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REGULATING ON-STREET PARKING

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Abstract*

Consider the choices available to a shopper driving to a city and trying to park downtown. One option, typical to many cities, is to follow the signposts to an off-street parking facility, which is often privately operated. Another option is to search for an on-street spot. If this proves unsuccessful, it is always possible to return to the off-street facility.

We formalise such a setting and examine optimal on-street parking policy in the presence of an off-street market. Not surprisingly, the amount of socially-wasteful searching behaviour is shown to depend on the prices of both the off- and on-street market. If the off-street market is run competitively, optimal on-street policy reduces to a simple and attractive rule: set the on-street price equal to the resource cost of off-street parking supply. Other pricing rules result in either excessive searching behaviour or excessive off-street investment costs. Time restrictions – a common alternative to on-street fees – are also shown to be inefficient.

In practice, however, off-street markets are unlikely to be competitive. We examine the case of a single off-street supplier playing as a Stackelberg follower to the government regulated on-street market. Based on a numerical example (calibrated to London), optimal on-street policy is shown to either involve setting a relatively high on-street price, such that the monopolist is induced to undercut and gain the entire parking demand, or setting a relatively low price, while the monopolist maximises profit on the residual demand curve. Which strategy is optimal is shown to be parameter dependent.

KEY WORDS: parking, transport pricing, publicly provided goods.

JEL Codes: R40, R48, L92, H42.

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Regulating on-street parking

I. Introduction

A small but growing literature is emerging on the economic analysis of urban parking policy. The growth in this literature is probably due to the recognition that, despite a large literature on urban road tolling, parking regulation continues to be one of the most widespread forms of intervention by urban governments in the transport market.

A natural benchmark in the discussion of parking policy is the first-best pricing of on-street parking spots, or, in other words, the optimal pricing of on-street space in the absence of other distortions, most notably those associated with road use[1].

Three recent papers (Verhoef *et al*., 1995, Arnott and Rowse, 2001, -henceforth AR and Anderson and de Palma, 2002 – henceforth AP) have considered the first best pricing of on-street parking. All these articles – and the non-formal seminal piece in this area by William Vickrey (1959) – analyse the problem abstracting from the presence of an off-street parking market.

This seems overly-restrictive. To fix ideas, consider a shopper driving downtown and deciding where to park. In many cities, a driver can follow clearly marked signs to a downtown off-street parking facility. These facilities are often run by the private sector and are subject to a per time unit fee. An alternative option is to search for a vacant on-street spot. This involves some extra effort, but, if successful, the driver may benefit from paying a lower price (or in a number of cities, a zero price) than the off-street market. If the search process is not successful, the driver can always return to the off-street market.

We construct a simple model to analyse optimal parking regulation in this type of situation. If the private off-street market is competitive, we derive the simple welfare maximising policy rule: set the per time unit on-street price equal to the resource cost of off-street parking. Pricing lower than this rate induces too many drivers to invest in socially wasteful searching. Pricing higher than this rate induces all drivers to use the off-street market, which results in excessive supply costs (compared with driver using the zero-resource cost on-street space).

This optimal 'matching' result differs qualitatively from the findings in the literature. Vickrey's discussion implicitly assumes an aggregate demand curve for on-street parking

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[1] As part of the wider road-pricing debate, several authors have concentrated on the second-best pricing of space: i.e. the optimal parking price in the absence of external charges on road use. See – for example, Arnott *et al*., 1991, Glazer and Niskanen, 1992 and Calthrop *et al*., 2000. Whilst this is an

defined over own-price alone, and hence suggests a policy of peak-load pricing[2]. The more realistic specification adopted here, however, suggests a qualitatively different (and simpler) policy response: just match the off-street price.

Both AR and AP-V[3] consider on-street pricing in explicitly spatial settings. Different parking spots are not perfect substitutes for one another. In both papers, the basic normative insight is qualitatively similar. First-best prices are set equal to marginal external cost. In AR, based on a symmetric homogeneous city located on the outside of a circle, this amounts to the additional searching time imposed upon other would-be parkers from the decision to park for an additional unit of time. Due to symmetry, the optimal parking fee is independent of location. In AP-V[4], adopting a linear city, spots nearer to the single point city centre are more desirable than those further away (due to reduced walk time). Marginal external cost at location x is equal to the increase in search costs imposed on other parkers at location x. Due to greater desirability of parking spots nearer the centre, and hence greater demand, the optimal parking fee increases with proximity to the CBD.

These findings are important and relate primarily to the explicitly spatial nature of the search process. However, as stressed above, they abstract from the presence of an off-street market. We focus on the on- off- street choice, but adopt a highly reduced form representation of space. We comment further on this specification issue in our conclusions (section VI).

We extend our basic model in two directions, both new to the literature. Firstly, in contrast to other authors, we examine both quantity and price regulation. In many urban areas, on-street time restrictions are a common alternative to meter fees. We show that a simple time restriction, in which the driver pays no charge as long as he parks for less than a given length of time, is less efficient than optimal price regulation – even when the restriction is set optimally. A time restriction distorts driver choice between the on- and off-street market, and too many drivers end up searching for a vacant on-street space.

Secondly, we consider the optimal on-street pricing policy in the realistic setting of a non-competitive off-street market. This can be considered as an example of optimal pricing

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important subset of the literature, we expect second-best pricing rules to emerge as deviations from first best price. Indeed, this is confirmed in the recent paper by Anderson and de Palma (2002).

[2] Vickrey's article does far more that suggest peak-load pricing. He goes on to consider the more realistic problem of peak-load pricing under demand uncertainty, and comes up with the appealing idea of allowing the meter fee rate to depend on the number of occupied spaces in the surrounding vicinity. AR suggest, however, that this will only result in a first-best allocation if drivers are informed of the entire time profile of demand.

[3] The article AP can be seen as a generalisation of the earlier paper by Verhoef et al., 1995. Both papers consider a linear city with both search and road use externalities. Both consider the optimal parking span. AP, however, also consider a monopolistically competitive market outcome and show, in generality, that this outcome does not decentralise the social optimum. We adopt the notation AP-V to refer to the two papers together.

taking into account pre-existing distortions on other markets à la Harberger, 1974. In a simple Stackelberg setting, in which the government (the leader) plays against a single off-street supplier (the follower), we show that optimal policy deviates from the matching result. Rather the optimal on-street price may be higher or lower than the off-street price. Setting a relatively low on-street price alleviates welfare losses from using the distorted off-street market, but at the cost of inducing too much searching. In contrast, setting a relatively high on-street price induces the off-street monopolist to undercut the on-street market. Search costs are minimised but welfare costs associated with the non-competitive price and higher supply costs of the off-street market remain. Surprisingly, for reference parameter values, the latter case applies: the optimal price is such that the on-street market is undercut.

The structure of the paper is as follows. In section II we set out the basic model. Section III examines the centralised allocation problem: how would demand be allocated to supply if government can control all driver decisions? This is an important benchmark. Section IV compares the optimal centralised allocation with that attainable under decentralised decision making, under the assumption that the off-street market is supplied by a perfectly competitive market. Both linear and non-linear pricing schedules are considered. Section V relaxes the competitive off-street market assumption. Results are derived analytically and optimal values are computed with a numerical example. Sensitivity analysis investigates the generality of findings. Section VI concludes.

