu^w}\right)\right]NP_i^w = 0 \tag{9}$$

where

$$\frac{dp_i}{dw_i} = \frac{dP_i^w}{dw_i} \bigg/ \frac{dP_i^d}{dp_i} = -\frac{\mu^d}{\mu^w}\frac{P_i^w(1-P_i^w)}{P_i^d(1-P_i^d)} = -\frac{\mu^d}{\mu^w} \tag{10}$$

for both symmetric and asymmetric conditions, using (6). Substituting from (10) in (9) and rearranging leads to a candidate equilibrium

$$p_i^e = w_i^e + c^1 + \alpha^h t_i + \frac{(\mu^w + \mu^d)}{(1-P_i^w)};\ \forall i = 1,\dots,n \tag{11}$$

Clearly (11) is not soluble analytically for the general asymmetric case in which P_i^w is given by (4). A numerical approach is required. The uniqueness of (11) is discussed in de Palma and Proost (2004).

In addition to effects on price, profit and market share, we are interested in the welfare implications of the asymmetric model. For the symmetric case, welfare per household can be calculated from

$$W(n) = \Xi - \frac{n}{N}(F+K) + (\mu^d + \mu^w)\log(n) \tag{12}$$

where $\Xi = -\beta + \theta(1-\beta) + h - c^1 - (\alpha^h + \alpha^d + \alpha^w)t$ and marginal cost pricing is assumed. The asymmetric version is derived later in Section 5.1.

3. Numerical Solution

3.1. Solution method

For the numerical solution we have to calculate both p_i and w_i. Thus we have *2n* unknowns and only *n* equations from the price equilibrium (11). However, the market clearing conditions, (6), provide *n* further relations between p_i and w_i. Unfortunately, these do not allow us to fully specify the problem, as, for the logit model, the *nth* relation, $P_n^w = P_n^d$, can be determined from the other *n-1* market clearing conditions; we then have *2n* unknowns and *2n-1* equations.

The uniqueness condition specifies that $p_i - w_i$ is determined uniquely from (11) for all *i*. For the symmetric case, it's clear that specifying *w* fixes *p*, although there are an infinite number of *w*, with corresponding *p*, which satisfy this condition. For the asymmetric case, it can also be shown that by adding some constant γ to all p_i and w_i, (11) is still satisfied and the probabilities remain unchanged for all firms. Thus, fixing the wage (or one price) of one firm leaves P_i^w and P_i^d unchanged but allows us to uniquely specify p_i and w_i for all *i*. Without loss of generality, we therefore fix the *nth* wage. To then determine p_i and w_i we use the Newton-Raphson approach, solving (11) and (6) as one system.

3.2. Limiting case

Testing the asymmetric model for the product and labour markets is not straightforward as there is no analytical relation between the equilibrium and optimum situations and we have no suitable empirical data at hand. We can of course easily test the model in the symmetric limit.

A further test is to consider the limit $\mu^w \to 0$. This effectively eliminates the labour market from the current model and reduces it to a single product market model, equivalent to the one presented in Anderson and de Palma (2001)[2].

4. Numerical Examples

4.1. Symmetric base-case parameters

It is clear that the model requires a significant amount of input information. Although the goal of the project is to gather data on realistic working and commuting situations, here, we limit our ambitions to a simple, stylised, symmetric example, based on an economy of one day. It is then possible to consider the effect of increasing consumer taste heterogeneity and of introducing asymmetries in travel time, product quality and disutility of labour in this economy.

We first assume that there are one million households in the city. Each household supplies eight hours of labour, of which one hour is spent on the differentiated good. The wage they earn for

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[2] The reader is referred to Dunkerley (2004) for details.

producing the non-differentiated good is arbitrarily set to one. They also make one commuting trip and one shopping trip per day, giving a total transport time of half an hour. The disutility of differentiated labour, β_i, for households is 0.2, reflecting a relatively high inclination to work. (Households would choose not to work if $\beta_i = 1$). The utility of consumption of the differentiated good, *h*, is 5. A sufficiently large value is chosen to ensure that consumers will buy the differentiated good. Further, truck deliveries are such that each truck contains sufficient intermediate good to produce 50 units of the differentiated good. One unit of the differentiated good requires an intermediate input that can be produced using 0.1 units of homogeneous labour. Finally, the total fixed costs faced by each firm represent some 45% of the total labour costs for the differentiated goods and that this fixed cost consists of 50% of public infrastructure and 50% of private infrastructure. In this simple case there is no head-tax. The model inputs for the symmetric case are summarised in Table 4-1.

