



**WORKING PAPER SERIES**  
**n° 2002-01**

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some further considerations**

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March 2002



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# Penalty and crime with lumpy choices: some further considerations

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## Acknowledgments

I would like to thank Matthew Braham, Winston Harrington, Luc Lauwers, Gerd Mühlheusser, Stef Proost, Erik Schokkaert and Frans Spinnewyn for helpful comments on an earlier version of this paper. I also benefited from the interaction with participants at the ERASMUS Programme in Law and Economics Annual Conference 2001 (Hamburg), at the 6th Spring Meeting of Young Economists (Copenhagen) and at the 2001 Annual Meeting of the European Public Choice Society (Paris). The usual disclaimer applies.

## Abstract

This paper clarifies an issue in the Hirshleifer and Rasmusen-Tsebelis controversy on the effects of penalties on crime : what is the effect of penalties if the transgression of law has a discrete nature and if the law enforcer cannot act as Stackelberg leader? We differentiate between technical (compliance costs) and institutional (penalties) parameters in the potential transgressor's payoff's functions. Depending on the penalty structure, we obtain equilibria either in pure or in mixed strategies. We confirm Hirshleifer and Rasmusen's results that small changes in the fine structure do not affect the equilibrium that is obtained, but that large changes do. In the equilibria with mixed strategies, we confirm Tsebelis's results that changes in the penalties do affect the potential's transgressor's strategies, but without affecting the probability that the individual chooses a particular element in the support of his strategies.

**Key words:** decentralized law enforcement; economic analysis of crime

**JEL Classification:** K140; K320; K420

# 1 Introduction

In the first modern economic analysis of crime and punishment, Becker (1968) suggested that optimal law enforcement would consist in combining the highest possible penalties with the lowest possible probabilities of apprehension.

In a series of challenging papers, Tsebelis (1990a, 1990b, 1993) has argued that this economic approach to crime is fundamentally flawed. According to Tsebelis, higher penalties would not lead to lower crime, but to lower crime enforcement. In equilibrium, the level of crime in society would then be independent from penalties.

This point has been criticized as well (see for instance Weissing and Ostrom (1991) and Hirshleifer and Rasmusen (1992)).

In this paper, we further investigate some of the points raised by Hirshleifer and Rasmusen (1992) and Tsebelis's (1993) answer. We show that a more explicit treatment of the payoff-structure of the game allows to clarify several of the issues raised in this discussion.

Hirshleifer and Rasmusen (1992) assert that Tsebelis's results (in their terminology, the Payoff Irrelevance Proposition - PIP) are due to some very specific assumptions used.

First, Tsebelis assumes that the police-criminal game is simultaneous. Therefore, he uses the Nash equilibrium (NE) solution concept: the criminals' actions must be optimal, *given the police's behavior*, but the police's behavior must also be optimal, *given the criminal's behavior*. Tsebelis then obtains an equilibrium in mixed strategies, where the probability of compliance does *not* depend on the magnitude of the penalty. Indeed, in a mixed strategy NE, criminals choose the probability of compliance to make the policy indifferent between enforcing and not enforcing the law. As long as the penalty has no intrinsic utility for the police, it should thus not play a role in the equilibrium strategy of the criminals<sup>1</sup>. Hirshleifer and Rasmusen argue that the police-criminal game is more appropriately modeled as a sequential game, where the police announces a policy, and sticks to it, even if it is not an *ex post* optimal response to the criminal's behavior. They show that PIP does not hold if the police can act as a Stackelberg leader.

A second point raised by Hirshleifer and Rasmusen (1992) is that in Tsebelis's approach, both criminal and police face lumpy choices. Hirshleifer and Rasmusen argue that it is more appropriate to assume that criminal and police actually choose their crime level and their enforcement effort along a continuum. They then show that PIP does not hold under this alternative approach, even using the NE as solution concept. Indeed, with continuous action spaces, they obtain equilibria in pure strategies. Moreover, they also consider a game with lumpy choices. They compare the equilibrium with the equilibrium in an equivalent game with continuous choices and obtain mixed results:

there is likely to be a pure strategy equilibrium when the available choices are asymmetrically placed -one of the strategy-pairs being

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<sup>1</sup>A similar result has been obtained by Holler (1993).

located close to and the others far away from the pure-strategy equilibrium choices of the underlying continuum. Conversely, when the available choices are more or less evenly distant from the equilibrium of the continuous game, a mixed-strategy equilibrium is likely.