A final word on the generality of the model. We conjecture that the findings in this paper (in the context of parking policies) may be relevant to the general theory on the public provision of private goods. The essential assumption of the model is such that the public sector has a cost advantage to the private sector, at least for an initial number of units. Rationing is random. This structure suggests a relevance to the health sector (with rationing by waiting times) or utility provision (with rationing by shortage).

II. The Model

II.1. Parking technology

There are two parking markets. The on-street market (indexed by X) has a fixed supply of spaces, such that demand equals on-street supply when each individual parks for Q

[4] AP consider the optimal pricing of on-street space with search congestion and cruising congestion. As we are not concerned with road congestion (or assumed that it is internalised via a fee), we refer to the results of AP on search congestion only.

units of time. These spots are assumed sufficiently close to one another, such that any spot on market X is a perfect substitute for any other. The off-street market (indexed by Y) can be thought of as a single off-street parking facility subject to constant returns to scale, with the resource cost of an additional unit of parking time given by C, where $0 < C < 1$.

II.2. Consumer payoffs

Assume a continuum of identical risk-neutral consumers. Each consumer enjoys a benefit from time parked in the urban centre during the peak-period. To keep matters simple, assume that utility is defined in a quasi-linear fashion. Utility from parking time is denoted by $u(t)$, where t denotes the time parked, and furthermore is specified by a quadratic form such that the marginal benefit from an additional unit of time parked, $u'(t)$ is given by $1-t$. Given this specification of consumer preferences, it is clear that the two parking markets are perfect substitutes[5].

On either market, a consumer decides how long to stay on the basis of a constant per time unit fee, p_i, where $i \in \{X, Y\}$. (In section IV.2 below, we explore the use of a non-linear fee schedule). Prices are assumed to be perfectly and costlessly enforceable[6]. Each consumer parks until the marginal benefit of an additional unit of stay equals the cost: $t_i^*(p_i) = 1 - p_i$. Consumer surplus[7], conditional on using a particular market i, is denoted by $v(p_i)$.

By assumption, a driver can proceed directly to the off-street market and enjoy surplus $v(p_Y)$. Alternatively, a driver may search for an on-street space, and finds a vacant on-street space with probability ρ, which is explicitly derived below. If the driver fails to find a vacant spot, however, he or she returns to the off-street market, incurring an additional cost d.

This cost can be interpreted literally as the driving costs (gasoline, time etc) of returning from the on-street to the off-street market. However, it is intended as a reduced from

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[5] This assumption can be relaxed. On-street parking may be closer to desired destination or may give a higher feeling of safety than the off-street market. In a model with identical individuals, however, this difference acts as a fixed term. Results, therefore, up to a fixed term, are not altered. Comments on preference heterogeneity are made in the concluding section.
[6] See Calthrop (2001) Chapter 4 for a model of optimal on-street parking pricing in the presence of costly enforcement.
[7] Given the quasilinear formulation, indirect utility of the consumer is equal to consumer surplus $v(p_i)$ plus income.

representation of a search cost. In expected terms, consumer surplus from searching on-street can be expressed as:

$$v_s = \rho v(p_X) + [1-\rho][v(p_Y) - d]$$

which re-arranges to give:

$$v_S = v(p_Y) + \rho[v(p_X) - v(p_Y)] - [1-\rho]d \qquad (1)$$

If he fails to find a vacant on-street spot, the driver can always park off-street and make a reservation level of utility, $v(p_Y)$. In addition, assuming that the price of on-street parking is lower than the off-street market, with probability ρ, he finds a vacant spot and makes the net consumer surplus of $v(p_X) - v(p_Y)$.

Expected search cost is given by the last term on the right hand side, $[1-\rho]d$. Increasing the on-street price reduces the optimal length of stay, t_X^*. This – as will be shown below- increases the probability of finding a vacant on-street spot, ρ. Expected search cost falls. This is the key mechanism derived by AR using a more structural representation of the search process based on stochastic queueing theory[8].

II.3. Rationing rule

The probability of finding a vacant spot is assumed to be given by a random-rationing rule. Denoting the percentage of drivers that choose to search for an on-street spot is denoted by λ, this is given by:

$$\rho(p_X, p_Y, \lambda) = Min\left\{\frac{Q}{\lambda t_X^*(p_X)}, 1\right\} \qquad (2)$$

This rule approximates a situation in which drivers arrive in a downtown area more or less at random. We consider this to be well-suited to modelling downtown areas used for shopping and leisure activities. It is far less relevant to workplace parking, where spots are often reserved.

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[8] Applying standard queueing theory, expected waiting time for a M/D/1 system is given by: $\lambda / 2(1-\rho)\mu^2$ where λ is the expected arrival rate, μ is the service rate (the inverse of the length of stay) and ρ is the utilisation rate or λ / μ. Increasing the length of stay acts to increase the expected wait time. In AR, the same mechanism exists: the expected cruising distance to find a vacant spot is given by 1/P, where P is the average density of vacant spaces. Increasing the density of vacant spaces reduces expected cruising distance (or search cost).

II.4. Equilbrium number of searchers

In equilibrium, no driver can increase expected consumer surplus by switching from searching to not searching or vice-versa. For a non-symmetric equilibrium (i.e. $0 < \lambda < 1$), this implies:

$$v_S(p_X, p_Y, \lambda) = v(p_Y) \tag{3}$$

We can solve for the equilibrium number of searchers, λ^*. Comparing (3) and (1) shows that, in equilibrium, a number of drivers choose to search such that the expected net consumer surplus from searching, $\rho[v(p_X) - v(p_Y)]$ equals the expected search cost, $[1-\rho]d$. Allowing for the possibility of a symmetric equilibrium, in which all drivers search or not, gives:

$$\lambda^*(p_X, p_Y) = \begin{cases} 1 & \text{if} \quad p_X \leq p_{\underline{X}}(p_Y) \\ \dfrac{Q}{[1-p_X]d}\{v(p_X) - v(p_Y) + d\} & \text{otherwise} \\ 0 & \text{if} \quad p_X > p_Y \end{cases} \tag{4}$$

Below a critical price level, $p_X < p_{\underline{X}}(p_Y) < p_Y$ all drivers choose to search[9]: $\lambda^* = 1$. The price of on-street parking is low in comparison with the on-street market, and hence the expected net consumer surplus from successfully finding a vacant on-street spot $\rho[v(p_X) - v(p_Y)]$ is sufficiently high, even when all drivers choose to search, to offset the expected search cost $(1-\rho)d$.

At a price slightly higher than this level, only a subset of drivers $\lambda^* < 1$ choose to search. The expected net consumer surplus is relatively small, such that the expected search cost is offset only when a subset of drivers choose to search. If the on-street price exactly matches the off-street price, the expected net consumer surplus is zero. Drivers may choose to search only if the expected search cost also equals zero – in other words, that the probability of finding a vacant spot equals 1. In this case[10], $\lambda^* = Q/[1-p_Y]$.