Table 4-1 Input parameters for symmetric base case

Parameter	Definition	Value	Units
$^*\theta$	Total labour time devoted to production of homogeneous good + transport	7.5	hours of labour per household
α^{ω}	No of commuting trips per unit of labour	1	Commuting trips per household
β_i	Disutility of labour in sub centre i	0.2	disutility
h_k	Utility of consumption per unit of differentiated good k	5	utility
α^d	No of shopping trips per unit of consumption	1	Shopping trips per household
α^h	No of freight trips per unit of production	0.02	
c^1	Intermediate input for differentiated good	0.1	homogeneous good per unit of differentiated good
μ^{ω}	Scale parameter for employment heterogeneity	0.2	-
μ^d	Scale parameter for consumption heterogeneity	0.2	-
t_i	Travel time to subcentre i	0.25	hours per trip
n	No of firms	5	
N	No of households	1,000,000	
*F	Fixed set-up cost for production of differentiated good at subcentre	75,000	Units of homogeneous good
*K	Fixed cost of road infrastructure for each subcentre	75,000	Units of homogeneous good
*T	Head tax per household	0	
*S	Fixed levy per firm	75,000	Units of homogeneous good

*not required for equilibrium calculation but for profit, welfare, etc

Assuming there are five firms in the market, we can calculate the short-term price and wage equilibrium, for an arbitrary wage of 1.0. These data are presented in Table 4-2. Profits are negative due to the fixed costs and we therefore consider gross profits only in the analysis throughout this section. As explained in Section 3.1, fixing the wage of one firm is necessary in order to calculate the price and wage equilibrium numerically. For the symmetric case, this means fixing the wage of all firms.

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Table 4-2 Symmetric base-case short-term equilibrium

Price	Wage	Profit	Gross profit	Market share (%)	Welfare
1.61	1.0	-0.05	0.10	20	10.089

This number of firms will clearly not be sustainable in the long-term because of the fixed costs and in fact only 3 firms are found in the free-entry equilibrium, while 2 firms are socially optimal. These numbers can be obtained from a symmetric version of the zero profit condition and by maximising the welfare equation (12) respectively[3]. Indeed it is shown that at most one more firm can be present in the free-entry equilibrium compared with the social optimum.

In this study we wish to investigate the effect on the short-term equilibrium of varying parameters from the symmetric base-case values. Firstly, we simply increase the degree of consumer taste heterogeneity via parameters μ^w and μ^d, keeping everything else unchanged. The results, shown in Table 4-3 for μ^w, are in fact identical for the two parameters as they have identical, independent distributions.

Table 4-3 Results for short-term equilibrium of increasing μ^w

μ^w	Price	Wage	Gross profit	Welfare
0.2	1,61	1,00	0,10	10.09
0.4	1,86	1,00	0,15	10.41
0.8	2,36	1,00	0,25	11.05
1.5	3,23	1,00	0,43	12.18
3	5,11	1,00	0,80	14.60
5	7,61	1,00	1,30	17.81

The solution remains symmetric, market share does not change and profits are affected by changes in prices, which increase with μ^w and μ^d. As the consumer tastes become more polarised, they have less substitution possibilities for consumption (or labour) choices. Each firm then has less need to compete with the other firms in the market and can charge a higher price for its product variant. Welfare is also increasing because consumers obtain greater utility from their product variant of choice.

4.2. Non-symmetric case – variation in a single parameter

In this section we consider the impact on the short-term equilibrium of varying the product quality, disutility of labour, and journey time between firms: parameters h_i, β_i and t_i respectively. While we initially look at the effect of varying these parameters independently, in a realistic setting it is likely that there will be asymmetries in all of them. A 'ranking' parameter $h_i - \beta_i - (\alpha^h + \alpha^d + \alpha^w)t_i$ is therefore introduced, which roughly represents the benefits accruing to households that choose to shop or work at subcentre *i*. We can then investigate the relationship between the ranking parameter (denoted rank) and (gross) profit and market share

[3] The reader is referred to de Palma and Proost for the calculation methods.

for different combinations of h_i, β_i and t_i. All other parameters from Table 4-1 are held constant in each case.