Using numerical simulations, Hirshleifer and Rasmusen show that for PIP to hold “the payoff parameter variations must be small enough so as not to cause a shift either to a pure-strategy equilibrium or to a mixed-strategy equilibrium involving different strategy elements”.

Hirshleifer and Rasmusen also provide an intuitive interpretation of the mixed nature of the equilibrium with lumpy choices:

Rational choice involves trade-offs, and lumpiness of the options available reduces what can be done in the way of trade-offs. Suppose a consumer initially finds apples too expensive to buy, but then the price falls. If the choice is between buying 50 apples or none, such a consumer may still take none - whereas, offered the opportunity of buying single apples, he might buy two or three instead (...) Intuition suggest that if the equilibrium strategy mixture is not to be affected, only payoff changes within a limited range are allowable.

In later paper, Tsebelis (1993) has answered Hirshleifer and Rasmusen. We shall not further explore the issues on the appropriateness of the NE as solution concept here - we refer to the original papers. For our purposes, it is only important to mention that Tsebelis has developed a credible model of crime enforcement where large changes in penalties do *not* affect the mixed strategies of criminals, thereby apparently contradicting Hirshleifer’s and Rasmusen’s conclusions. According to Tsebelis, Hirshleifer’s and Rasmusen’s results are due to the non-linearity of the considered payoff-functions.

The purpose of this paper is to clarify this specific issue: *what is the effect of changes in penalties in a enforcement model where the transgression of law has a discrete nature and where the law enforcer cannot act as Stackelberg leader?*

We consider two types of economic agents:

- Individuals who can choose between several levels of an activity that imposes external costs. Noncompliance can be detected with perfect accuracy by inspections of individual agents; inspections are however the only source of information for the agency<sup>2</sup>. Natural examples to think of are investments in pollution abatement or in workplace security.
- An enforcement agency that can determine freely the frequency with which it inspects individuals. We assume that in the constitutional division of power, the agency must take the legal standards and the penalties as given: the legislator determines, on the one hand, which activity level is the legal norm, and, on the other hand, the penalty structure.

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<sup>2</sup>In the terminology of Mookherjee and Png (1992), we consider enforcement through monitoring rather than through investigation.

Following Tsebelis’s approach, we shall assume that the enforcement agency cannot commit itself to an announced inspection probability.

Thus, so far, our model is very close to the models developed by Hirshleifer and Rasmusen (1992) on the one hand and Tsebelis’s (1993) on the other hand. The main difference is that we explicitly model how the payoff-functions are affected, on the one hand, by the penalty structure, and on the other hand by technical parameters.

Let us now turn to the notational assumptions.

An individual can choose between  $n = 1, \dots, i, \dots, n$  levels of expenditures  $\alpha_i$ , where  $\alpha_1 > \dots > \alpha_i > \dots > \alpha_n = 0$ . An individual is compliant if he spends  $\alpha_1$ .

If an individual spends  $\alpha_i$ , the agency faces a cost  $D_i$ . We shall not explicitly define this cost, but it can be given a wide variety of plausible interpretations. For instance, if the agency maximizes social welfare,  $D_i$  are the monetary value of external damages net of private compliance costs. In order to simplify terminology, we shall call this cost the “external cost of noncompliance”.

If an individual is found in noncompliance, he has to pay the fine  $\Psi_i$ .

We also assume that the agency derives *some* benefit from inspecting a non-compliant individual. For instance, the career perspectives of the agency’s staff may depend on the number of detected noncompliant individuals, or the staff may derive some moral satisfaction from fining noncompliant individuals. The only specific assumption we introduce here is that the fines are not redistributed to the agency and thus that this benefit does not depend on the collected fines.

For the agency, the benefit of inspecting an agent who spends  $\alpha_i$  is thus  $\Delta_{i,1}$ . We assume that  $\Delta_{n,1} \geq \dots \geq \Delta_{i,1} \geq \dots \geq \Delta_{1,1} = 0$ . In words, the lower the amount the individual spends, the higher the benefit of inspecting the individual. This needs not to be true for all conceivable types of criminal behavior<sup>3</sup>. Choosing this particular payoff structure however allows to avoid some pitfalls linked to the problems of marginal deterrence (see Friedman and Sjoström(1993)).

Inspecting an individual costs  $b$ .

Let  $p$  be the probability that the enforcement agency inspects the individual and let  $p_i$  be the probability that the individuals spends  $\alpha_i$ .

