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An Almost Ideal Sharing Scheme for Coalition Games with Externalities[+]

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Abstract

We propose a class of sharing schemes for the distribution of the gains from cooperation for coalition games with externalities. In the context of the partition function, it is shown that any member of this class of sharing schemes leads to the same set of stable coalitions in the sense of d'Aspremont et al. (1983). These schemes are "almost ideal" in that they stabilize these coalitions which generate the highest global welfare among the set of "potentially stable coalitions". Our sharing scheme is particularly powerful for economic problems that are characterized by positive externalities from coalition formation and which therefore are likely to suffer from severe free-riding.

JEL codes: C70, C71

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1. Introduction

The *"classical approach"* of studying the formation of coalitions assumes a transferable utility (TU)-framework and is based on the *characteristic function*.[1] This function assigns to every coalition a worth which is the aggregate payoff that a coalition can secure for its members, irrespective of the behavior of players outside this coalition. The main focus of the "classical approach" is the division of the worth among coalition members. Important research items include the axiomatic characterization of division rules like Shapley value or Nash bargaining solution (see, e.g., Thomson 1995) and existence proofs of core stable solutions for particular economic environments (see, e.g., Moulin 1988). A strength of the "classical approach" is the generality of results that can be often established using only some standard properties like superadditivity or convexity.[2]

Recently, most papers on coalition formation follow a *"new approach"* based on the *partition function*.[3] This function also assigns a worth to every coalition but this worth depends on the entire coalition structure, i.e., the partition of players inside and outside a coalition. The main focus of the "new approach" is the prediction of equilibrium coalition structures and the analysis how the presence of externalities affects the success of coalition formation. One of the strengths of the "new approach" is that it relates the success or failure of coalition formation to the kind of externality (positive versus negative externality).[4] For instance, it emerges from the literature that positive (negative) externalities provide an incentive for players to free-ride (cooperate), leading to small (large) stable coalitions. Typical examples of positive externalities are for instance output cartels and international environmental agreements. Firms not involved in an output cartel benefit from lower output by the cartel via higher market prices. Similarly, countries not involved in an international environmental agreement benefit from lower emissions by signatories via lower environmental damages. In

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1 An excellent overview of the "classical approach" and the "new approach" mentioned below is provided in Bloch (2003).

2 Roughly speaking, supperadditivity means that the joint worth of coalitions is higher when they merge than when they act separately (see section 2 for a formal definition). Convexity means that there are increasing returns to mergers: big coalitions gain more from mergers than small coalitions.

3 For an overview of the "new approach", see, apart from Bloch (2003), also Yi (1997 and 2003).

4 Roughly speaking, positive (negative) externality means that the worth of coalitions increase (decrease) if other coalitions,, i.e., outsiders, merge. See section 2 for a formal definition.

contrast, countries forming customs unions that abolish taxes among members but impose a common external tariff on outsiders exhibit a negative externality on non-members.[5]

Until now, however, the literature following the "new approach" has paid little attention to the division of the gains from cooperation. Due to the complexity of the partition function, most papers assume a fixed sharing rule.[6] One set of papers assumes ex-ante symmetric players in which case equal sharing emerges as "natural" division rule.[7] However, symmetry is a strong assumption that is difficult to justify in most economic environments. Another set of papers – mainly related to the analysis of international environmental agreements[8] - allows for asymmetric players but assumes a particular (exogenous) sharing rule of which most are solution concepts of the "classical approach" or modifications of them. Clearly, this alternative is also not satisfactory for at least three reasons. First, sharing rules are ad hoc and mostly lack a sound motivation in the context of the partition function. Second, the prediction of equilibrium coalition structures is sensitive to the specification of sharing rules. Third, no information is available whether there exist other sharing rules that could do better from a global point of view, letting alone whether there exists a sharing rule that is "optimal".

From the discussion of the shortcoming of the "new approach", two routes for future research seem suggestive. The first route could be in the tradition of a positive analysis, aiming at endogenizing the sharing rule in the process of coalition formation. This route is pursued for instance by Ray/Vohra (1999) and Maskin (2003). The second route could be in the tradition of a normative analysis and therefore also in the tradition of the classical approach (e.g., Chander and Tulkens 1997), searching for an optimal sharing rule.