Although, the comparative statics exercise can be performed in a number of ways, we have chosen to present results for scenarios in which all the firms exhibit some asymmetry, except firm 3, which is the control and always takes the symmetric case value. This approach can be used to look at the effect of simple increases or decreases in a parameter from the symmetric case but also provides additional interesting information on the effect of the distribution of the asymmetries.

Table 4-4 to Table 4-6 show the effect on the short-term equilibrium variables of varying h_i, β_i and t_i individually, by up to $\pm 20\%$ of the symmetric base-case values. For example, in Table 4-4, the product quality of firm 1 (h_1) is 20% less than that of firm 3 (h_3), the product quality of firm 2 (h_2) is 10% less than h_3, the product quality of firm 4 is 10% more than h_3 and the product quality of firm 5 is 20% more than h_3. Thus the asymmetries are distributed symmetrically around the base-case value. The entries for parameters β_i and t_i in Table 4-5 and Table 4-6 are obtained in an identical manner.

Table 4-4 Results for short-term equilibrium with asymmetry in h

Firm	h	β	t	Price	Wage	Gross profit	Market share	Rank
1	4,0	0,2	0,25	0,73	0,22	0,004	1,09	3,20
2	4,5	0,2	0,25	0,99	0,47	0,02	3,71	3,70
3	5	0,2	0,25	1,26	0,70	0,05	11,78	4,20
4	5,5	0,2	0,25	1,57	0,89	0,17	30,38	4,70
5	6,0	0,2	0,25	1,96	1,00	0,45	53,03	5,20

Table 4-5 Results for short-term equilibrium with asymmetry in β

Firm	h	β	t	Price	Wage	Gross profit	Market share	Rank
1	5	0,16	0,25	1,57	0,95	0,11	21,54	4,24
2	5	0,18	0,25	1,57	0,96	0,10	20,75	4,22
3	5	0,2	0,25	1,58	0,98	0,10	19,98	4,20
4	5	0,22	0,25	1,59	0,99	0,10	19,23	4,18
5	5	0,24	0,25	1,60	1,00	0,09	18,50	4,16

Table 4-6 Results for short-term equilibrium with asymmetry in t

Firm	h	β	t	Price	Wage	Gross profit	Market share	Rank
1	5	0,2	0,2	1,61	0,98	0,13	23,94	4,30
2	5	0,2	0,225	1,60	0,98	0,11	21,86	4,25
3	5	0,2	0,25	1,59	0,99	0,10	19,88	4,20
4	5	0,2	0,275	1,59	1,00	0,09	18,03	4,14
5	5	0,2	0,3	1,58	1,00	0,08	16,29	4,09

As discussed in Section 3.1, the wage of the fifth firm is fixed at the symmetric case value ($w_5 = 1$).We concentrate our analysis on the gross profit, since identical fixed costs are imposed for all firms. The fixed costs will of course affect the number of firms that can survive in the market in the long-term. This is discussed later.

From Table 4-4 it can be seen that the firm with the highest quality product has the largest profit and the greatest market share. It also has the largest differential between price and wage. Thus, a firm can charge a higher price because its product is more desirable and, because of its larger market share, it has to pay a higher wage than its competitors in order to attract workers. On the other hand, Table 4-5 shows that when households are less inclined to work for a particular firm (high disutility of labour), the firm has lower profits and a lower market share. It must pay a higher wage than its competitors to attract workers. Finally, it can be seen from Table 4-6 that the firm located closest to the city centre (shortest travel time) enjoys the largest profit and market share. The firm can not only charge the highest price but can also pay its workers less, as the reduced commuting time makes it the most desirable work place and shopping centre.

It should also be noted that a change in product quality has a more significant effect on the short-term equilibrium than comparable changes in disutility of labour and travel time. This is due to the larger change in absolute value of h_i, which therefore has a larger impact on the rank. Further, for all input parameters, the equilibrium values for gross profit, price and market share are not symmetric about the base-case. This can be explained by the formulation of the logit probabilities ((4),(5)). For $n \geq 3$ firms, perturbations to the probability about the symmetric value weight positive changes more than negative ones, since the probability is determined by the double exponential distribution.

As an illustrative example, gross profit and market share are also presented in Figure 4-1 as a function of rank for the data in Table 4-4 and additional scenarios, in which larger asymmetries are introduced. In each figure the $\pm 50\%$ case is calculated in the same way as the $\pm 20\%$ scenario but with 50% or 25% increases or reductions in the parameter value over the symmetric base-case (firm 3) value. The +100% case corresponds to 50% and 100% increases over the base-case for firms 4 and 5, while firms 1 and 2 have the same values as in the $\pm 50\%$ case. Figure 4-1 clearly shows that, for a given scenario, both gross profit and market share are increasing functions of rank. The rank reflects a positive dependence on h_i as borne out by the tabular results.