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[9] This requires a condition that the size of the search cost parameter d not be too large. If it were, the net gain from finding a vacant on-street space can be outweighed by the expected search cost, even at a very low on-street price. In the analytical model that follows, we rule this out by assuming that $\lambda(0, C) > 1$. Making this assumption simplifies the model, but results are not dependent upon it. Note that this condition is also met in the reference values for the numerical example.

[10] Given the perfect substitutability between markets, if the prices are equal, any number of drivers could search in the range $\lambda^* \in [0, Q/\{1-p_Y\}]$. We resolve this indeterminacy by assuming that, at the critical price level, drivers prefer to park on-street. This leads to a crisp optimal price result – but taking other assumptions would not alter the essence of the results below.

If the on-street price is set above the off-street price, however, all drivers proceed directly to the off-street market: $\lambda^* = 0$. This leads to discontinuity in the λ^* at the price $p_X = p_Y$.

II.5. Expected social welfare

Expected social welfare (per individual) is given by:

$$\begin{aligned} W(p_X, p_Y) = \lambda^* \left\{ v_S + \rho p_X t_X^* + [1-\rho][p_Y - C]t_Y^* \right\} \\ +[1-\lambda^*]\{v(p_Y) + [p_Y - C]t_Y^*\} \end{aligned} \tag{5}$$

Welfare is a weighted sum of the consumer surplus and expected net revenue from searching and proceeding directly to the off-street market. The weights, of course, correspond to the equilibrium percentage of searchers and non-searchers. This function is, naturally, also discontinuous at the price $p_X = p_Y$.

III. Centralised Allocation

Before turning to the maximisation of social welfare with respect to parking prices, consider the optimal allocation when government can control driver actions (consisting of a set of three variables (t_X, t_Y, λ) directly, rather than indirectly via two prices (p_X, p_Y). Clearly, equation (4) no longer applies.

It can never be efficient for searchers to fail to find an on-street spot. In any optimal allocation, therefore, $\lambda t_X \le Q$. The objective function for the government is given by:

$$\lambda u(t_X) + [1-\lambda][u(t_Y) - Ct_Y] \tag{6}$$

subject to three constraints (with multipliers shown in brackets):

$$\begin{aligned} \lambda \le 1 \quad (\gamma_1) \\ t_X \le \frac{Q}{\lambda} \quad (\gamma_2) \\ \lambda, t_X, t_Y \ge 0 \end{aligned} \tag{7}$$

The optimisation problem facing the government is the maximisation of (6) with respect to t_X, t_Y and λ, subject to the set of constraints (7).

The optimal solution[11] depends on two parameters: Q and C, though recall by assumption that $C<1$. Using the superscript C to denote the optimal Centralised allocation variable level: there are three qualitatively different optimal solutions:

Consider as a first case $Q>1$ (Case 1). With such a large on-street supply, there is no rationing problem. All parking demand ($\lambda = t_X = 1$) can be met (at zero resource cost) by the on-street market. Hence the off-street market is not used (and thus t_Y is indeterminate). The solution ($\lambda^C = 1; t_X^C = 1; t_Y^C$ indeterminate) is optimal, as is confirmed in Annex 1.

Now consider the case $Q<1$. One candidate solution is that all drivers are allocated to the on-street market such that demand is met by on-street supply. Thus, $\lambda = 1, t_X = Q < 1$. Imagine marginally increasing the length of stay of on-street parkers. Drivers gain (1-Q) in additional benefit. But due to the constraint $\lambda t_X \leq Q$, exactly $1/Q$ drivers must be re-allocated from the on-street to the off-street market. Given that the off-street market is priced at resource cost, the social cost of switching a driver is given by $u(Q)-v(C)$. The total change in social welfare is just:

$$(1-Q)-\frac{u(Q)}{Q}+\frac{v(C)}{Q}$$

Multiplying through by $-Q$, the sign of the welfare change is given by:

$$v(1-Q)-v(C) \tag{8}$$

Hence the critical condition for using the off-street market is whether $1-Q$ is greater than or less than C, i.e. whether shadow price of an additional unit of on-street parking time (given that all drivers park on-street) is less than or greater than the resource cost of off-street space.

This gives rise to two further cases. If $1-Q<C$ (Case 2), the optimal allocation of demand does not use the off-street market. The shadow price of additional on-street parking time is smaller than the resource cost of off-street parking. Annex 1 confirms that the optimal allocation in this case is given by ($\lambda^C = 1; t_X^C = Q; t_Y^C$ indeterminate).

Conversely, if $1-Q>C$ (Case 3), the shadow price of additional on-street parking time (given that the off-street market is not used) is greater than the resource cost of off-street parking. The off-street market is used. The optimal length of stay off-street is such that the

[11] Given the constraint $\lambda t_X \leq Q$, the set of feasible solutions is non-convex. Constraint qualification can be shown to hold, however, and hence the Kuhn-Tucker conditions are necessary but not sufficient for an optimum. However, there is a unique optimum.

marginal benefit of an additional unit of time, $1-t_Y$ equals the marginal cost, C. Hence, $t_Y^C = 1-C$.

If the off-street market is used, it follows that $t_X^C > Q$. Moreover, simple manipulations (similar to those used to derive equation (8)) show that at the optimal on-street stay, the social benefit from marginally increasing the length of on-street stay (net of the loss in on-street parking welfare to those drivers who, as a result, are forced to park off-street), which is given by $v(1-t_X^C)/Q$, must equal the social benefit from parking off-street $v(C)/Q$. Hence $t_X^C = 1-C = t_Y^C$, i.e. drivers park for an equal length of time on either parking market. Given this length of stay, the optimal number of on-street parkers equals $\lambda^C = Q/\{1-C\} \le 1$. Again, the formal derivation of this result appears in Annex 1.

Table 1 summarises the results under the three different assumptions concerning Q and C. Case 1 requires no policy response, and is essentially uninteresting.

	t_X^C	λ^C	t_Y^C
1. $Q > 1$	1	1	-
2. $1-C < Q \le 1$	Q	1	-
3. $Q \le 1-C$	$1-C$	$Q/\{1-C\}$	$1-C$

Table 1 Summary of central allocation results

Under Case 2, the on-street length of stay is limited, but all demand is allocated to the on-street market. Both Vickrey (1959) and Arnott and Rowse (1999) consider the optimal pricing of on-street parking in the absence of the off-street parking market. These papers can be considered as implicitly assuming that Case 2 applies. However, for large metropolitan areas, it is more realistic to consider Case 3. This justifies the use of an off-street market, which is observed in nearly all cities. Henceforth we assume Case 3 holds. For ease of comparison with later results, we collect the central finding in the form of a Lemma:

Lemma 1 *Assuming $Q \le 1-C$, the optimised central allocation is given by:*

$$t_X^C = t_Y^C = 1-C$$

$$\lambda^C = \frac{Q}{1-C}$$

Proof: *Annex 1*■

Figure 1 presents the results of a numerical example, calibrated to central London (see section V.1.1. for information on parameter values). Case 3 applies, as $Q = 0.17 < 1 - C = 0.74$. The shaded area gives the non-convex solution space, defined over the two control variables available to the central planner: λ and t_X. Iso-welfare functions are plotted ($W_1 > W_2 > W_3$) as a function of the two variables, given that the length of stay off-street is set (optimally) at $1 - C$.