In this paper, we take one modest step along this second route in the context of the "new approach". We illustrate our ideas with the well-known cartel formation game and the concept of internal and external stability of d'Aspremont et al. (1983): players have only the choice to remain at the fringe (i.e., singleton coalition) or joining the cartel (i.e., non-trivial coalition) and the cartel is called stable if no cartel member has an incentive to leave (internal stability) and no outsider has an incentive to join (external stability) the cartel.

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5 See Bloch (2003) and Yi (2003) for details and other examples.

6 The individual payoffs derived from a particular sharing rule are called valuations. See section 2 for details.

7 See the literature cited in Bloch (2003) and Yi (1997 and 2003).

8 See for instance Barrett (1997 and 2001), Bosello, Buchner and Carraro (2003), Botteon and Carraro (1997) and Eyckmans and Finus (2003).

Our analysis proceeds in five steps (relating to five propositions). First, we motivate our *"almost ideal sharing scheme"* (AISS) by introducing the notion of *"potentially internally stable coalitions"* (PISC). "Almost ideal" indicates that - in the presence of externalities - it is only possible to stabilize a subset of the set of coalitions through a sharing scheme, namely only those that are "potentially internally stable". This will be particularly relevant for positive externality games in which large coalitions are typically not potentially internally stable due to strong free-rider incentives.[9] "Sharing scheme" indicates that we do not propose a particular solution but a class of sharing rules that stabilizes all PISC. Second, we demonstrate robustness of our sharing scheme: even though AISS only aims at stabilizing all PISC, the set of stable coalitions (internally *and* externally stable coalitions) will be the same for any sharing solution belonging to AISS. Third, we show optimality of AISS in the context of positive externalities (implying that free-rider incentives are particular pronounced) and superadditivity. That is, the PISC with the highest global worth will be among the set of stable coalitions. Fourth, we relate AISS to individual rationality and, fifth, we proof existence of stable coalitions under AISS.

From our results in section 3, it will be apparent that they apply to many economic problems where the success of cooperation depends on the burden sharing arrangement among coalition members. Our sharing scheme is particular useful for improving upon the success of cooperation when coalition formation exhibits positive externalities on outsiders and therefore free-riding is an important problem. Examples of such economic problems are abundant and include for instance international agreements between governments in order to coordinate monetary policy (e.g., Kohler 2002), pollution control (e.g., Hoel 1992, Barrett 2001 and Bosello et al. 2003), the management of high seas fisheries stocks (e.g., Pintassilgo 2003), programs to eradicate contagious diseases (e.g., Arce M. and Sandler 2003) and efforts to combat international terrorism (e.g., Sandler and Enders 2004). However, before turning to our main results in section 3, we introduce some notation and definitions in section 2. Finally, section 4 raps up the main findings and points to some issues of future research.

2. Preliminaries

Let $\Gamma(N, \pi)$ be a coalition game between n (≥ 2) players. A coalition S is a subset of the set of players $N = \{1, ..., n\}$ and the set of all possible coalitions of N is denoted by 2^N, i.e., the power set of N. In this paper, we restrict attention to coalition structures consisting of no more

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9 For examples, see the literature cited in footnote 8 and the literature cited in Bloch (2003) and Yi (2003).

than one non-trivial or non-singleton coalition S (the cartel) while all other players $j \in N \setminus S$ are singletons (the fringe). In this setting, a coalition structure is fully characterized by coalition S. We define a partition function π that assigns a single real number $\pi_S(S)$ to coalition S and real numbers $\pi_j(S)$ to every singleton $j \in N \setminus S$ of the fringe:

$$[1] \qquad \pi : S \mapsto \pi(S) = (\pi_S(S), \pi_j(S)) \in \mathbb{R}^{1+(n-s)} \quad \text{with} \quad j \in N \setminus S.$$

The domain of this partition function is the power set of N. The image of this mapping is a vector of variable size (1+(n-s)), depending on s, i.e., the cardinality of coalition S, and on n, the total number of players. Thus, our partition function is a special case of the general definition of partition functions (see, e.g., Bloch 2003 and Yi 2003) since it disregards all partitions that consist of two or more non-trivial coalitions. In contrast to the characteristic function used in the "classical approach", the partition function assigns not only a worth to coalition S but also to the outsiders of this coalition. This is an important information for analyzing games with externalities.