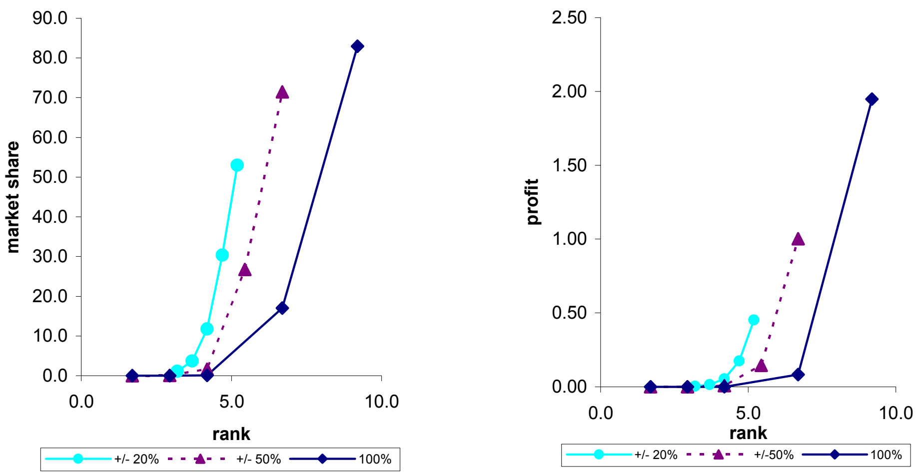

Figure 4-1 Market share and profit versus rank for asymmetries in h

4.3. Non-symmetric case – variations in multiple parameters

It is also interesting to consider the effect of a combination of asymmetries in h_i, β_i and t_i on the short-term equilibrium. The results for this scenario are presented in Table 4-7 and the corresponding graphs of gross profit and market share are presented in Figure 4-2. Only a $\pm 2\%$ change in h_i was adopted as this parameter otherwise has a dominant effect on the equilibrium variables. It is clear that the most profitable firm still has the largest market share and that these variables increase with rank[4]. The effect of the interaction of β_i, h_i and t_i on prices and wages is less apparent but clearly reflects the trade off between these parameters.

Table 4-7 Results for short-term equilibrium with asymmetry in h, β and t

Firm	h	β	t	Price	Wage	Gross profit	Market share	Rank
1	5	0,18	0,2	1,72	1,08	0,13	24,65	4,32
2	4,9	0,22	0,25	1,66	1,08	0,07	15,57	4,08
3	5,2	0,24	0,275	1,85	1,21	0,13	24,16	4,30
4	5,1	0,2	0,3	1,76	1,16	0,10	19,72	4,19
5	4,8	0,16	0,225	1,58	1,00	0,08	15,91	4,09

[4] A simple, approximate linear relationship between profit and rank can be derived

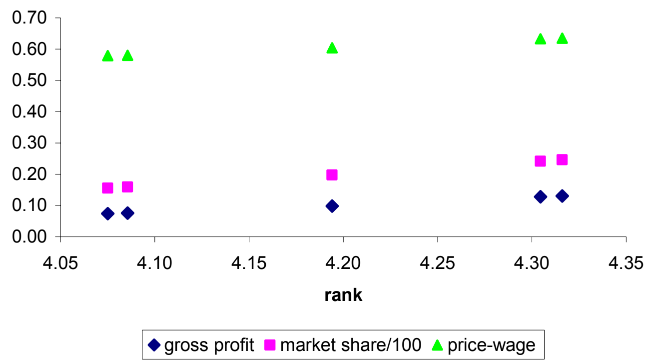

Figure 4-2 Short-term equilibrium results for asymmetries in h, β and t

We could also look at the effect of differences in fixed costs between firms. As noted earlier, these do not affect the short-term equilibrium calculation but change the magnitude of the profits by a non constant amount, which has consequences for the long-run equilibrium. For simplicity we do not consider asymmetries in the fixed costs further here.