INSERT FIGURE 1 ABOUT HERE

The highest welfare level is achieved at the point of tangency $t_X^C = 1 - C = 0.74$ and $\lambda^C = \dfrac{Q}{1-C} \simeq 0.23$.

IV. Perfect competition on-street

Can the optimal centralised allocation be decentralised via pricing instruments? We examine this problem under the assumption that the off-street market is perfectly competitive. The zero profit condition implies that $p_Y = C$ and thus $t_Y^*(C) = t_Y^C = 1 - C$. This is clearly a restrictive assumption and probably unrealistic. However, it establishes a benchmark case.

The government's problem is to price the on-street market in such a manner to induce driver behaviour in accordance with the two remaining conditions required for Lemma 1. We consider two cases: a linear and non-linear fee structure.

IV.1. Linear On-street price

If the unit on-street fee p_X is set equal to the optimal off-street price, it is clear that $t_X^*(C) = t_X^C = 1 - C = t_Y^*(C)$. Two of the three conditions of Lemma 1 are met. It is straightforward to show that the third and final condition is also met.

Recall that expected net consumer surplus from searching on-street is given by $\rho[v(p_X) - v(p_Y)]$. When prices are equal across markets, this equals zero. If expected search cost is strictly positive, all drivers are better off proceeding directly to the off-street market. Thus, in equilibrium, expected search costs, $[1-\rho]d$, must equal zero. In equilibrium, therefore, $\rho(C,C,\lambda^*) = Q\Big/\left\{\lambda^*[1-C]\right\} = 1$. Simple re-arrangement gives $\lambda^* = \lambda^C = Q\Big/\left\{1-C\right\}$. This solution is also seen directly from equation (4).

The main conclusion of this section is presented as a Proposition.

Proposition 1 *Given a perfectly competitive off-street parking market, a sufficient condition to maximise social welfare is to set the on-street price equal to the off-street price* $p_X^* = p_Y^* = C$ *. The resulting allocation is identical to the optimal centralised allocation given in Lemma 1.*

Proof: *In text* ■

The intuition for this result is closely related to that given under the centralised allocation under case 3: at the optimum, the marginal benefit from increasing the length of on-street stay equals the benefit from parking off-street. However, account needs to be taken of the equilibrium search condition. Recall that in the centralised allocation problem, the government has three control variables. In a decentralised solution, however, it has only two: the two price instruments. The number of searchers is set only indirectly, via the condition (4) . However, the optimal centralised allocation can be decentralised. Equation (4) binds at the optimum, as shown in Figure 2.

INSERT FIGURE 2 AROUND HERE

The Figure plots (as a heavy line) the equilibrium number of searchers function (equation (4)) over the feasible solution space from the centralised allocation problem. The $\lambda^*(p_X, C)$ function is rewritten as $\lambda^*(1-t_X, C)$ to aid comparability. This function is seen to pass through the optimal point, $t_X^*(C) = t_X^C; \lambda^*(C,C) = \lambda^C$. The optimal allocation can be decentralised.

Consider other pricing options. A greater-than-optimal price results in all drivers using the off-street market. The total cost of providing off-street parking is excessive (when compared to the optimum). Alternatively, a lower-than-optimal price results in too many drivers choosing to search. Total search costs are excessive (in comparison with the optimum).

IV.2. Non-linear on-street fee

Non-linear pricing schedules, including the limiting case of a time restriction, are commonly observed on the on-street parking market. Proposition 1 establishes that a linear fee is sufficient to implement the optimal centralised allocation. But it is clear that, at least in this model, any number of non-linear fee structures could do just as well.

An optimal fee structure needs to ensure that two conditions are simultaneously meet: that each driver parks for $1-C$ units of time and that only λ^C drivers choose to search. An infinite number of fee structures meet these conditions. Define the total on-street fee paid as $F_X(t)$. Consider, for instance, a 'one-part' fee structure given by:

$$F_X(t)=\begin{cases} C[1-C] & \text{if} \quad t\le 1-C \\ \infty & \text{otherwise} \end{cases}$$

No driver parks for longer than 1-C units of time. For all time units less than $1-C$, the marginal cost is zero. Therefore if a driver uses the spot at all, he or she stays for exactly $1-C$ units of time. Consumer surplus from a trip equals $v(C)$ and thus, as in the case of the linear fee, $\lambda^*=\lambda^C$.

It is common to observe a quantity restriction, which we term a *simple time restriction*. This is a fee schedule in which parkers do not pay a fee, but are subject to a time restriction. It is given by a form:

$$F_X(t)=\begin{cases} 0 & \text{if} \quad t\le r \\ \infty & \text{otherwise} \end{cases}$$

A simple time restriction[12] cannot decentralise the optimal allocation. Consider a candidate solution, $r=1-C$. On-street parkers remain for the optimal length of time: $t_X^*=t_X^C=1-C$. Conditional on finding a vacant spot, however, an on-street parker receives consumer surplus equal to $u(1-C)$, which re-arranges into $v(C)+C(1-C)$, in comparison with a payoff of $v(C)$ on the off-street market. Too many drivers choose to search as a result. The equilibrium number of searchers, denoted by λ_r^* is greater than λ^C.

To see this, re-write the percentage of searchers as:

$$\lambda_r^*=\frac{Q}{[1-C]d}\big(u(1-C)-v(C)+d\big)=\frac{QC}{d}+\frac{Q}{[1-C]}>\frac{Q}{[1-C]}=\lambda^C$$

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[12] Infact any strictly convex fee schedule in which $F_X(0)=0$ fails to decentralise the optimal allocation. A simple time restriction is, of course, just the limiting case of such a function.

An alternative candidate solution is a time restriction such that the optimal number of drivers choose to search. This occurs if consumer surplus is equal across markets, i.e. $u(r) = v(C)$. Simple manipulations show this to be the case if $r = 1 - \sqrt{C(2-C)}$. But then drivers stay for too short a period of time: $r < t_X^C = 1 - C$.

Annex 2 derives the optimal simple time restriction. Not surprisingly, the optimal value of r falls between the two extreme cases discussed above. For the reference parameter values, the optimal value of r equals 0.71, which is less than $t_X^C = 1 - C = 0.74$ but greater than the time restriction required to induce the optimal number of searchers, given by $r = 1 - \sqrt{C(2-C)} = 0.33$.

V. Off-street market power

It is probably not realistic to assume that the urban off-street parking market is perfectly competitive. Spatial differentiation, at the very least, implies a degree of market power. We investigate the benchmark case in which a single operator supplies the off-street market.

Moreover, we analyse the case of a simple sequential game, in which the government acts as a Stackelberg leader in the first stage[13]. This seems reasonable: governments are often constrained in their ability to change prices quickly[14]. The private off-street supplier observes this on-street price and responds by setting a profit-maximising off-street price. To simplify matters, both players are assumed to use a constant per time unit fee only.

Under perfect competition, government can pursue a matching policy. When the off-street market is non-competitive, however, the government will account for the reaction of the off-street supplier to any price set on-street. We derive the welfare-maximising government policy under these conditions and provide numerical results to highlight the findings.