As mentioned in the Introduction, most papers following the "new approach" base their analysis of coalition formation not on the aggregate payoff to a coalition (i.e., worth) but on the individual payoff to coalition members. A *valuation function* maps coalition structures into a vector of individual payoffs - called *valuations*. In our single coalition setting, a valuation function assigns to every coalition S of N a real-valued vector of length n, $v : 2^N \to \mathbb{R}^n : S \mapsto v(S)$, such that:

$$[2] \qquad \begin{cases} \sum_{i \in S} v_i(S) = \pi_S(S) \\ \quad v_j(S) = \pi_j(S) \quad \forall j \in N \setminus S. \end{cases}$$

For every coalition S, the valuation function v specifies how the worth of coalition S is allocated among its members. By construction, valuations $v_i(S)$ are *group rational*, meaning that the entire worth of coalition S is distributed among its members. For the outsiders to coalition S, the valuations $v_j(S)$ coincide with the worth $\pi_j(S)$ that are assigned by the partition function.

Note the difference between the concept of a valuation and an imputation known from the "classical approach". An imputation is usually only one vector of length n, listing the payoff to each player in the grand coalition (S=N) whereas a valuation function assigns vectors of length n to every possible coalition (structure), specifying individual payoffs to coalition members *and* outsiders. This more comprehensive view is necessary to capture externalities across coalitions and players (see Definition 2 below) and since - a priori - the "new approach" does not rule out inefficient coalition structures as potential equilibrium candidates. We subsequently turn to two properties that prove helpful for characterizing partition functions.

Definition 1: Superadditivity

A coalition game $\Gamma(N,\pi)$ is superadditive (SA) if and only if its partition function π satisfies: $\forall\, S \subseteq N, \forall\, i \in S: \ \pi_S(S) \geq \pi_{S\setminus\{i\}}(S\setminus\{i\}) + \pi_i(S\setminus\{i\})$.

Superadditivity means that the worth from cooperation cannot decrease. SA is a mild and standard assumption and is often motivated by arguing that "even if two coalitions merge, they always have the option of behaving as they did when they were separate, and so their total payoff should not fall" (Maskin 2003, p. 9).

Definition 2: Positive Externalities

A coalition game $\Gamma(N,\pi)$ exhibits positive externalities (PE) if and only if its partition function π satisfies: $\forall\, S \subseteq N, \forall\, j \neq i, j \notin S: \ \pi_j(S) \geq \pi_j(S\setminus\{i\})$ and $\exists\, k \neq i, k \notin S: \ \pi_k(S) >$ $\pi_k(S\setminus\{i\})$.

Positive externalities imply that outsiders to coalition S do not lose from the accession of player i to S. We assume a strictly positive effect for at least one outsider in order to rule out neutral effects of coalition formation. Negative externalities can be defined in a similar way. However, we omit this definition since it is not used in the following. Instead, we introduce the notion of stable coalitions according to d'Aspremont et al. (1983).

Definition 3: Internally and Externally Stable Coalitions

Let v be a valuation function for coalition game $\Gamma(N,\pi)$ and $v(S) \in \mathbb{R}^n$ the vector of valuations for the players in N when coalition S forms. Coalition S is stable with respect to the valuations v(S) if and only if:

- internal stability (IS): $v_i(S) \geq v_i(S\setminus\{i\}) \quad \forall\, i \in S$
- external stability (ES): $v_j(S) \geq v_j(S \cup \{j\}) \quad \forall\, j \in N\setminus S$.