5. Welfare Analysis

5.1. Theory

The symmetric welfare function (12) is derived from $W = \max E\left[U_{ik}\right]$ since profits are equally distributed among households (Anderson et al 1992). We can also determine a corresponding expression for welfare in the asymmetric case from the same starting point. Using the definition of utility (1) and substituting the random variables from (2) and (3) we obtain

$$U_{ik} = h_k - p_k - \alpha^d t_k + \mu^d \varepsilon_k + w_i - \beta_i - \alpha^w t_i - \mu^w \varepsilon_i + \theta(1-\beta) + \frac{1}{N}\sum_l \pi_l - T \qquad (13)$$

Then, because of the independence of the labour and consumption decisions in (13), we can write

$$W = \Psi + \max_i E\left[w_i - \beta_i - \alpha^w t_i + \mu^w \varepsilon_i\right] + \max_k E\left[h_k - p_k - \alpha^d t_k + \mu^d \varepsilon_k\right] \qquad (14)$$

where $\Psi = \theta(1-\beta) - \frac{1}{N}\sum_j (F_j + K_j)$ is constant. Since the ε_i are double exponentially distributed with zero mean, the distribution of the maximisation term for utility of labour (2nd term in(14)), has the form $H(x) = \prod_{i=1}^n G(x - (w_i - \beta_i - \alpha^w t_i))$, where *G* is the double exponential distribution. Then, by definition

$$E_i = \max_i E\left[w_i - \beta_i - \alpha^w t_i + \mu^w \varepsilon_i\right] = \int_{-\infty}^{\infty} x h(x) dx \qquad (15)$$

where *h(x)* is the density function corresponding to *H*. By means of Laplace transformation and the fact that the ε_i distribution has zero mean, we obtain

$$E_i = \mu^w \ln\left[\sum_j \exp\left(\frac{(w_j - \beta_j - \alpha^w t_j)}{\mu^w}\right)\right] \quad (16)$$

A similar expression to (16) can be derived in an identical manner for the utility of consumption (3^{rd} term in (14)) and combining these leads to the welfare formulation for the one day economy

$$W = \Psi + \mu^w \ln\left[\sum_j \exp\left(\frac{(w_j - \beta_j - \alpha^w t_j)}{\mu^w}\right)\right] + \mu^d \ln\left[\sum_j \exp\left(\frac{h_j - c_j - w_j - \alpha^d t_j}{\mu^d}\right)\right] \quad (17)$$

where $c_j = c^1 + \alpha^h t_j$ and marginal cost pricing is assumed.

We do not lose any generality by having w_i present in the formula for *W* since we could replace w_i by $w_i + \gamma$, where γ is the same constant for all *i*. Then because of the properties of the exponential and logarithm functions, the γ terms cancel. Also marginal cost pricing ensures that the relationship between the p_i and w_i does not change as firms are added, since $c^1 + \alpha^h t_i$ does not change.

5.2. Short-term equilibrium

It is interesting to compare the welfare in asymmetric short-term equilibria with the symmetric case. In general, an asymmetry in one parameter that leads to an overall increase (decrease) in utility also results in an increase (decrease) in welfare, as might be expected. The situation becomes more complicated when firms exhibit different combinations of asymmetries. This can be seen from the welfare calculations for each of the scenarios in Sections 4.2 and 4.3, which are presented in Table 5-1 below.

Table 5-1 Welfare corresponding to short-term equilibria

Asymmetry	-	h	β	t	h, β, t
Welfare	10.089	10.606	10.090	10.095	10.103

Introducing asymmetries of $\pm 20\%$, distributed evenly around the base-case value, in fact leads to an increase in welfare compared with the symmetric case, so the gains of households with increased utility more than offset the losses of those with decreased utility. This follows from the analysis of Section 4.2 regarding the formulation of the logit probabilities. Generalising to other scenarios, we find that this also holds true for the $\pm 50\%$ case but, for example, when a large increase in disutility of labour for some firm is not compensated for by an equal reduction in disutility for another firm, welfare decreases.

Although asymmetries in h_i, β_i and t_i can increase welfare, as shown in Table 5-1, some combinations of asymmetries will also reduce welfare relative to the symmetric case. Hence, it is difficult to discern a general trend in this case. Our results are interesting, however, and may potentially be important for a policy maker, who then may wish to encourage (or allow) certain asymmetries in location, quality etc. for welfare gains.