The enforcement agency’s expected costs are then:

$$\sum_{i=1}^n p_i D_i + p \{ b - \sum_{i=2}^n p_i \Delta_{i,1} \} \quad (1)$$

To see this, note that if the agency does not inspect the individual its expected costs are expected external costs:  $\sum_{i=1}^n p_i D_i$ . If the enforcement agency inspects the individual, its expected costs increase by  $b - \sum_{i=2}^n p_i \Delta_{i,1}$ .

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<sup>3</sup>Friedman and Sjoström (1993) have given the following counter-example: a gourmet may prefer stealing a lamb to stealing a sheep, although this imposes smaller costs on the victim of the theft.

The individual's expected costs are:

$$(1 - p) \sum_{i=1}^n p_i \alpha_i + p \sum_{i=1}^n p_i [\alpha_i + \Psi_i] \quad (2)$$

To see this,  $\sum_{i=1}^n p_i \alpha_i$  are the individual's expected costs if he is not inspected. If the individual is inspected and he has spent  $\alpha_i$ , his costs are  $\alpha_i + \Psi_i$ , this is the sum of compliance costs and of expected fines.

Finally, in what follows, we shall exclude cases where  $b = \Delta_{i,1}$ . The probability that these two technical parameters are equal to each other is zero. Considering these cases would be of very limited relevance, and, moreover, would lead to indeterminacies.

In Section 2, we show that for some parameter values, we obtain an equilibrium in pure strategies. This equilibrium is determined partly by the fine structure, but only "large" changes in the fine structure lead to a different equilibrium. In Section 3, we show that in all other NE, both the agency and the individual must play mixed strategies. We consider subsequently the individual's best response to any given strategy of the agency, and then the agency's best response to any given strategy of the individual. We show that the range of expenditure levels between which the individual will mix depends on the one hand on the fine structure and on the other hand on the relative magnitude of external damages and the cost of inspecting the individual. However, the probability of choosing one of the elements in the support of his mixed strategies is completely independent from the fine structure. Thus, the individual's strategies are insensitive to "small changes" in the fine structure. However, the probability of inspection depends on the fines corresponding to support of the individual's strategies, and is thus sensitive to "small changes". The only role the fines for the other transgressions play is that they need to induce the individual to mix between the "desired" expenditure levels.

## 2 Equilibrium in pure strategies

There is no strategic interaction between the individuals, and we can limit ourselves to the interaction between the agency and a representative individual.

First note that if  $b > \Delta_{n,1}$ , then  $b > \Delta_{i,1}$  for all  $i$ , and thus also  $b > \sum_{i=2}^n p_i \Delta_{i,1}$ . From Equation 1, the agency then always minimizes its expected costs by setting  $p = 0$ , but then Equation 2 shows that the individual minimizes his expected costs by choosing the highest level of noncompliance. We thus obtain immediately:

**Proposition 2.1** *If  $b > \Delta_{n,1}$ , the only strategy-pair that survives iterated elimination of dominated strategies is: the agency never inspects the individual and the individual spends nothing on compliance.*

In this case, both the agency's and the individual's behavior are completely independent from the fine structure, and the analysis becomes trivial. Thus, from now on, we shall assume that  $\Delta_{n,1} > b$ .

**Proposition 2.2** *Suppose there exists a  $j$  such that  $\Delta_{j,1} > b$  and that  $\alpha_j + \Psi_j < \alpha_k + \Psi_k$  for all  $k$ . The following strategy-pair is the unique NE: the agency inspects the individual with certainty, and the individual spends  $\alpha_j$ .*

**Proof**

First we show that this is indeed a NE.

If the agency inspects the individual with certainty ( $p = 1$ ) and if  $\alpha_j + \Psi_j < \alpha_k + \Psi_k$  for all  $k$ , then Equation 2 implies that the individual optimally always spends  $\alpha_j$ .

Now suppose that the individual spends  $\alpha_j$  with certainty. Equation 1 then reduces to:

$$D_j + p\{b - \Delta_{j,1}\}$$

$\Delta_{j,1} > b$  implies that the agency's best response is to inspect the individual.

To see that this equilibrium is unique, suppose first that the agency does not inspect the individual. The individual's best response is then to spend nothing, but then the agency's best response is to inspect the individual. There are thus *no NE where the agency does not inspect the individual*.