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[13] In choosing for a Stackelberg framework, we assume that the government can commit to the price chosen in stage 1. Alternatively, we could analyse the Nash-equilibrium in a simultaneous move game. This has been done in Calthrop (2001). However, as explained there, the payoff of players are not quasi-concave in own price nor upper semi-continuous in the joint price vector. Equilibria in pure-strategies need not exist (see Dasgupta and Maskin, 1986). Mixed strategy equilibria may exist in this game, but are awkward to compute. We prefer the Stackelberg formulation, which seems plausible and provides Nash equilibria in pure-strategies.

[14] One reader of this paper has suggested that the opposite formulation: that of the government as follower is just as reasonable. However, we feel it is harder to justify the implicit assumption of commitment for a private operator. In any event, an alternative assumption would still show that the

V.1. Analytics

As is standard, we solve the game backwards.

Stage 2: Off-street supplier sets profit-maximising price

For any given demand level, the off-street supplier maximises profit by setting the monopoly price, $p_Y^m = \frac{1}{2}(1+C)$. If the on-street price is set higher than this level ($p_X > p_Y^m$), the off-street supplier responds by setting the monopoly price and captures the whole market.

If the on-street price is lower than the monopoly price level, the supplier faces a simple choice. One option is to charge a price epsilon lower than the on-street market and, by capturing the whole market, earn profit $\Pi_1(p_X) = [p_X - \varepsilon - C][1 - p_X - \varepsilon]$. An alternative option is to charge the profit-maximising price, and earn maximum profit on the residual demand: $\Pi_2(p_X) = [1 - Q/\{1 - p_X\}][p_Y^m - C][1 - p_Y^m]$.

There exists an on-street price, denoted by $I_X(Q,C)$ below which the off-street supplier maximises profits by playing on the residual demand curve and above which he undercuts. This is given by: $\Pi_1(I_X) = \Pi_2(I_X)$, where it follows that $C < I_X < p_Y^m$.

The best-response function of the off-street supplier is given by[15]:

$$b_Y(p_X) = \begin{cases} p_Y^m & \text{if} \quad p_X < I_X \\ p_X - \varepsilon & \text{if} \quad I_X \le p_X \le p_Y^m \\ p_Y^m & \text{if} \quad p_X > p_Y^m \end{cases} \tag{9}$$

If the on-street price is relatively low $C < p_X < I_X$, undercutting is unattractive – for instance, if the on-street price is close to C, profit per unit of demand is small. Playing for the residual demand curve is a more profitable strategy. With a relatively low on-street price, any successful on-street parker remains for a relatively long period of time. The probability of finding a vacant spot is low, and hence the residual demand for the off-street market is relatively large. Charging the monopoly price on the residual demand curve is more profitable

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government may abandon a strategy of setting the on-street price equal to the off-street marginal resource cost.

[15] We assume that at the on-street price I_X, the off-street supplier plays aggressively and undercuts.

than undercutting. This scenario is reversed, of course, if the on-street price is relatively high: $I_X < p_X < p_Y^m$.

Using the implicit function theorem[16], we note that:

$$\frac{\partial I_X}{\partial Q} < 0 ; \quad \frac{\partial I_X}{\partial C} > 0 \tag{10}$$

Increasing the supply of on-street spaces acts to reduce the residual demand curve for off-street parking. Undercutting becomes a relatively more attractive option. If the resource cost of off-street space marginally increases, undercutting becomes a less attractive option.

Stage 1: Government sets welfare maximising on-street price

The government can compute the best-response function for the off-street supplier, given in equation (9). Using this information to maximise social welfare (in equation (5)), gives rise to the following optimisation problem:

$$\underset{p_X}{Max}\, W = \begin{cases} W(p_X, p_Y^m) & \text{if} \quad p_X < I_X \\ W(p_X, p_X - \varepsilon) & \text{if} \quad I_X \le p_X \le p_Y^m \\ W(p_X, p_Y^m) & \text{if} \quad p_X > p_Y^m \end{cases}$$

The optimal solution in the restricted parameter region $p_X \ge I_X$ is simple. For any on-street price set in this region, the off-street supplier reacts by undercutting. All drivers use the off-street market. Maximising welfare is therefore equivalent to maximising social surplus on the off-street market. Given that $I_X > C$, this occurs at the lowest price possible i.e. the corner solution, $p_X = I_X$. Social welfare is maximised on this restricted domain at the level:

$$W\Big|_{I_X} = v(I_X) + [I_X - C][1 - I_X] \tag{11}$$

Welfare consists of the sum of consumer surplus plus profit from the off-street market.

The optimal price in the region $p_X < I_X$ is less obvious. To simplify matters, assume that the critical indifference price, I_X is less than the price at which only a subset of drivers choose to search, $p_{\underline{X}}(p_Y^m)$ - this is the case for a wide range of parameter values[17].

---

[16] Define an equation $F(Q, C, I_X) \equiv \Pi_1(I_X) - \Pi_2(I_X) = 0$. Taking the ratio of derivatives, and using the constraint $C \le I_X \le p_Y^m$ gives the desired result.

All drivers choose to search. Social welfare, given in equation (5), simplifies to:

$$\begin{aligned} W(p_X) = &\rho[v(p_X) + p_X[1-p_X]] \\ &+\{1-\rho\}[v(p_Y^m) + [p_Y^m - C][1-p_Y^m] - d] \end{aligned} \tag{12}$$

Consider the impact of raising the on-street fee. The direct effect on social welfare of on-street parkers from reducing the length of stay, in expected terms, is $-\rho p_X$. However, reducing the length of stay increases the probability of finding a vacant spot. This increases welfare (in expected terms) by $\rho'[v(p_X) + p_X[1-p_X] - v(p_Y^m) - [p_Y^m - C][1-p_Y^m] + d]$.

Assuming an interior solution[18], the optimal price raises the on-street price until these two effects offset one another. Noting that $\rho' = \rho/[1-p_X]$, it is straightforward to derive an implicit expression for the optimal price level, denoted by p_X^1:

$$v(p_X^1) = v(p_Y^m) + [p_Y^m - C][1-p_Y^m] - d \tag{13}$$

At the optimal on-street price, the on-street consumer surplus equals the social surplus on the off-street market net of the search cost (which, given that all drivers search, is incurred with certainty).

For the purpose of comparative statics, we note:

$$\frac{\partial p_X^1}{\partial d} > 0, \frac{\partial p_X^1}{\partial C} > 0 \tag{14}$$

Raising either of the two parameters, C or d, acts to decrease the net social surplus from the off-street market (the right-hand side of equation (13)) and thus increase the optimal on-street price.

The optimality result derived in equation (13) can be used to simplify the expression for social welfare in equation (12). Social welfare evaluated at the optimal on-street price reduces to:

$$W\Big|_{p_X^1} = Qp_X^1 + v(p_Y^m) + [p_Y^m - C][1-p_Y^m] - d \tag{15}$$

---

[17] Infact this holds for all parameter values tested: repeated random drawings from the sets $C \in [0.05, 0.5], Q \in [0.05, 0.4], d \in [0.005, 0.03]$. The results can easily – if somewhat tediously –be extended to include that case that $p_{\underline{X}}(p_Y^m) < I_X$. The set of local optima needs to be extended somewhat. However, given that our numerical model suggests that this does not occur, we prefer to keep the text simple and ignore the possibility that the assumption is not met.