5.3. Long-term equilibrium

For the free-entry, long-term equilibrium, we require that the profit of the last firm to enter the market should not be less than zero. The results from the previous section indicate that the firm with the highest rank will always be the most profitable. Thus, we can find a long-term equilibrium, when market conditions are such that the highest ranked firms are able to enter the market first. We can then sort the firms by rank and, allowing firms to enter the market in decreasing rank order, calculate the short–term equilibrium profit obtained by each firm, until the profit of the m^{th} firm entering the market is negative. This gives us *m-1* firms in the free-entry equilibrium[5]. Clearly this system only works if the fixed costs are the same for all firms or, at least, the fixed costs result in profits which have the same rank order as the gross profits. We do not consider this aspect further here. Using the example presented in Table 4-7 (scenario w1 in Table 5-2 below) we find that there will be 3 firms in the long-run equilibrium.

For the symmetric case, the socially optimal number of firms in the market can be derived from the welfare formulation, as this is an analytical function of the number of firms. For the asymmetric case, using (17), we can calculate the welfare generated as each new firm is added to the market in the long-run equilibrium calculation (again in rank order). If welfare decreases when the p^{th} firm enters the market, then *p-1* firms are socially optimal. For scenario w1, welfare decreases as each additional firm enters the market, suggesting that 2 firms, the minimum possible in the market, would be socially optimal.

In the above example, the number of firms in the free-entry equilibrium and in the social optimum turns out to be the same as in the symmetric case (3 and 2 respectively). However, taking another set of firms with different parameters (h_i, β_i and t_i) as, for example, scenario w2 in Table 5-2, yields a different solution. In this case there are only 2 firms in both the long-run equilibrium and social optimum. Moreover, the firms themselves are different. For scenario w1, the rank indicates that firms 1, 3 and 4 would be present in the long-run equilibrium, whereas scenario 2 has firms 4 and 5 only. This is not to say that the 'wrong' firms will be present in the equilibrium but, rather, that different firms will be present in the long-run equilibrium depending on the range of values of h_i, β_i and t_i for a given set of firms competing in the marketplace. In each case, the rank correctly identifies the most profitable firms.

Table 5-2 Results for long-term equilibrium with asymmetry in h, β and t

firm	Scenario w1						Scenario w2					
	h	β	t	rank	Δprofit of m^{th} firm	Welfare of *m* firms	h	β	t	rank	Δprofit of m^{th} firm	Welfare of *m* firms
1	5	0,18	0,2	4,32			4	0.24	0.2	3.26	<0	-
2	4,9	0,22	0,25	4,08	<0	-	4.5	0.22	0.225	3.73	<0	-
3	5,2	0,24	0,275	4,30	0.25	10.288	5	0.2	0.25	4.20	-0.09	10.844
4	5,1	0,2	0,3	4,19	0.02	10.265	5.5	0.18	0.275	4.66	0.12	10.964
5	4,8	0,16	0,225	4,09	-0.05	10.191	6	0.16	0.3	5.13		

[5] In each case, the last firm entering the market has its wage set equal to one.

The free-entry equilibrium described above for scenario w1 is not necessarily the only equilibrium for this set of parameter values. If we now consider that there are barriers to entry, we can construct examples that have different firms present in the long-term equilibrium and social optimum. Firstly, for scenario w3, we assume that for some reason firm 2 is already established in the market and then allow the remaining firms to enter in rank order. In scenario w4 we assume that firms 2 and 5 are already present and then allow the other firms to again enter according to rank. The results are shown in Table 5-3 below.

Table 5-3 Examples of other possible long-run equilibria

Firm	h	β	t	Scenario w3			Scenario w4		
				Entry order (*m*)	Δprofit of m^{th} firm	Welfare of *m* firms	Entry order (*m*)	Δprofit of m^{th} firm	Welfare of *m* firms
1	5	0,18	0,2	2	0,34	10,198	3	0,13	10,1682
2	4,9	0,22	0,25	1			1		
3	5,2	0,24	0,275	3	0,08	10,237	4	0,02	10,1675
4	5,1	0,2	0,3	4	-0,03	10,189	5	-0,05	10,1026
5	4,8	0,16	0,225	5	<0	-	2	0,25	10,0576

In scenario w3, three firms (firms 1, 2 and 3) are able to exist in the long-term equilibrium and these are also socially optimal, since welfare only starts to decrease when firm 4 is added. In scenario w4, four firms are present in the long-term equilibrium but only three of these (firms 1, 2 and 5) are socially desirable. Clearly over-entry is possible, even when the most profitable firms enter the market first. Identifying limits to over-entry and possible under-entry is not obvious from the welfare and zero profit equations, (17) and (8). Some further work is needed in this area.