Suppose next that the individual spends  $\alpha_i \neq \alpha_j$  with certainty. Equation 1 then reduces to:

$$D_i + p\{b - \Delta_{i,1}\}$$

As we have excluded  $b = \Delta_{i,1}$ , the agency is never indifferent between inspecting and not inspecting the individual, and its best response is thus to play a pure strategy. We have shown above that there is no NE where the agency does not inspect the individual. Moreover, if the agency inspects the individual, we have shown the individual's best response is to spend  $\alpha_j$ .

Thus, there is *no NE in pure strategies where the individual spends  $\alpha_i \neq \alpha_j$* .

The individual will never mix between spending  $\alpha_j$  and spending  $\alpha_i$  with  $\alpha_i < \alpha_j$  (otherwise,  $\Delta_{i,1} > \Delta_{j,1} > b$  implies that the agency inspects with certainty and then the individual's best response is to spend  $\alpha_j$ ).

He will mix between spending  $\alpha_j$  and spending  $\alpha_i$  with  $\alpha_i > \alpha_j$  if he is indifferent between spending  $\alpha_j$  and spending  $\alpha_i$ :

$$\alpha_j + p\Psi_j = \alpha_i + p\Psi_i$$

or, equivalently:

$$p = \frac{\alpha_j - \alpha_i}{\Psi_i - \Psi_j}$$

$\alpha_i > \alpha_j$  implies that this is only a probability if  $\Psi_j > \Psi_i$  and if  $\Psi_j + \alpha_j > \Psi_i + \alpha_i$ , which contradicts the assumptions of this proposition.

Thus, there is no NE where the individual plays mixed strategies either. This immediately implies that the agency will never play mixed strategies either.  $\square$   
QED  $\square$

### Comments

Although we have a model with lumpy choices, we obtain here an equilibrium in pure strategies.

Clearly, the equilibrium strategies depend on the fine structure: the individual spends  $\alpha_j$  in equilibrium, even though he is certain to be inspected. The reason why he spends exactly this amount lies precisely in the fine structure, that deters him from spending another amount. It is clear that changes in the penalty structure can affect this equilibrium. For instance, suppose that all penalties are doubled.  $\alpha_j + \Psi_j < \alpha_k + \Psi_k$  for all  $k$  does not imply that  $\alpha_j + 2\Psi_j < \alpha_k + 2\Psi_k$  for all  $k$ .

For equilibria in pure strategies, we can confirm the position taken by Hirshleifer and Rasmusen: “small changes” do not change the equilibrium, but “large enough” changes do. Moreover, we have explicitly determined here what “large enough” means.

## 3 Equilibrium in mixed strategies

Thus, for equilibria in pure strategies, penalties do certainly matter. However, the Hirshleifer and Rasmusen-Tsebelis controversy concentrated on equilibria in mixed strategies. Whether or not such equilibria exist depends on the penalty structure. Given the structure of the game (both the number of players and their strategy spaces are finite), we know that there exists at least one NE for any given fine structure. Thus, if there is no NE in pure strategies, there must be at least one in mixed strategies (and vice versa). In this game, we can however go further:

**Lemma 3.1** *Suppose that for all  $j$  such that  $\Delta_{j,1} > b$ , there is at least one  $k$  such that  $\alpha_j + \Psi_j > \alpha_k + \Psi_k$ . If  $\Delta_{n,1} > b$ , then any equilibrium must be an equilibrium in mixed strategies.*

### Proof

In any equilibrium, the agency must mix between inspecting and not inspecting the individual. Indeed:



- If the agency inspects the individual with certainty, the individual will spend the amount  $\alpha_l$  that minimizes the sum of compliance cost and penalty. Under the assumptions of the lemma, if  $\Delta_{j,1} > b$ , then there is at least one  $k$  such that  $\alpha_j + \Psi_j > \alpha_k + \Psi_k$ . Thus, if spending  $\alpha_l$  minimizes the sum of compliance cost and penalty, then it *must* be that  $b > \Delta_{l,1}$ , which implies that the agency's best response is not to inspect the individual.
- If the agency does not inspect the individual, the individual's best response is to spend nothing, and then the agency's best response is to inspect the individual.

Suppose next that the individual spends  $\alpha_l$  with certainty. Equation 1 then reduces to:

$$D_i + p\{b - \Delta_{l,1}\}$$

As we have excluded  $b = \Delta_{l,1}$ , the agency is never indifferent between inspecting and not inspecting the individual, and its best response is thus to play a pure strategy, but we have shown above that this can never be part of a NE.