[18] To simplify exposition, we assume that $p_X^1 < I_X$ and hence the locally optimal price is p_X^1.This assumption holds for a wide range of values of model parameters, including the reference values. Moreover, the assumption can easily be relaxed, though little additional insight is gained, while there is a significant cost in terms of messier notation.

Social welfare can be evaluated as the sum of two components: the revenues generated from the on-street market plus the base level of social surplus gained if all drivers search and fail to find an on-street spot. This latter part consists of the consumer surplus plus profits from the off-street market, minus the search cost.

The government faces a choice. It can set a relatively low on-street price equal to p_X^1, in the knowledge that the off-street supplier maximises profit on the off-street market, and thus social welfare is given by (15). Alternatively, the government can set a relatively high on-street price, equal to I_X, thus inducing the off-street supplier to undercut the on-street market, and achieving a level of social welfare given by equation (11).

Setting a relatively low on-street price is desirable if $W\big|_{p_X^1} - W\big|_{I_X} > 0$, which is the case if:

$$\begin{aligned} Qp_X^1 - d > v(I_X) + [I_X - C][1 - I_X] \\ -v(p_Y^m) - [p_Y^m - C][1 - p_Y^m] \end{aligned} \tag{16}$$

i.e. if the revenue from the on-street market, net of the search cost d – the left-hand side of (16), exceed the net social surplus from inducing the off-street supplier to switch from monopoly pricing to undercutting the on-street market (the right-hand side).

V.2. Numerical Example

V.2.1. Reference parameter values

We define reference values for the three parameters in our model:

$$Q = 0.17$$
$$C = 0.26$$
$$d = 0.02$$

These values have been calibrated to the published literature on the on-street parking market in London for 1990. More information on this procedure can be found in Calthrop and Proost (2000).

V.2.2. Reference results

Table 2 presents the welfare levels associated with the two locally optimal policies, p_X^1 and I_X. If the government adopts price level p_X^1, drivers choose between an on-street price equal to 0.391 and an off-street price at the monopolist level, 0.63. As a result all drivers choose to search, even though the probability of finding a vacant spot is only approximately one-third.

	welfare level
p_X^1 =0.391	0.252
I_X =0.429	0.260

Table 2 Reference parameter value results

Alternatively, the government can set a somewhat higher on-street price, equal to I_X =0.429, and thus induce the off-street supplier to marginally undercut this price. All drivers proceed to the off-street market.

Table 2 shows that the welfare level from setting an on-street price equal to I_X exceeds that from p_X^1. Allowing the on-street market to be undercut induces all drivers not to search: search costs are saved. In addition, lower off-street prices result in higher social surplus gains on the off-street market. Against this, the on-street market is not used. More off-street parking is provided (at cost C per unit) than would be the case were some drivers using the on-street market.

The following section investigates the extent to which this result is determined by the choice of parameter values in the reference case.

V.3. Sensitivity tests

V.3.1. Search cost parameter, d.

Notice that the right-hand side of equation (16) does not depend on the value chosen for the search cost parameter d. The left hand side decreases in d, were, using equation (14), it can be shown that $Q\frac{\partial p_X^1}{\partial d} < 1$. Therefore the net benefit from setting a relatively low price, p_X^1 rather than I_X falls in d. This is confirmed in Table 3, where cells report welfare levels for differing assumptions on the magnitude of d.

	d=0.005	d=0.02	d=0.035
p_X^1	0.264	0.252	0.241
I_X	0.260	0.260	0.260

Table 3 Sensitivity tests on parameter d

For a low value of the search cost parameter, d=0.005, Table 3 confirms that the optimal on-street price is p_X^1 rather than I_X. The gain from using the on-street market spots (at zero resource cost) more than offset the low search costs resulting from inducing excessive numbers of drivers to search and the welfare loss from higher off-street prices.

V.3.2. Supply of on-street space, Q.

The impact of varying the supply of on-street space on welfare levels is shown in Table 4. Marginally increasing Q, increases the left-hand side of equation (16) by p_X^1. However the right-hand side also increases (by $-[I_X - C]\frac{\partial I_X}{\partial Q} > 0$ from equation (10)). For the numerical values shown in Table 4, it seems that the increase in the left-hand side outweighs that on the right-hand side, and increasing on-street supply increases the welfare from charging a relatively low price p_X^1 more than from I_X.

	Q=0.1	Q=0.17	Q=0.3
p_X^1	0.225	0.252	0.303
I_X	0.252	0.260	0.267

Table 4 Sensitivity tests on parameter Q

At a relatively high on-street supply, Q=0.3, we note that the government prefers a strategy of charging p_X^1 to I_X. Having a relatively large supply of space (at zero resource cost) ensures that the government is more likely to ensure that the spaces are used, and thus the positive cost of providing off-street space is saved.

V.3.3. Resource cost of off-street space, C.

Use of the relationships given in (14) and (10), show that a marginal increase in the off-street resource cost parameter C increases both the left-hand and right-hand side of equation (16). Table 5 presents results on the welfare levels associated with different assumptions regarding the magnitude of C. It shows that for the chosen range of values, marginally increasing the resource cost parameter reduces the welfare level from charging at p_X^1 by less than that at I_X .

	C=0.1	C=0.26	C=0.4
p_X^1	0.326	0.252	0.203
I_X	0.380	0.260	0.173

Table 5 Sensitivity tests on parameter C

Increasing the resource cost of off-street parking increases the opportunity cost from not using the on-street spots. Hence choosing the lower on-street price, and inducing use of the on-street market, becomes more preferable.

VI. Concluding remarks

The economic analysis of urban parking policy has abstracted from the presence of the off-street market. This misspecification of the problem is important, as the amount of on-street searching behaviour depends on both the on- and off-street market price.This paper has demonstrated that the off-street supply conditions have important implications for government regulation of the on-street market.

We stress four findings:

- The supply of urban off-street parking is optimal if the shadow cost of on-street parking (given that all drivers park on-street) is greater than the resource cost of off-street parking (i.e. Lemma 1). We expect this to be the case for large metropolitan areas, and hence, sound analysis of on-street parking policy should consider the presence of an off-street market.

- If the off-street market is competitive, Proposition 1 gives that a sufficient condition to maximise welfare is to set the on-street price equal to the off-street level. This is a striking result, with the attractive feature that it requires minimal information to implement.
- A common regulatory alternative to a meter fee, a simple time restriction, whereby drivers are allowed to park for free for any length of time up to a given limit, is not efficient. Even when set optimally, this non-linear price induces too many drivers to search for an on-street spot.
- If the off-street market is non-competitive, the optimal on-street policy deviates from the simple matching policy. In our set-up, the government acts as the Stackelberg leader, while a monopolistic off-street supplier is the follower. Optimal policy pursues one of two options: either set a relatively high on-street price, and induce the off-street monopolist to undercut the on-street market, or set a relatively low on-street price, in the knowledge that the monopolist profit maximises on the residual demand curve. The former strategy induces all drivers to proceed to the off-street market, and thus eliminates socially-wasteful search costs. In addition, there are welfare gains from inducing the off-street monopolist to set a lower price. The latter strategy, in contrast, induces all drivers to search, but requires a smaller supply of the higher resource cost off-street space. Which of the two strategies is optimally chosen is shown to be parameter dependent. For reference values, we find that government optimally sets a relatively high on-street price. But with small deviations in some parameters, the alternative option is optimal

The model is simple, transparent and tractable. Moreover, crisp and policy relevant results emerge. However, it is important to stress some limitations of the model.