6. The Model With Congestion

6.1. Model equations

The main difference in the model, when congestion is taken into account, is that travel times become endogenous. Instead of being constant, travel times increase with the number of road users, where the road users are shoppers, commuters and trucks delivering the intermediate input. de Palma and Proost assume that roads have a fixed capacity and that a bottleneck develops if the activity on a road exceeds its capacity. They use the bottleneck model developed by Arnott et al (1993), where road users choose their trip timing (with no congestion pricing). Generalising this we can define the endogenous travel time for the asymmetric model as $t_i = t_i^o + \delta \frac{N}{s_i} \alpha P_i^w$ where $\alpha = \alpha^d + \alpha^w + \kappa \alpha^h$ [6] and κ ensures that one truck trip has the same congestion effect as 2 shopping or commuting trips. In the absence of congestion t_i^o is the transport time from the centre to sub-centre *i* and s_i is the corresponding road capacity. The coefficient δ translates waiting time and schedule delays into equivalent costs.

[6] The $\alpha^x;\ x = d, w, h$ are defined in Section 4.1.

Setting s=750,000 car equivalents per hour for each subcentre, means an increase in travel time of approximately 8 minutes per 15 minute journey (i.e. a 50% increase), when there are 5 firms. The δ value is taken from the literature (Arnott et al 1993) and means that queuing and schedule delay costs are of the order of 25% of the wage.

6.2. Welfare analysis

The welfare equation in the presence of congestion can be obtained from (17) by substituting the congested travel time. It is given by

$$\begin{aligned} W = \Psi + \mu^w \ln\left[\sum_j \exp\left(\frac{(w_j - \beta_j - \alpha^w t^o{}_j - \Lambda^w P_j^w)}{\mu^w}\right)\right] \\ + \mu^d \ln\left[\sum_k \exp\left(\frac{h_k - w_k - c^1 - \alpha^h t^o{}_k - \alpha^d t^o{}_k - \Lambda^h P_k^d - \Lambda^d P_k^d}{\mu^d}\right)\right] \end{aligned} \quad (18)$$

Table 6-1 contains the welfare for $\pm 20\%$ asymmetries in β_i, h_i and t_i^o, as defined in Section 4.2, but now with fixed congestion, which is introduced via road capacity s_i.We also consider a similar perturbation in s_i.

Table 6-1 Welfare results for short-term equilibria with congestion

asymmetry	-	h	β	t	s
welfare	9.822	10.172	9.823	9.829	9.820

Comparing Table 6-1 with Table 5-1 indicates that congestion reduces welfare. The presence of congestion affects welfare in three ways. There are time costs (schedule delay costs), since the traffic is not able to travel at the free-flow speed ($t^o{}_i$) even if perfect congestion pricing can be imposed. In addition, if congestion is imperfectly priced, there are queuing costs. Congestion may also lead to over-entry in the longer term, since firms are able to make overly large profits. In the short-term this cost does not play a role as *n* is fixed. Table 6-1 and Table 5-1 are also consistent, in that $\pm 20\%$ changes in β_i, h_i and t_i^o increase welfare with respect to the symmetric case. However, the asymmetries in s_i reduce welfare. This is due to the inverse relation between s_i and congestion and the nature of the probability distribution, explained earlier. Performing the same exercise with $\pm 20\%$ changes in $1/s_i$ results in a welfare increase. The asymmetries in road capacity generate only modest changes from the symmetric case because their effect on welfare is determined by the magnitude of $\Lambda_i{}^w P_i^w$ relative to β_i (or $\Lambda_k{}^d P_k^d$ relative to h_k)and, for the values used in our study, $\Lambda_i{}^w P_i^w \approx O(0.1)$.

We next consider welfare changes from the symmetric case result when both β_i and s_i are varied. Welfare changes (dW) are presented in Figure 6-1 for a number of scenarios which are summarised in Table 6-2. In this case we see that their magnitude and sign depend on the relative magnitude of the asymmetries and whether they interact positively or negatively. In general, the effect of congestion dominates that of disutility of labour and welfare decreases, although this is not necessarily the case when the asymmetries are randomly paired, as would probably occur in a realistic setting.