Thus, both the individual and the agency must play mixed strategies.  $\square$   
QED  $\square$

Define now  $j^*$  as the natural number such that  $\Delta_{j^*,1} > b > \Delta_{j^*-1,1}$ . Thus,  $\alpha_{j^*}$  is the largest number such that if the individual spends  $\alpha_{j^*}$  or less, it is optimal for the agency to inspect the individual. Any possible equilibrium in mixed strategy consists in mixing between spending  $\alpha_{j^*}$  or less, and spending strictly more than  $\alpha_{j^*}$ . Indeed:

**Lemma 3.2** *Suppose that  $\Delta_{j,1} > b$  and that there is a  $k$  such that  $\alpha_j + \Psi_j > \alpha_k + \Psi_k$ . If  $k > j$ , then there is no equilibrium where the individual mixes between spending  $\alpha_j$  and  $\alpha_k$ .*

**Proof**

$k > j$  and  $\Delta_{j,1} > b$  imply that  $\Delta_{k,1} > b$ . Thus, if the individual mixes between spending  $\alpha_j$  and  $\alpha_k$ , the agency's optimal response consists in always inspecting the individual. But then  $\alpha_j + \Psi_j > \alpha_k + \Psi_k$  implies that is never optimal to spend  $\alpha_j$ .  $\square$  QED  $\square$

The fine structure then determines the particular NE in mixed strategies that is obtained:

**Proposition 3.1** *Suppose that  $\Delta_{k,1} > b > \Delta_{j,1}$ ,  $\Psi_k > \Psi_j$  and  $\Psi_l > \frac{\alpha_l - \alpha_k}{\alpha_j - \alpha_k} \Psi_j + \frac{\alpha_j - \alpha_l}{\alpha_j - \alpha_k} \Psi_k$  for all  $l \neq j, k$ . There is then a NE where the agency inspects the individual with probability  $p = \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j}$  and where the individual mixes between spending  $\alpha_j$  with probability  $\frac{b - \Delta_{k,1}}{\Delta_{j,1} - \Delta_{k,1}}$  and spending  $\alpha_k$  with probability  $\frac{\Delta_{j,1} - b}{\Delta_{j,1} - \Delta_{k,1}}$ .*

### Proof

Let us thus first verify for what parameter the individual mixes between two expenditure levels, say  $\alpha_j$  and  $\alpha_k$ . For the remainder of the discussion, assume without loss of generality that  $j < k$  and thus that  $\alpha_j > \alpha_k$  on the one hand, and that  $\Delta_{k,1} > \Delta_{j,1}$  on the other hand.

If the individual spends  $\alpha_j$ , its expected costs are:

$$(1-p)\alpha_j + p(\alpha_j + \Psi_j) = \alpha_j + p\Psi_j \quad (3)$$

The individual mixes between  $\alpha_j$  and spending  $\alpha_k$  if two conditions are fulfilled:

- First, the individual needs to be indifferent between spending  $\alpha_j$  and spending  $\alpha_k$ :

$$\alpha_j + p\Psi_j = \alpha_k + p\Psi_k$$

or, equivalently:

$$p = \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j} \quad (4)$$

Note that  $p$  can only be a probability if  $1 > \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j} > 0$ .

It is straightforward to verify that  $\alpha_j > \alpha_k$  implies that  $1 > \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j} > 0$  if and only if  $\Psi_k > \Psi_j$ . Otherwise, the individual will always spend  $\alpha_k$ . The intuition for this result is obvious: it is more expensive for the individual to spend  $\alpha_j$  than to spend  $\alpha_k$ . Thus, if the fine for spending  $\alpha_j$  is higher as well, the individual clearly has no reason to spend  $\alpha_j$ .

- Next, the individual must strictly prefer to spend  $\alpha_j$  to spending any other amount  $\alpha_l$  except  $\alpha_k$ :

$$\alpha_l + p\Psi_l > \alpha_j + p\Psi_j$$

Substituting  $p = \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j}$  in this condition, we obtain:

$$\alpha_l + \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j} \Psi_l > \alpha_j + \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j} \Psi_j$$

or, equivalently:

$$\Psi_l > \frac{\alpha_l - \alpha_k}{\alpha_j - \alpha_k} \Psi_j + \frac{\alpha_j - \alpha_l}{\alpha_j - \alpha_k} \Psi_k \quad (5)$$

Let us now turn to the *agency's best response to the individual's strategy*. The agency only plays a mixed strategy if it is indifferent between inspecting and not inspecting, and thus:

$$b = \sum_{i=2}^n p_i \Delta_{i,1} \quad (6)$$

Any combination of the  $p_i$  such that this condition is fulfilled will induce the agency to mix.