Definition 3 says that a coalition S is stable if no insider wants to leave and no outsider wants to join coalition S. Obviously, given a coalition game $\Gamma(N,\pi)$, there are many possible valuation functions which can be derived from its partition function. Consequently, a coalition S may be stable with respect to a particular valuation function v but may not be stable with respect to another valuation function v'. Therefore, we denote the set of coalitions that are internally stable with respect to valuation function v by $\Sigma^{IS}(v)$, the set of coalitions that are externally stable by $\Sigma^{ES}(v)$ and the set of *stable coalitions* by $\Sigma^{S}(v) = \Sigma^{IS}(v) \cap \Sigma^{ES}(v)$.

Since it is our ambition to study the impact of coalitional surplus sharing rules on the stability of coalitions in a more general framework, the use of one specific valuation function would be too restrictive. Therefore, we introduce a class of valuation functions and study the stability properties of its members. We call a member of this class of valuation functions an "almost ideal sharing scheme".

Definition 4: Almost Ideal Sharing Scheme

An *A*lmost *I*deal *S*haring *S*cheme (AISS) for coalition game $\Gamma(N,\pi)$ is a valuation function v^{λ} that satisfies:

$$\forall\, S \subseteq N: \quad \begin{cases} \forall\, i \in S: & v_i^{\lambda}(S) = \pi_i(S \setminus \{i\}) + \lambda_i(S)\,\sigma(S) \\ \forall j \in N \setminus S: & v_j^{\lambda}(S) = \pi_j(S) \end{cases}$$

with $\lambda(S) \in \Delta^{s-1} = \left\{ \lambda \in \mathbb{R}_+^s \,\middle|\, \sum_{j \in S} \lambda_j = 1 \right\}$ and $\sigma(S) = \pi_S(S) - \sum_{i \in S} \pi_i(S \setminus \{i\})$

where Δ^{s-1} denotes the set of all possible sharing weights for a coalition of s players and $\sigma(S)$ denotes the surplus (or deficit) of coalition S over the sum of free-rider payoffs $\pi_i(S \setminus \{i\})$ of its members. An AISS allocates to each coalition member its free-rider payoff, plus some share of the remaining surplus. The free-rider payoffs are associated with the scenario in which an individual coalition member leaves coalition S in order to become a singleton while the remaining members of coalition S continue to cooperate. These payoffs constitute lower bounds on the claims of individual coalition members with respect to the coalitional surplus in order *not* to leave the coalition. Thus, the free-rider payoff $\pi_i(S \setminus \{i\})$ may be regarded as the threat point of player i in coalition S and its weight $\lambda_i(S)$ may be interpreted as his bargaining power.

Note that there are as many AISS valuation functions for a coalition game $\Gamma(N,\pi)$ as there are ways to share in every possible coalition S of N the coalition surplus $\sigma(S)$ among its members. The set of all possible AISS valuation functions for game $\Gamma(N,\pi)$ will be denoted by $\mathcal{V}(\Gamma)$. It should also be noted that the surplus $\sigma(S)$ might be negative. That is, free-rider incentives may be so strong that the total sum of the free-rider payoffs is larger than the worth of the coalition. In this case, it is not possible to satisfy the claims of all coalition members and AISS will share the deficit according to weights $\lambda(S)$. We use the sign of $\sigma(S)$ to classify coalitions.

Definition 5: Potentially Internally Stable Coalitions

A coalition S is called *potentially internally stable* (PIS) for partition function π if and only if: $\pi_S(S) \geq \sum_{i \in S} \pi_i(S \setminus \{i\})$, i.e., $\sigma(S) \geq 0$.

We denote the set of coalitions that are PIS for a particular partition function π by $\Sigma^{PIS}(\pi)$. Note that PIS is a property of the partition function whereas internal stability (IS) - and also external stability (ES) - are properties of a specific valuation function.

3. Results

The construction of AISS valuation functions in Definition 4 suggests that they are designed to remedy free-riding in terms of internal stability. Therefore, Proposition 1 shows that every coalition that is potentially internally stable will be internally stable for any AISS valuation function.