Table 6-2 Summary of scenarios for asymmetries in both β and s

	s1	s2	s3	s4	s5
β	±50%	±50%	±50%	±50% reverse	±50% random order
s	±20%	±50%	+100%	+100%	±20%

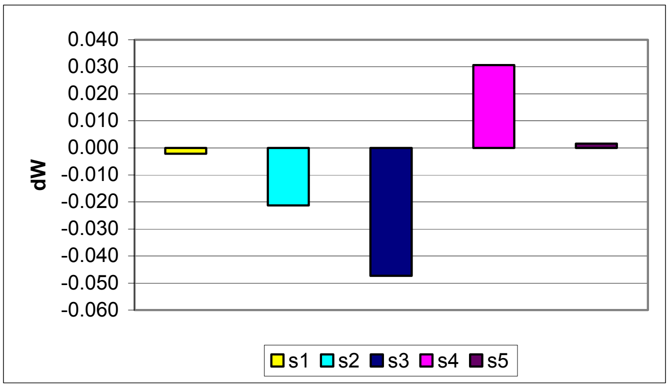

Figure 6-1 Differences in welfare from symmetric case for asymmetries in βand s

A similar analysis with h_i and s_i varying, indicates that, for the same magnitude of asymmetry, product quality effects dominate congestion effects and act to increase welfare but again, this does not generalise to the case, when pairings of asymmetries are randomised. It appears that the welfare effects of a particular set of asymmetric firms in the market will depend on the nature of the asymmetries. From the point of view of the policy planner, he may wish to encourage certain combinations of asymmetries in subcentre location, road capacity, product quality and amenities to attract workers, in order to increase social welfare.

7. Conclusions

In this study a model was developed to numerically calculate the short-term Nash equilibrium when there is product differentiation in an asymmetric oligopolistic market. The model was applied to the problem of shoppers and workers living in a city centre and commuting to subcentres. Asymmetries could occur in both product and labour markets via the parameters product quality, disutility of labour and travel times. Travel times were first assumed to be exogenous. It was then found that introducing asymmetries in the different parameters resulted in price equilibria that followed our economic intuition; a firm with higher product quality had larger profits and market share, for example. When combinations of asymmetries in product quality, disutility of labour and travel times were tested, a trade off between the parameters was seen. In all cases, the firm with the highest rank was the most profitable; rank being a rough measure of the benefits accruing to society from a firm. This ranking enabled us to calculate the number of firms in the long-term equilibrium and the socially optimal number of firms, assuming the highest ranked firm entered the market first. In contrast to the purely symmetric model, these numbers depend on the nature of the asymmetries of the different firms and, moreover, the long-term equilibrium is not unique. For comparison, for a given set of firms, we constructed scenarios in which there were barriers to entry, which lead to different total numbers and different combinations of firms being present in the long-run.

Congestion was also considered in the above approach using a bottleneck model, so that road capacity was limited and travel times endogenous. In the short-term welfare was reduced because of schedule delay costs and queuing costs. Asymmetries in product quality and disutility of labour followed the same pattern as for the no-congestion model, whereas travel times were dominated by changes in road capacity, which is inversely related to congestion

The model already provides a useful tool for the policy maker, as it allows him to assess the benefits to society of allowing firms to locate at various distances from the subcentre, sell products of different quality or provide amenities to attract workers. It would clearly be interesting and useful, however, to extend the model in a number of ways.

Firstly, we have only considered a rather stylised economy in this paper and, having established the main concepts, it would be sensible to look at a more realistic example. A more realistic economy could be constructed but obtaining empirical data for the city-subcentre set-up could be more problematic.

It would also be interesting to consider the effect of specific policy measures, such as congestion pricing. In their paper, De Palma and Proost calculate the short-term symmetric price equilibrium with congestion charging. With perfect road pricing, the total travel time cost, which depends on road capacity only and not on other potentially asymmetric parameters, is reduced by half. It should therefore be possible to generalise this approach to the asymmetric case, since road users do not have a choice of routes to their destination of choice and they are homogeneous in terms of their travel requirements (valuation of time, arrival time etc).

Finally we would also like to allow the product and labour markets to be uncovered. In the existing model, workers cannot choose not to work and shoppers have to buy one of the differentiated product variants. It would be interesting to introduce an outside option, as was considered for the simple one product market model but the generalisation is not straightforward. In particular, we currently require that each household consumes one unit of the differentiated good and supplies one unit of labour for its production. This leads to the market clearing condition that the proportion of workers and shoppers at any subcentre are equal. Clearly, allowing shoppers to choose an outside option, which could be produced in the city centre for example, or allowing workers to seek a different employment option would have consequences for the production and consumption of the differentiated good and for travel costs in general.

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