We model the on-street market as a single point in space. This differs from the explicitly spatial models of AP-V and AR, in which, for instance, on-streets spots closer to the desired destination are a higher quality product than those further away (due to reduced walk costs). As a consequence, on-street policy in our model is not concerned with the distribution of on-street parkers over available spaces. However, the key insight of our model will emerge in a spatial setting: the number of on-street searchers will depend on price and supply conditions on both markets. Relatively low on-street pricing will induce more and more drivers to search on-street until the equilibrium cost equals the off-street price. Both the parking-span and the distribution of searching will be distorted. We conjecture that a variant of Proposition 1 will emerge: in the optimum, the marginal social cost of an additional on-street parker must equal resource cost of off-street parking.

Other margins of driver behaviour, not captured in this model, may be important. In particular, drivers are loosely thought to be arriving at random: they cannot adjust departure time in order to increase the probability of finding a vacant spot by arriving in the 'shoulders' of the peak period[19]. Drivers are also assumed to be unable to gain information on the probability of finding a vacant space, nor is there a market allowing drivers to buy space in advance of arrival: see AR for some initial work on this topic.

We limit our investigation of non-competitive off-street supply to one particularly simple benchmark. Whilst this is sufficient to demonstrate that government may not wish to pursue a simple matching policy, one can imagine more realistic representations of the market, with a small number of spatially differentiated suppliers.

Finally, the model is greatly simplified by assuming identical consumers. New insights can potentially be gained from integrating heterogeneity. For instace, policy makers tend to think in terms of a long-term stay market (often at some distance from the city centre) and a short-term parking market (close to the centre), with special provision needed for those with a high cost of walking. Is this type of separating equilibrium desirable, and if so, what type of pricing and regulatory mix is best suited to decentralise it?

[19] Calthrop (2001) Chapter 5 uses a non-stationary stochastic queueing model to derive the optimal time profile of parking fees. Drivers choose when to schedule arrivals, given a common preferred time of facility usage. The results suggest that the optimal pricing schedule is non-linear. A time of arrival related component acts to spread arrivals efficiently throughout the peak period, while a length of stay component acts to limit parking time to the efficient level.

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Annex 1 – The centralised allocation problem.

The optimisation problem facing the government is the maximisation of (6) with respect to t_X, t_Y and λ, subject to the set of constraints (7).

Assume the in the optimal solution the on-street market is used: i.e. $\lambda > 0$, $t_X > 0$. The set of complementarity slackness conditions reduces to:

$$u(t_X) - [u(t_Y) - Ct_Y] - \gamma_1 - \gamma_2 \frac{Q}{\lambda^2} = 0 \qquad \text{(A1a)}$$

$$\lambda[1 - t_X] - \gamma_2 = 0 \qquad \text{(A1b)}$$

$$t_Y [1 - \lambda][1 - t_Y - C] = 0 \qquad \text{(A1c)}$$

$$\gamma_1[1 - \lambda] = 0 \qquad \text{(A1d)}$$

$$\gamma_2[\frac{Q}{\lambda} - t_X] = 0 \qquad \text{(A1e)}$$

Now we establish that the solutions presented are optimal.

Case 1

Setting $\lambda = 1$, implies, via (A1c) that t_Y is indeterminate. Assume $Q > t_X$, and hence (A1e) implies that the shadow price of an on-street parking spot $\gamma_2 = 0$, and hence from that $t_X^C = 1$. The remaining variable, $\gamma_1 \geq 0$, is solved from equation (A1a) for any value of $t_Y \in [0,1]$.

Case 2

Setting $\lambda = 1$, implies, via (A1c) that t_Y is indeterminate. Assume that $\gamma_2 > 0$, and hence, via (A1e), that $t_X = Q$. Equation (A1b) then gives that the shadow price of an additional unit of on-street parking, γ_2,is equal to $1 - Q$. Substituting the result into (A1a) gives that $\gamma_1 = v(1 - Q) - v(1 - t_Y)$, which is positive if $t_Y \in [0, Q]$.

Case 3 (Proof of Lemma 1)

Assume $\gamma_2 > 0$ and $\gamma_1 = 0$. Equation (A1c) implies that $t_Y^C = 1 - C$. From (A1e), $\lambda^C = \frac{Q}{t_X^C}$. Using (A1b), and substituting into (A1a) gives that $v(1 - t_X^C) - v(C) = 0$. Hence, $t_X^C = t_Y^C = 1 - C$.

Annex 2: The optimal simple time restriction

Preliminaries

To maintain notational consistency, it is more convenient to define the simple time restriction parameter, r, in terms of an implicit price, $p_X^r = 1 - r$. We also assume throughout that the off-street price is set equal to marginal resource cost, C.

We can write $u(r) = v(p_X^r) + p_X^r[1 - p_X^r]$ and define the expected payoff from choosing to search by:

$$v_S^r(p_X^r, C) = \rho\left\{v(p_X^r) + p_X^r[1 - p_X^r]\right\} + [1 - \rho]\left\{v(C) - d\right\}$$

Drivers search as long as $v_S^r \geq v(C)$, and thus we write the equation for the equilibrium number of searchers as:

$$\lambda_r^*(p_X^r, C) = \begin{cases} 1 & \text{if} \quad p_X^r \leq p_{\underline{X}}^r(C) \\ \dfrac{Q}{[1 - p_X^r]d}\left\{v(p_X^r) + p_X^r[1 - p_X^r] - v(C) + d\right\} & \text{otherwise} \\ 0 & \text{if} \quad p_X^r > p_{\overline{X}}^r(C) \end{cases} \tag{A2}$$

and a welfare function is given by:

$$W(p_X^r, C) = \lambda_r^* v_S^r(p_X^r, C) + [1 - \lambda_r^*]v(C) \tag{A3}$$

Optimisation

We proceed to maximise expression(A3). This welfare function is discontinuous (at a time restriction such that $p_X^r = p_{\overline{X}}^r(C)$ and non-differentiable at price $p_X^r = p_{\underline{X}}^r(C)$. We proceed by deriving local optima in the three critical regions: region 1 - $p_X^r < p_{\underline{X}}^r(C)$; region 2 - $p_{\underline{X}}^r(C) \leq p_X^r \leq p_{\overline{X}}^r(C)$; and, finally, region 3 - $p_X^r > p_{\overline{X}}^r(C)$. The globally optimal solution is then identified.

Region 1: $p_X^r < p_{\underline{X}}^r(C)$

In this region, the welfare function (A3) reduces to $v_S^r(p_X^r, C)$. The first-order derivative - the marginal benefit from a reduction in the time restriction – can be derived (via Roy's rule) and is given by:

$$\frac{Q}{[1-p_X^r]^2}\left\{v(p_X^r)-v(C)+d\right\}$$

Setting this condition to zero and solving for the implicit price gives:

$$p_X^{r1}=1-\sqrt{[1-C]^2-2d}$$

Hence the locally optimal price occurs either at an interior point, $p_X^r = p_X^{r1}$ or at the corner solution $p_X^r = p_{\underline{X}}^r(C)$.