However, if  $\Psi_l > \frac{\alpha_l - \alpha_k}{\alpha_j - \alpha_k} \Psi_j + \frac{\alpha_j - \alpha_l}{\alpha_j - \alpha_k} \Psi_k$  for all  $l \neq j, k$ , then the legislator has chosen a fine structure such that the individual mixes between spending  $\alpha_j$  and spending  $\alpha_k$ .

The agency will then play a mixed strategy if:

$$b = p_j \Delta_{j,1} + (1 - p_j) \Delta_{k,1}$$

thus, if:

$$p_j = \frac{b - \Delta_{k,1}}{\Delta_{j,1} - \Delta_{k,1}} \quad (7)$$

This is only a mixed strategy if  $1 > \frac{b - \Delta_{k,1}}{\Delta_{j,1} - \Delta_{k,1}} > 0$ . Because  $\Delta_{k,1} > \Delta_{j,1}$ , we require thus that  $\Delta_{k,1} > b > \Delta_{j,1}$ . Indeed, if  $\Delta_{j,1} > b$ , the agency's best response if the individual mixes between spending  $\alpha_j$  and spending  $\alpha_k$  is always to inspect, but we have seen that this can never be part of a NE. On the other hand, if  $b > \Delta_{k,1}$ , the agency's best response if the individual mixes between spending  $\alpha_j$  and spending  $\alpha_k$  is never to inspect, but we have seen that this cannot be part of a NE either.

We can thus conclude that whatever the fine structure, no NE is possible where the individual mixes between spending  $\alpha_j$  and spending  $\alpha_k$  unless  $\Delta_{k,1} > b > \Delta_{j,1}$ .  $\square$  QED  $\square$

### Comments

The most remarkable feature of this NE is probably how the fine structure affects the individual's strategy.

Indeed, the activity levels between which the individual mixes in equilibrium are determined simultaneously by:

- The relative magnitude of the fines and private compliance costs.
- The relative magnitude of the benefits and costs of inspecting an individual.

Equations 4 and 5 teach us is that *the fine structure determines between how many and between which expenditure levels the individual will mix in equilibrium*. Moreover, Equation 4 tells us that in an equilibrium where the individual mixes between spending  $\alpha_j$  and  $\alpha_k$ , the probability of inspection is:  $p = \frac{\alpha_j - \alpha_k}{\Psi_k - \Psi_j}$ .

However, Equation 6 shows that, although the fine structure affects *the composition of the support* of the individual's mixed strategies, the probability that the individual chooses a *particular element* in the support of his strategies is independent from the fines. Indeed, in a Nash equilibrium, the individual chooses the probability of spending a certain amount to make the agency indifferent between inspecting and not inspecting. If the fine has no intrinsic utility for the agency, then it should not play a role in the equilibrium strategy for the individuals: the probability that the individual chooses one particular element in the support of his strategies only depends on inspection costs and on the external benefits of bringing a noncompliant individual in compliance. This also means that *however large the fines, the individuals will never comply with certainty in equilibrium*.

For equilibria in mixed strategies, we can also confirm the position taken by Hirshleifer and Rasmusen: small changes do not affect the individual's behavior in equilibrium, but "large enough" changes do. Moreover, we have explicitly determined here what "large enough" means, and we have shown that even small changes affect the agency's behavior. On the other hand, we can confirm Tsebelis's point that even small changes in the penalty structure have a direct impact on the agency's behavior.

## 4 Conclusion

Apparently, the Hirshleifer and Rasmusen-Tsebelis controversy on the effects of higher penalties ended without clear conclusions. One possible explanation for this is that these authors do not differentiate between technical (compliance costs) and institutional (penalties) parameters in the individual's payoff's functions. We have explicitly introduced this distinction, and, globally speaking, we can confirm the results obtained by Hirshleifer and Rasmusen. For some parameter values, we obtain equilibria in pure strategies, and for other parameter values, we obtain equilibria in mixed strategies. Although small changes in the fine structure do not affect the equilibrium that is obtained, large changes do. In the equilibria with mixed strategies, we can confirm Tsebelis's results that changes in the penalties do affect the enforcer's strategies, but without affecting the probability that the individual chooses a particular element in the support of his strategies.

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