Proposition 1: AISS and Potential Internal Stability

Let $S \subseteq N$ be a coalition that is potentially internally stable, then S is internally stable for any AISS valuation function $v^{\lambda} \in \mathcal{V}(\Gamma)$.

Proof: Suppose that coalition S was PIS, implying $\sigma(S) \geq 0$ by Definition 5, but assume to the contrary that there existed a valuation $v^{\lambda} \in \mathcal{V}(\Gamma)$ such that coalition S would not be internally stable with respect to this valuation function. Then, $\pi_i(S \setminus \{i\}) + \lambda_i(S)\,\sigma(S) = v_i^{\lambda}(S) < v_i^{\lambda}(S \setminus \{i\}) = \pi_i(S \setminus \{i\})$ and hence $\sigma(S) < 0$ which contradicts the initial assumption. **QED**

The importance of Proposition 1 derives from three facts. First, internal stability is a necessary condition for a stable coalition, but is often violated for larger coalitions in positive externality games (see footnote 8). Second, any AISS valuation function ensures that every coalition that is potentially internally stable will actually be internally stable. As shown in Eyckmans and Finus (2003), other solution concepts may miss this potential substantially. Third, there is some degree of freedom in the choice of the sharing solution (through the choice of weights $\lambda(S)$) when aiming at stabilizing coalitions internally. The last observation is also supported by Proposition 2 below. On the way to this result, we establish first Lemma 1 which will turn out to be useful in the sequel because it establishes an important link between internal and external stability for valuations derived from AISS.

Lemma 1: AISS, Externally and Potentially Internally Stable Coalitions

Consider a coalition game $\Gamma(N,\pi)$ and the class of AISS valuation functions $\mathcal{V}(\Gamma)$. For any valuation function $v^{\lambda} \in \mathcal{V}(\Gamma)$, coalition S is not externally stable with respect to v^{λ} if and only if there exists a $j \in N \setminus S$ such that coalition $S \cup \{j\}$ is potentially internally stable.

Proof: Coalition S is not externally stable with respect to v^{λ} if and only if $\exists\, j \in N \setminus S:\ v_j^{\lambda}(S \cup \{j\}) > v_j^{\lambda}(S)$. This is equivalent to $\pi_j(S) + \lambda_j(S \cup \{j\})\,\sigma(S \cup \{j\}) > \pi_j(S)$ or $\sigma(S \cup \{j\}) > 0$, implying that $S \cup \{j\}$ is PIS. **QED**

An immediate corollary - the negation of Lemma 1 – is that a coalition S is externally stable if and only if for all $j \in N \setminus S$ coalition $S \cup \{j\}$ is not potentially internally stable. Another equivalent way is to say that coalition S is potentially internally stable if and only if for all $j \in S$ coalition $S \setminus \{j\}$ is not externally stable. It is important to note that Lemma 1 is a distinctive property of AISS valuation functions and may not hold for other valuation functions that are not AISS.

We now turn to our second result (Proposition 2) which shows that the set of stable coalitions is independent of the specific weights for any sharing solution in AISS. Hence, it constitutes some kind of invariance or robustness result. It contrasts with some of the literature (see footnote 8) that find different stable outcomes for various sharing rules.

Proposition 2: AISS and Robustness

Consider a coalition game $\Gamma(N,\pi)$ and the class of AISS valuation functions $\mathcal{V}(\Gamma)$. Let v^{λ} and $v^{\lambda'}$ be two AISS valuation functions in $\mathcal{V}(\Gamma)$, then i) $\Sigma^{IS}(v^{\lambda}) = \Sigma^{IS}(v^{\lambda'})$, ii) $\Sigma^{ES}(v^{\lambda}) = \Sigma^{ES}(v^{\lambda'})$ and iii) $\Sigma^{S}(v^{\lambda}) = \Sigma^{S}(v^{\lambda'})$.