Region 2: $p_{\underline{X}}^r(C) \le p_X^r \le p_{\overline{X}}^r(C)$

Rewrite the welfare function (A3):

$$\begin{aligned} W(p_X^r, C) &= \lambda_r^*[v_S^r - v(C)] + v(C) \\ &= v(C) \end{aligned}$$

where the second line follows from the definition of the search equilibrium. Under a time restriction, there is no 'wedge' between consumer surplus and social surplus on the on-street market. Hence, in equilibrium, drivers keep choosing to search until any expected net consumer surplus from choosing to search equals the expected search cost. The net gain in social welfare from choosing to search is zero. Each driver contributes $v(C)$ to social welfare regardless of whether they choose to search or not.

It is direct that the marginal benefit of increasing the implicit price (or reducing the time restriction) is zero. The set of optimal solutions is given by: $[p_{\underline{X}}^r(C), p_{\overline{X}}^r(C)]$.

Region 3: $p_X^r > p_{\overline{X}}^r(C)$

No one-uses the on-street market. As in region 2, the marginal benefit of increasing the implicit price is zero. The set of local optima is given by: $(p_{\overline{X}}^r(C), 1]$

Without loss of generality, the global optimum can be found from the set: $\left\{p_X^{r1}, p_{\underline{X}}^r(C), 1\right\}$. It is straightforward to establish that $W(1,C) < W(p_{\underline{X}}^r(C), C)$, and thus the search for the global optimum can be restricted to the interior and corner solution from region 1.

For reference parameter values, $p_X^{r1} < p_{\underline{X}}^{r}(C)$. Therefore the optimal implicit price is given by $p_X^r = 1 - r = p_X^{r1} = 1 - \sqrt{[1-C]^2 - 2d}$.

FIGURES

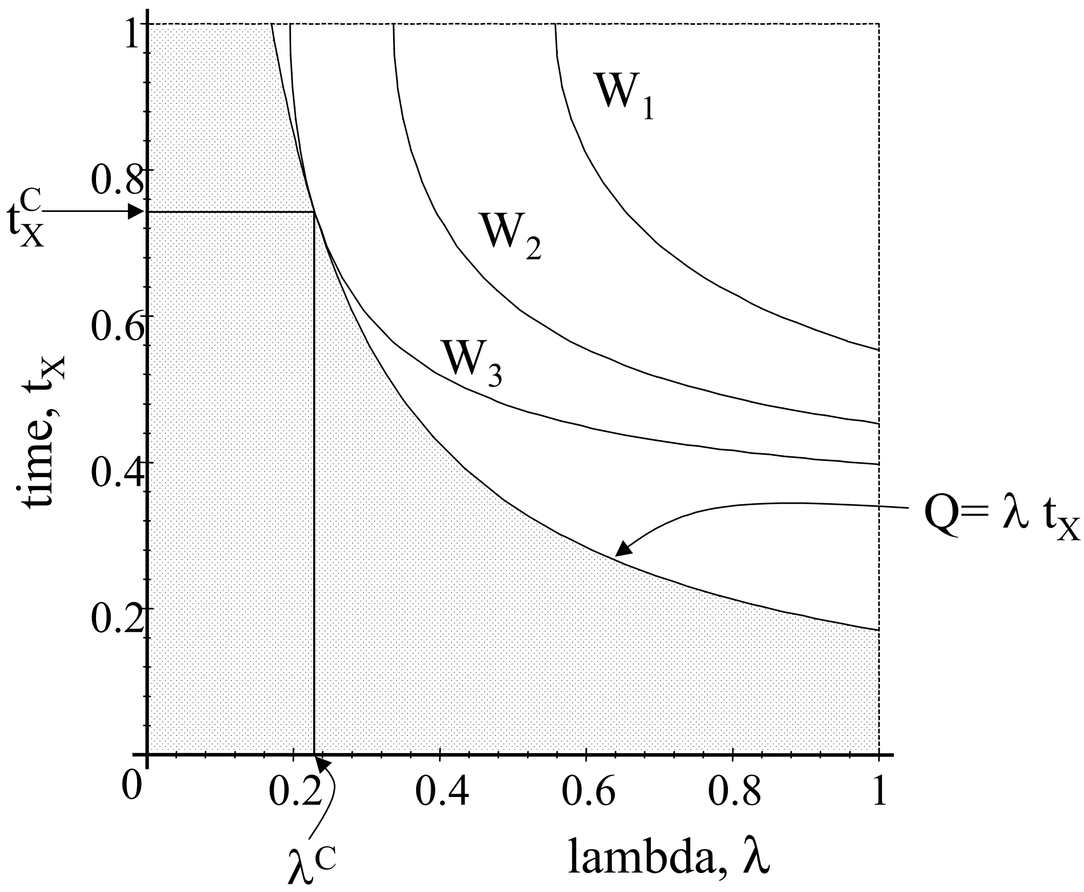

Figure 1 The optimal centralised allocation

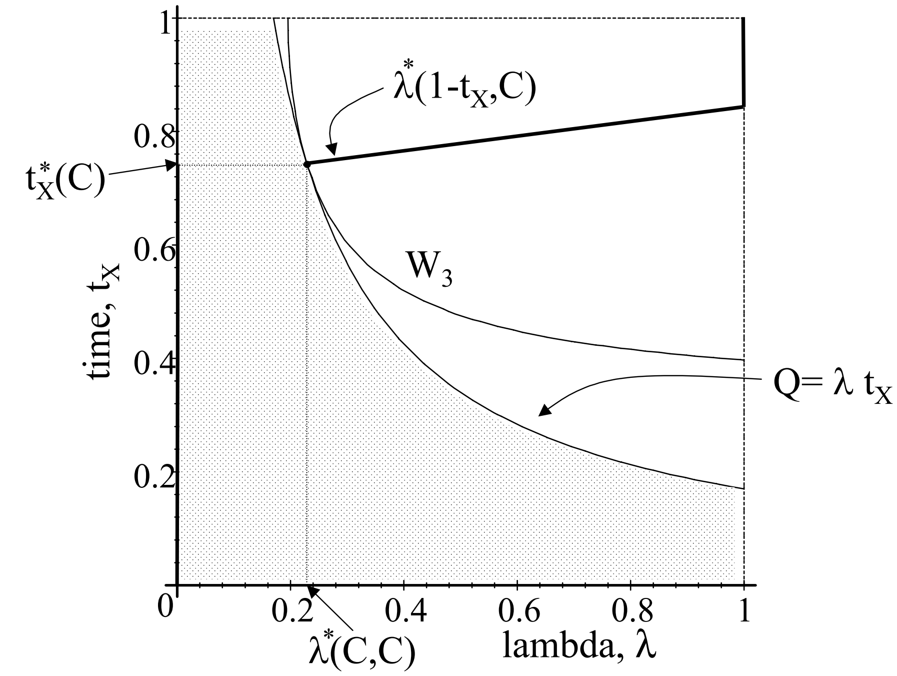

Figure 2 The decentralised allocation with a linear fee

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