Proof: i) Follows from Proposition 1. ii) We show that $S \in \Sigma^{ES}(v^{\lambda}) \Leftrightarrow S \in \Sigma^{ES}(v^{\lambda'})$. Note that $S \in \Sigma^{ES}(v^{\lambda})$ implies $\forall\, j \in N \setminus S:\ S \cup \{j\}$ is not PIS according to Lemma 1. Therefore, $\sigma(S \cup \{j\}) \le 0$ and hence $v_j^{\lambda'}(S \cup \{j\}) = \pi_j(S) + \lambda'_j(S \cup \{j\})\,\sigma(S \cup \{j\}) \le \pi_j(S) = v_j^{\lambda'}(S)$ which implies $S \in \Sigma^{ES}(v^{\lambda'})$. iii) Follows immediately from i), ii) and the definition of stability, i.e., $\Sigma^{S}(v) = \Sigma^{IS}(v) \cap \Sigma^{ES}(v)$. **QED**

We now establish our main result about the optimality of AISS. Clearly, any analysis about the implication of coalition formation on global welfare requires more structure of the underlying fundamentals of a model. Therefore, we assume apart from superadditivity also positive externalities for the partition function: positive externalities (PE) make the problem interesting since large coalitions are typically not internally stable due to free-rider incentives as mentioned above. Proposition 3 below says that adopting an AISS guarantees that the coalition which generates the highest global welfare among the potentially internally stable coalitions will not only be internally stable but also externally stable and therefore stable. The remarkable aspect of this result is that a sharing scheme that apparently is designed to foster internal stability is also capable of ensuring external stability for those coalitions that are most desirable in terms of global welfare.

Proposition 3: AISS and Optimality

Let $\Sigma^{PIS}(\pi)$ be the set of coalitions that are potentially internally stable in coalition game $\Gamma(N,\pi)$ that is superadditive and exhibits positive externalities and let S^* be the coalition with the highest global welfare in $\Sigma^{PIS}(\pi)$: $\forall\, S \in \Sigma^{PIS}(\pi), S \neq S^*$: $\pi_{S^*}(S^*) + \sum_{j\in N\setminus S^*} \pi_j(S^*) \geq \pi_S(S) + \sum_{j\in N\setminus S} \pi_j(S)$. Then, any valuation $v^\lambda \in \mathcal{V}(\Gamma)$ will make coalition S^* both i) internally and ii) externally stable and, hence, stable.

Proof: i) Follows from Proposition 1. ii) Assume to the contrary that $S^* \in \Sigma^{PIS}(\pi)$ generated the highest welfare among all coalitions that are PIS but would not be externally stable for some valuation function $v^\lambda \in \mathcal{V}(\Gamma)$. Hence, we know from Lemma 1 that there exists an outsider $j \in N \setminus S$ such that coalition $S^* \cup \{j\}$ is PIS. However, due to SA and PE, $\pi_{S^*\cup\{j\}}(S^* \cup \{j\}) + \sum_{k\in N\setminus(S^*\cup\{j\})} \pi_k(S^* \cup \{j\}) > \pi_{S^*}(S^*) + \sum_{k\in N\setminus S^*} \pi_k(S^*)$ which contradicts the initial assumption that S^* generates the highest welfare among all coalitions that are PIS.

QED

Proposition 3 can be interpreted as saying that we cannot do better in terms of global welfare than adopting an AISS if the agreement is required to satisfy stability in the sense of d'Aspremont et al. (1983). Since any AISS is a parametric valuation function depending upon some set of sharing weights $\lambda(S)$, there remains considerable flexibility how to allocate the surplus of the coalition without jeopardizing optimality. Any alternative set of sharing weights among AISS would also stabilize the coalition generating the highest global welfare.

We would like to finish with two more properties of AISS. The first property is individual rationality. That is, we show that for any potentially internally stable coalition every AISS leads to an individually rational solution in a coalition game with positive externalities. That is, every player receives at least her payoff than when all players act as singletons players. Note that this property may be violated for coalitions that are *not* potentially internally stable. However, this seems no problem because these coalitions will not emerge as stable outcomes (internally and externally stable coalitions) anyway.

Proposition 4: AISS and Individual Rationality

Consider a coalition game $\Gamma(N,\pi)$ with positive externalities and the set of AISS valuation functions $\mathcal{V}(\Gamma)$. If coalition S is potentially internally stable, then S is individually rational for all valuations $v^{\lambda} \in \mathcal{V}(\Gamma)$ and for all players $i \in N$: $v_i^{\lambda}(S) \geq v_i(\{i\})$.

Proof: If coalition S is potentially internally stable, then for every AISS it holds that $\forall\, i \in S$: $v_i^{\lambda}(S^*) \geq v_i^{\lambda}(S^* \setminus \{i\}) = \pi_i(S^* \setminus \{i\})$ and due to PE $\forall\, i \in N$: $\pi_i(S^* \setminus \{i\}) \geq \pi_i(\{i\})$. **QED**

The second property is existence. That is, we establish generally (without any particular assumption on the partition function) existence of a stable coalition under AISS. As a corollary, we show that AISS ensures the existence of at least one non-trivial stable coalition S ($s \geq 2$) provided the partition function satisfies superadditivity.

Proposition 5: AISS and Existence of a Stable Coalition

Consider a coalition game $\Gamma(N,\pi)$ and the set of AISS valuation functions $\mathcal{V}(\Gamma)$. For any $v^{\lambda} \in \mathcal{V}(\Gamma)$, there exists at least one stable coalition.

Proof: By definition, the trivial coalition S={i} is internally stable. If it is also externally stable, we are done. However, suppose the trivial coalition is not externally stable, then there exists at least one two-player coalition that is PIS by Lemma 1. Again, if one of the two-player coalitions is also externally stable, we are done. Continuing with this reasoning, it is evident that some coalition $S \subseteq N$ will be internally and externally stable, noting that $S = N$ is externally stable by definition. **QED**

Corollary 1: AISS and Existence of a Stable Non-Trivial Coalition

Consider a coalition game $\Gamma(N,\pi)$ and the set of AISS valuation functions $\mathcal{V}(\Gamma)$. For any $v^{\lambda} \in \mathcal{V}(\Gamma)$, there exists at least one stable non-trivial coalition provided $\Gamma(N,\pi)$ is superadditive.

Proof: Due to SA, $\pi_{\{i,j\}}(\{i,j\}) \geq \pi_i(\{i\}) + \pi_j(\{j\})$ holds for all possible pairs (i, j) of players in N which is equivalent to the condition of PIS. Hence, a proof in the spirit of the proof of Proposition 5 can be constructed, except that the starting point is not the trivial coalition but a coalition with two players. **QED**

Note that existence of an internally and externally stable coalition is not automatically guaranteed for other sharing solutions in the cartel formation game as examples for instance in Eyckmans and Finus (2003) demonstrate.

4. Conclusion and Suggestions for Further Research

This paper analyzed internally and externally stable coalitions in the cartel formation game of d'Aspremont et al. (1983). We proposed a sharing scheme for the distribution of the gains from cooperation where any particular solution belonging to this scheme leads to the same set of stable coalitions. Moreover, we showed that this scheme is an "almost ideal sharing scheme" in that it is capable of stabilizing these coalitions which generate the highest global welfare among the set of potentially stable coalitions. Our sharing scheme is particular powerful for economic problems that are characterized by positive externalities from coalition formation and which therefore suffer from free-riding (see the Introduction for examples). Our results improve in particular upon the existing literature that studied the impact of different sharing rules on the success of coalition formation (see footnote 8). In contrast to this literature, our scheme is robust to different specifications of sharing weights and realizes always the full global welfare potential.

For future research, we see at least three possible extensions. First, we have only considered a particular type of the partition of players with only one non-trivial coalition and all other players are singletons. A more general approach would allow for genuine partitions with several non-trivial coalitions. Of course, this would also imply to consider a more sophisticated stability concept than internal and external stability. A second extension could be to consider different stability concepts than internal and external stability. This could include concepts that define stability in terms of multiple deviation as for instance strong Nash equilibrium and coalition-proof Nash equilibrium. Third, our sharing scheme leaves the choice of surplus sharing weights open. Endogenizing the value of these weights, which may be interpreted as bargaining power, in games with heterogeneous players and externalities seems an interesting but also challenging topic for further research.

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